Spring017

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Control 1/2/2017

Dr. Omar Ghzawi Spring 017

* Review of the laplace transform

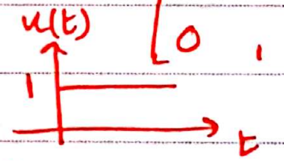
Def: given $f(t)$

$$\mathcal{L}f(t) = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

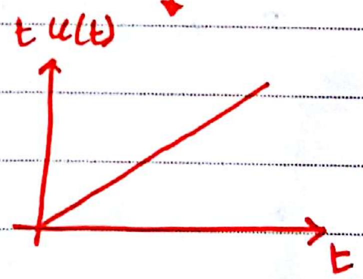
Based on the definition; it can be Shown that :-

$$\mathcal{L}\, u(t) = \frac{1}{s}$$

$$u(t) = \begin{cases} 1 , & t>0 \\ 0 , & t<0 \end{cases}$$



$$\mathcal{L}\, t\, u(t) = \frac{1}{s^2}$$ called ~~called~~ ramp



$\cancel{* \mathcal{L}\, t^n u(t) = \frac{n!}{s^{n+1}}}$

$$* \mathcal{L}\, t^n u(t) = \frac{n!}{s^{n+1}}$$

No.

$$\mathcal{L}\, e^{at} = \frac{1}{S-a} \;;\; \mathcal{L}\, e^{j\omega t} = \frac{1}{S-j\omega}$$

* Note: $S = \sigma + j\omega$

$$\mathcal{L} \cos\omega t = \mathcal{L}\, \tfrac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) = \frac{S}{S^2+\omega^2}$$

$$\mathcal{L} \sin\omega t = \mathcal{L}\, \tfrac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right) = \frac{\omega}{S^2+\omega^2}$$

*Properties:

* $\mathcal{L}\, e^{at} f(t) = F(s-a)$

* $\mathcal{L}\, \frac{d}{dt} f(t) = S F(S) - f(0)$

* $\mathcal{L}\, \frac{d^n}{dt^n} f(t) = S^n F(S) - S^{n-1} f(0) - S^{n-2} f'(0) - \dots$

* $\mathcal{L}\, f(t) * g(t) = F(s) \cdot G(s)$

Convolution

* $\mathcal{L} \int_0^\infty f(t)\, dt = \frac{1}{S} F(s)$

No.

$* \cancel{\mathcal{L}\, f(t)}$

$* \mathcal{L}\, t\, f(t) = -\frac{d}{ds} F(s)$

Ex : Determine

1) $\mathcal{L}\, e^{-2t} \cos(5t) = F(S+2)$

$\mathcal{L} \cos 5t = \frac{S}{S+\omega^2} = \frac{S}{S+25}$

$F(S+2) = \frac{S+2}{S+27}$

2) Solve $\frac{dy}{dt} + 4y = t\, u(t)$; $y(0) = 0$

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 29y = 58\, u(t)$

$y(0) = 0$

$y'(0) = 0$

FIVE APPLE

No.

3) $\mathcal{L} \int_0^\infty t e^{-4t} \sin 5t \, dt$

4) $\mathcal{L}^{-1} \dfrac{10s}{s^2+2s+5}$

No.

* Laplace transform Inversion (LT)

Sometimes the Inverse can be obtained by invoking the LT properties.

e.g: Obtain $\mathcal{L}^{-1}\frac{1}{(s+2)^2}$ using at least 3 properties

Complicated inverses requires the use of partial fractions.

Case One: no poles are distinct (different, not the same)

Use the cover up rule

No.

* Given $G(s) = \dfrac{S^m + \cdots}{S^n + \cdots} = \dfrac{S^m + \cdots}{(S-P_1)(S-P_2)\cdots(S-P_n)}$

$$= \frac{A_1}{S-P_1} + \frac{A_2}{S-P_2} \cdots + \frac{A_n}{S-P_n}$$

$$A_1 = \lim_{S \to P_1} (S-P_1)\, G(s)$$

$$\boxed{A_n = \lim_{S \to P_n} (S-P_n)\, G(S)}$$

Example: $\mathcal{L}^{-1} \dfrac{S^2+2S+5}{S^3+6S^2+11S+6}$

$$G(S) = \frac{S^2+2S+5}{(S+1)(S+2)(S+3)}$$

$P_1 = \cancel{4} -1$, $P_2 = -2$, $P_3 = -3$

$$\frac{A_1}{S+1} + \frac{A_2}{S+2} + \frac{A_3}{S+3}$$

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$$A_1 = \lim_{S \to 1} (S+1)\, G(S)$$

$$A_1 = \frac{4}{2} = 2 \;,\quad A_2 = \frac{5}{-1} = -5$$

$$A_3 = \frac{8}{2} = 4$$

~~If & G(s) distinct & the coefficient of the highest orders are 1 and the difference between the o~~

N.B: If the difference between the denominator and numerator ~~ex~~ in order is 1, then the Sum ~~between~~ of poles is 1.

Ex $\mathcal{L}^{-1} \dfrac{S^2+S+2}{(S-2)(S+3)(S-4)(S+5)}$

Prove that $A_1 + A_2 + A_3 + A_4 = \text{zero}$.

* The Poles must be distinct for these 2 rules.

No.

Case two: ~~the poles are identical (repeated), so~~

when Some of the poles are Identical.

~~Given~~

Given $G(s) = \dfrac{S^m + \cdots}{(S-P_1)^3 (S-P_2) \cdots (S-P_n)}$

$$= \frac{A_1}{(S-P_1)^3} + \frac{A_2}{(S-P_1)^2} + \frac{A_3}{(S-P_1)}$$

$$+ \frac{A_4}{(S-P_2)} + \cdots \frac{A_n}{(S-P_n)}$$

use the cover up-rule to determine A_1, A_4, ~~A_n~~ .. A_n

~~$A_2 = \lim (S-P_1)$~~

$$A_2 = \lim_{S \to P_1} \left(\frac{d}{ds} (S-P_1)^3 \, G(S) \right)$$

No.

$$\cancel{A_3 = \lim_{S \to P_1} \left[\frac{d^2}{ds^2} (S-P_1)^3 G(s) \right.}$$

$$A_3 = \lim_{S \to P_1} \left[\frac{1}{2!} \frac{d^2}{ds^2} (S-P_1)^3 G(s) \right]$$

Example: $\dfrac{S}{(S-2)^2 (S+4)}$

$$= \frac{A_1}{(S-2)^2} + \frac{A_2}{(S-2)} + \frac{A_3}{(S+4)}$$

$$A_1 = \frac{2}{6} = \frac{1}{3} \quad , \quad A_3 = \frac{-4}{36} = \frac{-1}{9}$$

$$A_2 = \lim_{S \to 2} \left[\frac{d}{ds} \cancel{(S-2)^2} \frac{S}{S+4} \right]$$

$$A_2 = \left. \frac{S+4-S}{(S+4)^2} \right|_{S=2} = \left. \frac{4}{(S+4)^2} \right|_{S=2}$$

$$A_2 = \frac{4}{36} = \frac{1}{9}$$

No.

~~Getting a real Solution when the poles are complex~~

* Getting a real Solution when the poles are complex

Given $G(s) = \dfrac{\omega_n^2}{(s^2+2\zeta\omega_n s+\omega_n^2)*s}$

zeta

$\zeta < 1 \rightarrow$ Gives complex Poles.

$$= \frac{\omega_n^2}{s*(s+\zeta\omega_n - j\omega_d)(s+\zeta\omega_n + j\omega_d)}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$= \frac{A}{(s-P_1)} + \frac{A^*}{(s-P_1^*)} + \frac{A_3}{s}$$

$$\mathcal{L}^{-1} G(s) = \underbrace{Ae^{P_1 t} + A^* e^{P_1^* t}}_{\text{Involve complex quantities}} + A_3\, u(t)$$

can be reduced to a real quantity after manipulation

No.

To get $\mathcal{L}^{-1} G(s)$ as real

$$G(s) = \frac{A}{s} + \frac{Bs+C}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$A(s^2+2\zeta\omega_n s+\omega_n^2) + Bs^2 + Cs = \omega_n^2$$

$$(A+B)s^2 + (C+2\zeta\omega_n)s + A\omega_n^2 = \omega_n^2$$

$A=1$, $B=-1$, $C=-2\zeta\omega_n$

$$G(s) = \frac{1}{s} + \frac{-s-2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

where $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\cancel{G(s) = \left(\frac{1}{s} - \frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}\right)}$$

$$G(s) = \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n \cdot \omega_d/\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}$$

No.

$$g(t) = u(t) - e^{-3\omega_n t} \cos \omega_d t$$

$$- \frac{3\omega_n}{\omega_d} e^{-3\omega_n t} \sin \omega_d t$$

$$= u(t) - e^{-3\omega_n t} \cos \omega_d t \; \frac{3}{\sqrt{1-3}} \; e^{-3\omega_n t}$$

$\sin \omega_d t$

No.

Ex: Determine 1) $\mathcal{L}^{-1} \dfrac{s+1}{(s+2)(s^2+4s+29)}$

by different methods

11) $\mathcal{L}^{-1} \dfrac{s^2+10}{(s-3)^3(s^2+3s+2)}$

No.

$$F(s) = \frac{S+1}{(S+2)(S^2+4S+29)}$$

find ~~f(s)~~ f(t) at t = 0

~~*Initial Value problem~~

* Initial Value theorem :-

$$\boxed{\lim_{t \to 0} f(t) = \lim_{S \to \infty} sF(s)}$$

$$\lim_{S \to \infty} S * \frac{1}{S+2} = \lim_{S \to \infty} \frac{1}{1 + \frac{2}{S}} = 1$$

* The final theorem

Gives the Steady State Value

~~$f_{ss} = \lim_{t \to \infty} f(t)$~~

$$\boxed{f_{ss} = \lim_{t \to \infty} f(t) = \lim_{S \to 0} S\,F(s)}$$

No.

* Provided the poles of $s\,F(s)$ are in the L.H.S of S-plane (-ve poles).
(الشرط)

Control

Book: Modern Control Engineering
5th ~~3rd~~ Edition

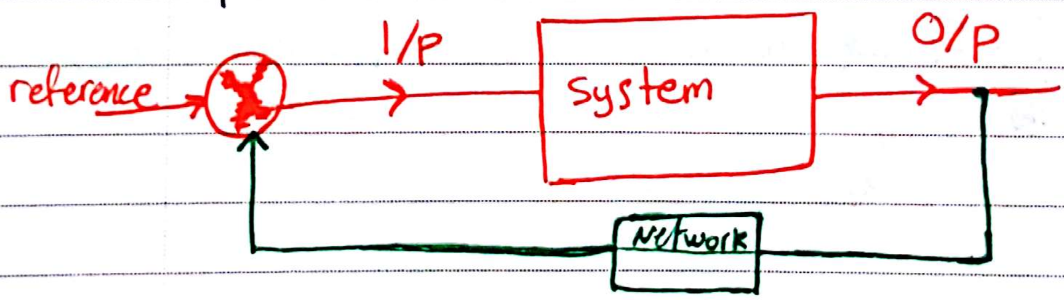
No.

Control Systems

Def: A System is a collection of objects (components, devices, subsystems) Put together to perform a certain objective.

The control meant in this Course is that using the concept of feedback.

With feedback the output or part of it is fedback (returned) to become part of the input.



$$O/P = f(ref, O/P)$$

* This is called feedback System or closed loop System.

Open loop Systems : are Systems where the Output does not affect the input.

(non feedback or Open loop Systems)

* Classification of systems as open or closed loop

i) a blind person walking : Open loop

ii) Filling a glass of water : closed loop

Input : water, rate of flow

Output : Filled glass

No.

~~iii) Microwave ovens~~

iii) Present-day design of Microwave ovens:

Open loop

IV) Washing Machines (~~used~~ Laundrette): Open loop

V) Heat Seeking Missiles : closed loop

VI) A rifle bullet : Open loop

VII) Electric Iron : Closed loop

No.

* Classifications as Positive or Negative feedback

With positive feedback the O/P Signal is returned to the input without Sign Inversion.

With negative feedback the O/P Signal is inverted prior to feeding it.

- Examples

1. filling a glass ~~of water~~ by a blind person (First time)

No feedback.

2. filling a glass by a Normal person

Negative feedback (Increase in level results in decrease in flow)

No.

3. Heat seeking missile

Negative feedback

4. Most read articles or Most viewed photo

Positive feedback

5. Two angry ~~person~~ men arguing

Positive feedback

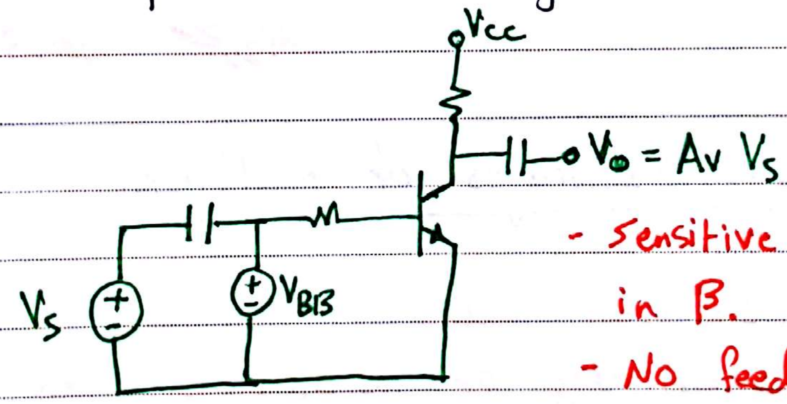
6. ~~Eo~~ Two people each is better than the other

Positive feedback

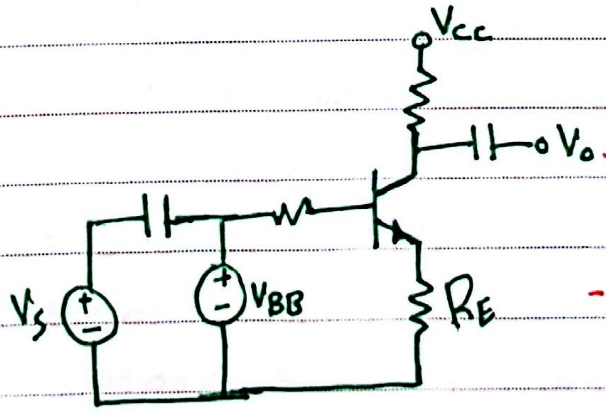
* Advantages of feedback :-

1. Increased Accuracy.
2. Increased Speed of response.
3. " Bandwidth.
4. *Insensitivity to parameter Variations.
5. Insensitivity to external disturbances.

Example on advantages 4:

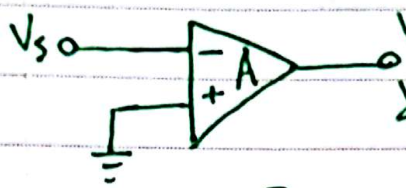


- Sensitive to variations in β.
- No feedback.



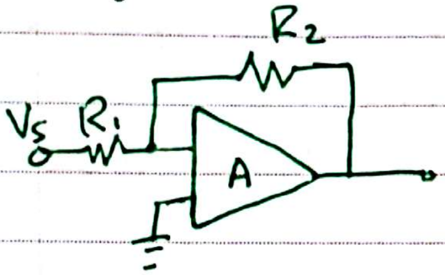
- Insensitive to variations in β.
- feedback is through RE.

$A(V^+ - V^-)$



$V_o = -AV_s$

- directly dependent on the variable A.
- No feedback



$V_o = -\frac{R_2}{R_1} V_s$

- Not dependent on the parameter A.
- feedback through R_2.

* Disadvantages of feedback:

1. reduced gain
2. Possibility of instability with careless design.
3. Increased complexity of the System, this entails: larger size: Extra weight Extra cost: added noise due to employement of ~~sensors~~ Sensors, All of these result in more envolved trouble shooting.

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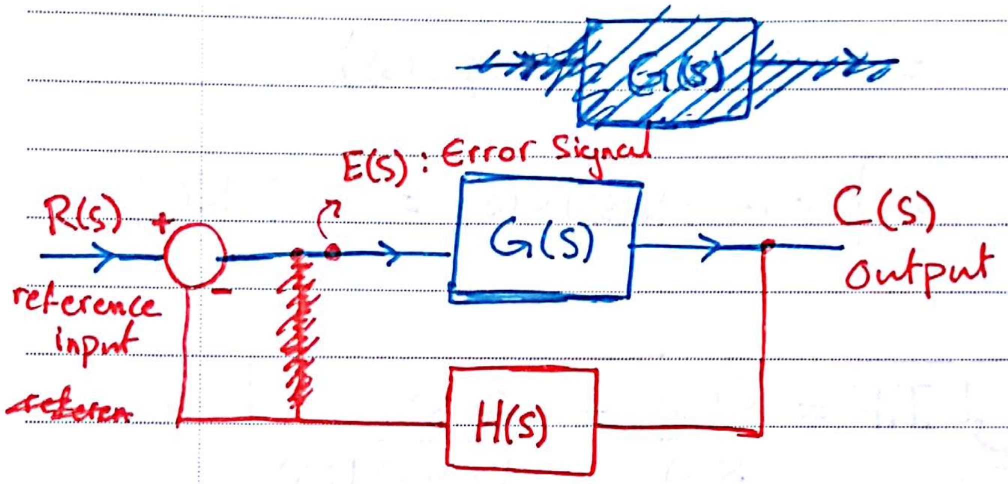
* Conclusion: feed back has to be justified.

Basic feedback loop



* reference input ≡ reference value ≡ Set value ≡ Set point ≡ desired value

$\boxed{G(s)}$: represents our System (Process). (Plant)

$\boxed{H(S)}$: represents feedback modification of the o/p.

↳ Does Energy conversion.

G(S): Forward Transfer function

G(s) H(S) : Open loop T.F

No.

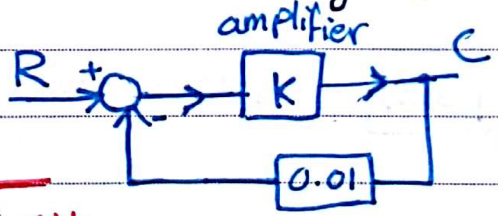
$$C(s) = G(s) \cdot E(s) \Rightarrow c(t) = g(t) \circledast E(t)$$

$$= G(s)\left(R(s) - H(s)\,C(s)\right)$$

$$\Rightarrow [1 + G(s)H(s)]\,C(s) = G(s)\,R(s)$$

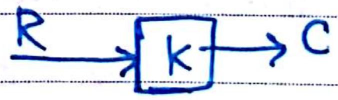
$$\boxed{\underbrace{C.L.T.F}_{\text{closed loop}} = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}}$$

Example: consider the following System



closed loop

$$CL.gain = \frac{C}{R} = \frac{K}{1 + 0.01K}$$



open loop

$$O.L\ gain = \frac{C}{R} = K$$

$k = 10^3$, $CL\ gain = \frac{1000}{11} = 90.9\ldots$

$OL\ gain = 1000$

$OL > CL$

$K = 10^4$, $CL\ gain = \frac{10000}{101} = 99.0099$

$OL\ gain = 10^4$

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$K=10^5$, CL gain $= \dfrac{10^5}{1001} = 99.90\ldots$

$K=10^6$, CL gain $= \dfrac{10^6}{10001} = 99.99\ldots$

CL gain $\longrightarrow$ 100 ;

H(s) مقلوب

reciprocal : مقلوب الرقم

* To prove:

$$\lim_{K\to\infty} \text{C.l gain} = \lim_{K\to\infty} \frac{K}{1+0.01K}$$

$$= \lim_{K\to\infty} \frac{1}{\frac{1}{K}+0.01}$$

$$= 100$$

* i.e the CL gain is insensitive to the parameter variations in the system attributed to K.

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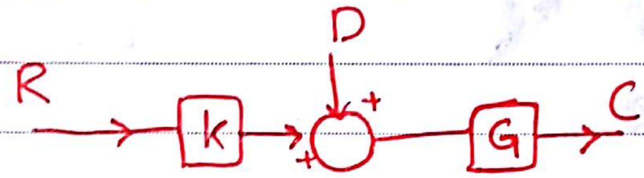
Note: also the cl gain has been reduced (considered as a disadvantage generally).

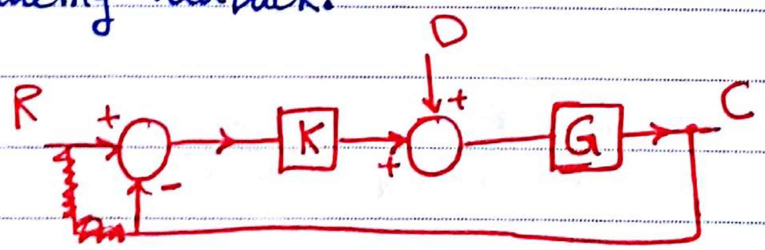
<u>Example</u>: Feedback reduces the effect of external ~~disadvantages~~ disturbances.



$C = KGR + DG$

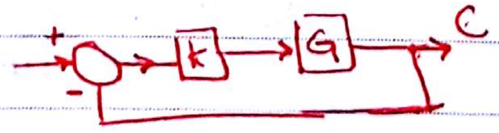
i.e the disturbance has direct influence on the output.

Consider ~~these~~ now no system after introducing feedback.

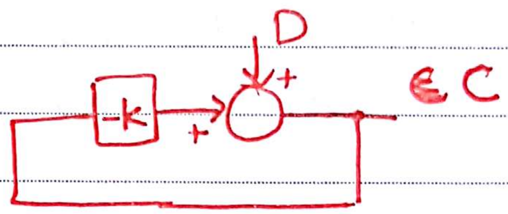


using superposition, let $D=0$

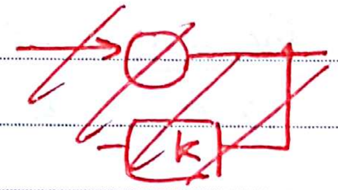
$$C_R = \frac{KG}{1+KG\cdot 1} R$$



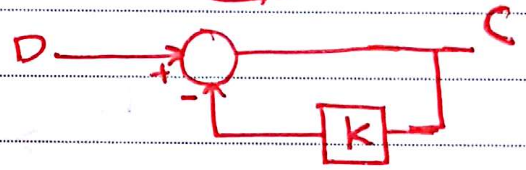
let $R = 0$



$$C_D = \frac{G}{1+GK} D$$



[redacted]



$$\boxed{C = \frac{kG}{1+kG} R + \frac{G}{1+Gk} D}$$

To make C highly dependent on R and not on D make $KG \gg 1$

In which case

$$\lim_{k \to \infty} \frac{G}{1+KG} = \lim_{k \to \infty} \frac{1}{k} = 0$$

resulting on

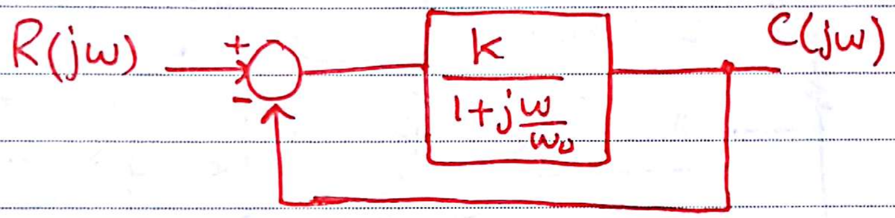
$$C = \lim_{k \to \infty} \frac{KG}{1+KG} R = R$$

No.

<u>Ex</u>: Prove that the bandwidth of a feedback System is increased

Hint: A system with $T.F = \dfrac{K}{1 + j\frac{\omega}{\omega_0}}$

has 3dB bandwidth $= \omega_0$

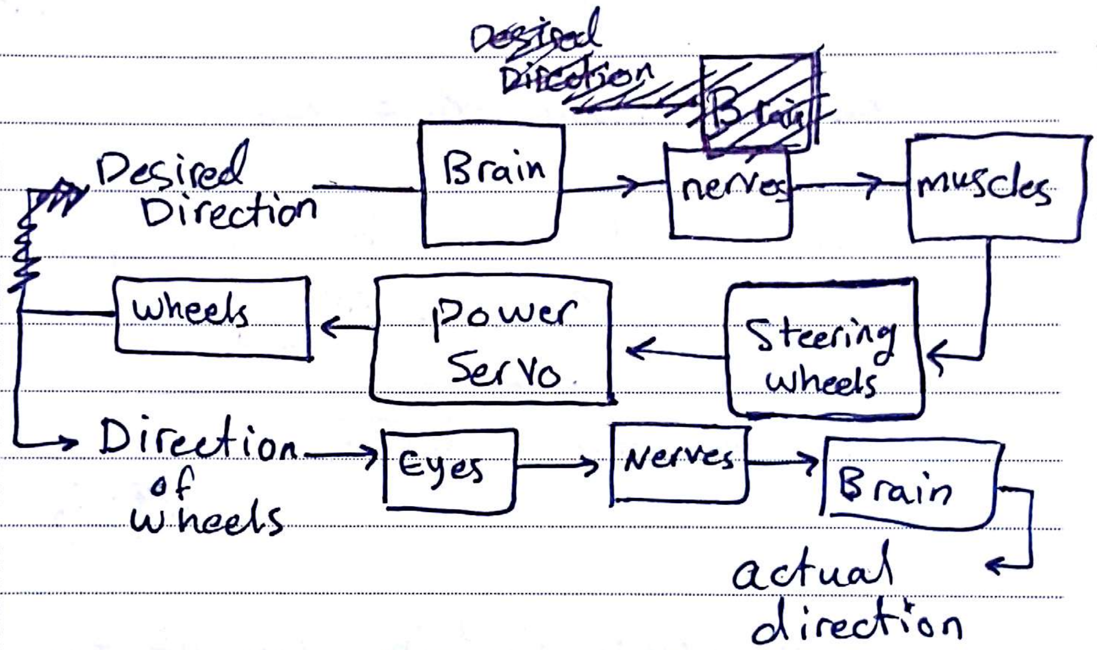


Determine the C.L.T.F $\dfrac{C(j\omega)}{R(j\omega)}$

Demonstrate that the gain of the bandwidth is constant

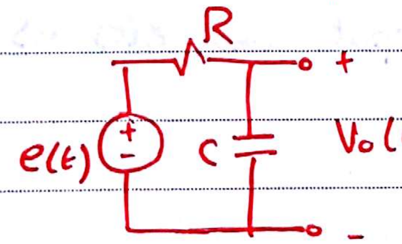
* Schematic diagrams

Consider the process of driving a car



* Block Diagram representation of systems

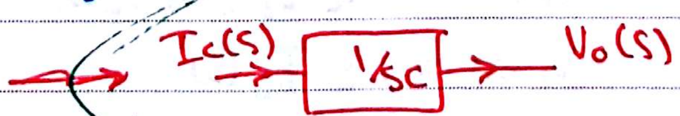
- Having modelled a system somehow in a certain domain.
- We use a transformation which facilitates block-gain multiplication and signal addition.
- A common Domain is the time domain and together with the laplace transform.
- the approach is best illustrated by a certain example



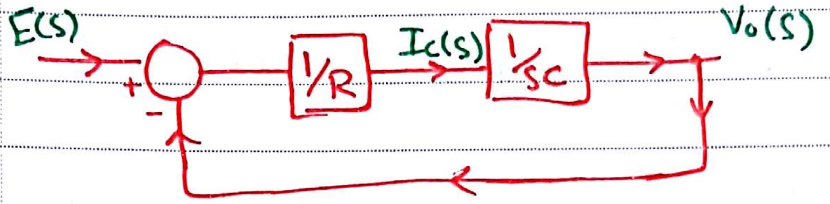
where e(t) is the set value ~~on~~.
and $V_o(t)$ is the output

$$i_c = \frac{e - V_o}{R} \Rightarrow i_c(s) = \frac{1}{R}\left[E(s) - V_o(s)\right]$$

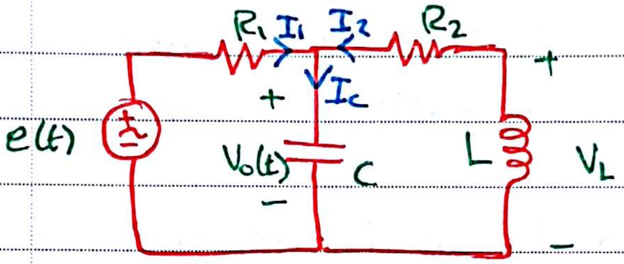
$$V_o = \frac{1}{C}\int_0^t i_c \, d\tau \Rightarrow V_o(s) = \frac{1}{SC} I_c(s)$$



$$\mathcal{L}\int_0^{10} f(t)\,dt = \frac{1}{s}F(s)$$



Consider another circuit:



• to obtain de a detailed block diagram using Integrators only (ie: 1/s) with $V_o(s)$ as output and $e(t)$ as set value.

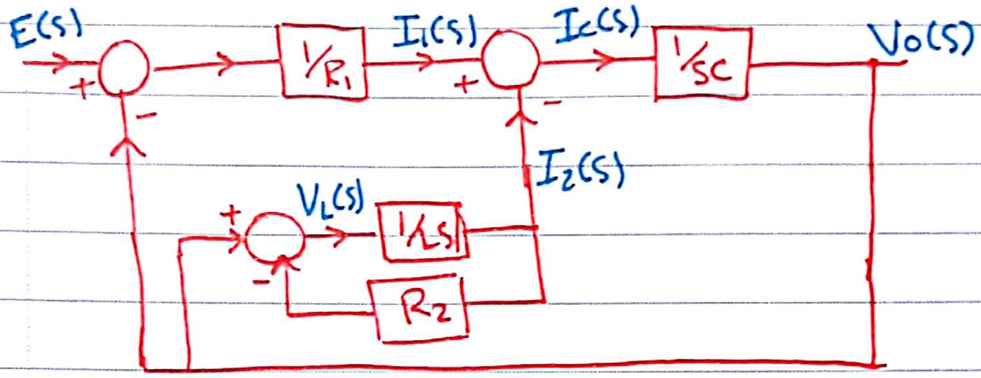
$$\cancel{I_1(s)}\quad I_1(S) = \frac{1}{R_1}\left[E(s) - V_o(s)\right]$$

$$V_o(S) = \frac{1}{SC} I_1(S)$$

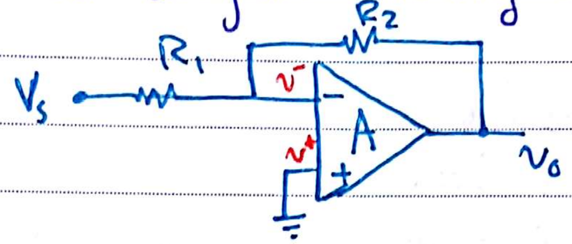
$$V_o(S) = R_2 I_2(s) + V_L(s)$$

$$V_L(S) = LS\, I_2(S)$$

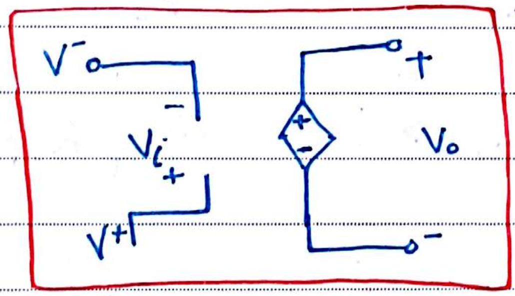
No.



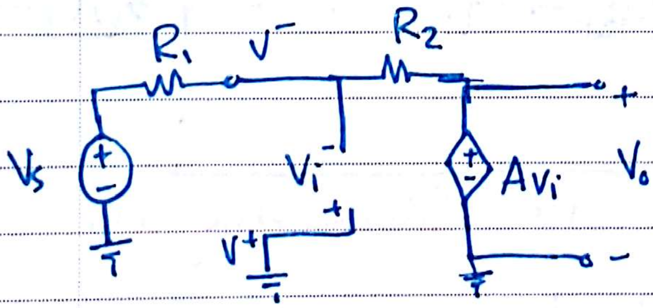
Example: Model the following circuit as a block diagram involving feedback

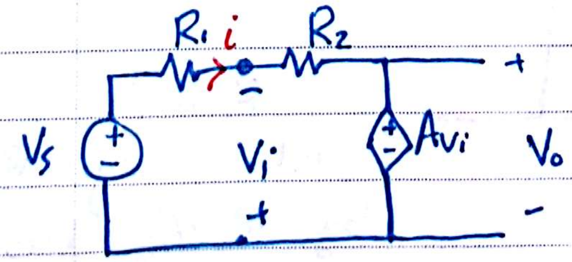


Using the ideal op-amp model where $R_i = \infty \Omega$ & $R_o = 0 \Omega$



Ideal Op-amp (memorize it)



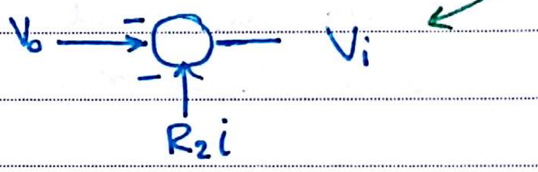


→ The Inverting op-amp

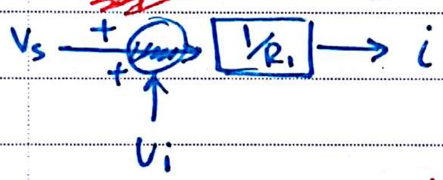
- $V_o = A V_i \longrightarrow$

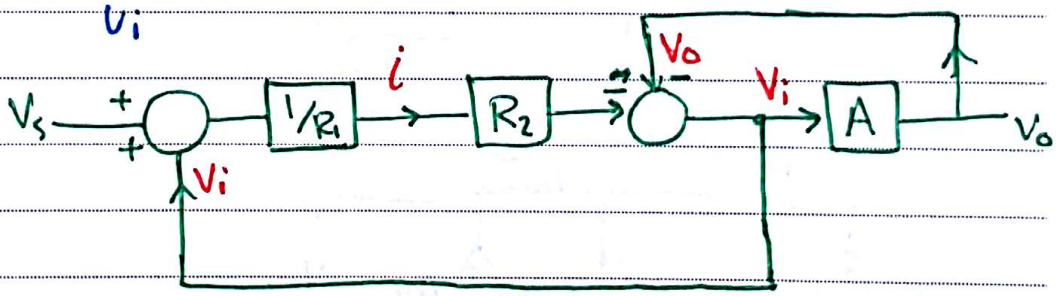
$-V_i - V_o - R_2 i = 0$

- $V_i + V_o + R_2 i = 0 \Rightarrow V_i = -V_o - R_2 i$



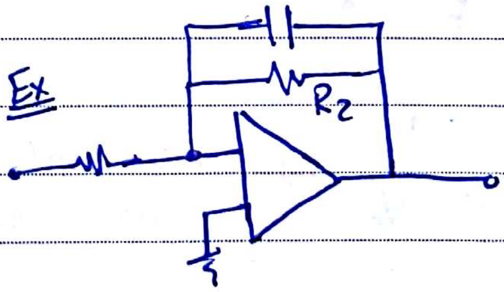
- $i = \dfrac{V_s + V_i}{R_1}$





This is the Block diagram.

<u>Ex</u>



$R_2 \,//\, \frac{1}{sC}$

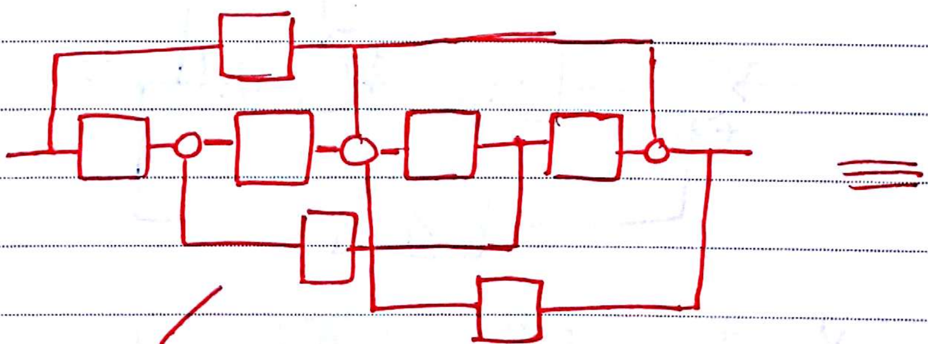
just change R_2 box by $R_2 // \frac{1}{sC}$ box

Exercise: Resolve the example differently by to get a different BD representation.

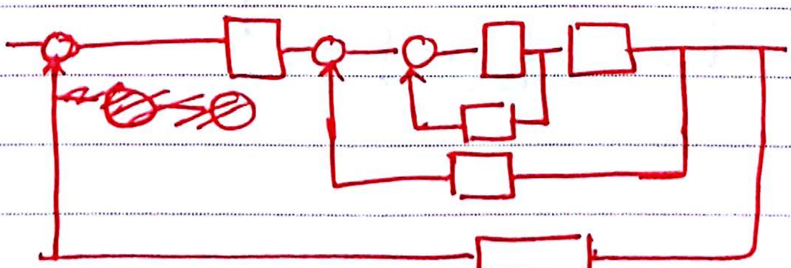
* Block Diagram Reduction

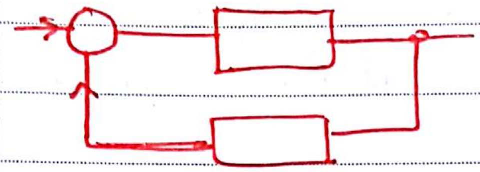
- Starting with a complicated block diagram, manipulate blocks and signals appropriately so as to get a basic feedback block within bigger ones.

- Such manipulative process ease the determination of the overall transfer function.



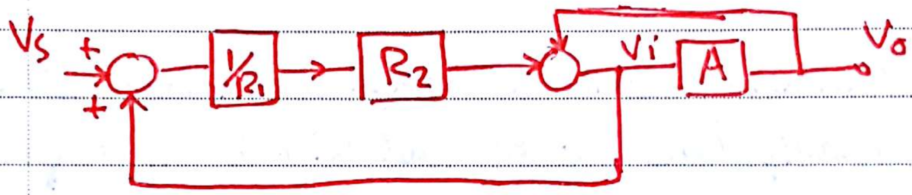
(complicated BD)

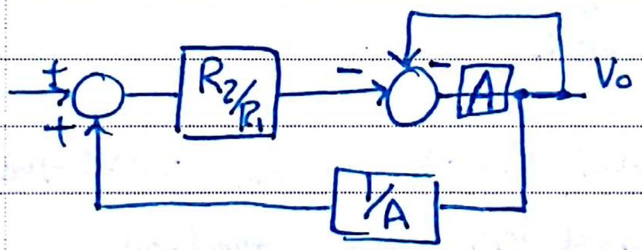


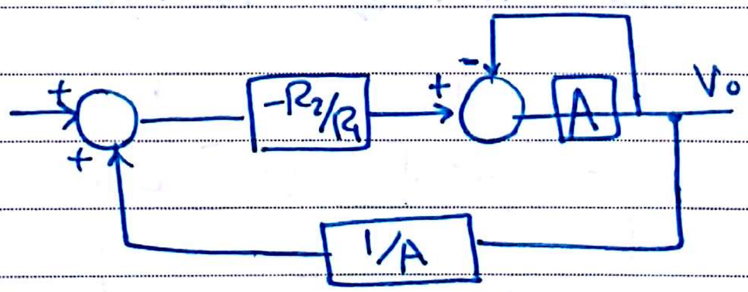


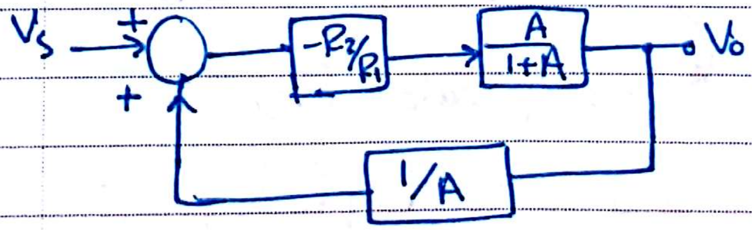
this will be the result.

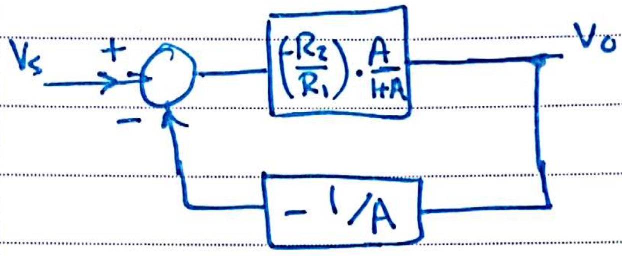
• Block diagram representing the Inverting Amplifier considered previously



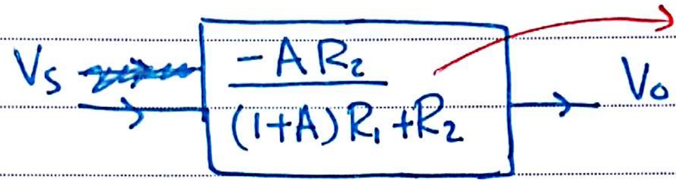








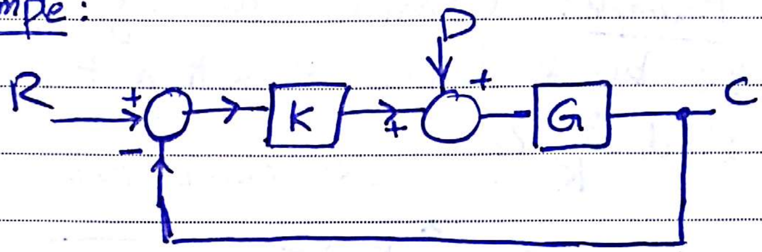
$$\frac{V_o}{V_s} = \frac{-\frac{A R_2}{(1+A)R_1}}{1 + \frac{R_2}{(1+A)R_1}}$$

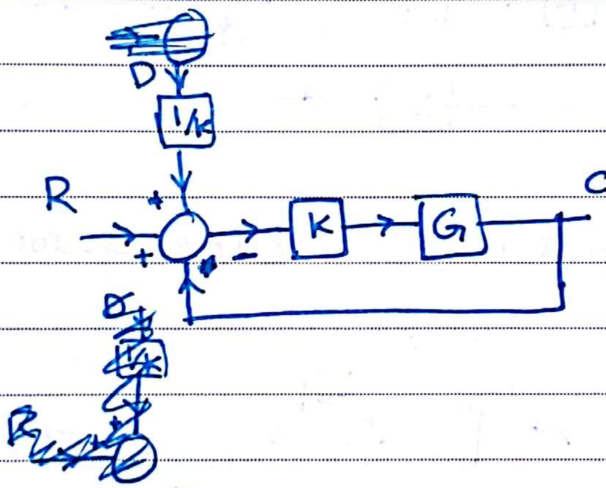


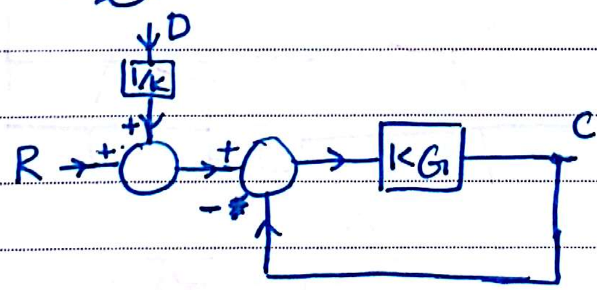
$$\frac{V_o}{V_s} = \frac{-AR^2}{(1+A)R_1 + R_2}$$

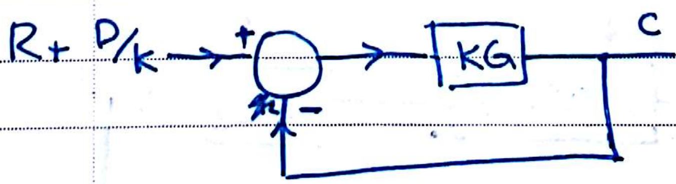
$$\lim_{A\to\infty} \frac{V_o}{V_s} = \frac{\frac{\overset{1}{\cancel{A}}}{\cancel{1+A}} R_2}{R_1 + \cancel{\frac{R_2}{1+A}}\ 0} = \frac{-R_2}{R_1}$$

Example:





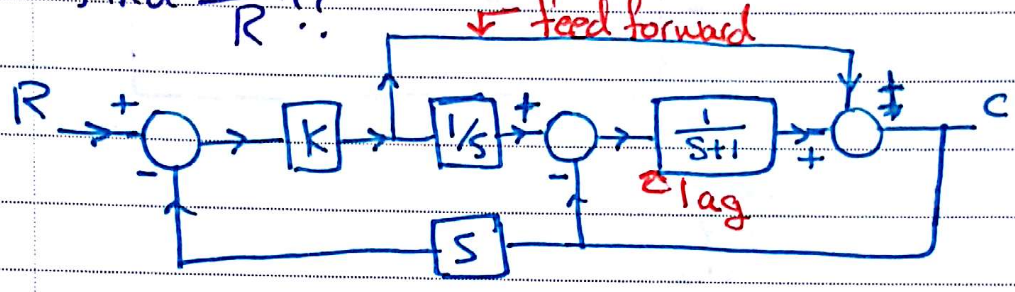




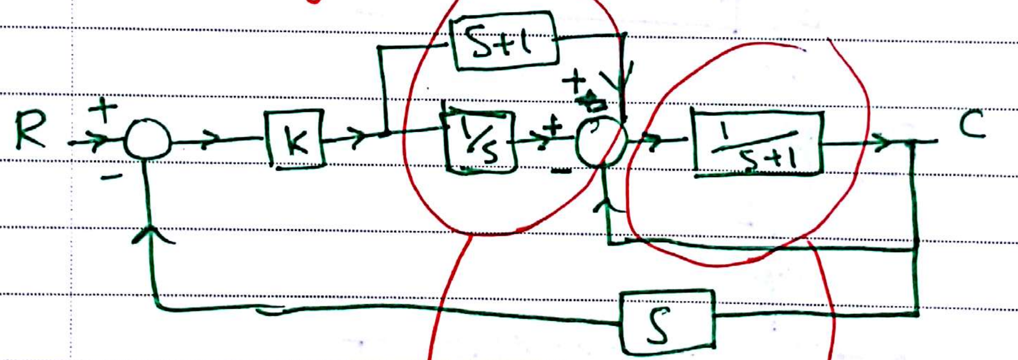
$$C = \frac{KG}{1+KG}\left(R + \frac{D}{K}\right)$$

$$= \frac{KG}{1+KG}R + \frac{G}{1+KG}D$$

Example: Consider the system shown, use the block diagram reduction techniques to find $\frac{C}{R}$??

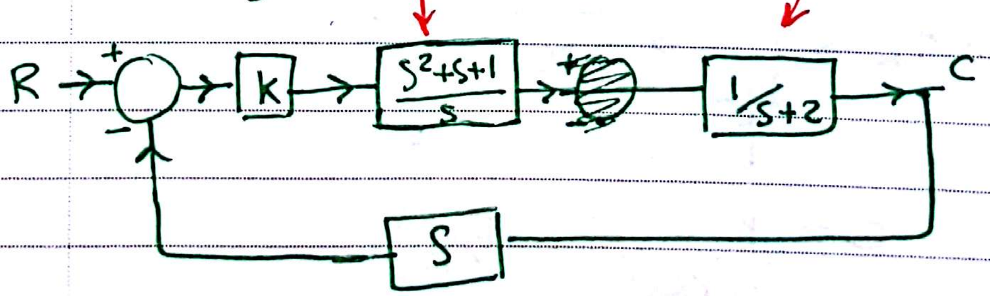


1/s: Integrator   S: differentiator



$$\frac{\frac{1}{S}}{1+(S+1)*\frac{1}{S}}$$

$$\frac{G(S)}{1+H(S)\,G(S)}$$



$$\frac{C}{R}=\frac{K\left(\frac{S^2+S+1}{S}\right)\left(\frac{1}{S+2}\right)}{1+K\left(\frac{S^2+S+1}{S}\right)\left(\frac{1}{S+2}\right)S}$$

$$\cancel{\frac{C}{R} = \frac{K(s^2+s+1)}{(s^2+s+1)+K}}$$

$$\frac{C}{R} = \frac{\dfrac{K(s^2+s+1)}{s(s+2)}}{\dfrac{s+2\cancel{s}+K(s^2+s+1)}{s+2}}$$

$$\frac{C}{R} = \frac{K(s^2+s+1)\cancel{(s+2)}}{[s+2+K(s^2+s+1)]\cdot(s\cancel{(s+2)})}$$

$$\frac{C}{R} = \frac{K(s^2+s+1)}{[s+2+K(s^2+s+1)]\cdot s}$$

Characteristics of First Order Systems

Characterization by a T.F of the form

$$G(s) = \cancel{\frac{K}{Ts+1}} \quad \frac{K}{TS+1}$$

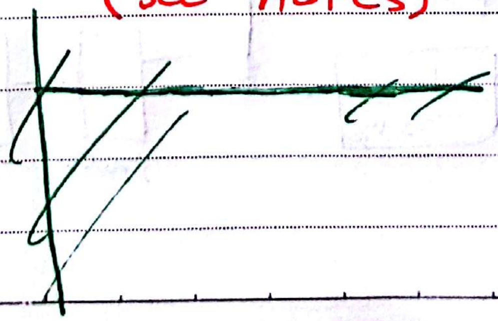
K is the gain.
T is the time constant.

Due to a Unit Step, the output is given by

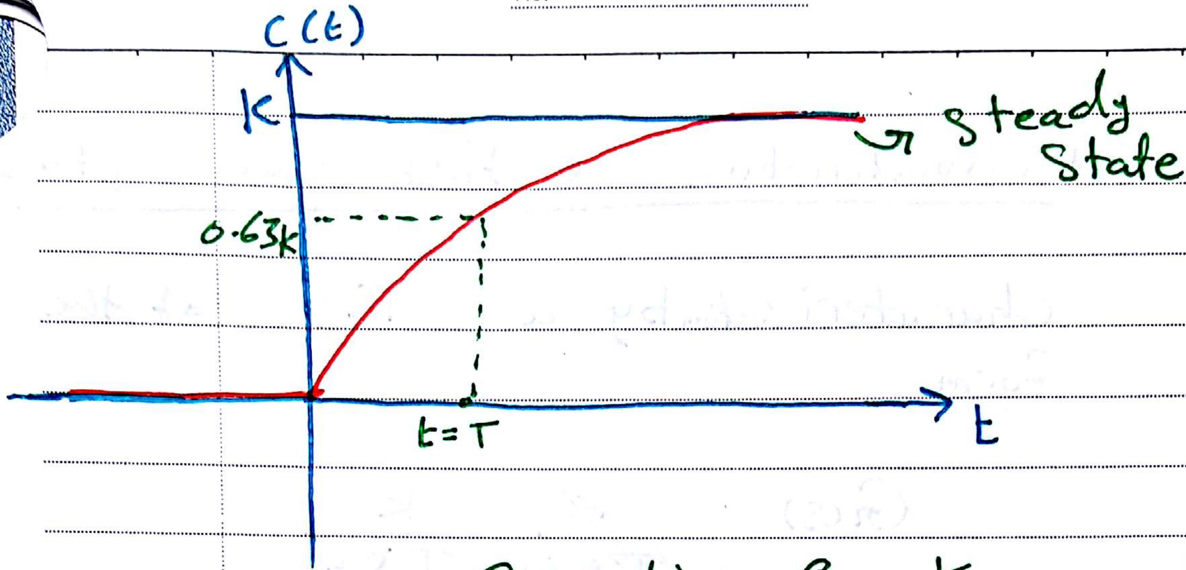
$$C(s) = G(s) \cdot \frac{1}{s}$$

$$= \frac{K}{s(Ts+1)}$$

after $\mathcal{L}^{-1} C(s)$, $C(t)$ looks like
(See notes)



No.



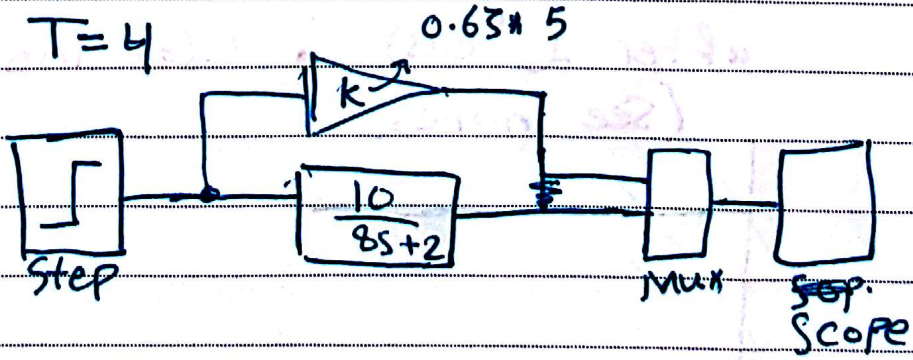
$$C_{ss} = \lim_{S \to \infty} S \cdot \frac{K}{S(TS+1)} = K$$

Simulink Determine C(t) when

$$G(S) = \frac{10}{8S+2}$$

$$= \frac{10}{2(4S+1)} = \frac{5}{4S+1}$$

K = 5
T = 4



* Second Order Systems

Our characterization is related to a Second Order System of a form given by

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \; ; \; \cancel{\omega_n \neq 0} \; \cancel{\zeta \neq 0}$$

zeta

$$0 < \zeta, \omega_n < \infty$$

~~Where ω_n~~

where ω_n is the natural undamped frequency.

ζ is the damping ratio.

• We get different responses dependent on the Value of ζ, assuming ω_n is kept fixed.

No.

• Underdamped System $(\zeta < 1)$

In this case it can be shown that the system response due to a unit step is

$$\cancel{c(t) = \left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \cos^{-1}\zeta)\right] \cdot u(t)}$$

$$c(t) = \left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \cos^{-1}\zeta)\right] \cdot u(t)$$

(See notes)

where ω_d is the underdamped natural frequency.

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \quad , \quad \omega_d < \omega_n$$

- Simulink

$$\boxed{\text{Step}} \rightarrow \boxed{\frac{25}{s^2+6s+25}} \rightarrow \boxed{\text{Scope}}$$

$\omega_n = \sqrt{25} = 5 \text{ rad/s}$

$6 = 2\xi(5) \Rightarrow \xi = 0.6$

$\xi \downarrow \Rightarrow$ more damping ratio
Oscillations (~~Sinusoi~~

$\omega_n \downarrow \Rightarrow$ becomes more Smooth

$\xi \downarrow, \omega_n \downarrow \Rightarrow$ more Sinusoidal

Dependance of the Overshoot on ζ

Given

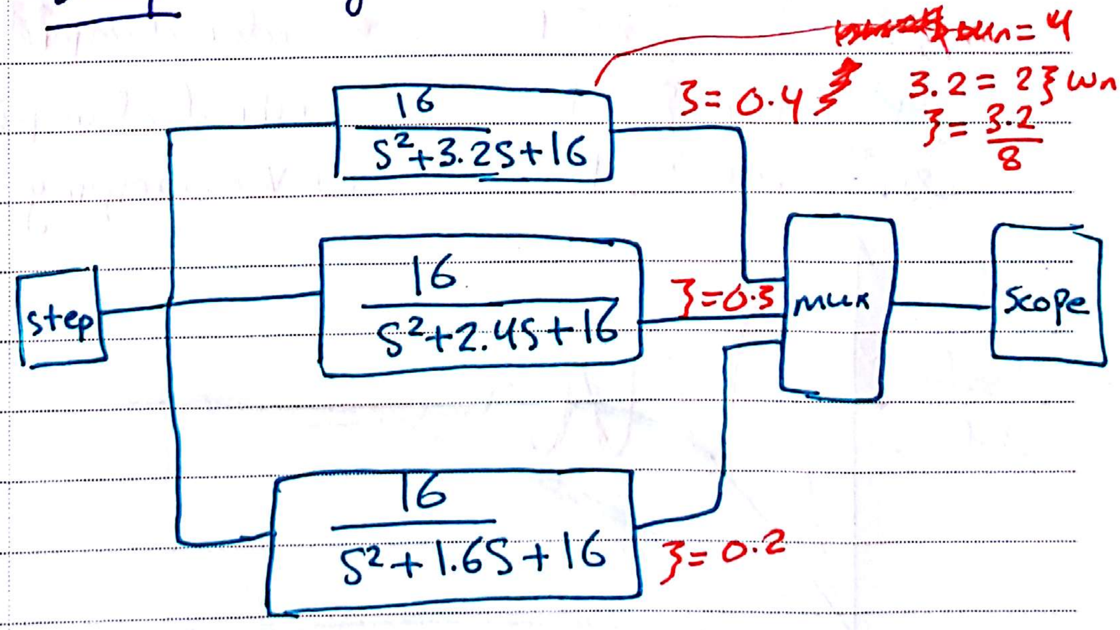
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The overshoot depends on ζ only given by

$$\cancel{e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times \text{Steady state output}}$$

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} \ * \ \text{steady state output}$$

Example: using Simulink



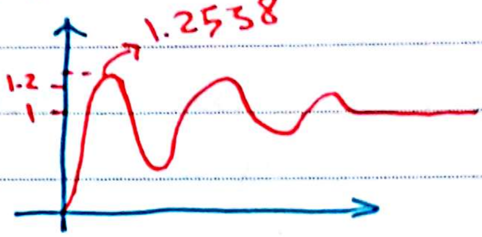
No.

For first transfer function the overshoot is

$$M = e^{-\zeta\pi/\sqrt{1-\zeta^2}} * \text{Steady state output}$$

$$= e^{-0.4\pi/\sqrt{1-0.4^2}}$$

$$= 0.2538$$



matlab

z=0.4 , M=exp(-z*pi/((1-z^2)^0.5))

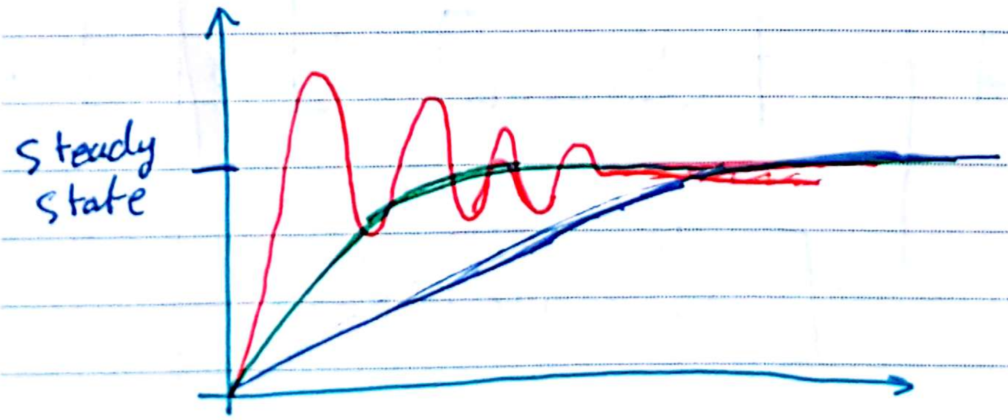
w_n [rad/s]

ζ [unitless]

الأحمر for $\zeta < 1$ under damping

الأخضر for $\zeta = 1$ critical damping

الأزرق for $\zeta > 1$ Over damping



FIVE APPLE

No.

for $\zeta < 1$ Poles are Complex

$\zeta = 1$ Poles are identical

$\zeta > 1$ Poles are distinct

$$C(t) = 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \cos^{-1}\zeta), \zeta < 1$$

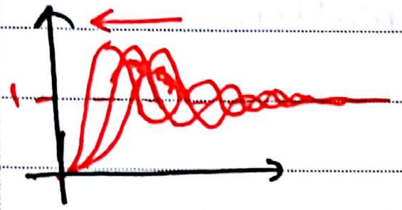
No.

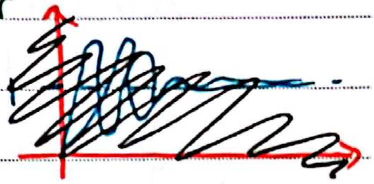
Simulink

Ex: $\omega_{n_1} = 4$, $\omega_{n_2} = 5$, $\omega_{n_3} = 6$, $\zeta = 0.3$ for all

⇒ they all have the same overshoot

the biggest ω_n will be faster





* Dependance of Speed of Response on ω_n

It depends on the natural frequency ω_n when ζ is fixed, where the bigger ω_n the faster the response.

re call that

$$C(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \cos^{-1}\zeta)$$

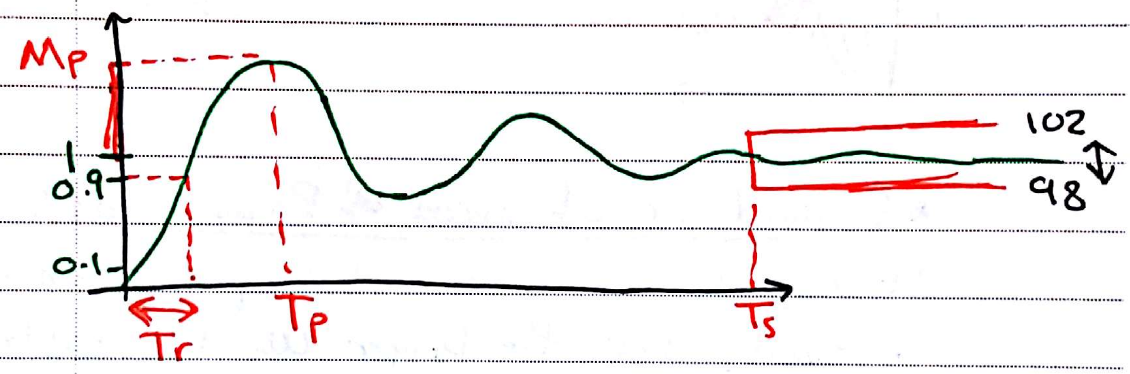
we can see that $e^{-\zeta\omega_n t}$ decays faster for larger ω_n.

* Time domain Specifications of a 2nd order System.

the Specifications apply to the following System.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad \zeta < 1 \quad \underline{\text{underdamped}}$$

* Side note: Steady state $\boxed{1}$ when ω_n^2 in denominator & numerator is the same & all the signs are the same in the denominator but the sign of ω_n^2 in numerator doesn't matter.



* Theoretically the oscillations keep going to infinity.

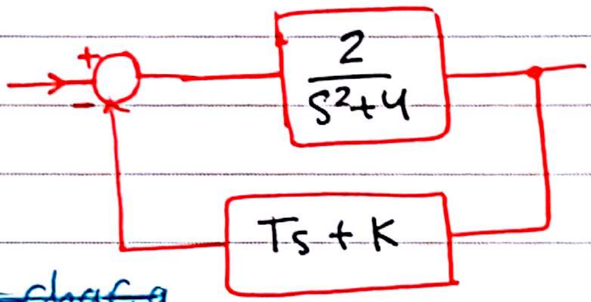
Tp: time of the first peak
Tr: rising time from 0.1 to 0.9
Ts: time when system reaches 2% of the steady state.
Mp: maximum overshoot.
* It can be shown that

$$\boxed{T_p = \frac{\pi}{\omega_d}} \quad , \quad \boxed{M_p = C_{ss} * e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)}}$$

$$\boxed{T_s \approx \frac{4}{\zeta\omega_n}} \quad , \quad \zeta < 0.8 \qquad T_r \approx$$

Ex given a System $G(s) = \frac{2}{s^2+4}$ design a controller Such that $\omega_n = 5\,rad/s$, $\zeta = 0.7$ $C_{ss} = 2$

→ but the System's response is Sinusoidal we introduce a controller of the form $H(s) = Ts + k$ in the feedback loop



~~the desired char. q~~

The desired characteristic eqn.

$$s^2 + 7s + 25$$

The System after control gives

$$CLTF = \frac{2}{s^2+4+2Ts+2k}$$

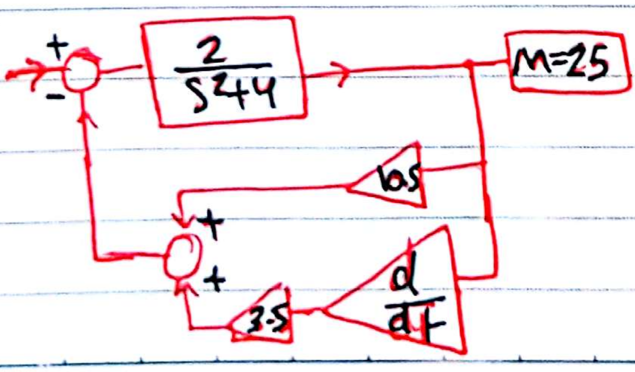
$2k+4 = 25 \Rightarrow k = 10.5$

$2T = 7 \Rightarrow T = 3.5$

here $C_{ss} = \frac{2}{2k+4}$

$C_{ss} = 0.08$

So multiply output by $\frac{2}{0.08} = 25$



20/3

No.

* Stability of a linear-time-Invariant System:

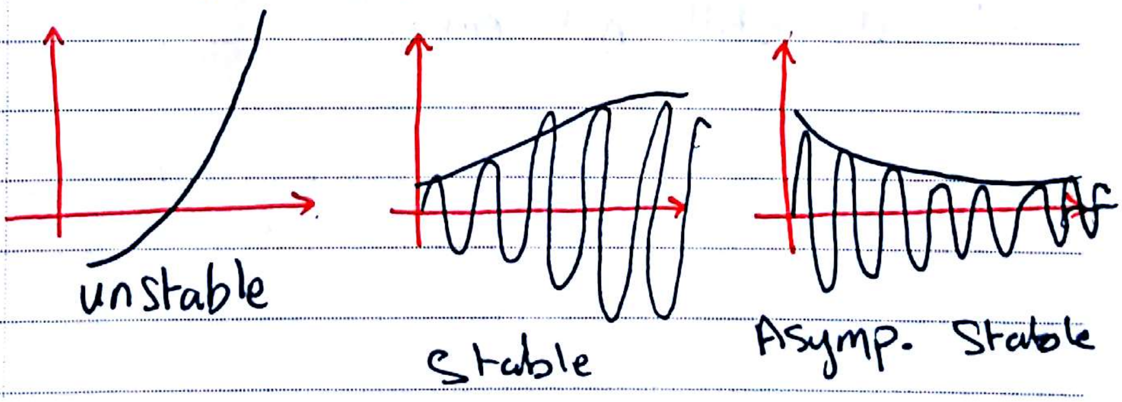
- consider the System output $C(t)$ due to zero excitation (i.e no forcing function, However the initial States are not all zeros).

- If $\lim_{t\to\infty} C(t) = \infty$, then the System is unstable.

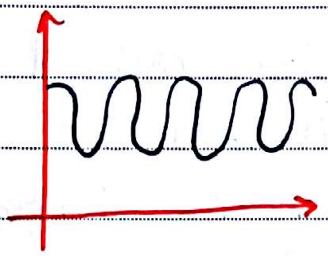
~~unstable~~

- If $\lim_{t\to\infty} C(t) = a$ (non-zero), then the System is Stable.

- If $\lim_{t\to\infty} C(t) = 0$, then the System is asymptotically Stable.



• Asymp. Stable is a stranger condition than the normal stable system.



the output is bounded
↳ Stable (not Steady)

* To determine the stability, find the roots of the characteristic equation
↳ Assume the equation is of the order 10, then it would be hard to determine the roots → we need another approach.

* Judging stability by $\lim_{t \to \infty} C(t)$ is cumbersome as it indicates finding the inverse L.T, Determining the roots of the C.E answers the question of stability, but it's still not easy.

يعني صعب

• The most convenient method is to use the Routh - Harwitz method.

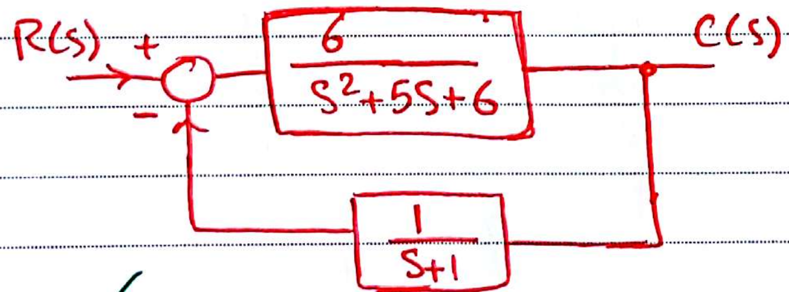
↓

$$C.E = 1 + G(s) \cdot H(s) = 0$$

• **The Routh Harwitz method:**

↳ Given the C.E, the method is best illustrated by an example.

Example: Consider the System Shown:-



$$\frac{C(s)}{R(s)} = \frac{\frac{6}{(s^2+5s+6)}}{1 + \frac{6}{s^2+5s+6} \cdot \left(\frac{1}{s+1}\right)} = \frac{6(s+1)}{s^3+6s^2+11s+12}$$

or $s^3 + 6s^2 + 11s + 12 = 0 \Rightarrow 1 + G(s) \cdot H(s)$

s^3	1	11
s^2	6	12
s^1	$\frac{6 \times 11 - 1 \times 12}{6} \to 9$	0
s^0	12	

No.

* What matters is the Signs of the values of the first column.

* No change in Signs → Stable

* Note: If we change H(s) into $\frac{66}{s+1}$ the system becomes unstable.

~~* Ex~~

• **Example:** Given the characteristic eqn.

$$C.E = 1 + G(s)H(s) = 0$$

$$= S^4 + 2S^3 + KS^2 + 10S + 5 = 0$$

Is the System Stable??

$$\begin{array}{c|ccc}
S^4 & 1 & K & 5 \\
S^3 & 2 & 10 & 0 \\
\hline
S^2 & K-5 & 5 & \\
S^1 & \frac{10K-60}{K-5} & 0 & \\
S^0 & 5\,\cancel{\left(\frac{10K-60}{K-5}\right)} \Big/ \cancel{\left(\frac{10K-60}{K-5}\right)} & &
\end{array}$$

$\left(\frac{2(5)-1(0)}{2}\right)$

$\left(\frac{2K-10}{2}\right)$

* For Stability

$K-5 > 0 \rightarrow K > 5$

& $\frac{10K-60}{K-5} > 0 \rightarrow K > 6$ & $K \neq 5$

$10K - 60 > 0$

$K > 6$

condition for Stability

$K > 6$

• Check using Matlab

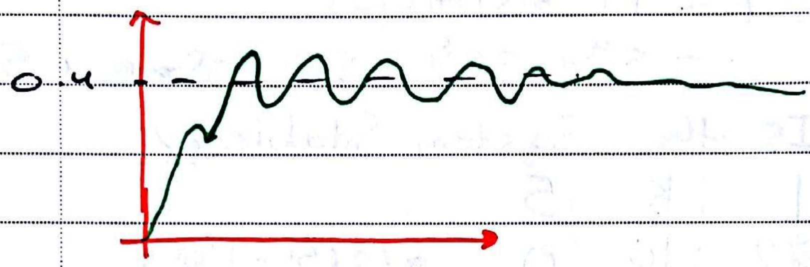
```
>> K = 7 ; CE = [1 , 2 , K , 10 , 5];
roots(CE)
```

~~-ve real~~

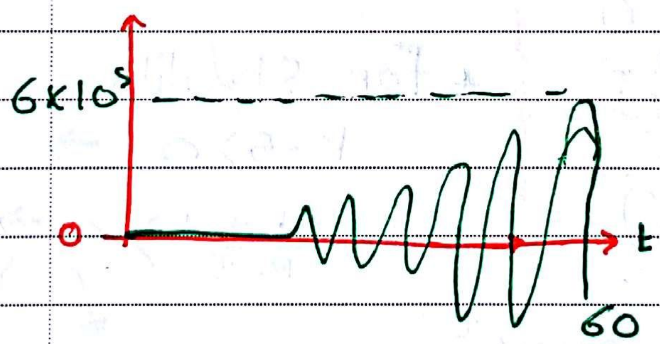
complex roots , all -ve real, but for K=4

it was unstable. (2 roots were complex with +ve real & 2 -ve real)

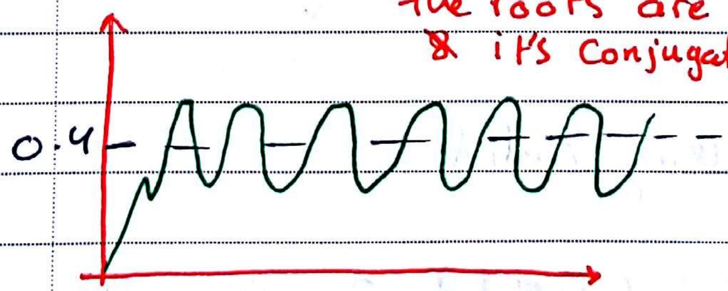
```
>> K=7;n=2;CE=[1 2 k 10 5]; roots(CE);
Step(n,CE)
```



For k=4



For K=6 (stable same as k=7 the roots are complex, -ve real & it's conjugate)



N.B: the number of poles of +ve real parts ~~are~~ ~~the~~ is the number of changes in sign in the first column.

No.

S^4	1
S^3	2
S^2	-1
S^1	-
S^0	5

2 Changes in Sign in the first columns.

2 poles with +ve real parts.

* The case where the first element is a zero:-

• Example: let ~~$S^4+2S^3+3S^2+6=0$~~

$$S^4+2S^3+3S^2+6S+4=0$$

S^4	+ 1	3	4
S^3	+ 2	6	0
S^2	+ ~~0~~ ε	4	
S^1	− $\frac{6\varepsilon-8}{\varepsilon}$ → −ve	0	
S^0	+ 4		

1) replace the zero by an ~~positive~~ ϵ of the Same Sign of the highest Power.

$$\lim_{\varepsilon \to 0} \frac{6\varepsilon-8}{\varepsilon} = -\infty$$

→ unstable, because there are changes in Sign.

No.

II) another method of solving
Replace S by $1/s$, Calculate the new CE, then apply rouths method.

III) multiply CE by $(S+1)$ then apply rouths method.

Exercize: apply the additional two methods to the previous example.

This is solved by me

method (I): replace by $\frac{1}{S}$

$$\frac{1}{S^4} + \frac{2}{S^3} + \frac{3}{S^2} + \frac{6}{S} + 4 = 0 \quad (* S^4)$$

$$1 + 2S + 3S^2 + 6S^3 + 4S^4 = 0$$

by Routh - Harwitz method
$\Rightarrow$ Unstable

method (2): ~~rep~~ multiply by $(S+1)$

$$S^4 + 2S^3 + 3S^2 + 6S + 4 = 0 \quad *(S+1)$$

$$(S^5 + 2S^4 + 3S^3 + 6S^2 + 4S) + (S^4 + 2S^3 + 3S^2 + 6S + 4)$$

$$S^5 + 3S^4 + 5S^3 + 9S^2 + 10S + 4 = 0$$

by routh - Harwitz method
$\Rightarrow$ unstable

Example :

$$C.E = S^4 + S^3 + 2S^2 + S + 1 = 0$$

$$\begin{array}{c|ccc}
S^4 & 1 & 2 & 1 \\
S^3 & 1 & 1 & 0 \\
\hline
S^2 & 1 & 1 & \\
S^1 & \cancel{0} & 0 & \\
S^0 & 1 & &
\end{array}$$

* Generate the Auxiliary equation

$$A(S) = 1 * S^2 + 1 * S^0 = S^2 + 1$$

$$\frac{d\, A(s)}{ds} = 2S$$

No change in the Sign $\rightarrow$ the System is Stable

* Check on matlab

roots([1 1 2 1 1])

No +ve real parts

The case of a complete row of zeros arises when the poles ⇒ Symmetric about the origin.

Example: $C.E = S^4 - 1 = 0$

S^4	1	0	-1
S^3	0	0	
S^2	ϵ	-1	
S^1	$4/\epsilon$	0	
S^0	-1		

$S^4 - 1$ has $\pm j, 1, -1$ as roots

* If a C.E change in Sign then the System is unstable.

* If a C.E has a missing Power ⇒ the System is unstable.

→ $S^6 + 2S^5 + 3S^3 - S^2 + 5S + 13 = 0$
(unstable → sign)

→ $S^6 + 2S^5 + 3S^3 + S^2 + 5S + 13 = 0$
(unstable → missing Power)

No.

matlab

```
>> n=1 ; d= [1 1 0] ; sys= tf(n,d);
step(sys)
```

---

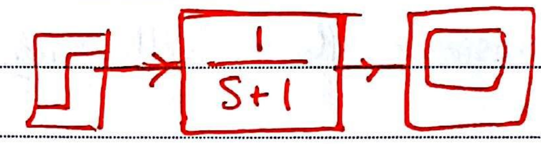
* for a second order system

$C.E = as^2 + bs + c = 0$

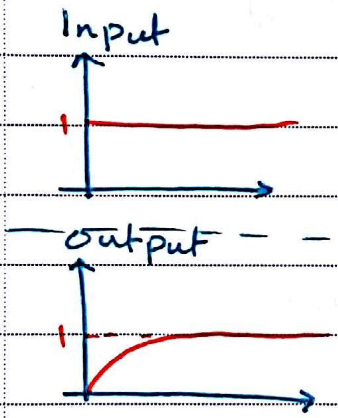
s^2	a	c
s^1	b	0
s^0	c	

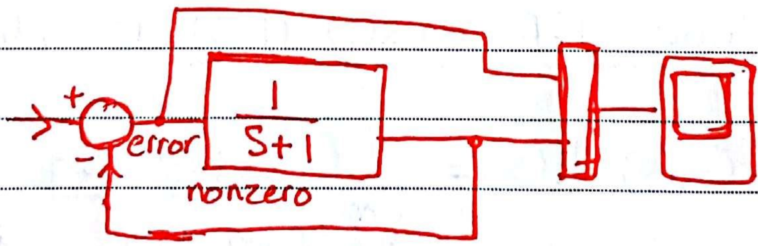
* Static error Coefficients

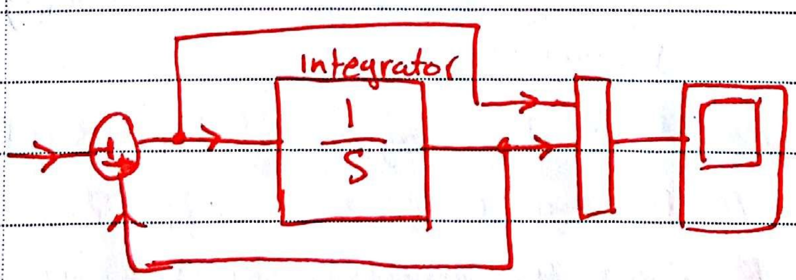
Motivation:



First order lag → output lags input



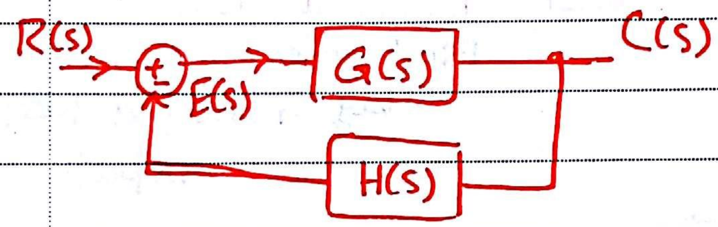




Impression:

Conclusion: An integrator results in a steady state error
$\downarrow$
zero

Consider the following System.



$$E(s) = \frac{1}{1+G(s).H(s)} . R(s)$$

where it is assumed that the open loop T.F $G(s)\,H(s)$ is described by

$$G(s)\,H(s) = \frac{K\left(s^m + b_1 s^{m-1} + \dots + b_m\right)}{s^N\left(s^p + a_1 s^{p-1} + \dots + a_p\right)}$$

Integrator

where N is the type of the System

$N=1$ type 1 , $N=2$ type 2 ...

(i.e: N gives the number of Integrators).

$$\frac{1}{s^N} = \left(\frac{1}{s}\right)^N \quad , \quad N \geq 0$$

↳ integer

Example: $G(s) = \dfrac{s+2}{s^3+4s^2}$

$H(s) = \dfrac{5}{s+3}$

What is K & N??

$$G(s).H(s) = \frac{5(s+2)}{(s^3+4s^2)(s+3)}$$

$$= \frac{5(s+2)}{s^2(s+4)(s+3)}$$

$$= \frac{5(s+2)}{s^2(s^2+7s+12)}$$

$K=5$, $N=2$ type 2 integrator.

Due to a unit step (i.e $R(s)=\frac{1}{s}$)

$\cancel{e_{ss} = \lim_{s\to 0} sE(s) = \lim_{t\to\infty} e(t)}$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{1}{1+K_p}$$

where K_p is known as the position error Coefficient.

where

$$K_p = \lim_{s\to 0} G(s).H(s)$$

No.

~~where~~

~~$K_p = \lim_{s \to 0} G(s) \cdot H(s)$~~

where

K_p = fininte Value if $N=0$

$\rightarrow$ e_{ss} = finite Value

If $N=1$

K_p = Infinity

$\rightarrow$ e_{ss} = Zero

*Conclusion :

$N \geq 1$

$K_p = \infty$

$e_{ss} = 0$

If $N=2$

K_p = Infinity

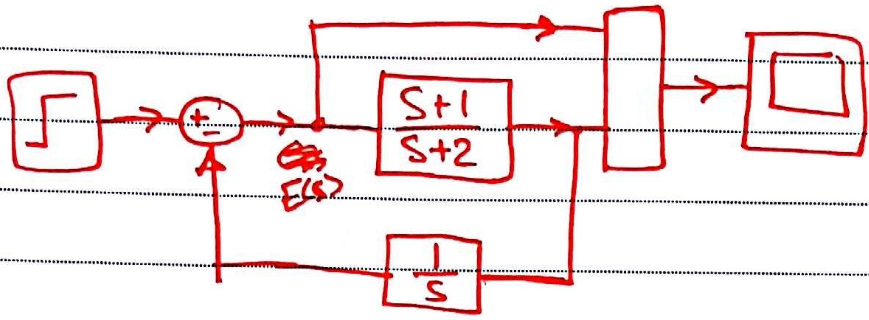
$\rightarrow$ e_{ss} = Zero

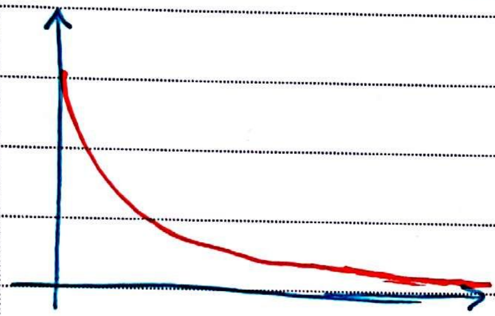
Conclusions are valid provided the closed loop System is Stable.

~~Example: Sim~~

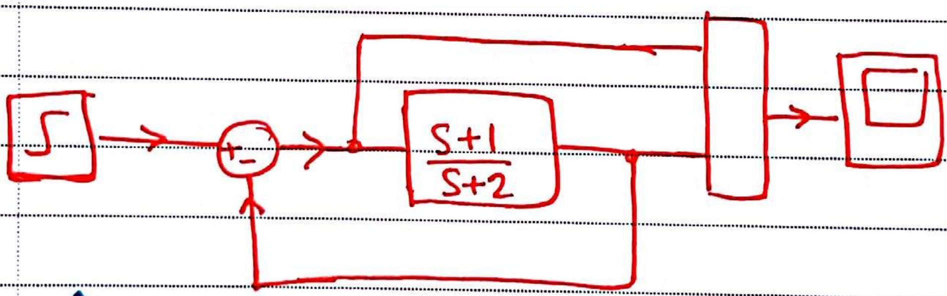
No.

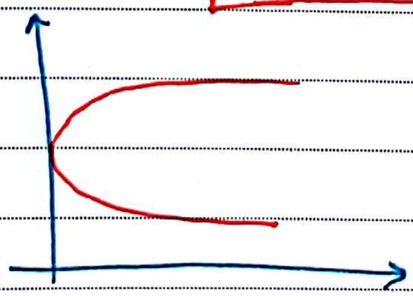
Example: Simulate



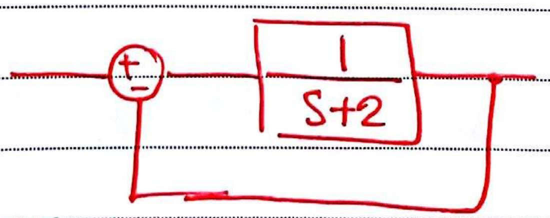


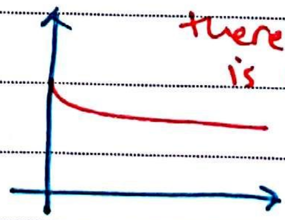
~~No~~ $e_{ss} \rightarrow$ zero





there is error





there is error

This is for stable systems.
(Steady state error)

Velocity error coefficient

Due to a unit ramp excitation

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s) = \frac{1}{K_v}$$

where,

$$K_v = \lim_{s\to 0} s\,G(s)\,H(s) = \lim_{s\to 0} s * \frac{1(s^2+\dots)}{s^N(s^2+\dots)}$$

Hence,

if $N=0$ then $K_v = 0 \Rightarrow e_{ss} = \frac{1}{0} = \infty$

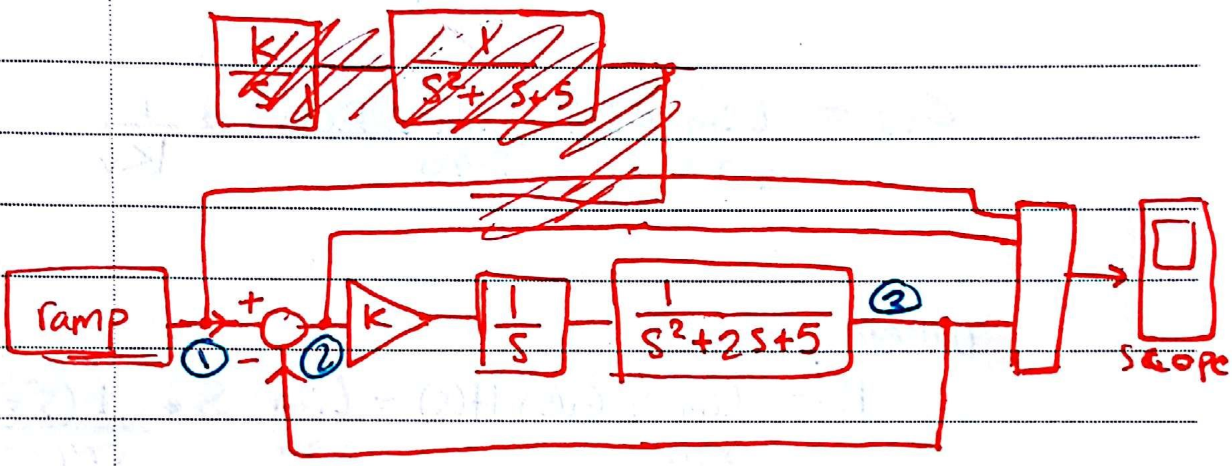
$N=1$ then $K_v = \text{finite value} \Rightarrow e_{ss} = \text{finite value}$

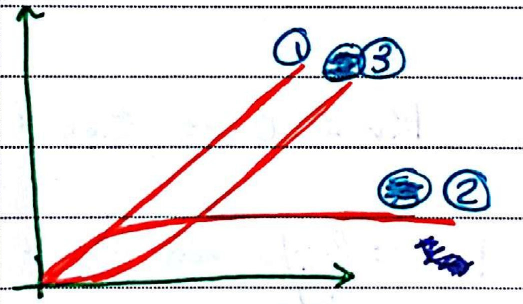
$N \geq 2$ then $K_v = \infty \Rightarrow e_{ss} = \text{zero}$

All this theory is valid provided the closed loop system is stable. (not the O.L system).

So check for CL stability.

Example Simulink

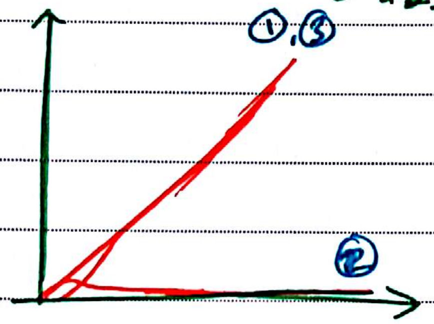




*If ~~when~~ T.F $\boxed{\frac{1}{S+1}}$

If double Integral $\boxed{\frac{1}{S^2}}$ ~~R.T.F~~

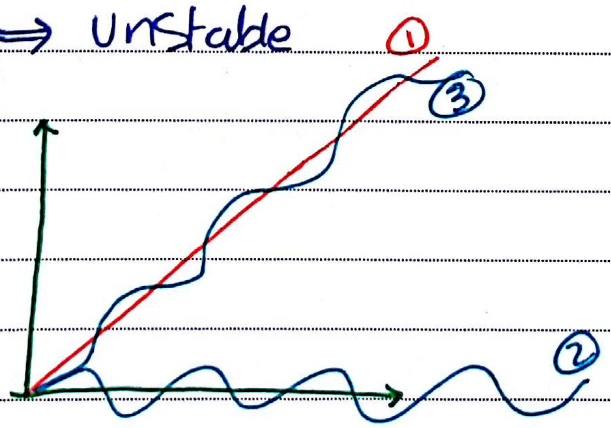
and T.F $\boxed{\frac{S+1}{S+5}}$



also add $\xrightarrow{4}$ between $\boxed{\frac{1}{S^2}}$ & $\boxed{\frac{S+1}{S+5}}$

* Change T.F to $\boxed{\frac{S+1}{S+0.5}}$

$\Rightarrow$ Unstable



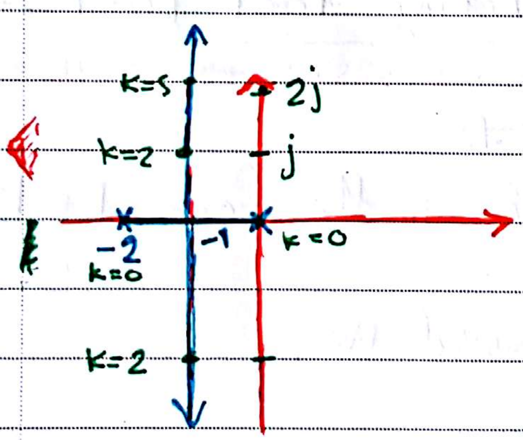
The Root Locus Method

المحل الهندسي

The RL is a pictorial exposition of the roots of a polynomial as a function of a certain parameter.

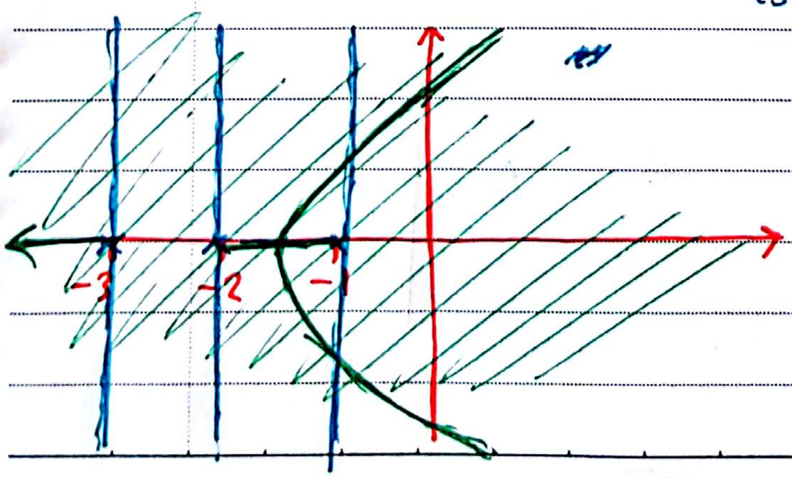
<u>Example:</u> If C.E $= S^2 + 2S + k = 0$, $k \geq 0$

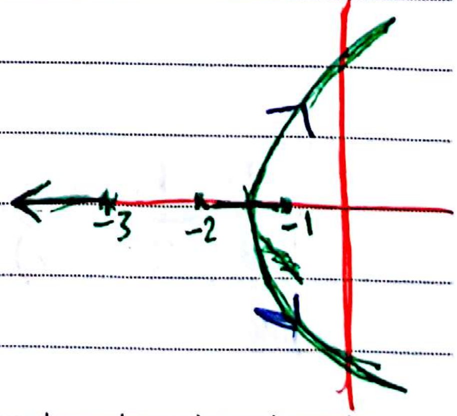
$$= 1 + \frac{k}{S(S+2)} = 0$$



<u>Example:</u> If C.E $= S^3 + 6S^2 + 11S + 6 + k = 0$

$$= 1 + \frac{k}{(S+1)(S+2)(S+3)} = 0$$





* The Idea of the RL is to get a Sketch of all the poles as a function of a parameter without factorization of polynomials.

This is possible following certain rules.

*Rules for Sketching root Loci :-

Considering $C.E = s^2+2s+ks+4k=0$

* ~~Represent the C.E~~

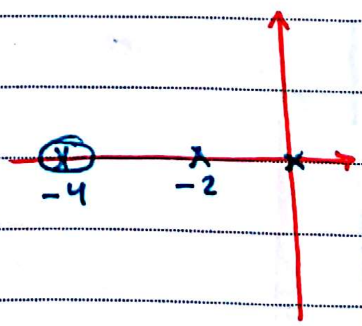
- Represent the C.E in the form $1+\frac{K\,Z(s)}{P(s)}=0$

where K is assumed +ve.

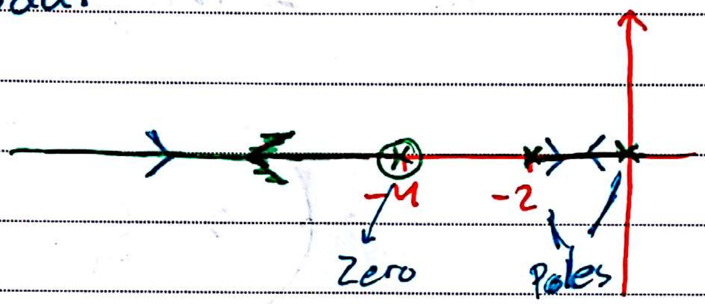
- Locate the poles ~~of~~ ~~P(s)~~ and zeros on the S-plane.

-

$$1+\frac{K(s+4)}{s(s+2)}$$

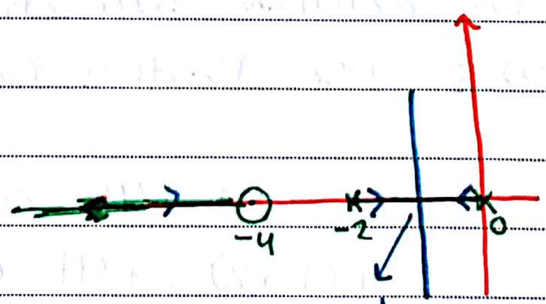


- A point lies on RL if the sum of ~~poles and zeros to the right is odd~~ the number of poles & zeros to the right is odd.



→ into the zeros : $K \uparrow$

→ out of the poles



- Breakaway Point (Break-in Point)

Calculated as the zeros of $\frac{dk}{ds} = 0$

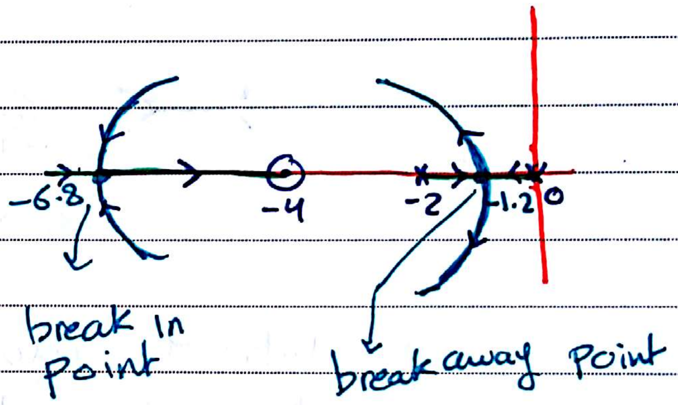
$$K = \frac{S^2+2S}{S+4}$$

$$-\frac{dk}{ds} = -\left[\frac{(2S+2)(S+4) - (S^2+2S)}{(S+4)^2}\right] = 0$$

No.

$$s^2+8s+8=0$$

$$s=\frac{-8\pm\sqrt{32}}{2}=-4\pm 2\sqrt{2}$$
$$\approx -1.2\,,\ -6.8$$



- To determine Intersection with the Imaginary axis use Routh's Criterion.

$$s^2+2s+ks+4k=0$$
$$s^2+(k+2)s+4k=0$$

s^2	1	4k
s^1	k+2	0
s^0	4k	

Determine a k which results in a complete row of zeros

In our case $k=0$
NO Intersection with the Imaginary axis.

FIVE APPLE

No.

Asymptotes

$$\boxed{\text{angle of asymptotes} = \frac{(1 \pm 2h) * 180}{\text{No. of Poles} - \text{No. of Zeros}}}$$

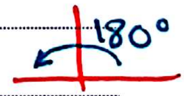
$h \rightarrow$ integer, So we are taking odd multiples of 180

$h = 0, 1, 2 \ldots n_p - n_z$

In our example

$$\frac{180}{2-1} = \underset{h=0}{\underset{\downarrow}{180}},\ \underset{h=1}{\underset{\downarrow}{540}}$$

$540^\circ = 180^\circ$ asymptote on locus



we measure angles CCW

asymptote are on real axis

Point of intersection with real axis

$$\boxed{\sigma = \frac{\sum \text{poles} - \sum \text{zeros}}{n_p - n_z}}$$

In our example

$$\frac{(0 + (-2)) - (-4)}{2-1} = 2$$

The Angle & Magnitude conditions

The C.E $= 1 + G(s)H(s) = 0$

$$G(s)H(s) = -1 = -1 + j0$$

i.e : $|G(s)H(s)| = 1$ ← Known as the magnitude condition

$\angle G(s)H(s) = (1 \pm 2h)180°$ ← Known as the ~~magnitude~~ Angle condition

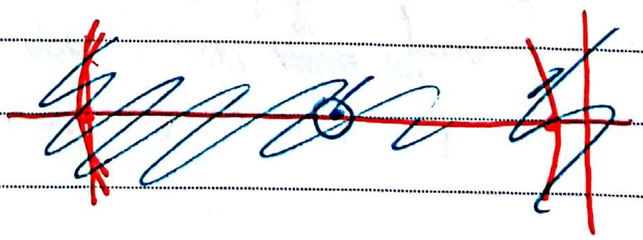
---

- The rules to draw the RL are based on the angle condition.

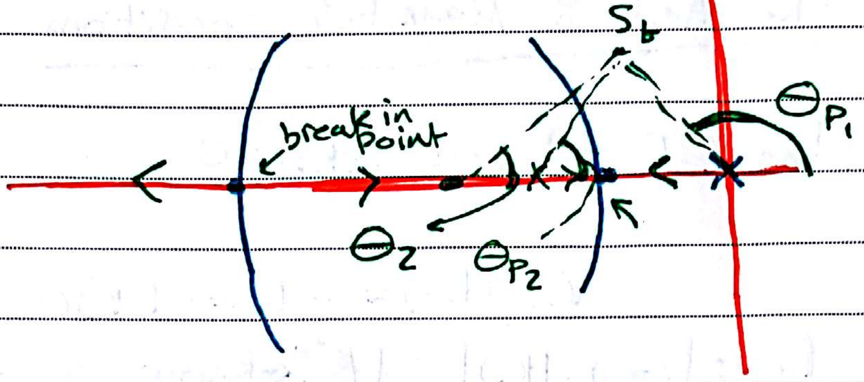
- The ~~angle~~ magnitude condition is used to calibrate the RL.

- The angles are measured from the poles and zeros to the point under consideration. ~~(Suspession)~~ suspesion

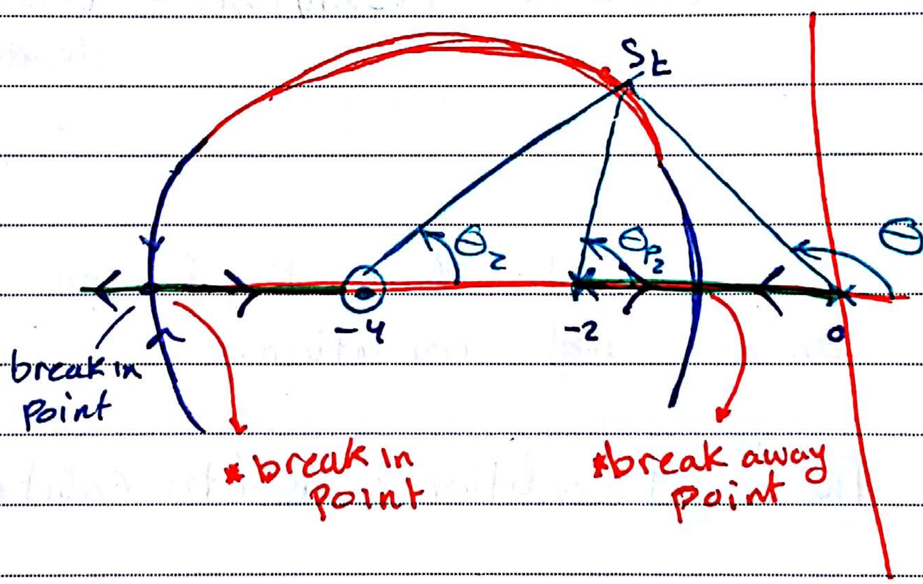
$$\left(\text{i.e is } \left(\sum \text{angle due to zeros}\right) - \left(\sum \text{angles due to poles}\right) \overset{?}{=} (1 \pm 2h) \cdot 180° \right)$$



No.



↓ Same figure



- RL is Symmetrical on the Real axis

matlab

>> n= [1 4] ; d=[1 2 0] ; rlocus(n,d)

* numerator   break away point   ~~zeros~~ zero   poles   * denominator

```
>>n=[1 4]; d=[1 -2 0]; rlocus(n,d)
```

No.

```
>>K=1.8 ; dc= feedback(K*n,d,1,1)
; Sysc= tf(n,dc) ,Step(Sysc)
```

$$T.F = \frac{S+4}{1.8S+7.2}$$

```
>> K=1.8 ; Sys= tf(n,d), sysc=feedback
Sysc=feedback(K*Sys,1),Step(Sysc)
```

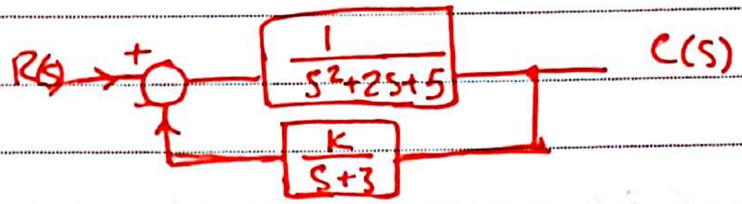
$$T.F = \frac{S+4}{S^2+2S}$$

$$T.F = \frac{1.8S+7.2}{S^2+3.8S+7.2}$$

FIVE APPLE

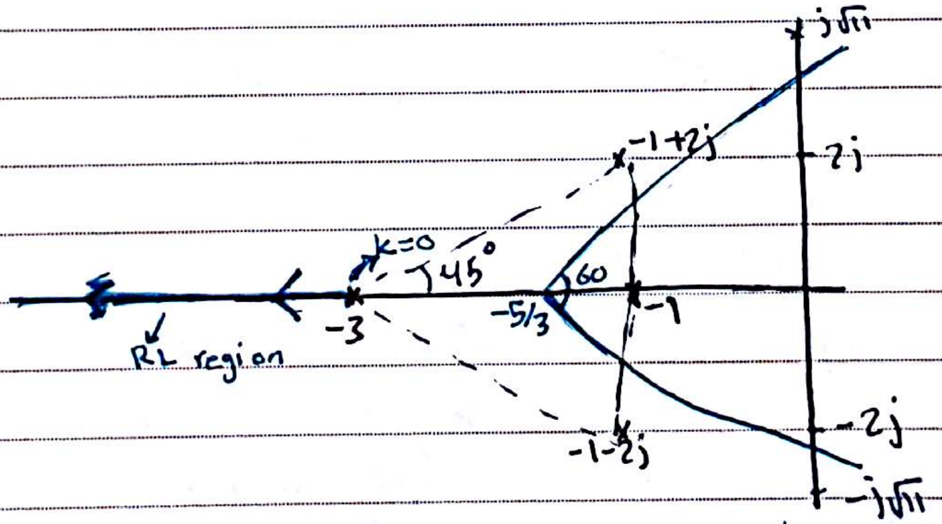
No.

Example: consider the following system



$$C.E = 1 + G(s) \cdot H(s) = 0$$

$$1 + \frac{K}{(s+3)(s^2+2s+5)} = 0$$



~~@ K=0 closed loop poles open loop poles~~

@ K=0 the C.L poles are the O.L poles.

$$\text{angle of asymptote} = \frac{(1 \pm 2h) * 180}{\text{No. of Poles} - \text{No. of zeros}}$$

$\rightarrow h = 0, 1, 2 \cdots N$ integers

$$= \frac{180}{3-0} = 60°, 180°, 300° \ldots\ldots$$

No.

@ $\sigma = \dfrac{(-3-1+2j-1-2j) - (0)}{3-0}$

$= -5/3$

@ $k=\infty$ the RL approaches the asymptotes

* Intersection with jw axis
Use Routh's on the C.E

$(s+3)(s^2+2s+5)+k=0$

$s^3+5s^2+11s+15+k=0$

s^3	1	11
s^2	5	15+K
s^1	$8-K/5$	0
s^0	15+K	

→ complete Row of zeros (for Intersection with Img axis)

* for RL K>0 (always)

$40-k=0 \Rightarrow k=40$

* RL intersects jw axis @ $k=40$
at $5s^2+55=0$
$s=\pm j\sqrt{11}$

A(S)

FIVE APPLE

No.

*Departure angle

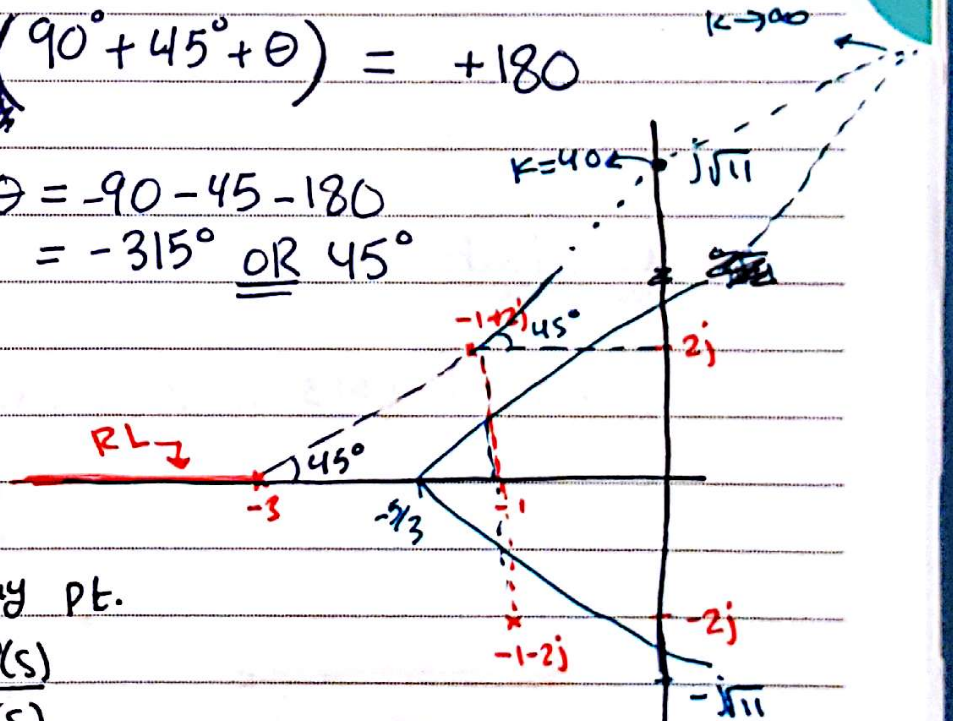
$$\begin{pmatrix}\text{angles from}\\ \text{zeros to a complex}\\ \text{pole under}\\ \text{consideration}\end{pmatrix} - \begin{pmatrix}\text{angles from the remaining}\\ \text{poles to the pole under}\\ \text{consideration} + \theta\end{pmatrix}$$

$$= (1 \pm 2h) \cdot 180^\circ$$

$$0 - (90^\circ + 45^\circ + \theta) = +180$$

$$\theta = -90 - 45 - 180$$

$$= -315^\circ \ \underline{\text{OR}}\ 45^\circ$$



*break away pt.

$$K = \frac{-P(s)}{Z(s)}$$

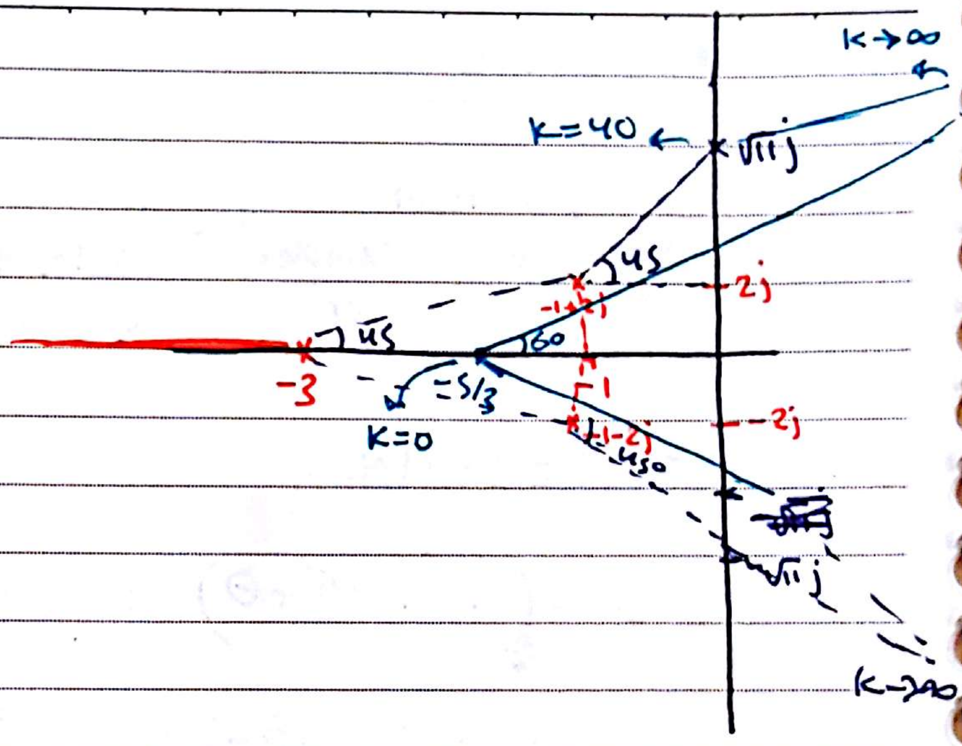
$$-K = \underline{s^3 + 5s^2 + 11s + 15}$$

$$\frac{d\,{-K}}{ds} = 3s^2 + 10s + 11 = 0$$

$$s = \frac{-10 \pm \sqrt{-32}}{6} = -\frac{10}{6} \pm j\sqrt{32}$$

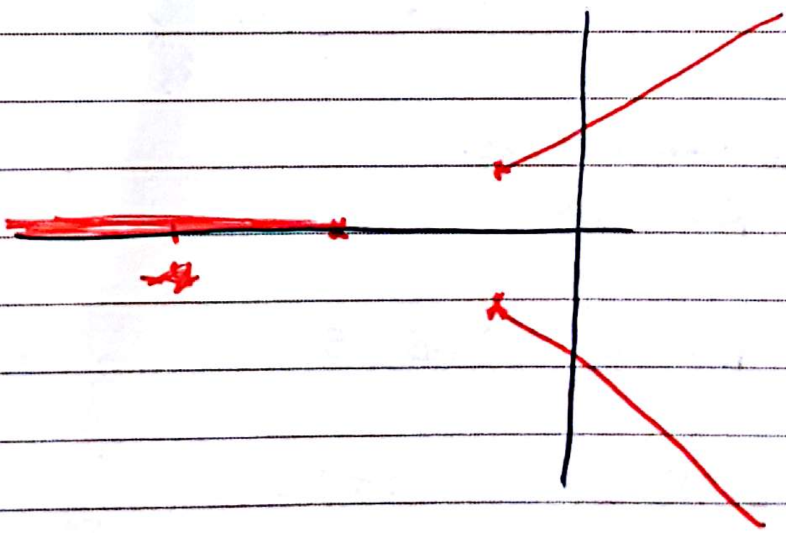
⇒ complex ⇒ no breakaway point

No.



* matlab

```
>> n=1 ; d=[1 5 11 15]; rlocus (n,d)
```



*Calibration of the RL

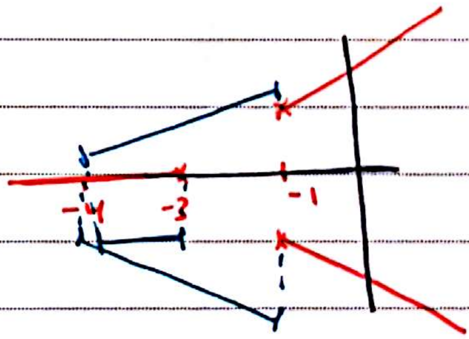
The magnitude condition is used.

$$\left| K \frac{Z(s)}{P(s)} \right|_{s=-4} = 1$$

$$K \frac{1}{1 \times \sqrt{13} \times \sqrt{13}} = 1 \Rightarrow K = 13$$

no zero $\Rightarrow 1$

distances from poles



find K @ $s = j\sqrt{11}$

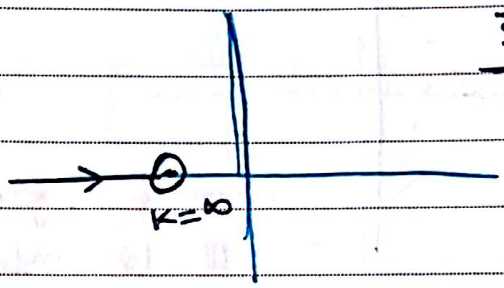
$$K \frac{1}{\left(\sqrt{1+(\sqrt{11}-2)^2}\right)\left(\sqrt{1+(\sqrt{11}+2)^2}\right)\left(\sqrt{20}\right)} = 1$$

$\Rightarrow K \approx 40$

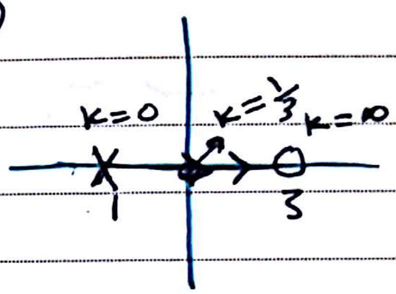
No.

*Qualatitive Sketching of Root loci

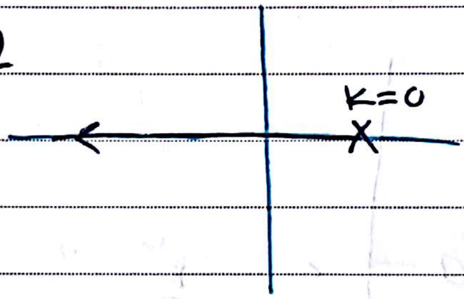
1)



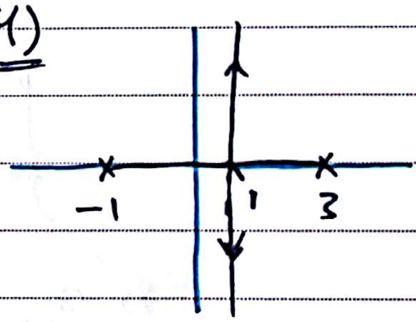
3)



2)

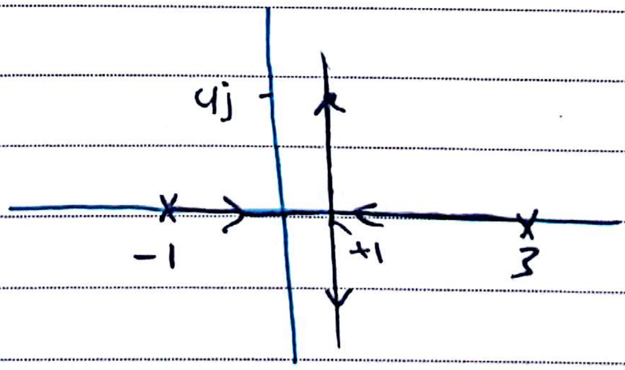


4)



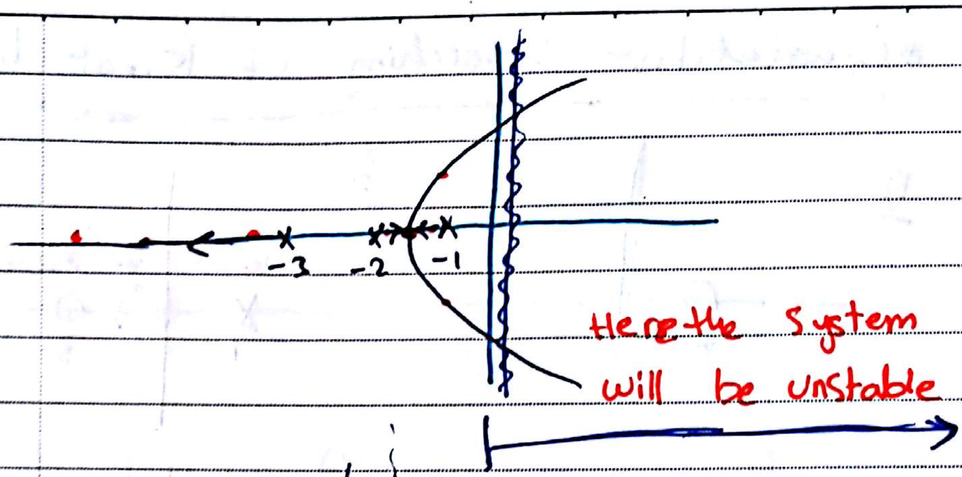
System unstable ~~with +ve real~~ if it's poles ~~have are~~ is or ~~is~~ are +ve real.

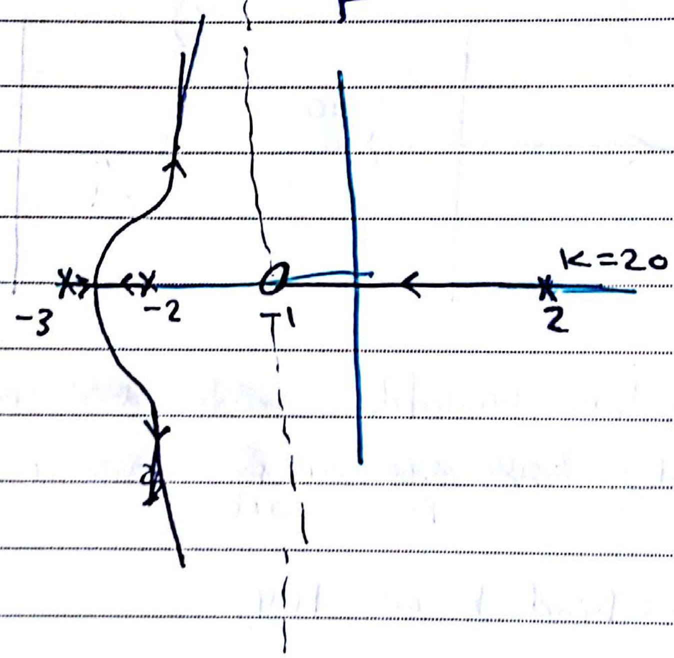
Ex: Find k @ 1+4j



@ 4j → $k\left(\frac{1}{\sqrt{20}\,\sqrt{20}}\right)=1 \Rightarrow k=20$

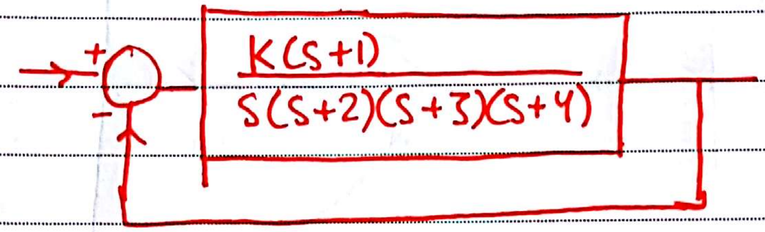
No.





* The RL indesign

Given the System Shown



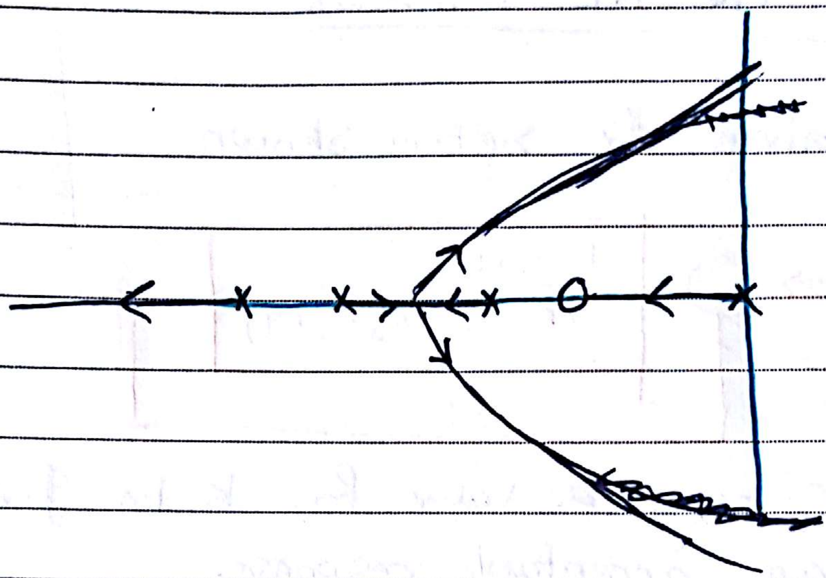
Choose a value for K to given an acceptable response.

$$s(s+2)(s+3)(s+4) = s^4 + 9s^3 + 26s^2 + 24s$$

matlab m-file

```
n=[1 1]; d=[1 9 26 24 0];
Sys=tf(n,d);
figure(1)
rlocus(n,d)
K=rlocfind
sysf=feedback(K*Sys,1)
figure(2)
step(sysf)
```

No.

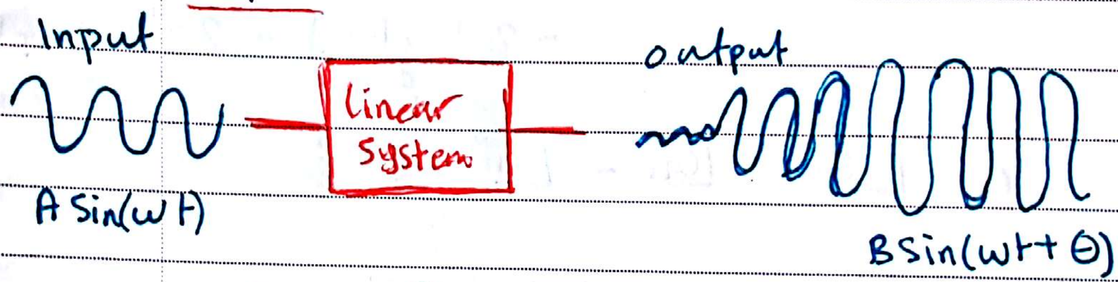


FIVE APPLE

~~Frequency Response~~

The Frequency Response (FR)

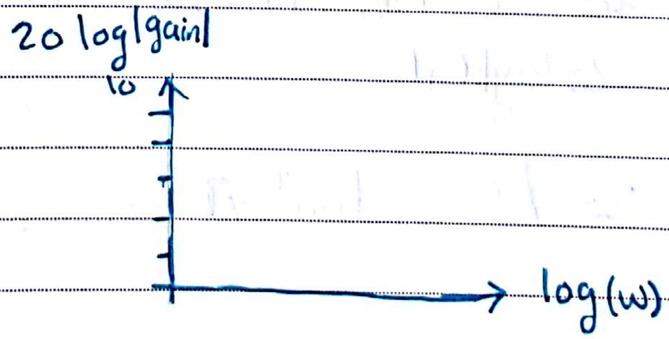
FR is the steady state output response of a linear System due to a Sinusoidal excitation (input).

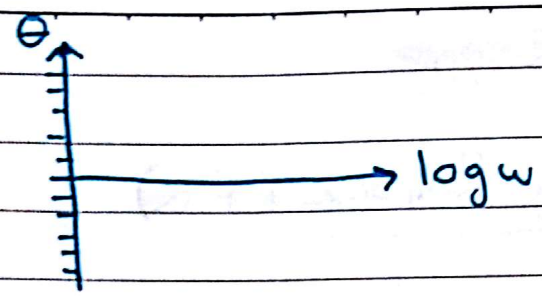


$$\text{Gain} = \frac{B}{A}, \quad \text{Phase} = \theta$$

The Bode diagram

A convenient plotting method for frequency response data.





$$20\log_{10}\frac{-10\,G_1}{G_2 G_3} = 20\log_{10}|-10| + 20\log_{10}|G_1|$$
$$-20\log_{10}|G_2| - 20\log_{10}|G_3|$$

$$\theta = \angle -10^\circ + \angle G_1^\circ - \angle G_2^\circ - \angle G_3^\circ$$

$$= -180^\circ + \angle G_1^\circ - \angle G_2^\circ - \angle G_3^\circ$$

* $20\log|KG| = 20\log|K| + 20\log|G|$

i.e the Same plot of $20\log|G|$ shifted up or down by $20\log|K|$

* $20\log|Ge^{j\omega T}|$ has the same magnitude plot as $20\log|G|$

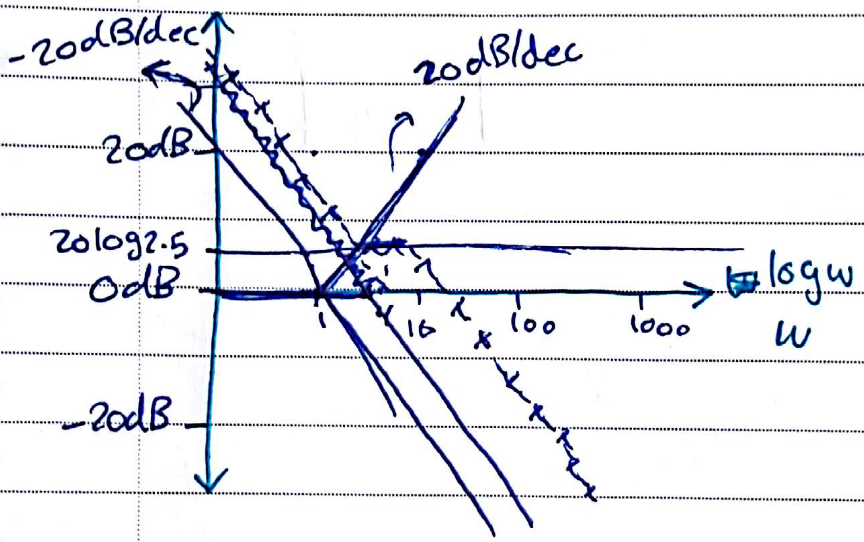
i.e $\angle Ge^{j\omega T} = \angle G + \tan^{-1}\omega T$

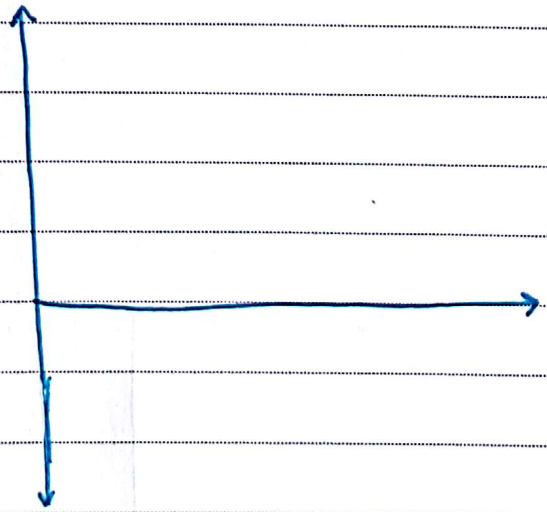
No.

Example: $G(s) = \dfrac{10(1+s)}{s(s+4)} \Rightarrow G(s) = \dfrac{10(1+jw)}{jw(jw+4)}$

$$G(jw) = \frac{10\left(1+j\frac{w}{1}\right)}{j\frac{w}{1} \times 4\left(j\frac{w}{4}+1\right)}$$

$$= \frac{2.5\left(1+j\frac{w}{1}\right)}{jw\left(1+j\frac{w}{4}\right)}$$





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26/4

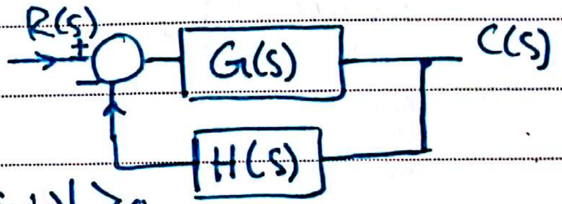
No.

*Stability using the Bode diagram (BD)

For the system shown

I) Draw the BD

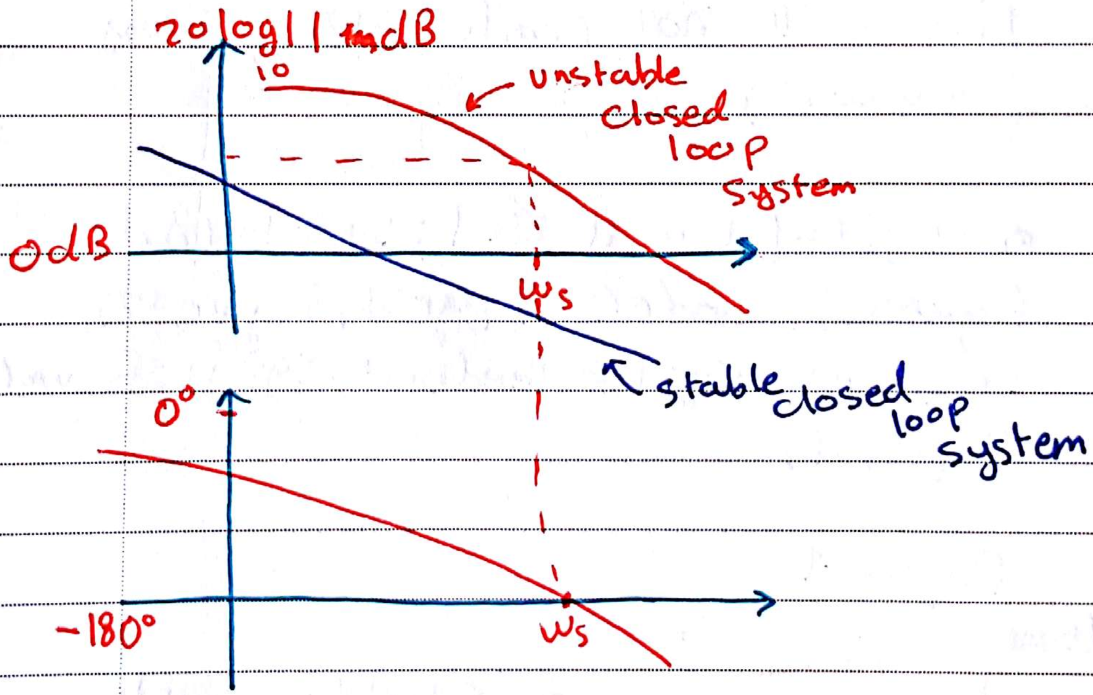
~~II) If the 20log~~



II) If the $20\log_{10}|G(j\omega_s)H(j\omega_s)| > 0$ dB

where

$\angle G(j\omega_s)H(j\omega_s) = -180^\circ$ then the closed loop system is unstable.



No.

matlab

```
n=1;d=[1 6 11 70];sys=tf(n,d);
figure(1), Bode(sys),grid
```

بيعطيني رسمتين

Bode Diagram

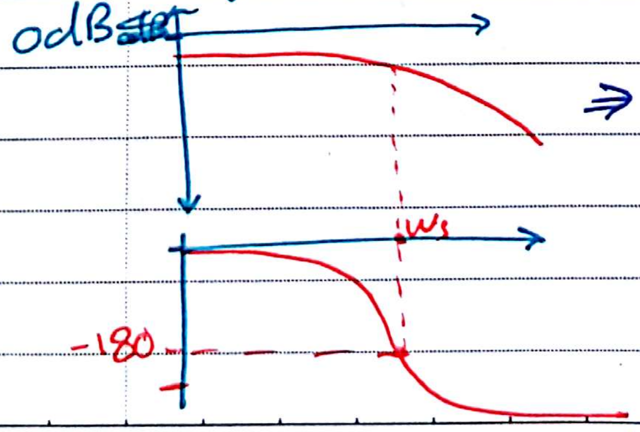
magnitude

phase

the system shown is always stable (i.e will not reach -180, always below it)

```
n=1;d=[1 6 11 60];sys=tf(n,d);
figure(2), Bode(sys),grid,figure(2),
step(sys); sysf = feedback(sys,1),figure(3)
step(sysf)
```

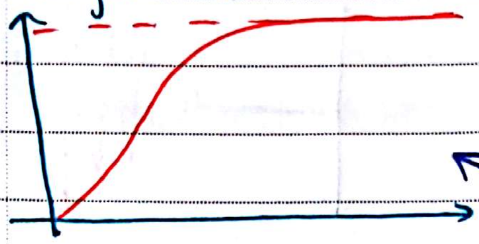
Figure 1



⇒ Stable System

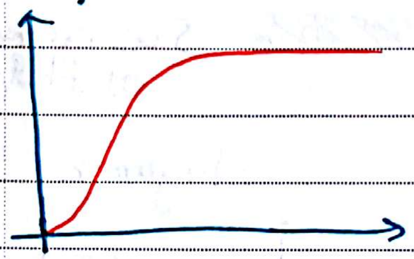
No.

Figure (2)



Shows that the system is stable

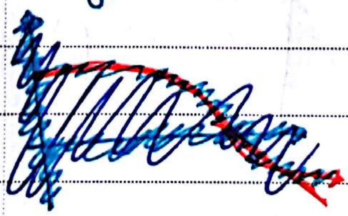
Figure(3)



---

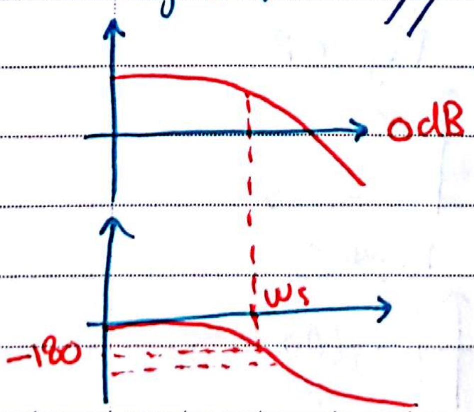
* Same as the previous program but for n=70

~~Figure 4~~



Unstable system

Figure 1



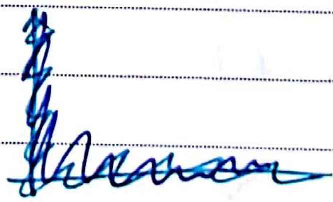


Figure 2

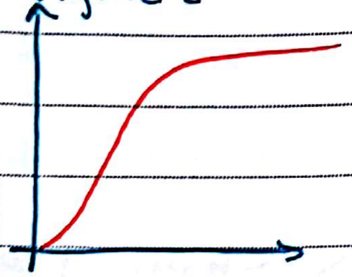
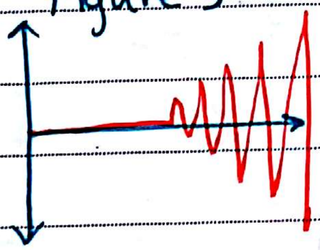


Figure 3



For $n=60$ $\Rightarrow$ Sustained oscilation

Figure 1

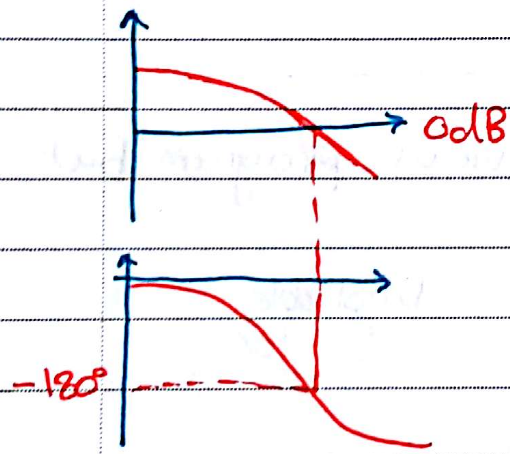


Figure 2

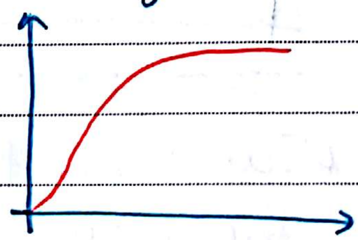
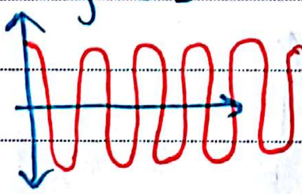


Figure 3



For $n=40$

Figure 1

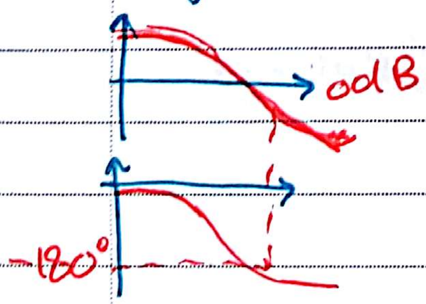
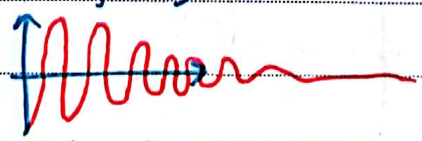


Figure 2

Figure B



No.

~~the change in gain k will differ~~
~~For the stability here, we are concerned~~
~~with G(s) H(s).~~

$$C.E = S^3 + 6S^2 + 11S + 6 + K$$

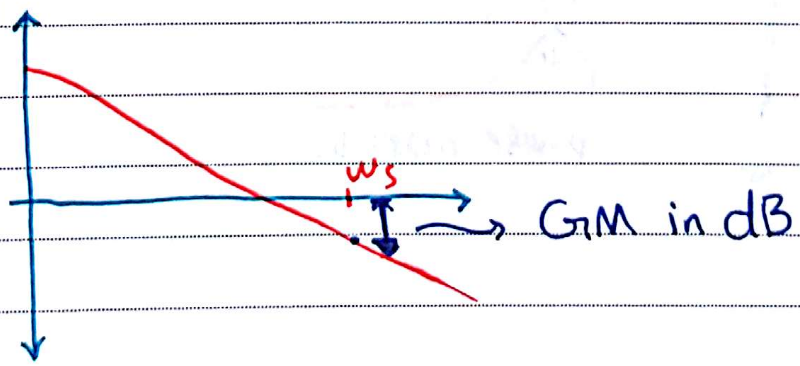
S^3	1	11
S^2	6	$6+K$
S^1	$\frac{60-K}{6}$	0
S^0	$K+6$	

* K is n

for $K < -6$
the system is unstable
& for $K > 60$
the system is also
unstable.

* Gain Margin

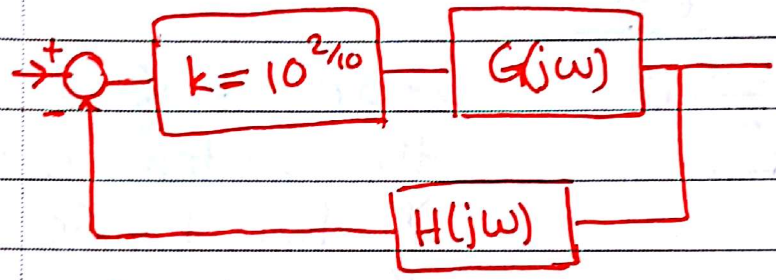
The amount of gain introduced into the system before the onset of instability.

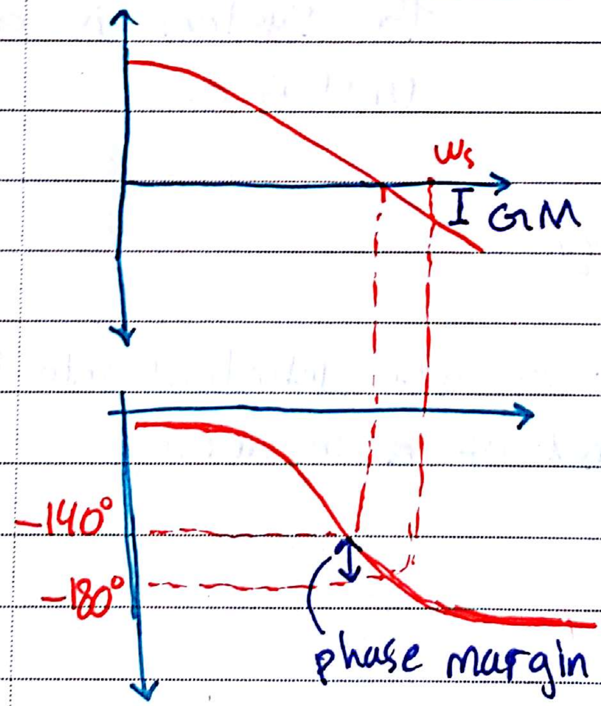


e.g If GM=4dB then linear gain is given by $4 = 20\log_{10} K$

$$K = 10^{4/20}$$

$$K = 10^{0.2}$$





No.

*Phase Margin

The amount of phase Introduced Into the System before the onset of Instability.
(بداية)

matlab
~~#~~ Margin (sys)

the previous program add $\boxed{\text{margin(sys)}}$ for n= 10

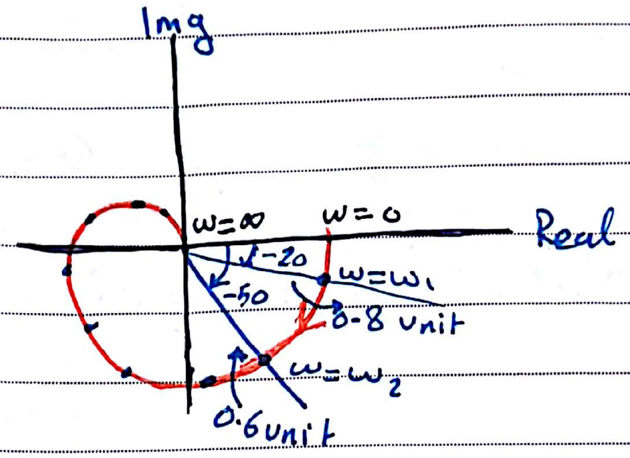
this will show the Gain margin & phase margin.

No.

~~* The Nyquist~~

* The Nyquist Diagram (ND)

ND is a polar plot of magnitude & phase as a function of a certain parameter(ω).



* the angle & magnitude will be given from a table or measured.

* Every 90° is an order.

→ this system is a third order System.

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No.

How to Draw ND

» Given $G(s)\,H(s) = \dfrac{10(s+2)}{s(s+8)}$

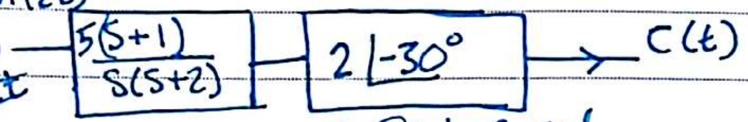
w	$10(\overset{jw}{s}+2)$	$\frac{1}{s}$	$\frac{1}{s+8}$	$G(s)\,H(s)$
1	$10 * \sqrt{5} \angle 26^\circ$	$1 \angle -90$	$\frac{1}{\sqrt{65}} \angle -7^\circ$	$10\sqrt{5} * 1 * \frac{1}{\sqrt{65}} \angle {-90-7 \atop +26}$
				$2.77 \angle -71^\circ$
2				
4				
⋮				

& we finish the table.

No.

Ex: A System is given by

$u(t) = 8\sin(2t)$

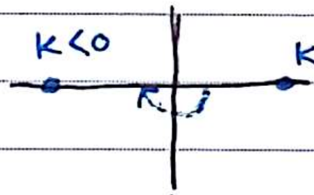


@ $\omega = 2$ rad

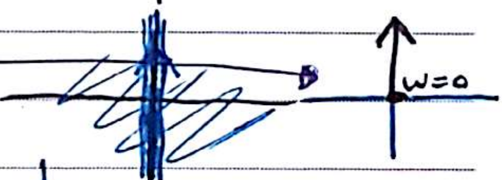
Determine $C(j\omega)$, also determine $c(t)$ & Find the gain??

*ND of certain T.Fs

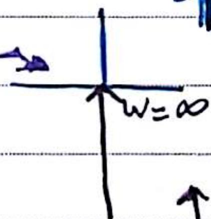
$G(j\omega)\,H(j\omega) = K$ →



" " $= j\omega$ →

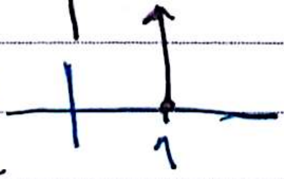


" " $= \dfrac{1}{j\omega}$ →



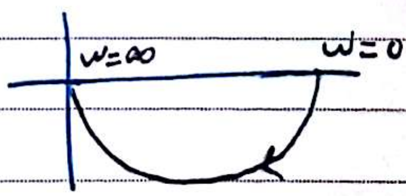
" " $= 1 + j\omega T$ →

time constant



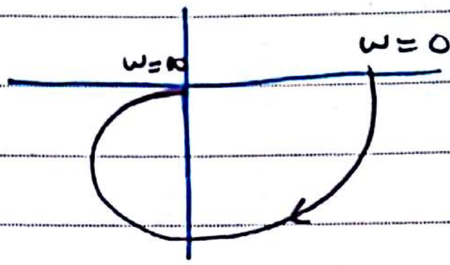
" " $= \dfrac{1}{1+j\omega T}$ →

first order lag

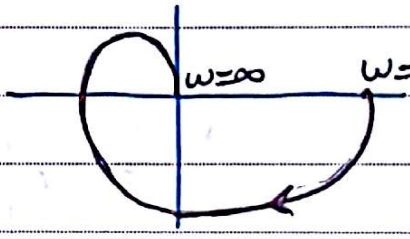


No.

$$G(s)H(s) = \frac{\omega_n^2}{s^2 + \zeta\omega_n s + \omega_n^2}$$



$G(s)H(s) =$ A third order System



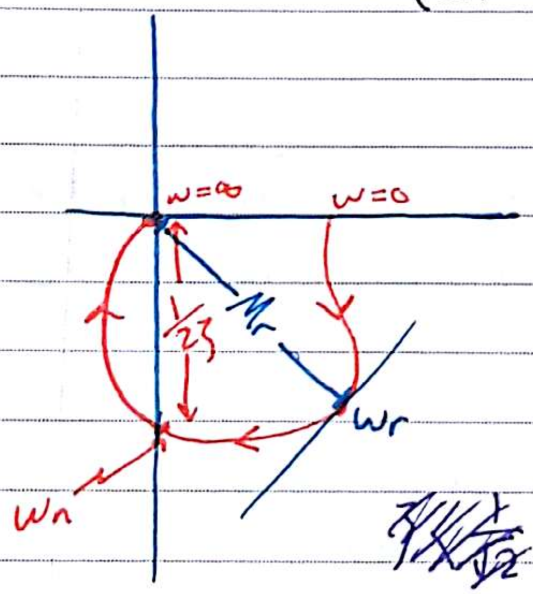
*ND of a particular Second Order System

Consider:-

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

so

$$G(j\omega) = \frac{\omega_n^2}{(\omega_n^2 - \omega^2) + j2\zeta\omega_n\omega}$$



* أكبر فتحة للفرجار

⇐ مماس

* Cm ⇒ Units

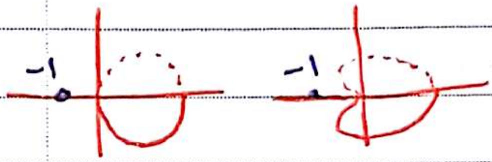
* $\zeta < \frac{1}{\sqrt{2}}$

It can be shown that $\boxed{\omega_r = \omega_n\sqrt{1-2\zeta^2}}$

& $\boxed{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$, $\zeta < \frac{1}{\sqrt{2}}$

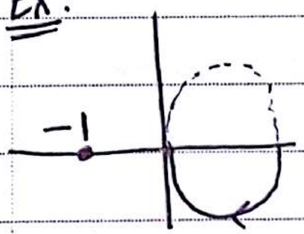
* Stability Using ND :

Having the ~~Nyquist~~ ND of $G(s)\,H(s)$ (OL)
the Stability of the closed loop System (CL)
can be determined as follows:-



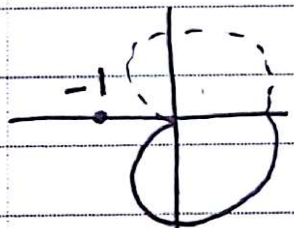
* If the ND for $-\infty < \omega < \infty$ encircles the -1 point then the system is Unstable.

Ex:



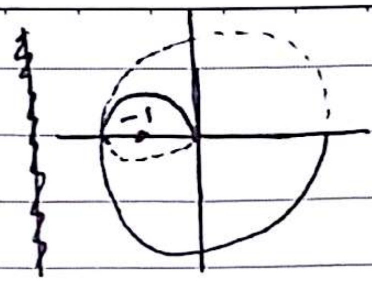
~~closed loop~~ ⇒ ~~stable~~

⇒ closed loop is Stable

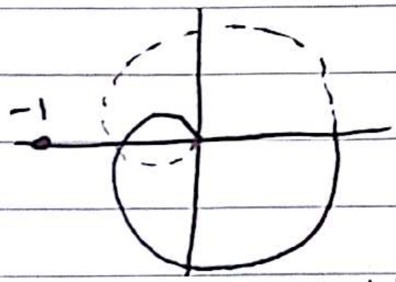


⇒ closed loop is Stable

No.



⇒ closed loop is unstable



⇒ CL is Stable

matlab

```
>> n=30 ; d= [1 6 11 6] ; sys= tf(n,d);
Figure (1), Nyquist (sys), figure (2),
sysf = feedback(sys,1) ; Step (sysf)
```

Figure (1)

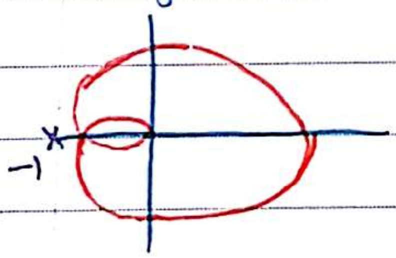
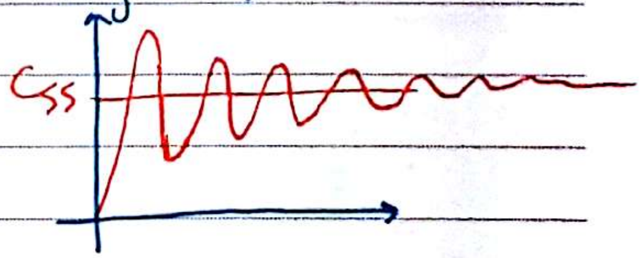


Figure (2)



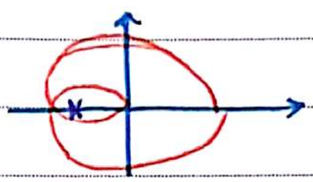
>> n=10

Figure(1) ⇒ nyquist is far from −1 ⇒ Stable

& less oscillations in Figure (2)

>> n=70

Unstable



Figure(1)

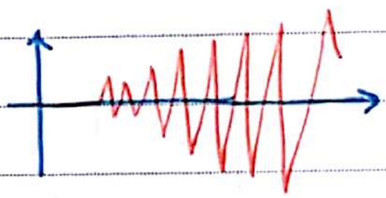


Figure (2)

>> n=60

Sustainably Stable

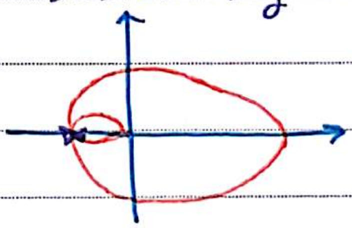
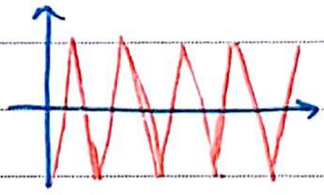


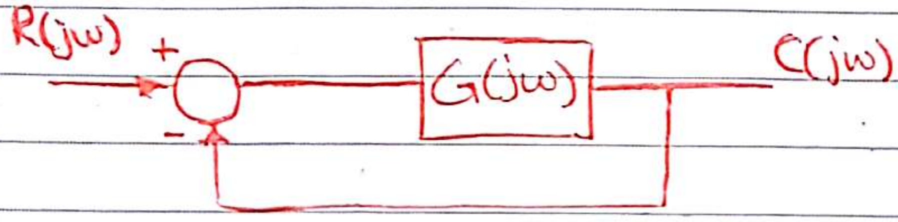
Figure (1)



Figure(2)

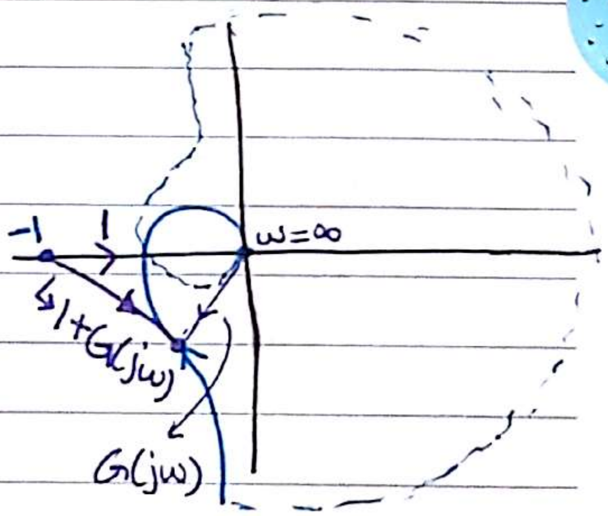
* Obtaining the Closed loop Response Graphically.

Consider the following Particular System.

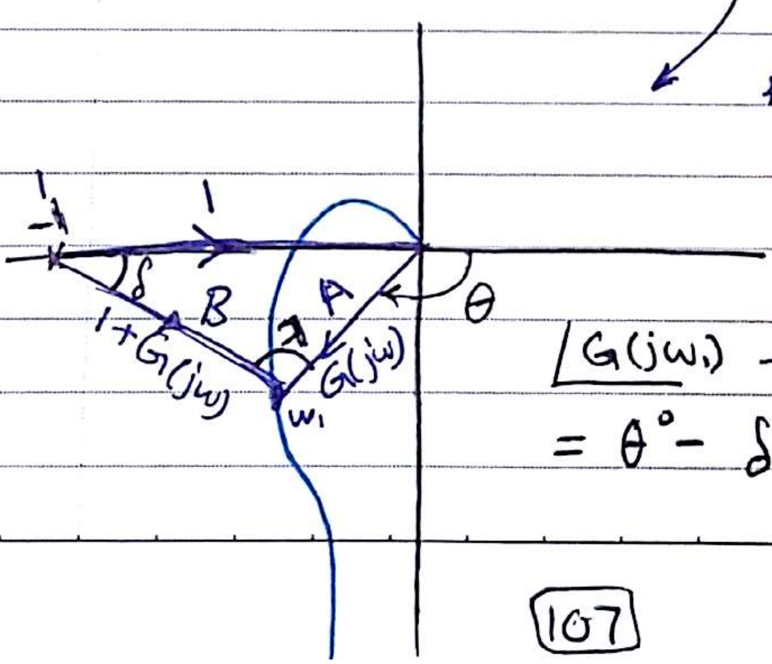


$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)}$$

CL gain response



$$\left|\frac{C(j\omega)}{R(j\omega)}\right| = \left|\frac{G(j\omega)}{1+G(j\omega)}\right| = \frac{A}{B}$$



* This is for Stable Systems.

$$\angle G(j\omega_1) - \angle 1+G(j\omega_1) = \theta^\circ - \delta^\circ = \lambda^\circ$$

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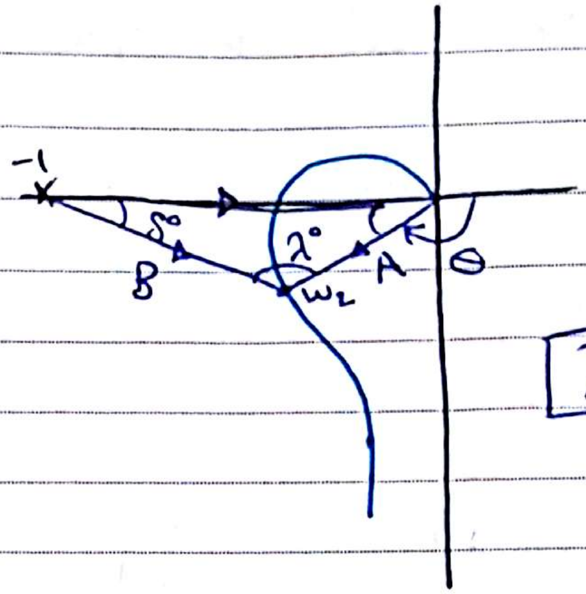


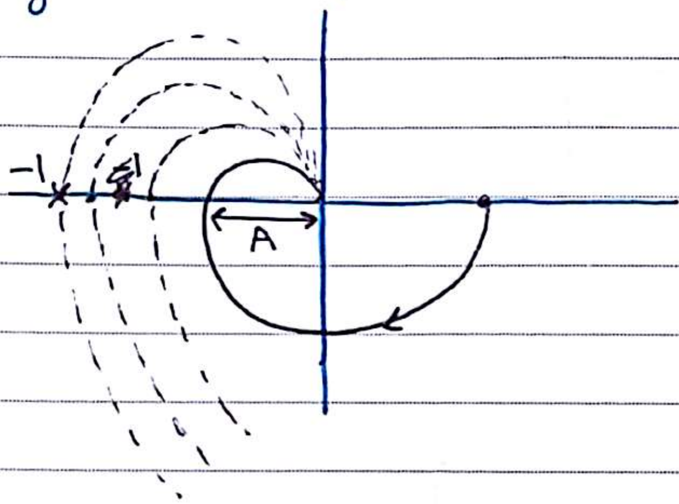
$$\boxed{\frac{A}{B}\angle\lambda}$$

$$\boxed{\lambda = \theta - \delta}$$

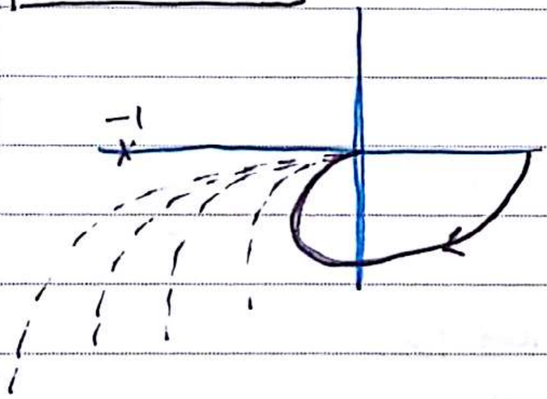
<u>N.B</u>: This graphical method applies for systems which are CL stable and Unity feedback.

* Gain & phase margins

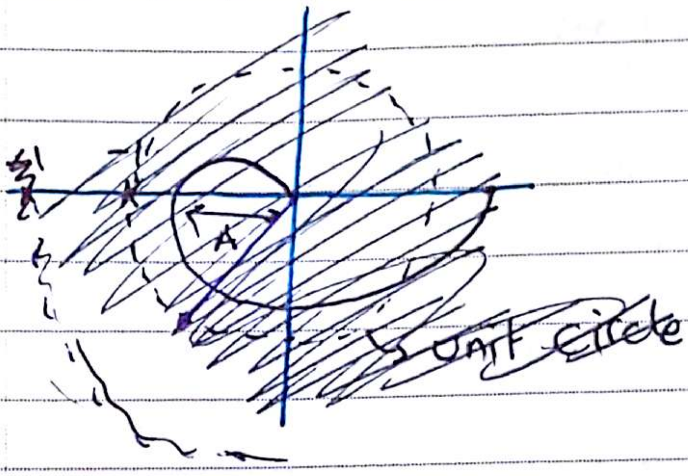
They are meaningful only when the CL system is stable.

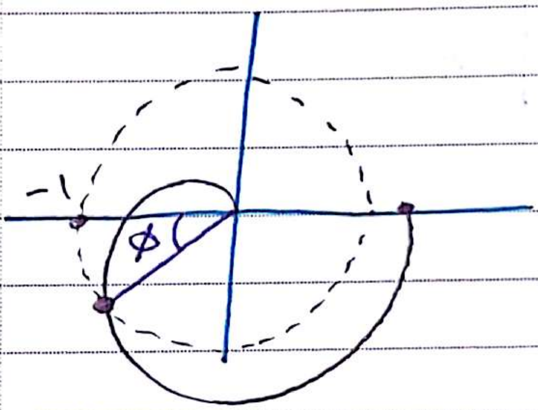


$$\boxed{GM = \frac{1}{A}}$$



* for 2nd order will not ~~include~~ include -1.
* will always be a stable system.





$\boxed{PM = \phi}$

* linear gain is ~~20log 10~~

gain in dB $\rightarrow 6dB = 20\log_{10}|G|$

$\rightarrow |G| = 2$ ← linear gain

No.

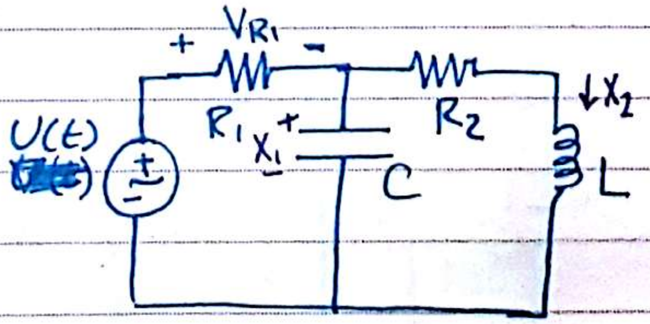
* State Space Representation

Why the State Space Representation is needed ~~for~~ ??

i) for multi-Input multi-Output Systems.
ii) for time-Varying Systems.
iii) for Systems with time-delay.
iv) for non-linear systems.
v) for Optimally-controlled Systems.
vi) for Systems with non-zero Initial conditions.

Modeling of Systems Using the States

Best illustrated by an example :-



let $V_c = X_1$
let $I_L = X_2$ → States

let $V_{R_1} = y$ → output

$$C\dot{x}_1 = \frac{U - X_1}{\cancel{R_2}R_1} - X_2$$

$$\dot{X}_1 = -\frac{1}{R_1C}X_1 - \frac{1}{C}X_2 + \frac{1}{R_1C}u \text{ -------- } (1)$$

$$X_1 = R_2X_2 + L\dot{X}_2$$

$$\dot{X}_2 = \frac{1}{L}X_1 - \frac{R_2}{L}X_2 \text{ - - - -- } (2)$$

$$y = U - X_1 \text{ - - - } (3)$$

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} \dot{X}_1 \\ \\ \dot{X}_2 \end{bmatrix}$$

In matrix form ←

$$\dot{X} = \begin{bmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1C} \\ 0 \end{bmatrix} U$$

$$y = \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 1 \cdot u$$

i.e

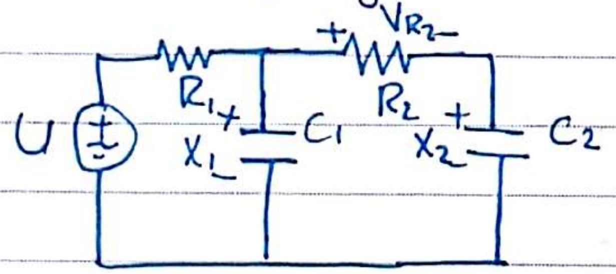
state equations $\rightarrow$ $\dot{x} = Ax + Bu$ ~~x belongs~~

$y = Cx + Du$, $x \in R^n$, $u \in R^m$ ← output equations

$y \in R^p$

$p \leq n$

*Exercice: obtain A, B, C, D for the following circuit.



choosing $x_1 = V_{c_1}$, $x_2 = V_{c_2}$, $y = V_{R2}$

*Exercize: Remodel the circuit when $R_2 = 0\,\Omega$, choose $y = V_{R_1}$

* Stability Determination:

Stability is determined by the eigenvalues of A.

If atleast one eigenvalue has positive real part, then the system is unstable.

Example: given $\dot{x} = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} X + \begin{bmatrix} 1 \\ 5 \end{bmatrix} u$

$$y = [6 \quad 7] X + 8u$$

for stability $\lambda[A]$ is given by $|A - \lambda I| = 0$

$$\begin{vmatrix} -1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = -4 + \lambda - 4\lambda + \lambda^2 \cancel{=0} - 6 = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$\lambda_1 = -2, \quad \lambda_2 = 5$$

this Value Shows that the System is Unstable.

No.

Exercize: Determine the Stability of

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} X$$

* Time Response

It can be shown that

$$\boxed{X(t) = e^{At} X(t_0) + \int_{t_0}^{t} e^{A(t-\tau)} \cdot B\, u(\tau)\, d\tau}$$

Where e^{At} is known as the exponential Matrix. ~~evaluated as~~ evaluated as

$$e^{At} = \mathcal{L}^{-1}[sI_n - A]^{-1}$$ → to get a closed form Solution

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{At} = I_n + At + \frac{A^2 t^2}{2!} + \cdots$$ Convenient Numerically

No.

*** Properties of e^{At}**

1. $e^{At}\Big|_{t=0} = I_n$

2. $e^{A(t_1+t_2)} = e^{At_1} e^{At_2}$

3. $e^{(A+B)t} = e^{At} e^{Bt}$, if $AB = BA$

~~4.~~ $\cancel{[e^{At}]^{-1} = e^{A(-t)}}$

4. $(e^{At})^{-1} = e^{A(-t)}$

Example: let $\dot{X} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

$X(0) = 0$

* $u(t)$ is a unit step.

$$e^{At} = \mathcal{L}^{-1} \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}^{-1}$$

$$= \mathcal{L}^{-1} \frac{1}{s(s+2)} \begin{bmatrix} s+2 & 1 \\ 0 & s \end{bmatrix}$$

$$\cancel{= \mathcal{L}^{-1} [\, 1/s \quad 1/s(s+}$$

No.

$$= \mathcal{L}^{-1}\left[\begin{bmatrix} 1/s & 1/(s(s+2)) \\ 0 & 1/(s+2) \end{bmatrix}\right]$$

$$= \begin{bmatrix} 1 & \frac{1}{2}-\frac{1}{2}e^{-2t} \\ 0 & e^{-2t} \end{bmatrix}$$

* Check by $e^{At}\big|_{t=0} = I$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$X(t) = e^{At}\,\overset{0}{X(0)} + \int_0^t e^{A(t-\tau)} B\,u(\tau)\,d\tau$$

$$= 0 + e^{At}\int_0^t e^{-A\tau} B * 1\,d\tau$$

$$= \begin{bmatrix} 1 & \frac{1}{2}-\frac{1}{2}e^{-2t} \\ 0 & e^{-2t} \end{bmatrix} * \int_0^t \begin{bmatrix} 1 & \frac{1}{2}-\frac{1}{2}e^{2\tau} \\ 0 & e^{2\tau} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} d\tau$$

$$= \begin{bmatrix} 1 & \frac{1}{2}-\frac{1}{2}e^{-2t} \\ 0 & e^{-2t} \end{bmatrix} \int_0^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} d\tau$$

$$= \begin{bmatrix} \quad & \quad \end{bmatrix}\begin{bmatrix} t \\ 0 \end{bmatrix} = \begin{bmatrix} t \\ 0 \end{bmatrix}$$

No.

i.e $X_1(t) = t$, $X_2(t) = 0$

Exercize: Resolve using $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Exercize: Resolve using $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

*Steady State Value

Given $\dot{X} = AX + BU$, then provided the System is stable and $|A| \neq 0$ then due to a unit step

$$\boxed{X_{ss} = \lim_{t \to \infty} X(t) = -A^{-1}B}$$

The transfer function Matrix $G(s)$

It can be shown that

$Y(s) = G(s)\,U(s)$ where

$G(s) = C[SI - A]^{-1}B + D$

FIVE APPLE

No.

<u>Exercize</u>: given $\dot{x} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$$y = [2 \ \ 1]\, x + 1u$$

Determine $G(s)$??

انتهت المادة

بالتوفيق

#Power_Unit
Control_Spring 017