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[redacted]

Q1: Solve the following system of ODE's

$$Y' = \begin{pmatrix} 0 & 1 \\ -4 & 4 \end{pmatrix} Y + \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{2t}}_{=0}$$

$y_h \Rightarrow$

$$\begin{pmatrix} 0-\lambda & 1 \\ -4 & 4-\lambda \end{pmatrix} \Rightarrow \det = (-\lambda)(4-\lambda)+4=0$$

$$-4\lambda+\lambda^2+4=0$$

$$\lambda^2-4\lambda+4=0$$

$$(\lambda-2)(\lambda-2)=0$$

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$(2=\lambda_1=\lambda_2=\lambda)$ repeated $\quad (\lambda-2)^2=0 \quad \boxed{\lambda=2}$

$$X_\lambda \Rightarrow \begin{pmatrix} -2 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{array}{l} -2a+b=0 \\ -4a+2b=0 \end{array}\right\} \rightarrow b=2a \text{ (a is free vector } \rightarrow a=t \rightarrow b=2t)$$

$$X_\lambda \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} t \\ 2t \end{pmatrix} = t\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$y_\lambda \Rightarrow \begin{pmatrix} -2 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{array}{l} -2c+d=1 \Rightarrow -d=-2c-1 \Rightarrow \boxed{d=2c+1} \text{ let } c=2 \text{ then } \boxed{\begin{pmatrix} 2 \\ 5 \end{pmatrix}=d} \\ -4c+2d=2 \qquad\qquad\qquad\qquad\qquad\quad d=5 \end{array}$$

$$y_h = C_1\begin{pmatrix} 1 \\ 2 \end{pmatrix}e^{2t} + C_2\left[t\begin{pmatrix} 1 \\ 2 \end{pmatrix}e^{2t} + e^{2t}\begin{pmatrix} 2 \\ 5 \end{pmatrix}\right]$$

تكملة الحل

خلف هذه الورقة

Q2: Find two series solutions for the following differential equation

$$(1+x^2)\,y''-4xy'+6y=0,$$

Near $x_o=0$.

Q3: Find the form of the particular solution following differential equation

$y^{(5)} - 2y^{(4)} + 4y''' - 8y'' = (2x-3)e^{2x}$.

$$y^{(5)} - 2y^{(4)} + 4y''' - 8y'' = 0$$

$y = e^{rx}$

$y' = re^{rx}$

$y'' = r^2 e^{rx}$

$y''' = r^3 e^{rx}$

$y^{(4)} = r^4 e^{rx}$

$y^{(5)} = r^5 e^{rx}$

$$r^5 - 2r^4 + 4r^3 - 8r^2 = 0$$

$$r^2(r^3 - 2r^2 + 4r - 8) = 0$$

$$(r-2)(r-2)(r-2) = 0$$

$r = 2$

$r_2 = r_3 = r_1 = 2$ (repeated)

$$y_h = c_1 e^{2x} + c_2 x e^{2x} + c_3 x^2 e^{2x}$$

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$$y_p \Rightarrow y^{(5)} - 2y^{(4)} + 4y''' - 8y'' = (2x-3)e^{2x}$$

$\cancel{y_p = e^{2x}(Ax + B_0)}$

$\cancel{y_p = xe^{2x}(Ax + B) = Axe^{2x} +}$

$$W = \begin{vmatrix} e^{2x} & xe^{2x} & x^2 e^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} & 2x^2 e^{2x} + 2xe^{2x} \\ 4e^{2x} & 4xe^{2x} + xe^{2x} + 2e^{2x} & \text{نشتق} \end{vmatrix}$$

Q4: Let A and B be any 3×3 matrices such that $|B| = 8$ and $AB = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$

(a) Find $|A^{-1}|$.

(b) Solve $B\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

$\cancel{|A \cdot B| = 3(2) - 2(20) + 1(5) = 6 - 40 + 5 = -29 \neq 0}$ 1

$\cancel{|A \cdot B| = |A| \cdot |B|}$

$\cancel{-29 = |A| \cdot 8}$

$\cancel{|A| = \frac{-29}{8}}$

$|A \cdot B| = 3(2) - 2(20) + 1(5) = -29 \neq 0$

$\det(A^{-1}) \cdot \det(B^{-1}) = \det(AB)^{-1}$

$(AB)^{-1} = \frac{1}{\det} C^T$

$C^T = (C_{ij})$

$C_{ij} = (-1)^{i+j} M_{ij}$

1- $1\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix} = 2 \rightarrow M_{11}$

2- $-1\begin{bmatrix} 5 & 2 \\ 0 & 4 \end{bmatrix} = -20 \rightarrow M_{12}$

3- $1\begin{bmatrix} 5 & 1 \\ 0 & 1 \end{bmatrix} = 5 \rightarrow M_{13}$

4- $-1\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} = -7 \rightarrow M_{21}$

5- $1\begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix} = 12 \rightarrow M_{22}$

6- $-1\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} = -3 \rightarrow M_{23}$

7- $1\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = 3 \rightarrow M_{31}$

8- $-1\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = -1 \rightarrow M_{32}$

9- $1\begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} = -4 \rightarrow M_{33}$

$C^T = \begin{bmatrix} 2 & -20 & 5 \\ -7 & 12 & -3 \\ 3 & -1 & -4 \end{bmatrix}$

$\frac{1}{\det} C^T = \begin{bmatrix} \frac{-2}{29} & \frac{20}{29} & \frac{-5}{29} \\ \frac{7}{29} & \frac{-12}{29} & \frac{3}{29} \\ \frac{-3}{29} & \frac{1}{29} & \frac{4}{29} \end{bmatrix} = (AB)^{-1}$