THE UNIVERSITY OF JORDAN

0301101 Calculus I | Second test | Dec. 5, 2015

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For instructor use only, please do not write in this table.

Q1	Q2	Q3	Grade
15	5	7	27

Q1) Fill in the blanks with the answers only. Each part is worth 1.5 marks.

1) The vertical asymptote(s) of $f(x) = \frac{x-3}{x^2-x-6}$ is (are) $x = -2$ ✓

2) $\lim_{x\to -2} \frac{x^2-|x|-2}{x+2} = -3$ ✓

3) $\lim_{x\to 0}(2 + x^2 \sin\left(\frac{2}{x^2}\right)) = 2 + 0 = 2$ ✓

4) If $f(x) = 3x^{\sqrt{3}}$, then $\lim_{x\to 1} \frac{f(x)-3}{x-1} =$ $f(x) = 3x^{\sqrt{3}}$ ✓

5) If the tangent line to $f(x)$ at $x = 4$ is $y - 3x = 1$, then $f'(4) = 3$ ✓

6) $\lim_{x\to\infty} e^{x-x^2} = e^{-\infty} = 0$ ✓

7) If $f(x) = \tan^{-1}(e^{3x})$, then $f'(x) = \frac{e^{3x} \cdot 3}{1 + (e^{3x})^2}$ ✓

8) If $f(x) = 5^{\sin(x)}$, then $f'(x) = 5^{\sin x} \cdot \ln 5 \cdot \cos x$ ✓

9) If $f(x) = (3x+1)^{x^2}$, then $f'(x) = (3x+1)^{x^2}\left(\frac{3x^2}{3x+1} + 2x \ln(3x+1)\right)$ ✓

10) $\lim_{a\to 0} \frac{\ln(x^2+a)-\ln(x^2)}{a} = \frac{d}{dx} \ln|x^2| \Big|_{x=0} \Rightarrow \frac{2x}{x^2} = \frac{0}{0}$ dne. $\ln 0$ dne ✓

11) Let $f(x) = \frac{1}{\sin(x)-3} - \frac{x}{(x-2)}$. Then f is discontinuous at $x = 2$ ✓

12) If $f(x) = g(x^4 + 4)$ and $g'(5) = -2$, then $f'(1) = -1000$ ✓

13) $\lim_{x\to 4} \frac{\sin(x^2-16)}{x-4} = 8$ ✓

Q2) (5 points) Find $\lim_{x\to-\infty} f(x) = \sqrt{x^2+2x} - \sqrt{x^2+1}$. × $\frac{\sqrt{x^2+2x} + \sqrt{x^2+1}}{\sqrt{x^2+2x} + \sqrt{x^2+1}}$

$$\lim_{x\to-\infty} fx = \frac{x^2+2x-x^2+1}{\sqrt{x^2+2x}+\sqrt{x^2+1}} = \lim_{x\to-\infty} \frac{2x+1}{|x|\sqrt{1+\frac{2}{x}} + |x|\sqrt{1+\frac{1}{x^2}}}$$

$$= \lim_{x\to-\infty} \frac{2x+1}{-x\left(\sqrt{1+\frac{2}{x}} + \sqrt{1+\frac{1}{x^2}}\right)} = \lim_{x\to-\infty} \frac{x\left(2-\frac{1}{x}\right)}{-x\sqrt{1+\frac{2}{x}} + \sqrt{1+\frac{1}{x^2}}}$$

$$= \lim_{x\to-\infty} -\frac{2-\frac{1}{x}}{\sqrt{1+\frac{2}{x}} + \sqrt{1+\frac{1}{x^2}}} = -\frac{2}{2} = \boxed{-1}$$

(5)

Q3) (7 points) Find the points on the curve $2x^2 + xy + y^2 = 14$ where the normal line is parallel to the line $y = x$.

by $y = x$

the slope of normal line is = 1

*the slope of the tangent $\Rightarrow$ $1 = \frac{-1}{f'x}$ $\Rightarrow$ $f'x = -1 = y'$

Diff.

$2x^2 + xy + y^2 = 14$

$4x + xy' + y + 2yy' = 0$

$4x - x + y + 2y = 0$

$3x - y = 0$

$\boxed{x = \frac{y}{3}}$

$2x^2 + x\cdot x + x^2 = 14$

$2x^2 + x^2 + x^2 = 14$

$4x^2 = 14 \Rightarrow x^2 = \frac{14}{4} \Rightarrow x = \pm\sqrt{\frac{14}{4}}$

$2\frac{9y^2}{25} + \frac{3y^2}{5} + y^2 = 14$

$2\frac{y^2}{9} + \frac{y^2}{3} + y^2 = 14$

$y^2\left(\frac{2}{9} + \frac{1}{3} + 1\right) = 14$

$y^2\frac{14}{9} = 14$

$y^2 = 9 \quad y = \pm 3$

$\left(\frac{y}{3}, 3\right)\left(\frac{y}{3}, -3\right) \Rightarrow \boxed{(1,3)(-1,-3)}$

(7)