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The University of Jordan
Electrical Eng. Dept.
First Exam

Electromagnetism EE251 & EE253
Student Name in Arabic
Answers should be written in ink.

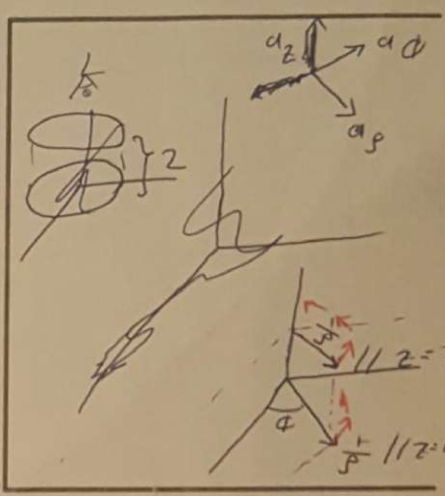
Exam Duration:50 min

Sun. 2/11/2014
Registration #:
Mark out of 20:

Question 1 a) A vector field is given in **cylindrical** coordinates by $A = \frac{1}{\rho} a_\phi$.

i)Sketch the field in the plane **z=0** and in the plane **z=2**.

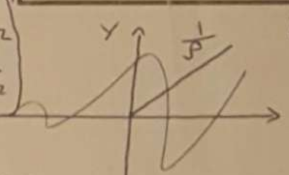
ii)Represent **A** in **Cartesian** coordinates.



$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ +\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{\sqrt{x^2+y^2}} \\ 0 \end{bmatrix}$$

$A_x = \frac{-\sin\phi}{\sqrt{x^2+y^2}}$ $\quad A_y = \frac{\cos\phi}{\sqrt{x^2+y^2}}$ $\quad A_z = 0$

Note that: $\sin\phi = \frac{y}{\sqrt{x^2+y^2}}$, $\cos\phi = \frac{x}{\sqrt{x^2+y^2}}$



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Down

$$\vec{A} = \frac{-\sin\phi}{\sqrt{x^2+y^2}} \hat{a_x} + \frac{\cos\phi}{\sqrt{x^2+y^2}} a_y$$

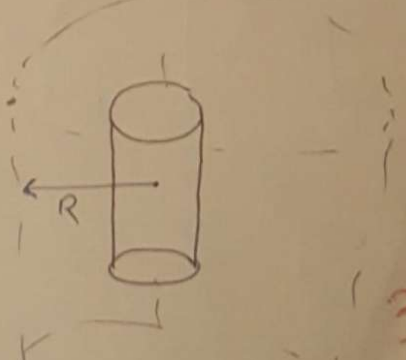
Ans.

$$\vec{A} = \frac{-\left(\frac{y}{\sqrt{x^2+y^2}}\right)}{\sqrt{x^2+y^2}} a_x + \frac{\left(\frac{x}{\sqrt{x^2+y^2}}\right)}{\sqrt{x^2+y^2}} a_y \Rightarrow \boxed{\frac{-y}{x^2+y^2} \hat{a_x} + \frac{x}{x^2+y^2} \hat{a_y}}$$

b) An open-ended infinite hollow cylinder carries charge of density ρ_s Cm^{-2}. Determine the electric field intensity by any method you prefer.

ρ_s

Power Unit



$Q = \oint \vec{D} \cdot \vec{ds}$

S was charges

radius

$\rho_s \cdot (2\pi / L) \cancel{2\pi} = D_\rho \int_0^{2\pi} \int_0^{L} \rho \, d\phi \, dz \; a_\rho$

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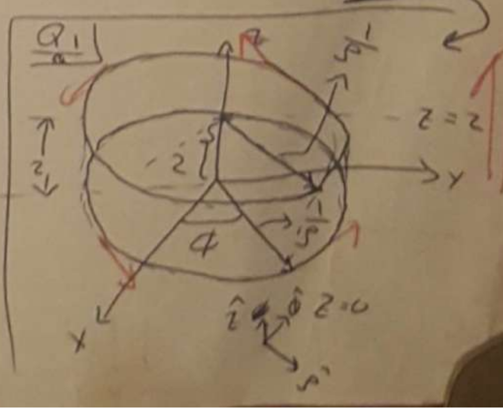
$\rho_s \cdot 2\pi L = D_\rho \cdot R \cdot (L-0) \int_0^{2\pi} d\phi$

ρ_s radius $2\pi L = D_\rho \cdot R \, L \cdot 2\pi$

$\vec{D}_\rho = \frac{\text{radius} \, \rho_s}{R} \hat{a_\rho} \Rightarrow \boxed{\vec{E} = \frac{\rho_s}{\epsilon_0 R} \hat{a_\rho}}$

$D_\rho \quad D = \epsilon \cdot E$

Q1 / a(i)



Q_2 / b

$$V = \frac{\rho_s}{2\epsilon_0}\left(\sqrt{\rho^2+h^2} - h\right)$$

Ans

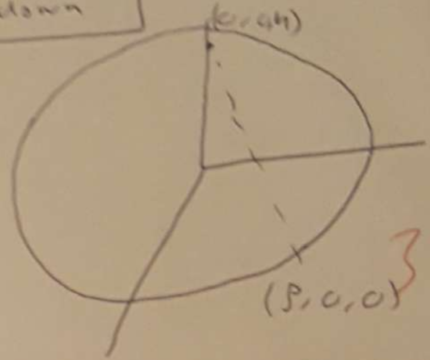
Question 2 $\vec{E} = \frac{\rho_s}{2\epsilon}\left[\frac{1}{2}(\rho^2+h^2)^{-\frac{1}{2}} \cdot 2\rho\right] a_\rho + 0_\phi + 0_z = \frac{\rho_s}{2\epsilon} \cdot \rho \cdot \frac{1}{\sqrt{\rho^2+h^2}}$

a) A circular disk of radius **a** situated in the z=0 plane with its axis along the z-axis, is charged with $\rho_s = \frac{1}{\rho}$ C/m². **Formulate the solution procedure** to determine the **electric field intensity** at (0, 0, h). Q_2 / b

it's answer down

$$DQ = \rho_s\, ds \Rightarrow \boxed{\rho_s\, \rho\, d\phi\, d\rho\, a_z}$$

$$a_R = \frac{\langle -\rho, 0, h\rangle}{\sqrt{\rho^2+h^2}} \Rightarrow \frac{-\rho\, a_\rho + h\, a_z}{\sqrt{\rho^2+h^2}}$$



3.5

$$\vec{E} = \int_S \frac{\rho_s\, \rho\, d\phi\, d\rho \cdot (-\rho\, a_\rho + h\, a_z)}{4\pi\epsilon\, (\rho^2+h^2)^{\frac{3}{2}}}$$

$E_\rho = 0$ due to symmetry $\Rightarrow$ $\vec{E} = \iint$ $\boxed{\vec{E} = \frac{\frac{1}{\rho}\rho_s\, h}{4\pi\epsilon}\int_0^{2\pi}\int_{\rho=0}^{a}\frac{\rho\, d\rho\, d\phi}{(\rho^2+h^2)^{\frac{3}{2}}}\, a_z}$

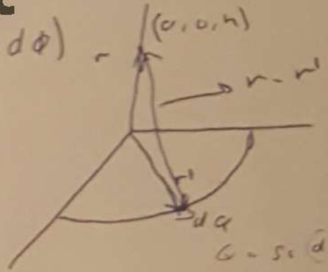
look down

b) **A circular disk of radius R** situated in the z=0 plane with its axis along the z-axis and carries a charge ρ_s Cm^{-2}. Calculate the **potential** at (0, 0, h). **Use the potential to determine the electric field intensity** at (0, 0, h).

it's answer up

$dQ = \rho_s\, ds$

$$Q(r') = \rho_s\,(d\rho\, \rho\, d\phi)$$



$$\boxed{V = \int \frac{Q(r')}{4\pi\epsilon\, |r - r'|}}$$

$$\int_S \frac{\rho_s \cdot \rho\, d\rho\, d\phi}{4\pi\epsilon\, |-\rho\, a_\rho + h\, a_z|} \Rightarrow \int_S \frac{\rho_s}{4\pi\epsilon}\, \frac{\rho'\, d\rho'}{(\rho^2+h^2)^{\frac{1}{2}}}$$

$$\frac{\rho_s}{4\pi\epsilon}\int_S \frac{\rho\, d\rho\, d\phi}{\sqrt{\rho^2+h^2}} \Rightarrow \boxed{\frac{\rho_s}{4\pi\epsilon}\int_0^{R}\int_0^{2\pi}\frac{\rho\, d\rho\, d\phi}{\sqrt{\rho^2+h^2}} = V}$$

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$$\vec{E} = -\nabla V \Rightarrow \boxed{\vec{E} = \frac{\partial V}{\partial \rho}\, a_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\, a_\phi + \frac{\partial V}{\partial z}\, a_z}$$

$$\frac{dV}{d\rho}\, a_\rho$$

Now $\Rightarrow V = \frac{\rho_s}{4\pi\epsilon} \cdot 2\pi \int_0^{R} \rho\, (\rho^2+h^2)^{-\frac{1}{2}}\, d\rho \Rightarrow \frac{\rho_s}{2\epsilon} \cdot \frac{1}{2}\,(\rho^2+h^2)^{\frac{1}{2}}\Big|_0^{R}$

Q_2 / a Ans

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

Ans

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$

$$\boxed{\frac{\rho_s}{2\epsilon}\left(\sqrt{R^2+h^2} - h\right)}$$

Q_2 / a

$$\vec{E} = \frac{1\, h}{\rho\, 4\pi\epsilon}\int_0^{2\pi}\int_0^{a}\frac{\rho\, d\rho\, d\phi}{(\rho^2+h^2)^{\frac{3}{2}}}\,\hat{z}$$

$$\vec{E} = \frac{\rho_s\, h}{4\pi\epsilon}\int_0^{2\pi}\int_0^{a}\frac{\rho\, d\rho\, d\phi}{(\rho^2+h^2)^{\frac{3}{2}}}\cdot \hat{a}_z$$