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The University of Jordan Summer 2013 **Electromagnetics I EE251**

Student name [illegible] **Student No.** 0127430

Question 1

A parallel plate capacitor device contains a dielectric slab as shown below, with plate spacing d, and area of dielectric A_d, and free space A_f. Using field theory (no circuit methods, assume electric field is uniform):

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Show that the electric field in free space is the same as that in dielectric

$$E = \frac{P}{\epsilon_0} =$$

Find the capacitance using the energy definition for capacitance

$$w = \frac{1}{2}CV^2 = \frac{1}{2}|E|^2 \epsilon V = \frac{1}{2}\frac{V^2}{d^2}\times A\,\epsilon$$

$$\frac{1}{2}CV^2 = \frac{1}{2}\frac{V^2 A\epsilon}{d} \rightarrow C = \frac{A\epsilon}{d}$$

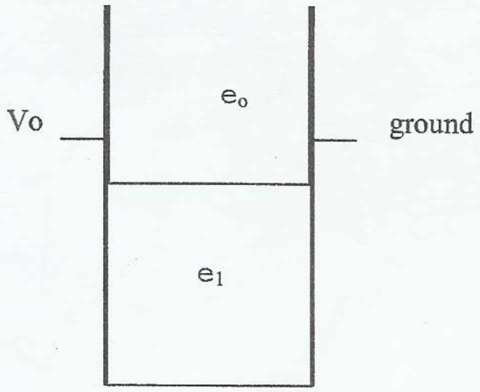
Find the capacitance using the charge definition for capacitance

$$C = \frac{Q}{V} = \frac{\oint \epsilon E \cdot ds}{\int E \cdot dl} = \frac{E\,\epsilon A}{E \times d} = \frac{\epsilon A}{d}$$

Find the energy stored in the capacitor, then find the energy if the dielectric is removed (the work done in removing the dielectric)

$$w = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\frac{\epsilon A}{d}\times V_0^2$$



Question 2

A cylindrical conductor of radius a, and length l, with conductivity s. If a voltage V_o is applied across the ends, find (in terms of the given variables):

The resistance of the rod

$$R = \frac{V}{I} = \frac{\int E \cdot dl}{\int \sigma E \cdot ds} = \frac{\cancel{E} L}{\sigma \cancel{E} A} = \frac{L}{\sigma A} = \frac{L}{s \pi a^2} \boxed{= \frac{L}{s \pi a^2}}$$

$J = sE$

The current

$J = sE = \frac{sV}{L}$

$$I = \int_s J \cdot ds = \int \frac{sV}{L} \cdot ds = \frac{sV \times \pi a^2}{L} = \boxed{\frac{\pi a^2 s V}{L}}$$

The current density

$$I = \frac{J}{A} = \frac{sV}{L}$$

The current density as a function of charge density and charge velocity (vector form)

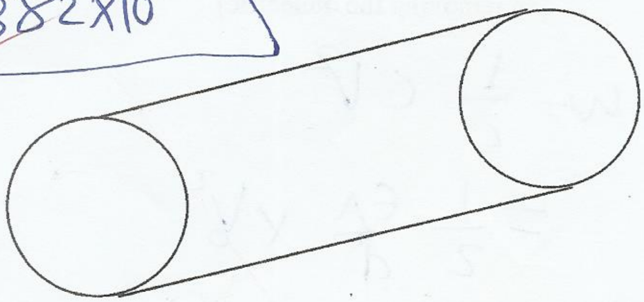
$J = n \mu_e$ =

Consider copper, with resistivity of 1.7 10^{-8} Ohm-meter. Find the resistance if the length is 1 m, and radius 100 squared micrometer. Find R

$$\frac{1.7 \times 10^{-8} \times 1}{100 \times 10^{-6}} = \boxed{1.7 \times 10^{-4} \text{ ohm}}$$

The quality factor is defined as $Q = \omega L / R$. Find Q for an inductor if inductance is 100 mH, frequency is 10 MHz and resistance as above

$$Q = \frac{100 \times 10^{-3} \times 10 \times 10^{6}}{1.7 \times 10^{-4}} = \boxed{0.5882 \times 10^{2}}$$



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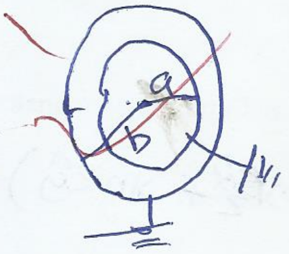
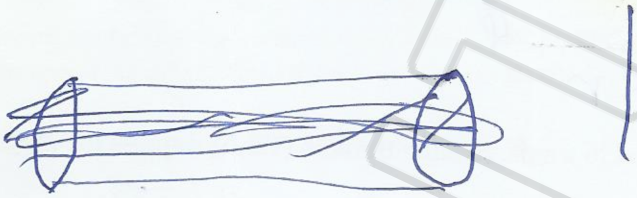
Question 3

A spherical capacitor is made from two conducting shells, of inner one with radius a, and outer of radius b. A voltage is applied to the outer shell, and inner is grounded.

Sketch the device





Assuming charge Q is on the outer shell, find surface charge density on outer and an inner shells

outer $\rho_s = \dfrac{Q}{A} = \dfrac{Q}{\frac{4}{3}\pi b^3}$ inner $\rightarrow \rho_s = \dfrac{Q}{\frac{4}{3}\pi a^3}$ volume

Find the Electric field using Gauss law

$\oint D \cdot ds = Q$

$\int_0^{b}\int_0^{2\pi}\int_0^{\pi} D\hat{R}\cdot R^2 \sin\theta \, d\theta \, d\phi \, dR = D \times \dfrac{R^3}{3} \times 2\pi \times -\cos\theta \Big|_0^{\pi}$

$= \dfrac{4\pi R^3 E\epsilon}{3} = Q \rightarrow E = \dfrac{3Q}{4\pi R^3 \epsilon}\hat{R}$

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Find the electric field if relative permittivity is 3

$E = \dfrac{Q}{4\pi R^2}$ R

Find the capacitance of the device from flux relation

$C = \dfrac{Q}{V} = \dfrac{Q}{3 \cdot \int E \cdot dl} = \boxed{\dfrac{2R^2 \times 4\pi\epsilon}{9}}$

$\boxed{\int E \cdot dl = -\int \dfrac{3Q}{4\pi R^3 \epsilon}\hat{R}\cdot\hat{R} = \dfrac{3Q}{2R^2 \times 4\pi\epsilon} =}$

$= \dfrac{2(b^2 - a^2) \times 4\pi\epsilon}{9}$

Find capacitance as the radius of outer shell goes to infinity

$C = \infty$

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Question 4

The magnetic field of an infinite length straight line of current I on z axis is

$$H = \frac{I}{2\pi r}\hat{\phi}$$

If line is parallel to z axis, passing through (1,2,0), express the magnetic field in Cartesian coordinates

$$H = \frac{I}{2\pi r}\left(-\cos\phi\,\hat{x} + \sin\phi\,\hat{y}\right) = \frac{I}{2\pi\sqrt{x^2+y^2}}\left(-\cos\phi\,\hat{x} + \sin\phi\,\hat{y}\right)$$

If line is parallel to x axis, passing through origin, express the magnetic field in Cartesian coordinates

$H =$

If line is parallel to x axis, passing through (1,2,0), express the magnetic field in Cartesian coordinates

If line equation is **r** = s (1 **x** + 1 **y**), where s is a real number. Express the field in Cartesian coordinates