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Mid 3/4/2017
@ 9:00

Engineering Economy.

Second Semester (2017)

Dr. Hanan.

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CHAPTER (2) & (3):

* Decision Making:

1- Recognition of problem.
2- Definition of the goal.
3- Assembly of Data.
4- Identification of alternatives.
5- Selection of criteria for judging the best alternative →
- do nothing
- do nothing now → but Later.
- Brain storming.

$\eta \equiv$ efficiency $= \frac{\text{output}}{\text{input}}$

→ Could be: (Always we need max. efficiency).

fixed output → minimum input.
fixed input → maximum output.
input & output not fixed → choose what give you max. η.

Example: A concrete aggregate mix is required to contain at least 31% sand for proper batching.

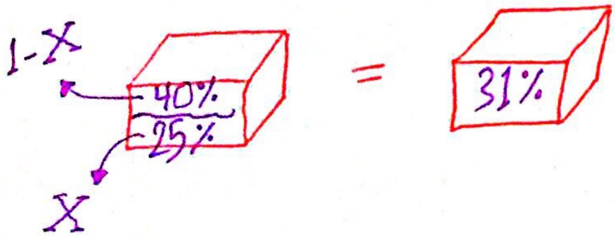
What do you choose if it is found that:

one source → 25% sand at 3 JD/m^3.
Another source → 40% sand, 60% coarse aggregate at 4.4 JD/m^3.

⟹

* we use both sources as follow:

assume 1 m^3.



$X * 0.25 + (1-X) * 0.40 = 0.31$

Solving:

$X = 0.6$

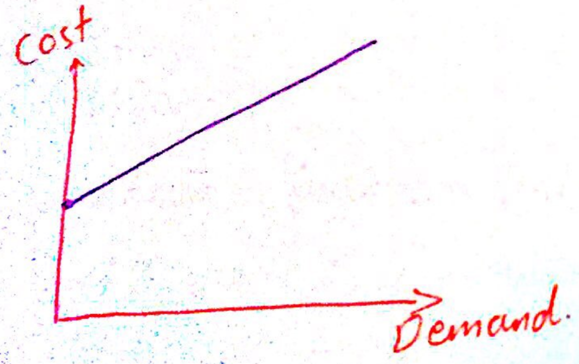
Now for the Cost:

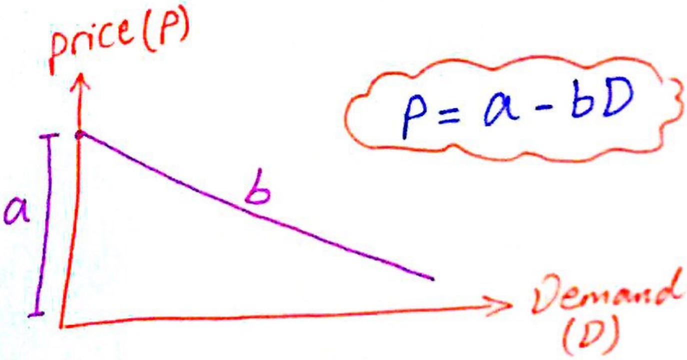
$0.6 * 3 + 0.4 * 4.4 = 3.56\ JD/m^3$

※ Cost:

1. fixed Cost: ex. like pay for the rent.
2. Variable Cost: it is changing with respect to what you want to produce.
3. Direct Cost: has a relation with what you produce.
4. Indirect Cost: has No relalation with what you produce.

Total Cost = fixed cost + variable Cost $\Rightarrow$ $\boxed{C_T = C_F + C_V}$





$P = a - bD$

※ Cost, Revenue, Profit:

Profit = Revenue − Cost

BreaKeven = Cost = Revenue.

Example: Given that:

$C_F = 39{,}000$ $\quad C_V = 55$ per unit

selling price $= 160 - 0.03D$ per unit

max output $= 3000$ unit / month.

$D \equiv$ Number of units.

Find the following:

a) Total Revenue? total revenue $= \underline{\underline{(160 - 0.03D)\,D}}$

b) Optimum Revenue?

$$\frac{\partial R}{\partial D} = 0 \Longrightarrow 160 - 0.03D * 2 = 0 \longrightarrow \boxed{D = 2666} \text{ unit.}$$

c) Profit & max Profit?

Profit = Revenue − Cost $= (160 - 0.03D)D - (39{,}000 + 55D)$

max profit:

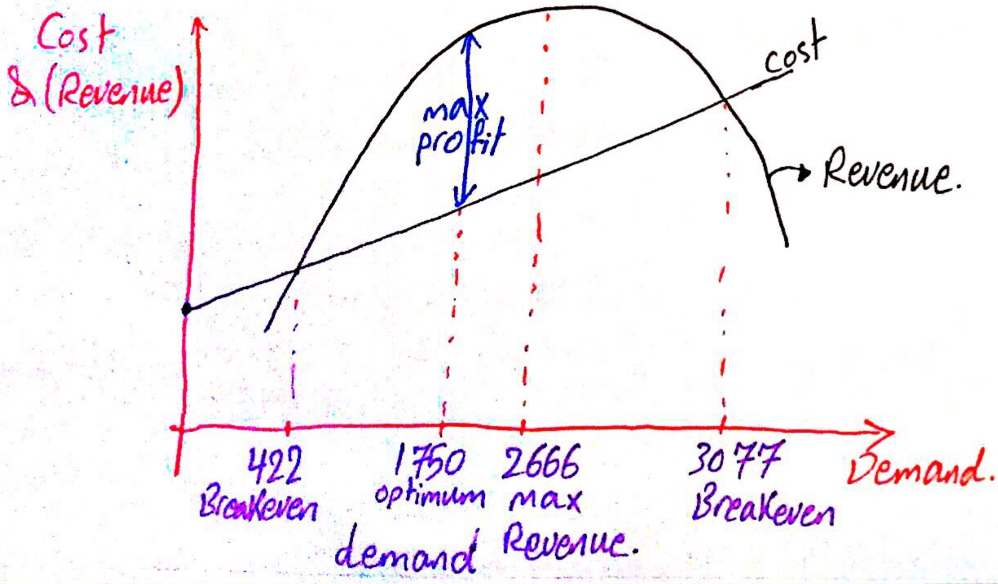
$$\Longrightarrow \frac{\partial P}{\partial D} = 0 \Longrightarrow \text{solving: } \boxed{D = 1750}$$

This called optimum demand "max profit"

d) Revenue?

BreaKeven = Revenue = Cost

$(160 - 0.03D)D = 39{,}000 + 55D$ solving: $D = 422$ & 3077



* Profit ≡ distance between Cost & Revenue.

※ Top-down:

15750 including fees, housing, books increasing 6% annually
+5000

<u>year:</u>

1 15750 * 1.06 = 16,695 + 5000 = 21,695

2 16,695 * 1.06 = 17,697 + 5000 = 22,694

3 17,697 * 1.06 + 5000 = 23,759

4 18,759 * 1.06 + 5000 = 24,884

The sum = 93,036

※ Bottom-up:

Add all things together to Know how much money you need.

※ Index:

⇒ An index is a dimensionless number that indicate how a cost or price has changed with time.

$$\boxed{C_n = C_K\left(\frac{I_n}{I_K}\right)}$$

C_n: Cost at time (n).

C_K: cost at time (K).

<u>Example</u>: A company bought machines in 2010 for 500,000 when the index had a value = 325

⇒ How much does it cost to replace the machines in 2017 if the index is 400.

$$C_n = 500{,}000\left[\frac{400}{325}\right] \Longrightarrow \boxed{C_n = 615{,}385}$$

※ Unit Technique:

Total price = unit price * number of units

Total price = $\sum$ direct costs + $\sum$ unit price * number of units

Ex.

direct costs	unit price		# of units
120	0.1	*	100
2 * 100	0.05	*	500
25	2.5	*	2
1000			
Σ = 1345	Σ = 40		

Total price = 40 + 1345 = $\boxed{1385}$

※ Power sizing:

$$C_A = C_B \left(\frac{S_A}{S_B}\right)^x$$

where x: cost capacity factor (it reflects economies of scale)
↓
from books, people, experience.

Example:

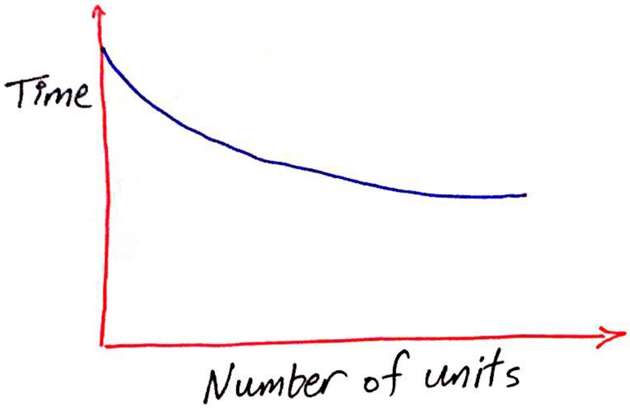
install a 600KW fuel plant a 200KW cost 100 million 20 years ago cost index was 400.
Now cost index = 1200
cost capacity factor = 0.79

find the cost ?

cost now (200KW) $= 100 * 10^6 * \left(\frac{1200}{400}\right) = 300 * 10^6$

$C_{600} = 300 * 10^6 \left(\frac{600}{200}\right)^{0.79} = 714 * 10^6$

※ Learning & improvement:



This called: "Learning curve"

$S \equiv$ slope of the learning curve.

i.e: for 90% learning:
$\Rightarrow \underline{\underline{S = 0.9}}$

** with experience things take less time to be produced.

$$Z_u = K(u)^n$$

$K \equiv$ time for unit 1.
$u \equiv$ number of units.

$$\boxed{n = \frac{\log S}{\log 2}}$$

Example: The time required to produce the first car is 100 hrs Learning rate = 80%. Determine the time required to assemble the 10^{th} car?

solution: $Z_u = K(u)^n = 100\,(10)^{\frac{\log 0.8}{\log 2}} = \boxed{47.6\text{ hrs}}$

Example: For the previous example what is the time required to assemble 10 cars?

$$\sum_1^{10} = 100 + 100(2)^n + 100(3)^n + \cdots = 100(1 + 2^n + 3^n + 4^n + \cdots + 47.6)$$
$$= \boxed{631\text{ hrs}}.$$

*if we need the average time:

$$\frac{631}{10} = 63.1\text{ hrs}.$$

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Example: 1000 hrs require to produce the first unit.
600 hrs require to produce the fifth unit.

⇒ How long would it take to produce the 20th unit?

$600 = 1000\,(5)^n \implies \boxed{n = -0.317}$

$Z_u = 1000\,(20)^{-0.317} \implies \boxed{Z_u = 386.9 \text{ hrs.}}$

if we want to find S:

$n = \dfrac{\log S}{\log 2} = -0.317 \implies \boxed{S = 0.803}$

End of CH3

CHAPTER (4):

⇒ Definitions:

* purchase cost: ⇒ "first Cost"
* Principel: ⇒ رأس المال.
* Anticipated life of an asset: ⇒ الأثر الموجود.
* Anticipated maintenance cost.
* Resale value "Salvage value".
* Period: ⇒ interval of time at which you pay.
* Cashflow (CF): money movement.

⇒ money coming in (+) [receipts].
⇒ money going out (−) [disbursements].

* Time value: ⇒ The change in the amount of money over a period of time.

* Interest = total amount accumulated − original invenstment.

* Interest Rate $= \dfrac{\text{interest accuired per unit time}}{\text{original amount}}$

Example: A person bought something worth 30,000 JD to be paied in two different ways:

① Pay now for discount 3%.

OR

② Pay 5000 now, at the end of year 1 pay 8000, then 6000/year for 4 years.

⇒ List CF:

minus sign since the money going out.

year	CF ①	CF ②
now	−29,100	−5000
1	0	−8000
2	0	6000
3	0	6000
4	0	6000
5	0	6000

Example: A man borrowed 1000 JD from a bank at 8% interest rate, he agreed to repay the loan in two end of year payments. At the end of the first year he will repay half of the 1000 + interest. At the second year he will repay the remaining half + interest of second year.

List the CF table?

year	CF
0	1000
1	−580
2	−540

$8\% * 1000 = 80$

$\Rightarrow 500 + 80 = \underline{\underline{580}}$ JD.

$8\% * 500 = 40$

$\Rightarrow 500 + 40 = \underline{\underline{540}}$ JD.

* Simple Interest:

↳ earned on the Capital or principal (Not applicable).

Interest = Principal * number of periods * interest/period rate.

$\Rightarrow$ $\boxed{Interest = P * n * i}$

Example: If you borrowed 1000 JD fo 2.5 year at 14% per year (simple interest), how much money will you owe at the end of the 2.5 years?

$1000 * \frac{14}{100} * 2.5 = 350 \Rightarrow 1000 + 350 = 1350$ JD.

year	CF amount borrow	interest	amount owed	amount paid
0	+1000	0	1000	0
1	0	140	1140	0
2	0	140	1280	0
2.5	0	70	1350	-1350

※ Compound Interest:

⇒ interest on top of interest reflecting the time value of money.

Assume $P = 1000$
$i\% = 10\%$

period	amount due at begin of year	interest during the year	amount at end of year
1	1000	100 ← $10\% * 1000 * 1$	1100
2	1100	110	1210
3	1210	121	1331

if it was simple interest:

$$1000 + 1000 * \frac{10}{100} * 3 = \boxed{1300}$$

As variables:

period	amount due at begin of year	interest during the year	amount at end of year
1	P	Pi	$P + Pi$ OR $P(1+i)$
2	$P(1+i)$	$iP(1+i)$	$Pi(1+i) + P(1+i) = P(1+i)^2$
⋮			⋮
10			$P(1+i)^{10}$

* Some Definitions:

$P \equiv$ Sum of money at present.
$F \equiv$ Sum of money at future.
$n \equiv$ Number of interest period.
$i \equiv$ interest rate per interest period.
$A \equiv$ Series of equal consecutive amounts of money per period.

* Cashflow Diagram:

* receipts → income.
* disbursements → (cost)

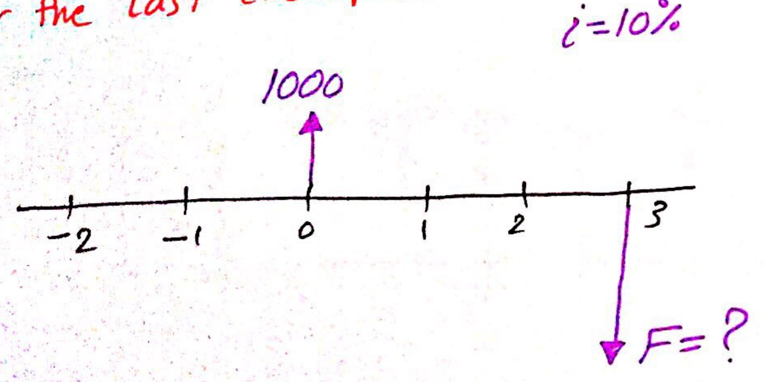
⇒ Net cashflow → receipts − disbursements.

* Cashflow diagram: it is a graphical representation of cashflows on time scale.

$-1 \xrightarrow{\text{means}}$ one period ago.
$0 \longrightarrow$ Now.
$1 \longrightarrow$ one period from now.
and so on.

↑ + (Receipts). ↓ − (Disbursements)

for the last example:

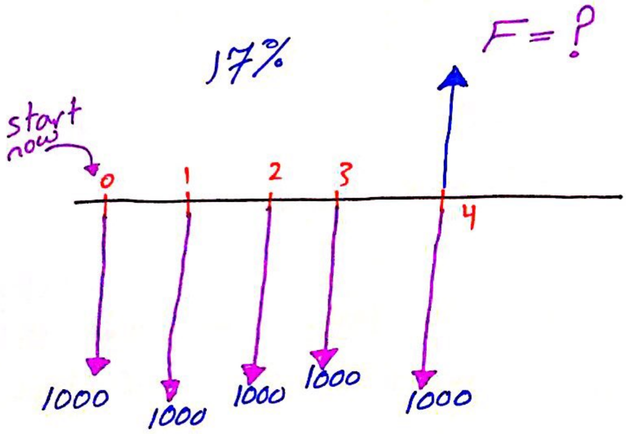


⇒ $F = 1331$

Example:

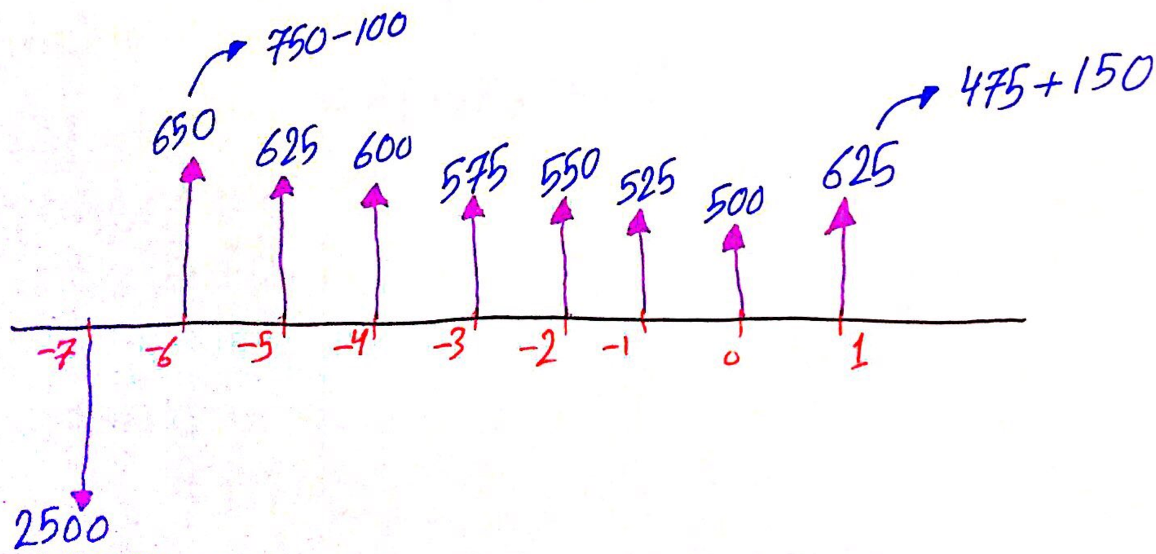
Start now & make 5 deposits of 1000JD per year. in 17% per year account. How money will be accumulated and can be immediately with drawn after the last payment.

* Construct the cashflow Diagram ?



Example: A company invested 2500 JD in a new machine 7 years ago. Annual income was 750 JD. During the first year 100 JD was spent for maintenance. a cost which increased by 25 JD each year. The machine is to be sold next year with a salvage value of 150 JD.

⇒ Construct the Cashflow Diagram ?



* going back to interest equation:

$$F = P(1+i)^n$$

Future sum as single payment of a present value P at i% interest rate & n periods.

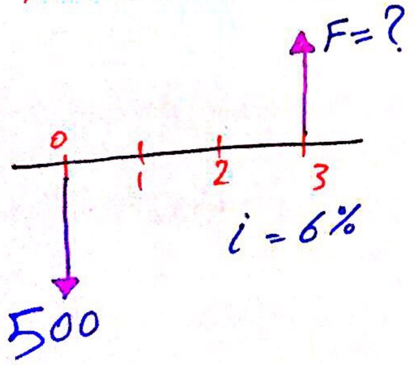
⇒ In function notation:

$$F = P(\underline{F/P}, i\%, n)$$

↳ we read it find F given P.

<u>Example</u>: if 500 were deposited in a bank. How much would be in the account 3 years from now if $i=6\%$?

* cash flow diagram:



* method(1):
By the rule $F = P(1+i)^n$

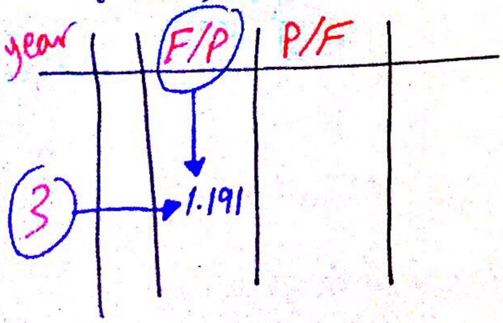
$F = 500 * (1.06)^3$

$= \boxed{595.5}$ JD

* method (2): By using the table:

as Function Notation: $F = P(F/P, 6\%, 3)$

$i = 6\%, n = 3, P = 500$



⇒ we do this:

$F = 500 * 1.191$

$= \boxed{595.5}$ JD

※Rules:

1. Cash flows cannot be added or subtracted unless They occur at the same time.

2. To move cash flows one period in the time forward ⇒ [$*(1+i)$].
   ↳ due to the main rule: $F = P(1+i)^n$

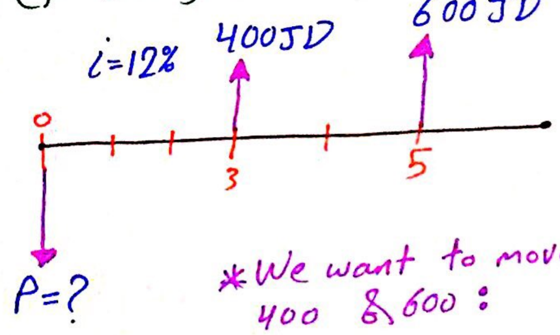
Example: suppose at year "0" you were offered a piece of paper that guaranteed that you would be paid 400 JD at the end of 3 years & 600 JD at the end of 5 years if $i = 12\%$

* How much would you pay for it?

⇒ CF Table:

year	CF
0	−P
1	0
2	0
3	+400
4	0
5	+600

⇒ CF Diagram:



* We want to move 400 & 600:

$$P = \frac{F}{(1+i)^n}$$

$$\Rightarrow P_1 = \frac{400}{(1.12)^3} = 284.7$$

$$P_2 = \frac{600}{(1.12)^5} = 340.5$$

→ $P = P_1 + P_2$

$P = 625.2$ JD

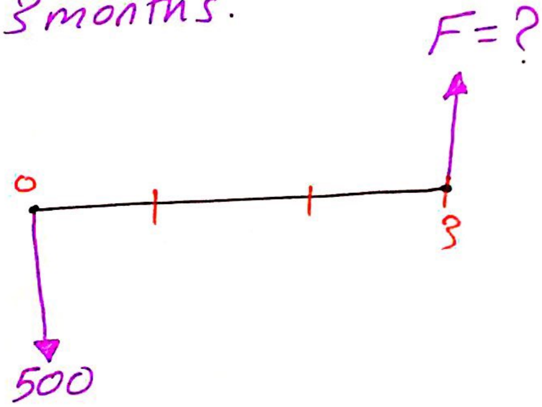
OR from the Table:

$$P = 400(P/F, 12\%, 3) + 600(P/F, 12\%, 5) = 400(0.7118) + 600(0.5674) = 625.16 \text{ JD}$$

Example: Suppose a Bank gives 6% Annual interest compound quarterly.

* How much it would be in the account after 3 years?

Compound Quarterly ≡ four interest periods per year so each = 3 months.



$F = P(F/P, i\%, n)$

6% annually

⇒ every 3 months $\frac{6\%}{4} = 1.5\%$

& number of periods through 3 years ≡ 12

$F = 500(F/P, 1.5\%, 12) = 598$ JD

* Finding (i) given P, F, N !?

Example:

if the price in 2017 is 2.03
& the price in 2005 was 1.07

* Find (i)?

continue. ⇒

Solution:

$N = 2017 - 2005 = 12$ years.

By Calculations:

$F = P(1+i)^n \Rightarrow 2.03 = 1.07\,(1+i)^{12}$

$\Rightarrow i = \sqrt[12]{\frac{2.03}{1.07}} - 1 \Rightarrow \boxed{i = 5.48\%}$

By Table:

$\frac{F}{P} = \frac{2.03}{1.07} = 1.897$

we search this number in the table.

$i = 5\%$

year	F/P
12	1.7959

$i = 6\%$

year	F/P
12	2.0122

so our number between them.

$i = 5\% \rightarrow 1.7959$

$i = ?\% \rightarrow 1.897$

$i = 6\% \rightarrow 2.0122$

$\Rightarrow$ Do an interpolation to find i:

$\boxed{i = 5.47\%}$

※ Finding N given P, F, i !?

Example: How long would it take for 500 JD investment today to become 1000 JD at 5% ?

$F = P(1+i)^n$

$2 = (1.05)^n \Rightarrow n = \dfrac{\log 2}{\log 1.05} = 14.2$ years.

* Rule:

For money to double :

* Compound interest $\longrightarrow \boxed{\dfrac{72}{i}}$

* Simple interest $\longrightarrow \boxed{\dfrac{100}{i}}$

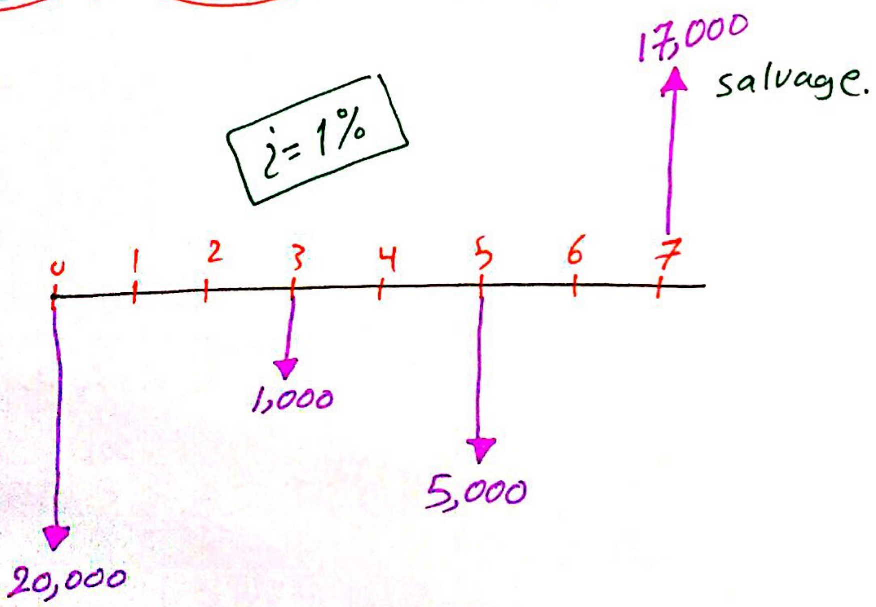
Example: if we want to double a 1000 JD in a bank use 5% compound interest :

How long it would take ?

for compound interest $\Rightarrow n = \dfrac{72}{i} = \dfrac{72}{5} = 14.4$ years.

※ Equivalence :

Example:



1 Need the CF @ year 7.

2 Need the CF @ year 4.

$\Longrightarrow$

1

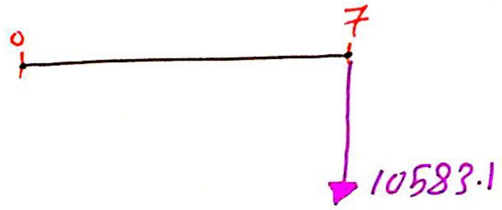
⇒ To add them together we need to shift them all to year 7.

$$F = P(1+i)^n$$

$$\Rightarrow 20{,}000\underbrace{(F/P, 1\%, 7)}_{1.0721} + 1000\underbrace{(F/P, 1\%, 4)}_{1.0406} + 5000\underbrace{(F/P, 1\%, 2)}_{1.0201} - 17000$$

$$= \boxed{10583.1}\ \text{JD}$$

⇒ The CF equivalence diagram

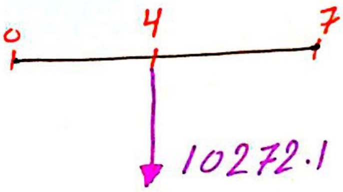


2

$$F = P(1+i)^n$$

$$\Rightarrow 20000\underbrace{(F/P, 1\%, 4)}_{1.0406} + 1000\underbrace{(F/P, 1\%, 1)}_{1.01} + 5000\underbrace{(P/F, 1\%, 1)}_{0.9904} - 17000\underbrace{(P/F, 1\%, 3)}_{0.9767}$$

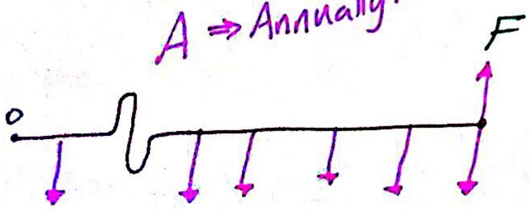
$$= \boxed{10272.1}\ \text{JD}$$



* We can check our answers by:

$F = P(1+i)^n = 10272.1\ (1 + 0.01)^3$ = it must give 10583.1 JD.

$A \Rightarrow$ Annually.



* For future:

$$F = A + A(1+i) + A(1+i)^2 + \cdots\cdots + A(1+i)^{n-1} \quad \cdots \boxed{1}$$

$\ast(1+i)$

$$F(1+i) = A(1+i) + A(1+i)^2 + \cdots\cdots + A(1+i)^n \quad \cdots \boxed{2}$$

⇒

$\Rightarrow$ subtract $\boxed{1}$ from $\boxed{2}$:

$$F(1+i) - F = A\left[(1+i)^n - 1\right]$$

$$\Rightarrow \quad F = A\left[\frac{(1+i)^n - 1}{i}\right] \quad \ldots (*)$$

* For Present:

from equ. (*): $P(1+i)^n = F = A\left[\frac{(1+i)^n - 1}{i}\right]$

$$\Rightarrow \quad P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Example:

$F = 1000$, $i = 6\%$ annually, compounded monthly, $n = 1$ year

equal payments each month find A?

Number of periods $n = 12$.

$i = 6\%$ annually $\Rightarrow i_{(month)} = 0.5\%$

method (1):

using $F = A\left[\frac{(1+i)^n - 1}{i}\right]$

$$1000 = A\left[\frac{(1+0.005)^{12} - 1}{0.005}\right]$$

solving:

$$A = 81.1$$

method (2):

using Notation:

$$A = F(A/F, 0.5\%, 12)$$

$\Downarrow$

find it from the table.

Quiz.

* Answer the following:

Q1| List two situations that engineering economics analysis can play a role ?

Answer: * payment method in any trading (Know the best way to pay).
* Calculating a company (needed money) before starting.

Q2| Define (fixed, variable, incremented) cost, provide your answer with example ?

Answer: fixed cost is not affected.
variable cost is affected.

Q3| List 3 fundemental principles of engineering economy ?

Answer: ① develope alternate.
② common unit measure.
③ use a consistent view point.

Q4| Name two fundemental approaches ?

Answer: ① Bottom-up. ② top-down.

Q5| What CER stands for ?

Answer: Cost Estimating Relationship.

Q6| The time required to assemble the 2nd car = 100 hrs.
The time required to assemble the 5th car = 70 hrs.
⇒ Find the Learning curve slop parameter (S)?

Answer: $Z = K(u)^n$

$$\Rightarrow \quad 100 = K(2)^n \;---① \qquad 70 = K(5)^n \;--②$$

$$\frac{①}{②}: \quad \frac{100}{70} = \frac{K}{K}\left(\frac{2}{5}\right)^n$$

$$\Rightarrow (0.4)^n = \frac{10}{7} \Rightarrow \log_{0.4}(0.4)^n = \cancel{\log}_{0.4}\frac{10}{7} = n$$

$$n = \log_{0.4}\frac{10}{7} \Rightarrow n = -0.389$$

$$n = \frac{\log S}{\log 2} \Rightarrow \log S = -0.117 \Rightarrow S = 0.76$$

Q7| How many years we need to double 800 JD if $i = 3\%$ & intrest is compound?

Answer: for a compound:
$$\text{number of years} = \frac{72}{i} = \frac{72}{3} = 24 \text{ year.}$$

Q8| $C_F = 39{,}000$ JD
$C_V = 55$/unit
selling price $= (160 - 0.07D)$/unit
max output = 4500 unit
D # of units.

Find the revenue?

⇒ Answer:

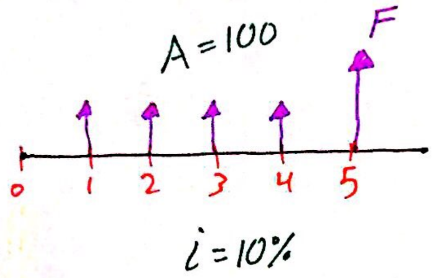
Total revenue $= \boxed{(160 - 0.07D)D}$

Revenue = BreaKeven = Cost

$(160 - 0.07D)D = 39{,}000 + 55D$

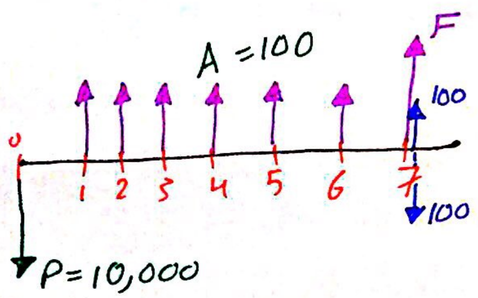
$$\boxed{D = 677 \ \& \ 823}$$

Ex. Find F ?



$\Rightarrow F = 100\,(F/A, 10\%, 5)$

Ex. Find F ?

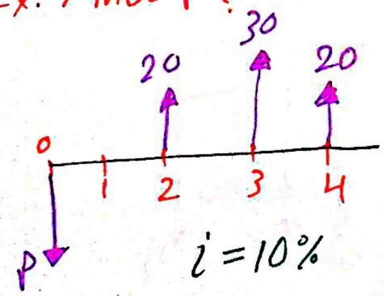


$\Rightarrow F = 10{,}000\,(F/P, 7\%, 7) - 100\,(F/A, 7\%, 7) + 100$

Another method:

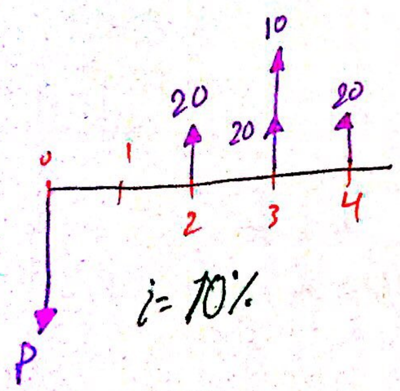
$F = 10{,}000\,(F/P, 7\%, 7) - 100\,(F/A, 7\%, 6) * (F/P, 7\%, 1)$

Ex. Find P ?



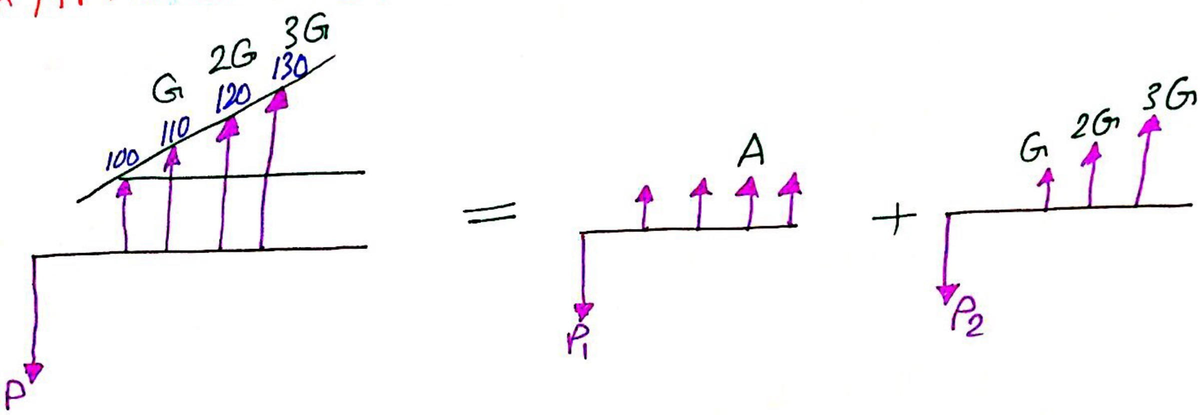
$\Rightarrow P = 20(P/F, 10\%, 2) + 30(P/F, 10\%, 3) + 20(P/F, 10\%, 4)$

Ex. Find P ?



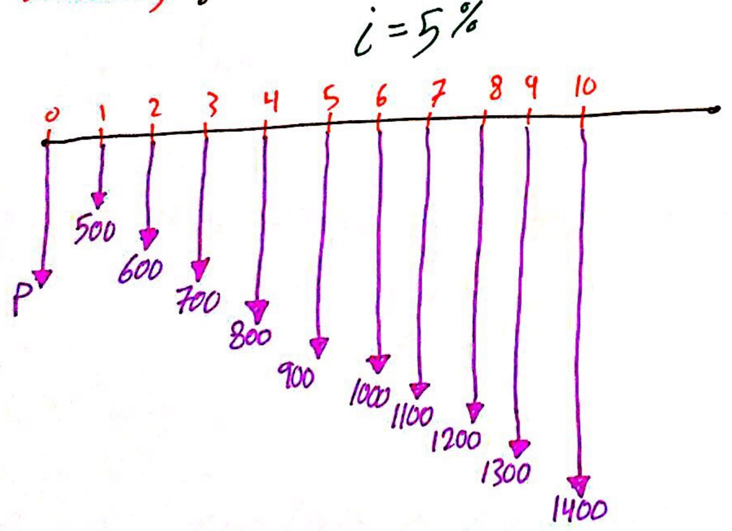
$\Rightarrow P = 20(P/A, 10\%, 3)(P/F, 10\%, 1) + 10(P/F, 10\%, 3)$

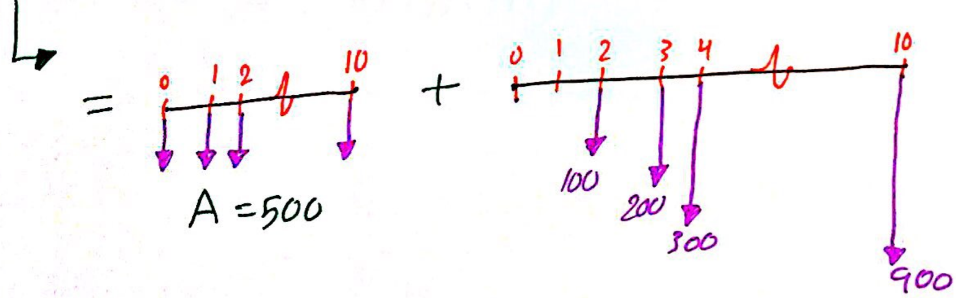
* Arithmatic Gradient:



$$\Rightarrow P = P_1 + P_2 = A\,(P/A \cdots\cdots) + G\,(P/G \cdots\cdots)$$

Example: A man decided to save 500 JD by the end of the year. He expects to increase his savings 100 JD each year for 9 years thereafter what would be the present worth of the investment $i = 5\%$ annually ?

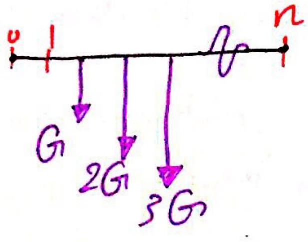


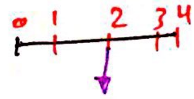


$$\Rightarrow P = P_1 + P_2 = 500\,(P/A, 5\%, 10) + 100\,(P/G, 5\%, 10)$$

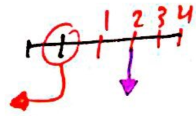
*Arithmatic Gradient:

general form:

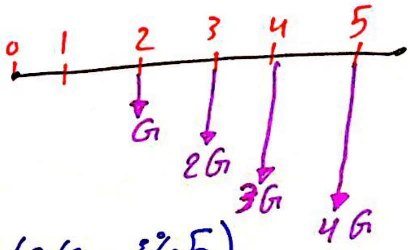




$\Rightarrow$ $n = 4$

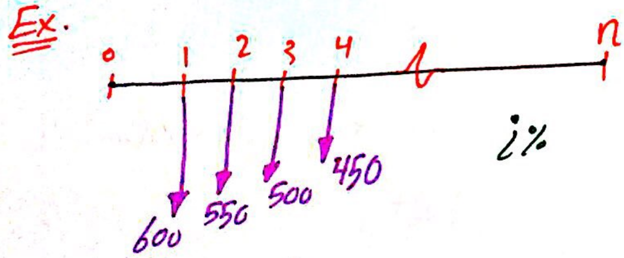


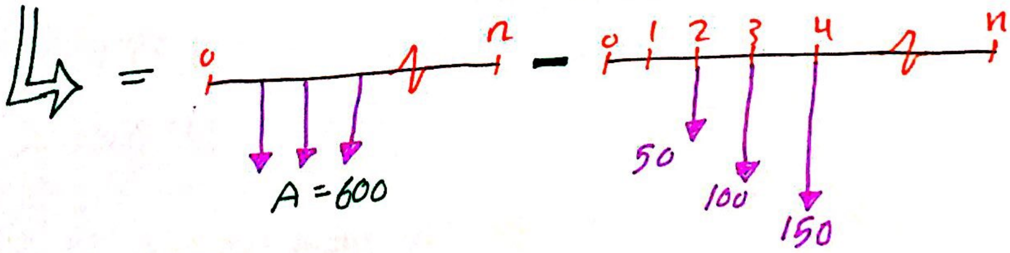
in general:



$P(P/G, i\%, 5)$

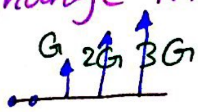
Ex.





$$\Rightarrow P = 600(P/A, i\%, n) - 50(P/G, i\%, n)$$

* The Arithmetic gradient is applicable where a period by period change in cash flow is a uniform amount.



* Sometimes the period by period change is a uniform rate (g). → could be (+) or (−).

<u>Example</u>: the maintenance of a machine costs 100JD the first year, and increasing 10% per year, $i=8\%$, find P if $n=5$ & List the cash flow table?

year	CF
1	100
2 → $100+\frac{10}{100}*100$ →	110
3 ⋮	121
4 ⋮	133.1
5	146.41

$$P = 100 + 110(P/F, 8\%, 1) + 121(P/F, 8\%, 2) + 133.1(P/F, 8\%, 3) + 146.41(P/F, 8\%, 4)$$

⇒ for more easier way using the following relations:

* if $i \neq g$:

$$\boxed{P = A\left[\frac{1-(1+g)^{n}(1+i)^{-n}}{i-g}\right]}$$

$$\Rightarrow P = 100\left[\frac{1-(1+0.1)^{5}(1+0.08)^{-5}}{0.08-0.1}\right]$$

$$\Rightarrow P = 480.43$$

* if $i = g$: $\boxed{P = An(1+i)^{-1}}$

* Nominal & Effective interest rates:

Nominal Rate: The declaired rate written or verbal, usually annual interest rate without considering compounding.

Effective Rate: we calculate taking into account the effect of any compounding during the year.

$r \equiv$ Nominal interest rate per year.

$i_e \equiv$ effective interest rate per interest period.

$i_a \equiv$ effective interest rate per year (annual).

$m \equiv$ number of compounding subperiods per time period (year).

* 1 JD compounded once @ 10%

$$i_a = (1+r)^1 - 1 = r = i = \underline{\underline{10\%}}$$

* 1 JD compounded twice $\underline{\underline{m=2}}$ @ 10% → $i = \frac{r}{2}$

$$i_a = \left(1 + \frac{10\%}{2}\right)^2 - 1 = \underline{\underline{10.25\%}}$$

* 1 JD compounded 3 times @ 10%

$$i_a = \left(1 + \frac{10\%}{3}\right)^3 - 1 = \underline{\underline{10.34\%}}$$

Example: Nominal rate 1.5% per month:

[1] Find Nominal rate per year? $1.5 * 12 = 18\%$

[2] Find effective per year if compounded monthly?

$$i_a = \left(1 + \frac{1.5\%}{1}\right)^{12} - 1 = 19.56\%$$

Example: if i effective $= 20\%$ annually compounded monthly:

1. find Nominal rate/year. ?

$$20\% = \left(1+\frac{r}{12}\right)^{12} - 1 \Longrightarrow \boxed{r = 18.37\%}$$

2. find Nominal rate/quarter?

$$\frac{r}{4} = \frac{18.37\%}{4} = \boxed{4.59\%}$$

3. find effective rate/quarter?

$$i_e = \left(1+\frac{18.37\%}{12}\right)^{3} - 1 \Longrightarrow \boxed{i_e = 4.66\%}$$

Example: Nominal Rate $= 10\%$ compounded monthly:

find the following:

1. i_e per year ?
2. i_e per 6-months ?
3. nominal rate/month. ?
4. i_e per 3-years ?

Solution:

① $i_e = \left(1+\frac{10\%}{12}\right)^{12} - 1$

$\boxed{i_e = 10.47\%}$

② $i_e = \left(1+\frac{10\%}{12}\right)^{6} - 1$

$\boxed{i_e = 5.1\%}$

③ $\frac{10\%}{12} = \boxed{0.833\%}$

④ $i_e = \left(1+\frac{10\%}{12}\right)^{36} - 1$

$\boxed{i_e = 34.8\%}$

** $F = P(1+i)^n$

$\Rightarrow F = P\left(1+\frac{r}{m}\right)^{n \cdot m}$

Let $m \rightarrow \infty$

$\Rightarrow \frac{r}{m} \rightarrow 0$

$m \cdot n \rightarrow \infty$

Assume $\frac{r}{m} = x$.

$m = \frac{r}{x} \Rightarrow m \cdot n = \frac{r \cdot n}{x}$

$F = P\left[\lim_{x \to 0} (1+x)^{1/x}\right]^{r \cdot n} \Rightarrow \lim_{x \to 0} (1+x)^{1/x} = 2.718 = e$

$\Rightarrow$ * $F = P e^{rn}$ *

* $P = F e^{-rn}$ * Continuous Compounding.

Example: if you deposited 1000JD in a Bank @ 6% nominal rate compounded continuously, how much would be in your account at the end of 3-years?

$F = P e^{r \cdot n} \Rightarrow F = 1000\, e^{0.06*3}$

$\Rightarrow F = 1197.2$ JD

*** For money to double @ $i = 10\%$

using:

simple interest $\Rightarrow n = \frac{100}{i} = \frac{100}{10} = 10$ years.

compound $\Rightarrow n = \frac{72}{i} = \frac{72}{10} = 7.2$ years.

Continuous $\Rightarrow F = P e^{r \cdot n} \Rightarrow$ we have $F = 2P$.

$\Rightarrow 2P = P e^{0.1*n} \Rightarrow \ln 2 = 0.1 n$

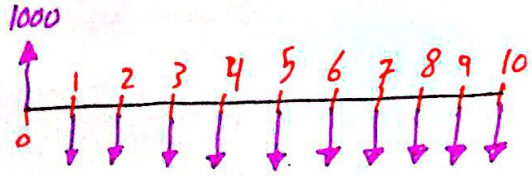
$\Rightarrow n = 6.93$ years.

Example: suppose you take a loan of 1000JD with 15% compounded monthly. Repay debt at uniform payments at the end of each month for 10 months, starting one month from now.

1] find the uniform monthly payments ?
2] find the balance after the fifth payment. ?
3] find the balance after the sixth payment ?

solution:

1]

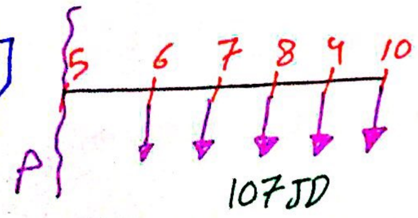


$\Rightarrow A = P(A/P, i, n)$ — i: 1.25%

$= 1000(A/P, \frac{15\%}{12}, 10)$ — this stand for years so we divide by 12.

$= \boxed{107\ JD}$

2]



$\Rightarrow P = 107(P/A, 1.25\%, 5)$

3] $P = 107(P/A, 1.25\%, 4)$

* * *

End of Mid Material

Good LucK

CHAPTER (5):

* All decision criteria should incorporate a measure of equivalence or basis of comparison so that, apparent differences become real differences that are comparable with time value of money considered.

* Basis of comparison:
  - present worth.
  - Future worth
  - Annual worth.
  - Internal rate of return (IRR).
  - External rate of return (ERR).

* Concept of equivalence:
  → resolve all C.F to P, F, A.

* efficiency: $\eta = \frac{output}{input}$.

situation	criteria
* fixed output	min. input.
* fixed input	max. output.
* Neither input nor output are fixed	$\frac{output}{input}$ max.

* Minimum Attractive Rate of Return (MARR):

Example: A bond for sale at 800JD, The buyer gets 100 JD at the end of each year for 10 years, MARR = 10%
is it suitable or Not ?

⟹

⇒ Solution:

There are 3-statuses:

1) $P = 100\,(P/A, 10\%, 10)$
$= 614$ JD
$614 < 800$ (reject).

2) $A = 800\,(A/P, 10\%, 10)$
$= 130.1$
$130.1 > 100$
(reject).

3) $800 = 100\,(P/A, i\%, 10)$
solve from table:
$i = 4.28\%$
$4.28\% < 10\%$
(reject)

---

* Different Lives:

When the PW (present worth) method is used for comparing alternatives having different lives, the same procedure is used except that the alternatives must be compared over the same number of years (??)

* Compare the alternatives over a period of time equal to the least common multiple for their lives.

* Compare the alternatives using a planning horizon which does not take into consideration the lives of alternatives.

Example:

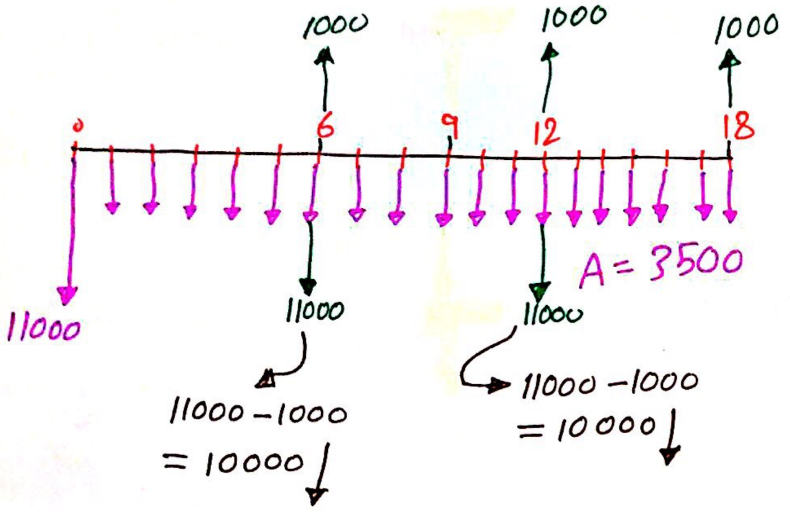
Two machines:

	A	B
First cost.	11,000	18,000
operating cost.	3500/year	3100/year
salvage value.	1000	2000
Life (years).	6	9

$i = 15\%$

Solutions for the two parts ⇒

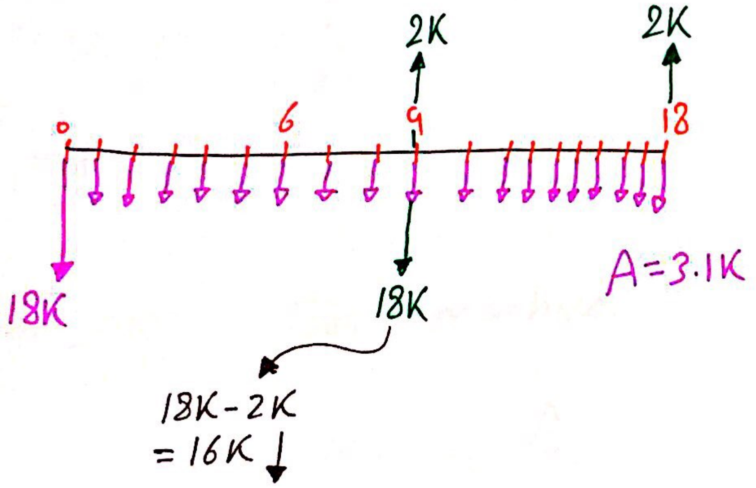
⟹ part(1):



$$PW = 3500\,(P/A, 15\%, 18) + 11000 + 10000\,(P/F, 15\%, 6) + 10000\,(P/F, 15\%, 12) - 1000\,(P/F, 15\%, 18)$$

$$= \boxed{38559 \text{ JD}}$$

⟹ Part(2):



$$PW = 18K + 3.1K\,(P/A, 15\%, 18) + 16K\,(P/F, 15\%, 9) - 2K\,(P/F, 15\%, 18)$$

$$= \boxed{41384 \text{ JD}}$$

* So now we have 38559 & 41384

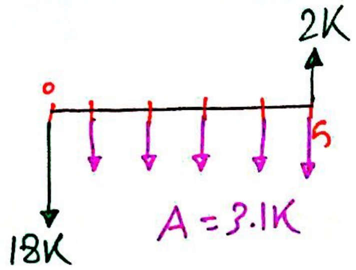
we choose the lowest cost

⟹ choose 38559 (choice 1).

* For the same previous example:

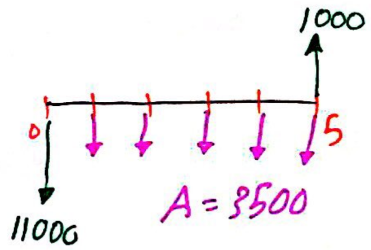
consider a planning horizon = 5 years.

⇒ for machine B:



$$PW = 18K + 3.1K\,(P/A, 15\%, 5) - 2K\,(P/F, 15\%, 5)$$

⇒ for machine A:



$$PW = 11000 + 3500\,(P/A, 15\%, 5) - 1000\,(P/F, 15\%, 5)$$

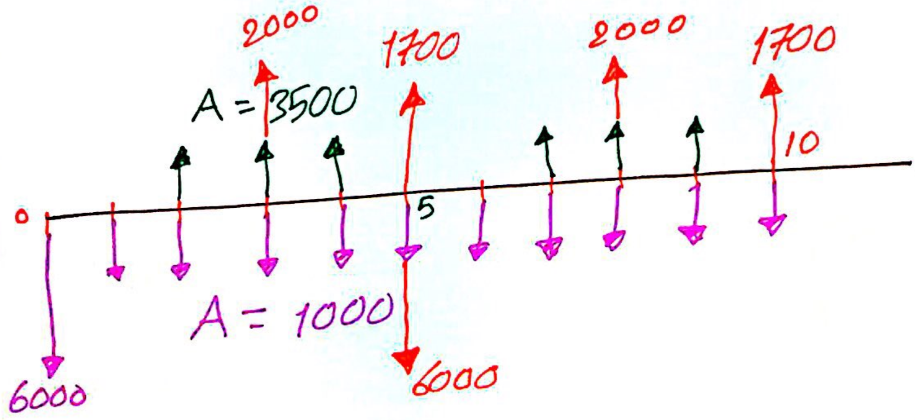
Homework: Two machines: $i = 8\%$

	A	B
initial cost	6000	8400
operation cost	1000	1200
Annual income starting end of second year.	3500	4000
income at the end of 3rd year	2000	3000
salvage	1700	2100
years	5	10

⇒ solution:

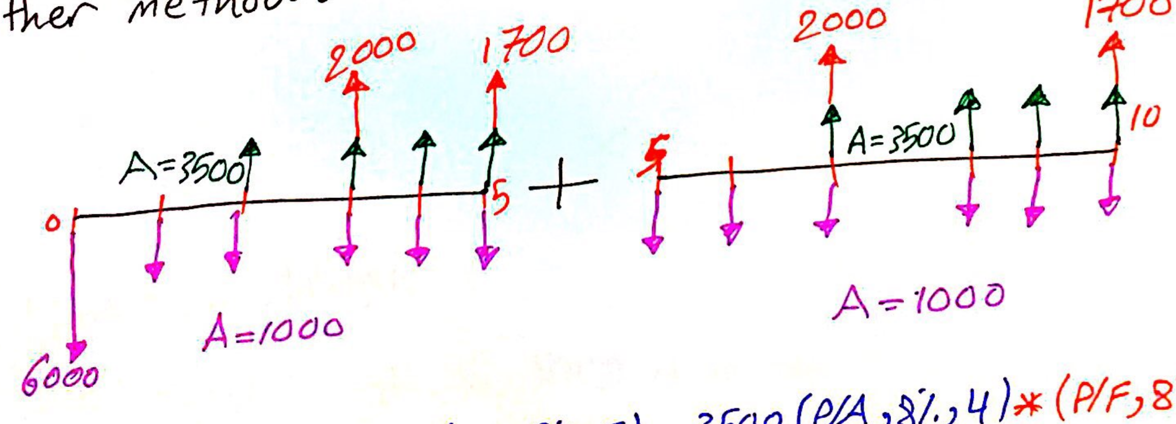
LCM(5,10) = 10

for machine (A):



$$PW = 6000 - 6000(P/F,8\%,5) - 1000(P/A,8\%,10) + 3500(P/A,8\%,4) * (P/F,8\%,1)$$
$$+ 3500(P/A,8\%,4) * (P/F,8\%,6) + 2000(P/F,8\%,3)$$
$$+ 2000(P/F,8\%,8) + 1700(P/F,8\%,5) + 1700(P/F,8\%,10)$$

Another method:

⇒ 

$$PW_1 = 6000 + 1000(P/A,8\%,5) - 3500(P/A,8\%,4) * (P/F,8\%,1)$$
$$- 2000(P/F,8\%,3) - 1700(P/F,8\%,5)$$

$$\Rightarrow PW = PW_1 + PW_1 * (P/F,8\%,5)$$

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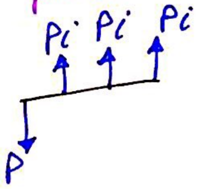
※ Capitalized Cost:

it is the present sum of money that would need to be set a side at some interest rate, to yield funds required to provide service indefinitly.

$P + Pi$



$\Rightarrow \boxed{A = Pi} \Rightarrow P = \frac{A}{i}$ CW%

Example:

$i = 0.07$

	A	B	AHC
* initial cost	6800	12000	7500
* annual operating	2000	—	—
* annual maintainance starting at end of 3rd year.	—	1600	2100
* income (every 3rd year)	—	1400	1100
* salvage	1800	—	—
* extra cost at end of 3 year	500	700	—
* Lives.	9	∞	6

* Annual Cash Flow Analysis:

Example: a man bough a car for 5000JD if he expects it to last 10 years what will be his equivalent annual uniform cost (EAUC) if $i = 7\%$? $A = 5000(A/P, 7\%, 10) = \boxed{712\ JD}$

↳ suppose he wants to sell the car after 10 years for 3000JD find EAUC ? $A = 5000(A/P, 7\%, 10) - 3000(A/F, 7\%, 10)$

$= \boxed{667\ JD}$

***

$PW_C = PW_B$ (cost, Benefit)

$PW_C - PW_B = 0$

Net present worth $= 0$

$EAUC = EAUB$

$\frac{PW_C}{PW_B} = 1$

⇒ interest rate that makes these equations TRUE is "the Rate of Return" [ROR]

it is also called:

"Breakeven rate of return"

OR. "profitability index."

it has the symbol: $i^*\%$

Example: pay 1500 now & get 500 each year for 10 years.

what is the ROR?

selecting one of the equations:

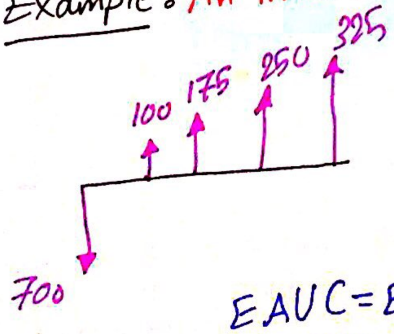
$PW_C = PW_B \Rightarrow 1500 = 500\,(P/A, i^*\%, 10)$

$\Rightarrow 3 = (P/A, i^*\%, 10)$

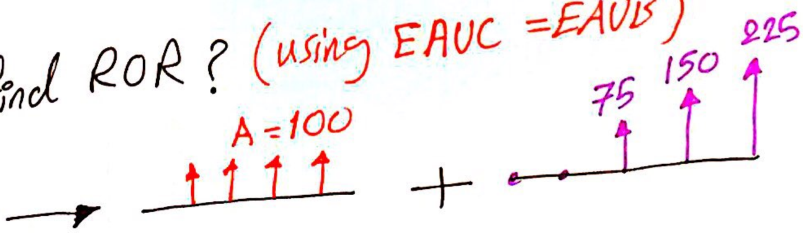
Try: $i = 30\% \Rightarrow 3.092$ → Interpolation: $i = 30.007\%$

$i = 33\% \Rightarrow 2.715$

Example: An investment resulted in the following cashflow:



find ROR? (using EAUC = EAUB)



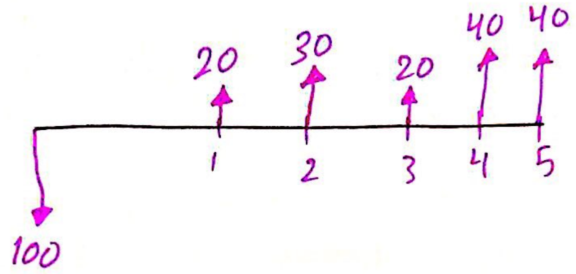
$EAUC = EAUB$

$\Rightarrow 700\,(A/P, i\%, 4) = 75\,(A/G, i\%, 4) + 100$

Try $i = 5\%$:	197	208 → Δ11
$i = 8\%$:	205	211 → Δ−6
Need Δ0 $i = 7\%$:	206	206

So $i^*\% = 7\%$

Example: find internal rate of return (IRR) ?

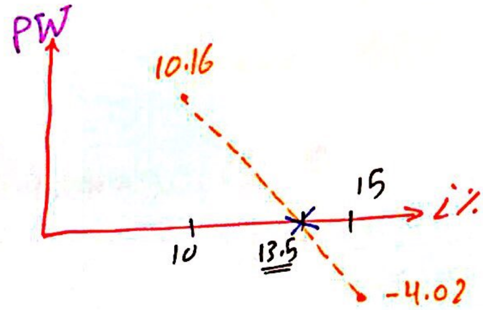


net $PW = 0$

$AEUB = AEUC$
$PW_c = PW_B$
$\frac{PW_c}{PW_B} = 1$

$PW = 0 = -100 + 20(P/F, i\%, 1) + 30(P/F, i\%, 2) + 20(P/F, i\%, 3) + 40(P/F, i\%, 4) + 40(P/F, i\%, 5)$

Trying: $i = 10\% \Rightarrow -100 + 110.16 = 10.16 \neq 0$
$i = 15\% \Rightarrow -100 + 95.98 = -4.02 \neq 0$



$\Rightarrow$ from the figure find i when $PW = 0$

so $i = 13.5\%$

$\Rightarrow$ Cashflow table: $\rightarrow$ column 5 = column 4 + 3
column 3 = column 2 * 13.5%

year [1]	uncovered investment begining of year [2]	interest [3]	cash flow [4]	investment re-payment at end of year removal of un-recovered balance [5]	unrecovered investment end of year [6]
1	-100	-13.5	20	6.5	-93.5
2	-93.5	-12.6	30	17.4	-76.1
3	-76.1	-10.3	20	9.7	-66.4
4	-66.4	-8.9	40	31.1	-35.3
5	-35.3	-4.8	40	35.2	$\simeq 0$

Example:

10 JD → 15 JD   IRR 50% ⇒ 25 JD.

20 JD → 28 JD   40% ⇒ 28 JD

from this we can notice that iRR is not a scale to choose the best project.

Example: A company has MARR = 16%   90,000 variable for ivestment:

project		
A	50,000	@ 35%
B	85,000	@ 29%

After one year:

A ⇒ 50,000 (1+0.35) + 40000 * (1+0.16) = 113,900

B ⇒ 85,000 (1+0.29) + 5000 (1+0.16) = 115,450

average(i):

$$A \Rightarrow \frac{50,000 * 0.35 + 40,000 * 0.16}{90,000} = 26.6\%$$

$$B \Rightarrow \frac{85,000 * 0.29 + 50,000 * 0.16}{90,000} = 28.6\%$$

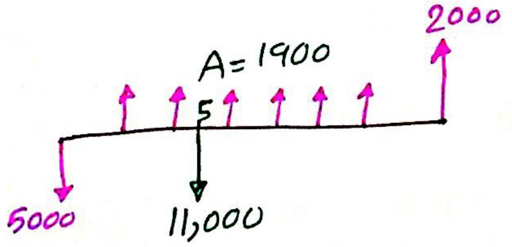
---

* Incremental Investment rate of returns
  * Tabulate the cash flow for the two alternatives:
  * if equal lives 0 → n
  * for not equal lives 0 → least common multiplier.
  * Alternative B is the one with higher initial cost.
    * ⇒ if $i_* >> MARR$ choose the higher initial cost.
    * ⇒ if $i_* << MARR$ choose the lower initial cost.

Example: Two machines with MARR = 15%

	I	II
first cost	13,000	8,000
annual cost	1600	3500
salvage	2000	0
life time	5	10

year	A	B	$\Delta(B-A)$
0	-8000	-13000	-500
1	-3500	-1600	-1900
2	-3500		1900
3			1900
4			1900 − 11,000
5			1900
6			1900
7			
8			
9			
10		+2000	+2000



$$0 = -5000 + 1900\,(P/A, i^*, 10) - 11000\,(P/F, i^*, 5) + 2000\,(P/F, i^*, 10)$$

By Trial for:

$i = 12\% \Rightarrow +11.2$

$i = 15\% \Rightarrow -3.1$

So i^* between $12\% \rightarrow 15\%$

since $i^* << MARR$

$\Rightarrow$ Choose the lower capital investment.

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※ External Rate of Return (ERR):

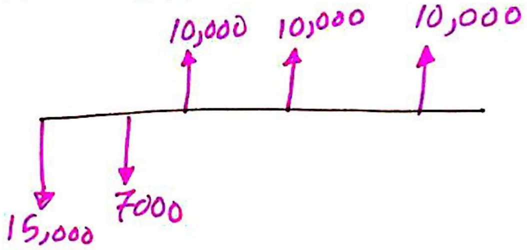
* Discount outflows to time 0 at $\epsilon\%$ per compounding period.
* Compaint all inflows to period N at $\epsilon\%$

$\Rightarrow$ find the interest rate which establishes equilibrium between the two quantities.

Example:

cash flow
−15,000
−7,000
+10,000
10,000
1000

$\epsilon\% = MARR = 12\%$

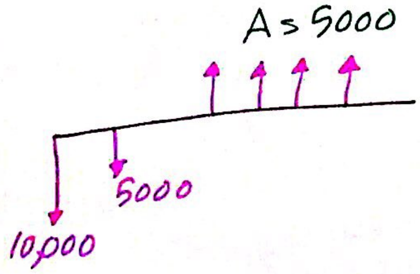


outflows $= 15{,}000 + 7000(P/F, 12\%, 1) = 21{,}250.$
inflows $= 10{,}000(F/A, 12\%, 3) = 33{,}744$

$\Rightarrow 21250(F/P, i^{*}_{ERR}\%, 4) = 33744$

$\Rightarrow i = 16.67\% > MARR.$

Example: $\epsilon = 15\%$, $MARR = 20\%$. find ERR?

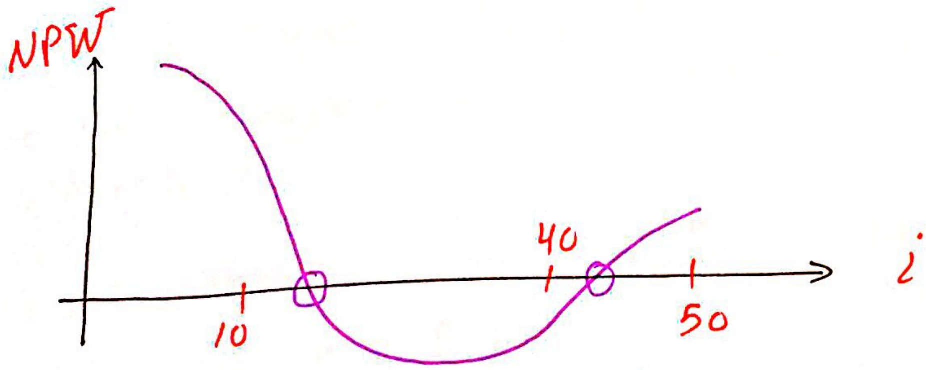


$10{,}000 + 5000(P/F, 15\%, 1) = 5000(F/A, 15\%, 5)$
$= \text{expenses}(F/P, i^{*}_{ERR}, 6)$

$i^{*} = 14.2\% < MARR$
(Reject).

year	cashflow	$i=0\%$	$i=10\%$	$i=20\%$	$i=40\%$	$i=50\%$
0	+19	19	19	19		
1	+10	10	9.1	8.3		
2	−50	−50	−41.3			
3	−50	−50	−32.6			
4	+20	+20	+13.7			
5	+20	+60	+37.3			
		+9	0.2	−2.6	−1.2	+0.6

$$\Rightarrow 19+10(P/F,i,1)-50(P/F,i,2)-50(P/F,i,3)+20(P/F,i,4)$$
$$+\,20(P/F,i,5)$$



number of sign changes	number of i^*
0	0
1	0, 1
2	0, 1, 2
3	0, 1, 2, 3

Examples:

−50	+20	−50	+50
−20	+20	+20	−20
−30	+10	+20	−20
−40	+30		
−10	+40		

→ i = Zero.

※ Payback period:

↳ it is the time required for the benefits to equal the cost of the investment.

* In all situations the criteria is:

To minimize the payback period

[how fast an investment can be recovered without considering the time value].

*Notes:

* it is an approximate analysis.

* all costs & all benefits prior (before) payback are included.

* all costs & all benefits after payback are completly ignored.

* It may or maynot lead to the correct selection between alternatives.

⟹ Alternatives:

	cost	annual benefits	salvage	years
Alt 1 →	2000	450	100	8
Alt 2 →	3000	600	700	8

$i = 8\%$

⟹ Alt 1 : $\frac{2000\ JD}{450\ JD/year} = 4.4$ years payback.

Alt 2 : $\frac{3000\ JD}{600\ JD/year} = 5$ years.

Example: A machine is purchased for 18,000 JD and expected to produce annual return of 3000 salvage value is 3000 if sold at any time during the anticipated ownship (10 years) if $i=15\%$ find the pay back?

$$18000 = 3000(P/A, 15\%, n) + 3000(P/F, 15\%, n)$$

$\Rightarrow$ By trial $n=10 \rightarrow 15799$.
$n=15 \rightarrow 17541$.
$n=16 \rightarrow 18182$.

in between 15 & 16 but the time 10 years so Reject.

year	cash flow	comulative PW at 0%	$i=20\%$ PW	comulative PW
0	-25000	-25000	-25000	-25000
1	8000	-17000	6667	-18333
2	8000	-9000	5556	-12777
3	8000	-1000	4630	-8147
4	8000	+7000 ≃ 4 years	3858	-4289
5	8000	+20000	5223	+934 ≃ 5 years

* * *

End of CH5.

CHAPTER (6):

Comparison & Selection among Alternatives:

*Objective:

To evaluate correctly capital investment alternatives when time value of money is important.

Budget → furniture.
→ PC & printers.
→ cameras & pens.

⇒ each one is mutually exclusive.

* Examine on basis of economic considerations all alteratives provide comparable usefulness quality, performance, etc.

* The alternative that requires minimum investment is chosen (Base Alternative).
  ↳ Unless the incremental increase in capital is justified.

* There are Two basic Alternatives:

1. * investment alternative: initial capital investmentment that produces positive cash flows.

2. * Cost Alternative: all cash flow is negative except salvage.

Scanned by CamScanner

※ Cost Alternative:

$i = 10\%$

year	Alt. (A)	Alt (B)	Δ (B−A)
0	-38000	-415000	-35000
1	-38100	-27400	+10700
2	-39100	-27400	+11700
3	-40100	-27400	12700
salvage	0	+2600	2600
	PW = -477,077	PW = -463,607	PW = 13470

choose Alt (B).

from it, B is justified.

※ Investment Alternative:

	Alt (A)	Alt (B)
capital invest.	-60000	-73000
annual Rev.	22000	26225

MARR = 10%

n = 4

*Take Alt (A) as base alternative.
Then we compare with Alt (B).

$PW_A = -60000 + 22000(P/A, 10\%, 4) = 9738$ JD.

$PW_B = -73000 + 26225(P/A, 10\%, 4) = 10131$ JD

$10131 - 9738 = \underline{393}$

* we choose for the highest PW so choose Alt B.

Another method.

⇒ Another method:

$\Delta(B-A)$

-13,000

+4,225

⇒ $PW_\Delta = -13000 + 4225(P/A, 10\%, 4)$

$= +393$

* if the answer +ve choose B.

so we Choose Alt B.

Rule(1): when revenues are present choose the alternative with maximum profit.

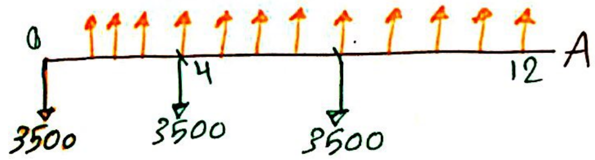
Rule(2): when revenues are NOT present minimize the Cost.

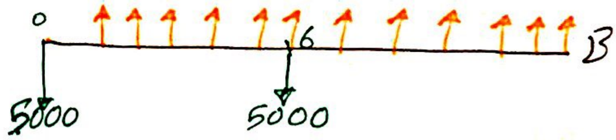
* For equal Lives use PW, FW, AW, - - - .
* For unequal lives use infinity or least common multiplier.

Example:

MARR = i = 10%

	A	B
capital input	3500	5000
annual income	1255	1480
useful Life	4	6





LCM(4,6) = 12 ⇒ $\boxed{n = 12}$

$PW_A = -3500 + 1255(P/A, 10\%, 12) - 3500(P/F, 10\%, 4) - 3500(P/F, 10\%, 8) = 1028$ JD.

$PW_B = -5000 + 1480(P/A, 10\%, 12) - 5000(P/F, 10\%, 6) = 2262$ JD.

since $PW_B > PW_A$ choose Alt B.

⇒ for the example in page 45:

if we want to use the IRR:

$PW = 0$

$IRR_A \Rightarrow 60000 = 22000(P/A, i^*, 4) \Rightarrow \boxed{i^* = 17.3\%}$

$IRR_B \Rightarrow 73000 = 26225(P/A, i^*, 4) \Rightarrow \boxed{i^* = 16.3\%}$

Here we choose A since $IRR_A > IRR_B$ But the rule say DO NOT compare IRR together, compare with MARR.

⇒ so $IRR_A > MARR$ & $IRR_B > MARR$ (so Both successful)

IRR_Δ: $13000 = 4225(P/A, i^*, 4) \Rightarrow \boxed{i^* = 11.4\%} > MARR$

so choose **B**.

Therefore, it is justified to add more investment because you will get $\Delta PW \geq 0$ & $IRR_\Delta > MARR$.

※ The Incrementa analysis procedure to avoid incorrect rating of MEA using rate of return method:

1) Arrange alternatives based on increasing capital investment.

2) Establish a base alternative:

   a) Cost alternative (least capital investment).

   b) Investment alternative. if the first alternative is acceptable select it as base, or go to the next one. (acceptable ≡ $IRR \geq MARR$ & $PW \geq 0$)

3) Use iteration to evaluate differences (Δ, increment cashflow).

If the next (higher investment alt.) is acceptable choose it as base alt.

$\Delta(B-A)$ — A, B: if B better it is base.

↳ if $IRR < MARR$ (cancel it)

$\Delta(C-B)$ — C, D: ↳ if C better Now it is base.

$\Delta(D-C)$ ↳ if D better then it is the base.

Example: which one is the best:

$i = MARR = 20\%$

	①	②	③	④
capital investment	140000	122000	100000	168200
annual expenses	16900	22100	29000	14800
salvage value.	14000	14000	10000	25600
useful Life.	5	5	5	5

⇒ * Re-arranging:

	A	B	C	D
	−100000	−122000	−140000	−168200
	−29000	−22100	−16900	−14800
	10000	14000	14000	25600
PW	−182708	182465.6	−184914	−201972

Δ	(B−A)	(C−B)	(D−B)
Δ cap. invest.	−22000	−18000	46200
Δ annual	6900	5200	7300
Δ salvage	4000	0	11600
Δ IRR	20.51%	12.3%	< 20.3
justified!	✓	X (cancel C)	X

$\Rightarrow 22000 = 6900\,(P/A, iRR\%, 5) + 4000\,(P/F, iRR\%, 5)$

$iRR = 20\% \rightarrow 21300$

$iRR = 25\% \rightarrow 23400$

we have: $20\% < iRR < 25\%$ regardless to its value $\Rightarrow iRR > MARR$.

⇒ Best To choose **B**.

Example: (investment alt.).

MARR = 10%	A	B	C	D	E	F
capital invest.	−900	1500	2500	4000	5000	7000
net annual Rev.	150	276	400	925	1125	1425
IRR	10.6%	13%	9.6%	19.1%	18.3%	15.6%
justified!	✓	✓	X	✓	✓	✓

$-900 + 150\,(P/A, i^*, 10) = 0 \Rightarrow IRR$ ✓

⇒

Δ	(B−A)	(D−B)	(E−D)	(F−E)
Δ cap. inv.	−600	−2500	⋮	⋮
Δ Rev.	126	649	⋮	⋮
Δ IRR.	10.6%	22.6%	15.1%	8.1%
justified!	✓	✓	✓	×

choose $\underline{\underline{E}}$.

Alt. E is choosen (it is the last invest. for which the last increment of capital invest is justified).

DO NOT CHOOSE:

1] The highest IRR on total cashflow (D).

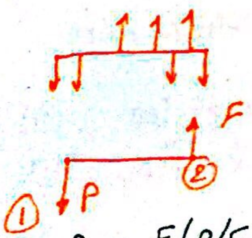
2] The highest IRR on incremental inv. (D).

3] The highest capital inv. that has IRR ≥ MARR (F).

***using incremental ERR:**

<u>review</u>:

① discount all outflow cash to present at ϵ% = MARR.
compound all inflows to future at ϵ% = MARR.



for more than one CF ⇒ ΔP.

$P = F(P/F, ERR, n)$

*similer to IRR: ΔPW of −ve CFs @ $i = \epsilon\%$ usually MARR.
ΔFW of +ve CFs @ $i = \epsilon\%$ usually MARR.

find $i = ERR$ $\Delta P = \Delta F(P/F, i, n)$ if $i > MARR$.

<u>Example:</u> MARR = 15%

	A	B	C
initial cost.	−6000	−7000	−9000
annual. CF	2000	3000	3000
salvage.	0	200	300
Life time.	3	4	6

⟹

⇒ LCM(3,4,6) = 12 so $\underline{n=12}$

$-6000 + 2000(P/A, ERR\%, 3) = 0$

we have 2000 every year so No interest ⇒ $i^* = 0\%$

CANCEL Alt $\boxed{A}$.

year	B → defender	C → challenger.	Δ(C−B)
0	−7000	−9000	−2000
1	3000	3000	0
2	3000	3000	0
3	3000	3000	0
4	3000+200−7000	3000	6800
5	3000	3000	0
6	3000	3000+300−9000	−8700
7	3000	3000	0
8	3000+200−7000	3000	6800
9	3000	3000	0
10	3000	3000	0
11	3000	3000	0
12	3000+200	3000+300	100

using IRRΔ:

⇒ $-2000 + 6800(P/F, i^*, 4) - 8700(P/F, i^*, 6) + 6800(P/F, i^*, 8) + 100(P/F, i^*, 12)$

⇒ find IRRΔ if > MARR choose challanger.

using ΔERR:

(P) out flows $= -2000 + -8700(P/F, 15\%, 6)$

(F) inflows $= 6800(F/P, 15\%, 8) + 6800(F/P, 15\%, 4) + 100$

$\Rightarrow P = F(P/F, ERR_\Delta, 12)$

$\Rightarrow ERR_\Delta > MARR.$

$\frac{B}{C}$ B → Benefits. C → Cost

for more than one project: $\frac{\Delta B}{\Delta C} \geq 1$ choose the higher cost, otherwise choose the lower cost alt.

Example:

	Route N	Route S
initial Cost.	10000000	15000000
maintenance.	35000	55000
Roadwer./year.	200000	450000
life time.	30	30
$EUAW_{cost}$.	685500	1030750
$EUAW_B$	200000	450000

$685500 = 10000000(A/P, 5\%, 30) + 35000$

B/C:

Route N: $\frac{200}{685.5} \simeq 0.3$

Route S: $\frac{450}{1030} \simeq 0.4$

we don't choose here

⇒

$\Delta(S-N)$

$\Delta B = 250000$ $\qquad \frac{\Delta B}{\Delta C} = 0.74 < 1$ $\qquad$ choose Route N.

$\Delta C = 345250$

Example: you have 20.

−5	−20
10	30

$\frac{\Delta B}{\Delta C} = \frac{30-10}{20-5} = \frac{20}{15} = 1.33 > 1$

B/C $\frac{10}{5} = 2$ $\qquad \frac{30}{20} = 1.5$

Example: MARR = 10%

	C	A	B	D
Bulding cost.	−190000	−200000	−257000	−350000
cash flow year	19000	220000	35000	42000
	30	30	30	30
present worth of benefit.	183260	207394	329945	345934.
B/C	0.97	1.03	1.2	1.13

	B − A	D − B
ΔB	329945 − 207394 = 12255	65989
ΔC	75000	75000
$\frac{\Delta B}{\Delta C}$	1.64	0.88 < 1

choose B.

end of CH6.

CHAPTER (7):

property − tangible. → personal, real.

* Intangible:

1 − physical life. $\qquad$ 2 − useful life.

3 − Depreciable Life. $\qquad$ 4 − economic life.

* **Book Value:** represents remaining undepreciated investment after depreciation is subtracted = cost - depreciation

* **Market Value:** The cost of the asset if it is to be sold.

* **Depreciation:** The decrease in value of physical property with the passage of time (For Tax Purposes).
  * Land is a NON-depreciable asset.
  * The depreciation affects the cashflow of taxes but it is NOT a cashflow.

* The money I have is for buying a new asset.

income (I) = Benefits - Cost.

$(\ ) = I - D \rightarrow$ Depreciation.

X tax rate.

$\text{tax} = X(I-D)$

$$\Rightarrow ATCF = I - X(I-D)$$
$$= I(1-X) + XD$$

* **Methods of depreciation:**

$\boxed{1}$ straight line (ST) depreciation.

$\Rightarrow$ a constant dep. charge is made anually $= \frac{1}{N}(P-S)$

N: number of years, P: price, S: salvage.

Example:

an asset 50000, salvage 10000, n=5

a) Calculate the annual dep.?

$$\frac{50000-10000}{5} = 8000$$

b) Compute BV (Book value) each year?

50000

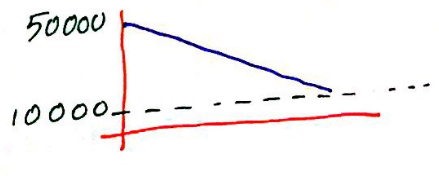
$BV_1 = 42000$

$BV_2 = 34000$

$BV_3 = 26000$

$BV_4 = 18000$

$BV_5 = 10000 \equiv$ salvage value.



2 Declining Balance Depreciation (DB)

↳ Double Declining Balance (DDB)

⇒ The dep. is determined by multiplying a uniform pecentage by BV at the end of the year.

⇒ The max percentage allowed is 200% (DDB).

$$\text{Dep.} = \frac{2}{N}(BV) = \frac{2}{N}(\text{cost} - \text{dep. charges to date}).$$

<u>Example:</u> cost 900, Life 5, salvage 70

find dep. using DDB?

1^{st} year: $dep = \frac{2}{5}(900-0) = 360$

2^{nd} year: $dep = \frac{2}{5}(900-360) = 216$

3^{rd} year: $dep = \frac{2}{5}(900-576) = 130$

4^{th} year: $dep = \frac{2}{5}(900-706) = 78$

5^{th} year: $dep = \frac{2}{5}(900-784) = 46$ < salvage.

---

3 Unit production method.

⇒ dep. is based on activity not on time.

$$D = \frac{B - SU}{\text{estimated life time production.}}$$

<u>Example:</u> cost 50000
salvage 10000
when repacked after 30000 hrs of use

find dep. rate per hour & the BV after 10000 hrs of operation?

$$D = \frac{50000 - 10000}{50000} = 1.33 \text{ JD/hr.}$$

after 10 000 hrs: $10000 * 1.33$

$BV = 50000 - 10000 * 1.33 = 36700$

---

4 Sum of years digits method.

$$S = N + (N-1) + (N-2) + \cdots = \frac{N(N+1)}{2}$$

$$D = \frac{N-(t-1)}{S}(BV_0 - BV_N)$$

Example: $BV_0 = 80000$, $BV_N = 20000$, $N = 6$

$S = \frac{6(7)}{2} = 21 \Rightarrow D_1 = \frac{6-(1-1)}{21}(80000 - 20000) = 17142$

$D_2 = \frac{6-(2-1)}{21} * 60000$ and so on for the rest.

⇒

⇒ Compare between the methods in the ex.

Dep.

S.O.Y.D ①	SL ②	DDB ③
17142	10000	26666
14285.7	10000	17777
⋮	⋮	7901
⋮	⋮	⋮
D ⇒ 2857	10000	0

⇒ we choose the one with higher dep.
at the begining DDB was the best, at the end SL is the best.

---

4 MACRS : modified accelerated cost recovery system.

⇒ two systems → General dep. S (GDS).
→ Alternative dep. S (ADS).

each property is categorized into asset classes in each class the property is assigned a recovery period.

Table 7.2:

asset class	Description of assets	class life	GDS	ADS
00.11	furniture	10	7	10
00.12	computers.	6	5	5

Table 7.3: DDB = 200%

year	3-years	5-years	7-years.
1	0.3333		
2	0.4445		
3	0.1481		
⋮			

Example: A firm is purchased a semi-conductor manufacturing equipment
cost = 100000 find dep. charge permissible in the fourth year?
* go to table 7.2 the asset has a classlife = 6 recovery = 5
* " " " 7.3 under recovery = 5 years you will find:

1	0.2
2	0.32
3	0.19
4	0.1152
5	0.1152

dep. or cost of recovery = 0.1152 * 100000 = 11520.

BV at the end of the fourth year:

$100000 - 100000\,[0.2 + 0.32 + 0.19 + 0.1152]$

$= 172801.$

Two alternative:

	S_1	S_2
capital investment	100000	200000
useful life	7	6
annual income.	20000	40000
SV	30000	60000
GDS recovery	5	5

$i = 10\%$, income tax = 40%.
MARCS (GDS) is required.

for S_1:

from table 7.3

year.	BTCF	depreciation rate.	dep. value.	taxable income	income tax	ATCF
0	-100000	—	—	—	0	-100000
1	20000	0.2	20000	0	0	20000
2	20000	0.32	32000	-12000	-4800	24800
3	20000	0.192	19200	800	+320	19680
4	⋮	0.1152	11520	8480	3392	166800
5	⋮	0.1152	11520	8480	3392	166800
6	⋮	0.0576	5760	14240	5690	14310
7	20000	0	0	20000	8000	12000
7(salvage)	30000	0	0	30000	12000	18000

$\ast$ 100000

H.W ⇒ Do the same for S_2.

---

tax rate = 20%

	BTCF	dep.	taxable income	tax	ATCF
i	-1000000	—	—	0	-1000000
1	300000	120000	180000	36000	264000
2	300000	120000	180000	36000	264000
3	300000	120000	180000	36000	264000
4	300000	120000	180000	36000	264000
5	300000 + 400000 (salvage)	120000	180000 + 400000 = 580000	116000	520000

straight line dep.

$$\frac{1000000 - 400000}{5} = 120000$$

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End of Material.