16

The University of Jordan
Department of Mathematics
Calculus III, First Exam

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Instructor's Name: د. منال غانم Lecture's time: ___________

Q1) (2 points) If a = 2i + j + k and b = 3i + j − 2k. Then find $\text{proj}_a b$

$\text{proj}_{\vec a} b = \text{comp}_{\vec a}\vec b \left(\frac{\vec a}{|a|}\right)$

$\vec a = \langle 2, 1, 1\rangle$

$\vec b = \langle 3, 1, -2\rangle$

$= \left(\frac{\vec a \cdot b}{|\vec a|}\right)\frac{\vec a}{|\vec a|} = \frac{\langle 2,1,1\rangle\langle 3,1,-2\rangle}{\sqrt{4+1+1}} \cdot \frac{\vec a}{|\vec a|}$

$= \frac{6+1-2}{\sqrt 6} \times \frac{\langle 2,1,1\rangle}{\sqrt 6} = \frac{5\langle 2,1,1\rangle}{6}$

$= \left\langle \frac{10}{6}, \frac{5}{6}, \frac{5}{6}\right\rangle$

2

Q2) (4 points) Find the distance between the two lines:

L_1 : $x = 2 + t,\ y = 4 - 2t,\ z = 1 + 4t,$

L_2 : $x = 1 - 2s,\ y = 4s,\ z = 1 - 8s.$

Let P_1 point on $L_1 \Rightarrow P_1 = (2, 4, 1)$

Let P_2 point on $L_2 \Rightarrow P_2 = (1, 0, 1)$

Are they Parallel?

$\vec V_1 = \langle 1, -2, 4\rangle$

$V_2 = \langle -2, 4, -8\rangle$

$\Rightarrow \vec V_1 = \alpha V_2 \therefore$ Parallel

then $D = \frac{|\vec{P_1P_2} \times \vec V_1|}{|\vec V_1|} \Rightarrow \frac{|\langle -16, 4, -2\rangle|}{\cancel{\sqrt{\ }}\ \sqrt{1+4+16}}$

$\vec{P_1P_2} \times \vec V_1 = \begin{vmatrix} -1 & -4 & 0 \\ 1, & -2, & 4 \end{vmatrix}$

$= \frac{\sqrt{16^2+4^2+2^2}}{\sqrt{21}}$

$= (-16-0) - (-4-0) + (2+4)$

$= \frac{\sqrt{256+16+4}}{\sqrt{21}}$

$= -16i + 4j - 2k$

$= \frac{\sqrt{276}}{\sqrt{21}}$

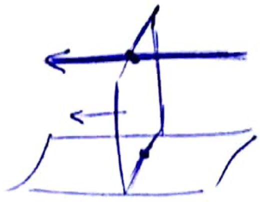
$t = y + 3$ $\quad t = 2x - 1$

$t = 3z + 6 \qquad y = -3 + t$

$\frac{x - \frac{1}{2}}{2} = \frac{y+3}{1} = \frac{z+2}{3} \qquad 2x = t + 1$

$t - 6 = 3z \qquad x = \frac{t}{2} + \frac{1}{2}$

$z = \frac{t}{3}$



Q3) (4 points) Find the equation of the plane that is parallel to the line: $2x - 1 = y + 3 = 3z + 6$, perpendicular to the plane: $4x - 2y + z = 4$ and passes through the point $p(3, -1, 5)$

~~Find point on the line $P_1 (\frac{1}{2}, 3, 2)$~~

$n_{\pi_2} = n_{\pi_1} \times V_L$

$n_{\pi_1} = (4, 2, 1)$

$V_L = (\frac{1}{2}, +1, \frac{1}{3})$

$n_{\pi_2} = \begin{vmatrix} 4 & -2 & 1 \\ \frac{1}{2} & 1 & \frac{1}{3} \end{vmatrix} = (\frac{2}{3} - 1) + (\frac{4}{3} - \frac{1}{2}) + (4 - 1)$

$\frac{-1}{3} i - \frac{5}{6} j + 3 k$

3.5

equ of plane is

$\frac{-1}{3}(x-3) + \frac{-5}{5}(y+1) + 3(z-5) = 0$

Q4) (4 points) Identify and sketch the following surfaces

a) $x^2 - 2y + 5z^2 = 3$.

b) $z = \sqrt{x^2 + y^2 - 4}$.

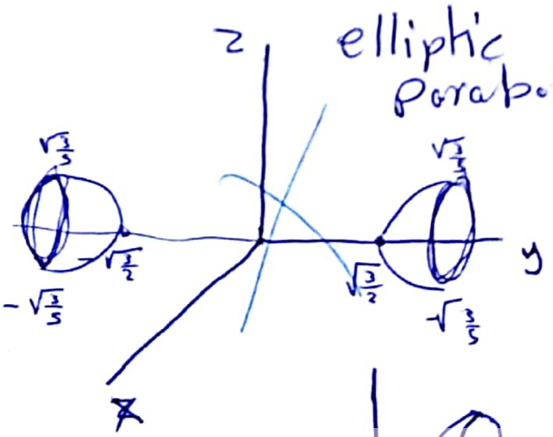
a) $\frac{x^2}{3} - \frac{2y}{3} + \frac{5z^2}{3} = \frac{3}{3}$

$\frac{x^2}{3} - \frac{y}{\frac{3}{2}} + \frac{z^2}{\frac{3}{5}} = 1$

$+\frac{x^2}{3} + \frac{z^2}{\frac{3}{5}} - (\frac{y}{\frac{3}{2}} + 1) = 0$

3

elliptic Parabolid

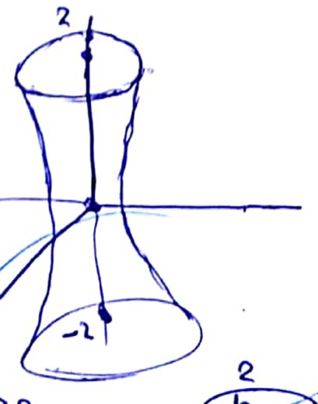


but $c < 0 \Rightarrow$

$z^2 = x^2 + y^2 - 4$

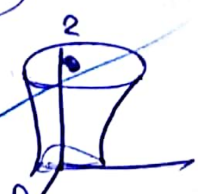
$x^2 + y^2 - z^2 = 4$

$\frac{x^2}{4} + \frac{y^2}{4} - \frac{z^2}{4} = 1$



center (0,0,0)

but $z > 0$ then upper half



Hyperbolic of one sheet

Q5) (3 points) Let a and b be two vectors such that $|a| = 2$, $|b| = 4$ and $|a+b| = 6$. Find $a \cdot b$.

$\cancel{a \cdot b = |a||b| \cos\theta}$

$\cancel{|a+b|^2 = |a|^2 + 2a \cdot b + |b|^2}$

$\cancel{\cos\theta = \frac{a \cdot b}{|\cdot|}}$

$|a+b|^2 = (a+b) \cdot (a+b)$

$36 = |a|^2 + 2\, a \cdot b + |b|^2$

$36 = 4 + 2\, a \cdot b + 16$

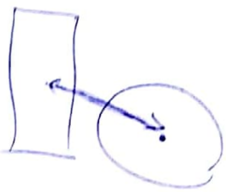
3

$36 - 4 - 16 = 2\, a \cdot b$

$\Rightarrow 2\, a \cdot b = 16 \Rightarrow \boxed{a \cdot b = 8}$

Q6) (3 points) Find the shortest distance between the plane: $4x - 2y + z = 4$ and the sphere: $x^2 + y^2 + z^2 - 4x + 8y - 10z + 36 = 0$.

1



$x^2 - 4x + y^2 + 8y + z^2 - 10z = -36$

$(x-2)^2 + (y+4)^2 + (z-5)^2 = -36 + 4 + 16 + 25$

$(x-2)^2 + (y+4)^2 + (z-5)^2 = 9$ Let $x = z = 0$

$0 + (y+4)^2 + 0 = 9 \Rightarrow (y+4)^2 = 9 \Rightarrow y + 4 = 3$

$y = -1$

$\therefore (0, -1, 0)$ point on the sphere

$$D_{pP} = \frac{|4x - 2y + z - 4|}{\sqrt{16 + 4 + 1}} = \frac{|0 + 2 + 0 - 4|}{\sqrt{21}}$$

$$= \frac{2}{\sqrt{21}}$$