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Jordan University
Mathematics Department
Calculus III, Second Exam, 7/8/2016

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Lecture Time: ______________ . Instructor's Name: [illegible] .

1) ( 4 points ) Find the arclength of $r(t) = < t, \quad \frac{2}{3}(t+1)^{\frac{3}{2}}, \quad \frac{2}{3}(t-1)^{\frac{3}{2}} >$, $1 \le t \le 2$.

$$r'(t) = \langle 1 , (t+1)^{\frac{1}{2}} , (t-1)^{\frac{1}{2}} \rangle$$

$$|r'(t)| = \sqrt{1 + (t+1)^{\frac{1}{2}\,2} + (t-1)^{\frac{1}{2}\,2}}$$

$$|r'(t)| = \sqrt{1 + t + \cancel{1} + t - \cancel{1}}$$

$$r'(t) = \sqrt{1 + 2t}$$

$$\int_1^2 \sqrt{1+2t}\, dt = \int_1^2 (1+2t)^{\frac{1}{2}} = \frac{2}{3}\cdot\frac{1}{2}(1+2t)^{\frac{3}{2}} \Big|_1^2$$

$$= \frac{2}{3}(5)^{\frac{3}{2}} - \frac{2}{3}(3)^{\frac{3}{2}}$$

$$= \frac{3}{2}\sqrt[2]{5^3} - \frac{2}{3}\sqrt[2]{3^3} = \frac{2}{3}\left(\sqrt[2]{125} - \sqrt[2]{27}\right)$$

3.5

2) ( 5 points ) Sketch the graph of

a) $r(t) = < 3, \quad \cos(t), \quad \sin(t) >, \quad 0 \le t \le 2\pi$.

Assume $y = \cos t$, $z = \sin t$ $\quad \sin t^2 + \cos t^2 = 1$

$z^2 + y^2 = 1$ and $x = 3$

~~sphere~~ ~~circle~~ with
sircle radus 1
and center (0,0)

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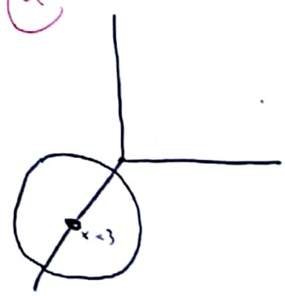
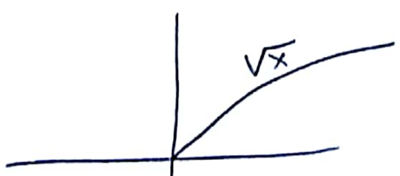


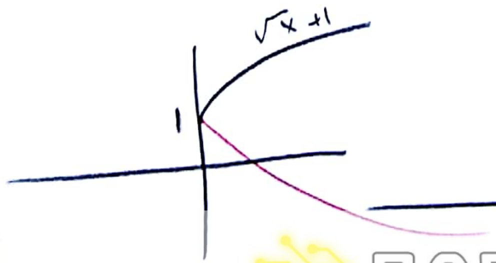

b) $r(t) = < t^2, \quad t+1 >, \quad t \in \mathbb{R}$.

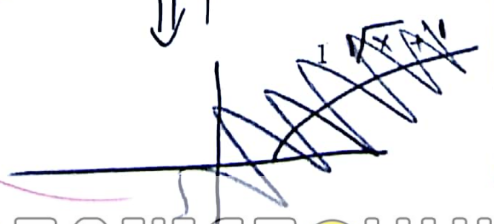
$x = t^2$
$y = t + 1$

$t = \sqrt{x}$

$y = \pm\sqrt{x} + 1$



3) ( 6 points ) Find and classify all critical points of $f(x, y) = y^2 - 2y\cos(x)$, $-1 < x < 2$.

$f(x,y) = y^2 - 2y\cos x$

$f_x = 0 + 2y\sin x = 0 \Rightarrow +2y\sin x = 0$

$\sin x = 0$ $\boxed{x = 0}$

$f_y = 2y - 2\cos x = 0$ $\quad 2\cos x = 2y$ $\boxed{\cos x = y}$ $\boxed{y=1}$

$\Rightarrow \boxed{y = \cos x}$ $\boxed{y=1}$

$-1 < x < 2$

$-1 < x < 2$, $y = 1$

$x = -1$

$f_x = 2y\sin -1 = 0$

$\boxed{y = 0}$

$f(x, 1) = 1 - 2\cos x$

$f_x = 2\sin x = 0$ $\boxed{x=0}$

$f_x = 2y\sin x = 0$

$2\sin x = 0$

$\sin x = 0$

$\boxed{x = 0}$

$f_y = 2 - 2\cos x = 0$

$\cos x = 1$

$\boxed{x = 0}$

$f_y = 2y - 2\cos -1$

$2(y - \cos -1) = 0$

$y - \cos -1 = 0$

$y = \cos -1$

$(-1, 1)$

$(2, 1)$

$f_{xx} = 2y\cos x$

$f_{yy} = 2$

$f_{xy} = (2\sin x)^2$

$(0, 1)$

| $x$ | $y$ | $f_{xx}$ | $f_{yy}$ | $f_{xy}^2$ | $D$ | |
|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 2 | 0 | 2 | $>0$ $f_{xx}>0$ min value |
| $-1$ | 1 | 1 | 2 | $4\sin^2(-1)$ | $2+4\sin^2(-1)$ | $>0$ min |
| ~~2~~ | ~~1~~ | 1 | 2 | $4\sin^2(+)$ | $2+4\sin^2(+)$ | $>0$ min |

4) ( 5 points ) Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point (4,2,0).

$D = \sqrt{(x-4)^2 + (y-2)^2 + z^2}$

$D^2 = f(x,y) = (x-4)^2 + (y-2)^2 + z^2$

$\nabla f = \lambda \nabla g$

$f_x = \lambda g_x$

$f_y = \lambda g_y$

$f_z = \lambda g_z$

$2(x-4) = \lambda 2x$

$2(y-2) = \lambda 2y$

$2z = \lambda - 2z$

$\boxed{\lambda = -1}$

$2(x-4) = (-1)2x$

$x - 4 = -x$

$2x = 4$

$\boxed{x = 2}$

$2(y-2) = \lambda 2y$

$y - 2 = -y$

$2y = 2$

$\boxed{y = 1}$

$z^2 = x^2 + y^2$

$z^2 = 4 + 1$

$z = \pm\sqrt{5}$

points are

$(2, 1, \sqrt{5})$

$(2, 1, -\sqrt{5})$

$f(2, 1, \sqrt{5}) = 4 + 1 + 5 = 10$

$f(2, 1, -\sqrt{5}) = 4 + 1 + 5 = 10$

so since the two points are equal

Then Both are closest.

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$\|\nabla c\|$

5) ( 3 points ) Find the maximum rate of change in $f(x,y,z) = x^2yz + y^3z$ at $(1,2,-1)$.

$\nabla f = f_x i + f_y j + f_z k$

$\nabla f = 2xyz\, i + (x^2 z + 3zy^2) j + (x^2 y + y^3)$

$\|\nabla f\|_{(1,2,-1)} = \sqrt{(2xyz)^2 + (x^2 z + 3zy^2)^2 + (x^2 y + y^3)^2}$

$= \sqrt{(4)^2 + (-13)^2 + (1)^2}$ 2 $= \sqrt{16 + 1 + 169} = \sqrt{186}$

2.5

6) ( 4 points ) Show that the following limit does not exist. $\lim_{(x,y)\to(0,0)} \frac{x\,y^3}{x^2 + y^6}$

along $x = y^3$

$\lim_{y \to 0} \frac{y^3 y^3}{y^6 + y^6} = \frac{y^6}{2y^6} = \frac{1}{2}$

along $x = 2y^3$

$\lim_{y\to 0} \frac{2y^6}{4y^6 + y^6} = \frac{2y^6}{(4+1)y^6} = \frac{2}{5}$

Since there are two path with two different value of the limit

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$y^3 \le y^6 + x^2$

~~since there is squared~~

~~$0 \le \frac{y^3}{x^2 + y^6} \le 1$~~

~~$0 \le \frac{xy^3}{x^2+y^6} \le y$~~

~~$\lim 0 = 0$ Bd $\lim_{x,y\to 0} y = 0$~~

~~since it's Bd then lim 0 y = 0~~

7) ( 3 points ) Suppose that $F(x,y,z) = 0$ and $z = f(x,y)$ is a differentiable function. Show that $z_x = \frac{-F_x}{F_z}$.

$F_x = \frac{\partial F}{\partial x}$

$\frac{dz}{dx} = F_z = \frac{\partial F}{\partial z}$

$\frac{\partial F}{\partial z}$

$\nabla f = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$

$\frac{\partial z}{\partial x} = \nabla f - \frac{\partial z}{\partial y}$

$\frac{\partial F}{\partial x} = \frac{\partial z}{\partial x} \cdot \frac{\partial F}{\partial z}$

$\frac{\partial F}{\partial z}$

$-\frac{F_x}{F_z} = \frac{\frac{\partial z}{\partial x} \cdot \frac{\partial F}{\partial z}}{\frac{\partial F}{\partial z}}$

$\therefore -\frac{F_x}{F_z} = \frac{\partial z}{\partial x} = z_x$

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