

Your Name: $\qquad$ Sample Solution $\qquad$
Your Student ID: $\qquad$
Your Instructor Name: $\qquad$
Your Section \#: $\qquad$ ; Your Lecture Times: $\qquad$

## Read The Following Instructions Carefully:

1. Write your name, ID, date and sign the attached Honor Pledge. Your exam will NOT be graded if you do not.
2. This exam booklet has 6 numbered pages and 5 problems. Check that your exam includes all 6 pages. Show ALL of your work on these pages. Two blank pages are added at the end for your scratch work.
3. WRITE your name (in Arabic) and student number in the spaces above. Also, make sure to write your instructor's name, section \# and lecture times.
4. You are NOT permitted to use notes, books, calculators, or mobile phones during this exam.
5. This exam lasts for 75 minutes. Point values are listed for each problem to assist you in best using your time.

| Problem | Max Points | Score |
| :---: | :---: | :---: |
| 1 | 16 |  |
| 2 | 10 |  |
| 3 | 24 |  |
| 4 | 24 |  |
| 5 | 26 |  |
| Total | 100 |  |

## Problem 1. (16 points)

A. (7 points) Perform each of the following conversions. Write your answers in the boxes at the right edge.
(i) $\quad \mathbf{1 0 1 0 1 1 0 0 1}_{\mathbf{2}}=2^{8}+2^{6}+2^{4}+2^{3}+2^{0}=256+64+16+8+1=\mathbf{3 4 5}_{\mathbf{1 0}}$
(ii) $\quad \mathbf{2 1 8}_{\mathbf{1 0}}=128+64+16+8+2=2^{7}+2^{6}+2^{4}+2^{3}+2^{1}=\mathbf{1 1 0 1 1 0 1 0}_{\mathbf{2}}$
(iii) $\quad \mathbf{1 1 0 1 0 1 0 1}_{\mathbf{2}}=011010101_{2}=\mathbf{3 2 5}_{\mathbf{8}}$
(iv) $\quad \mathbf{5 7 4}_{\mathbf{8}}=5 \mathrm{x} 8^{2}+7 \mathrm{x} 8^{1}+4 \times 8^{0}=5 \mathrm{x} 64+7 \mathrm{x} 8+4 \mathrm{x} 1=320+56+4=\mathbf{3 8 0}_{\mathbf{1 0}}$
(v) $\quad \mathbf{6 B 4} \mathbf{1 6}^{6}=01101011 \mathbf{0 1 0 0}_{2}=\mathbf{1 1 0 1 0 1 1 1 0 1 0 0}_{\mathbf{2}}$
(vi) $\quad \mathbf{7 6 2}_{\mathbf{1 0}}=2 \times 256+15 \times 16+10=2 \times 16^{2}+15 \times 16^{1}+10 \times 16^{0}=\mathbf{2 F A}_{16}$
(vii) $\quad \mathbf{C A 7 5}_{16}=1100101001110101_{2}=001100101001110101_{2}=\mathbf{1 4 5 1 6 5}_{8}$
B. (9 points) In the blank in front of each expression in the left hand column, write the letter ( $\mathrm{P}-\mathrm{Z}$ ) corresponding to the equivalent expression in the right hand column. Not all answers in the right hand column will be used and some may be used more than once.

$$
(A+\bar{B})(\bar{A}+C)(\bar{B}+\bar{C})=(A \bar{A}+A C+\bar{B} A+\bar{B} C)(\bar{B}+\bar{C})
$$

(i) $\mathrm{S}:=A C \bar{B}+A C \bar{C}+\bar{B} A \bar{B}+\bar{B} A \bar{C}+\bar{B} C \bar{B}+\bar{B} C \bar{C}$

$$
=A \bar{B} C+A \bar{B}+A \bar{B} \bar{C}+\bar{B} C=A \bar{B}(C+1+\bar{C})+\bar{B} C=A \bar{B}+\bar{B} C
$$

(ii) R: $\overline{A+\bar{C}}+\overline{\bar{B}+(D+\bar{A})}=\bar{A} \cdot \overline{\bar{C}}+\overline{\bar{B}} \cdot(\overline{D+\bar{A}})=\bar{A} C+B(\bar{D} \cdot \overline{\bar{A}})=\bar{A} C+A B \bar{D}$

$$
(\bar{A}+\overline{C+\bar{D}})(B+\overline{C \cdot D})=(\bar{A}+\bar{C} \cdot \overline{\bar{D}})(B+\bar{C}+\bar{D})
$$

(iii) U: $=\bar{A} B+\bar{A} \cdot \bar{C}+\bar{A} \cdot \bar{D}+\bar{C} \cdot D \cdot B+\bar{C} \cdot D \cdot \bar{C}+\bar{C} \cdot D \cdot \bar{D}$

$$
=\bar{A} \cdot B+\bar{A} \cdot \bar{C}+\bar{A} \cdot \bar{D}+\bar{C} \cdot D \cdot(B+1)=\bar{A} \cdot B+\bar{A} \cdot \bar{C}+\bar{A} \cdot \bar{D}+\bar{C} \cdot D
$$

## Problem 2. ( 10 points)

A. (4 points) Given $\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C})=\mathbf{\Sigma m}(\mathbf{1}, \mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{7})$, express the function in $\underline{\text { algebraic }}$ sum-of-minterm and product-of-maxterm form.
$\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})=\frac{\bar{A} \cdot \bar{B} \cdot C+\bar{A} \cdot B \cdot C+A \cdot \bar{B} \cdot \bar{C}+A \cdot \bar{B} \cdot C+A \cdot B \cdot C}{\text { (sum-of-minterms) }}$
$\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})=\quad \frac{(A+B+C)(A+\bar{B}+C)(\bar{A}+\bar{B}+C)}{\text { (product-of-maxterms) }}$
B. (4 points) Given the function $\mathbf{G}(\mathbf{A}, \mathbf{B}, \mathbf{C})$ defined by the Karnaugh map below, complete the truth table and the short-hand SOP and POS expressions for this function.

| C |  |  |
| :---: | :---: | :---: |
| AB | 0 | 1 |
| 00 | 0 | 0 |
| 01 | 1 | 1 |
| 10 | 1 | 0 |
| 11 | 0 | 1 |


| A | B | C | G |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

$$
\begin{aligned}
& \mathrm{G}(\mathrm{~A}, \mathrm{~B}, \mathrm{C})=\sum \mathrm{m}\left(\ldots \quad 2,3,4,7 \_\right) \\
& \mathrm{G}(\mathrm{~A}, \mathrm{~B}, \mathrm{C})=\Pi \mathrm{M}\left(\ldots \quad 0,1,5,6 \_\_\right)
\end{aligned}
$$

C. (2 points) Complete the Karnaugh map for

$$
\mathrm{H}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\Pi \mathrm{M}(0,2,5,11,14)+\mathrm{d}(1,4,7,9,12)
$$



## Grading note:

For this part all students were given +2 points.

Students who had the correct answer were given an additional 2 points

## Problem 3 (24 points)

For the follow expression, derive a simplified sum of products expression using a Karnaugh Map. List the prime implicants, indicating which are essential.

$$
F(A, B, C, D)=\bar{A} \cdot C \cdot D+A \cdot B \cdot D+\bar{B} \cdot C+\bar{A} \cdot C \cdot \bar{D}+A \cdot B \cdot \bar{C} \cdot \bar{D}
$$

Essential?

simplified SOP expression $\quad \underline{\bar{A} \cdot C+\overline{\bar{B}} \cdot C+A \cdot B \cdot \bar{C}+C \cdot D}$

## Grading note:

For each correct prime implicant: +1

For each correct yes/no: +1
For each incorrect prime implicant: -1
Correct simplified SOP expression +2
B. (12 points) For the follow expression, derive a simplified product of sums expression using a Karnaugh Map. List the prime implicants, indicating which are essential.
$F(A, B, C, D)=(\bar{A}+\bar{B}+C+D) \cdot(\bar{A}+\bar{C}+\bar{D}) \cdot(A+C+D) \cdot(A+\bar{C}) \cdot(B+C+D)$

simplified POS expression: $(C+D) \cdot(\bar{C}+\bar{D})(A+\bar{C})$ or $\underline{(C+D) \cdot(\bar{C}+\bar{D}) \cdot(A+D)}$

## Grading note:

For each correct prime implicant: +1.5

For each correct yes/no: +1

For each incorrect prime implicant: -1

Correct simplified POS expression +2

## Problem 4 (24 points)

A. (14 points) Consider the following circuit below. Assume the encoder has the following input priority: $\mathrm{I}_{2}>\mathrm{I}_{0}>\mathrm{I}_{3}>\mathrm{I}_{1}$. Determine Out for the following Input patterns.

B. (10 points) Consider five possible definitions for the block below left. One block input is the symbolic value $\mathbf{A}$. The other input, $\boldsymbol{C}$ is a control value. The output behavior for each of the five definitions is given. Give the logical (gate) name for each definition. Note that $\mathbf{A}^{\prime}$ means not $\mathbf{A}$.

| In | C | (1) | (2) | $(3)$ | (4) | (5) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 0 | A | A | 1 | HiZ |
| A | 1 | A | $\mathrm{~A}^{\prime}$ | 1 | 0 | A |

(1)
$\qquad$ (2) $\qquad$ (3) $\qquad$
(4) $\qquad$
(5) $\qquad$ Tri-State

## Problem 5 (26 points)

Design a combinational circuit that takes a 3-bit binary number $n=\left(n_{2} n_{1} n_{0}\right)$ and computes its "modulo 3", i.e., the remainder from n/3 operation. Assume that complemented inputs are available.
A. (6 points) Build the truth table.

| $n_{2}$ | $n_{1}$ | $n_{0}$ | $O_{1}$ | $O_{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |

B. (10 points) Design the circuit using the minimum number of 4-to-1 multiplexers

C. (10 points) Design the circuit using the minimum number of 2-to-4 decoders with enable and gates.


