15

[redacted]

يتكون الامتحان من ٤ اسئلة في ٤ ورقات.

Q1. (5 points) Use Gauss elimination to solve the system

$$x_1 - x_2 - x_3 = -3$$
$$2x_1 + 3x_2 + 5x_3 = 7$$
$$x_1 - 2x_2 + 3x_3 = -11$$

$A = \begin{bmatrix} 1 & -1 & -1 \\ 2 & +3 & +5 \\ 1 & -2 & 3 \end{bmatrix}$

$B = \begin{bmatrix} -3 \\ 7 \\ -11 \end{bmatrix}$

By Gaussian elimination

$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 2 & 3 & 5 & 7 \\ 1 & -2 & 3 & -11 \end{array}\right]$

$2R_1 + R_3 \to R_3$

$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 4 & 1 & 3 & 7 \\ 0 & +1 & -4 & -8 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 2 & 3 & 5 & 7 \\ 1 & -2 & 3 & -11 \end{array}\right]$

$2R_1 + R_2 \to R_2$

$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 4 & 1 & 3 & 1 \\ 1 & -2 & 3 & -11 \end{array}\right]$

$2R_1 + R_3 \to R_3$

$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 4 & 1 & 3 & 1 \\ 3 & -4 & 1 & -17 \end{array}\right]$

~~$4R_1 + R_2 \to R_2$~~ $R_1 \leftrightarrow R_2$

$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 1 \\ 1 & -1 & -1 & -3 \\ 3 & -4 & 1 & -17 \end{array}\right]$

$R_2 - R_3 \to R_1$

$\left[\begin{array}{ccc|c} 1 & -5 & 2 & -12 \\ 1 & -1 & -1 & -3 \\ 3 & -4 & 1 & -17 \end{array}\right]$

$3R_2 - R_3 \to R_3$

$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 1 \\ 1 & -1 & -1 & -3 \\ 0 & 1 & -4 & -8 \end{array}\right]$

$R_2 + R_3 \to R_3$

$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 1 \\ 1 & -1 & -1 & -3 \\ \frac{0}{5} & \frac{0}{5} & \frac{-5}{5} & \frac{-11}{5} \end{array}\right]$

$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 1 \\ 1 & -1 & -1 & -3 \\ 0 & 0 & -1 & \frac{-11}{5} \end{array}\right]$

$R_1 - 4R_2 \to R_2$

$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 1 \\ 0 & \frac{5}{5} & \frac{7}{5} & \frac{-11}{5} \\ 0 & 0 & -1 & \frac{-11}{5} \end{array}\right]$

$\left[\begin{array}{ccc|c} \frac{4}{4} & \frac{1}{4} & \frac{3}{4} & 1 \\ 0 & 1 & \frac{7}{5} & \frac{-11}{5} \\ -1\times\ 0 & 0 & -1 & \frac{-11}{5} \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ 0 & 1 & \frac{7}{5} & \frac{-11}{5} \\ 0 & 0 & 1 & \frac{11}{5} \end{array}\right]$

$\boxed{z = \frac{11}{5}}$

$y + \frac{7}{5}z = \frac{-11}{5}$

$y + \frac{7}{5} \times \frac{11}{5} = \frac{-11}{5}$

$y = \frac{-11}{5} - \frac{77}{5}$

$\boxed{y = \frac{-22}{5}}$

$x + \frac{1}{4}y + \frac{3}{4}z = \frac{1}{4}$

$x + -\frac{22}{5} + \frac{33}{20} = \frac{1}{4}$

$x = \frac{1}{4} - \frac{22}{5} + \frac{33}{20}$

$$\begin{bmatrix} -3 & -3 & -3 \\ 12 & 25 & 18 \\ 14 & 18 & 12 \end{bmatrix}$$

Q2. (6 points) If $|A| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 5$, find the determinants of the following matrices:

a) $B = \begin{bmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$ $= 5$ (as we change the rows the det still as it)

b) $C = \begin{bmatrix} a_1 - b_1 & a_2 - b_2 & a_3 - b_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 2c_1 & 2c_2 & 2c_3 \end{bmatrix}$ $= 0$ as we substruct any two rows it will get zero determin

c) $M = \begin{bmatrix} 4a_1 & 4a_2 & 4a_3 \\ b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$ $= 5$ as we multi Ply By a constat still as it

Q3. (7 points) Given a system

$$Y' = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix} Y + \begin{bmatrix} e^{-2t} \\ 5 \end{bmatrix} \quad \Rightarrow$$

a) Solve the corresponding homogeneous system .

b) Find a suitable form for the particular solution Y_p, if the method of undetermined coefficients to be used.

$\underline{\underline{a}}$ $\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$\vec{X}(A - \lambda I) = 0$

$\begin{bmatrix} 2-\lambda & 4 \\ 1 & -1-\lambda \end{bmatrix} = 0$

$(2-\lambda)(-1-\lambda) - 4 = 0$

$-2 + \lambda - 2\lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 1\lambda - 6 = 0$

$\lambda = 3$

$\lambda = -2$

6

when

$\lambda = 3$

$\vec{X}\lambda = \begin{pmatrix} a \\ b \end{pmatrix}$

$\begin{bmatrix} -1 & 4 \\ 1 & \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-a + 4b = 0$

~~a = 4b~~

$b = \frac{a}{4}$ $\quad$ $a \Rightarrow$ free variable

$a = t$

$b = \frac{t}{4}$

$\therefore X\lambda_1 = t\begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix}$

$X\lambda_2 \Rightarrow \begin{pmatrix} c \\ d \end{pmatrix}$

$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$4c + 4d = 0$

$d = -c$ $\quad$ $c = t$

$d = -t$

$\therefore X\lambda_2 = t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$Y_h = C_1 \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix} e^{3t} + C_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$

$\underline{\underline{b}}$ $Y_p = (\vec{U}t + \vec{V})e^{-2t}$

$+ \, t$

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Q4. (7 points) Find a power series solution for $(x-1)y''+y'=0$ about $x_0=0$.

By $X-1\Big|_{x_0=0} = 0-1 = -1 \neq 0 \Rightarrow$ ordinary

$y = \sum_{0}^{\infty} a_n x^n$

$y' = \sum_{1}^{\infty} a_n n x^{n-1}$

$y'' = \sum_{2}^{\infty} a_n (n)(n-1) x^{n-2}$

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Sub in the eqn.

$(x-1)\sum_{2}^{\infty} a_n (n)(n-1) x^{n-2} + \sum_{1}^{\infty} a_n (n) x^{n-1} = 0$

$\sum_{2}^{\infty} a_n(n)(n-1) x^{n-1} - \sum_{2}^{\infty} a_n(n)(n-1) x^{n-2} + \sum_{1}^{\infty} a_n (n) x^{n-1} = 0$

$\sum_{1}^{\infty} a_n(n)(n-1) x^{n-1} - \sum_{1}^{\infty} a_{n+1}(n+1) n x^{n-1} + \sum_{1}^{\infty} a_n (n) x^{n-1} = 0$

[1] powers are equal

[2] radius are equal

$\sum_{1}^{\infty} \left[a_n (n)(n-1) - a_{n+1}(n+1)(n) + a_n(n) \right] x^{n-1} = 0$

$a_n (n)(n-1) + a_n(n) = a_{n+1}(n+1)(n)$

$\frac{a_n (n)(n-1+1)}{n+1\,(n)} = a_{n+1}$

$\frac{a_n x(n-1)}{n+1\,(n)} + \frac{a_n (n)}{n+1}$

$\boxed{\frac{a_n (n)}{n+1} = a_{n+1}}$

$\frac{a_n(n-1) + a_n}{n+1}$

$n \geq 1$

$n=0 \Rightarrow a_1 = \frac{a_0 \cdot 0}{0+1} = 0$

$n=1 \Rightarrow a_2 = \frac{a_1 \cdot 1}{2} = \frac{a_1}{2} = 0$

n