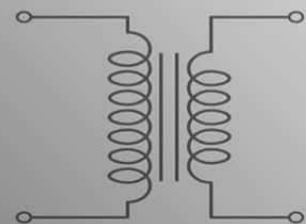


Circuits I

Fall 017



Dr. Ghazi Alsukkar

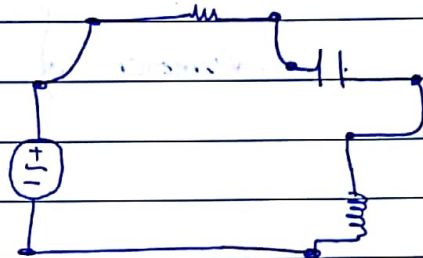


Powerunit-ju.com

Circuit Analysis:

Circuit : CKT

Linear Circuits \longleftrightarrow Non linear Circuits
↓
System.



Physical System $\xleftrightarrow{\text{Modeling}}$ Mathematical equations.

Circuit analysis:

- 1- DC analysis : Direct current (steady state) ثابتة
- 2- Transient analysis : Switching on & off انتقالية
- 3- AC analysis : Alternating current
- 4- Frequency analysis

Analyze the CKT P

Find : Voltage & Current

- Power
- Energy
- Impedance, Admittance.

Resistance

$$\text{Voltage} = \text{Current} \times \text{Impedance}$$

Units:

• يحتاج ٢ أو أكثر ← رقم + وحدة + اقلام

To measure any physical quantity : we need 2 or more things.

Temperature → Number & Unit

20°C

75°F

10 K.

SI units : International system of units.

7 basis units :

Quantity	unit	Appreviation.	
length	meter	m	20 m
mass	Kilogram	Kg	
time	Second	s (sec)	
electrical current	Ampere	A	20 A
temperature	Kelvin	K	
amount of substance	mole	mol	
luminone intensity	Candela	cd	

Derived units :

ex

Energy → Joule (J)

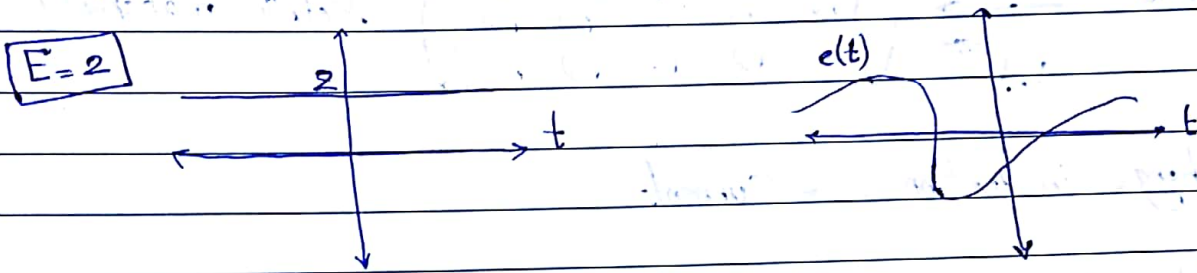
Power → Watt (W)

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} = \text{Joule} / \text{sec} = \text{Watt}$$

* Physical Quantities:

Energy: $E \rightarrow \bar{a} = \text{volts}$
 $e(t)$ is ~~variable~~

amount of work that can be performed by a force.



Unit = Joule (J)

calorie (Cal)

1 calorie = 4.187 J

1 mile \rightarrow 1.6 kilometers

Power: $P \rightarrow P(t)$

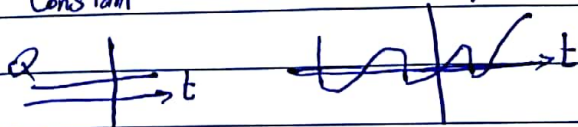
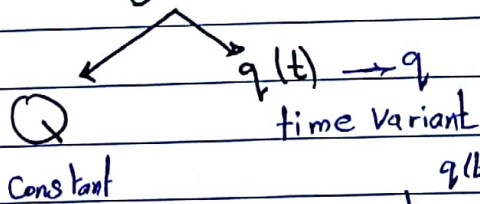
- amount of work done per unit of time.
- Rate at which energy is expended.

$P = \frac{\Delta W}{\Delta t} \quad [\text{Watt} = \text{Joule/sec}]$

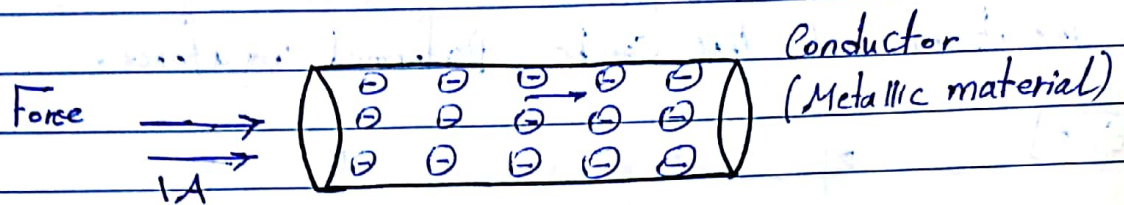
$P = \frac{d e(t)}{dt}$

* Basic Electric Quantities:

- Charge \rightarrow Coulomb [C]



Charge $\left\{ \begin{array}{l} \text{Positive (+ve) ; Proton} \\ \text{Negative (-ve) ; Electron.} \end{array} \right.$



Charge in motion \rightarrow Current.

Coulomb :

Number of electrons that passes through an arbitrary cross-section of a wire during an interval of one second.

1A \rightarrow 1 Coulomb = 6.242×10^{18} electron.

$$\text{Electron Charge} = \frac{1}{6.242 \times 10^{18}} = -1.602 \times 10^{-19} \text{ C}$$

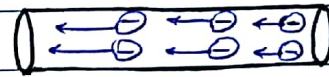
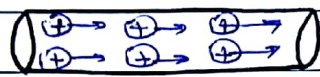
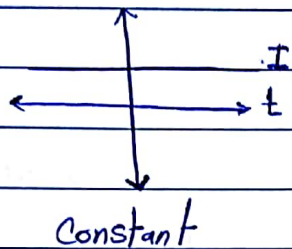
$$\text{Proton Charge} = +1.62 \times 10^{-19} \text{ C}$$

* Charge Conservation:

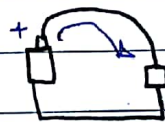
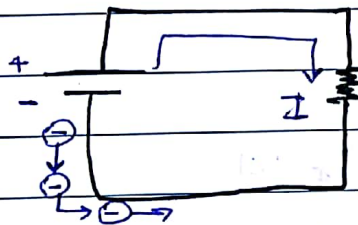
Total amount of Charge is constant.

* Current: [A] \rightarrow Number, Unit, direction 5A

I \rightarrow time varying (Ac)
 Dc \rightarrow Direct Current Alternating Current



I
 Current Direction
 (by convention)



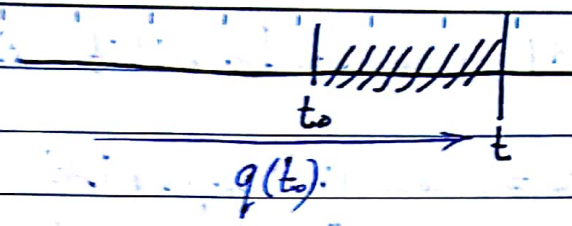
Current Density:

A measure of the rate at which charges are moving.

$$i = \frac{dq}{dt} \quad [C/sec] \equiv A$$

Total Charge transferred between t_0 & t

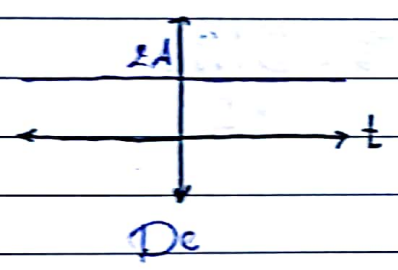
$$\int_{q(t_0)}^{q(t)} dq = \int_{t_0}^t i(t) dt$$



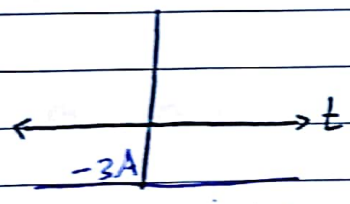
- The overall (total) charge over all time is:

$$\int_{-\infty}^t i(t) dt = q(t_0) + \int_{t_0}^t i(t) dt$$

$$I_1 = 2A$$

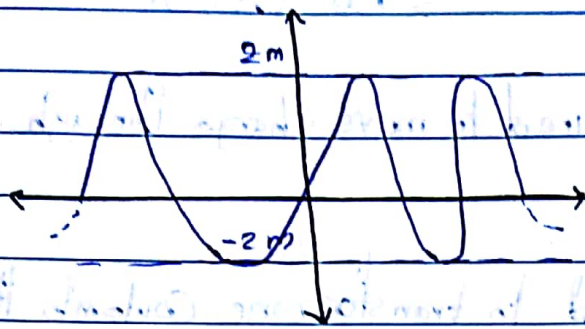


$$I_2 = -3A$$



دے ایشی ثابت ویکس
 دے آے

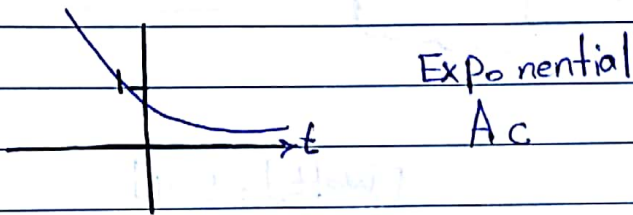
$$i_1 \equiv i_1(t) = 2 \sin(5t) \text{ mA}$$



$$\omega = 5$$

$$\frac{2\pi}{T} = \frac{2\pi f}{1} = 5$$

$$i = e^{-2t} \text{ A}$$



- * 5C of +ve Charge is moving in 1 second
- * -5C of -ve Charge " " " " "

Voltage (Potential Difference)

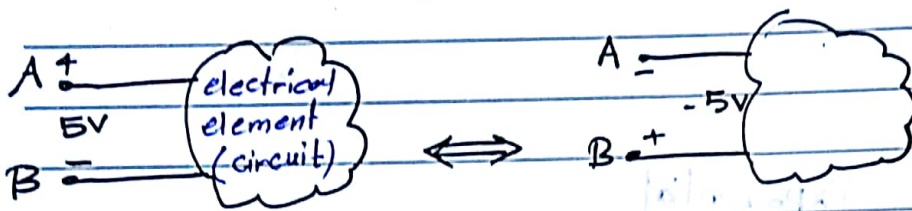
$$[\text{Volt}] = [\text{V}]$$

$$\begin{matrix} \rightarrow V(\text{Dc}) & \text{ثابت} \\ \rightarrow V(t) & (\text{Ac}) \text{ متغير} \end{matrix}$$

- is a measure of the work required to move charge through an element.

→ The amount of energy needed to transfer one Coulomb through an element.

$$1 \text{ Volt} = 1 \text{ J/1C}$$



* Power

→ P : Constant Power

$$[\text{Watt}] = [\text{W}]$$

→ p = P(t) Instantaneous Power

$$P = \frac{\Delta E}{\Delta t}, \quad P(t) = \frac{dE(t)}{dt}$$

$$\text{Watt} = \text{Joule/sec}$$

$$P = \frac{\text{Joule}}{\text{Coulomb}} \cdot \frac{\text{Coulomb}}{\text{Sec}}$$

V A

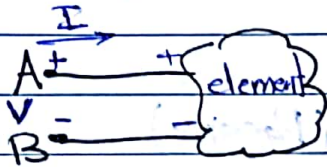
$P =$ number of coulombs transferred per second \times energy needed to transfer one coulomb through the element.

$P =$ Current \times Voltage.

$$P = IV = VI$$

$$P(t) = w(t) \cdot i(t)$$

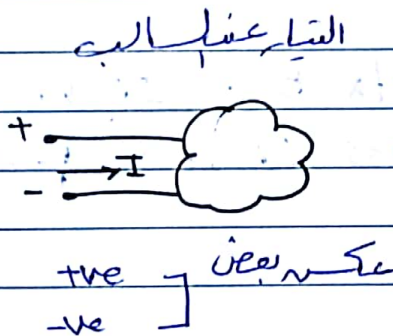
Passive sign Convention:



$$P = VI$$

$+ve$: amount of power absorbed by the element (passive)
 $-ve$: amount of power delivered by the element (active)

active sign convention:

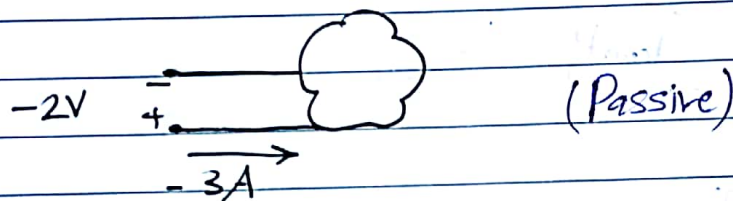


* If one Joule of energy is expended in transferring one Coulomb of charge through the device in one second, then the rate of energy transfer is one watt.

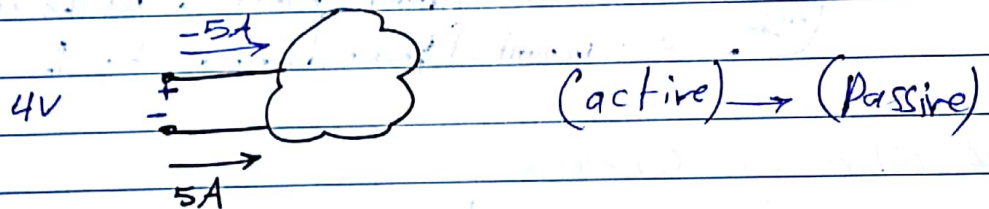
* Calculate the absorbed Power:



$$P = 2 \times 3 = 6 \text{ W (absorb)}$$



$$P = -2 \times -3 = 6 \text{ W (absorb)}$$



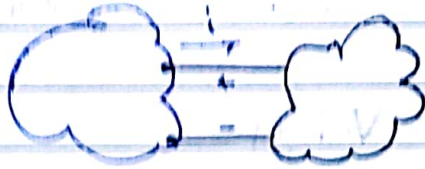
active
↓
passive

$$P = 4 \times -5 = -20 \text{ W (deliver)}$$

This device absorbs -20 W of power.

* Passive sign convention:

$P = P_p$ Absorbed by ...
 Delivered by ...



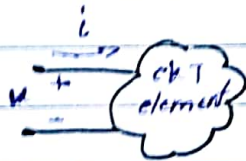
$P = Vi$

$P = Vi$

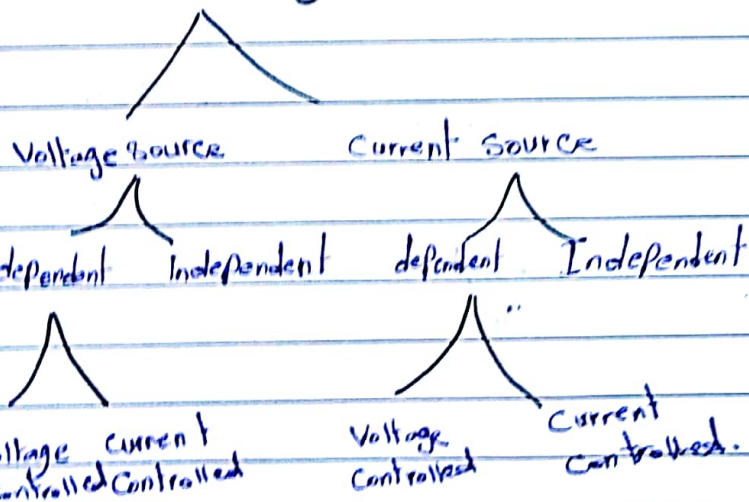
→ +ve (absorb)
 → -ve (deliver supply)

* CKT elements :-

- Any element is characterized by the relation between the voltage across its terminals and the current passing through it.



* Power Supply :-

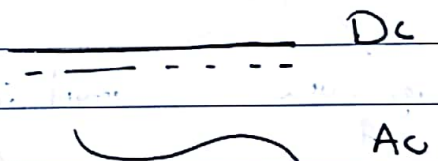
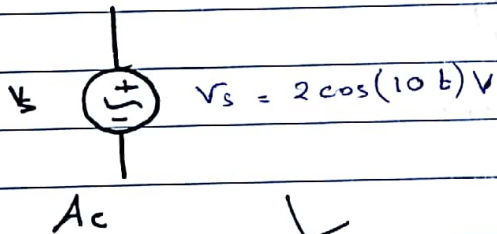
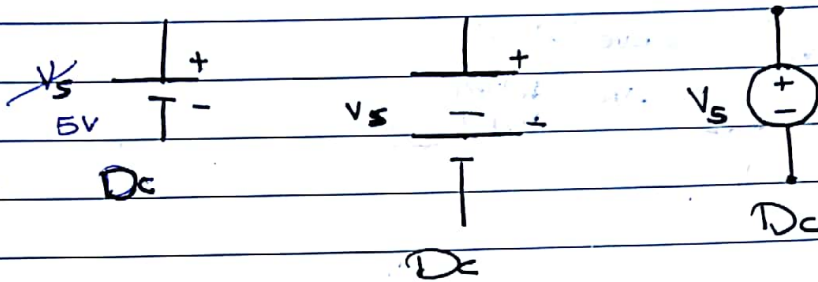


* Independent Voltage source :

- It is characterized by a terminal voltage which is completely independent of the current passing through it \Rightarrow (Ideal)

zero current \leftarrow 7.6V Volt *

Ideal voltage sources \Rightarrow constant voltage .



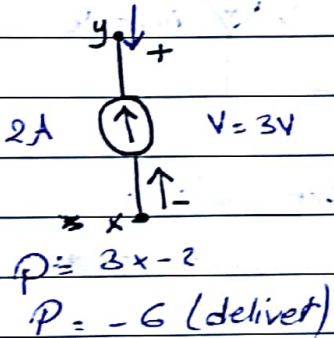
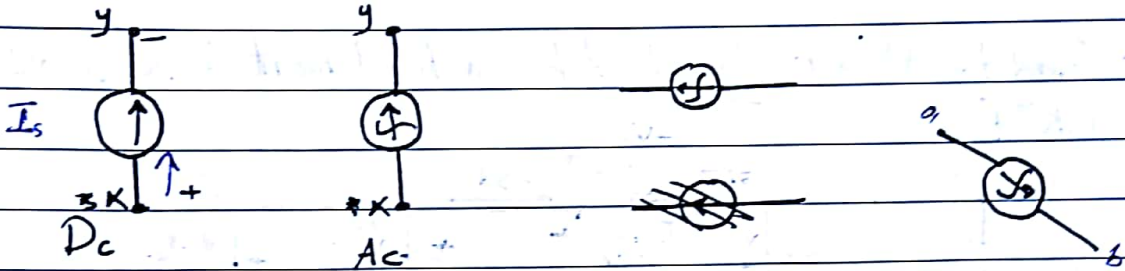
red/Blue (+)
Black (-)

رصيد التيار على الجهد

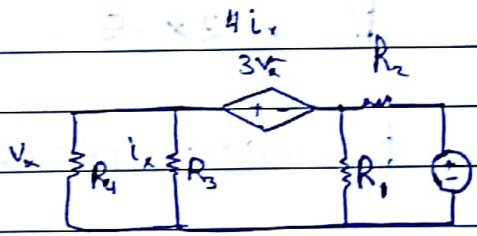
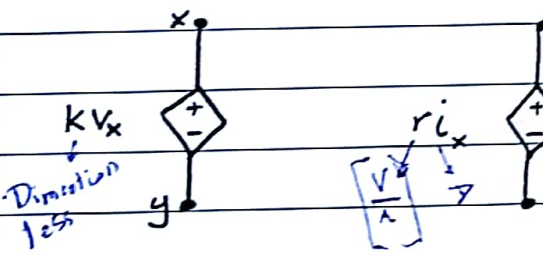
* Independent current source :

- The current through this element is completely independent of the voltage across it.

Ideal Current source \Rightarrow Current constant ..



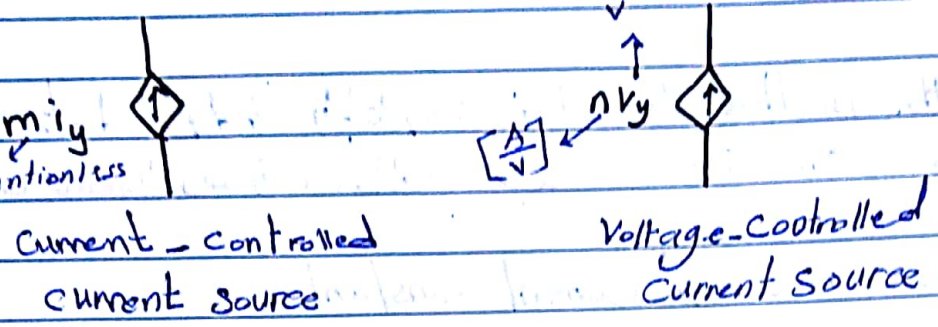
- Dependent sources (controlled) :



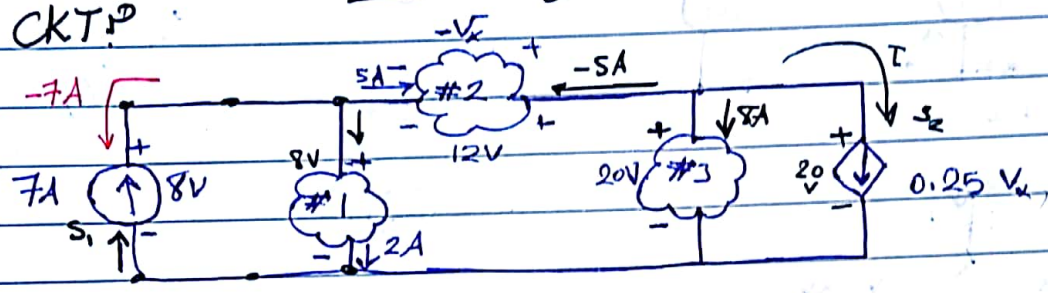
Voltage controlled voltage source.

Current controlled voltage source.

المقاومة
20Ω
Dimensionless



Ex: Find the Power absorbed by each element in the show CKTP



$P_{S_1} = 8 \times -7 = 56 \text{ watt (absorb)}$ deliver $\rightarrow (+56)$

على توازي V مقبول

$P_1 = 8 \times 2 = 16 \text{ watt}$

$P_2 = 12 \times -5 = -60 \text{ watt (deliver)}$

$P_3 = 20 \times 8 = 160 \text{ watt}$

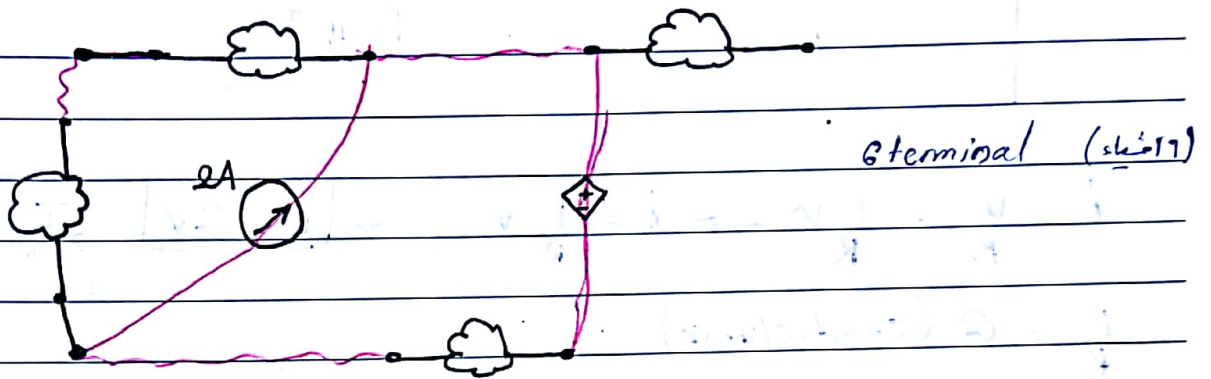
$P_{S_2} = 20 \times (0.25 V_x)$
 $= 20 \times (0.25 \times -12)$
 $= -60 \text{ watt}$

$-V_x = 12$
 $V_x = -12$

قانون حفظ الطاقة = + + +
 = صفر

* Networks & CKT:

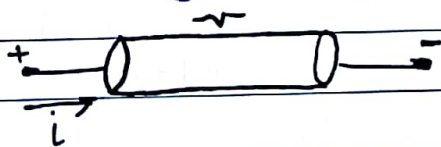
- Two or more CKT elements connected together are called Network.
- A Network that contains at least one closed path is called a circuit.



- Any CKT is a network.
- Any Network is not necessarily a CKT.
- A Network with at least one active device.
 - active network.
- A Network without any active devices
 - passive network.

Ohm's Law:

- voltage across conducting materials is directly proportional to the current flowing (passing) through the material.



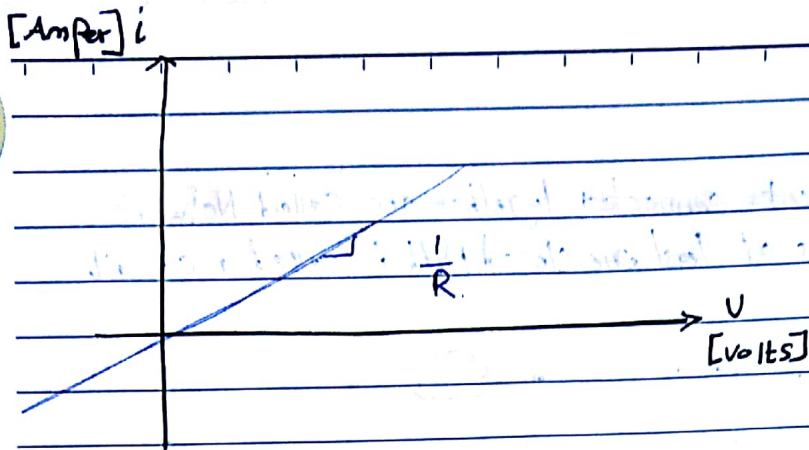
Resistor.

$V \propto i$ → Proportionality constant.
 $V = Ri$

Resistance → $R = \frac{V}{i} \left[\frac{V}{A} \right] = [\Omega]$
ohm

$V = RI$ DC

$\vec{V} = Z\vec{I}$
impedance



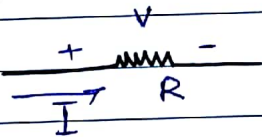
$$i = \frac{V}{R} = \frac{1}{R} V \rightarrow i = \frac{1}{R} V$$

$$i = GV \quad \text{I = GV}_{DC}$$

$$\frac{1}{R} = G \text{ (conductance)}$$

$$[\text{siemens}] = [S]$$

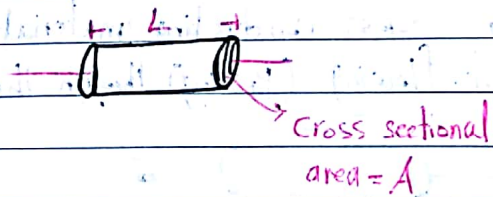
$$1 S = \frac{1 A}{1 V} = \frac{1}{\Omega} = \Omega^{-1} = \text{mho}$$

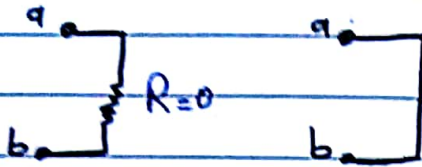


$$V = RI$$

$$R = \frac{\rho L}{A}$$

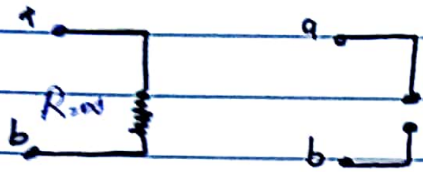
Resistivity $[\Omega \cdot m]$





Short circuit
S.C.

هنا شحنت غير دورنا تخسر طاقتنا



Open circuit
O.C.

التي هنا لا يمر (الشحنت لا تمر)

$$P = VI$$

$$V = RI$$

$$P = (RI)I = RI^2$$

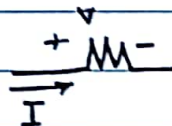
$$P = RI^2 \quad (\text{power absorbed by the resistor})$$

$$P = V \left(\frac{V}{R} \right) = \frac{V^2}{R}$$

(R دائما موجبة)

$$P = \frac{V^2}{R}$$

Ex:



① $R = ?$ if $I = -1.6 \text{ mA}$, $V = -6.3 \text{ V}$

$$V = RI \rightarrow R = \frac{V}{I} = \frac{-6.3}{-1.6 \text{ mA}} = 3.94 \text{ k}\Omega = 3940 \Omega$$

② $V = -6.3 \text{ V}$, $R = 21 \Omega$

Find the absorbed power by R :-

$$P = \frac{V^2}{R} = \frac{(-6.3)^2}{21} = 1.89 \text{ watt.}$$

والتي هنا
absorb لتي
الموجبة
(Passive)

③ $v = -8V$, if R absorbing $0.24W$, Find I ?

$$P = 0.24 \text{ watt}$$

$$v = -8V$$

$$P = vI$$

$$I = \frac{P}{v} = \frac{0.24}{-8} = -30 \text{ mA}$$
$$= -30 \times 10^{-3} \text{ A}$$

$$i(t) = e^{2t}$$

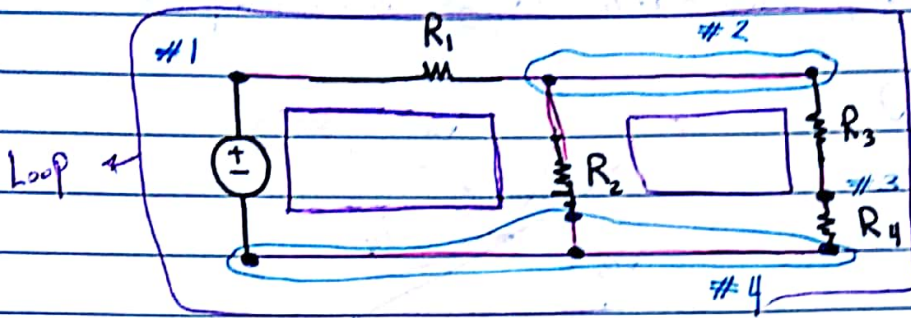
$$v(t) = \cos(5t) \quad \text{Find } P:$$

so) $p(t) = e^{2t} \cos(5t) \text{ watt}$

CH 2: KVL, KCL, Series, Parallel.

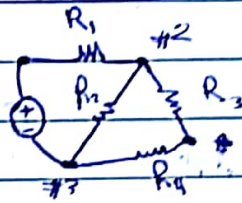
نقطة اتصال مشتركة

Node: a point at which two or more elements have a common connection.



How many element? 5

How many Branches? 5



Node و عناصر في س.ج

إذا فروع

Path: a set of nodes and elements passed.

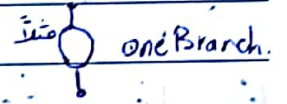
Loop: Closed Path.

مسار مغلق يمر من العقد

- Loop is a Path.

- Path is not necessarily a loop.

Branch: one element and the node at each and fit.

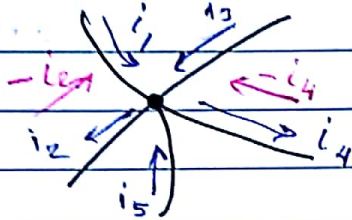
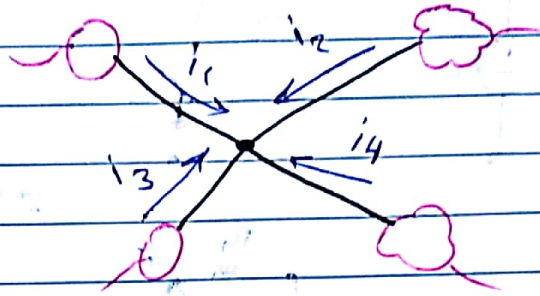


Kirchhoff's Current Law (KCL):

The algebraic sum of the currents entering (leaving) any node is zero \rightarrow Charge conservation
 مجموع التيارات الداخلة = مجموع التيارات الخارجة

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\sum_{n=1}^N i_n = 0$$



* ينزل كل شيء *

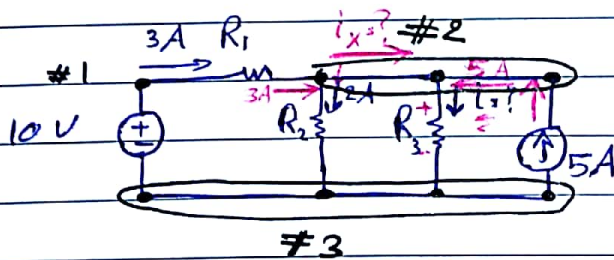
$$i_1 - i_2 + i_3 - i_4 + i_5 = 0$$

مجموع I الداخلة = مجموع I الخارجة

$$i_1 + i_3 + i_5 = i_2 + i_4$$

The sum of currents going in equal to the sum of currents going out. $\sum i_{in} = \sum i_{out}$

Ex: Find the current in R_3 , $i = ?$, $P_3?$

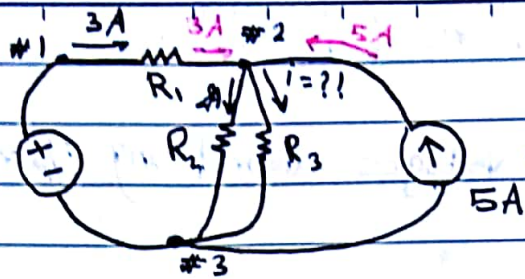


5 Branch.
3 Node
6 loops

$$3 = 2 + i_x \rightarrow i_x = 1A$$

$$i_x + 5 = i$$

$$1 + 5 = i \rightarrow i = 6A$$



Node #2 :

$$3 + 5 = 2 + i$$

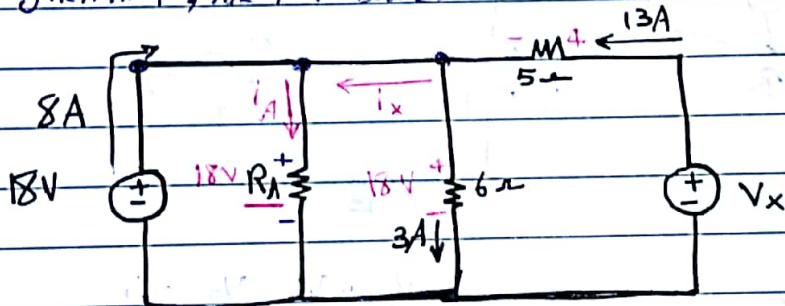
$$i = 6A$$

$P_3 = P_1$

$$P_3 = i^2 R_3$$

$$= (6)^2 R_3$$

ex: Count the number of branches & find $R_A = ?$
 Given that, the 18V source delivers 8A of current.



$$13 = 3 + i_x \rightarrow i_x = 10A$$

$$8 + 10 = i_A \rightarrow i_A = 18A$$

$$R_i = V$$

$$R_A(18) = 18 \rightarrow R_A = 1\Omega$$

5 Branches.

$$P_A = IV = (18)^2 \text{ watt}$$

Kirchhoff's Voltage Law (KVL):

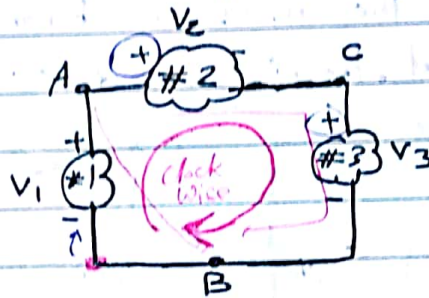
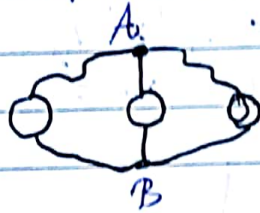
- The algebraic sum of the voltages around any closed path is zero.

$$\sum_{n=1}^N V_n = 0$$

$$V_1 + V_2 + V_3 + \dots + V_N = 0$$

- energy conservation.

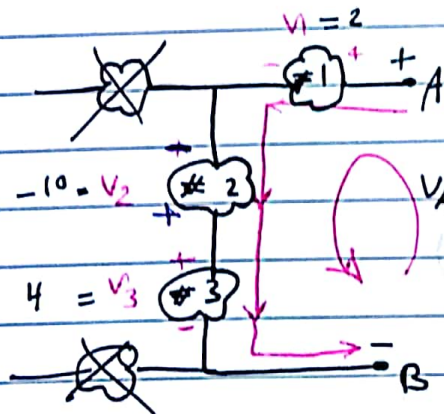
- The energy required to move a unit of charge from point A to point B is independent of the path chosen to go from A to B.



$$V_1 = V_2 + V_3$$

$$-V_1 + V_2 + V_3 = 0$$

$$V_2 + V_3 = V_1$$



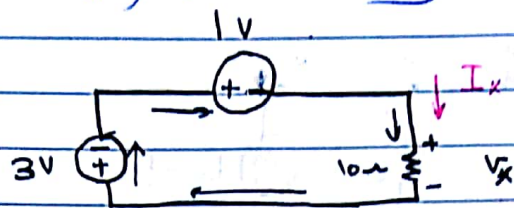
$$-V_{AB} + V_1 - V_2 + V_3 = 0$$

$$V_{AB} = V_1 - V_2 + V_3$$

$$= 2 - (-10) + 4$$

$$= 16V$$

ex: Determine I_x & V_x in the following circuit.



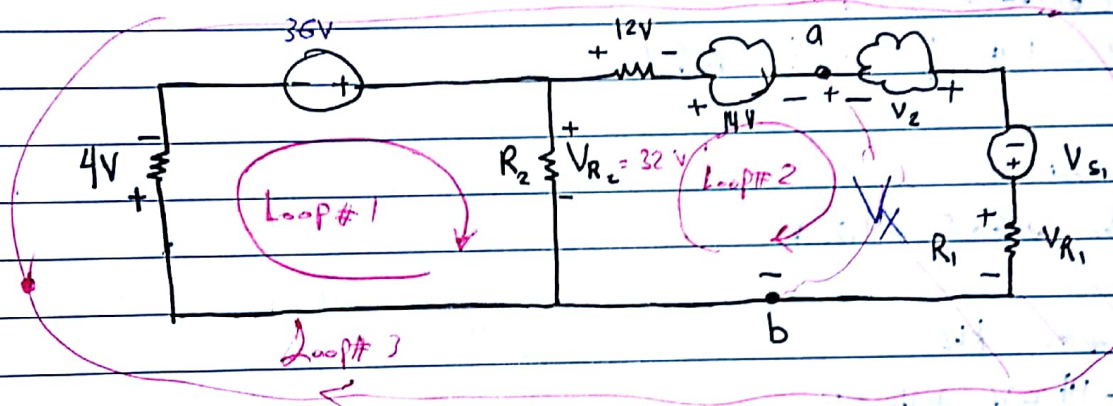
in series → $I_x = I_{R_1}$

$$+3 + 1 + V_x = 0$$

$$V_x = -4V$$

$$10 I_x = -4 \rightarrow I_x = \frac{-4}{10} = -0.4A = -400mA$$

ex: Determine V_{R_2} & V_x ?



① Loop #1: KVL

$$+4 - 36 + V_{R_2} = 0$$

$$V_{R_2} = 32V$$

$$V_x = V_{ab}$$

$$-V_x - V_2 - V_{s1} + V_{R1} = 0$$

$$V_x = -V_2 - V_{s1} + V_{R1}$$

② Loop #2: KVL

$$-32 + 12 + 14 + V_x = 0$$

$$V_x = 32 - 12 - 14 = 6V$$

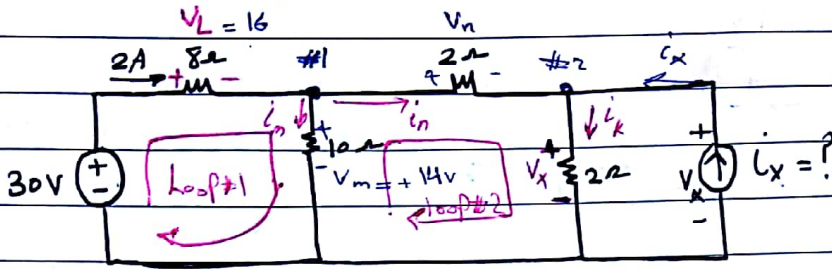
or

③ Loop #3

$$+4 - 36 + 12 + 14 + V_x = 0$$

$$V_x = 6V$$

Determine V_x of I_x :-



$$V_L = 2 \times 8 = 16 \text{ V}$$

① Loop #1 : KVL

$$-30 + 16 + V_m = 0 \longrightarrow V_m = +14 \text{ V}$$

$$i_m = \frac{14}{10} = 1.4 \text{ A}$$

KCL @ node #1 :

$$2 = 1.4 + i_n \longrightarrow i_n = 0.6 \text{ A}$$

Ohm Law :

$$V_n = 0.6 \times 2 = 1.2 \text{ V}$$

② Loop #2 : KVL

$$-14 + V_n + V_x = 0$$

$$-14 + 1.2 + V_x = 0 \longrightarrow V_x = 12.8 \text{ V}$$

$$i_k = \frac{12.8}{2} = 6.4 \text{ A}$$

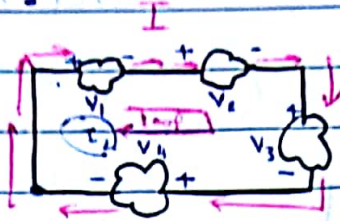
③ Node #2 : KCL

$$i_x + 0.6 = 6.4 \longrightarrow i_x = 5.8 \text{ A}$$

$$P_x = 12.8 \times (-5.8) \text{ deliver}$$

$$P_{30\text{V}} = 30(-2) \text{ deliver}$$

* Single Loop CKT :



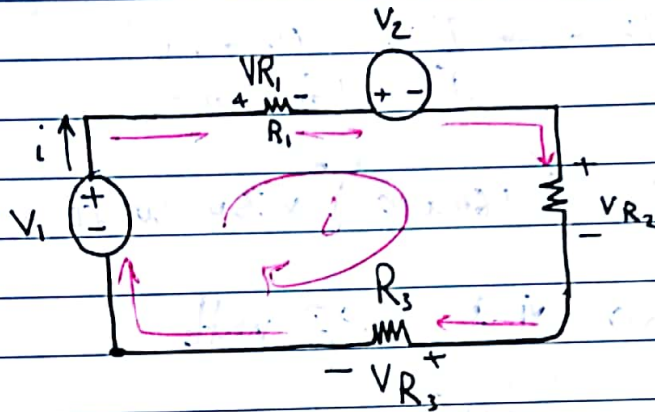
KVL :

$$V_1 + V_2 + V_2 + V_4 = 0$$

connected in series. \rightarrow same current.

- Element carrying the same current, are said to be connected in series.

$$I = P$$



$$-V_1 + V_{R_1} + V_2 + V_{R_2} + V_{R_3} = 0$$

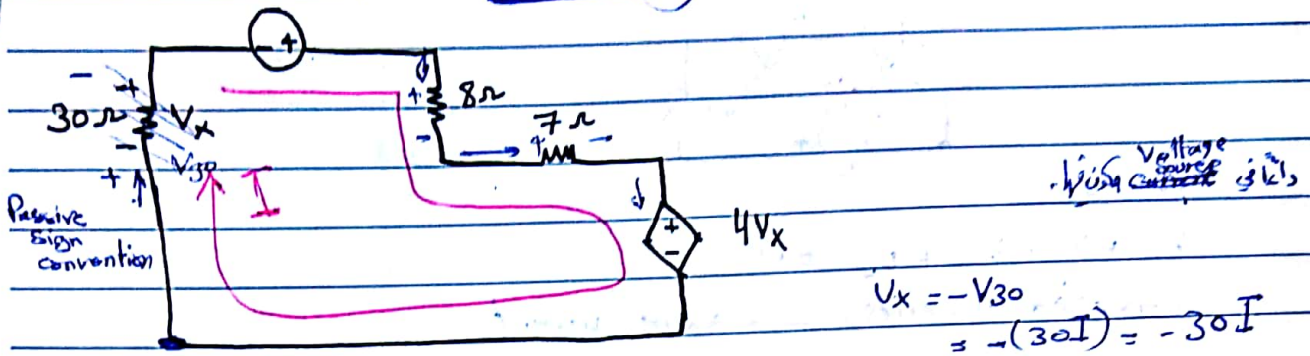
$$-V_1 + iR_1 + V_2 + iR_2 + iR_3 = 0$$

$$iR_1 + iR_2 + iR_3 = V_1 - V_2$$

$$i(R_1 + R_2 + R_3) = V_1 - V_2$$

$$i = \frac{V_1 - V_2}{(R_1 + R_2 + R_3)}$$

ex: Find the 12V power absorbed by each element:



$$V_x = -V_{30} = -(30I) = -30I$$

KVL:

$$30I - 12 + 8I + 7I + 4V_x = 0$$

$$30I - 12 + 8I + 7I + 4(-30I) = 0$$

$$-75I = 12 \rightarrow I = -0.16 \text{ A}$$

$$I = -160 \text{ mA}$$

$$P_{30\Omega} = I^2(30) = (-160 \times 10^{-3})^2 \times 30 \text{ watt} = 0.768 \text{ W}$$

$$P_{12V} = -(-160 \times 10^{-3}) \times 12 = -1.92 \text{ watt}$$

$$P_{8\Omega} = I^2(8) = (-160 \times 10^{-3})^2 \times 8 = 0.2048 \text{ watt}$$

$$P_{7\Omega} = (-160 \times 10^{-3})^2 \times 7 = 0.1792 \text{ watt}$$

$$P_{4V_x} = (4V_x)(-160 \times 10^{-3})$$

$$= (4 \times -30 \times -160 \times 10^{-3})(-160 \times 10^{-3}) = -3.072 \text{ W}$$

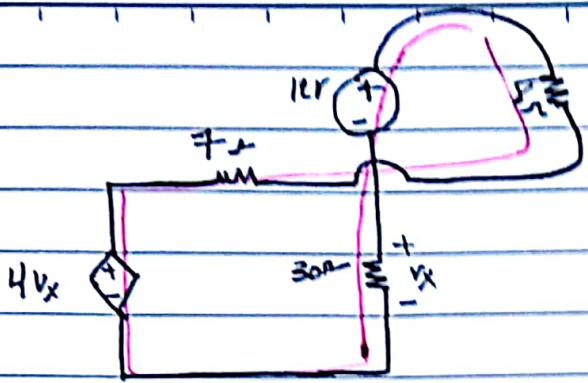
$$P_{30} + P_{12V} + P_{8\Omega} + P_{7\Omega} + P_{4V_x} = 0$$

- The sum of absorbed power by each element of a SKT is Zero.

$$\sum P_{\text{absorbed}} = \sum P_{\text{supplied}}$$

القانون الأول من كيرشوف

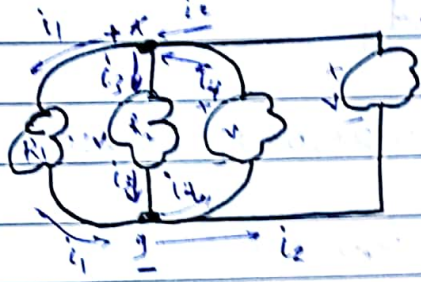
Single loop



* Single Node - Pair CKT :

القانون الثاني من كيرشوف

KCL :



$$i_2 + i_4 = i_1 + i_3$$

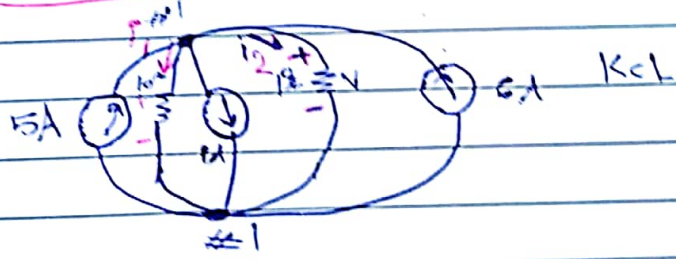
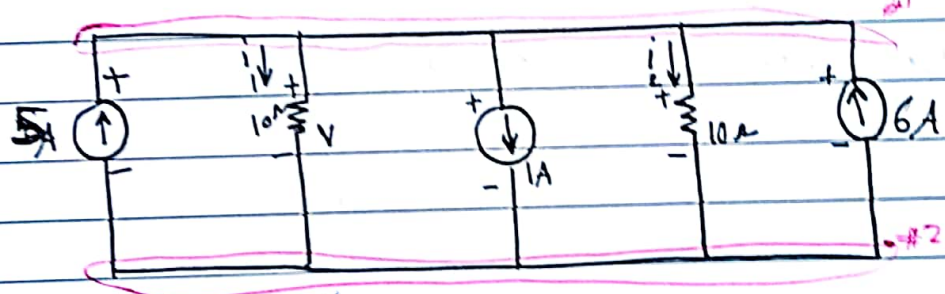
$$V = ?$$

- elements connected in a CKT having a common voltage across them are said to be connected in parallel.

$$i_1 = \frac{V}{R_1}$$

$$i_3 = \frac{V}{R_3}$$

Voltage and Current source



$$5 + 6 = 1 + i_1 + i_2$$

$$10 = i_1 + i_2$$

$$10 = \frac{V}{10} + \frac{V}{10}$$

$$10 = \frac{2V}{10}$$

$$\boxed{V = 50V}$$

$$\boxed{i_1 = i_2 = \frac{50}{10} = 5A}$$

$$P_{5A} = 50 \times -5 = -250W \text{ deliver}$$

$$P_{6A} = 50 \times -6 = -300W$$

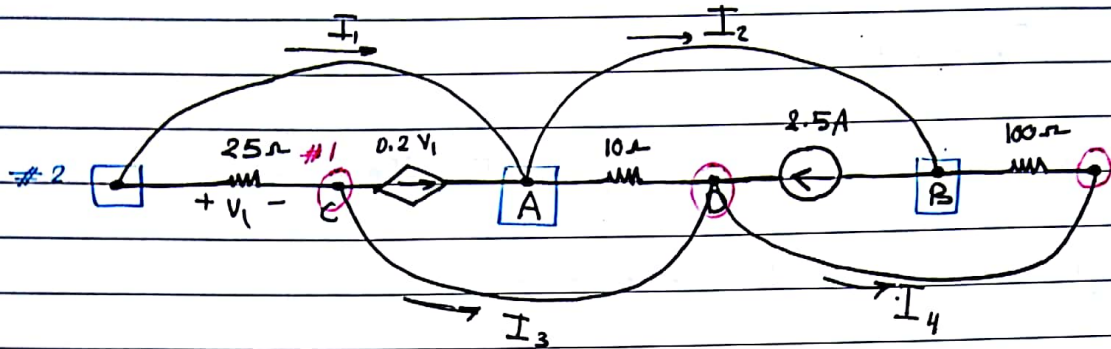
$$P_{1A} = 50 \times 1 = 50W$$

$$P_{10\Omega} = \frac{V^2}{10} = \frac{(50)^2}{10} = 250$$

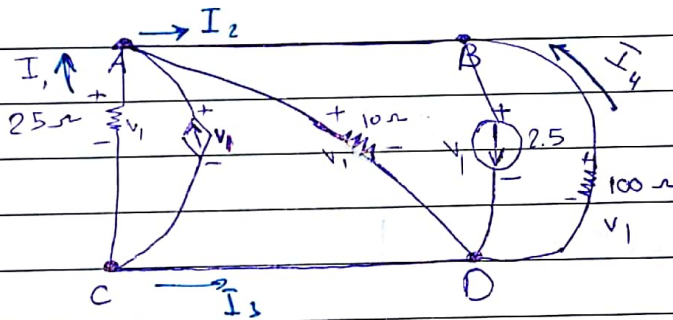
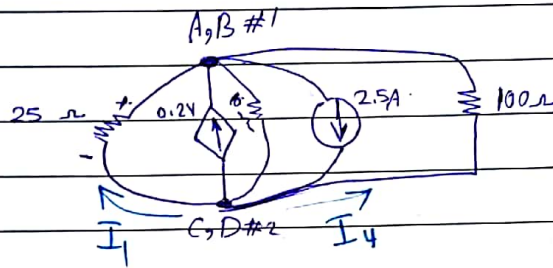
$$P_{10\Omega} = \frac{V^2}{10} = \frac{(50)^2}{10} = 250$$

$$500 + 50 - 300 - 250 = 0$$

ex: I_1, I_2, I_3, I_4 ?



2 Node:



Node A:

$$I_1 + 0.2V_1 = I_{10} + I_2$$

$$-\frac{V_1}{25} + 0.2V_1 = \frac{V_1}{10} + I_2$$

$$-\frac{V_1}{25} + 0.2V_1 - \frac{V_1}{10} = I_2 \dots \textcircled{1}$$

⑨ Node B:

$$I_2 + I_4 = 2.5$$

$$I_2 + \frac{-V_1}{100} = 2.5 \rightarrow I_2 = 2.5 + \frac{V_1}{100} \quad (2)$$

$$-\frac{V_1}{25} + 0.2V_1 - \frac{V_1}{10} = 2.5 + \frac{V_1}{100}$$

$$V_1 = 50V$$

$$I_1 = \frac{-V_1}{25} = \frac{-50}{25} = -2A$$

$$I_2 = 2.5 + \frac{50}{100} = 3A$$

$$I_4 = \frac{-50}{100} = -0.5A$$

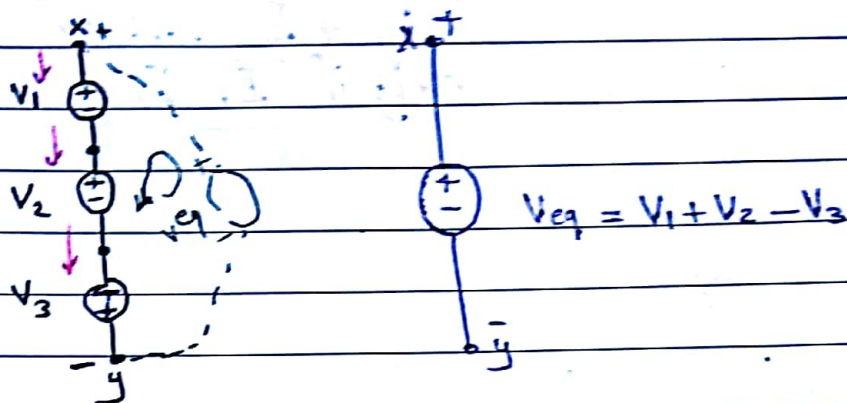
⑨ Node C:

$$I_1 + 0.2(50) + I_3 = 0$$

$$-2 + 0.2(50) = -I_3$$

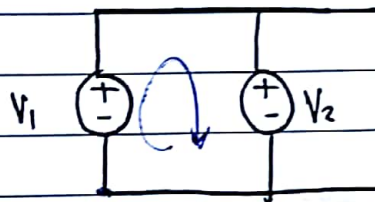
$$I_3 = -8A$$

* Series & Parallel Connection of Sources :



$$-V_{eq} + V_1 + V_2 - V_3 = 0$$

$$V_{eq} = V_1 + V_2 - V_3$$

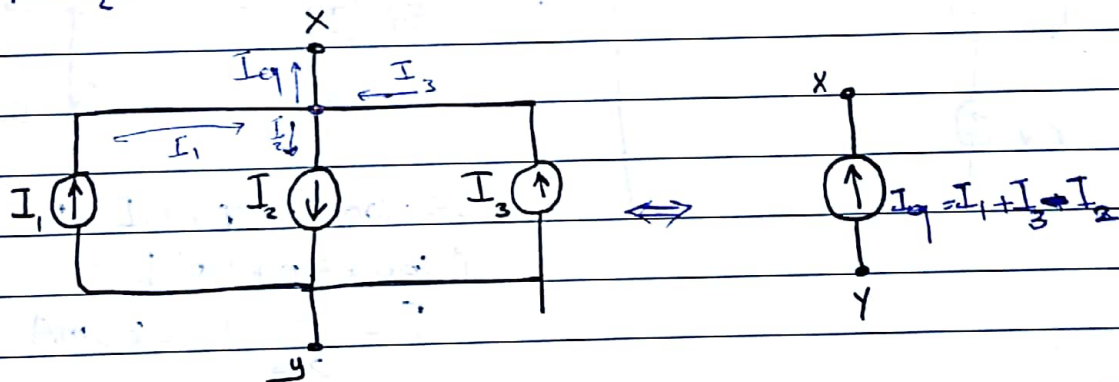


$V_2 \neq V_1$ Problem
only when $V_1 = V_2$

بالقوة التي يجب أن يكون لها نفس الجهد

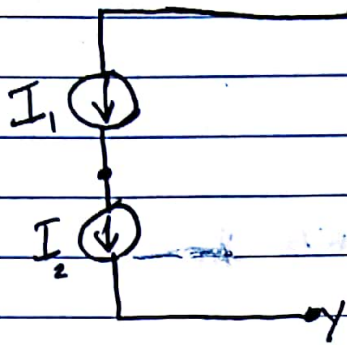
$$-V_1 + V_2 = 0$$

$$V_1 = V_2$$



$$I_1 + I_3 = I_{eq} + I_2$$

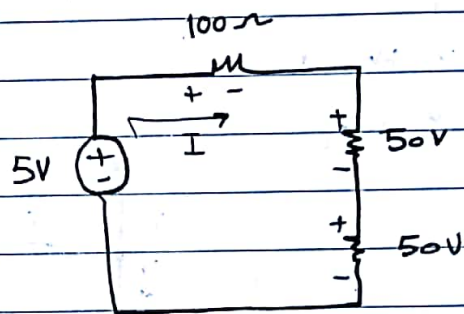
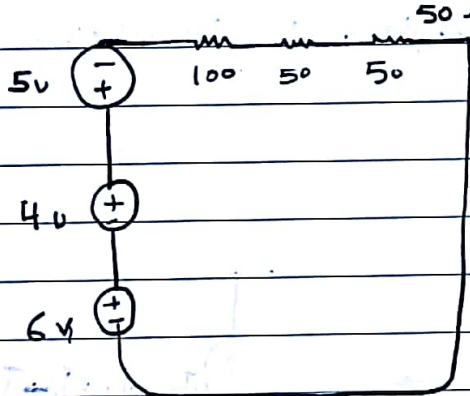
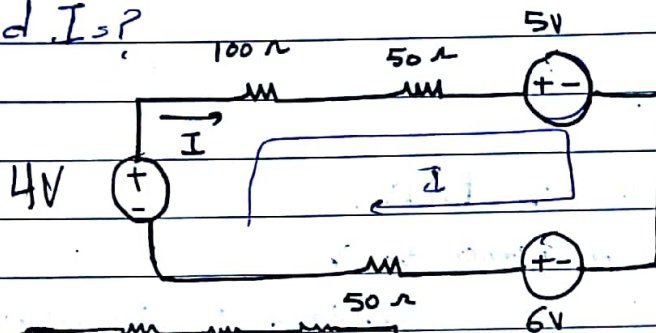
$$I_1 + I_3 - I_2 = I_{eq}$$



If $I_1 \neq I_2$ Problem
 Only if $I_1 = I_2$

$$I_1 = I_2$$

ex: find I ?

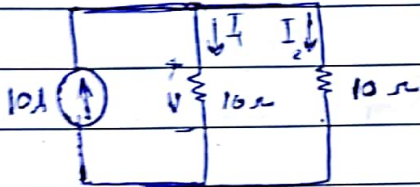
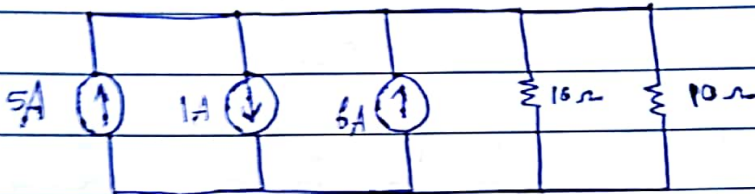
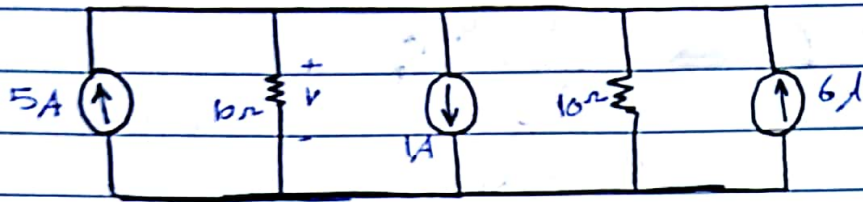


$$-5 + 100I + 5I + 5I = 0$$

$$(100 + 50 + 50)I = 5$$

$$I = \frac{5}{200} \text{ A} = 25 \text{ mA}$$

ex: find $V = ?$



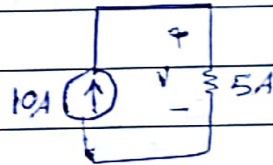
KCL:

$$10 = I_1 + I_2$$

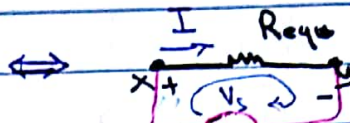
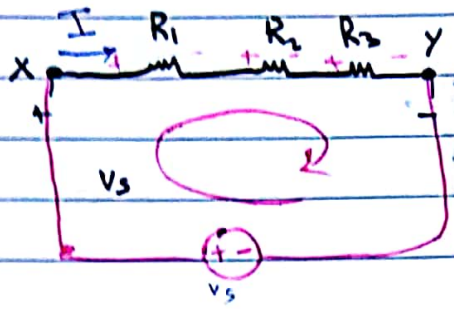
$$10 = \frac{V}{10} + \frac{V}{10}$$

$$10 = V \left(\frac{1}{10} + \frac{1}{10} \right)$$

$$V = 50 \text{ V}$$



* Resistors in series & Parallel :



$$R_{eq} = R_1 + R_2 + R_3$$

$$-V_s + I R_{eq} = 0$$

$$V_s = I R_{eq}$$

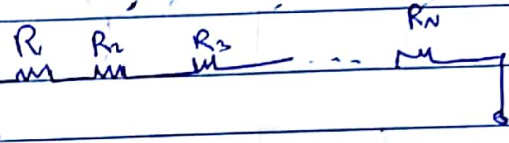
KVL :

$$-V_s + V_1 + V_2 + V_3 = 0$$

$$V_s = V_1 + V_2 + V_3$$

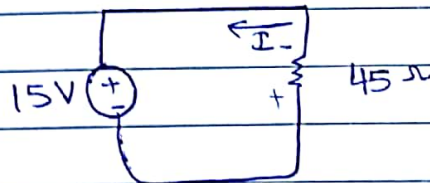
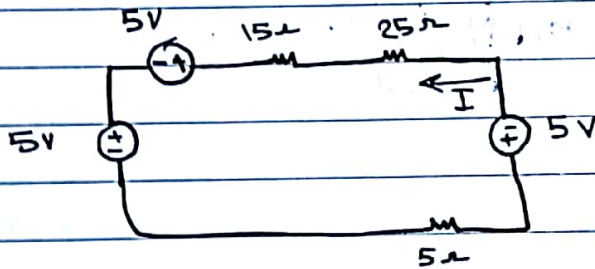
$$= I R_1 + I R_2 + I R_3$$

$$V_s = I (R_1 + R_2 + R_3)$$



$$R_{eq} = \sum_{n=1}^N R_n$$

ex: Determine I in the shown CKT of P15

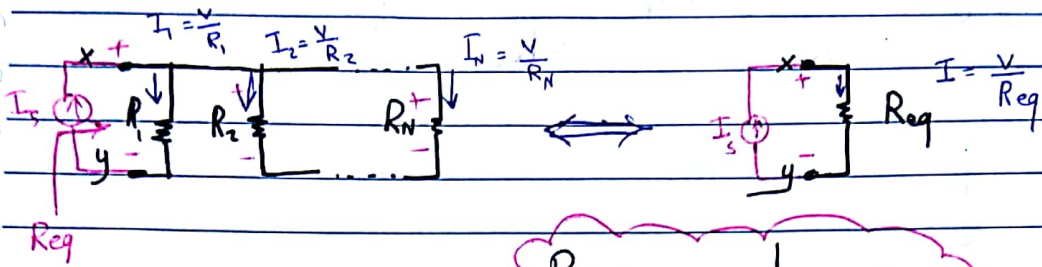


$$-15 - I(45) = 0$$

$$I = \frac{-15}{45}$$

$$P_{15\Omega} = \left(\frac{-15}{45} \right)^2 (15) \text{ W.}$$

* Parallel Resistors :



KCL :

$$I_s = I_1 + I_2 + \dots + I_N$$

$$I_s = \frac{V}{R_1} + \frac{V}{R_2} + \dots + \frac{V}{R_N}$$

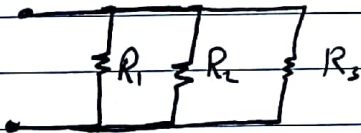
$$I_s = V \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} \right)$$

$$I_s = V \left(\frac{1}{R_{eq}} \right)$$

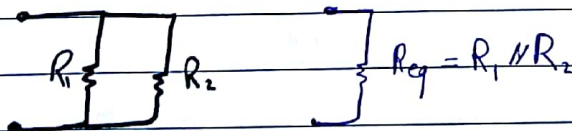
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

$$G_{eq} = G_1 + G_2 + \dots + G_N$$

$$G_{eq} = \sum_{n=1}^N G_n$$



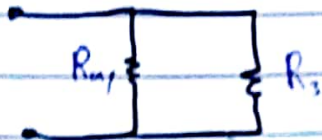
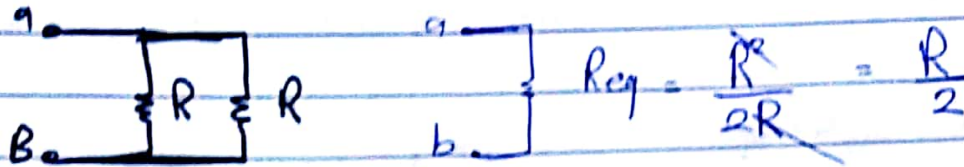
$$R_1 \parallel R_2 \parallel R_3 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

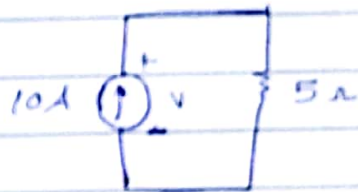
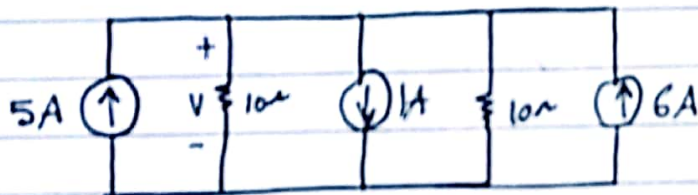


$((R_1 // R_2) // R_3) =$

$R_{eq1} = \frac{R_1 R_2}{R_1 + R_2}$

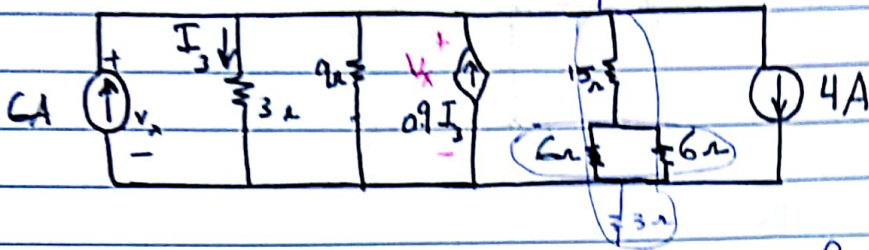
$R_{eq2} = \frac{R_{eq1} \cdot R_3}{R_{eq1} + R_3}$

ex: Determine V in the CKT:



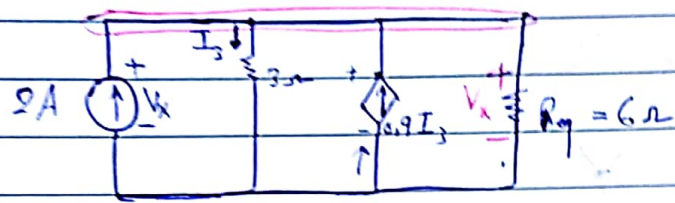
$V = 10 \times 5 = 50V$

Ex: Calculate the Power & Voltage of the dependent source:



15Ω + 3Ω Series
15 + 3 = 18Ω

$$R_{eq} = \frac{18 \times 9}{18 + 9} = 6\Omega$$



KCL:

$$I_3 = \frac{V_x}{3}$$

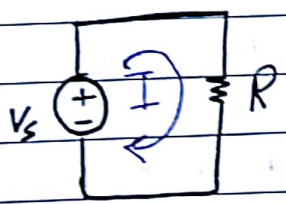
$$2 + 0.9 I_3 - I_3 + \frac{V_x}{6}$$

$$2 + 0.9 \left(\frac{V_x}{3} \right) - \frac{V_x}{3} + \frac{V_x}{6}$$

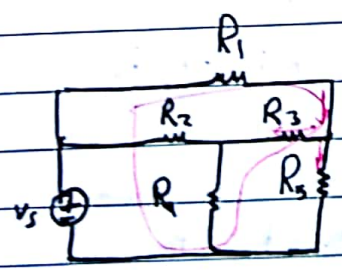
$$\boxed{V_x = 10V} \quad , \quad I_3 = \frac{10}{3} A$$

$$P = -(0.9 \times \frac{10}{3}) (10)$$

$$= -30 \text{ Watt (deliver)}$$



Series & Parallel P
Series and Parallel $\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{S_{eq}}$



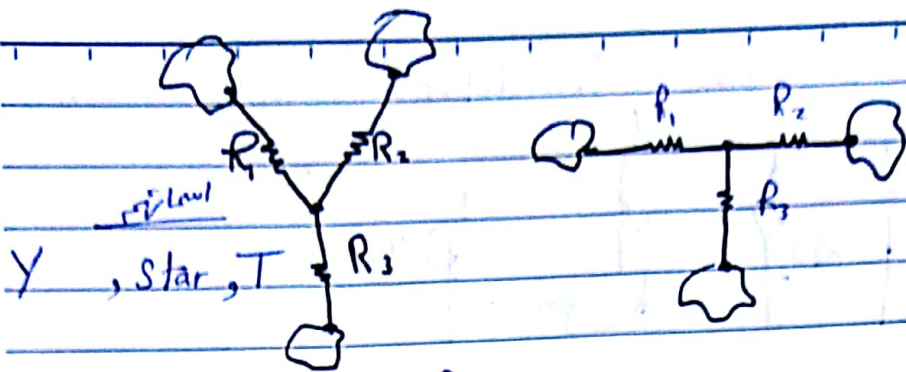
Series & Parallel \times 1000

$$R_3, R_2, R_1 \rightarrow T$$

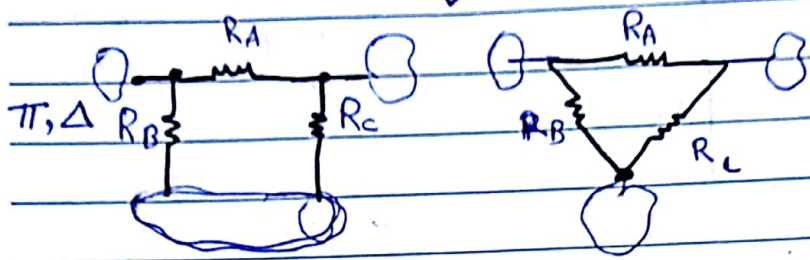
$$R_1, R_2, R_5 \rightarrow T$$

$$R_3, R_4, R_5 \rightarrow \Pi$$

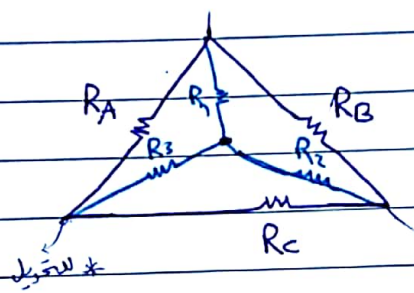
$$R_1, R_2, R_3 \rightarrow \Pi/A$$



Y, Star, T



Π, Δ



$\Delta \rightarrow Y :$

$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$Y \rightarrow \Delta :$

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

در صورتی که $R_1 = R_2 = R_3 = R$

if $R_A = R_B = R_C = R$

$$R_1 = \frac{R}{3}$$

$$R_2 = \frac{R}{3}$$

$$R_3 = \frac{R}{3}$$

در صورتی که $R_1 = R_2 = R_3 = R$

if $R_1 = R_2 = R_3 = R$

$$R_A = 3R$$

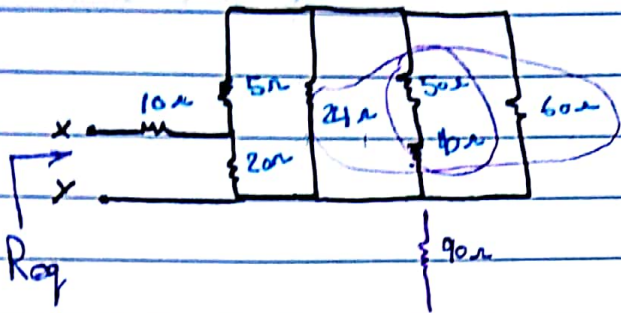
$$R_B = 3R$$

$$R_C = 3R$$

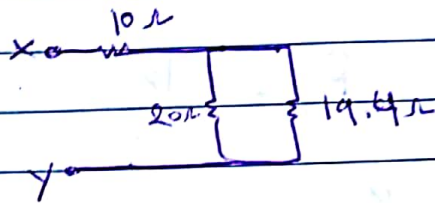
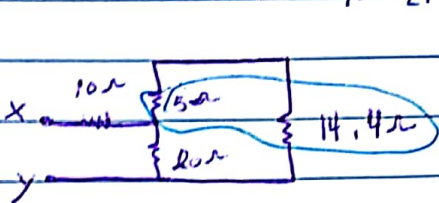
R_{eq} Independent

ex: Find R_{eq} between x & y :

Shorting ckt open leads ← current source
 " Short ckt " ← Voltage

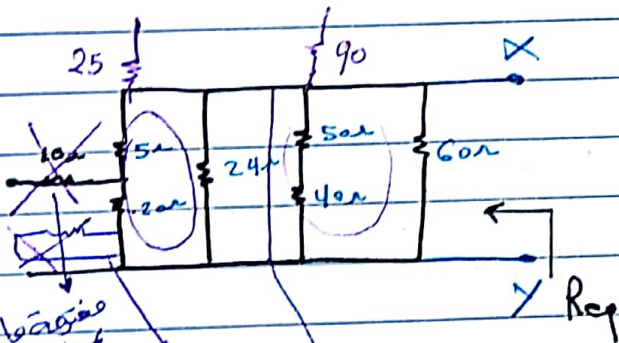


$$R = \frac{1}{\frac{1}{60} + \frac{1}{90} + \frac{1}{24}} = 14.4 \Omega$$



$$R = \frac{20 \times 19.4}{20 + 19.4} = 9.85 \Omega$$

$$R_{eq} = 10 + 9.85 = 19.85 \Omega$$



مفتوحة
 بمضخة I
 طرفين متساويين
 على نفس المكان
 Short ckt.

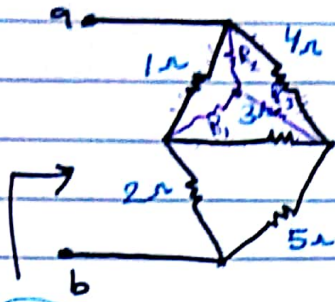
$$R_{eq} = \frac{1}{\frac{1}{25} + \frac{1}{24} + \frac{1}{90} + \frac{1}{60}}$$

=

دائماً نروح الى آخر الدارة لنبعد عن
 المكان .

بم I فيها ولطيفي كطبع Short ckt
 الحقا ومارس

بقی series و Parallel کے اصول استعمال کرو۔
 اسے سنبھالو Δ کی شکل میں



R_{ab}

$$R_1 = \frac{3}{8} = 3 \parallel 8 \parallel 4 \parallel 1 \text{ کے مجموعہ}$$

$$R_2 = \frac{4}{8} = \frac{1}{2} \Omega$$

$$R_3 = \frac{12}{8} = \frac{3}{2} \Omega$$

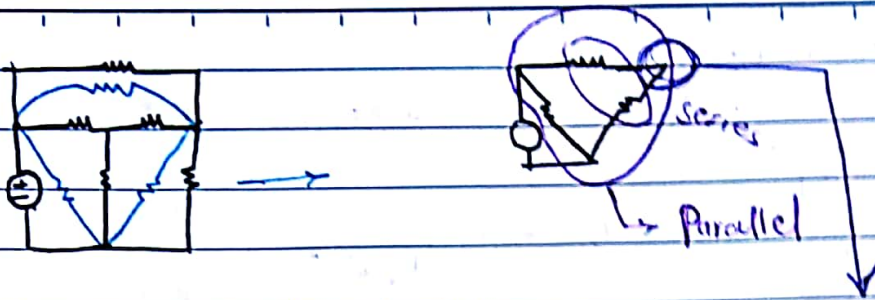
$R_1, 2\Omega \rightarrow$ Series
 $2 + \frac{3}{8} = \frac{19}{8} \Omega$

$R_3, 5\Omega \rightarrow$ Series
 $\frac{3}{2} + 5 = 6.5 \Omega$

$6.5, \frac{19}{8} \rightarrow$ Parallel

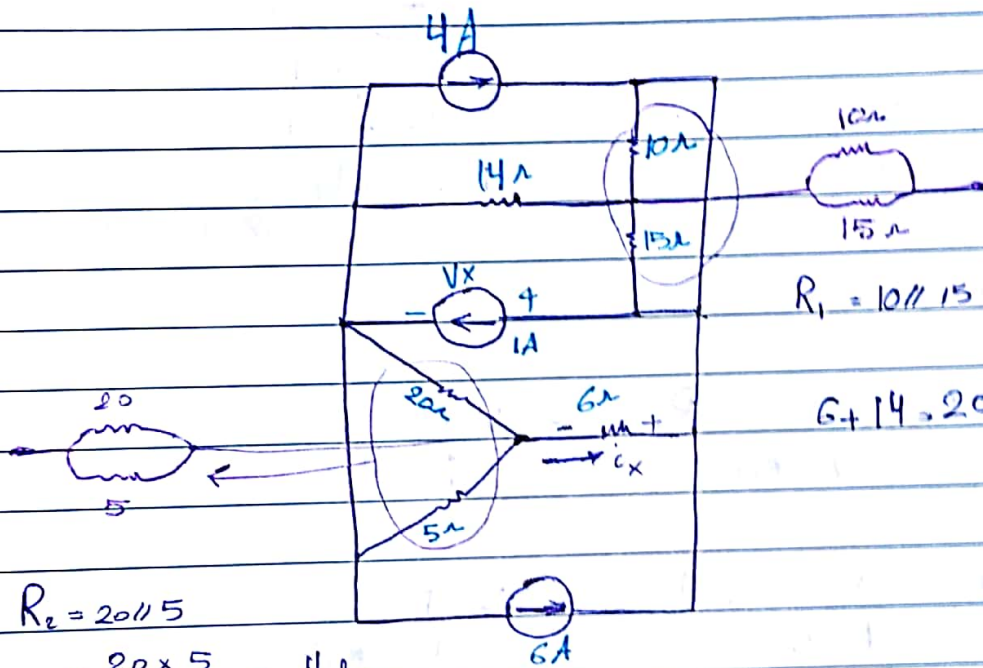
$$R_1 = \frac{19 \parallel 6.5}{8} \rightarrow \frac{\frac{19 \times 6.5}{8}}{\frac{19}{8} + 6.5}$$

$R_1, R_2 \rightarrow$ Series
 $R_1 + R_2 = R_{eq}$
 $+ 2 = R_{eq}$



المقاومة المكافئة للتيار الكهربائي

ex: Find V_x and i_x :



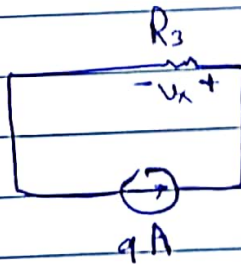
$$R_1 = 10 \parallel 15 = \frac{10 \times 15}{10 + 15} = 6 \Omega$$

$$6 + 14 = 20 \Omega$$

$$R_2 = 20 \parallel 5 = \frac{20 \times 5}{20 + 5} = 4 \Omega$$

$$4 \Omega + 6 \Omega \rightarrow \text{Series} = 10 \Omega$$

$$R_3 = 20 \parallel 10 = \frac{20 \times 10}{20 + 10} = \frac{20}{3} \Omega$$

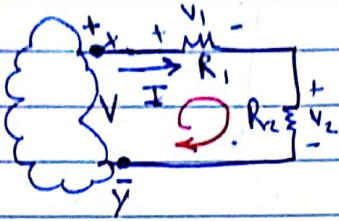


$$V_x = 9 \times \frac{20}{3} = 60 \text{ Volt}$$

$$i_x = \frac{-60}{10} = -6A$$

* Voltage Division :-

Voltage Divider



$$V_1 = V \cdot \frac{R_1}{(R_1 + R_2)}$$

$$V_2 = V \cdot \frac{R_2}{(R_1 + R_2)}$$

$$V_1 = ?$$

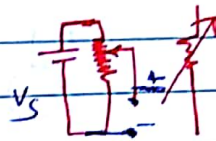
$$V_2 = ?$$

$$V = V_1 + V_2$$

$$V = IR_1 + IR_2$$

$$V = I(R_1 + R_2)$$

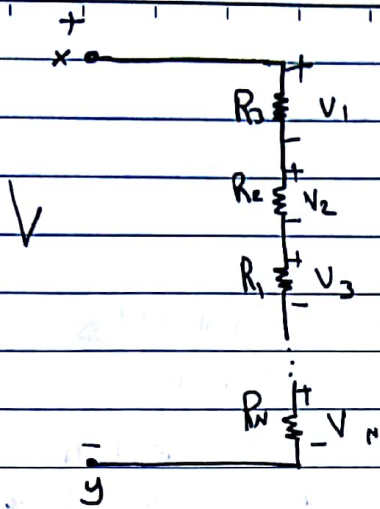
$$I = \frac{V}{(R_1 + R_2)}$$



Potentiometer.

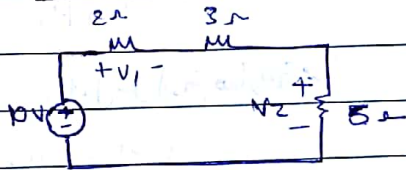
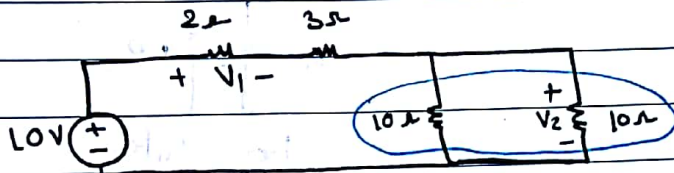
$$V_1 = IR_1 = \frac{V}{(R_1 + R_2)} \cdot R_1 = \frac{V R_1}{(R_1 + R_2)}$$

$$V_2 = IR_2 = \frac{V}{(R_1 + R_2)} \cdot R_2 = \frac{V R_2}{(R_1 + R_2)}$$



$$V_k = V \frac{R_k}{\sum_{n=1}^N R_n}$$

ex: Find V_1 & V_2 :

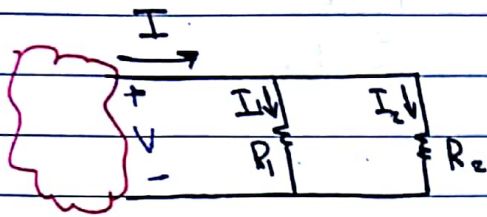


$$V_1 = 10 \cdot \frac{2}{2+3+5} = 2 \text{ volt}$$

$$V_2 = 10 \times \frac{5}{10} = 5 \text{ volt}$$

$$V_{2\Omega} = 3 \text{ volt} \rightarrow V_{5\Omega} = 10 \cdot \text{Lisjo}$$

* Current Division:



هذا المقاومة بأخذها في العالجه الثانيه مشه

المقاومه الموجوده على التيار البتاليه .

التيار عكس مع المقاومات

* بالجهد متردي مع المقاومات .

Voltage Division

$$I_1 = I \cdot \frac{R_2}{(R_1 + R_2)}$$

$$I_2 = I \cdot \frac{R_1}{(R_1 + R_2)}$$

$$V = I (R_1 \parallel R_2)$$



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$V = I \frac{R_1 R_2}{R_1 + R_2}$$

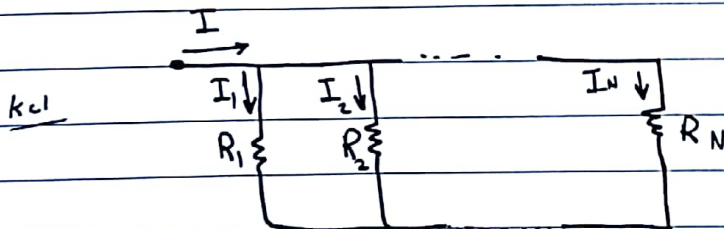
المقاومه المكافئه اقل من اقل مقاومه فيها

(هذا اقل من ٤ و٥)

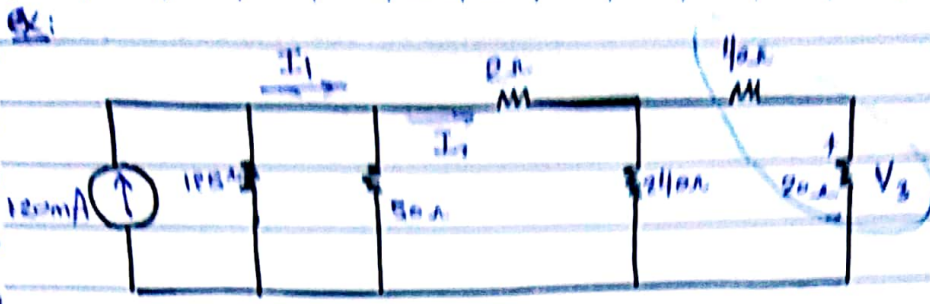
$$I_1 = \frac{V}{R_1} = \frac{I R_1 R_2}{(R_1 + R_2)} \cdot \frac{1}{R_1}$$

$$I_1 = I \frac{R_2}{(R_1 + R_2)}$$

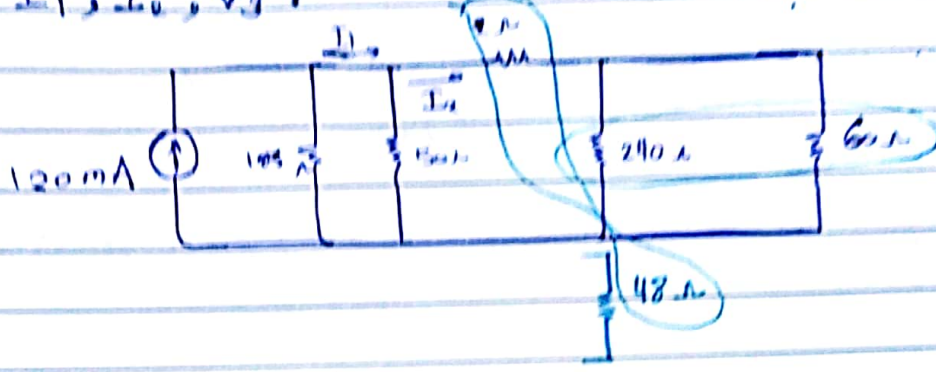
$$I_2 = \frac{V}{R_2} = \frac{I R_1 R_2}{(R_1 + R_2)} \cdot \frac{1}{R_2} = I \frac{R_1}{(R_1 + R_2)}$$



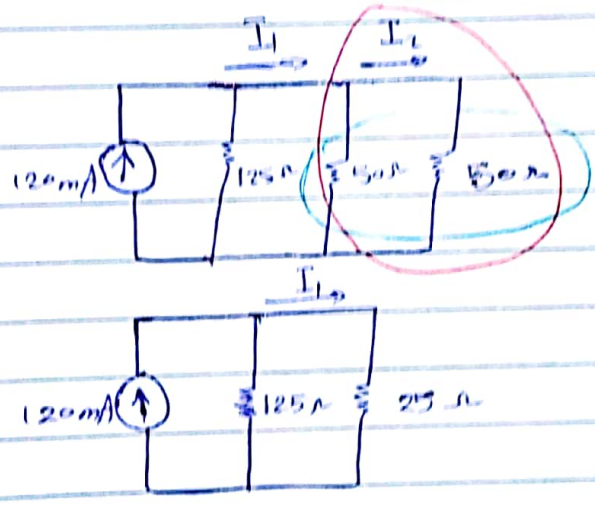
$$I_k = \left(\frac{1/R_k}{\sum_{n=1}^N \frac{1}{R_n}} \right) I = \frac{G_k}{\sum_{n=1}^N G_n} I$$



find I_1, I_2, V_3 .



$$60 \parallel 240 = \frac{60 \times 240}{60 + 240} = 48 \Omega$$

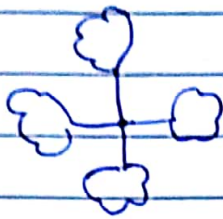


$$I_1 = 120 \times 10^{-3} \times \frac{125}{125 + 50} = 100 \text{ mA}$$

$$I_2 = I_1 \cdot \frac{50}{50 + 50} = 100 \text{ m} \cdot \frac{1}{2} = 50 \text{ mA}$$

$$V_3 = 48 \times 50 \text{ m} \cdot \frac{20}{60} = 0.8 \text{ V}$$

CH3: Nodal Analysis:

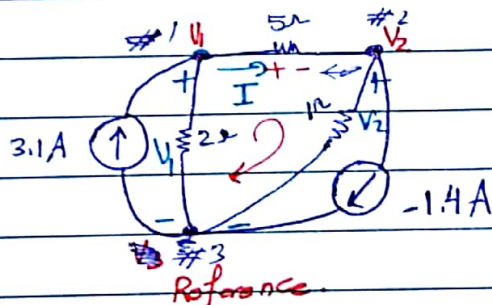
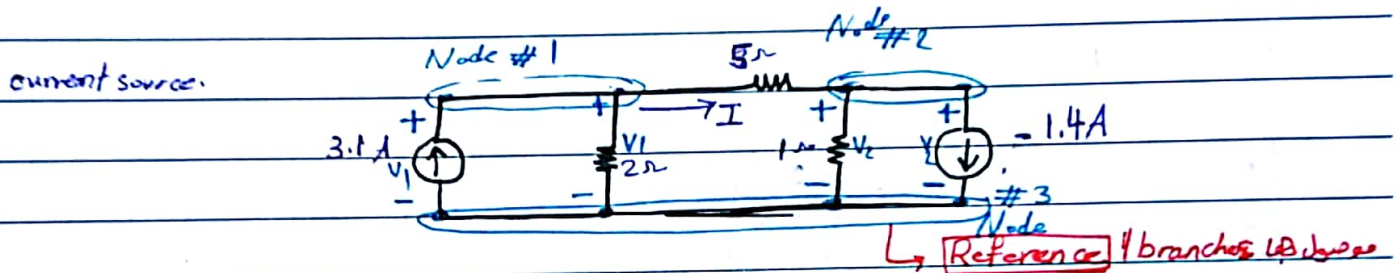


kel:

- N nodes \rightarrow we need $(N-1)$ equations (eq.)
 unknown are voltages \rightarrow (Nodal) Voltages

Reference

Reference Node: the node with the greatest number of branches connected to it.



2 eq / 2 unknown voltage.

Reference in circuit

$$V = I(5)$$

$$-V_1 + V_1 - V_2 = 0$$

$$V = V_1 - V_2$$

$$I = \frac{V - V_2}{5}$$

↓

التيار الخارج عن المقاومة

Q1 node #1 :

$$\frac{V_1 - V_2}{5} + \frac{V_1 - 0}{2} = 3.1$$

$$0.7 V_1 - 0.2 V_2 = 3.1 \dots (1)$$

Q2 node #2 :

$$\frac{V_2 - V_1}{5} + \frac{V_2 - 0}{1} = 1.4 = 0$$

$$-0.2 V_1 + 1.2 V_2 = 1.4 \dots (2)$$

Cramer's Rule:

$$\begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3.1 \\ 1.4 \end{bmatrix}$$

Solve:

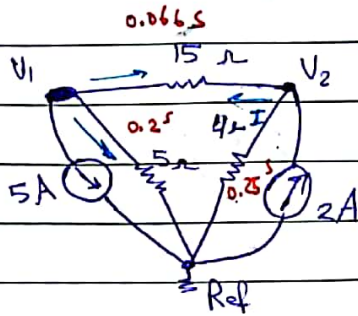
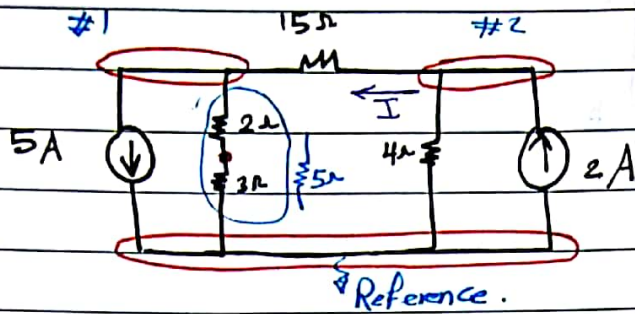
$$V_1 = 5V \quad V_2 = 2V$$

Nodal Voltages.

$$I = \frac{V_1 - V_2}{5} = \frac{3}{5} A$$

$$P_{5\Omega} = \left(\frac{3}{5}\right)^2 \times 5 = \frac{9}{5} \times 5 \text{ watt}$$

ex: Find $I = ?$



① Node # 1 :

$$5 + \frac{V_1}{5} + \frac{V_1 - V_2}{15} = 0$$

$$5 + 0.2 V_1 + 0.066 (V_2 - V_1) = 0$$

$$\frac{4}{15} V_1 - \frac{1}{15} V_2 = -5 \quad \dots \textcircled{1}$$

إذا امكننا الكتابة بالـ S Conductance

$$I = \frac{V}{R} = GV$$

② Node # 2 :

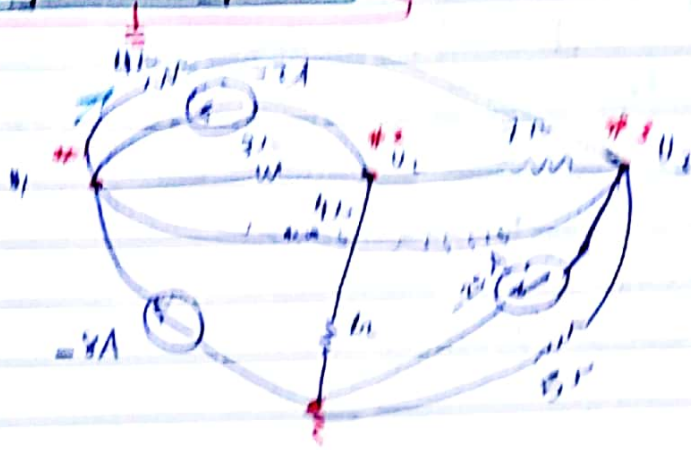
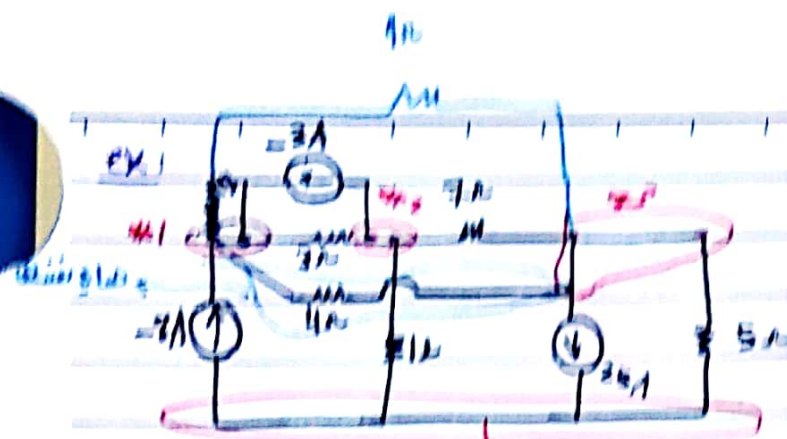
$$\frac{V_2 - V_1}{15} + \frac{V_2}{4} = 2$$

$$0.066 (V_2 - V_1) + 0.25 V_2 = 2$$

$$-\frac{1}{15} V_1 + \frac{19}{60} V_2 = 2 \quad \dots \textcircled{2}$$

$$V_1 = -\frac{145}{8} V \quad V_2 = \frac{5}{2} V$$

$$I = \frac{V_2 - V_1}{15} = 2.578 A$$



Node #1 :

$$\frac{V_1 - V_2}{4} + \frac{V_1 - V_2}{3} = -3 + -6$$

$$\frac{7}{12} V_1 - \frac{1}{3} V_2 = -\frac{1}{4} V_2 = -11 \quad \text{--- (1)}$$

Node #2 :

$$-3 + \frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_2 - V_3}{1} = 0$$

$$-\frac{1}{3} V_1 + \frac{21}{21} V_2 - \frac{1}{1} V_2 = 3 \quad \text{--- (2)}$$

Node #3 :

$$\frac{V_3 - V_2}{7} + \frac{V_3 - V_1}{4} + \frac{V_3}{5} = 25 = 0$$

$$-\frac{1}{4} V_1 - \frac{1}{7} V_2 + \frac{76}{140} V_3 = 25 \quad \text{--- (3)}$$

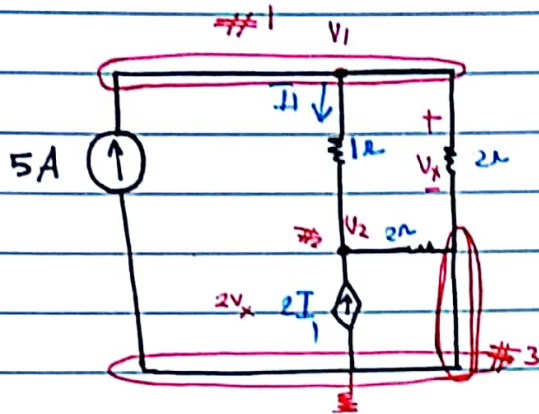
Cramer's Rule:

$$V_1 = 5.412 \text{ V}$$

$$V_2 = 7.736 \text{ V}$$

$$V_3 = 46.32 \text{ V}$$

ex:



$$V_x \leftarrow I_1 \text{ dir. Lilit } \overline{\text{net Lilit}}$$

Node #1:

$$\frac{V_1 - V_2}{1} + \frac{V_1}{2} = 5$$

$$\frac{3}{2} V_1 - V_2 = 5 \dots (1)$$

Node #1:

$$\frac{V_1 - V_2}{1} + \frac{V_1 - 0}{2} = 5$$

$$\frac{3}{2} V_1 - V_2 = 5 \dots (1)$$

Node #2:

$$\frac{V_2}{2} + \frac{V_2 - V_1}{1} = 2 V_x$$

$$V_x = V_1$$

$$-3 V_1 + 3 V_2 = 0 \dots (2)$$

Node #2:

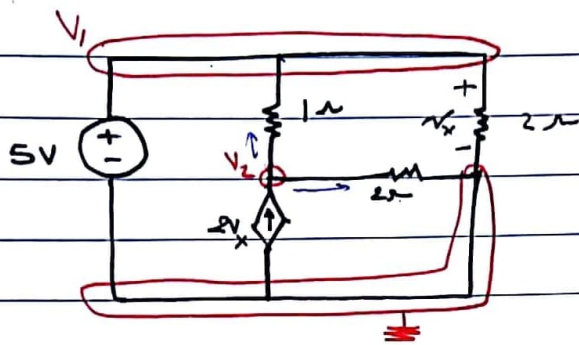
$$V_1 = -10 \text{ V}, V_2 = -20 \text{ V}$$

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} = 2 I_1$$

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} = 2 \left(\frac{V_1 - V_2}{1} \right)$$

$$-3 V_1 + \frac{7}{2} V_2 = 0 \dots (2)$$

$$V_1 = \frac{70}{9}, V_2 = \frac{20}{3}$$



① Node #1

$$V_1 = 5V$$

② Node #2: kcl

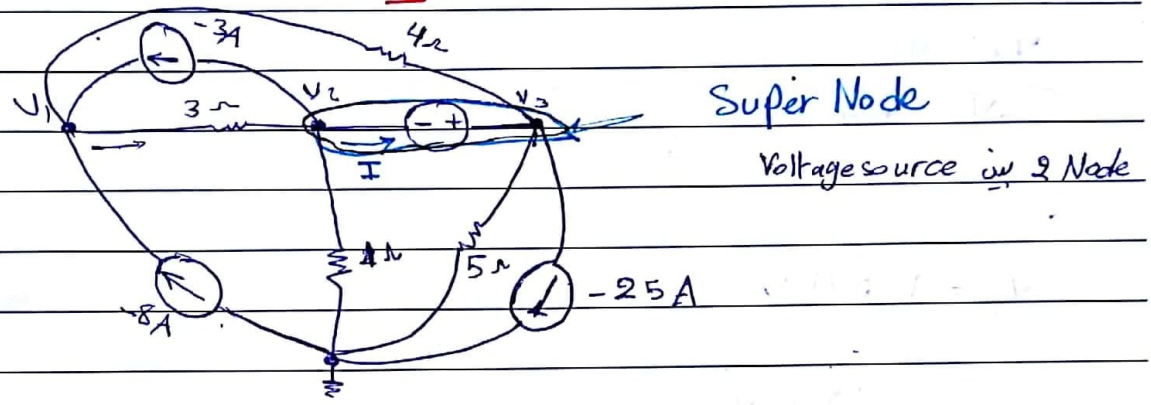
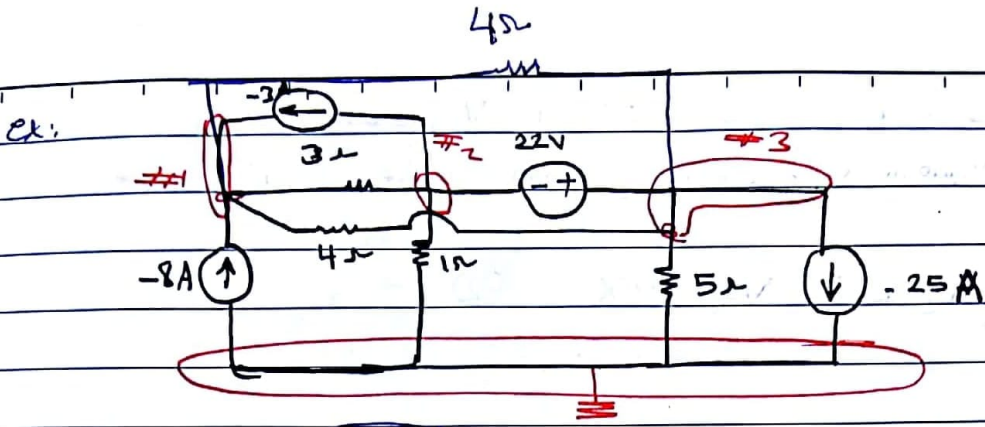
$$\frac{V_2}{2} + \frac{V_2 - 5}{1} = 2V_x$$

$$V_x = V_1 = 5$$

$$\frac{V_2}{2} + V_2 - 5 = 10$$

$$V_2 = \frac{30}{3} V$$

Nodal Voltage is a Voltage source.



③ Node #1 :-

$$\frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4} = -3 - 8$$

$$\boxed{\frac{7}{12} V_1 - \frac{1}{3} V_2 - \frac{1}{4} V_3 = -11} \quad \dots (1)$$

④ Node #2 ;

$$\frac{V_2}{1} + \frac{V_2 - V_1}{3} + -3 + I = 0 \dots *$$

∴ is not Super Node.

⑤ Node #3 :-

$$\frac{V_2}{5} + -25 + \frac{V_3 - V_1}{4} + -I = 0 \dots *$$

$$\frac{V_2 - V_1}{3} + \frac{V_2}{1} + \frac{V_3}{5} + -25 + \frac{V_3 - V_1}{4} = 0 \dots (2)$$

$$V_2 - V_1 + V_2 - 3 + V_3 + -25 + V_3 - V_1 = 0$$

مجموع I الكارمين 4 + مجموع I الكارمين 5 = مجموع I الكارمين 3 + مجموع I الكارمين 1

$$\boxed{-\frac{7}{12} V_1 + \frac{4}{3} V_2 + \frac{9}{20} V_3 = 28} \dots (2)$$

$$V_3 - V_2 = 22$$

$$\boxed{-V_2 + V_3 = 22} \dots (3)$$

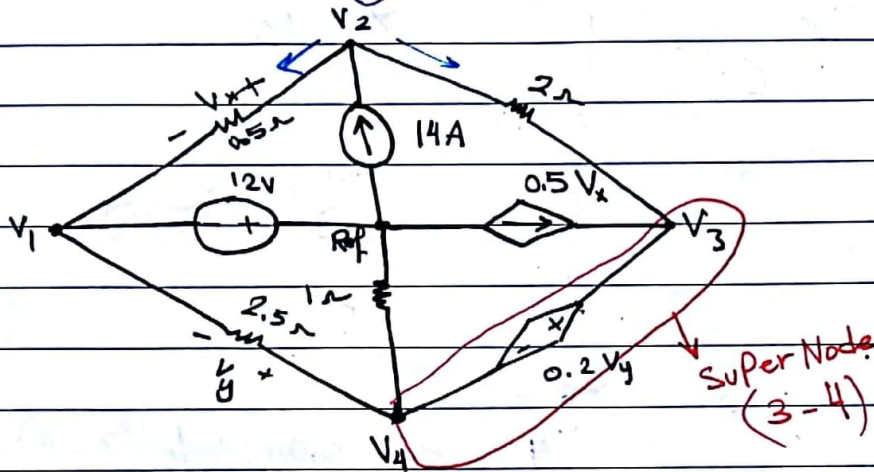
Sol :

$$V_1 = 1.071 V$$

$$V_2 = 10.5 V$$

$$V_3 = 32.5 V$$

ex: Determine the Nodal Voltage in the CKT:



⑧ Node #1 :

$$V_1 = -12 \text{ Volt.}$$

⑨ Node #2 : Kcl

$$\frac{V_2 - (-12)}{0.5} + \frac{V_2 - V_3}{2} = 14$$

$$2.5 V_2 - 0.5 V_3 = -10 \dots \textcircled{1}$$

⑩ ^{super} Node # (3-4) :

$$\frac{V_3 - V_2}{2} + \frac{V_4}{1} + \frac{V_4 - (-12)}{2.5} = 0.5 V_x$$

$$V_x = V_2 - V_1$$

$$V_x = V_2 + 12 \textcircled{*}$$

$$\frac{V_3 - V_2}{2} + V_4 + \frac{V_4 + 12}{2.5} = 0.5 (V_2 + 12)$$

$$-V_2 + \frac{1}{2} V_3 + \frac{7}{5} V_4 = 1.2 \dots \textcircled{2}$$

$$V_3 - V_4 = 0.2 V_y$$

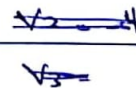
$$V_y = V_4 - V_1$$

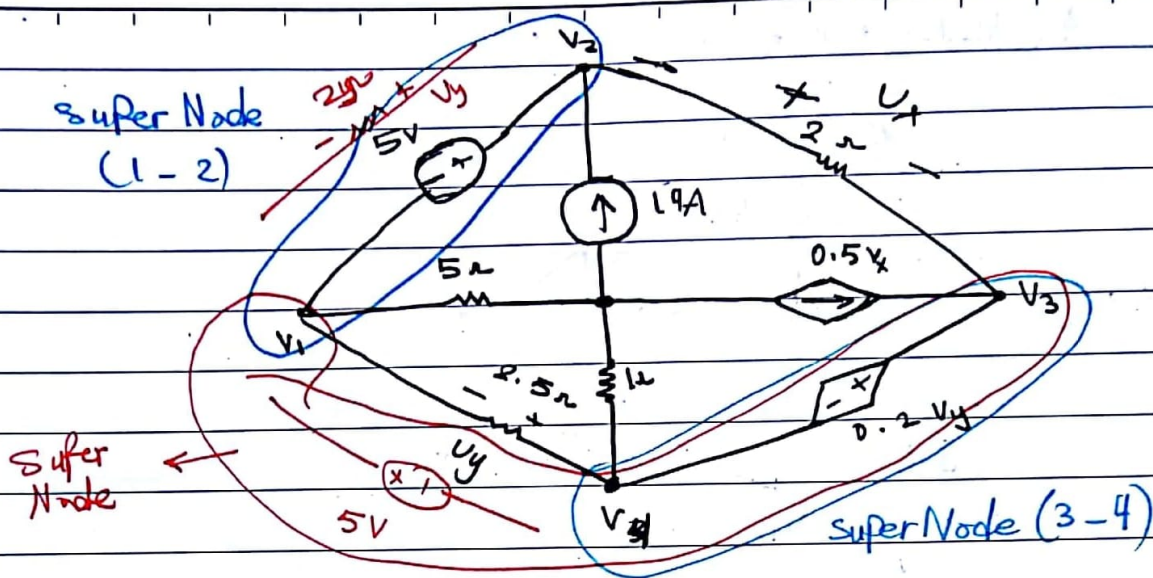
$$V_3 - V_4 = 0.2 (V_4 + 12)$$

$$V_y = V_4 + 12$$

$$V_3 - \frac{6}{5} V_4 = 2.4 \dots \textcircled{3}$$

super
Node





Super Node (1-2):

$$\frac{V_1}{5} + \frac{V_1 - V_4}{2.5} + \frac{V_2 - V_3}{2} = 14 \dots (1)$$

$$V_2 - V_1 = 5 \dots (2)$$

Super Node (3-4):

$$\frac{V_3 - V_2}{2} + \frac{V_4}{1} + \frac{V_4 - V_1}{2.5} = 0.5(V_2 - V_3) \dots (3)$$

$$V_3 - V_4 = 0.2(V_4 - V_1) \dots (4)$$

* CKT. Jib. binc

Super Node (1,3,4):

$$\frac{V_1 - V_2}{2} + \frac{V_1}{5} + \frac{V_4}{1} + \frac{V_3 - V_2}{2} = 0.5(V_2 - V_3) \dots (1)$$

$$V_3 - V_4 = 0.2(V_2 - V_1) \dots (2)$$

$$V_1 - V_4 = 5 \dots (3)$$

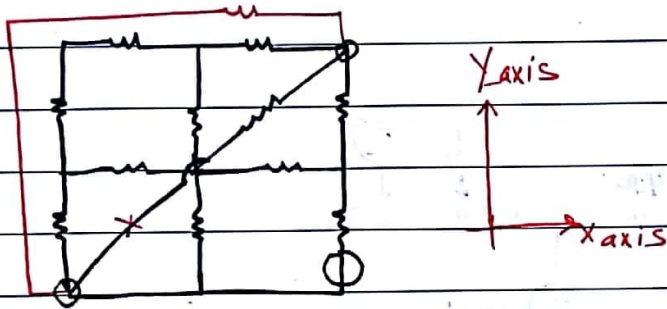
Node #2:

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{2} = 14 \dots (4)$$

* Mesh Analysis:
(KVL)

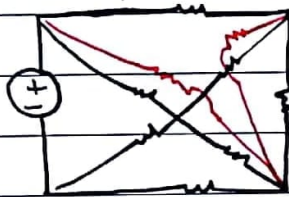
في شروط الاستخدام Mesh كل CKT
بالترتيب

Planar CKT: a CKT which can be drawn in a plane surface.

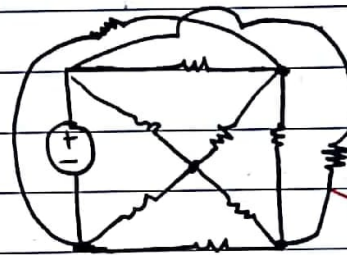


Planar.

4-Meshes → 4 دوائر مغلقة داخلية



Planar.



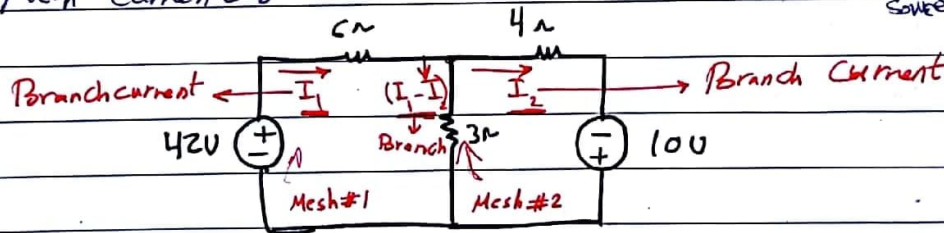
هناك بهارتيرب على z-axis

Non planar.

Mesh: a loop that does not contain any other loop within it.

* Mesh Current :-

Current Mesh ← Current source



Mesh #1 :

$$-42 + 6I_1 + 3(I_1 - I_2) = 0$$

$$9I_1 - 3I_2 = 42 \dots (1)$$

9) Mesh #2 :

$$3(I_2 - I_1) + 4I_2 - 10 = 0$$

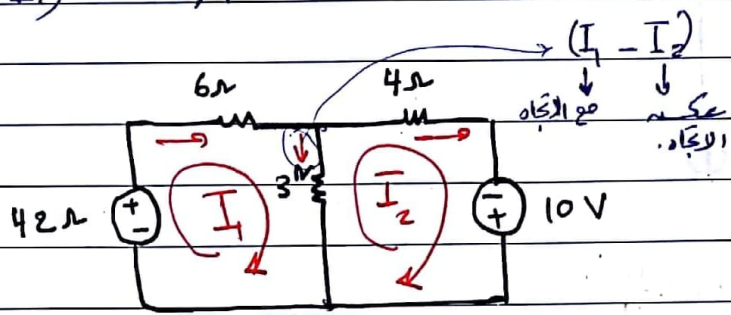
$$-3I_1 + 7I_2 = 10 \dots (2)$$

$$I_1 = 6A$$

$$I_2 = 4A$$

$$(I_1 - I_2) = 2A$$

ex:



I_1 mesh current.

I_2 mesh current.

9) Mesh #1 :

$$-42 + 6I_1 + 3(I_1 - I_2) = 0$$

$$9I_1 - 3I_2 = 42 \dots (1)$$

9) Mesh #2 :

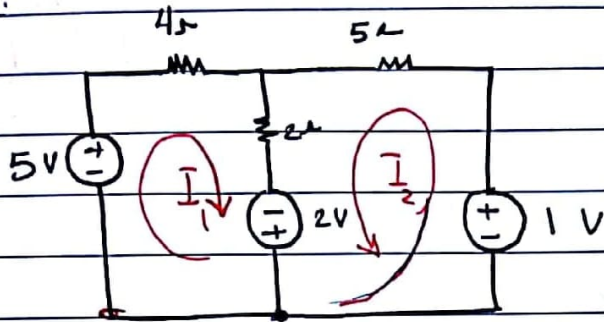
$$3(I_2 - I_1) + 4I_2 - 10 = 0$$

$$-3I_1 + 7I_2 = 10 \dots (2)$$

$$P_{3\Omega} = (I_2 - I_1)^2 \times 3$$

$$V_{3\Omega} = (I_2 - I_1) \times 3$$

ex:



⑨ Mesh #1:

$$-5 + 4I_1 + 2(I_1 + I_2) + 2 = 0$$

$$6I_1 + 2I_2 = 7 \quad \dots \textcircled{1}$$

⑨ Mesh #2:

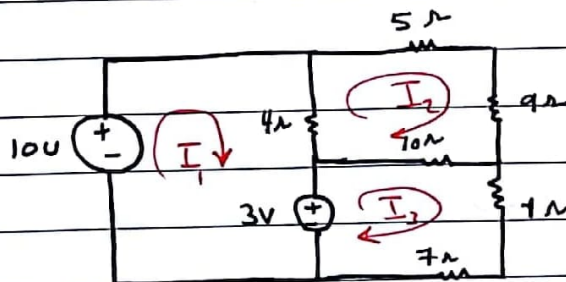
$$-1 + 5I_2 + 2(I_2 + I_1) - 2 = 0$$

$$2I_1 + 7I_2 = 3$$

$$I_1 = \frac{43}{38} \text{ A}, \quad I_2 = \frac{2}{19} \text{ A}$$

$$P_{1V} = -I_2(1)$$

ex: Determine I_1, I_2, I_3 :



9) Mesh #1 :

$$-10 + 4(I_1 - I_2) + 3 = 0$$

$$4I_1 - 4I_2 = 7 \dots \textcircled{1}$$

Mesh = 3 Eq₄

Node = 7 - 1

↓
6 eq₄

9) Mesh #2 :

$$4(I_2 - I_1) + 5I_2 + 9I_2 + 10(I_2 - I_3) = 0$$

$$-4I_1 + 28I_2 - 10I_3 = 0 \dots \textcircled{2}$$

9) Mesh #3 :

$$-3 + 10(I_3 - I_2) + 1I_3 + 7I_3 = 0$$

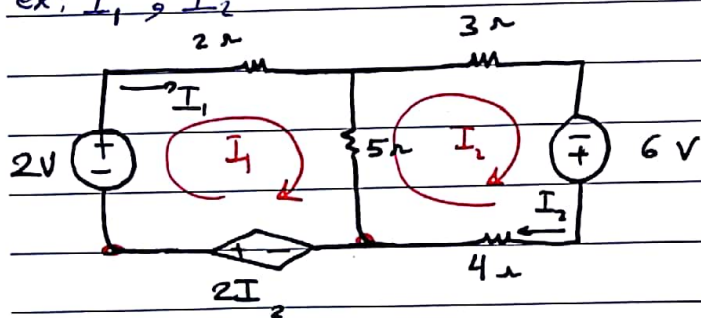
$$-10I_2 + 18I_3 = 3 \dots \textcircled{3}$$

$$I_1 = 2.22 \text{ A}$$

$$I_2 = 47 \text{ mA}$$

$$I_3 = 427.7 \text{ mA}$$

ex: I_1 & I_2



9) Mesh #1 :

$$-2 + 2I_1 + 5(I_1 - I_2) - 2I_2 = 0$$

$$7I_1 - 7I_2 = 2 \dots \textcircled{1}$$

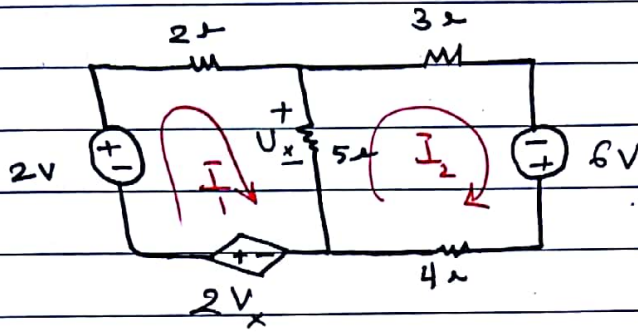
9) Mesh #2 :

$$5(I_2 - I_1) + 3I_2 - 6 + 4I_2 = 0$$

$$-5I_1 + 12I_2 = 6 \dots \textcircled{2}$$

$$I_1 = \frac{16}{49} \text{ A} \quad I_2 = \frac{56}{49} \text{ A}$$

ex:



① Mesh #1:

$$\begin{aligned} -2 + 2I_1 + 5(I_1 - I_2) - 2(U_x) &= 0 & U_x &= 5(I_1 - I_2) \\ -2 + 2I_1 + 5(I_1 - I_2) - 2(5(I_1 - I_2)) &= 0 \\ -3I_1 + 5I_2 &= 2 \quad \text{--- ①} \end{aligned}$$

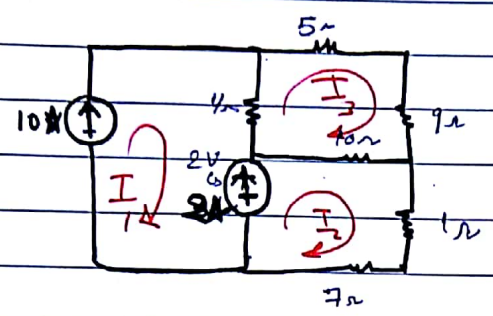
② Mesh #2:

$$\begin{aligned} 5(I_2 - I_1) + 3I_2 - 6 + 4I_2 &= 0 \\ -5I_1 + 12I_2 &= 6 \quad \text{--- ②} \end{aligned}$$

$$I_1 = \frac{6}{11} \text{ A} \quad , \quad I_2 = \frac{8}{11} \text{ A}$$

ex 1:

Reference direction ← Current Source
يسهل علينا الحل



KVL:

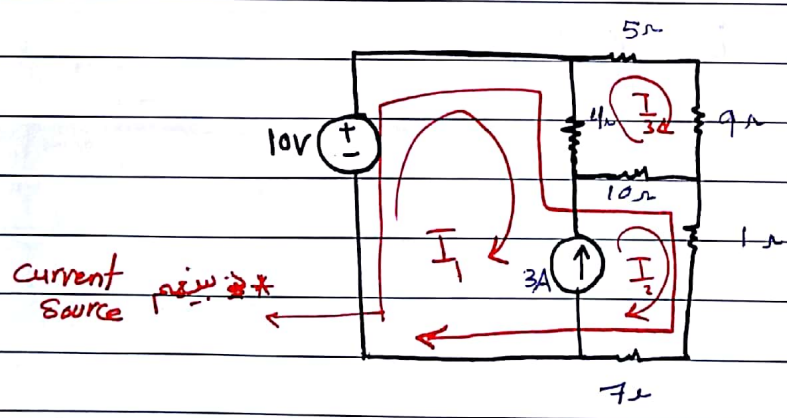
Mesh #1: $I_1 = 10A$

Mesh #2:

$$-2 + 10(I_2 - I_3) + I_2 + 7I_2 = 0 \dots (2)$$

Mesh #3:

$$4(I_3 - I_1) + 5I_3 + 9I_3 + 10(I_3 - I_2) = 0 \dots (3)$$



Mesh #1:

$$-10 + 4(I_1 - I_3) = 0 \dots (1)$$

Mesh #2:

$$-3 + 10(I_2 - I_3) + I_2 + 7I_2 = 0 \dots$$

$$-10 + 4(I_1 - I_3) + 10(I_2 - I_3) + I_2 + 7I_2 = 0 \dots (1)$$

Mesh #3:

$$4(I_3 - I_1) + 5I_2 + 9I_3 + 10(I_3 - I_2) = 0$$

$$-4I_1 - 10I_2 + 3I_3 = 0 \dots (2)$$

$$3 = I_2 - I_1 \rightarrow \boxed{-I_1 + I_2 = 3} \dots (3)$$

Super Mesh (1-2):

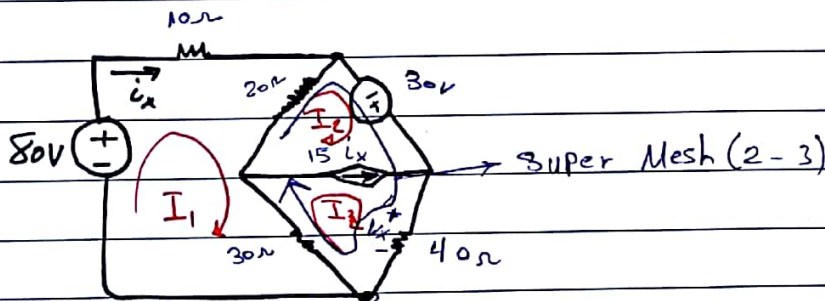
KVL:

$$-10 + 4(I_1 - I_3) + 10(I_2 - I_3) + 1I_2 + 7I_2 = 0$$

$$4I_1 + 18I_2 - 14I_3 = 10 \dots (1)$$

$$-I_1 + I_2 = 3 \dots (2)$$

$$i_x = V_x = P$$



3 unknown \rightarrow 3 Equ

Mesh #1:

$$-80 + 10I_1 + 20(I_1 - I_2) + 30(I_1 - I_2) = 0$$

$$60I_1 - 20I_2 - 30I_3 = 80 \dots (1)$$

$$V_x = 40I_3$$

$$= 104.16 \text{ Volt}$$

Super

Mesh (2-3):

$$30(I_3 - I_1) + 20(I_2 - I_1) + 30 + 40I_3 = 0$$

$$-50I_1 + 20I_2 + 70I_3 = 30 \dots (2)$$

$$15i_x = I_3 - I_2 \quad (i_x = I_1)$$

$$\frac{I_1 = 87}{149} \text{ A}$$

$$\frac{I_2 = -917}{149} \text{ A}$$

$$\frac{I_3 = 388}{149} \text{ A}$$

$$15I_1 + I_2 - I_3 = 0 \dots (3)$$

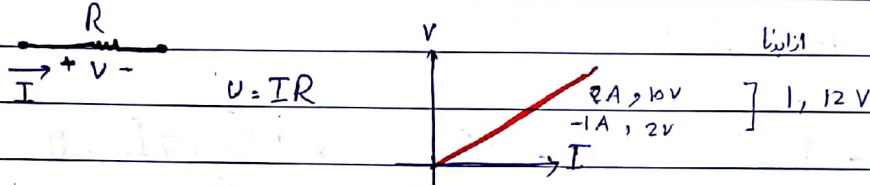
* Linearity & Superposition:

Linear & simple circuit

* Linear CKT: Composed of linear elements

* Linear element: an element that has a linear voltage-current relationship.

* Resistor: (Linear element)



Let $I_{new} = kI$, k constant.

$$\rightarrow V_{new} = RI_{new}$$

$$= R(kI) = k(RI)$$

$$V_{new} = kV$$

$$V_1 = RI_1, \quad V_2 = RI_2$$

$$I_{new} = I_1 + I_2$$

$$V_{new} = R(I_{new})$$

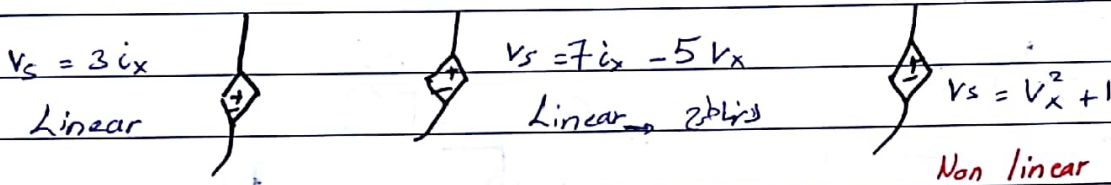
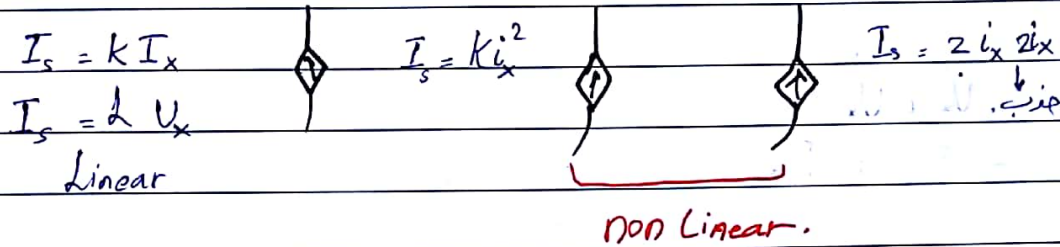
$$= R(I_1 + I_2) = RI_1 + RI_2$$

$$V_{new} = V_1 + V_2$$

* Independent Source \rightarrow Linear.

* Dependent Sources: \rightarrow linear
 \rightarrow non-linear.

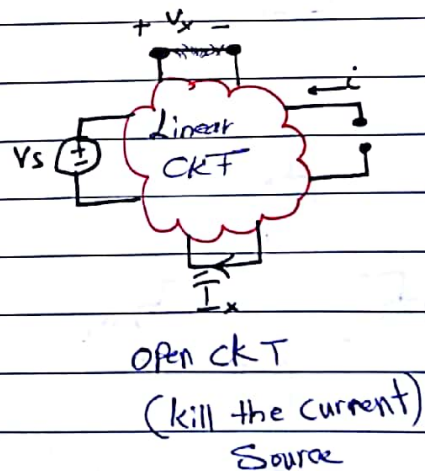
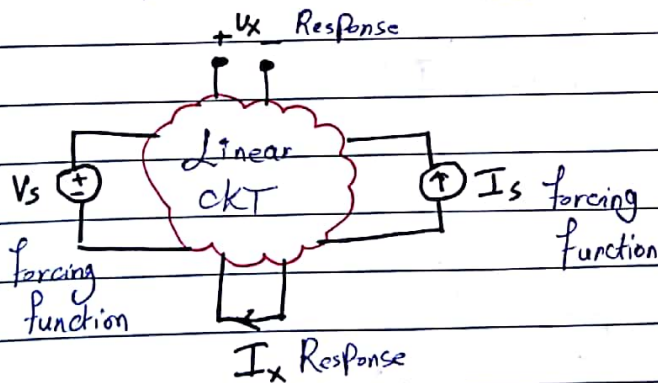
Linear $V_s \propto$ First power of the Variable
 I_s

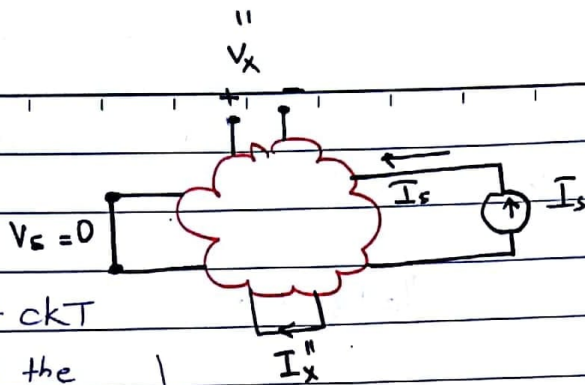


\rightarrow Linear CKT: Then we can apply Superposition

* Superposition:

The response in a linear CKT having more than one independent source can be obtained by adding the responses caused by the separate independent sources acting alone.



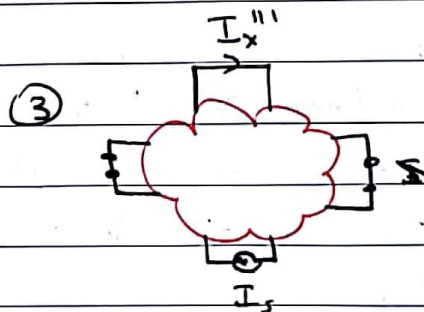
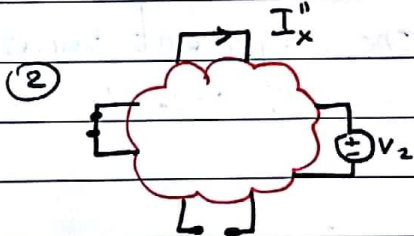
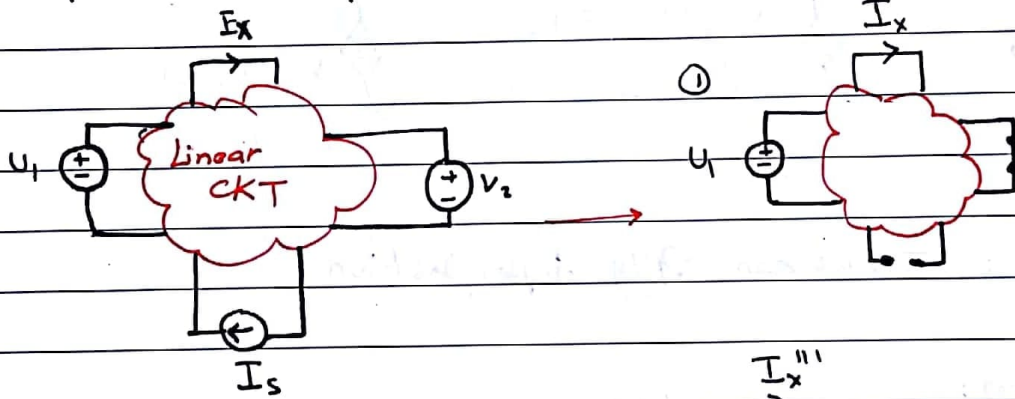


Short ckt
(kill the voltage source)

$$V_x = V_x' + V_x''$$

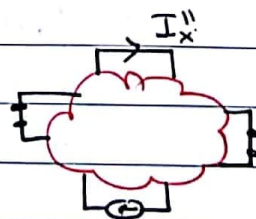
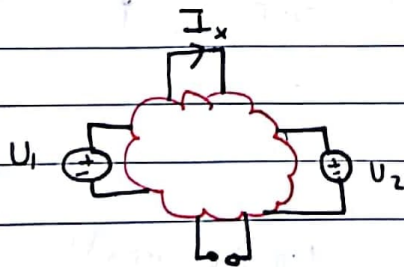
$$I_x = I_x' + I_x''$$

* Super position Principle :

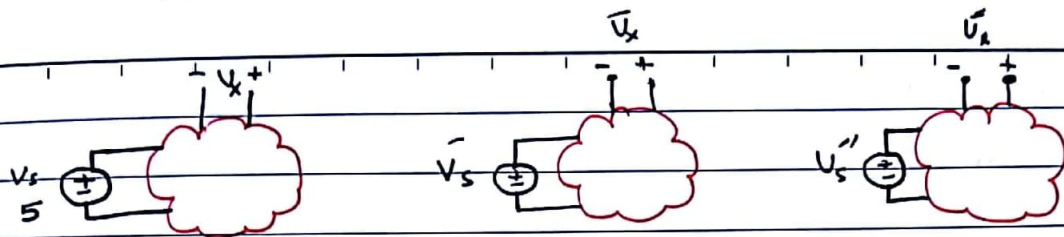


$$I_x = I_x' + I_x'' + I_x'''$$

* ممکنہ اسٹیل گل source و سیرل دوسر

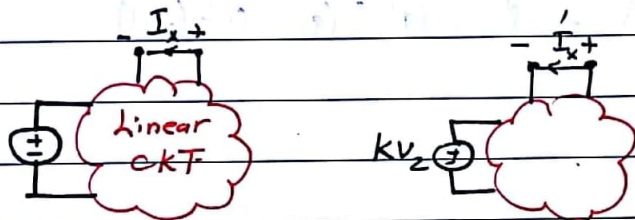


$$I_x = I_x' + I_x''$$



$$V_s = V_s' + V_s''$$

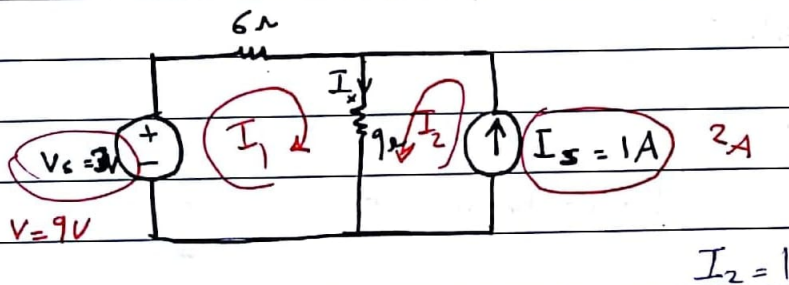
$$V_x = V_x' + V_x''$$



بفرض كخطوة بجزء على جزء الآخر

$$I_x' = \underline{k} I_x$$

ex: I_x , P_x



$$I_2 = 1$$

Mesh #1:

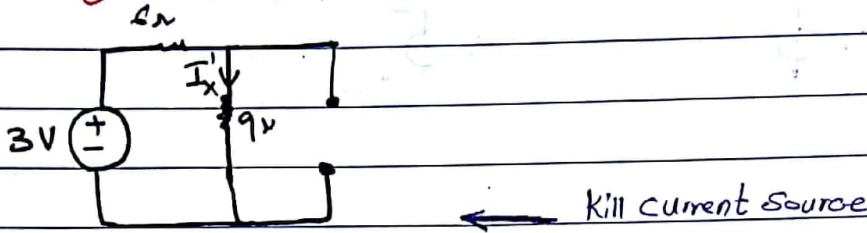
$$-3 + 6I_1 + 9(I_1 + 1) = 0$$

$$15I_1 = -6 \rightarrow I_1 = \frac{-6}{15} \text{ A}$$

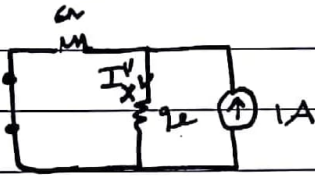
$$I_x = I_1 + 1 = \frac{-6}{15} + 1 = 0.6 \text{ A}$$

$$P_x = 9I_x^2 = 9(0.6)^2 = 3.24 \text{ W}$$

by Superposition:



$$I'_x = \frac{3}{6+9} = \frac{3}{15} = 0.2A \rightarrow P'_x = (I'_x)^2 (9) = 0.36W$$



$$I''_x = 1 \times \frac{6}{6+9} = 0.4A \rightarrow P''_x = (I''_x)^2 (9) = 1.44W$$

$$I'_x + I''_x = 0.6A \checkmark$$

$$P'_x + P''_x \stackrel{?}{=} P_x$$

$$\times 0.36 + 1.44 \stackrel{?}{=} 3.24 \rightarrow \text{non linear} \quad \text{مساواة غير متساوية}$$

Voltage and Current \rightarrow Linear

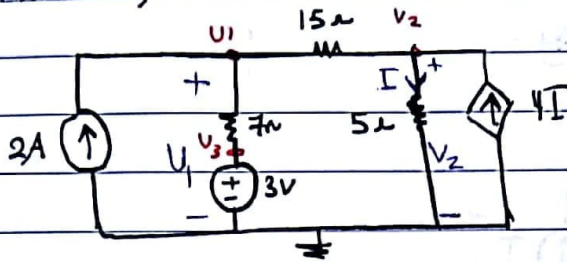
Find $I_{x_{\text{new}}}$ When $V_s = 9V$ & $I_s = 2A$

$$I_{x_{\text{new}}} = 3(I'_x) + 2(I''_x)$$

$$= 3(0.2) + 2(0.4) = 1.4A$$

ex: Find V_1 & V_2 :

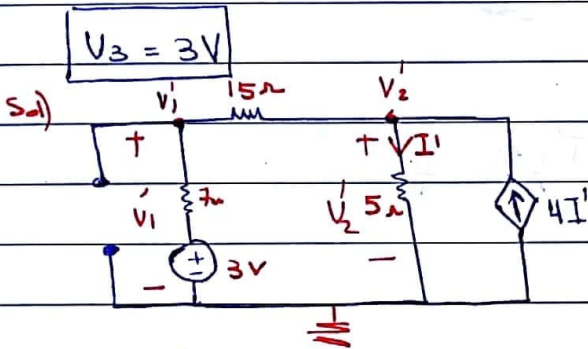
kill $\frac{V_3}{3}$ dependent



الحل الاسهل والمختصر :-

$$\frac{V_1 - 3}{7} + \frac{V_1 - V_2}{15} = 2 \dots (1)$$

$$\frac{V_2 - V_1}{15} + \frac{V_2}{5} = 4 \left(\frac{V_2}{5} \right) \dots (2)$$



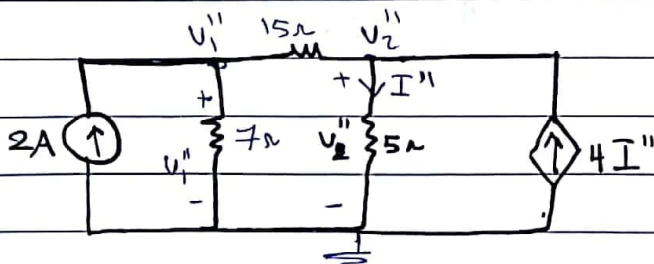
Node #1:

$$\frac{(V_1' - 3)}{7} + \frac{V_1' - V_2'}{15} = 0 \dots (1)$$

Node #2:

$$\frac{(V_2' - V_1')}{15} + \frac{V_2'}{5} = 4 I'$$

$$\frac{V_2' - V_1'}{15} + \frac{V_2'}{5} = 4 \left(\frac{V_2'}{5} \right) \dots (2)$$



Node 1:

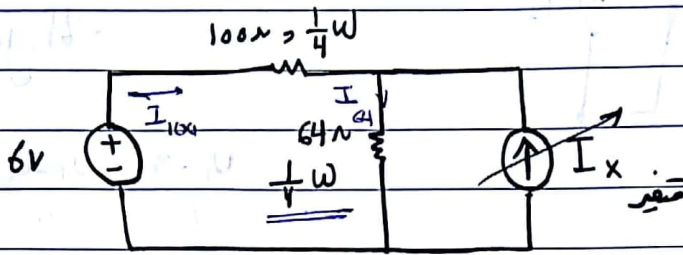
$$\frac{V_1''}{7} + \frac{(V_1'' - V_2'')}{15} = 2 \dots (1)$$

$$\frac{V_2''}{5} + \frac{V_2'' - V_1''}{15} = 4 \left(\frac{V_2''}{5} \right) \dots (2)$$

$$V_1 = V_1' + V_1''$$

$$V_2 = V_2' + V_2''$$

ex: Find the maximum current (I_x) such that the resistors do not exceed their power rating. Power rating \leq Power rating



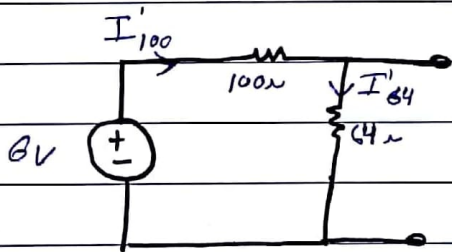
$$P_{100} = \frac{1}{4} = I^2(100)$$

$$P_{64} = \frac{1}{4} = \frac{I^2(64)}{64}$$

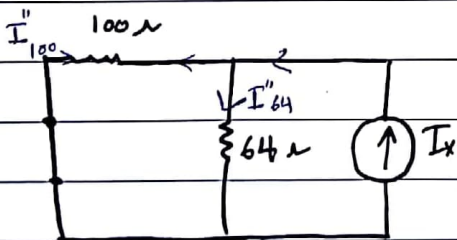
Power rating \rightarrow max Power.

$$I_{100 \text{ max}} = \sqrt{\frac{1}{400}} = 50 \text{ mA}$$

$$I_{64 \text{ max}} = \sqrt{\frac{1}{4 \times 64}} = 62.5 \text{ mA}$$



$$I'_{100} = I'_{64} = \frac{6}{164} = 36.59 \text{ mA}$$



$$I''_{64} = I_x \frac{100}{164}$$

$$I''_{100} = -I_x \frac{64}{164}$$

$$I'_{64} + I''_{64} \leq 65.5 \text{ mA}$$

$$I''_{64} \leq 62.5 \text{ mA} - 36.59 \text{ mA}$$

$$I_x \frac{100}{164} \leq 25.91 \text{ m}$$

$$I_x \leq 42.49 \text{ mA}$$

$$I'_{100} + I''_{100} \leq 50 \text{ m}$$

$$I''_{100} \leq 50 \text{ m} - 36.59 \text{ m}$$

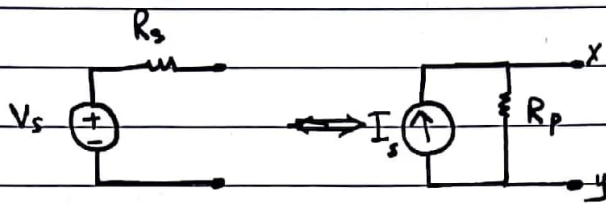
$$- I_x \frac{64}{164} \leq (50 - 36.59) \text{ m}$$

$$- I_x \leq 34.36$$

$$I_x \geq -34.36 \text{ m}$$

$$I_{x \text{ max}} = 42.49 \text{ mA}$$

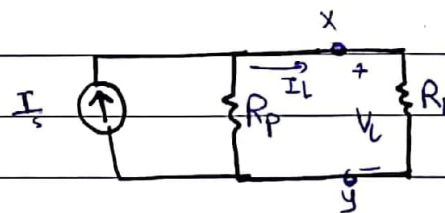
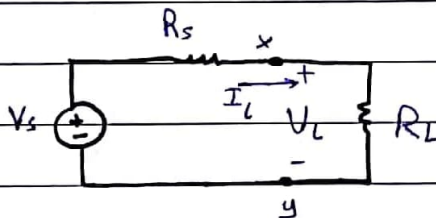
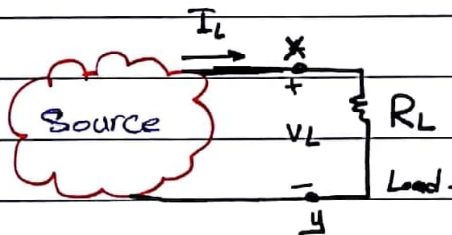
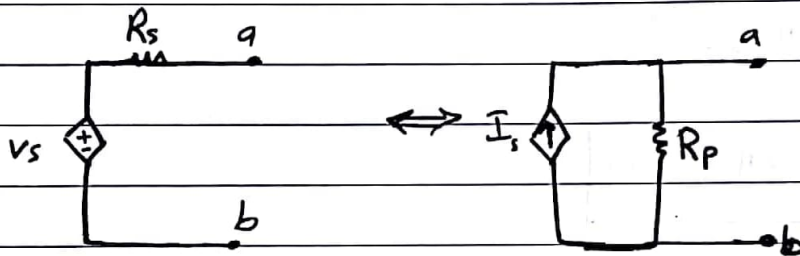
* Source Transformation:



(+ ↑) الكيناه للسرعة

$$R_s = R_p$$

$$V_s = I_s R_s$$



$$V_L = V_s \frac{R_L}{R_s + R_L}$$

$$V_L = I_L R_L$$

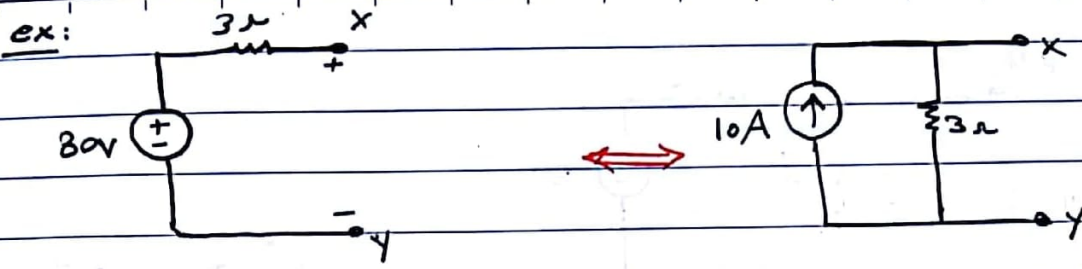
$$V_L = I_s \left(\frac{R_p}{R_p + R_L} \right) \cdot R_L$$

if $V_s = I_s R_p$ & $R_s = R_p$

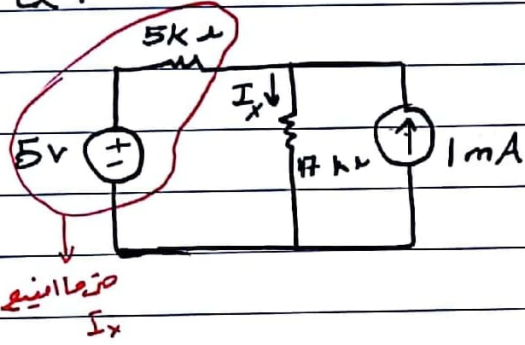
$$= I_s R_p \left(\frac{R_L}{R_p + R_L} \right)$$

voltage source $\xrightarrow{\text{insert}}$ with resistors

current source $\xrightarrow{\text{in parallel}}$ with R

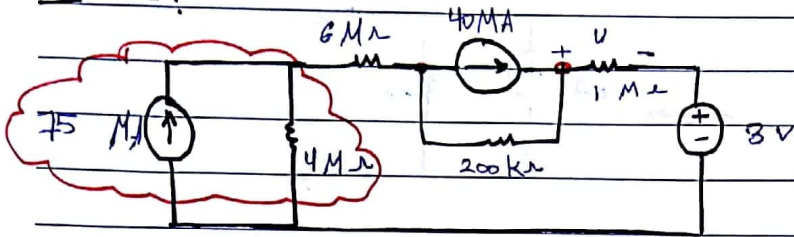


ex: $I_x = ?!$



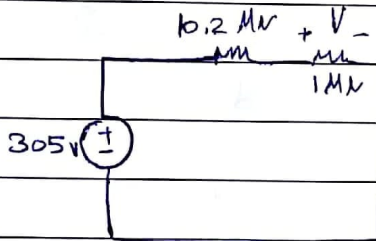
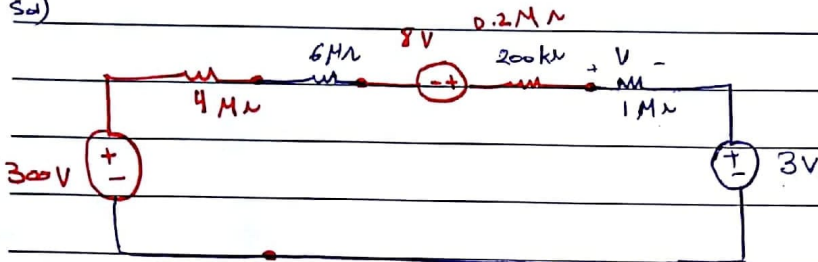
$$I_x = 2 \times 10^{-3} \frac{5}{5+47} = 1.92 \times 10^{-4} \text{ A} = 192 \mu\text{A}$$

ex: $V = ?$



$40 \times 200k = 8 \text{ Volt}$

Sol)

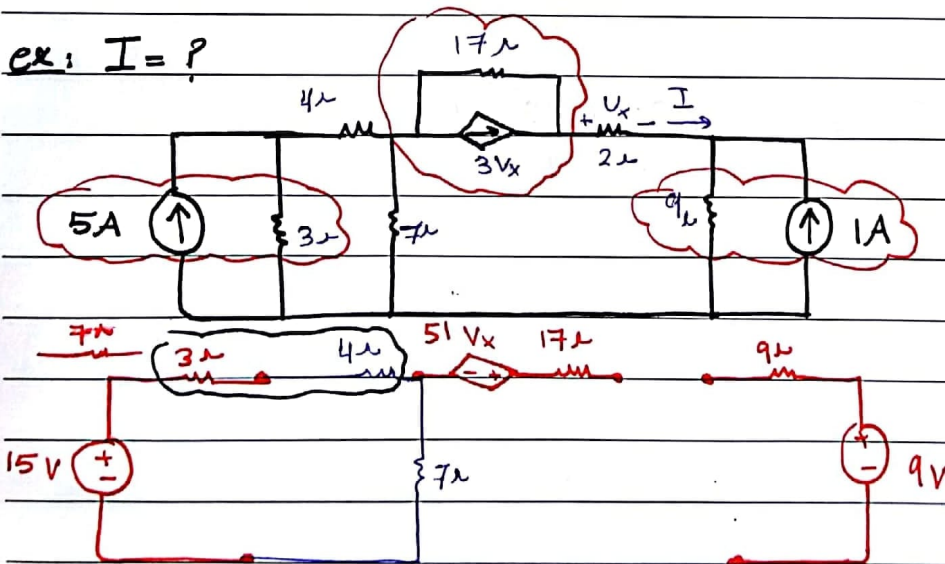


جودى كى ت بسىة
Source و R

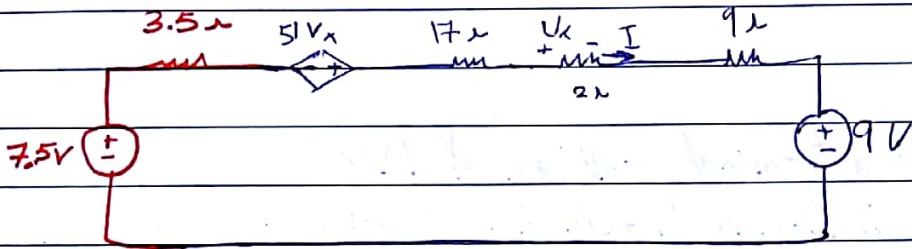
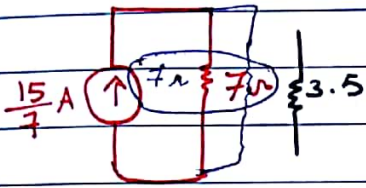
$$V = 305 \times \frac{1}{(10.2 + 1)} = 27.23 \text{ V}$$

Single loop \rightarrow voltage source

ex: $I = ?$



I الموجب مع الاتجاه \uparrow



$$-7.5 + 3.5I - 5V_x + 17I + 2I + 9I + 9 = 0$$

$$-7.5 + 3.5I - 5(2I) + 17I + 2I + 9I + 9 = 0$$

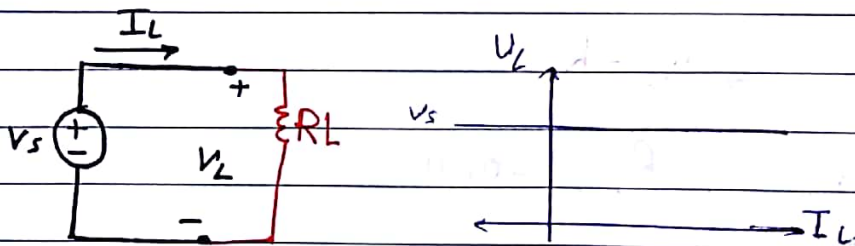
$$I = 21.276 \text{ mA}$$

$$U_x = 2I$$

* Practical Sources:

* Practical Voltage Source

Ideal: a device where terminal voltage is independent of the current through it.



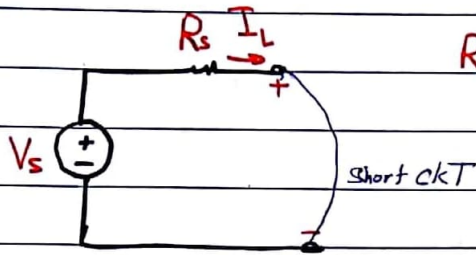
Let $v_s = 1 \text{ V}$

if $R_L = 1 \Omega \rightarrow I_L = 1 \text{ A} \rightarrow P = 1 \text{ W}$.

if $R_L = 1 \text{ M}\Omega \rightarrow I_L = 1 \mu\text{A} \rightarrow P = 1 \times 10^{-6} \text{ W}$
Mega
= 1 \mu\text{W}

Provide unlimited amount of Power.

- Practical:



R_s = Internal Resistance.
(output).

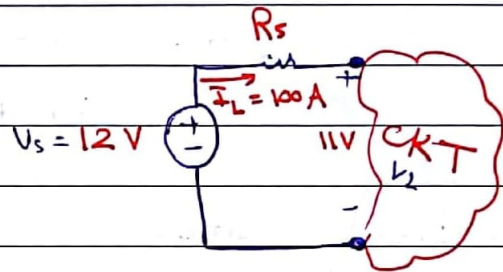
ex: Car battery has a terminal voltage of 12V.

When no current is flowing through it. Open ckt

This terminal voltage reduced to 11V when 100A is flowing.

$$-V_s + R_s I_L + V_L = 0$$

$$V_L = V_s - R_s I_L$$



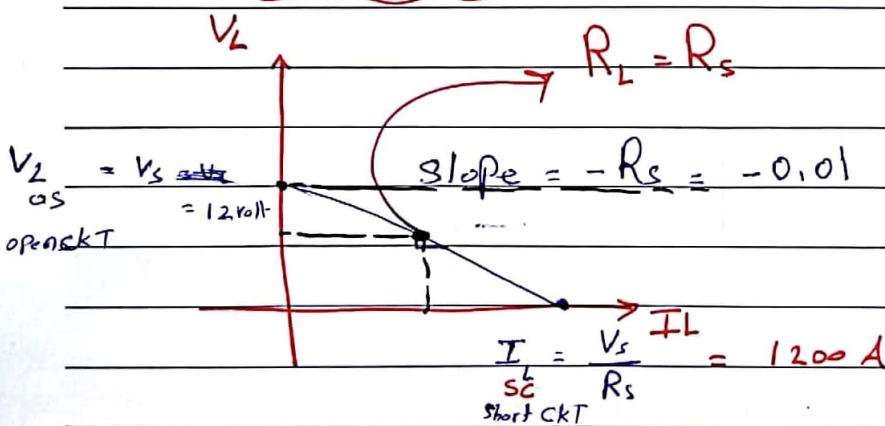
$$11 = 12 - R_s (100)$$

$$R_s = 0.01 \Omega$$

$$V_L = 12 - 0.01 I_L$$

~~U_L~~

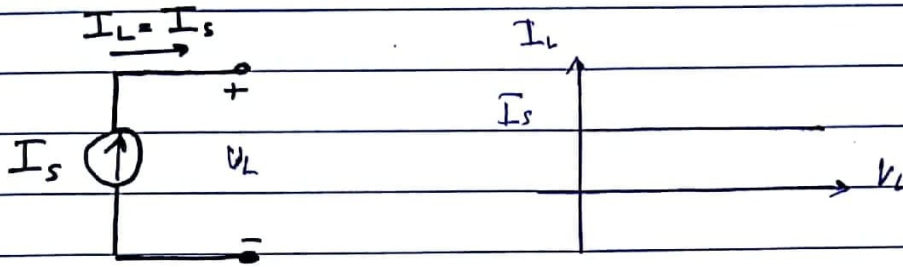
$$U_L = 0 \rightarrow I_L = \frac{V_s}{R_s}$$



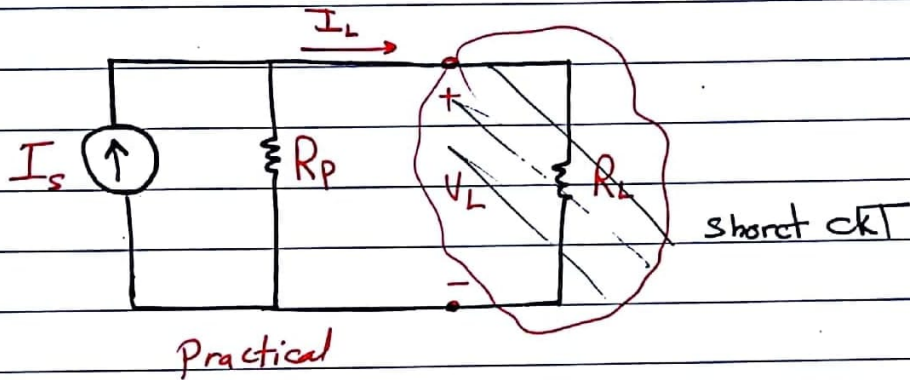
$I_L = 0$ when $R_L = \infty$ open ckt $\rightarrow V_{L,oc} = V_s$

$V_L = 0$ when $R_L = 0$ short ckt $\rightarrow I_{L,sc} = \frac{V_s}{R_s}$

* Practical Current Source Representation:



Ideal



Kcl:

$$I_L = I_s - \frac{V_L}{R_p}$$

