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The university of Jordan Calculus II Second exam 10 April 2016

[redacted]

مدرس المادة: وقت المحاضرة:

Q1: Find the area of the surface obtained by rotating the curve

$y = \frac{1}{6}x^3 + \frac{1}{2}x^{-1}, \quad 1 \le x \le \sqrt{2}$ about x-axis

$$\int_1^{\sqrt{2}} 2\pi\left(\frac{x^3}{6} + \frac{1}{2x}\right)\sqrt{1 + \left(\frac{x^8 - 2x^4 + 1}{4x^4}\right)}$$

$$2\pi\int_1^{\sqrt{2}} 2\pi\left(\frac{x^3}{6} + \frac{1}{2x}\right)\sqrt{4}$$

2.5

Q2: The base of the solid S is the region bounded by $y = x^2 - 1$ and $y = 1 - x^2$. **Find the volume** of S if every cross section perpendicular to the x-axis is a right triangle(مثلث قائم الزاوية) with its hypotenuse(الوتر) on the base and one angle equal to 30°.

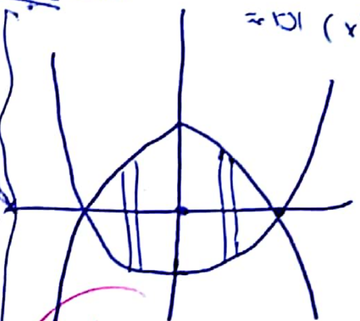
$\sin x = \frac{المقابل}{الوتر}$

$\frac{1}{2} = \frac{المقابل}{}$

الوتر $= x^2 - 1 - 1 + x^2$

$2x^2 - 2$

$= 2(x^2 - 1)$



$$\int_0^1 \frac{1}{2}\,\frac{S^2}{4}\,dx + \int_0^{-1} \frac{1}{2}\,\frac{S^2}{4}\,dx$$

$$\int_0^1 \left(\frac{x^2 - 1}{4}\right) + \int_0^{-1} \left(\frac{x^2 - 1}{4}\right)$$

$$\int_0^1 \frac{x^2}{4} - \frac{1}{4} + \int_0^{-1} \frac{x^2}{4} - \frac{1}{4} \Rightarrow \frac{3x^3}{4}\Big|_0^1 - \frac{1}{4} + \frac{3x^3}{4} - \frac{1}{4} =$$

1

$1 - x^2 = x^2 - 1$

$1 + 1 = x^2 + x^2$

$\frac{2}{2} = \frac{2x^2}{2}$

$x^2 = 1$

$x = \pm 1$

Q3: Find the sum

$$\sum_{n=1}^{\infty}\left(\frac{1}{(n+1)(n+3)}+\frac{5}{7^{n+2}}\right) \qquad \sum \frac{1}{(n+1)(n+3)} + \sum \frac{5}{7^{n+2}}$$

$$\frac{a}{n+1} + \frac{b}{n+3}$$

$1 = a(n+3) + b(n+1)$

$n=-3 \Rightarrow b = -\frac{1}{2}$

$n=-1 \Rightarrow a = \frac{1}{2}$

$$\sum_{1}^{\infty} \frac{\frac{1}{2}}{n+1} + \frac{\frac{-1}{2}}{n+3}$$

2

$$\sum \frac{5}{7^n \cdot 7^2} = \frac{5}{49}\sum \frac{1}{7^n} \Rightarrow \left(\frac{1}{7}\right)^n$$

$|r|<1$

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$$\frac{5}{49}\left(\frac{\frac{1}{7}}{1-\frac{1}{7}}\right) = \frac{5}{49}\,\frac{\frac{1}{\cancel{7}}}{\frac{6}{\cancel{7}}}$$

??

$$\boxed{= \frac{5}{49\times 6} + \frac{1}{8}}$$

$$\left(\frac{1}{2}+\frac{-1}{8}\right)+\left(\frac{1}{4}+\frac{-1}{10}\right)+\left(\frac{1}{8}+\frac{-1}{12}\right)+\left(\frac{1}{10}+\frac{-1}{14}\right)\cdots\left(\frac{\frac{1}{2}}{(n-1)+1}+\frac{\frac{-1}{2}}{(n-1)+3}\right)\left(\frac{\frac{1}{2}}{n+1}+\frac{\frac{-1}{2}}{n+3}\right)$$

$$\frac{1}{2}+\frac{1}{4}+\frac{\frac{1}{2}}{n}-\frac{\frac{1}{2}}{n+2} \Rightarrow \frac{1}{2}+\frac{1}{4}+\lim\frac{1}{2n}-\lim\frac{1}{2n+2} = \frac{1}{2}+\frac{1}{4}+0+0 = \frac{1}{8}$$

Q4: Set up the integral that gives the volume of the solid generated by revolving the region bounded by $y = x^3$, $y = 27$, $x = 0$ about the line $x = -3$.

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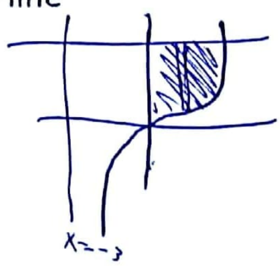

(a) Using washer method, (Do not evaluate the integral).

$$\int_0^{27} 2\pi (r)^2\, dy = \int_0^{27} 2\pi \left(\sqrt[3]{y}+3\right)^2 dy$$

??

?

(b) Using cylindrical shell method, (Do not evaluate the integral).

$$\int_0^{3} 2\pi r(h)\, dx$$

$$\int_0^{3} 2\pi (x+3)(27-x^3)\, dx$$

4

Test

$\int \frac{n}{e^{n^2}} \quad \frac{1}{e^{n^2}} \cdot n$

$n\, n e^{n^2} - e^{n^2}$

(a) $\sum_{n=1}^{\infty} n e^{-n^2} = \sum \frac{n}{e^{n^2}}$

$\lim \int_1^{\infty} \frac{x}{e^{x^2}}\, dx \Rightarrow \int_1^{t} \frac{x}{e^{x^2}}\, dx$

$du = x \quad v = e^{x^2}$

$\downarrow \qquad \searrow -2xe^{-x^2}$

$\frac{x^2}{2}$

$\lim x - 2xe^{x^2} - \int \frac{x^2}{2} - 2xe^{x^2} = \lim \cancel{t}\, 2 \cancel{t} e^{-t^2} + \frac{1}{2} e^{-t^2}$

$0 + 0 = 0$

div

1

(b) $\sum_{n=1}^{\infty} \frac{(n+1)(5n+2)}{(3n+2)(n+3)(2n+7)(n+10)} < \frac{n^2}{n^4} = \sum \frac{1}{n^2} \to$ P series

??

$P > 1$

so by $\cancel{L}$ C.T, $\sum_{n=1}^{\infty} a_n$ is convergent   convergent

Since $\frac{1}{n^2}$ conve

(c) $\sum_{n=1}^{\infty} n^2 \sin^2 \frac{5}{n} \Rightarrow$ $b_n = \frac{1}{n^2} \Rightarrow$ P series $2 > 1$ conv

$\lim n^2 \sin^2 \frac{5}{n} \cdot \frac{1}{n^2}$ $\lim \sin^2 \frac{5}{n} = \lim \sin 0 = 0$

cuz' $\lim \frac{a_n}{b_n} = 0 \overset{and}{\Rightarrow}$ since b_n is conv so a_n is convergent

Q6: A sequence $\{a_n\}_{n=1}^{\infty}$ whose sequence of partial sums is $\{S_n\}_{n=1}^{\infty}$, where $S_n = \frac{3n}{n+1}$, find a_n

2

~~$S_n = a_{n-1} + a_n$~~

$S_n = a_n + S_{n-1}$

$a_n = S_n - S_{n-1}$

$= \frac{3n}{n+1} - \frac{3(n-1)}{(n-1)+1}$

$\frac{3n}{n+1} - \frac{3n-3}{n}$

$\frac{3n^2 - (3n - 3(n+1))}{n(n+1)} = \frac{3\cancel{n^2} - \cancel{3n^2} - \cancel{3n} + \cancel{3n} + 3}{n^2+1}$

$\boxed{a_n = \frac{3}{n^2+n}}$