Sol to First Exam:23/3/2014 Electromagnetics I: EE251

Prob. # 1[5 PTS]: *In a given region whose permittivity $\varepsilon = 4\varepsilon_0$ F/m, the electric flux density $D = 4\pi x\, a_x - 3\, a_y + 5\pi z\, a_z$ C/m², then:*

*i. At the origin (0, 0, 0) write down **E** in spherical coordinate.*

$E(0,0,0) = -(3/4\varepsilon_0)a_r$ V/m

ii. Determine the volume charge density in this region.

$\nabla \cdot D = \rho_v = 4\pi + 5\pi = 9\pi$ C/m³

Prob. # 2 [5 PTS]: *A good conducting spherical shell whose radius = a is located in a medium whose permittivity $\varepsilon = 2\varepsilon_0$. The potential of this shell = V_0 V. Then find the electric field, and the potential for this arrangement everywhere.*

Assuming that the total charge on the shell = q then

the potential of the spherical shell $=V(r)=q/(8\pi\varepsilon_0 r)$ and at r=a, $V(r=a) = V_0$ then $q = 8\pi\varepsilon_0 a V_0$ C.

$E_r = q/(8\pi\varepsilon_0 r^2) = aV_0/r^2$ V/m for $r \geq a$, and = zero for $r<a$. $V(r) = aV_0/r$ Volt for $r>a$, and $= V_0$ for $r \leq a$

Prob. # 3 [5 PTS]: *Derive the potential and the electric field **E** at a point (r, θ, ∅) far away from a short electric dipole whose moment $m_e = 10^{-9}/9\, a_z$ Cm.*

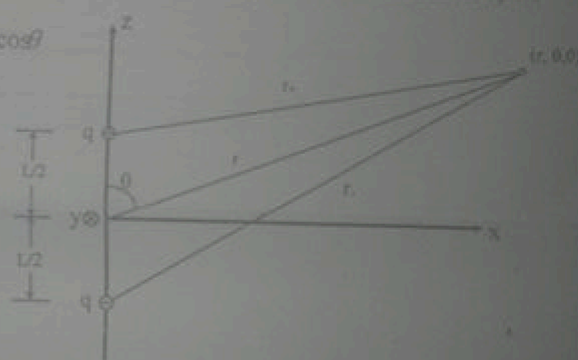
$V(r,\theta) = \frac{q}{4\pi\varepsilon}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$; $r_\pm = r\left[1 \mp (L/2r)\cos\theta + \ldots\right] \cong r \mp (L/2)\cos\theta$

$V(r,\theta) = \frac{q}{4\pi\varepsilon}\frac{L\cos\theta}{r^2 - (L/2)^2\cos^2\theta}$; $V(r,\theta) = \frac{qL}{4\pi\varepsilon r^2}\cos\theta$;

$E = -\nabla V$; $E = E_r a_r + E_\theta a_\theta = \frac{qL}{4\pi\varepsilon r^3}\left[2\cos\theta\, a_r + \sin\theta\, a_\theta\right]$ V/m

$m_e = qL = qL\, a_z$ Cm

$qL = 10^{-9}/9$

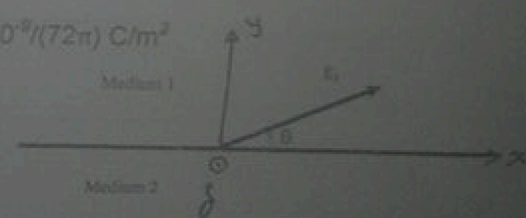


Prob. # 4 [5 PTS]: *In the figure shown below medium 1 permittivity = 1.5 ε_0 F/m and medium 2 is a grounded perfect conductor. The magnitude of the electric field E_1 in medium 1 = 1 V/m. Add your Cartesian coordinate system on this figure and accordingly determine the components of **E** and **D** in both regions. Determine the surface charge densities everywhere and identify their types.*

In medium 2 is good conducting then $E_2 = 0 = D_2$ or $E_{2tang} = 0$ then from B. C. $E_{1tang} = 0$, accordingly $\theta = 90^\circ$, so $E_1 = 1a_y$ V/m. $D_1 = 1.5\varepsilon_0\, a_y = 10^{-9}/(24\pi)\, a_y$ C/m².

free surface charge density at $y=0^-$ $\rho_{sf} = 10^{-9}/(24\pi)$ C/m²

bounded surface charge density $y=0^+$ $\rho_{sp} = -(\varepsilon-\varepsilon_0)E_{1n} = -10^{-9}/(72\pi)$ C/m²



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Prob. # 1[5 PTS]: *In a given region whose permittivity $\varepsilon = 4\varepsilon_0$ F/m, the electric flux density* $\mathbf{D} = 4\pi x\, \mathbf{a}_x - 3\, \mathbf{a}_y + 5\pi z\, \mathbf{a}_z$ *C/m^2; then:*

*i. At the origin (0, 0, 0) write down **E** in spherical coordinate.*

$E(0,0,0) = -(3/4\varepsilon_0)\mathbf{a}_r$ V/m

ii. Determine the volume charge density in this region.

$\nabla . D = \rho_v = 4\pi + 5\pi = 9\pi$ C/m^3

Prob. # 2 [5 PTS]: *A good conducting spherical shell whose radius = a is located in a medium whose permittivity $\varepsilon = 2\varepsilon_0$. The potential of this shell = V_0 V. Then find the electric field, and the potential for this arrangement everywhere.*

Assuming that the total charge on the shell = q then

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Prob. # 3 [5 PTS]: *Derive the potential and the electric field **E** at a point (r, θ, ø) far away from a short electric dipole whose moment $\mathbf{m}_e = 10^{-9}/9\, \mathbf{a}_z$ Cm.*

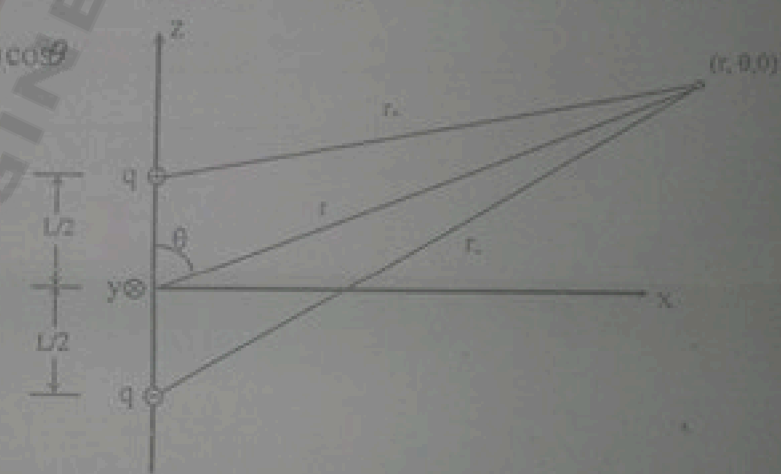
$$V(r,\theta) = \frac{q}{4\pi\varepsilon}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) \quad r_\pm = r\left[1 \mp (L/2r)\cos\theta + ..\right] \approx r \mp (L/2)\cos\theta$$

$$V(r,\theta) \approx \frac{q}{4\pi\varepsilon}\frac{L\cos\theta}{r^2 - (L/2)^2\cos^2\theta}, \quad V(r,\theta) = \frac{qL}{4\pi\varepsilon r^2}\cos\theta \quad V$$

$$\mathbf{E} = -\nabla V, \quad \mathbf{E} = E_r\mathbf{a}_r + E_\theta\mathbf{a}_\theta = \frac{qL}{4\pi\varepsilon r^3}\left[2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta\right] \text{ V/m}$$

$$m_e = qL = qL\,\mathbf{a}_z \quad \text{Cm}$$

$qL = 10^{-9}/9$



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free surface charge density at y=0⁻ $\rho_{sf} = 10^{-9}/(24\pi)$ C/m^2

bounded surface charge density y=0⁺ $\rho_{sp} = -(\varepsilon - \varepsilon_0)E_{1n} = -10^{-9}/(72\pi)$ C/m^2