Problem 1. Solve the following short problems. **(9 points)** 8

a) (32.25)$_{10}$ is equal to (35.22)$_9$ 1

Note: in problem (a) round your answer to two digits to the right of the radix point.

35.

0.25 × 9 = 2.25 → 2

0.25 × 9 = 2.25 → 2

0.25 × 9 → 2.25 → 2

b) (752.52)$_8$ is equal to (1EA.A8)$_{16}$ 1

0001 1110 1010 . 1010 1000

1 E A . A

c) (1230)$_4$ is equal to (0001 0000 1000)$_{BCD}$ 1

$1\times4^3 + 2\times4^2 + 3\times4^1 + 0\times4^0$

64 + 32 + 12 + 0

(108) =

0001 0000 1000

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d) The number of students attending the Digital Logic course is 215. How many digits are needed to encode the students using a numbering system with radix=6?

$6^n = 215$ $\boxed{3 \text{ digits}}$ 1

6×6×6

6×36

36

6×

216

e) What is the minimum radix of a numbering system that can be used to represent the number of days in a year using two digits only?

$r^2 = 365$

$\boxed{r = 20}$ 1

$\begin{array}{r} 19 \\ 19 \\ \hline 171 \\ 190+ \\ \hline 361 \end{array}$

f) Given $F(A, B, C) = \sum_m(0, 4, 5, 7)$. Determine:

$$F(A,B,C) = \prod_M(1, 2, 3, 6)$$

1

g) Given $F(A, B, C) = \bar{A}B + \overline{AC}$. Determine:

$$\bar{F}(A,B,C) = \sum_m(1, 4, 5, 6, 7)$$

$\bar{f} = \overline{\bar{A}B + \overline{AC}}$

$= (A + \bar{B}) \cdot (A + C)$

$(A + \bar{B} + C\bar{C}) \cdot (A + C + B\bar{B})$

$(A + \bar{B} + C) \cdot (A + \bar{B} + \bar{C}) \cdot (A + B + C)$

010 $\boxed{2}$ $\quad$ 011 $\boxed{3}$ $\quad$ 000 $\boxed{0}$ $\quad$ ~~$(A + \bar{B} + C)$ 010~~

$\bar{F} = \prod_m(0, 2, 3)$

h) What is the even parity bit for the number $(860)_{16}$? $\boxed{1}$

$(1000\ 0110\ 0000)_2$ $\quad$ 1

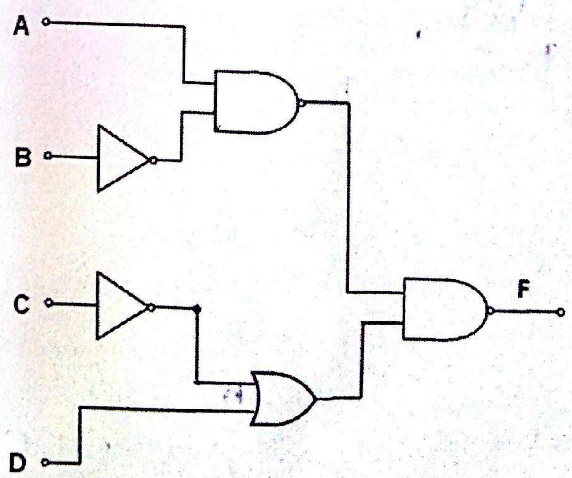
$\boxed{1}\ 1000\ 0110\ 0000$

i) Write the Boolean expression of the function F(A, B, C, D) implemented in the following circuit diagram.

Note: Do not simplify or modify F.

$$F(A,B,C,D) = \overline{(\overline{A\bar{B}} \cdot (\bar{C} + D))}$$

1



$\overline{A\bar{B}}$

$\bar{C} + D$

$\overline{(\overline{A\bar{B}} \cdot (\bar{C} + D))}$

Problem 2. Using Boolean algebra only, prove that:

(3 points) 2.5

$$(AB + \bar{B}C) + B(\bar{C} + \bar{A}) = B + C$$

$AB + \bar{B}C + B\bar{C} + \bar{A}B = B + C$

$AB + \bar{A}B + \bar{B}C + B\bar{C} = B + C$

$B(A + \bar{A}) + \bar{B}C + B\bar{C} = B + C$

$B + \bar{B}C + B\bar{C} = B + C$

$B + B\bar{C} + \bar{B}C =$?

$B(1 + \bar{C}) + \bar{B}C =$

$B + B\bar{C}$

$B + \bar{B}C$

$\bar{B}C + B\bar{C}$

$\bar{B}B + \bar{B}\bar{C} + BB + \bar{C}C$

$B + \bar{B}\bar{C} + BC$

$B(1 + C) \quad B + \bar{B}\bar{C}$

$AB\bar{C} + \bar{A}B\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$

$\bar{B}(AC + \bar{A}C)$

$\bar{B}C(A + \bar{A})$

$\bar{B}C$

$\bar{C}(B(A + \bar{A}))$

$\bar{C}(AB + \bar{A}B)$

Problem 3: Given the K-map of $F(A, B, C, D)$, write the optimized Boolean expression of F as a Product of Sums (PoS).

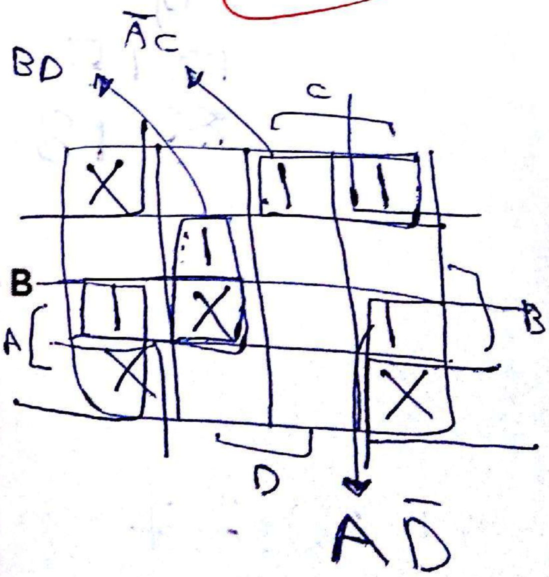
(3 points) 1

$\bar{F} = \bar{A}C + BD + A\bar{D}$

$\bar{\bar{F}} = \overline{\bar{A}C + BD + A\bar{D}}$

$= (A + \bar{C}) \cdot (\bar{B} + \bar{D}) \cdot (\bar{A} + D)$

AB \ CD			C	C
	X	1	0	0
B	1	0	1	1
A, B	0	X	1	0
A	X	1	1	X
		D	D	



F(A,B,C,D)= $(A + \bar{C}) \cdot (\bar{B} + \bar{D}) \cdot (\bar{A} + D)$

Problem 4: Consider the following k-map for the function F(A, B, C, D). Identify the expressions of its essential prime implicants. 1.5 **(1.5 points)**

AB \ CD			C	C
	1			1
B			1	
A B	1	1	1	1
A	1			1
		D	D	

Essential prime implicants:

AB

BCD

$\bar{B}\bar{D}$

$AB + BCD + \bar{B}\bar{D}$

Problem 5: Given the following function F: **(3.5 points)** 3.5

$$F(A, B, C, D) = ((\bar{A} + \bar{C}).(BCD + \bar{A}\,\bar{B})) + \bar{C}D$$

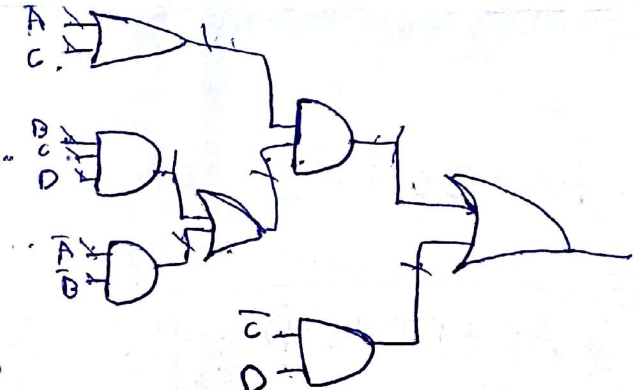
a. What is the literal cost (L), the gate-input cost (G) and the gate-input cost with invertors counted (GN), of F?

L = 9

G = 15

GN = 18

1.5



b. Fill-in the K-map of F.

0, 1, 2, 3, 5, 7
9, 13, 15

AB \ CD			C	C
	1	1	1	1
B		1	1	
A B		1	1	
A		1		
		D	D	

2

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

$\bar{A}BCD + \bar{A}\bar{A}\bar{B} + B\bar{C}CD + \bar{A}\bar{B}C + \bar{C}D$

$\bar{A}BCD + \bar{A}\bar{B} + \bar{B}CD + \bar{A}\bar{B}C + \bar{C}D$