

Your Name (in Arabic): \_\_\_\_\_

Your Student ID: \_\_\_\_\_

Your Instructor Name: \_\_\_\_\_

Your Section #: \_\_\_\_\_ ; Your Lecture Times: \_\_\_\_\_

**Read The Following Instructions Carefully:**

1. This exam booklet has 7 numbered pages and 6 problems. Check that your exam includes all 6 pages. Show ALL of your work on these pages. Two blank pages are added at the end for your scratch work.
2. WRITE your name (in Arabic) and student number in the spaces above. Also, make sure to write your instructor's name, section # and lecture times.
3. You are NOT permitted to use notes, books, calculators, or mobile phones during this exam.
4. This exam lasts for 75 minutes. Point values are listed for each problem to assist you in making the best use of time.

| Problem | Max Points | Score |
|---------|------------|-------|
| 1       | 6          |       |
| 2       | 5          |       |
| 3       | 5          |       |
| 4       | 5          |       |
| 5       | 4          |       |
| 6       | 5          |       |
| Total   | 30         |       |

**Problem 1 (6 pts) Multiple Choice & Short Answer**

A. **(.5 pts)** A supermarket stocks 9000 items and you want to refer to each item by a unique binary number (ID). What is the minimum number of bits you need to assign one binary number (ID) to each item?

14 bits

B. **(1.5 pts)** Circle True or False

(a) The expression  $F = X'YZ + ZX' + XY'Z$  is in minimal SOP form.

True      False

(b) "Don't Care" values in a K-map will always yield a more minimal form.

True      False

(c) A NOR gate can be represented using an AND gate along with NOT gates.

True      False

C. **(1 pts)** The following selection is a: (circle all that apply)

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   |   |   |   | Y |   |
|   | X | X | 0 | 0 |   |
|   | 0 | 1 | 1 | 0 |   |
|   | 0 | 1 | 1 | 1 | X |
| w | 0 | 1 | 0 | 1 |   |
|   |   |   |   |   | z |

i) Prime Implicant      ii) Essential Prime Implicant

D. **(1.5 pts)** Convert 75.125 to binary, hexadecimal, and octal. Show each step

i. Binary = 1001011.001<sub>2</sub>

ii. Hexadecimal = 4B.2<sub>16</sub>

iii. Octal = 113.1<sub>8</sub>

E. **(.5 pts)** Which of the following gate combinations can be used to represent any logic function? (Circle all that apply)

(a) AND, OR, NOT

(b) NAND, NOR

(c) XNOR

(d) NOR

- F. **(1 pts)** Given  $F(x,y,z) = \Sigma m(0,1,6,7)$
- (a) What is the product of maxterms for F using the  $\prod$  notation ?  
 $\prod M(2,3,4,5)$
- (b) Write out the function F(x,y,z) as a product of maxterms using the “(x+y+z)...” notation. You do not need to simplify.  
 $(X+Y'+Z)(X+Y'+Z')(X'+Y+Z)(X'+Y+Z')$

**Problem 2 Boolean Algebra (5 pts)**

Use Boolean algebra to simplify the following expression as much as possible:

$$(b(acb + a' + ab))' + ((a+c+ba)(a'+b')(b'+c+b'a))'$$

Show the steps of your derivations and state which axioms (or numbers in the table below) you are using at each step. If you do all the possible simplifications then your final answer will be

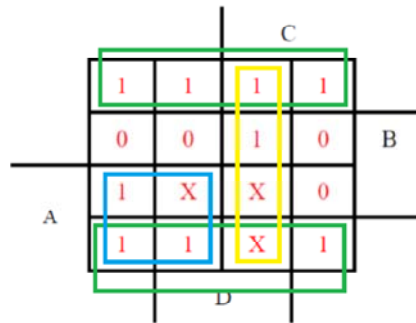
$$b' + a + c'$$

- (.5 pts)**  $(b(acb + a' + ab))' + ((a+c+ba)(a'+b')(b'+c+b'a))'$
- (.5 pts)**  $(b(ab + a'))' + ((a+c+ba)(a'+b')(b'+c+b'a))'$  (10)
- (.5 pts)**  $(b' + a(a' + b')) + ((a+c+ba)(a'+b')(b'+c+b'a))'$  DeMorgan's (9)
- (.5 pts)**  $(b' + ab') + ((a+c+ba)(a'+b')(b'+c+b'a))'$  (12)
- (.5 pts)**  $b' + ((a+c+ba)(a'+b')(b'+c+b'a))'$  (10)
- (.5 pts)**  $b' + ((a+c)(a'+b')(b'+c+b'a))'$  (10)
- (.5 pts)**  $b' + ((a+c)(a'+b')(b'+c))'$  (10)
- (.5 pts)**  $b' + ((a+c)(a'+b'))'$  Consensus Theorem (13)
- (.5 pts)**  $b' + a'c' + ab$  DeMorgan's (9)
- (.5 pts)**  $b' + a + a'c'$  (12)
- (.5 pts)**  $b' + a + c'$  (12)

|                                |  |                     |
|--------------------------------|--|---------------------|
| 1. $x+0 = x$                   | $x*1 = x$                                  |                     |
| 2. $x+1 = 1$                   | $x*0 = 0$                                  |                     |
| 3. $x+x = x$                   | $x*x = x$                                  |                     |
| 4. $x+x' = 1$                  | $x*x' = 0$                                 |                     |
| 5. $(x')' = x$                 |  |                     |
| 6. $x+y = y+x$                 | $xy = yx$                                  | (Commutative)       |
| 7. $x+(y+z) = (x+y)+z$         | $x(yz) = (xy)z$                            | (Associative)       |
| 8. $x(y+z) = xy+xz$            | $x+yz = (x+y)(x+z)$                        | (Distributive)      |
| 9. $(x+y)' = x'y'$             | $(xy)' = x'+y'$                            | (DeMorgan's)        |
| 10. $x+xy = x$                 | $x(x+y) = x$                               |                     |
| 11. $xy+xy' = x$               | $(x+y)(x+y') = x$                          |                     |
| 12. $x+x'y = x+y$              | $x(x'+y) = xy$                             |                     |
| 13. $xy + x'z + yz = xy + x'z$ | $(x + y)(x' + z)(y + z) = (x + y)(x' + z)$ | (Consensus Theorem) |

**Problem 3 Karnaugh Maps (5 pts)**

Consider the function  $F(A,B,C,D) = \sum m(0,1,2,3,7,8,9,10,12)$  with don't care terms  $d(A,B,C,D) = \sum m(11,13,15)$ .



(a) **(2 pts)** List all of the prime implicants.

$B', AD, AC', CD$

(b) **(2 pts)** List all of the essential prime implicants.

$B', AC', CD$

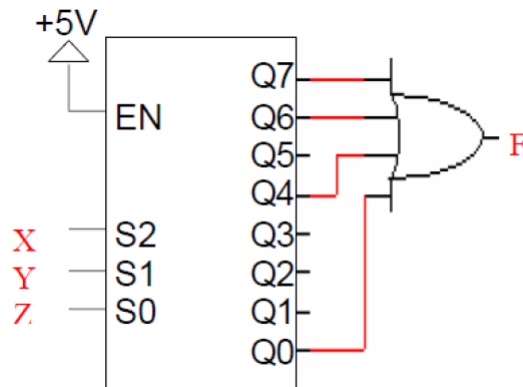
(c) **(1 pts)** Give a minimal sum-of-products expression for F.

$F = B' + AC' + CD$

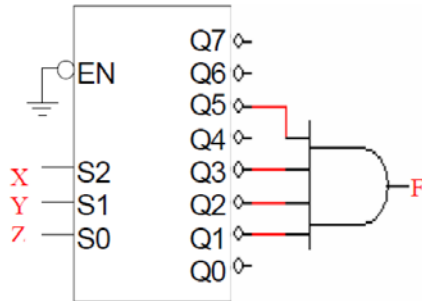
**Problem 4: Decoders and Multiplexers (5 pts)**

Assume we want to implement a function  $F(X,Y,Z) = \sum m(0,4,6,7)$ .

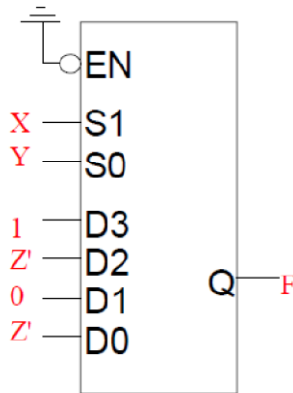
(a) **(1 pts)** Show how to implement the function F using only the decoder below and one additional



(b) (2 pts) Next, implement F using the decoder shown here and one additional gate

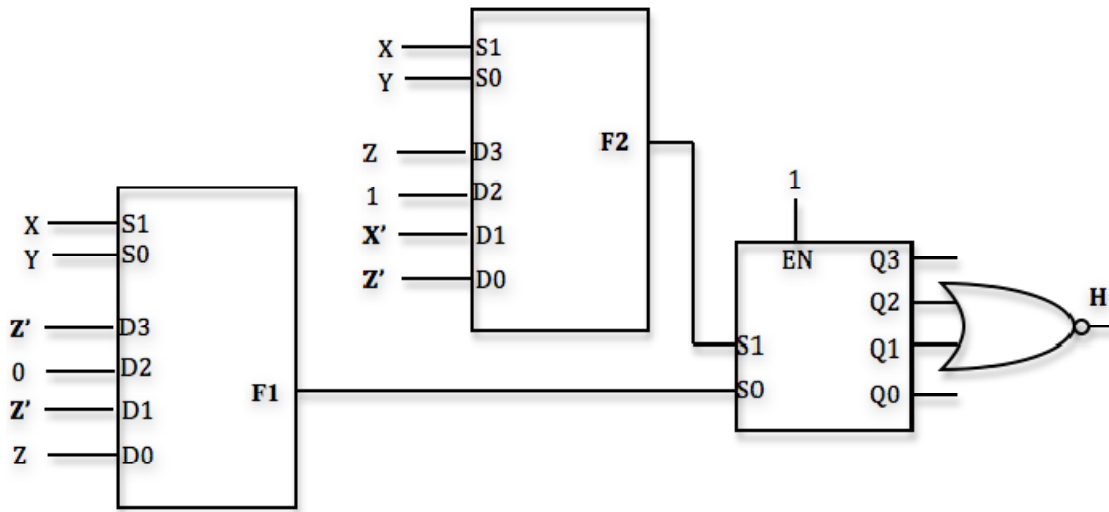


(c) (2 pts) Build a circuit for the same function using a 4-to-1 multiplexer and any additional gates you like.



**Problem 5: Circuit Analysis (4 pts)**

Consider the following circuit:



(a) **(3 pts)** Derive the truth table for the function  $H(X, Y, Z)$ .

*Intermediate Columns, if necessary*

| X | Y | Z | F1 | F2 |  |  | H |
|---|---|---|----|----|--|--|---|
| 0 | 0 | 0 | 0  | 1  |  |  | 0 |
| 0 | 0 | 1 | 1  | 0  |  |  | 0 |
| 0 | 1 | 0 | 1  | 1  |  |  | 1 |
| 0 | 1 | 1 | 0  | 1  |  |  | 0 |
| 1 | 0 | 0 | 0  | 1  |  |  | 0 |
| 1 | 0 | 1 | 0  | 1  |  |  | 0 |
| 1 | 1 | 0 | 1  | 0  |  |  | 0 |
| 1 | 1 | 1 | 0  | 1  |  |  | 0 |

(b) **(1 pt)** Express  $H(X, Y, Z)$  as a sum of minterms, based on your above truth table from (a)

$H(X, Y, Z) = \sum m(2)$

Problem 6 (5 pts)

$x$  is a decimal number that can have one of the following values  $\{ 0, 1, 2, 4, 6, 7 \}$ .  
A function,  $f(x)$ , is defined as follows:

$$f(x) = x^2 \quad \text{when } x \text{ is equal to } 0, 1, 2$$

$$f(x) = x - 1 \quad \text{when } x \text{ is equal to } 4$$

$$f(x) = 2x - 11 \quad \text{when } x \text{ is equal to } 6, 7$$

You have been requested to design a combinational logic circuit to generate  $f(x)$  represented in binary.  
 $x$  is also available in binary for your circuit.

(a) What is the number of input and output lines of your circuit?

**3 input. (0.5 mark)**  
**3 output (0.5 mark)**

(b) Show the truth table of your circuit. Make sure you label the binary variables of the input and output columns

**(1 mark)**

| <b>X</b> | <b>Y</b> | <b>Z</b> | <b>Q2</b> | <b>Q1</b> | <b>Q0</b> |
|----------|----------|----------|-----------|-----------|-----------|
| <b>0</b> | <b>0</b> | <b>0</b> | <b>0</b>  | <b>0</b>  | <b>0</b>  |
| <b>0</b> | <b>0</b> | <b>1</b> | <b>0</b>  | <b>0</b>  | <b>1</b>  |
| <b>0</b> | <b>1</b> | <b>0</b> | <b>1</b>  | <b>0</b>  | <b>0</b>  |
| <b>0</b> | <b>1</b> | <b>1</b> | <b>x</b>  | <b>x</b>  | <b>x</b>  |
| <b>1</b> | <b>0</b> | <b>0</b> | <b>0</b>  | <b>1</b>  | <b>1</b>  |
| <b>1</b> | <b>0</b> | <b>1</b> | <b>x</b>  | <b>x</b>  | <b>x</b>  |
| <b>1</b> | <b>1</b> | <b>0</b> | <b>0</b>  | <b>0</b>  | <b>1</b>  |
| <b>1</b> | <b>1</b> | <b>1</b> | <b>0</b>  | <b>1</b>  | <b>1</b>  |

(c) Find the simplified output equation(s).

**Q0 = X+Z (1 mark)**

**Q1 = XY'+YZ (1 mark)**

**Q2 = X'Y (1 mark)**