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الجامعة الأردنية	0301202 رياضيات هندسية 1	الامتحان الأول: 2016/11/9
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يتكون الامتحان من 5 أسئلة في 3 ورقات.

[1] (5 marks) Find a function $M(x,y)$ so that the given ODE is exact

$$\underbrace{M(x,y)}_{M}dx + \underbrace{(xe^x + 2xy)}_{N}dy = 0$$

$M_y = M(y)$
$N_x = xe^x + e^x + 2y$] exact $\quad M_y = xe^x + e^x + 2y$

~~$U_x = M_y$~~ $\quad$ ~~$U_x M(x,y) = Z$~~ $\quad U_y = N$

~~$U(x,y) = M(x,y)$~~ $\quad$ ~~$U(x,y) = \int M(x,y)$~~ $\quad U = \int xe^x + 2xy\, dy$

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$U = xe^x y + \cancel{x} y^2 + f(x)$

$U_x = M$

$M(x,y) = y(xe^x + e^x) + y^2 + f'(x)$

[2] (5 marks) Solve the ODE: $(1+y)y'' = (y')^2$

X missing

~~$(1+y) z z' = z^2$~~

~~$y' = z$~~
~~$y'' = z z'$~~

$\frac{yy''}{y} + \frac{y''}{y} = \frac{(y')^2}{y}$

$y'' + \frac{1}{y}y'' = \frac{(y')^2}{y}$

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$y' = z$
$y'' = z\frac{dz}{dy}$

$z\frac{dz}{dy} + \frac{1}{y}z\frac{dz}{dy} = \frac{z^2}{y}$

$\frac{z\,dz}{dy}\left(1+\frac{1}{y}\right) = \frac{z^2}{y}$

$\frac{1}{z}\frac{z}{z^2}\frac{dz}{dy} = \frac{1}{\left(1+\frac{1}{y}\right)y}$ $\Rightarrow$

$y+1$

$\int\frac{dz}{z} = \int\frac{dy}{y+\left(\frac{1}{y^2}\right)}$

$\ln(z) = \cos^{-1}\frac{1}{y}$

$\frac{1}{y} = \cos\ln(z)$

$\frac{1}{y} = \cos\ln y'$

[3](5 marks) Find a suitable form for the particular solution y_p of the given ODE if it is to be solved using the undetermined coefficients method

$$y''' - y'' - 4y' + 4y = 5 - e^x + xe^{2x}$$

$\boxed{1+1} \leftarrow \lambda^3 - \lambda^2 - 4\lambda + 4 = 0$

$(\lambda - 1)(\lambda^2 - 4)$

$(\lambda - 1)(\lambda - 2)(\lambda + 2)$

$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = -2$

$y_h = c_1 e^x + c_2 e^{2x} + c_3 e^{-2x}$

y_p

$y_1 x = A x$

$y_2 x = B e^x \cdot x$

$y_3(x) = (cx + D) e^{2x} \cdot x$

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$\therefore y_p = (Ax) + Bxe^x + (cx + D)e^{2x}$

	λ^3	λ^2	λ	c
	1	-1	-4	4
1+		1	0	-4
	1	0	-4	0

$y = c_1 e^x + c_2 e^{2x} + c_3 e^{-2} + Ax + Bxe^x + (c_4 x + D) e^{2x}$

[4] (5 marks) Solve the initial value problem:

$$\underbrace{(2xy + 2x)}_{M}dx - \underbrace{y}_{N}dy = 0, \qquad y(0) = -2.$$

$\left.\begin{array}{l} M_y = 2x + 0 \\ N_x = 0 \end{array}\right\}$ not exact

$\dfrac{M_y - N_x}{N} = \dfrac{2x - 0}{y} = \dfrac{2x}{y}$ X

$\dfrac{N_x - M_y}{M} = \dfrac{-2x}{2xy + 2x} = \dfrac{-\cancel{2x}}{\cancel{2x}(y+1)} = \dfrac{-1}{(y+1)} = F(y)$

~~$\frac{-1}{(y+1)}(2xy + 2x) - \frac{-1}{(y+1)} y\,dy = 0$~~

$\dfrac{-1\,(2x)\cancel{(y+1)}}{\cancel{(y+1)}} + \dfrac{y}{(y+1)} = 0$

~~$M_y = \frac{(2xy+2x)(1) + 2xy + 2x (2x) + c}{(y+1)^2} \Rightarrow 2x(y+1) + (y+1)\,2(x)$~~

~~$N_x = \frac{-y(1) + (y+1)}{(y+1)^2}$~~

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$-2x\,dx + \dfrac{y}{y+1}\,dy = 0$

$\int 2x\,dx = \int \dfrac{y}{y+1}\,dy$

$x^2 = \int$ ~~...~~

$c \rightarrow x^2 = \int \dfrac{y}{y+1}\,dy$

$\left.\begin{array}{l} M_y = 0 \\ N_x = 0 \end{array}\right\}$ exact

$U_x = M \Rightarrow U = \int -2x\,dx$

$= x^2 + y(y)$

$U_y = N$

$2x + g' = \dfrac{y}{y+1} \Rightarrow g(y)' = \dfrac{y}{2x(y+1)} \Rightarrow g(y) =$

[5] (5 marks) Solve the ODE: $x^2y'' - 3xy' + 4y = x^3$

$y = y_h + y_p$

$y_h \Rightarrow$

~~r (r-1)~~ let ~~y~~

$r(r-1) - 3r + 4 = 0$

$r^2 - r - 3r + 4 = 0$

$r^2 - 4r + 4 = 0$

$(r-2)(r-2) = 0$

$r_1 = 2 \Rightarrow$

$r_2 = 2 \Rightarrow$

$y_h = c_1 e^{2x} + c_2 x e^{2x}$

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$$W = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix}$$

$e^{2x}(2xe^{2x} + e^{2x}) - 2e^{2x} \cdot e^{2x}$

$2xe^{4x} + e^{4x} - 2e^{4x}$

$e^{4x}(2x + 1 - 2)$

$e^{4x}(2x - 1)$

y_p :-

$y = -y_1 \int \frac{y_2\, g(x)}{w} + y_2 \int \frac{y_1\, g(x)}{w}$

$y = -e^{2x} \int \frac{x e^{2x} x^2}{e^{4x}(2x-1)} + x e^{2x} \int \frac{e^{2x} x}{e^{4x}(2x-1)}$

$-e^{2x} \int \frac{x^2}{e^{2x}(2x-1)} + x e^{2x} \int \frac{x}{e^{2x}(2x-1)}$

y_p :-

$y_p = Ax^3 + Bx^2 + Cx + D$

$y'_p = 3Ax^2 + 2Bx + C$

$y''_p = 6Ax + 2B$