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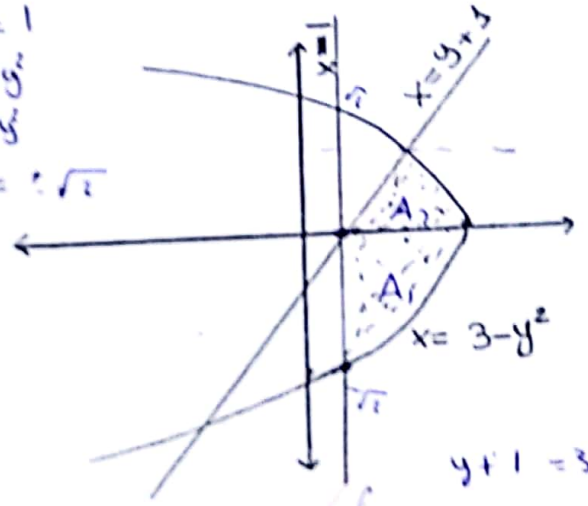
وقت المحاضرة: 8-9 مدرس المادة: صال عانص

Q1: Find the area of the shaded region

$$A_1 = \int_{-\sqrt{2}}^0 (3-y^2) - (1) dy$$

$$A_2 = \int_0^{\sqrt{2}} (3-y^2) - (y+1) dy$$

Intersection  
 $3-y^2=1$   
 $3-1=y^2$   
 $2=y^2$   
 $y=\pm\sqrt{2}$



$$A_{tot} = \int_{-\sqrt{2}}^0 3-y^2-1 + \int_0^{\sqrt{2}} 3-y^2-y+1$$

$$= \int_{-\sqrt{2}}^0 2-y^2 + \int_0^{\sqrt{2}} -y^2-y+2$$

$$2y - \frac{y^3}{3} \Big|_{-\sqrt{2}}^0 + \left[ -\frac{y^3}{3} - \frac{y^2}{2} + 2y \right]_0^{\sqrt{2}}$$

$$-2\sqrt{2} + \frac{\sqrt{2}}{3} + \left[ -\frac{2}{3} - \frac{1}{2} + 2 \right] =$$

Q2: Find (if exists)  $\int_0^1 x(\ln x) dx$

$\ln 0 = \infty$   $x$

$$\lim_{t \rightarrow 0^+} \int_t^1 x(\ln x) dx \Rightarrow \int x(\ln x) dx$$

$$u = \ln x \quad dv = x$$
$$\downarrow \quad \downarrow$$
$$du = \frac{1}{x} \quad \frac{x^2}{2}$$

$$\Rightarrow \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}$$

$$= \ln x \cdot \frac{x^2}{2} - \frac{1}{2} \int x dx$$

$$= \ln x \cdot \frac{x^2}{2} - \frac{1}{2} \cdot \frac{x^2}{2}$$

$$\left( \ln(1) \cdot \frac{1}{2} \right) - \left( \ln t \cdot \frac{t^2}{2} \right) - \left( \frac{1}{4} - \frac{t^2}{4} \right)$$
$$= - \ln t \cdot \frac{t^2}{2} - \frac{1-t^2}{4}$$

$$\lim_{t \rightarrow 0^+} - \ln(t) \frac{t^2}{2} - \frac{1-t^2}{4}$$
$$= (-\infty) \cdot \frac{0}{2} - \left( -\frac{1}{4} \right)$$
$$\infty \cdot \infty + \frac{1}{4}$$
$$\infty + \frac{1}{4} = \infty$$

divergent

Q3: Find  $\int \frac{x dx}{\sqrt{x^2-4x}}$   $\Rightarrow \int \frac{x dx}{\sqrt{x^2-4x+4-4}} = \int \frac{x dx}{\sqrt{(x-2)^2-4}}$

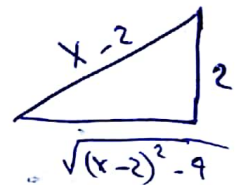
$\Rightarrow x = a \sec \theta$   
 $x-2 = 2 \sec \theta \Rightarrow \sec \theta = \frac{x-2}{2}$   
 $dx = 2 \sec \theta \tan \theta d\theta$

$\sqrt{x^2-a^2} \leftarrow = 2 \tan \theta$

$\int \frac{(2 \sec \theta + 2) \cdot (2 \sec \theta \tan \theta) d\theta}{2 \sec \theta \tan \theta} = 4 \int (\sec \theta + 1) d\theta$  (2.7)

$= 4 \ln |\sec \theta + \tan \theta| + \theta$

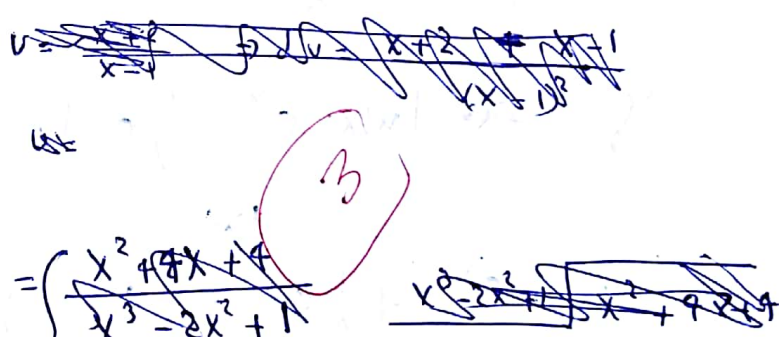
$= 4 \ln \left| \frac{x-2}{2} + \frac{2}{\sqrt{(x-2)^2-4}} \right| + \sec^{-1} \frac{x-2}{2}$



Q4:  $\int \frac{1}{x} \left( \frac{x+2}{x-1} \right)^2 dx$

$\int \frac{1}{x} \frac{(x+2)^2}{(x-1)^2} dx$

$\int \frac{(x+2)^2}{x(x-1)^2} = \int \frac{(x+2)^2}{x(x^2-2x+1)}$



~~$\frac{(x+2)^2}{x(x-1)^2}$~~   $= \int \frac{x^2+4x+4}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

$(x+2)^2 = A(x-1)(x-1)^2 + Bx(x-1) + Cx$

~~$x=1$~~   $x=1 \Rightarrow (1+2)^2 = 0 + 0 + C \Rightarrow C = 9$

$x=0 \Rightarrow 4 = -A \Rightarrow A = -4$

$x=-1 \Rightarrow 1 = -8A + 2B + -C$

$1 = (-8)(-4) + 2B - 9 \Rightarrow -32 - 9 - 1 + 2B = 0 \Rightarrow 2B = 42 \Rightarrow B = 21$

$= \int \frac{-4}{x} + \frac{21}{x-1} + \frac{9}{(x-1)^2} \Rightarrow -4 \ln|x| + 21 \ln|x-1| + \frac{9}{x-1}$

Q5: Find  $\int \frac{(x-1)e^x}{x^2} dx$

~~$w = e^x$   
 $\frac{dw}{dx} = e^x$   
 $dx = \frac{dw}{e^x}$~~

$w = e^x$   
 $\frac{dw}{dx} = e^x$   
 $dx = \frac{dw}{e^x}$

~~$\int \frac{(x-1)e^x}{x^2} dx$~~

~~$\int \frac{(u(w)+1)}{(u(w))^2}$~~

~~$\int \frac{(u(w)-1)e^x}{(u(w))^2} \cdot \frac{dw}{e^x}$~~

~~$\int \frac{1}{u(w)^2} \Rightarrow \int \frac{1}{u(w)^2} - \int \frac{1}{u(w)^2}$   
 $\int \frac{1}{u(w)} - \int \frac{1}{u(w)^2}$~~

~~$\frac{dz}{dw} = \frac{1}{w}$~~

~~$\int \frac{x-1}{x^2} \cdot e^x$~~

~~$u = e^x$   
 $du = e^x$   
 $v = \ln x - \frac{1}{x}$   
 $\frac{dv}{dx} = \frac{1}{x} - \frac{-1}{x^2}$~~

~~$\int \frac{xe^x - e^x}{x^2} = \frac{xe^x}{x^2} - \frac{e^x}{x^2} (\ln x + \frac{1}{x}) - \int e^x (\ln x - \frac{1}{x})$~~

~~$z = e^x$   
 $\frac{dz}{dx} = e^x$   
 $z = \ln x$   
 $\frac{dz}{dx} = \frac{1}{x}$   
 $w = x \cdot d$   
 $x =$~~

~~$\int \frac{e^x}{x} - \int \frac{e^x}{x^2}$~~

~~$\int \frac{1}{\ln x} - \int \frac{1}{(\ln x)^2}$~~

