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EE 251: Electromagnetic 1
Second Exam (Summer 2016)
July 31th, 2016

Notes:

$\varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho \mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} = \begin{vmatrix} a\rho & a\phi & az \\ \frac{\partial}{\partial \rho} & \frac{1}{\rho}\frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A\rho & A\phi & Az \end{vmatrix}$$

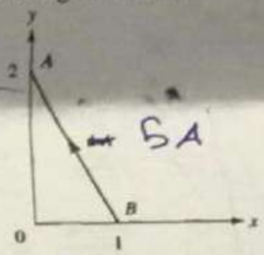
$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

$a\phi\left(\frac{\partial}{\partial \rho} Az - \frac{\partial}{\partial z} A\rho\right)$

5

Question 1 (6 pts)

Find **H** at (0, 0, 5) due to a conductor AB carrying 5 A of current as in the figure shown.



$\bar{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\, a\phi$ A/m

$AB = (-1, 2, 0)$
$|AB| = \sqrt{5}$

$BC = (0, -2, 5)$
$|BC| = \sqrt{29}$

$Ac = (-1, 0, 5)$
$|Ac| = \sqrt{26}$

$BA = (1, -2, 0)$
$|BA| = \sqrt{5}$

* $AB \cdot Ac = 1$

$\cos\alpha_1 = \frac{AB \cdot Ac}{|AB||Ac|} = 0.08$ (0.5)

* $BA \cdot Bc = 4$

$\cos\alpha_2 = \frac{BA \cdot Bc}{|BA||Bc|} = 0.33$ (1)

$a\phi = al \times a\rho$ (1)

$al = \left(\frac{-ax + 2ay}{\sqrt{5}}\right) \times \left(\frac{-ax + 5az}{\sqrt{26}}\right)$

$a\phi = al \times a\rho$

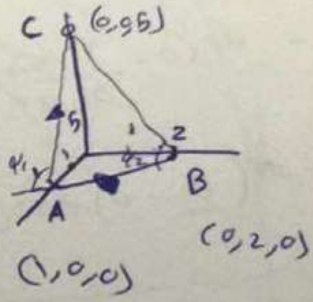
$a\phi = \frac{1}{\sqrt{5}\sqrt{26}}(5ay + 2az)$ (1)

$a\phi = 0.08(5ay + 2az)$

$\bar{H} = \frac{5}{4\pi(1)}(0.33 - 0.08)\, a\phi$

$\boxed{\bar{H} = 0.039\, ay + 0.015\, az}$

(0.5)



Question 2 (9 pts)

An infinitely long conductor of radius 'a' carries a uniform current with $\mathbf{J} = J_o\, \mathbf{a}_z$. Find the magnetic vector and scalar potentials everywhere.

7.5

$B = \nabla \times A$

$B = \mu_o H$

$\boxed{\rho < a}$

$\oint H \cdot dl = I_{enc} = \int J \cdot ds$ (1)

$\int_0^{2\pi} H_\phi \,\rho\, d\phi = \int_0^{a}\int_0^{2\pi} J_o \,\rho\, d\phi\, d\rho$

$H_\rho 2\pi = J_o \frac{2\pi}{2} \rho^2 \Big|_0^a$

$\boxed{\bar{H} = \frac{J_o a^2}{2\rho} \hat{a}_\phi}$ $\boxed{\bar{B} = \mu_o H}$

$\boxed{\bar{B} = \frac{\mu_o J_o a^2}{2\rho} a_\phi}$ (0.5)

$B = \nabla \times A$

$\frac{\mu_o J_o a^2}{2\rho} a_\phi = \left(\frac{\partial A_z}{\partial \rho} - \frac{\partial}{\partial z} A_\rho\right) \hat{a}_\phi$ (-)

$\frac{\partial A_z}{\partial \rho} = \int_0^a \frac{\mu_o J_o a^2}{2\rho} d\rho \rightarrow \boxed{A_z = \frac{\mu_o J_o a^2}{2} \ln a + C_1}$

wb/m

$\int \frac{-\partial}{\partial z} A_\rho = \int \frac{\mu_o J_o a^2 dz}{2\rho}$

$\boxed{A_\rho = \frac{\mu_o J_o a^2}{2\rho} z + C_2}$ Wb/m   (0.5)

$A = (A_\rho, A_\phi, A_z)$ = (✓, 0, ✓)

$H = -\nabla V_m$ (1)

$-\frac{J_o a^2}{2\rho} = \frac{dV_m}{\partial \phi} a_\phi$

$\boxed{V_m = \frac{-J_o a^2 \cancel{2}\pi}{\cancel{2}\rho}}$ (0.5)

$\boxed{\rho > a}$

$\oint H \cdot dl = I_{en} = \int \bar{J} \cdot d\bar{s}$

$H_\rho 2\pi = \int_0^{\rho}\int_0^{2\pi} J_o \,\rho\, d\phi\, d\rho$

$\odot\ az$

$\odot\ I$

$H_\rho 2\pi = \frac{2\pi J_o \rho^2}{2}$

$\boxed{\bar{H} = \frac{J_o \rho}{2} \hat{a}_\phi}$ (0.5)

$\boxed{\bar{B} = \frac{\mu_o J_o \rho}{2} a_\phi}$

$B = \nabla \times A$ (1)

$\frac{\mu_o J \rho}{2} = \left(\frac{\partial}{\partial \rho} A_z - \frac{\partial}{\partial z} A_\rho\right)$

$A_z = \int_0^{\rho} \frac{\mu_o J \rho}{2}$

$\boxed{A_z = \frac{\mu_o J \rho^2}{4} + C_1}$ (1)

wb/m

$\frac{\mu_o J \rho}{2} = -\frac{\partial}{\partial z} A_\rho$

$A_\rho = \int -\frac{\mu_o J \rho}{2}$

$\boxed{A_\rho = \frac{-\mu_o J \rho}{2} z + C_2}$

wb/m

$H = -\nabla V_m$

$-\frac{J_o \rho}{2} = \frac{d}{\partial \phi} V_m a_\phi$

$V_m = -\frac{J_o \rho \cancel{2}\pi}{\cancel{2}}$ (1)

Question 3 (9 pts)

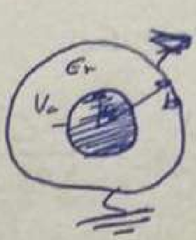
Conducting spherical shells with radii 'a' and 'b' where a < b, are maintained at a potential difference of 'V_0' such that $V(r = b) = 0$ and $V(r = a) = V_0$. If the region between the two conductors is filled with a dielectric material 'ε_r'.

Determine:

a) The V and **E** in the region between the shells.

b) The total charge induced on the shells.

c) The capacitance of the capacitor.



7.5

a)

$\nabla^2 V = 0$

$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dV}{dr}\right) = 0$

$\frac{dV}{dr} = \frac{A}{r^2}$

$\boxed{V = -\frac{A}{r} + B}$

@ $V(r=b) = 0$

$\boxed{B = \frac{A}{b}}$

@ $V(r=a) = V_0$

$V_0 = \frac{-A}{a} + \frac{A}{b}$

$\boxed{A = \frac{V_0}{\left[\frac{1}{b} - \frac{1}{a}\right]}}$

$\cancel{E = -\nabla V}$

$\cancel{E = A \ln r \; \hat{a}_r}$

$\cancel{E = \frac{V_0}{\left[\frac{1}{a} - \frac{1}{b}\right]} \ln r \; \hat{a}_r}$

$\bar{E} = -\nabla V$

$\bar{E} = -A \ln r \; \hat{a}_r$

$\boxed{\bar{E} = \frac{V_0 \ln r}{\left[\frac{1}{a} - \frac{1}{b}\right]} \hat{a}_r}$ Volt/m

0.5

$\boxed{V = \frac{-V_0}{r\left[\frac{1}{b} - \frac{1}{a}\right]} + \frac{A}{b}}$ Volt

1

b) $Q = \oint \bar{D} \cdot \bar{ds} = \int \epsilon \bar{E} \cdot ds$

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$

$Q = \int_0^{2\pi}\int_0^{\pi} \frac{\epsilon V_0 \ln r}{\left[\frac{1}{a} - \frac{1}{b}\right]} r^2 \sin\theta \, d\theta \, d\phi$

$\boxed{Q = \frac{\epsilon V_0 r^2 \ln r \, 4\pi}{\left[\frac{1}{a} - \frac{1}{b}\right]}}$ C

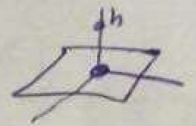
c) $C = \frac{Q}{V_0} = \frac{\epsilon V_0 r^2 \ln r \, 4\pi}{\left[\frac{1}{a} - \frac{1}{b}\right]} \cdot \frac{\left[\frac{1}{b} - \frac{1}{a}\right]}{V_0}$

$\boxed{C = \epsilon r^2 \ln r \, 4\pi \; F}$

#method of images

(4.5)

$Q = \int \epsilon E \cdot ds$



Question 4 (6 pts)

Consider a point charge 'Q' placed at a distance 'h' from a perfect conducting plane of infinite extent. Find the induced charge on the conductor surface.

we find E

$r_1 = (0,0,0) - (0,0,h) = (0,0,-h)$

$|r_1| = h$

$r_2 = (0,0,0) - (0,0,-h) = (0,0,h)$

$|r_2| = h$

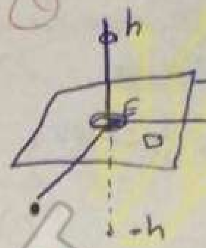
$\rho_s\, ds$

$\int \epsilon E \cdot ds$

$\bar{E} = E_+ + E_-$

$= \dfrac{Q\,\bar{r}_1}{4\pi\epsilon r_1^3} + \dfrac{-Q\,\bar{r}_2}{4\pi\epsilon r_2^3}$

(1)



$= \dfrac{Q\,(0,0,-h)}{4\pi\epsilon h^3} - \dfrac{Q\,(0,0,h)}{4\pi\epsilon h^3}$

(1)

$\boxed{\bar{E} = \dfrac{-2Q}{4\pi\epsilon h^3}\, a_z}$ V/m

(0.5)

#doctor says net Coulombs Law

$Q = \int \bar{D} \cdot \bar{ds} = \int \epsilon \bar{E} \cdot \bar{ds}$

$\boxed{\int ds = S}$ → since it's in cartesian

$Q = \iint_{-\infty}^{\infty} \dfrac{-2Q}{4\pi h^3}\, dx\, dy$

$\boxed{Q = \dfrac{-2QS}{4\pi h^3} = \dfrac{-QS}{2\pi h^3}}$ C

(1)