The University of Jordan
Electrical Eng. Dept.
First Exam

Electromagnetism I EE251
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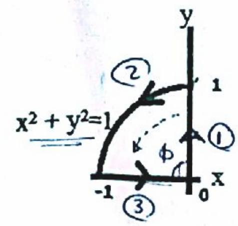
Answers should be written in ink.
Exam Duration: 50 min

Sun. 20/7/2014
Registration #: 0110494
Mark out of 20: 11

Question 1 a) An electric field is given by $A = x\, a_x + y\, a_y$. **Evaluate** $\int_L A.dl$ where L is as shown



line ①: $x = 0$, ~~$dx = 0$~~,

$dl_1 = dy \rightarrow \int_0 \cancel{(x a_x)}(y a_y)\, dy = \frac{1}{2} a_y$

line ②: By cylindrical $(x = 1, y = 1) \Rightarrow \rho = 1$,

$dl_2 = -d\phi$

$\rightarrow -\int_0^{\frac{\pi}{2}} (\rho\, a_\rho)\, d\phi = -(\rho \times \frac{\pi}{2} - 0) = \frac{-\pi}{2} \rho\, a_\rho$

$x^2 + y^2 = 1$

we transform to cylindrical

$A = x a_x + y a_y$

$A = \rho a_\rho$

line ③: $\phi = 0$ $\quad dl_3 = -d\rho$

$-\int_{-1}^{0} \rho\, d\rho = -\frac{\rho^2}{2}\Big|_{-1}^{0} = -\frac{1}{2} a_\rho$

b) **Check** your answer by **resolving** the problem through representing **A** in **cylindrical** coordinates. You may use

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} a_\rho \\ a_\phi \end{bmatrix}$$

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} a_\rho \\ a_\phi \end{bmatrix}$$

$x^2 + y^2 = 1$

$\therefore |x^2 + y^2| = |\rho|^2$

& $x a_x + y a_y = \rho a_\rho$

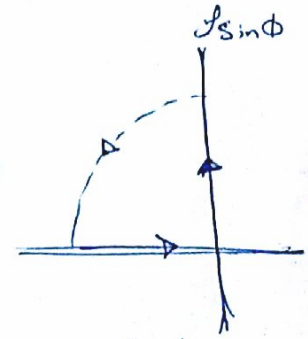
$\therefore A = \rho a_\rho$

$\rho = \sqrt{x^2 + y^2}$

how

line ①: $\phi = 0$

$dl_1 = d\rho \quad \Rightarrow \int_1^0 \rho\, d\rho = \frac{1}{2} \rho^2 \Big|_1^0$



(ρ, ϕ)

باقي الأفرع
نفس حل
فرع (a)

Question 2 : a) Given a uniform distribution of charges $\rho_S\ Cm^{-2}$ over the surface of a sphere of radius **R**. Use **Gaussian surfaces** to determine the **electric field intensity** for $0 \le r < \infty$.

we have ~~2~~ 3 Cases

first one $r > R$

$\oint \vec{D} \cdot \vec{ds} = \iiint \cancel{\rho_v} \cdot dv$

$\vec{D} \iint r \sin\theta \, d\theta \, d\phi \quad = \rho_v \iiint R^2 \sin\theta \, d\theta \, d\phi \, d\rho$

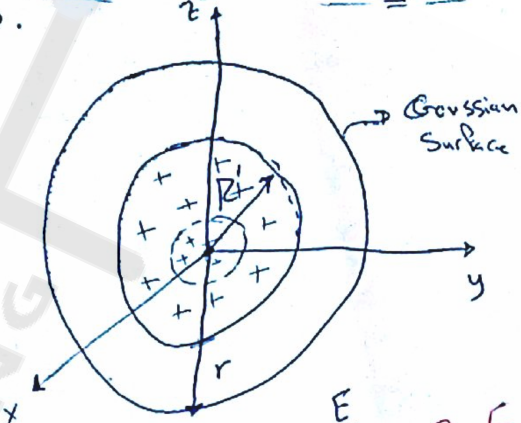
$D\,(4\pi r^2) = \rho_v \dfrac{4\pi R^3}{3} \hat{a}_R$

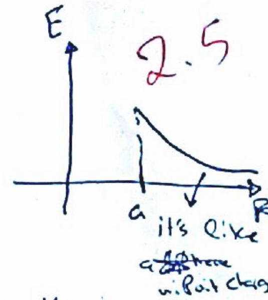
$\therefore D = \rho_v \dfrac{R^3}{3r^2} \hat{a}_R$

~~when r > R it's like a point~~

when $r < R$

then $E = 0 \Rightarrow D = 0$ because the charge distribution over the surface then there's No electric field inside it.





2.5

b) Calculate the **overall force** on a charge **q** situated at point (8,0,0) due to the **combined** electric field resulting from an infinite surface charge ρ_S situated in the **xy plane**, an infinite line charge $-\rho_l$ situated in the **xy plane** and passing through the point (-2,0,0) , and a point charge **Q** situated at point (0,-8,0).

(*) $F_{Tot} = F_1 + F_2 + F_3$

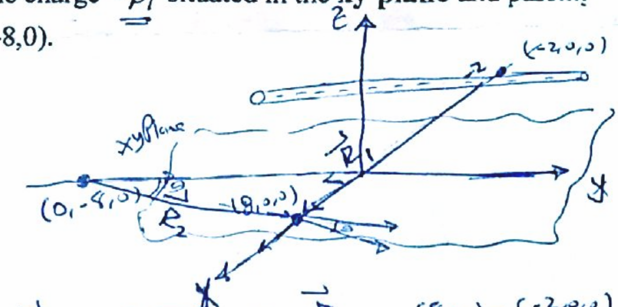
(1) F_1 @ Surface $=$ Zero because E at any Point on the Surface $=$ Zero the $F = EQ \Rightarrow F = 0$

(2) $F_2 = E\,q = \dfrac{q \int -\rho_L \, dl}{4\pi\epsilon_0 |R_1|^2} \hat{a}_{R_1} = \dfrac{-q\,\rho_L \int dy\, \hat{a}_x}{4\pi\epsilon_0 \times 8 \times 10^{?}}$

$F_2 = -9 \times 10^{?} \times q \times \rho_L \, \hat{a}_x$

(3) $F_3 = \dfrac{Q\,q}{4\pi\epsilon_0 |\vec{a}_{R_2}|^2} \vec{a}_{R_2} = \dfrac{9 \times 10^{9}\, Q\, q}{128} (0.7\hat{a}_y + 0.7\hat{a}_x)$

$= 4.92 \times 10^{7}\, Q\, q\, \hat{a}_x + 4.92 \times 10^{7}\, Q\, q\, \hat{a}_y$



$|\vec{R}_2| = 8\sqrt{2}$

$\hat{a}_{R_2} = \dfrac{\vec{R}_2}{|\vec{a}_{R_2}|}$

$\hat{a}_{R_2} = \vec{R}_2 \sin\theta\, \hat{a}_y + \vec{R}_2 \cos\theta\, \hat{a}_x$

$\vec{R}_1 = (8,0,0) - (-2,0,0)$

$\vec{R}_1 = 8 - (-2)\, \hat{a}_x$

$\therefore \vec{R}_1 = 10\, \hat{a}_x$

$|\vec{R}_1| = 10$

$\hat{a}_{R_1} = \dfrac{10\, \hat{a}_x}{10} = \hat{a}_x$

2.5

c) For any vectors A and B, what is $A \bullet (A \times B)$ as a number ? prove.

$\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\hat{a}_x - (A_x B_z - A_z B_x)\hat{a}_y + (A_x B_y - A_y B_x)\hat{a}_z$

$A \cdot (\vec{A} \times \vec{B}) = (A_x, A_y, A_z) \cdot (\vec{A} \times \vec{B})$

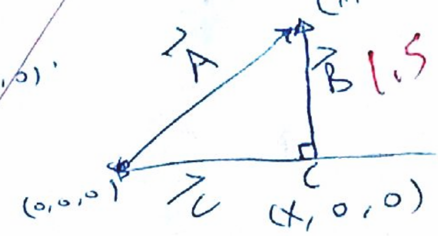
$= A_x A_y B_z - A_x A_z B_y - A_y A_x B_z - A_y A_z B_x + A_x B_y A_z - A_z A_y B_x$

$=$ Zero ✓

d) Use operations defined on vectors to prove Pythagoras theorem

$\vec{A}, \vec{B}, \vec{C}$

$\vec{B} = (x, y, z) - (0,0,0)$



1.5

By distance (short distance) theorem