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الجامعة الأردنية	الرياضيات الهندسية١: الامتحان الأول	
[redacted]		مدرس المادة: د. منال غانم وقت المحاضرة: 9:30 – 11:00

Q1: Solve the following differential equation

$xy' = y^2 + y$

$y = vx$

$y' = v + xv'$

$x(v + xv') = v^2x^2 + vx$

$\cancel{xv} + x^2 v' = v^2 \cancel{x^2} \cancel{+ vx}$

$v' = v^2$

$\frac{dv}{dx} = v^2$

$dv = v^2 dx$

$\int \frac{dv}{v^2} = \int dx$

$\frac{v^{-1}}{-1} = x + c$

$-\frac{1}{v} = x + c$

$v = \frac{y}{x}$

$-\frac{1}{\frac{y}{x}} = x + c$

$-\frac{x}{y} = x + c$

$-\frac{y}{x} = \frac{1}{x+c}$

$-y = \frac{x}{x+c}$

$y = \frac{-x}{x+c}$

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Q2: Solve the following differential equation

$xy'' + 2y' + xy = 0$

Given a solution, $y_1 = \frac{\cos x}{x}$.

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$\cancel{\frac{xy'' + 2y' = xy}{x}}$

$\frac{xy''}{x} + \frac{2y'}{x} + \frac{xy}{x} = \frac{0}{x}$

$y'' + \frac{2}{x} y' + y = 0$

$y_2 = y_1 U$

$\int \frac{1}{y_1^2} e^{-\int \frac{2}{x} dx} dx$

$\int \frac{x^2}{\cos^2(x)} e^{\int \frac{2}{x} dx} dx$

$\int \frac{x^2}{\cos^2(x)} e^{2\ln x} dx = \int \frac{x^2}{\cos^2(x)} e^{\ln x^2} dx = \int \frac{x^4}{\cos^2(x)} dx$

$= \int x^4 \sec^2(x) dx$

by parts

$\cancel{u = x^4}$ $\cancel{dv = \sec^2(x)dx}$

$\cancel{du = 4x^3 dx}$ $\cancel{v = \tan(x)}$

$\cancel{x^4 \tan(x) - \int}$

$\cancel{u = \sec^2(x)}$ $\cancel{dv = x^4 dx}$

$\cancel{du = 2\sec^2(x)\tan(x)}$ $\cancel{v = \frac{x^5}{5}}$

$u = x^4$ $dv = \sec^2(x) dx$

$du = 4x^3 dx$ $v = \tan(x)$

$x^4 \tan(x) - 4\int x^3 \tan(x) dx$

$\int x^3 \tan(x) dx$

$u = \tan(x)$ $dv = x^3 dx$

$du = \sec^2(x) dx$ $v = \frac{x^4}{4}$

$\frac{x^4}{4} \tan(x) - \int \frac{x^4}{4} \sec^2(x) dx$

$u = \frac{x^4}{4}$ $du = \sec^2(x) dx$

$du = \frac{x^3}{4} dx$ $v = \tan(x)$

$\cancel{\frac{x^4}{4} - \frac{1}{4}\int x^3 \tan(x) dx}$

$\frac{x^4}{4} - \frac{1}{4}\int x^3 \tan(x) dx = \int x^3 \tan(x) dx$

$\frac{x^4}{4} = 1.25 \int x^3 \tan(x) dx \cdot \frac{1}{1.25}$

$\frac{1}{1.25} \cdot \frac{x^4}{4}$

$\cancel{x^4 \tan(x) - 4\left(\frac{x^4}{4} - \frac{1}{4}\int x^3 \tan(x) dx\right)}$

$x^4 \tan(x) - \cancel{4}\left(\frac{x^4}{\cancel{4}(1.25)}\right)$

$U(x) = x^4 \tan(x) - \frac{x^4}{1.25}$

$y_2 = \frac{\cos(x)}{x}\left(\cancel{x}^4 \tan(x) - \frac{x^4}{1.25}\right)$

$y_2 = \frac{\cos(x)}{\cancel{x}} x^{\cancel{4}3}\left(\tan(x) - \frac{1}{1.25}\right)$

$y_2 = x^3 \cos(x) \tan(x) - \frac{\cos(x)}{1.25}$

$\boxed{y = y_1 + y_2 = \frac{\cos(x)}{x} + x^3 \cos(x) \tan(x) - \frac{\cos(x)}{1.25}}$

Q4: Solve the following differential equation

$x^2y'' - 3xy' + 3y = 3\ln x - 4.$

$x^2y'' - 3xy' + 3y = 0$

~~$y_h \Rightarrow \frac{x^2y''}{x^2} - \frac{3xy'}{x^2} + \frac{3y}{x^2} = \frac{0}{x^2}$~~

~~$y'' - \frac{3}{x}y' + \frac{3}{x^2}y = 0$~~

$y_h = 0$

$y = x^r$

$y' = rx^{r-1}$

$y'' = r(r-1)x^{r-2}$

$(r^2-r)x^{r-2} \cdot x^2 - 3x\, rx^{r-1} + 3x^r = 0$

$r^2x^r - rx^r - 3rx^r + 3x^r = 0$

$r^2x^r - 2rx^r + 3x^r = 0$

$x^r(r^2 - 2r + 3) = 0$

~~$(r - \ )(r - \ ) = 0$~~

$\Delta = b^2 - 4ac$

$4 - 4(1)(3)$

$\boxed{\Delta = 4 - 12 = -8}$

$r_1 = \frac{2 + \sqrt{8}i}{2(1)} = \underbrace{1}_{\alpha} + \underbrace{\frac{\sqrt{8}}{2}}_{\beta} i$

$r_2 = 1 - \frac{\sqrt{8}}{2} i$

$\boxed{y_h = c_1 \underbrace{x^1 \cos\left(\frac{\sqrt{8}}{2}x\right)}_{y_1} + c_2 \underbrace{x^1 \sin\left(\frac{\sqrt{8}}{2}x\right)}_{y_2}}$

$y_p = -y_1 \int \frac{y_2 G(x)}{W} dx + y_2 \int \frac{y_1 G(x)}{W} dx$

$G(x) = \frac{g(x)}{a} = \frac{3\ln|x| - 4}{x^2}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$

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Q3: Solve the following differential equation

$\underbrace{(x^2 + y^2)}_{M}dx - \underbrace{2xy}_{N}dy = 0.$

$M_y = 2y \quad N_x = 2y \longrightarrow$ Exact

$N_x = M_y$

$\int U_x = M = \int x^2 + y^2 \, dx$

$\boxed{U = \frac{x^3}{3} + xy^2 + g(y)} \cdots\cdots *$

$U_y = N = \cancel{2xy} + g'(y) = \cancel{2xy}$

$g'(y) = 0$

$g(y) = K \longrightarrow$ const.

$\boxed{U = \frac{x^3}{3} + xy^2 + K = 0}$

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