University of Jordan

School of Engineering

Student Name: Hamza abedrabbo حمزة عبدربه

Student Number: 014075

محمد عبدالعزيز

$$\frac{5.5}{20}$$

Electrical Engineering Department

First Exam, Spring Semester: 2015/2016

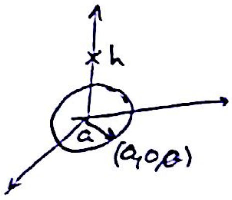
Course Title: Electromagnetics I	Date: 9/3/2016
Course No: (0903251)	Time Allowed: 60 Min.
Instructors: Dr. Mohamed Adbelazzez & Dr. Yanal Faouri	

Note that bold letters are vectors

Problem 1 (8 points)

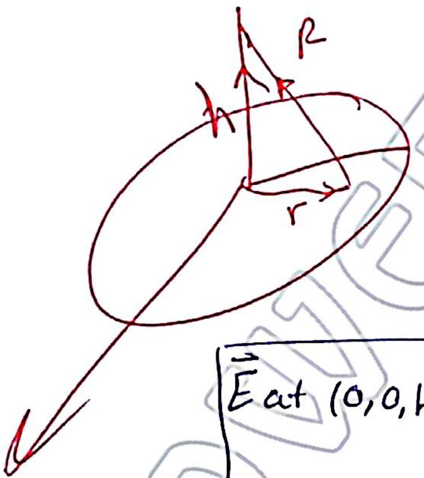
A disk of radius "a" with surface charge density of ρ_s C/m². Calculate:

A. **E** at (0, 0, h)



$$\vec{E} = \int \frac{\rho_s}{4\pi\epsilon_0} \frac{r-r'}{(r-r')^3}\, ds$$

$$= \int_0^{2\pi}\int_0^{a} \frac{\rho_s}{4\pi\epsilon_0} \cdot \frac{(0,0,h)-(a,0,0)}{(\sqrt{a^2+h^2})^3}\, d\rho\, d\phi\, \vec{a}_z$$



$$= \frac{\rho_s}{4\pi\epsilon_0} \int_0^{2\pi}\int_0^{a} \frac{-a\vec{a}_\rho + h\vec{a}_z}{(\sqrt{\rho^2+h^2})^3}\, \rho\, d\rho\, d\phi\, \vec{a}_z$$

$$= \frac{\rho_s}{4\pi\epsilon_0} \int_0^{2\pi}\int_0^{a} h\, d\rho\, d\phi\, \vec{a}_z$$

$$= \frac{\rho_s}{2\,4\pi\epsilon_0} * h a * 2\pi\ \vec{a}_z$$

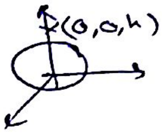
$$\boxed{\vec{E}\text{ at }(0,0,h) = \frac{\rho_s}{2\epsilon_0} * h a\ \vec{a}_z}$$

6

$$\vec{R} = h\vec{a}_j - r\vec{a}_r$$

$$|\vec{R}| = (h^2 + r^2)^{\frac{1}{2}}$$

B. V at (0, 0, h)



$E = -\nabla V$

$V = -\int E \cdot dl$

$= -\int_{\infty}^{h} \frac{\rho_s}{2\epsilon_0} h a \, dz$

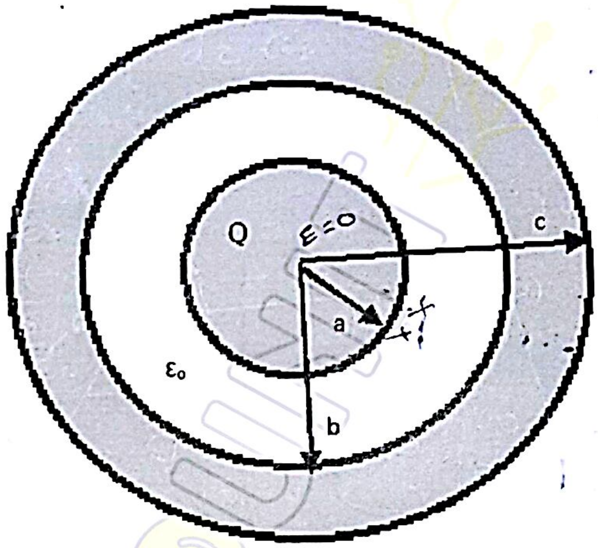
$= -\frac{\rho_s a}{2\epsilon_0} \int_0^h h \, dz$

$$\boxed{V_{\substack{at \\ (0,0,h)}} = -\frac{\rho_s}{4\epsilon_0} a h^2}$$

you must start from a point you know, you don't know its potential

Problem 2: (12 points)

A solid good conductor sphere of radius "a" has a positive net charge Q is enclosed by a conducting spherical shell of inner radius "b" and external radius "c" (c > b) has the same center with the solid sphere.



Determine:

A. **E** and **D** everywhere.

$\boxed{\vec{D} = \varepsilon_o E}$

[1] $r<a$

$E = 0$ [Good conductor]

$\vec{D} = 0$

$\cancel{\varepsilon_o \vec{E} * 4\pi b^2} = \rho_v \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi$

$\cancel{\left[\int_0^b r^2 dr - \int_0^a r^2 dr\right.}$

$4\pi \cancel{\varepsilon_o b^2 \vec{E}} = \rho_v \left[\cancel{-\cos\theta}\right]_0^{\pi} * 2\pi * \left(\frac{1}{3}(b\right.$

[2] $a<r<b$

$\oint_s \varepsilon_o \vec{E} \cdot ds = Q_{enc}$

-3

$\varepsilon_o \int_s \vec{E} \cdot ds = Q$ [illegible]

$\varepsilon_o \vec{E}_r \oint_s \cdot ds = \rho_v \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \int_a^b r^2 dr$

$\cancel{\varepsilon_o E \; 4\pi b^2 = \rho_v \int_0^{\pi}}$

$\varepsilon_o \vec{E} * 4\pi b^2 = Q$

$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_o b^2}\vec{a_r}}$ $\boxed{\vec{D} = \frac{Q}{4\pi b^2}\vec{a_r}}$

at $r = b$.

at any r

$\cancel{\vec{E} = \frac{\frac{4\pi}{3}(b^3-a^3)\rho_v}{4\pi\varepsilon_o b^2}}$

(2) $\cancel{\vec{E} = \frac{Q(b^3-a^3)}{3\varepsilon_o b^2}\vec{a_r}}$

$\cancel{\vec{D} = \frac{Q(b^3-a^3)}{3b^2}\vec{a_r}}$

B. The potential everywhere.

$$V = \int E \cdot dl$$

① $r < a$

$V = 0$

② $a < r < b$

$$V = -\int_0^b \frac{Q(\cancel{b^3 - a^3})}{\cancel{\varepsilon_0 b^2}\, 4\pi\varepsilon_0 b^2}\, dr\, \vec{a_r}$$

$$\cancel{= -\frac{Q}{\varepsilon_0}\int_0^b b - \frac{a^3}{b^2}\, dr\, \vec{a_r}}$$

$$\cancel{\boxed{V = \frac{-Q}{3\varepsilon_0}\left(+\frac{1}{2}b^2 + \frac{a^3}{b}\right)}}$$

$\int b^2$

$$\boxed{V = \frac{-Q}{4\pi\varepsilon_0}\int_0^b \frac{1}{b^2}\, dr\, \vec{a_r} = \frac{Q}{4\pi\varepsilon_0 b}}$$

③ $b < r < c$

$$V = -\int_0^c E \cdot dl$$

$$= -\frac{Q}{4\pi\varepsilon_0}\int_0^c \frac{(c-a)}{|c-a|^3}\, d\vec{a_r}$$

$$\boxed{V = -\frac{Q\left(\frac{1}{2}c^2 - ac\right)}{4\pi\varepsilon_0 |c-a|^3}}$$

C. The surface charge density everywhere.

$\rho_v = \nabla \cdot V$

① $r < a$

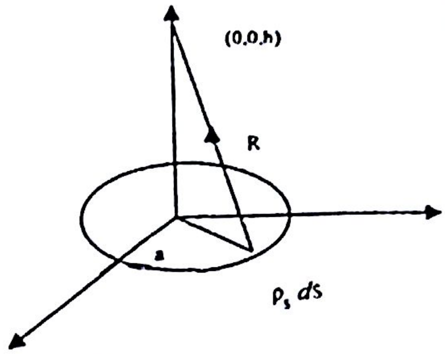
$\rho_v = 0$

② $a < r < b$

$\rho_v = ($

③ $b < r < c$

Electromagnetics (I): First Exam Solution



Problem 1:

(a)

$$\mathbf{E}=\int_S \frac{\rho_S dS}{4\pi\varepsilon_o R^2}\hat{\mathbf{a}}_r=\int_S \frac{\rho_S dS\mathbf{R}}{4\pi\varepsilon_o R^3}$$

$$dS=\rho d\rho d\phi$$

$$\mathbf{R}=(0,0,h)-(\rho,0,0)=-\rho\hat{\mathbf{a}}_\rho+h\hat{\mathbf{a}}_z$$

$$R=\sqrt{\rho^2+h^2}$$

$$\mathbf{E}=\int_0^{2\pi}\int_0^{a}\frac{\rho_S(h\hat{\mathbf{a}}_z)}{4\pi\varepsilon_o(\rho^2+h^2)^{3/2}}\rho d\rho d\phi \Rightarrow \rho-\textit{component}\text{ will add up to zero due to symmetry}$$

$$\mathbf{E}=\frac{\rho_S h}{4\pi\varepsilon_o}2\pi\int_0^a\frac{\rho}{(\rho^2+h^2)^{3/2}}d\rho\hat{\mathbf{a}}_z \Rightarrow \mathbf{E}=\frac{\rho_S h}{2\varepsilon_o}\left(\frac{1}{h}-\frac{1}{\sqrt{a^2+h^2}}\right)\hat{\mathbf{a}}_z \Rightarrow \mathbf{E}=\frac{\rho_S}{2\varepsilon_o}\left(1-\frac{h}{\sqrt{a^2+h^2}}\right)\hat{\mathbf{a}}_z\ \text{V/m}$$

(b)

$$V=\int_S\frac{\rho_S dS}{4\pi\varepsilon_o R}=\int_0^{2\pi}\int_0^{a}\frac{\rho_S}{4\pi\varepsilon_o(\rho^2+h^2)^{1/2}}\rho d\rho d\phi=\frac{\rho_S}{4\pi\varepsilon_o}2\pi\int_0^a\frac{\rho}{(\rho^2+h^2)^{1/2}}d\rho=\frac{\rho_S}{2\varepsilon_o}\left(\sqrt{a^2+h^2}-h\right)\text{V}$$

$$V=-\int_l \mathbf{E}\cdot d\mathbf{l}=-\frac{\rho_S}{2\varepsilon_o}\int_\infty^h\left(1-\frac{z}{\sqrt{a^2+z^2}}\right)dz=-\frac{\rho_S}{2\varepsilon_o}\left(\left(z-\sqrt{a^2+z^2}\right)\Big|_\infty^h\right)=\frac{\rho_S}{2\varepsilon_o}\left(\sqrt{a^2+h^2}-h\right)\text{V}$$

Problem 2:

(a)

$$0<r<a:\oint_S\mathbf{D}\cdot d\mathbf{S}=Q_{enc}=0\Rightarrow\mathbf{E}=\mathbf{D}=0$$

$$a<r<b:\oint_S\mathbf{D}\cdot d\mathbf{S}=Q_{enc}=Q\Rightarrow\int_0^{2\pi}\int_0^{\pi}D_r r^2\sin\theta d\theta d\phi\Rightarrow 4\pi r^2 D_r=Q\Rightarrow\mathbf{D}=\frac{Q}{4\pi r^2}\mathbf{a}_r\ \text{C/m}^2,\ \mathbf{E}=\frac{Q}{4\pi\varepsilon_o r^2}\mathbf{a}_r\ \text{V/m}$$

$$b<r<c:\oint_S\mathbf{D}\cdot d\mathbf{S}=Q_{enc}=0\Rightarrow\mathbf{E}=\mathbf{D}=0$$

$$c<r:\oint_S\mathbf{D}\cdot d\mathbf{S}=Q_{enc}=Q\Rightarrow 4\pi r^2 D_r=Q\Rightarrow\mathbf{D}=\frac{Q}{4\pi r^2}\mathbf{a}_r\ \text{C/m}^2,\ \mathbf{E}=\frac{Q}{4\pi\varepsilon_o r^2}\mathbf{a}_r\ \text{V/m}$$

(b)

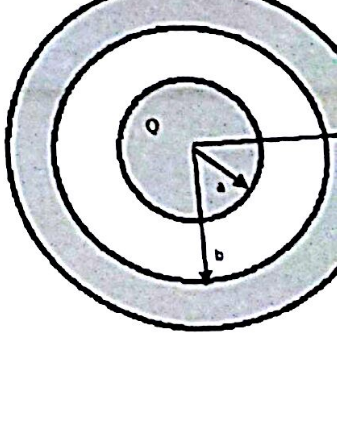
$$0<r<a:V=-\int_l\mathbf{E}\cdot d\mathbf{l}\Rightarrow V=-\int_\infty^c\frac{Q}{4\pi\varepsilon_o r^2}dr+-\int_c^b\frac{Q}{4\pi\varepsilon_o r^2}dr+-\int_b^a\frac{Q}{4\pi\varepsilon_o r^2}dr-\int_a^r\frac{Q}{4\pi\varepsilon_o r^2}dr\Rightarrow V=\frac{Q}{4\pi\varepsilon_o c}+\frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{a}-\frac{1}{b}\right.$$

$$a<r<b:V=-\int_l\mathbf{E}\cdot d\mathbf{l}\Rightarrow V=-\int_\infty^c\frac{Q}{4\pi\varepsilon_o r^2}dr+-\int_c^b\frac{Q}{4\pi\varepsilon_o r^2}dr+-\int_b^r\frac{Q}{4\pi\varepsilon_o r^2}dr\Rightarrow V=\frac{Q}{4\pi\varepsilon_o c}+\frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{r}-\frac{1}{b}\right)\text{V}$$

$$b<r<c:V=-\int_l\mathbf{E}\cdot d\mathbf{l}\Rightarrow V=-\int_\infty^r\frac{Q}{4\pi\varepsilon_o r^2}dr=-\int_\infty^c\frac{Q}{4\pi\varepsilon_o r^2}dr+-\int_c^r\frac{Q}{4\pi\varepsilon_o r^2}dr\Rightarrow V=\frac{Q}{4\pi\varepsilon_o c}\text{V}$$

$$c<r:V=-\int_l\mathbf{E}\cdot d\mathbf{l}\Rightarrow V=-\int_\infty^r\frac{Q}{4\pi\varepsilon_o r^2}\mathbf{a}_r\cdot dr\mathbf{a}_r\Rightarrow V=\frac{Q}{4\pi\varepsilon_o r}\text{V}$$

(c)



$$\text{At r}=\text{a}\Rightarrow\rho_S=D_n=\frac{Q}{4\pi a^2}\text{C/m}^2$$

$$\text{At r}=\text{b}\Rightarrow\rho_S=D_n=\frac{-Q}{4\pi b^2}\text{C/m}^2$$

$$\text{At r}=\text{c}\Rightarrow\rho_S=D_n=\frac{Q}{4\pi c^2}\text{C/m}^2$$