POWER UNIT

Electromagnatics I

NoteBook

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بأفكارنا_نُبدع

1st Exam Material

24/6

ex. use Dimensional Analysis (DA) to show that

$$d = ut + \frac{1}{2} at \quad \text{is wrong.}$$

Notes all the quantites should be of the same dimension.

e.g distance = distance + distance

sol.

$$[d] = L$$

$$[ut] = \frac{L}{T} T = L$$

$$[\tfrac{1}{2} at] = \frac{L}{T^2} T = \frac{L}{T} \longrightarrow \text{wrong} \left(\begin{array}{l}\text{should} \\ \text{be } L\end{array}\right)$$

dimension less

$\therefore$ the given equation is wrong.!!

*The ~~not~~ wrong equation: $[\frac{1}{2} a t^2] = \frac{L}{T^2} T^2 = L$

*but if it was $[2 a t^2] = \frac{L}{T^2} T^2 = L$ (Wrong but we can't know that)

ex (next page)

Example: from observation

$T_P \propto l_g$ → gravity

$[T_P] = [K\, l\, g]$ K is dimensionless

$$T = L^n \left(\frac{L}{T^2}\right)^d = L^{n+d}\, T^{-2d}$$

Thus:

$n + d = 0 \Rightarrow n = -d$

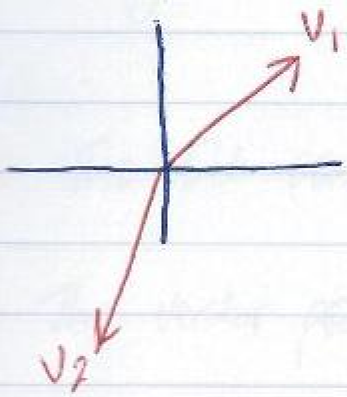
$-2d = 1 \Rightarrow d = -\frac{1}{2} \Rightarrow n = \frac{1}{2}$

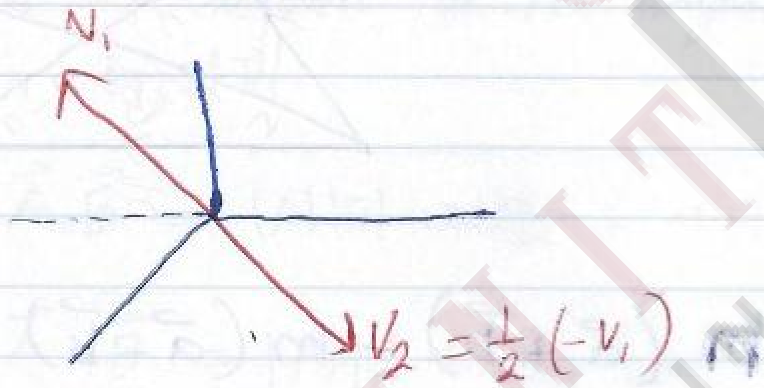
$$\boxed{T_P = K\, l^{\frac{1}{2}} g^{-\frac{1}{2}} = K\sqrt{\frac{l}{g}}}$$

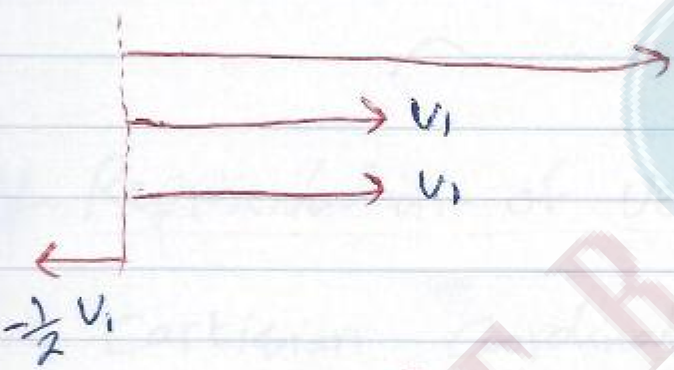
* DA doesn't help in determining K.

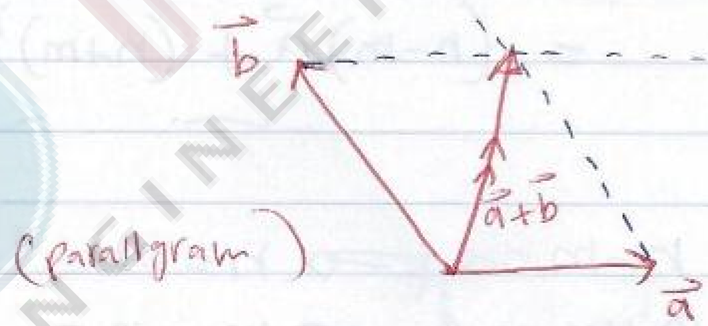
Vectors:

Quantites with magnitude and direction.

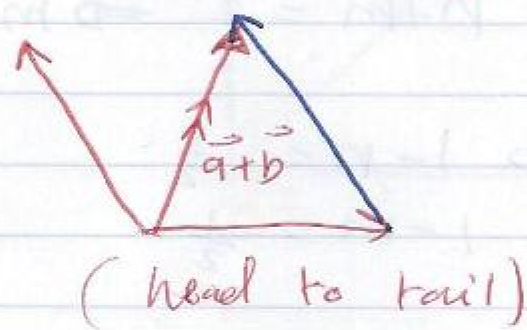


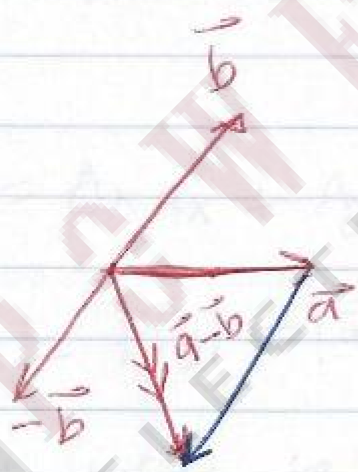






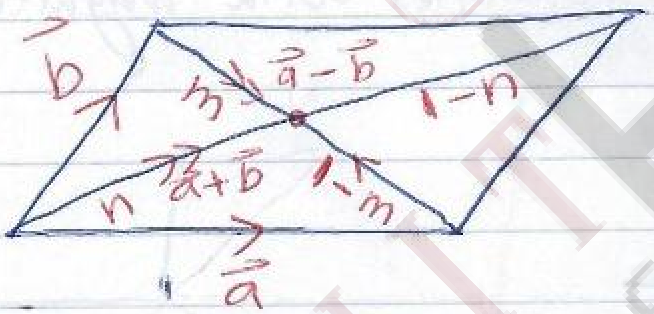
or





$$\vec{a}-\vec{b} = \vec{a}+(-\vec{b})$$

Example:



Sol. $\vec{b} = n(\vec{a}+\vec{b}) - m(\vec{a}-\vec{b})$

$= (n-m)\vec{a} + (n+m)\vec{b}$

$\Rightarrow n-m=0 \Rightarrow n=m$

$n+m=1 \Rightarrow m=n=\frac{1}{2}$

$\Rightarrow 1-n=\frac{1}{2}$

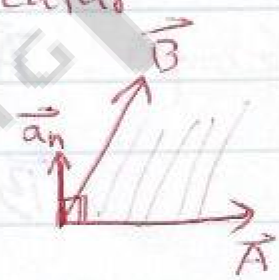
$1-m=\frac{1}{2}$

* Operations on Vectors:

1 Magnitude of a vector: A measure of its length, denoted by $|\vec{v}|$

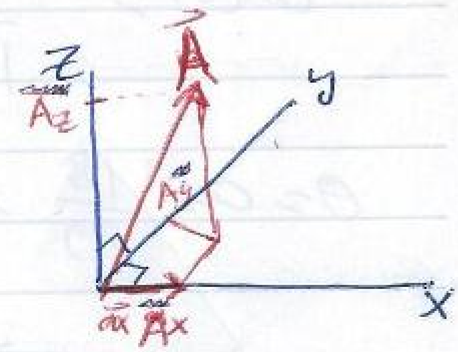
2 The Dot product: $\vec{A} \cdot \vec{B} = |A||B| \cos\theta_{AB}$; a Scalar

3 The vector product: $\vec{A} \times \vec{B} = |A||B| \sin\theta_{AB} \, \vec{a}_n$



* Representation of Vectors:

1 Cartisian Coordinate systems.



$$\vec{A} = A_x a_x + A_y a_y + A_z a_z$$

$$|\vec{a}_x| = |\vec{a}_y| = |\vec{a}_z| = 1$$

* if two vectors are orthogonal (perpindicular to each other), then:

* $\vec{A} \cdot \vec{B} = |A||B| \cos 90^\circ = 0$

* Using the fact that $\vec{a}_x, \vec{a}_y, \vec{a}_z$ are mutually orthogonal, then:

$$a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$$

Based on this; $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

* $\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\vec{a}_x + (A_x B_z - A_z B_x)(-\vec{a}_y) + (A_x B_y - A_y B_x)\vec{a}_z$

Examples

$\vec{A} = 2a_x - 3a_y + 4a_z$

$\vec{B} = 5a_x + 2a_z$

sol.

$$\cos(\theta) = \frac{\vec{A} \cdot \vec{B}}{|A||B|} = \frac{10 + 0 + 8}{\sqrt{29}\,\sqrt{29}} = \frac{18}{29}$$

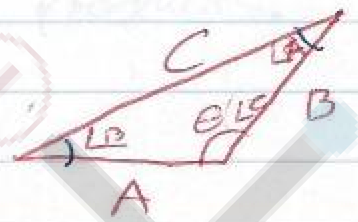
$$\theta = \cos^{-1}\frac{18}{29} =$$

$$\vec{a}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{2}{\sqrt{29}} a_x - \frac{3}{\sqrt{29}} a_y + \frac{4}{\sqrt{29}} a_z$$

$\frac{\vec{A}}{|\vec{A}|}$

ex. Determine the unit vector normal (orthognal) to $\vec{A}$ & $\vec{B}$

ex. For the following triangle with lengths A, B and C. prove:



i) $C^2 = a^2 + b^2 - 2AB\cos\theta$

ii) $\dfrac{A}{\sin A} = \dfrac{B}{\sin B} = \dfrac{C}{\sin C}$

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* Properties of the scalar and vector products:

* $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ commutative. تبديلي

* $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$ distrutative توزيعي

* $\vec{A}\cdot\vec{A} = |\vec{A}|^2$

* $\vec{A}\cdot\vec{B} = 0$ if $\theta = 90^\circ$

* $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$.

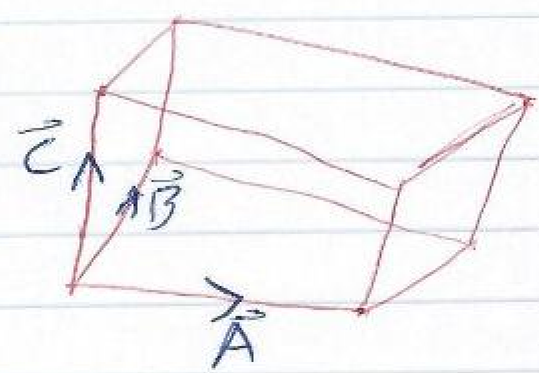
* $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

* $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

* $\vec{A}\times\vec{A} = 0$ if $\theta = 0^\circ$

* $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{C}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{C}\times\vec{A})$ Scalar triple product

→ it gives the volume of the parallelopiped generated by $\vec{A}, \vec{B}$ & $\vec{C}$.



* calculated as:

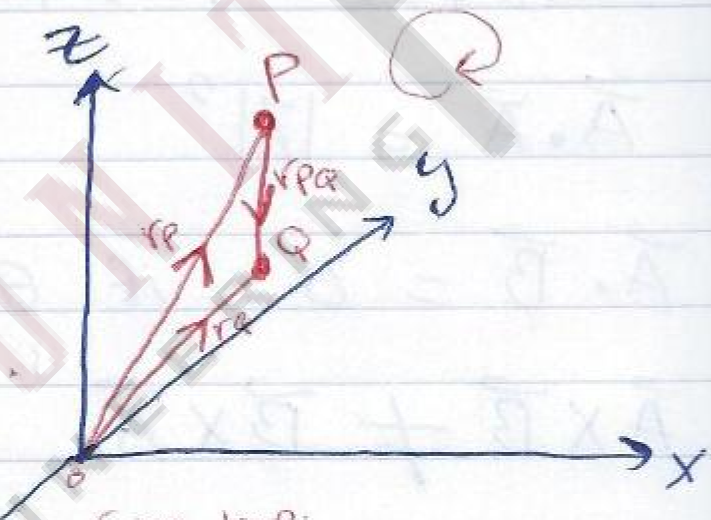
$$\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

bac − cab

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot B)$$

" Vector triple product "

The Position vector :



$$\vec{r}_P = x_P \vec{a}_x + y_P \vec{a}_y + z_P \vec{a}_z$$

$$\vec{r}_Q = x_Q \vec{a}_x + y_Q \vec{a}_y + z_Q \vec{a}_z$$

from loop:

$r_P + r_{PQ} - r_Q = 0$

$r_{PQ} = r_Q - r_P$

$\vec{r}_{PQ} = \vec{r}_Q - \vec{r}_P$ (distance vector) or (displacement vector)

ex.

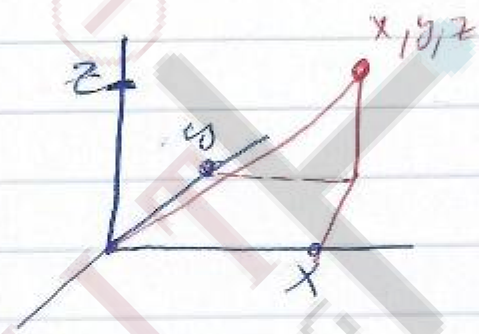
$a + b + c + d - e + f + g = 0$

*

Ch.2

* Coordinate systems:

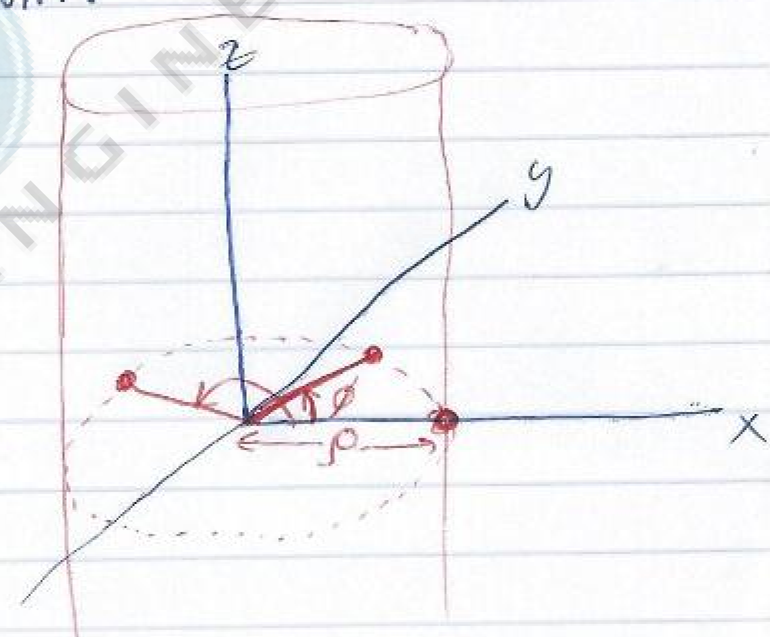
* cartesian Coordinate system.



$$\vec{r}_p = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$$

$$|r_p| = \sqrt{x^2+y^2+z^2}$$

* Circular Cylindrical system:



$x = \rho \cos \phi$

$y = \rho \sin \phi$

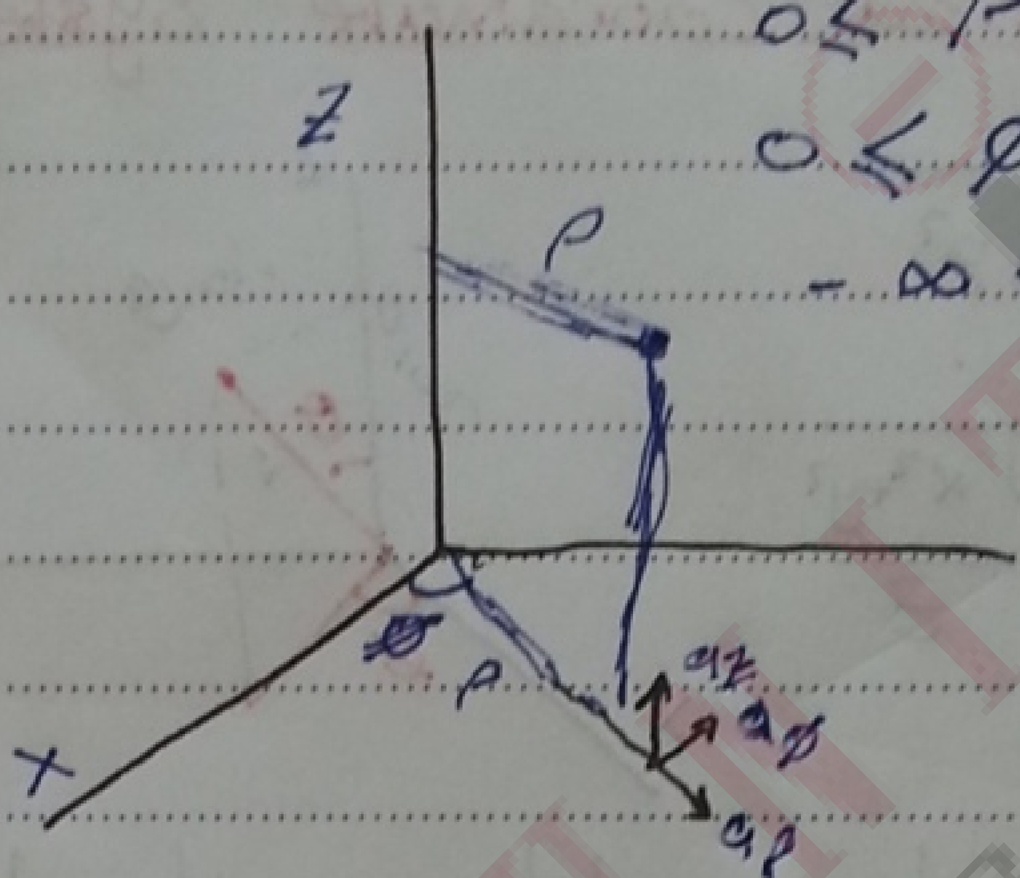
$z = z$

OR

$\rho = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1} \dfrac{y}{x}$

$z = z$



$0 \leq \rho < \infty$

$0 \leq \phi < 2\pi$

$-\infty < z < \infty$

$$A = A_\rho \vec{a}_\rho + A_\phi \vec{a}_\phi + A_z a_z$$

$$|\vec{A}| = \left[A_\rho^2 + A_\phi^2 + A_z^2\right]^{1/2}$$

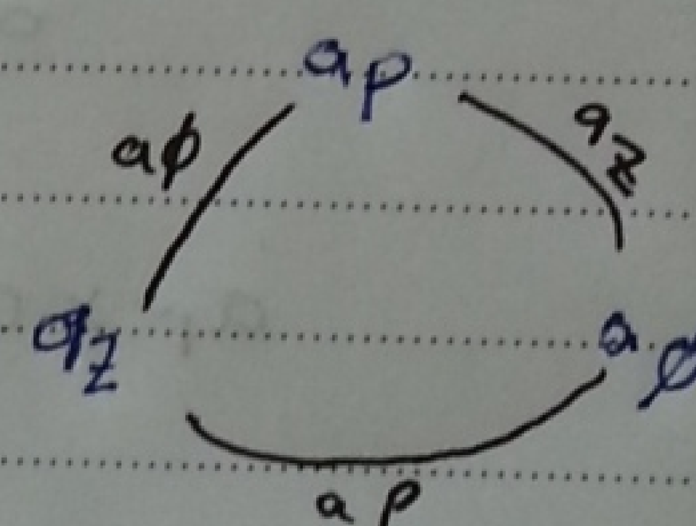
$$\vec{a}_\rho \cdot \vec{a}_\rho = \vec{a}_\phi \cdot \vec{a}_\phi = \vec{a}_z \cdot \vec{a}_z = 1$$

$$\vec{a}_\rho \cdot \vec{a}_\phi = \vec{a}_\rho \cdot \vec{a}_z = 0$$

$$\vec{a}_\rho \times \vec{a}_\phi = \vec{a}_z$$

$$a_\phi \times a_z = a_\rho$$

$$a_z \times a_\rho = a_z$$

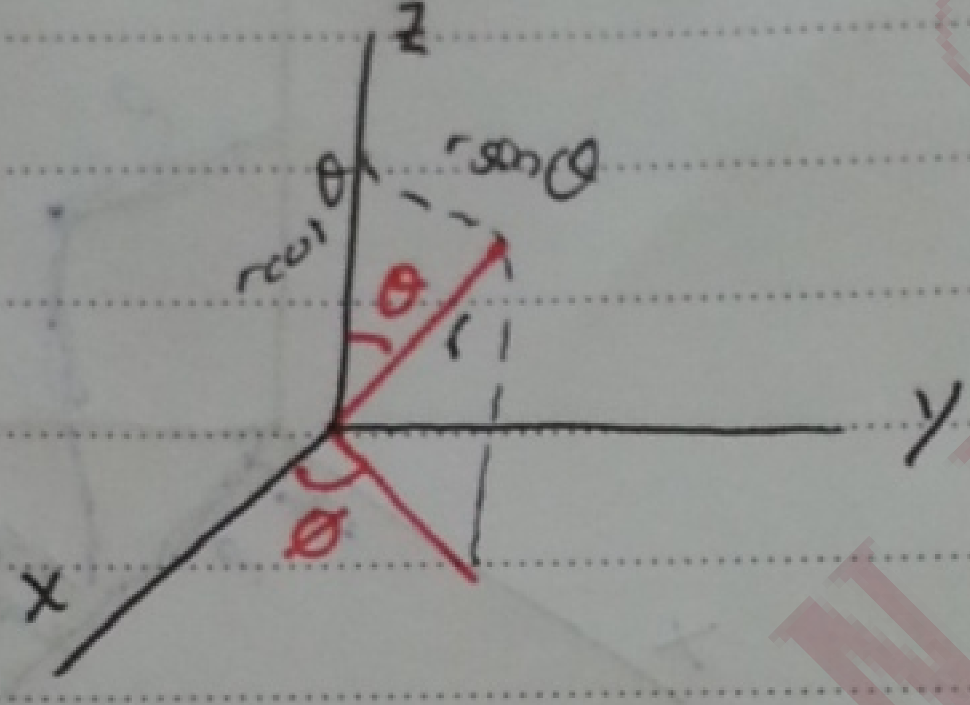


Spherical coordinate system:-

$r = \sqrt{x^2+y^2+z^2}$

$\theta = \tan^{-1} \dfrac{\sqrt{x^2+y^2}}{z}$

$\phi = \tan^{-1} \dfrac{y}{x}$



$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

A point is determined by a distance r. and two angles θ, ϕ.

$0 \leq r < \infty$

$0 \leq \theta \leq \pi$

$0 \leq \phi < 2\pi$

$\vec{A} = A_r \vec{a_r} + A_\theta \vec{a_\theta} + A_\phi \vec{a_\phi}$

$= (A_r, A_\theta, A_\phi)$

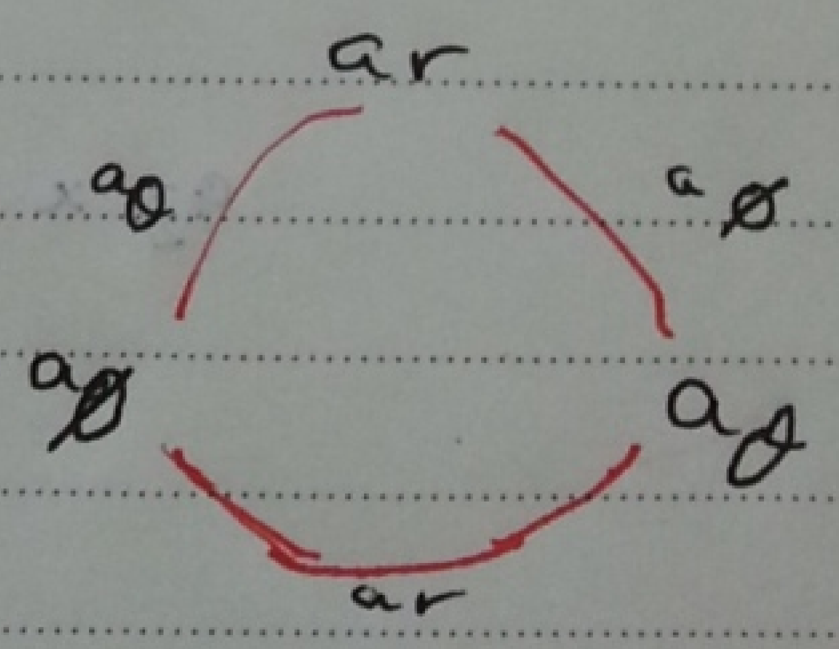
$\vec{a_r} \cdot \vec{a_r} = \vec{a_\theta} \cdot \vec{a_\theta} = \vec{a_\phi} \vec{a_\phi} = 1$

$a_r a_\theta = a_r a_\phi = 0$

$a_r \times a_\theta = a_\phi$

$a_\theta \times a_\phi = a_r$

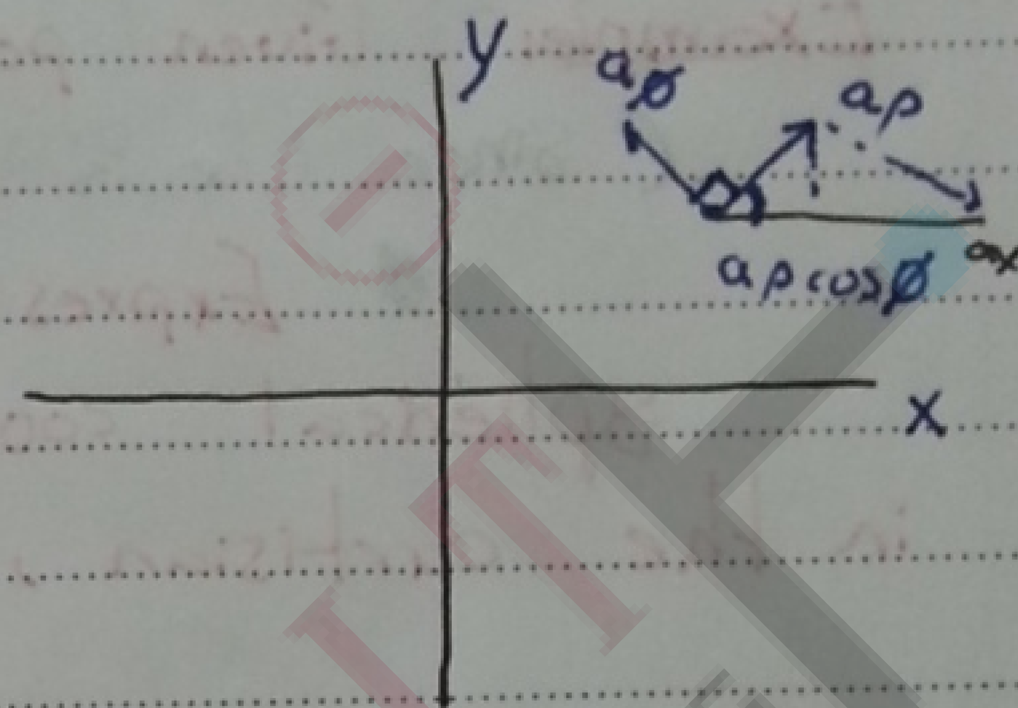
$a_\phi \times a_r = a_\theta$



Coordinate Transformation:

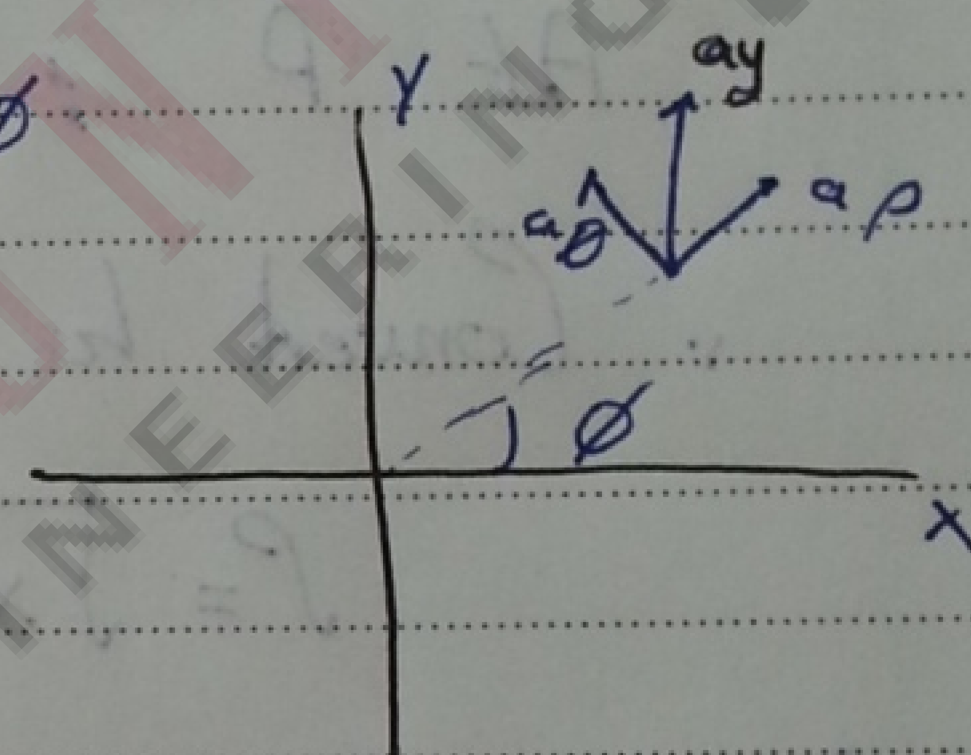
$a_x = \cos\phi \, a_\rho + \sin\phi \, (-a_\phi)$

$a_x = a_\rho \cos\phi - a_\phi \sin\phi$



Similarly:

$a_y = \sin\phi \, a_\rho + \cos\phi \, a_\phi$



In matrix form:

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}}_{Q} \begin{bmatrix} a_\rho \\ a_\phi \end{bmatrix}$$

$$= Q \begin{bmatrix} a_\rho \\ a_\phi \end{bmatrix}$$

Where $QQ^T = I_n$ i.e. Q is orthogonal matrix.

$$\Rightarrow Q^{-1} = Q^T$$

$$\begin{bmatrix} a_\rho \\ a_\phi \end{bmatrix} = Q^{-1} \begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix}$$

$$\therefore QQ^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Sphyrical أو Cylindrical (بما إنها سالب إذاً لا يمكن أن تكون

Subject Date No.

Ex] $P(-2, 6, 3)$ and Vector field $\vec{A} = y a_x + (x+z) a_y$

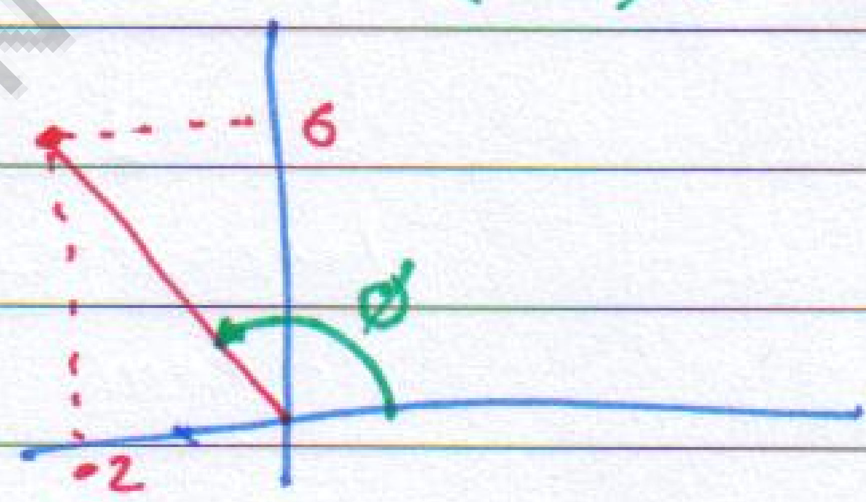
So:-

$\bar{A}$ at $P \Longrightarrow x = -2 \quad y = 6 \quad z = 3$

#1 Convert to cylindrical:

$$\rho = \sqrt{x^2+y^2} = 6.32$$

$$\phi = \tan^{-1}\left(\frac{y}{z}\right) = \tan^{-1}\left(\frac{6}{-2}\right) = 108.34^\circ$$



#2 Convert to sphyrical:

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{\rho^2+z^2} = 7$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = 64.62^\circ$$

$$\phi_s = \phi_c = 108.43^\circ$$

No. continued

P is $(-2, 6, 3)$

or $(6.32, 108.43^{\circ}, 3)$

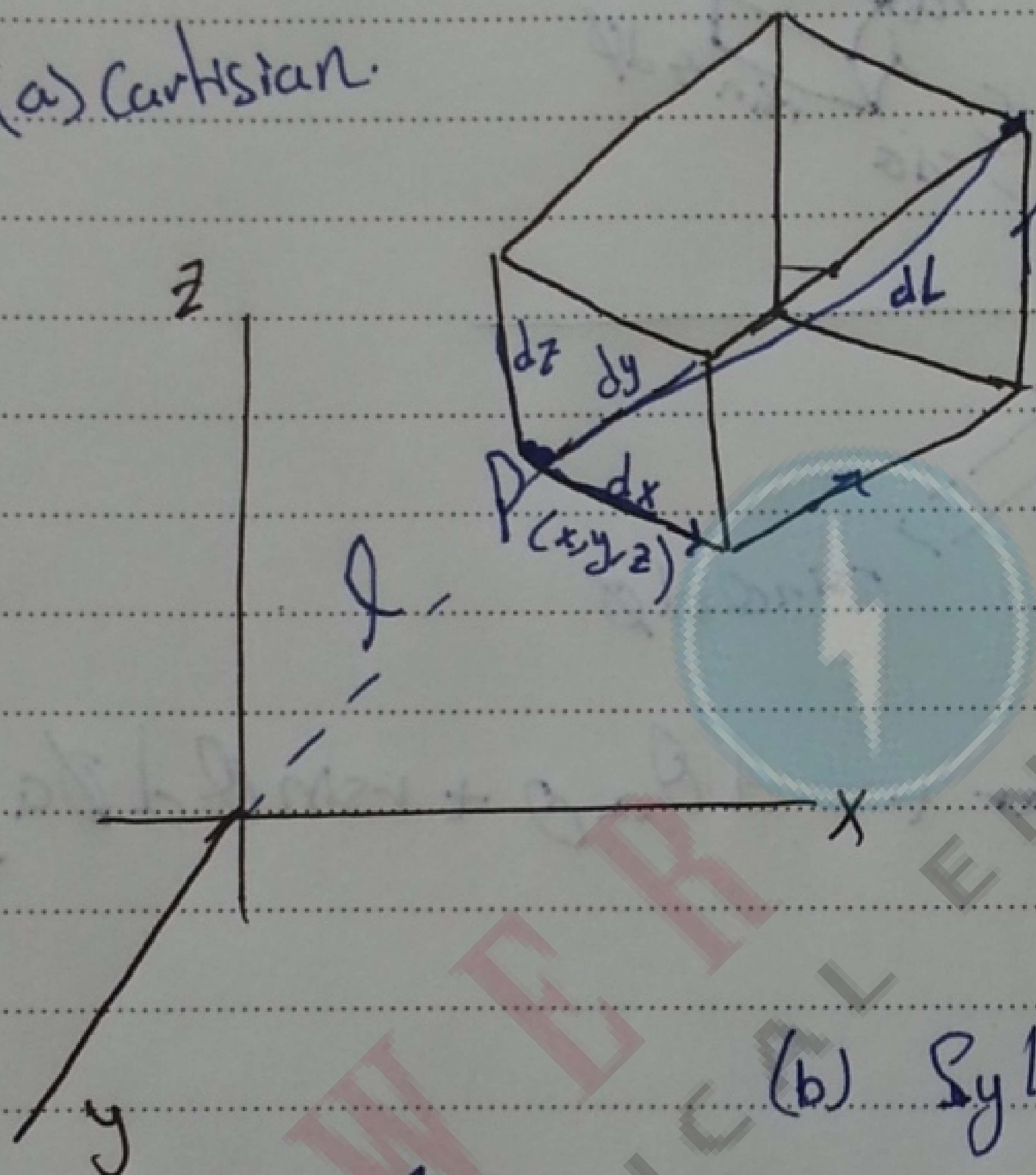
or $(7,$

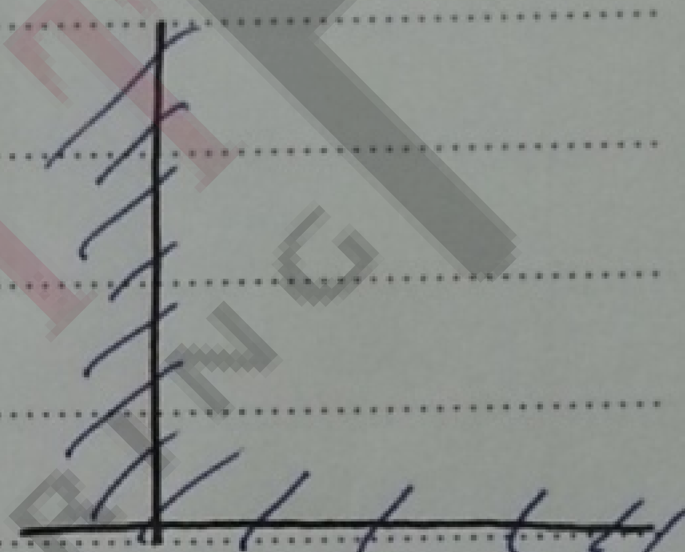
Vector calculus:

(1) Differential line (length):

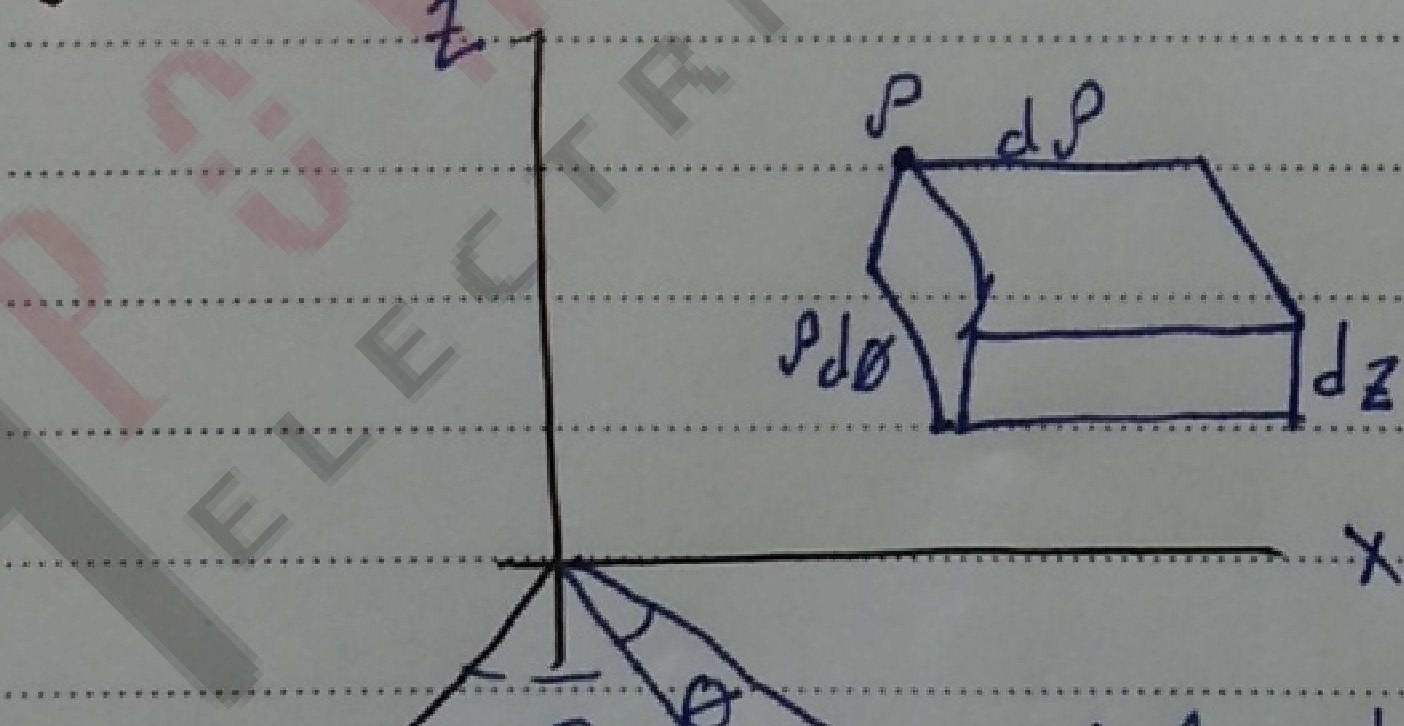
$d\ell = dx\, a_x + dy\, a_y + dz\, a_z$

(a) Cartesian





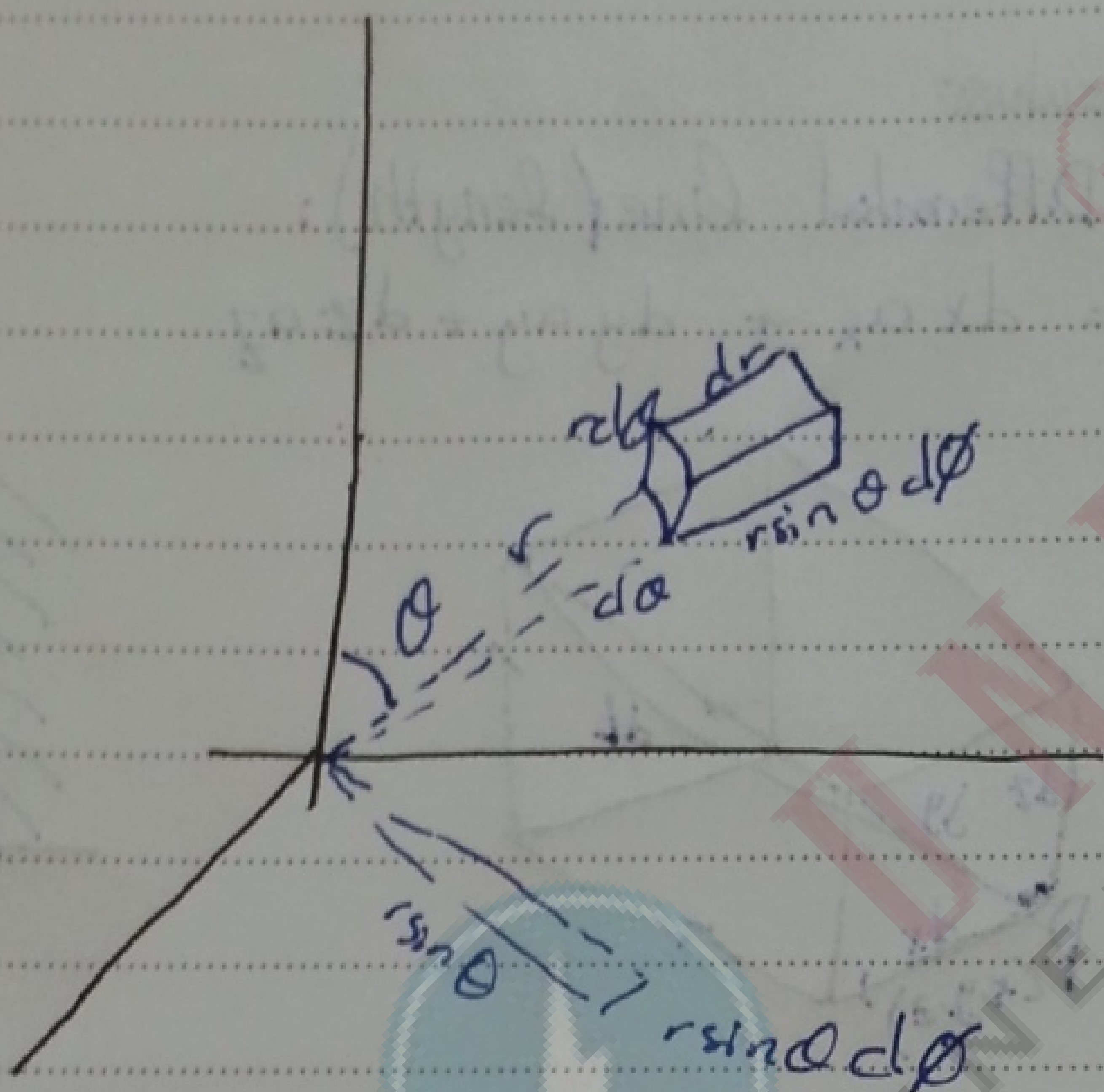
(b) Sylindrical



$d\ell = d\rho\, a_\rho + \rho\, d\phi\, a_\phi + dz\, a_z$

$d\theta$

No. continued

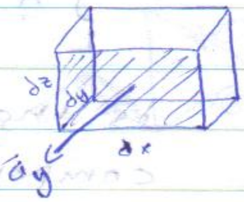


$$dl = dr\,a_r + r\,d\theta\,a_\theta + r\sin\theta\,d\phi\,a_\phi$$

[2] differential area

* defined as a vector quantity $\partial \bar{S} = S\,\bar{a}_n$
  where $\bar{a}_n$ is the normal to the area.
* consider the differential lines' equations:

cartesian ←
$$\begin{cases} \partial s_1 = \partial y\, \partial z\ \bar{a}_x \\ \partial s_2 = \partial x\, \partial z\ \bar{a}_y \\ \partial s_3 = \partial x\, \partial y\ \bar{a}_z \end{cases}$$



cylindrical ←
$$\begin{cases} \partial s_1 = \rho\, \partial \phi\, \partial z\ \bar{a}_\rho \\ \partial s_2 = \partial \rho\ \partial z\ \bar{a}_\phi \\ \partial s_3 = \rho\, \partial \rho\, \partial \phi\ \bar{a}_z \end{cases}$$

spherical ←
$$\begin{cases} \partial s_1 = r^2 \sin\theta\ \partial\theta\, \partial\phi\ \bar{a}_r \\ \partial s_2 = r\, \partial r \sin\theta\ \partial \phi\ \bar{a}_\theta \\ \partial s_3 = r\ \partial r\ \partial\theta\ \bar{a}_\phi \end{cases}$$

[3] differential volume

* it is a scalar

$\partial V_L = \partial x\, \partial y\, \partial z$

$\partial V_c = \rho\, \partial \rho\, \partial \phi\, \partial z$

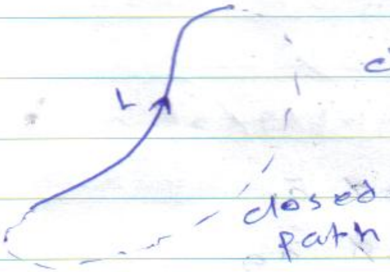
$\partial V_s = r^2\, \partial r\, \partial \theta\, \partial \phi \sin\theta$

* line integral

* by line we mean the path along a curve in space.

$$\int_L \vec{A} \cdot \partial L \rightarrow |A| \cos\theta * |\partial L|$$

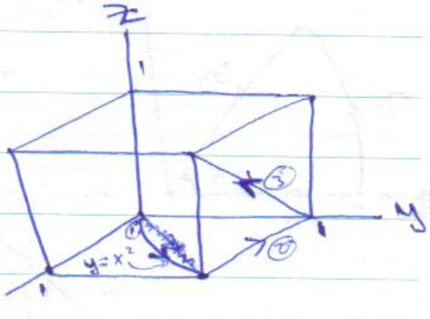
is the integral of the tangential component of $\vec{A}$ along the curve L



circulation is $\oint_L \vec{A} \cdot \partial L$

ex: let $\vec{F} = x^2 a_x - xz\, a_y - y^2 a_z$

determine $\int_L \vec{F} \cdot d\vec{l}$ where L is as shown



* along L_1, $y = x^2$, $z = 0$

$\Rightarrow dL_1 = dx\, a_x + dy\, a_y + 0\, a_z$

$\vec{F} \cdot dL = x^2 dx - 0 - 0$

$\int_{L_1} \dots = \int_0^1 x^2\, dx = \frac{1}{3}$

* along L_2, $y = 1$, $z = 0$

$F = x^2 a_x - 0 - a_z$

$dL_2 = dx\, a_x + 0\, a_y + 0\, a_z$

$\vec{F} \cdot d\vec{L} = x^2\, dx$

$\int_{L_2} \dots = \int_1^0 x^2\, dx = -\frac{1}{3}$

* along L_3, $y = 1$, ~~$x = z$, $dx = dz$~~

$F = x^2 a_x - xz\, a_y - a_z$

$dL_3 = dx\, a_x + dz\, a_z$

$\vec{F} \cdot d\vec{L}_3 = x^2\, dx - dz$

$\int_{L_3} \dots = \int_0^1 x^2\, dx - \int_0^1 dz \rightarrow -\frac{2}{3}$ $\boxed{\frac{1}{3} - 1}$

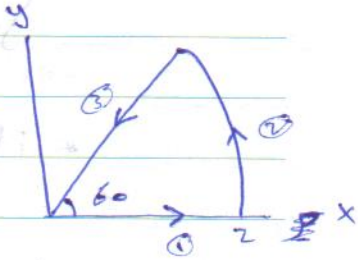
$= \int x^2\, dx - dx$

$x = z$
$dx = dz$

$= \int_0^1 (x^2 - 1)\, dx$

$= -\frac{2}{3}$, $\int_L F \cdot dL = \frac{1}{3} + \frac{1}{3} - \frac{2}{3} = \frac{-2}{3}$

ex: let $F = \rho \cos\phi \, a_\rho + z \sin\phi \, a_z$



* along 1 $z = 0, \phi = 0$

$F = \rho * 1 \, a_\rho + 0 \, a_\phi + \cancel{a_z} 0 \, a_z$

$dL = d\rho \, a_\rho$

$\int_0^2 \rho \, d\rho = 2$

* along 2 $\rho = 2, z = 2$

$F = 2 \cos\phi \, a_\rho$

$dL = \rho \, d\phi \, a_\phi$

$F \cdot dL = 0$

* along 3 $z = 0, \phi = 60$

$F = \rho * \frac{1}{2} a_\rho$

$dL = d\rho \, a_\rho$

$\int_2^0 \frac{1}{2} \rho \, a\rho = -1$

em1

8/7/14 tues

* surface integral

$$\int \vec{A} \cdot d\vec{s}$$

أحمد عمر محمد الغزاوي

* if the surface area enclosed a volume.

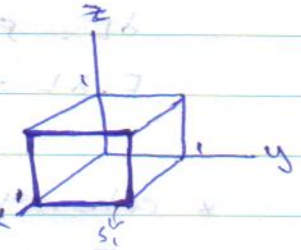
$\oint \bar{A} \cdot d\bar{s}$ * it gives the total flux.

ex: let $\vec{A} = x^2 a_x - xz a_y - y^2 a_z$

$x = 1$

$d\bar{s}_1 = dy\, dz\, \hat{a}_x$

$\bar{A} \cdot d\bar{s}_1 = x^2\, dy\, dz = dy\, dz$



$$\int \bar{A} \cdot d\bar{s} = \int_{y=0}^{1} \int_{z=0}^{1} dy\, dz$$

$$= 1$$

ex: * obtain the circumference, ~~of~~ area of the circle

* obtain area, volume of the cylinder
* obtain area, volume of the sphere

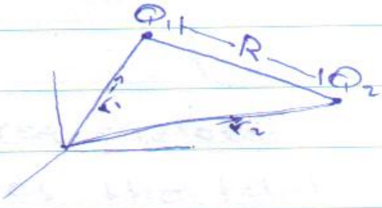
using diff. length, area, volume.

احمد عمر محمد الخزاوي

* chapter 4: electrostatic Fields

* columbs law

$$|\vec{F}| = K \frac{Q_1 Q_2}{R^2}$$



$$= \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}$$

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} a_{R_{12}} \qquad * \frac{R_{12}}{R_{12}}$$

$$\boxed{* \vec{R} = |R| \, a_R}$$

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^3} \vec{R}_{12}$$

$$\vec{F}_{12} = \frac{Q_1 Q_2 (\bar{r}_2 - \bar{r}_1)}{4\pi\epsilon_0 |\bar{r}_2 - \bar{r}_1|^3}$$

$$\vec{F}_{21} = \frac{Q_1 Q_2 (\bar{r}_1 - \bar{r}_2)}{4\pi\epsilon_0 |\bar{r}_2 - \bar{r}_1|^3}$$

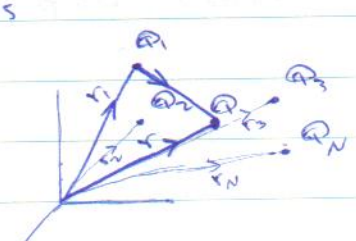
So $\bar{F}_{12} = -\bar{F}_{21}$

*

* Force due to Assembly of charges

* let $Q_1, Q_2, \ldots Q_N$ be static charges with position vectors $\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N$
* the force on a test charge Q with position vector $\bar{r}$ is

$$\vec{F} = \frac{Q}{4\pi\epsilon_0} \sum_{j=1}^{N} \frac{Q_j (\bar{r} - \bar{r}_j)}{|r - r_j|^3}$$



ex. 4.2 (see book)
important

* the electric field intensity (strength)

defind as $\vec{E} = \lim_{Q \to 0} \frac{\vec{F}}{Q}$

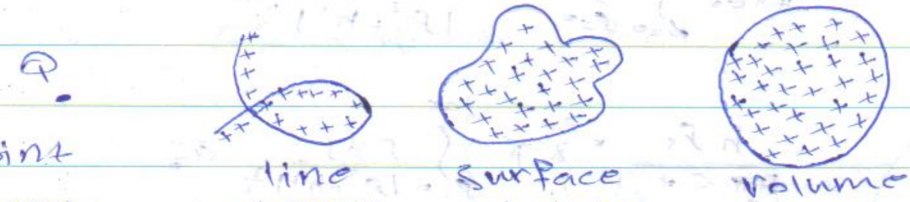
* $\bar{E}$ due to a charge Q_1 is $\bar{E} = \frac{Q Q_1 \vec{R}_{12}}{4\pi\epsilon_0 |R_{12}|^3}$

now $\div \frac{1}{Q} \Rightarrow \bar{E} = \frac{Q_1}{4\pi\epsilon_0 |R_{12}|^3} \vec{R}_{12}$

* $\vec{E}$ due to Assembly of charges

$$\bar{E} = \frac{1}{4\pi\epsilon_0} \sum_{j=1}^{N} \frac{Q_j (r - r_j)}{|r - r_j|^3}$$

* electric field due to ~~continaues~~ continnons charge distribuation



point charge

line charge

surface charge

volume charge

distribution

$dQ = \rho_L \, dL$ / $dQ = \rho_s \, ds$ / $dQ = \rho_v \, dv$

$$\vec{E} = \int \frac{\rho_l \, dL}{4\pi\epsilon_0 |R|^2} \bar{a}_R = \int \frac{\rho_s \, ds}{4\pi\epsilon_0 |R|^2} \bar{a}_R = \int \frac{\rho_v \, dv}{4\pi\epsilon_0 |R|^2} \bar{a}_R$$

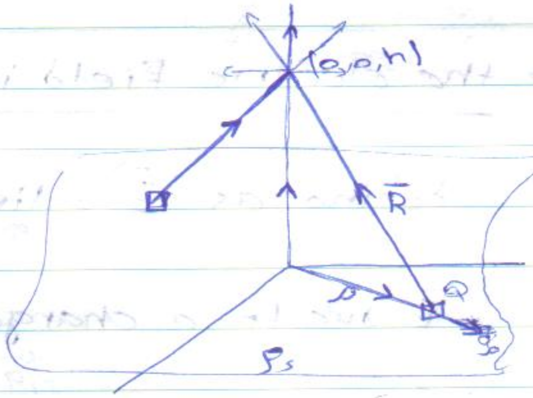
line surface volume

$\vec{R} - \vec{h} + \vec{\rho} = 0$

$\vec{R} = h\,\bar{a}_z - \rho\,\bar{a}_\rho$

$|\vec{R}| = \sqrt{(-\rho)^2 + h^2}$

$= \sqrt{\rho^2 + h^2}$



$dQ = \rho_s \, ds$

$= \rho_s \, \rho \, d\rho \, d\phi$

$d\phi = \frac{x}{\rho}$

$$d\vec{E} = \frac{\rho_s \, \rho \, d\rho \, d\phi \, (-\rho\bar{a}_\rho + h\bar{a}_z)}{4\pi\epsilon_0 \, (\rho^2 + h^2)^{\frac{3}{2}}}$$

* due to symmetry $\vec{E}$ is in the direction of $\bar{a}_z$

$$\vec{E} = \frac{\rho_s}{4\pi\epsilon_0} \int_{\phi=0}^{2\pi} \int_{\rho=0}^{\infty} \frac{\rho \, d\rho \, d\phi \, h \, \bar{a}_z}{(\rho^2 + h^2)^{\frac{3}{2}}}$$

$$= \frac{h\rho_s}{\cancel{4\pi}\epsilon_0} * 2\pi \int_{\rho=0}^{\infty} \frac{\rho \, d\rho}{(\rho^2+h^2)^{\frac{3}{2}}} \bar{a}_z$$

$$= \frac{h\rho_s}{2\epsilon_0} \bar{a}_z \left(-(\rho^2+h^2)^{-\frac{1}{2}} \Big|_0^\infty \right)$$

$$= \frac{\rho_s}{2\epsilon_0} \bar{a}_z \, h \left(\frac{-1}{\sqrt{\rho^2+h^2}} \Big|_0^\infty \right)$$

$$= \frac{\rho_s}{2\epsilon_0} \bar{a}_z \, h \left(0 + \frac{1}{h} \right) = \frac{\rho_s}{2\epsilon_0} \bar{a}_z$$

$\frac{-\rho}{(\rho^2+h^2)^{\frac{3}{2}}}$

$\frac{\partial}{\partial\rho} (\rho^2+h^2)^{-\frac{1}{2}}$

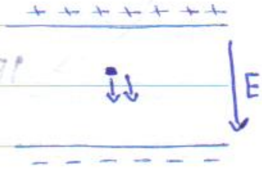
$-\frac{1}{2}(2\rho)(\rho^2+h^2)^{-\frac{3}{2}}$

* generally $\vec{E} = \frac{\rho_s}{2\epsilon_0} \vec{a}_n$,

a_n is the unit vector normal to the plain that the $\vec{E}$ resides in.

* $\vec{E}$ within a parallel plate capacitor

$$\vec{E} = \frac{\rho_s}{2\epsilon_0} a_n + \frac{-\rho_s}{2\epsilon_0} (-\bar{a}_n)$$



$$= \frac{\rho_s}{\epsilon_0} a_n$$

ex: a ~~sphere of radius~~ charge of density ρ_v C/m^3 is uniformally distributed over a sphere of radius R. calculate the total charge.

$$dL = dr\,\bar{a}_r + r\,d\theta\,\bar{a}_\theta + r\sin\theta\,d\phi\,\bar{a}_\phi$$

$$dv = r^2 \sin\theta\,dr\,d\theta\,d\phi$$

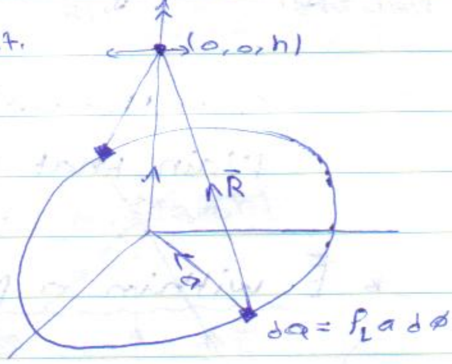
$$dQ = \rho_v\,dv$$

$$Q = \int_v \cancel{E_v}\, dQ \quad ??$$

$$= \int_{r=0}^{R} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho_v r^2 \sin\theta\,dr\,d\theta\,d\phi$$

$$= \rho_v \frac{4}{3} \pi R^3$$

ex: line charge
circle , a is const.



$\vec{R} = a(-a_\rho) + h\, a_z$

$dQ = \rho_L\, a\, d\phi$

$d\vec{E} = \frac{\rho_L\, a\, d\phi}{4\pi\epsilon_0 |\vec{R}|^3}\, \vec{R}$

$|\vec{R}| = (a^2+h^2)^{\frac{1}{2}}$

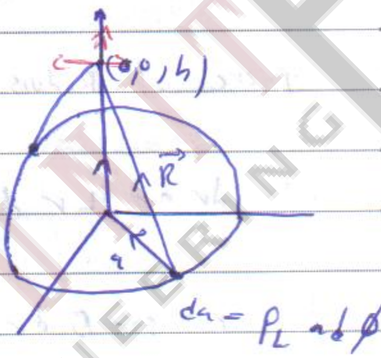
$d\vec{E} = \frac{\rho_L\, a\, d\phi\, (-a\, \bar{a}_\rho + h\, \bar{a}_z)}{4\pi\epsilon_0 (a^2+h^2)^{\frac{3}{2}}}$ due to symmetry

$\vec{E} = \frac{\rho_L\, a\, h\, \bar{a}_z}{4\pi\epsilon_0 (a^2+h^2)^{\frac{3}{2}}} \int_{\phi=0}^{2\pi} d\phi$

$= \frac{\rho_L\, a\, h\, \bar{a}_z}{2\epsilon_0 (a^2+h^2)^{\frac{3}{2}}}$

ex : line charge

circle . a is const



$\vec{R} = a(-a_\rho) + h\, a_z$

$dQ = \rho_L\, a\, d\phi$

$$d\vec{E} = \frac{\rho_L\, a\, d\phi}{4\pi\epsilon_0 |R|^2}\vec{R}$$

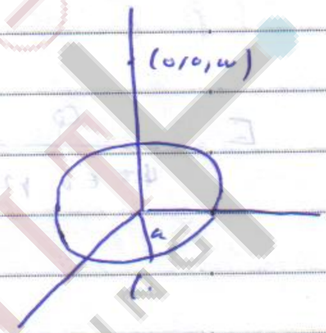
$$|\vec{R}| = (a^2 + h^2)^{\frac{1}{2}}$$

$$d\vec{E} = \frac{\rho_L\, a\, d\phi\,(-a\,\vec{a}_\rho + h\,\vec{a}_z)}{4\pi\epsilon_0 (a^2+h^2)^{3/2}}$$

$$\vec{E} = \frac{\rho_L\, a\, h\, \vec{a}_z}{4\pi\epsilon_0 (a^2+h^2)^{3/2}} \int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{\rho_L\, a\, h\, \vec{a}_z}{2\epsilon_0 (a^2+h^2)^{3/2}}$$

$$\frac{d\vec{E}}{dh} = 0 \Rightarrow h = \pm \frac{a}{\sqrt{2}}$$



when

$$\vec{E} = \frac{\rho_L \, a h}{2 \epsilon_0 (a^2 + h^2)^{3/2}} \, a_n$$

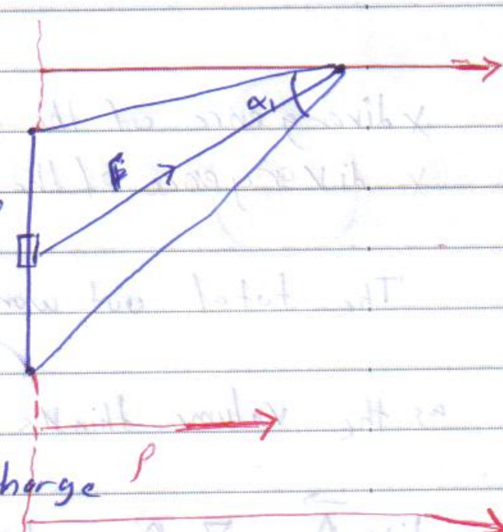
$a/2$

$$\lim_{a \to 0} \vec{E} = \frac{Q \quad h}{4 \pi \epsilon_0 \, [a^2 + h^2]^{3/2}} \, a_n \qquad Q = \rho_L \, 2 \pi a$$

$$\rho_L \, a = \frac{Q}{2\pi}$$

$$\vec{E} = \frac{Q}{4 \pi \epsilon_0 \, h^2} \, a_n$$

Example



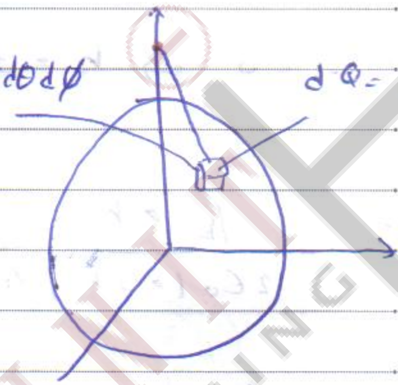
$$\vec{E} = \frac{\rho_L}{4 \pi \epsilon_0 \rho} \left[(-\sin\alpha_2 - \sin\alpha_1) \, a_\rho + (\cos\alpha_2 - \cos\alpha_1) \right] a_z$$

For an infinite line of charge

$$\vec{E} = \frac{\rho_L}{2 \pi \epsilon_0 \rho} \, a_\rho$$

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

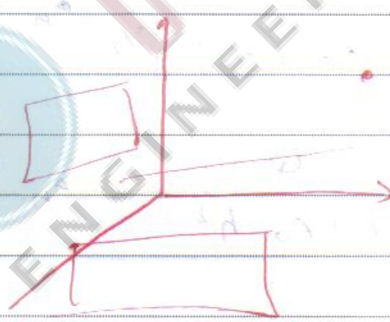
$dQ = \rho_v \, dv$



$$E = \frac{Q}{4\pi \epsilon_0 r^2} a_r$$

Ex:

Don't solve it from first principles, Utiliz solution already obtained



* divergence of the field.

* divergence of the field:

The total out word flux per unit volume as the volume thinks to ~~300~~ zero (about a point)

$$\operatorname{div} \vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta V \to 0} \frac{\oint \vec{A} \cdot d\vec{s}}{\Delta V}$$

18/17.5

Based is this definition, if

$$\vec{A} = A_x a_x + A_y a_y + A_z a_z$$

It can be show (see book)

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

* Electric flux Density $(\vec{D})$

$$\vec{D} = \epsilon_0 \vec{E}$$

ψ is the Electric flux.

colomb columb.m^{-2}

$$\Psi = \int_S \vec{D} \cdot d\vec{s}$$

$$\Psi = \oint \vec{D} \cdot d\vec{s}$$

$$\psi = \oint \vec{D} \cdot d\vec{s} = Q_{enc}.$$

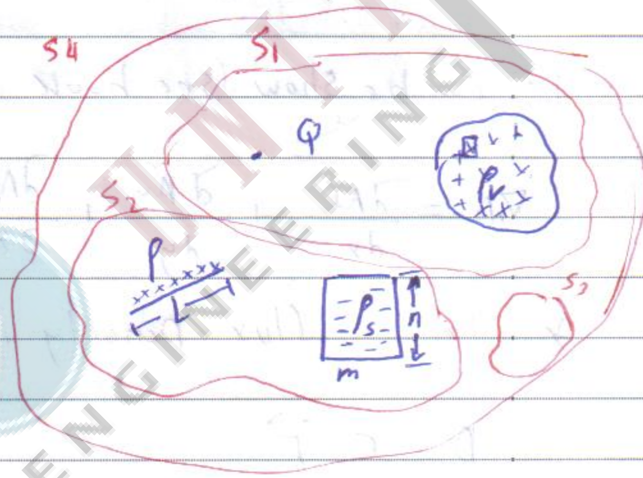
$$\int_V \rho_v \, dv = \int_V \nabla \cdot \vec{D} \, dv$$

$$\Rightarrow \nabla \cdot \vec{D} = \rho_v$$

$$\oint \vec{D}\cdot d\vec{s} = \int \rho_v \, dv ,$$

eassier to calculate than $\oint \vec{D}\cdot d\vec{s}$

Example:



$$\Psi_{s_1} = Q + \int_v \rho_v$$

$$\Psi_{s_2} = \rho_L l + (-\rho_s) MN$$

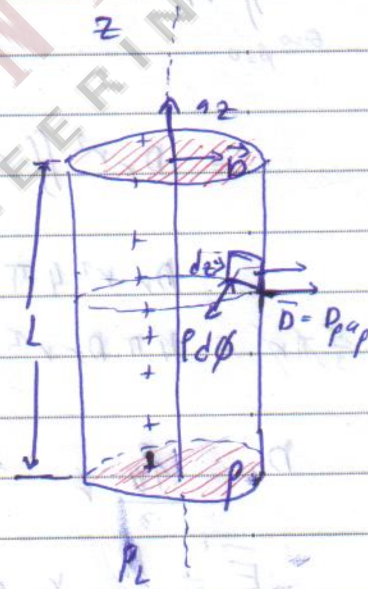
$$\Psi_{s_3} = 0$$

$$\Psi_4 = Q + \int_v \rho_v + \rho_L L + (-\rho_s) MN$$

* Gauss Surface :

Used to simplify calculation of $\vec{E}$ & $\vec{D}$ when symmetry exists,

Example :



$$\rho_L \cdot L = \int_{z=0}^{L}\int_{0}^{2\pi} \underbrace{\vec{D}}_{D_\rho a_\rho} \cdot \rho \, d\phi \, dz \; a_\rho$$

$$= \iint D_\rho \, \rho \, d\phi \, dz$$

$$\rho_L \cancel{L} = \rho D_\rho \, 2\pi \cancel{L}$$

$$D_\rho = \frac{\rho_L}{2\pi \rho}$$

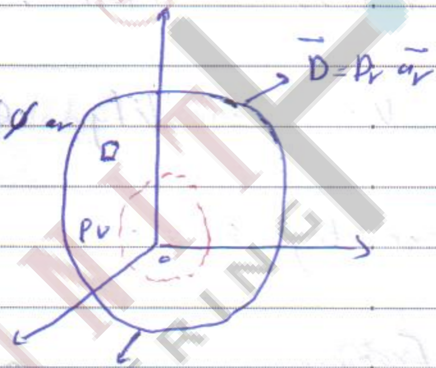
$$\vec{D} = \frac{\rho_L}{2\pi \rho} a_\rho$$

$$\vec{E} = \frac{\rho_L}{2\pi \epsilon_0 \rho} a_\rho$$

Example:

$$\rho_v \frac{4}{3}\pi r^3 = \iint D_r\, a_r \cdot r\,d\theta\, r\sin\theta\, d\phi\, a_r$$

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} D_r a_r \cdot r\,d\theta\, r\sin\theta\, d\phi$$



$$= D_r r^2 \iint \sin\theta\, d\theta\, d\phi$$

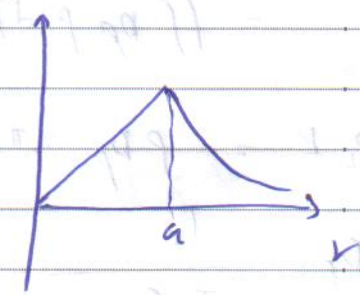
radius = a

$$= D_r r^2 4\pi$$

$$\rho_v \frac{4}{3}\pi r^3 = 4\pi D_r r^2$$

$$D_r = \frac{\rho_v}{3} r = \frac{\rho_v}{3} r$$

$$\vec{E} = \frac{\rho_v}{3\epsilon_0} r\, a_r$$



EM 1 Notebook

Dr: Omar Ghazawi

By : Amr Aljada'

بأفكارنا_نُبدع

Second Material

Subject

Exercise: Use the general formula for the resistance to obtain the resistance of a uniform conductor.

منتظم المقطع

Subject Date No.

Power:

Power = force * velocity $\theta = 0°$

$\vec{F}, \vec{v}$ are in the same direction

= force . velocity "dot product" when $\vec{F}, \vec{v}$ are not in the same direction $\theta \neq 0°$

$$P = q\vec{E} \cdot \vec{u}$$ Power: scaler

$$dP = \rho_v \, dv \, \vec{E} \cdot \vec{u}$$

$$dP = \vec{E} \cdot \rho_v \vec{u} \, dv$$

$$dP = \vec{E} \cdot \vec{J} \, dv$$

$$P = \int_v dP = \int_v \vec{E} \cdot \vec{J} \, dv$$ *

$$= \sigma \int_v |\vec{E}|^2 \, dv \quad \left\{ \vec{J} = \sigma \vec{E} \right.$$

Subject Date No.

$$\left\{\begin{aligned} \text{energy} &= \text{force} \times \text{displacement} \\ \text{Power} &= \text{force} \times \frac{d}{dt}(\text{displacement}) \\ &= \text{Power} \times \text{velocity} \end{aligned}\right.$$

$$\left\{\begin{aligned} w &= F.d \\ \frac{w}{t} &= F.\frac{d}{t} \\ &= F.V \end{aligned}\right\}$$

Exercise: ① Use the general formula for the power to obtain the power in a uniform conductor.

② prove that $P = I\,V$ "in uniform"

Solution:-

$$P = \int_V E.J\,dv$$

$$= \int_L E.dl \int_S J.ds$$

$$= V \quad I$$

Subject Date No.

Example:

let $\vec{J} = \frac{1}{r^3}\left(2\cos\theta \, a_r + \sin\theta \, a_\theta \right) \; A/m^2$

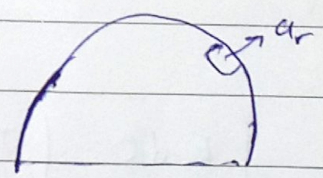
calculate current through :

i) hemispherical shell of radius 20 cm
ii) spherical shell of radius 10 cm

solution:

$$I = \int_S \vec{J} \cdot \vec{ds}$$

$$\vec{ds} = r^2 \sin\theta \, d\theta \, d\phi \, a_r$$



hemisphere ← $\frac{\pi}{2}$

$$I = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \frac{2}{r} \cos\theta \sin\theta \, d\theta \, d\phi$$

Subject Date No.

$$I = \frac{2}{0.2}(2\pi)\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta$$

$$= 20\pi \left. \frac{\sin^2\theta}{2} \right|_0^{\pi/2}$$

$$= 10\pi \text{ A}$$

ii) second case :

$$I = \int_0^{\pi}\int_0^{2\pi} \frac{2}{r} \cos\theta \sin\theta \, d\theta \, d\phi$$

$$= \frac{2}{0.1} \times 2\pi \int_0^{\pi} \cos\theta \sin\theta \, d\theta$$

$$= 20\pi \left. \frac{2\sin^2\theta}{2} \right|_0^{\pi} = 0 \text{ A}$$

check : use $\nabla \cdot \vec{J} = \lim \dfrac{\oint \vec{J} \cdot d\vec{S}}{\Delta V}$

NOTEBOOK

Subject Date No.

$$\Rightarrow \int_S \vec{J} \cdot d\vec{s} = \int_V \nabla \cdot \vec{J} \, dv$$

$$I = \int_V \nabla \cdot \vec{J} \, dv = 0A$$

$$= 0 \quad A$$

Subject Date No.

Polarisation of Dielectrics :

استقطاب

→ موصليتها ضعيفة

لا توصل التيار الكهربائي

insulators "عازل"

electrons are not free to move

عدد الالكترونات الطليقة قليلة جدا"

electrons are bound to the nucleus

To keek them off : تحتاج الالكترونات لطاقة كبيرة

عندما يؤثر مجال كهربائي على ذرة الكتروناتها غير حرة يحدث استقطاب

In dielectrics , charges are tightly bound to the crystaline structure , unable to move freely as is the case in metals

If an external electric field is applied , the positive & negative charges are displaced of their original positions , resulting in the creation of dipoles.

$$\vec{P} = Q\,\vec{d}$$

dipole moment.

Polarisation is defined as :

$$\vec{P} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{n} Q_k \, \vec{d}_k}{\Delta V}$$

الاشتقاقة غير مطلوب

It can be shown that

$$\vec{D} = \epsilon . \vec{E} + \underbrace{\vec{P}}_{\text{Polarisation}}$$

"هذه المعادلة في حالة Dielectric"

$\vec{P} = 0$ in free space & in metals

in free space
$\vec{D} = \epsilon . \vec{E}$
$\vec{P} = 0$ $\{ Q = 0$ no charges

also $\vec{P}$ depends on $\vec{E}$ through :

$$\vec{P} = \epsilon . \chi_e \vec{E}$$

χ_e : is the susceptability of the dielectric

القابلية للتغير من المجال

hence:

$$\vec{D} = \epsilon_0 (1 + \chi_e) \vec{E}$$

$$= \epsilon_0 \epsilon_r \vec{E}$$

$\left\{ \begin{array}{l} \epsilon_r = 1 + \chi_e \\ \epsilon_r \geq 1 \end{array} \right.$

also: $\epsilon_r = \dfrac{\epsilon}{\epsilon_0} \Rightarrow \epsilon = \epsilon_0 \epsilon_r$

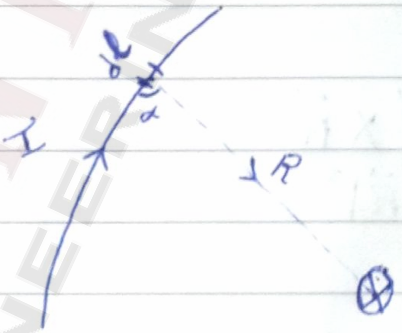
Subject Date No.

Magnetostatics (Ch7)

The Biot-Savart's law

The magnitude of the magnetic field due to an elemental current:

$$dH \propto \frac{I\,dl\,\sin\alpha}{|R|^2}$$



Since the field has direction:

Idl: elemental current

$$d\vec{H} = k\,\frac{I|d\vec{l}|\sin\alpha}{|\vec{R}|^2}\,a_n$$

$$d\vec{H} = \frac{k\,I|d\vec{l}|\sin\alpha\,|a_R|}{|\vec{R}|^2}\,a_n = kI\,\frac{d\vec{l}\times a_R}{|R|^2}$$

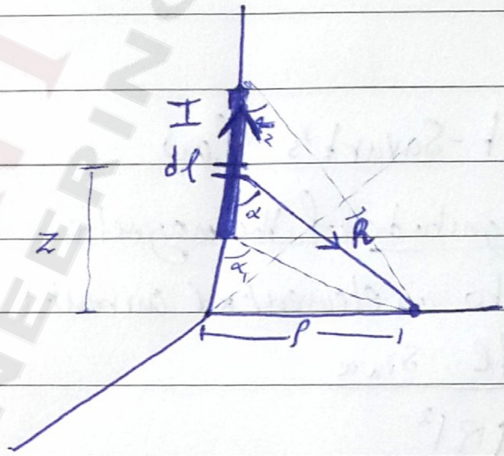
a_n: Perpendicular to the plane

$$= kI\,\frac{d\vec{l}\times|\vec{R}|\,a_R}{|\vec{R}|^3} = \frac{1}{4\pi}\,I\,\frac{d\vec{l}\times\vec{R}}{|\vec{R}|^3}$$

$k = \frac{1}{4\pi}$ in SI unit system

Subject Date No.

Example:



$$d\vec{H} = \frac{I \, d\vec{l} \times \vec{R}}{4\pi \, |\vec{R}|^3}$$

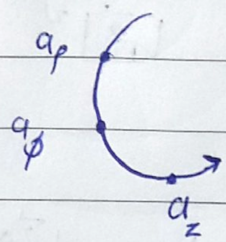
$$= \frac{I}{4\pi} \frac{dz\, a_z \times (\rho a_\rho - z a_z)}{(\rho^2 + z^2)^{\frac{3}{2}}}$$

$dl = dz \, a_z$

$\vec{R} = \rho a_\rho - z \, a_z$

$|\vec{R}| = (\rho^2 + z^2)^{\frac{1}{2}}$

$$\vec{H} = \int_L d\vec{H} = \int_L \frac{I}{4\pi} \frac{\rho \, dz \, a_\phi - 0}{(\rho^2 + z^2)^{\frac{3}{2}}}$$



$$= \frac{I}{4\pi} \int_L \frac{\rho \, dz}{(\rho^2 + z^2)^{\frac{3}{2}}} a_\phi$$

$$= \frac{I \rho \, a_\phi}{4\pi} \int_L \frac{1}{(\rho^2 + z^2)^{\frac{3}{2}}} dz$$

Subject Date No.

By trigonometric substitution:

let $z = \rho \cot\alpha$

$$dz = -\rho \csc^2\alpha \; d\alpha$$

$$\rho^2 + z^2 = +\rho^2(1+\cot^2\alpha) = +\rho^2 \csc^2\alpha$$

$$\Rightarrow \vec{H} = \frac{-I \rho a_\phi}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho \csc^2\alpha}{\rho^3 \csc^3\alpha} d\alpha$$

$$\vec{H} = \frac{-I a_\phi}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \sin\alpha \; d\alpha = \frac{-I}{4\pi\rho}(\cos\alpha_1 - \cos\alpha_2)\, a_\phi$$

$$\boxed{\vec{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\, a_\phi}$$

N O T E B O O K

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Special case : Semi-infinite wire :

$\alpha_1 = 90^\circ$ $\alpha_2 = 0$

$$\vec{H} = \frac{I}{4\pi\rho} a_\phi$$

Special case: infinite wire:

$\alpha_1 = +180^\circ$ $\alpha_2 = 0$

$$\vec{H} = \frac{I}{2\pi\rho} a_\phi$$

NOTEBOOK

Ampere's Circuit Law :

Briefly : the circulation of $\vec{H}$ is equal to the sum of enclosed currents.

$\vec{H}$: magnetic field intensity

Circulation : line integral over a closed path

$$\oint_L \vec{H} \cdot d\vec{l} = I_{enc} = \sum_{i=1}^{N} I_i$$

المجموع الجبري

I is scaler

using stokes theorem

$$\oint_L \vec{H} \cdot d\vec{l} = \int_S (\nabla \times \vec{H}) \cdot ds = I_{enc}$$

$$I_{enc} = \int_S J \cdot ds = \int_S (\nabla \times \vec{H}) \cdot ds$$

$$\Rightarrow \nabla \times \vec{H} = \vec{J}$$

Maxwells third equation in point form

Subject Date No.

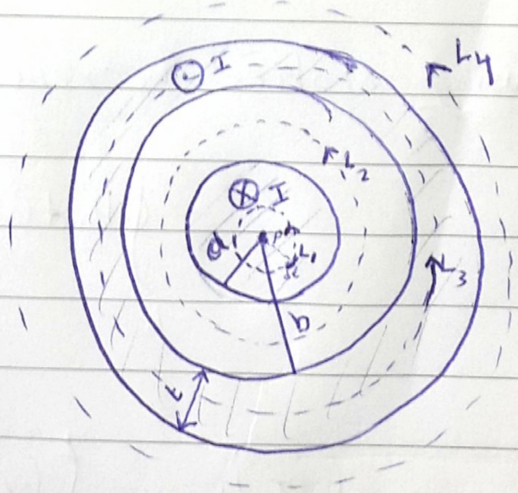
$$\nabla \times \vec{H} = \vec{J}$$

$$J \neq 0 \Rightarrow \nabla \times \vec{H} \neq 0$$

i.e the magnetic field is not conservative

اذا أخذت (closed path) ليس شرطاً أن يكون ($\Delta V \neq 0$) Magnetic field is
potential

Example: consider Infinite length Coaxial Transmission Line.



Infinite length: uniform magnetic field

$\otimes$: z axis

for
$0 \leq \rho \leq a$

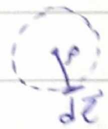
$$\oint_{L_1} H \cdot d\vec{l} = I_{enc} = \int_s \vec{J} \cdot \vec{ds}$$

Subject Date No.

$$\vec{J} = \frac{I}{\pi a^2}\, a_z$$

$$d\vec{s} = d\rho\ \rho\, d\phi\ a_z$$

$$d\vec{l} = \rho\, d\phi\ a_\phi$$



$$\vec{H} = H_\phi\, a_\phi$$

$$\int_{L_1} \vec{H} \cdot d\vec{l} = \int_s \vec{J} \cdot d\vec{s}$$

$$\int_{\phi=0}^{2\pi} H_\phi a_\phi \cdot \rho\, d\phi\, a_\phi = \int_{\rho=0}^{\rho} \int_{\phi=0}^{2\pi} \frac{I}{\pi a^2}\, a_z \cdot \rho\, d\rho\, d\phi\, a_z$$

$$H_\phi\, \rho\, (2\pi) = \frac{I}{\pi a^2}\, (2\pi)\, \frac{\rho^2}{2}$$

$$H_\phi = \frac{I}{2\pi a^2}\, \rho$$

$$\vec{H} = \frac{I\rho}{2\pi a^2}\, a_\phi$$

N O T E B O O K

Subject Date No.

when $\rho \geq b+t$

$$\oint H_\phi a_\phi \cdot \rho d\phi \, a_\phi = I + (-I) = 0$$

$$2\pi\rho H_\phi = 0$$

Since $\rho \neq 0$

$\Rightarrow H_\phi = 0$

for the case $b \leq \rho \leq b+t$

$$\oint H_\phi a_\phi \cdot \rho d\phi \, a_\phi = I + \int_s \vec{J_b} \cdot d\vec{s}$$

$$2\pi\rho H_\phi = I - \frac{I}{\pi} \int_b^\rho \int_0^{2\pi} \frac{1}{t^2+2bt} a_z \cdot d\rho \, \rho d\phi \, a_z$$

$$J_b = \frac{I}{\pi(b+t)^2 - \pi b^2} (-a_z)$$

لأن التيار سالب "اتجاهه معكوس"

$$= \frac{-I}{\pi(t^2+2bt)} a_z$$

N O T E B O O K

Subject Date No.

$$2\pi\rho H_\phi = I - \frac{2I}{2}\,\frac{1}{t^2+2bt}\,\rho^2\Big|_b^\rho$$

$$2\pi\rho H_\phi = I\left(1 - \frac{(\rho^2-b^2)}{t^2+2bt}\right)$$

$$H_\phi = \frac{I}{2\pi\rho}\left(1 - \frac{(\rho^2-b^2)}{t^2+2bt}\right)$$

NOTEBOOK

Subject Date No.

Magnetic flux Density : $(\vec{B})$

Defined as

$$\vec{B} = \mu_0 \vec{H}$$

μ_0 is the permeability of free space.
النفاذية

The magnetic flux through a surface S is

$$\Psi = \int_S \vec{B} \cdot d\vec{s}$$

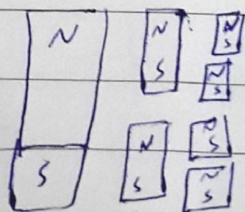
also

$$\Psi = \oint_S B \cdot ds = 0$$

لايوجد مغناطيس بقطب واحد

"لايوجد مغناطيس بشحنة واحدة
لا يتوجب وجود الشحنة" ⇐ المحصلة = صفر

implying the non-existance of separate magnetic charges



Subject Date No.

Using the divergence theorem:

$$\oint_s B . ds = \int_v (\nabla . \vec{B}) . ds = 0$$

$\nabla . \vec{B} = 0$ one of Maxwell equations

Electromagnatics I

NoteBook

By: Ahmad Ghzawi
Dr. Omar Ghzawi

بأفكارنا_نُبدع

5th Week

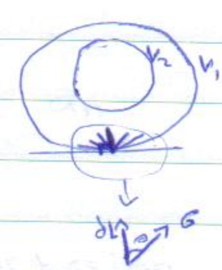
* the gradient of a scalar field

$$\partial V = \frac{\partial V}{\partial x}\partial x + \frac{\partial V}{\partial y}\partial y + \frac{\partial V}{\partial z}\partial z$$

$$= \underbrace{\left(\frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z\right)}_{\bar{G}} \cdot \underbrace{\left(\partial x\,\bar{a}_x + \partial y\,\bar{a}_y + \partial z\,\bar{a}_z\right)}_{\partial \bar{L}}$$

so $\partial V = \bar{G} \cdot \partial \bar{L} = |\bar{G}||\partial \bar{L}| \cos\theta$

$$\frac{\partial V}{|\partial L|} = |\bar{G}| \cos\theta$$



$\max \left|\frac{\partial V}{|\partial L|}\right|$ when $\theta = 0°$ $= G$

* the gradient $\vec{G}$ gives the direction of maximum increase in V.

so $\bar{G} \cdot \partial \bar{L} = \nabla V = \left(\frac{\partial}{\partial x}\partial x + \frac{\partial}{\partial y}\partial y + \frac{\partial}{\partial z}\partial z\right) V$ must be a scalar field

* The Electric potential :

Defind as work per unit charge.

If a charge Q is moved with in an electric field.

then along a certain path L.

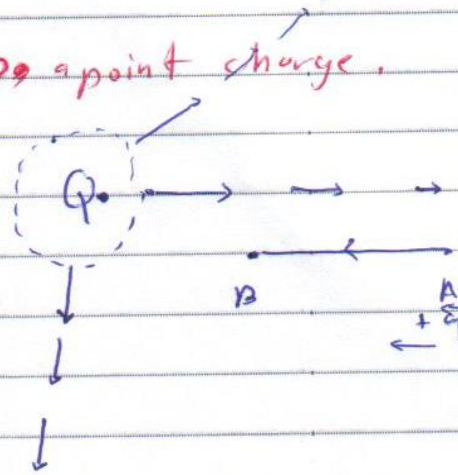
Then work done is :

$$dW = \cancel{QE} - Q\vec{E} \cdot d\vec{L}$$

$$W = -Q \int_L \vec{E} \cdot d\vec{L}$$

$$V = \frac{W}{Q} = -\int_L \vec{E} \cdot d\vec{L}$$

Ex: Consider potential due to a point charge.



$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\vec{a_r}$$

$$d\vec{l} = +dr\,\vec{a_r}$$

$$V(r) = -\int_A^B E\cdot dl$$

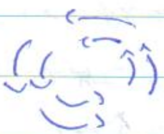
$$V(r) = -\int_A^B \frac{Q}{4\pi\epsilon_0 r^2}\,a_r \cdot dr\,a_r$$

$$V(r) = -\int_{r=r_A}^{r=r_B} \frac{Q}{4\pi\epsilon_0 r^2}\,dr$$

$$V(r) = \frac{Q}{4\pi\epsilon_0}\cdot\frac{1}{r}\Big|_{r_A}^{r_B} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_B}-\frac{1}{r_A}\right]$$

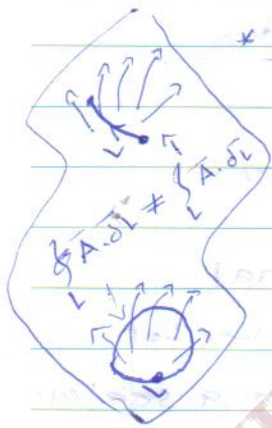
* **curl of a vector field**
  * an operation on vector field results in a vector quantity
  * it gives a measure of rotation in the field, so curl $\bar{E} = 0 \Leftarrow$ $\uparrow\uparrow\uparrow\uparrow\uparrow$ E

    (no rotation)



$\Rightarrow$ curl $\bar{E} \neq 0$

* **Definition**



$$\text{curl } A = \nabla \times A = \lim_{\Delta s \to 0} \frac{\oint_L \bar{A}\cdot\bar{dl}}{\Delta s}\, \bar{a}_n$$

$$\text{rot } (A) = \nabla \times A$$

* it can be shown that in cartesian coordinate

$$\text{curl } A = \nabla \times A = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

ex: let $A = K\,\bar{a}_x$

so

$$\text{curl} A = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ K & 0 & 0 \end{vmatrix} = 0$$

div $A = 0$

ex: ~~$R\bar{a}_r$~~ $A = r\bar{a}_r$

curl $A = 0$

div $A \neq 0$

$\Rightarrow$ curl $A \neq 0$

div $= 0$

curl $A \neq 0$

div $A \neq 0$ $\Leftarrow$

* it can be shown that

curl $E = \nabla \times E = 0$ irrotational

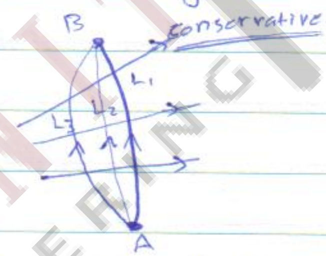
& $\bar{E} = -\nabla V$ getting a vector field through a scalar field

* V is easier to calculate than $\bar{E}$

em

24/7/14 thurs

* in conservative fields, the potential ~~diff~~ difference does not depend on the path taken.
it depends on the starting & ending points.



* V_{AB} is the same irrespective of the path taken.
so, in calculation choose a path which simplifies the calculations.

* $V_{AB} = V_B - V_A$, $V_{AB} = -V_{BA} \Rightarrow V_{AB} + V_{BA} = 0$

i.e. $\oint_L \bar{A} \cdot \bar{dl} = 0$ if the field is conservative.

ex: let $V_{(r)} = \frac{10}{r^2} \sin\theta \cos\phi$

consider that $\nabla V = \frac{\partial v}{\partial r} a_r + \frac{1}{r}\frac{\partial v}{\partial \theta} a_\theta + \frac{-1}{r\sin\theta}\frac{\partial v}{\partial \phi} a_\phi$

$E = -\nabla V$

$= -\left[\frac{-20}{r^3} \sin\theta \cos\phi \, \bar{a}_r + \frac{10}{r^3} \cos\theta \cos\phi \, \bar{a}_\theta - \frac{10 \sin\theta}{r^3 \sin\theta} \sin\phi \, \bar{a}_\phi\right]$

POWER UNIT

ElectroMagnaticsI

NoteBook

Dr. Omar Ghzawii

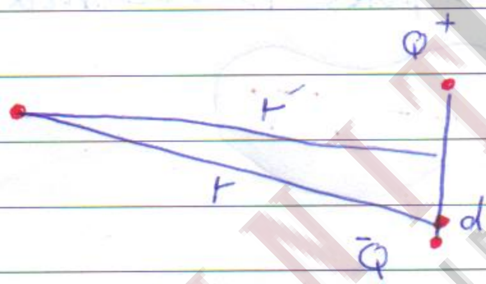
بأفكارنا_نُبدع

NoteBook

6th week

Subject   Date   No.

* Electric Dipole momont * "P142"



* A dipole is two equale and opposite changes displasment by a distance "d"

* The product $\bar{P} = Q\bar{d}$ is shown as it can be shown « see book » that:

$$\boxed{V(r) = \frac{\bar{P} \cdot \bar{a}_r}{4\pi \epsilon_\circ r^2}}$$

* If the center of Dipole is the position at $\bar{r}$. then:

$$\boxed{V(r) = \frac{P \cdot (\bar{r} \cdot \bar{r}')}{4\pi\epsilon_\circ |\bar{r} \cdot \bar{r}'|^3}}$$

* The Electric field intersity due to a dipoles is obtained through:

$$\boxed{E = -\nabla V}$$

EX) calculate "E" For the dipole ((see book))

Finding: $E \propto \frac{1}{r^3}$

* EquiPotential surfaces *

Are contours of equal Potential

all Point distance r have the same Potial

$V_3 > V_2 > V_1$

$\vec{E}$ line are Perpendicular to the eqipotential surface

EX) study the electric field line due to dipole ((see book))

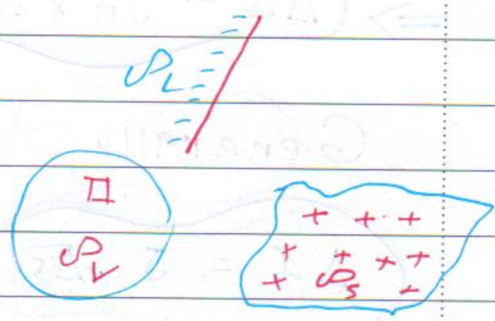
* Energy Stored in an Electric field Due to distance charge

$$W = \frac{1}{2} \sum_{k=1}^{N} Q_k V_k$$

* Due to Continus distribution of charges

$$W = \frac{1}{2} \int \bar{D} \cdot \bar{E} \, dv$$

$$= \frac{1}{2} \int \epsilon \cdot |E|^2 \, dv$$



* energy density we is:

$$\frac{dw}{dv} = \frac{1}{2} \vec{D} \cdot \bar{E}$$

* These distribution of charges altogether result in a certain electric field "E"

CH 5

Subject Date No.

* condition in Material Space *

for conductor $\sigma >> 1$
for issulater $\sigma << 1$

Current : $I = \frac{dQ}{dt}$

Normal current density : $J_n = \frac{\Delta I}{\Delta s}$

$\Rightarrow \Delta I = J_n * \Delta s$

Generally

$\Delta I = \bar{J} \cdot \overline{\Delta s}$

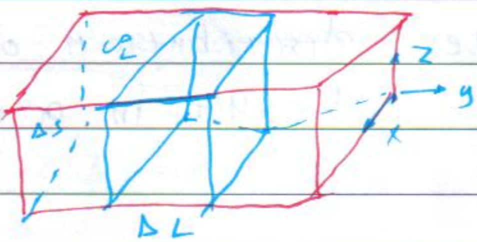
$I = \int_s \bar{J} \cdot ds$

* consider a uniform material

$\Delta I = \frac{\Delta Q}{\Delta t}$

$= \rho_v \Delta s \frac{\Delta L}{\Delta t}$

$J_{ny} = \frac{\Delta I}{\Delta s} = \rho_v u_y$



Subject Date No.

$$J_{ny} = \rho_v u_y$$

Generally:

$$\bar{J} = \rho_v \bar{u}$$

* In the Presenence of an electric field

$$F = -e\bar{E}$$

$$m \frac{\bar{u}}{\tau} = -e\bar{E}$$

$$u = \frac{-e\tau}{m} E$$

also $J = \rho_v \bar{u}$

$$= -en_* \frac{-e\tau}{m} \bar{E}$$

$$J = \boxed{\frac{ne^2\tau}{m}} E$$

σ for materal $= 10^{7}$

σ // inslater $= 10^{-17}$

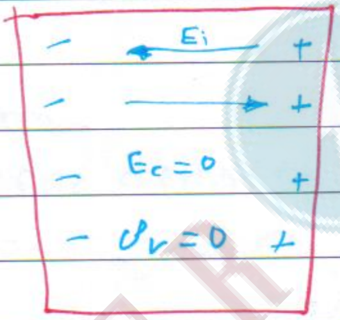
$$J = \sigma \bar{E}$$

$\sigma \rightarrow$ conductivity of materail

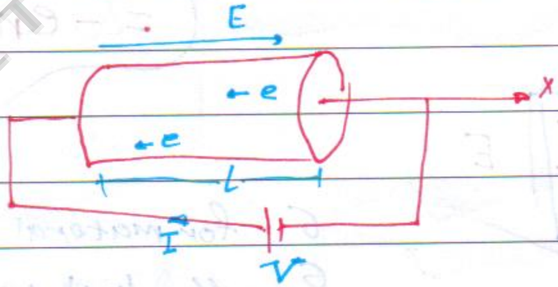
$$\bar{J} = \sigma \bar{E}$$

is Kawn as the Point form of ohm's Law.

* Electric Field in Conductors:



* Defining the Resistance:



* for uniform Conductor:

$$E_x = V/L$$

also

$$J_x = \frac{I}{S} = \sigma E_x \Longrightarrow \sigma * \frac{V}{L} = I/S$$

Subject Date No.

* The resistanee is "R" defind as:

$$R = \frac{V}{I}$$

$$R = \frac{1}{\sigma}\frac{L}{s} = \frac{\rho L}{s}$$

where "ρ" is the resitivity

* In the case where the conducter is not uniform:

$$R = \frac{V}{I} = \frac{\rho \bar{E}\cdot \bar{dL}}{\sigma \oint \bar{E}\cdot d\bar{s}}$$

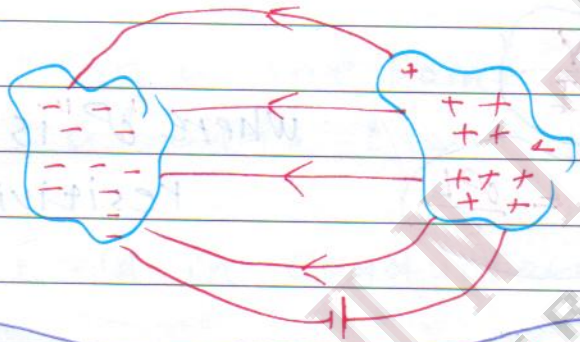
* it can be shown that that the power is given by "see book"

$$P = \oint_v \bar{E}\cdot\bar{J}\, d_v$$

* for a uniform conducters "derive"

$$P = VI = I^2 R = \frac{V^2}{R}$$

* Defining the Capacitance *



$$\boxed{\text{capacitance}(C) = \frac{Q}{V} = \frac{\epsilon \oint E \cdot ds}{\int E \cdot dL}}$$

Ex) shown that:

$$D = -\rho_s a_x = \rho_s(-a_x)$$

Sol.,,

assuming the fringing is negligible

Due to uniformity $\rho_s = \frac{Q}{S}$

$$E = \frac{-\rho_s}{\epsilon} a_x = \frac{Q}{\epsilon S} a_x$$

$$V = -\int E \cdot dL = -\int_0^d \frac{-Q}{\epsilon S} a_x \cdot dx\, a_x$$

$$\boxed{= \frac{Qd}{\epsilon S} = V}$$

$$C = \frac{Q}{V} = C = \frac{Q}{\frac{Qd}{\epsilon S}} = \boxed{\frac{\epsilon S}{d}}$$

Subject Date No.

* Energy Stored in a capacitor

$$\boxed{w = \frac{1}{2}\int D \cdot E \, dv}$$

$$= \frac{1}{2}\epsilon \int |E|^2 dv$$

$$= \frac{1}{2}\epsilon \int \frac{Q^2}{\epsilon^2 S^2} dv$$

* due to uniformalty of the Parallel plate capacitr:

$$= \frac{1/2\,\epsilon\, Q^2}{\epsilon^2 S^2}\int dv$$

$$= \frac{1}{2}\epsilon \frac{Q^2}{\epsilon^2 S^2} = \frac{1}{2}\frac{Q^2}{\frac{\epsilon S}{d}}$$

$$= \frac{1}{2}\frac{Q^2}{C}$$

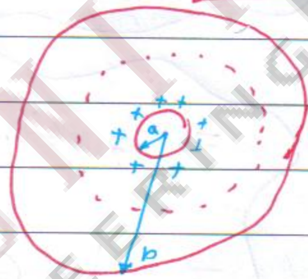
$$= \boxed{\frac{1}{2} C V^2} \; ; \; \boxed{\frac{1}{2} Q V}$$

Subject Date No.

* Coaxil capacitor "cylindrical capacitor"

* it can be shown "see book" $\boxed{P\,227}$

$$C = \frac{2\pi\epsilon L}{\ln\left(\frac{b}{a}\right)}$$



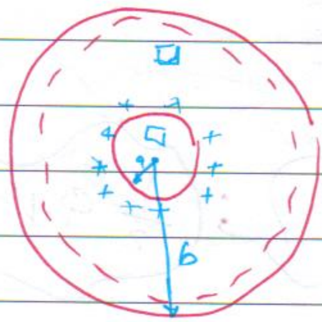
* Spherical Capacitor *

* It can be shown: "P 228"

$$C = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}$$

AE93

* If "$b \to \infty$" $\Rightarrow$ $C = 4\pi\epsilon a$



Subject Date No.

* RC Product *

$$RC = \frac{\epsilon}{\sigma}$$

* It can be used to calulate:

"C" "R" if "R" "C" is Known

Ex)

Electromagnatics I

NoteBook

Dr. Omar Ghzawi

By: Abdelrahman Bahboh

بأفكارنا_نُبدع

10th. Aug. 2014

Maxwell's Equation :- (CH#7)

A compact formulations for the EM.

So far they are differential (point) form or Integral form.

differential (point) form	Integral form
1 $\nabla \cdot \vec{D} = \rho_v$	$\oint \vec{D} \cdot \vec{ds} = \int_v \rho_v \, dv$

Gauss's Law

2 $\nabla \cdot \vec{B} = 0$	$\oint \vec{B} \cdot \vec{ds} = 0$

non-Existance of magnatic monopole.

* Magnatic field lines are closed
* $\vec{B}$:- magnatic flux density.

3 $\nabla \times \vec{E} = 0$	$\oint \vec{E} \cdot \vec{dl} = 0$

conservation of the electrostatic field.

4 $\nabla \times \vec{H} = \vec{J}$	$\oint \vec{H} \cdot \vec{dl} = \oint \vec{J} \cdot \vec{ds}$

Ampere's Law

10th. Aug. 2014

※ Magnetic Forces :- (CH # 8)

Ex :- what are the differences between electrostatics and magnetostatic fields ??

$\vec{F_e} = Q\vec{E}$ → Force in the same direction of $\vec{E}$

$\vec{F_m} = Q\vec{u} \times \vec{B}$ → Force is perpendicular to $\vec{u}$.

* Due to Both Fields :-

$\vec{F} = \vec{F_e} + \vec{F_m} = Q\vec{E} + Q\vec{u} \times \vec{B}$

$$\boxed{\vec{F} = Q(\vec{E} + \vec{u} \times \vec{B})}$$

→ Force on a current element :-

$I\vec{dl} = \vec{K}\,d\vec{s} = \vec{J}\,dv$

also $\vec{J} = \rho_v \vec{u}$

→

10th. Aug. 2014

$$\underline{\underline{I\,d\vec{l}}} = \rho_v \vec{u}\, dv = \rho_v\, dv\, \vec{u} = \underline{\underline{dQ\,\vec{u}}}$$

* Hence :-

$$d\vec{F}_m = dQ\,\vec{u} \times \vec{B} = I\,d\vec{l} \times \vec{B}$$

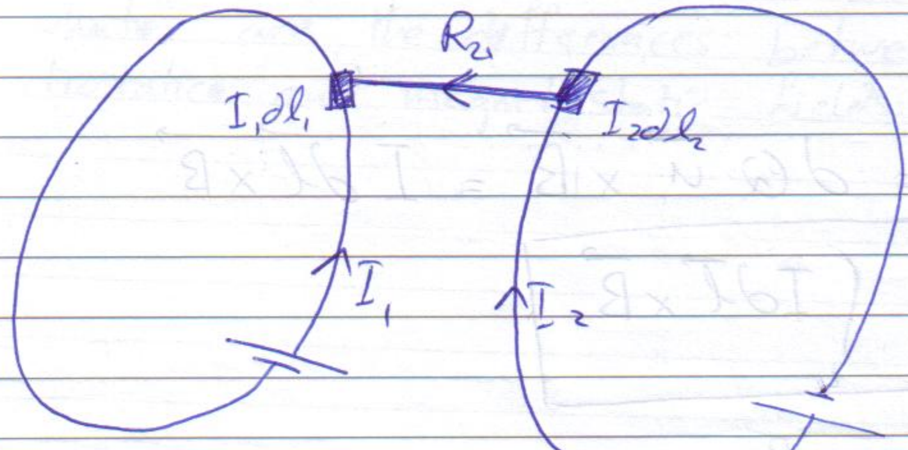
$$\boxed{\vec{F}_m = \int_L I\,d\vec{l} \times \vec{B}}$$

* giving the force imposed on a conductor due to an external magnatic field.

Five Apple

11th. Aug. 2014

Force Between Two Current Elements :-



$$d(dF_{m_1}) = I_1 \, d\vec{l}_1 \times \vec{B}_2$$

$$d\vec{B}_2 = \frac{\mu_0 I_2 d\vec{l}_2 \times \vec{a}_{R_{21}}}{4\pi |\vec{R}_{12}|^2}$$

$$F_m = \oint \oint \frac{\mu_0 I_1 I_2 \, d\vec{l}_1 \times (d\vec{l}_2 \times \vec{a}_{R_1})}{4\pi |\vec{R}_{21}|^2}$$

Example 8.1 :- a charged particale of mass 2 Kg and a charge 3 C starts at point (1, -2, 0) with velocity $4 a_x + 3 a_z$ & Electric field $12 a_x + 10 a_y$ V/m. At time t = 1 sec. determine

1. the acceliration of particle
2. the velocity.
3. it's Keinatic energy.
4. it's position

11th. Aug. 2014

$\boxed{1}$ $\vec{F}_e = Q\vec{E} = m\vec{a}$

$\vec{a} = \frac{Q\vec{E}}{m} = \frac{3}{2}(12a_x + 10a_y)_z$

$\boxed{\vec{a} = 18a_x + 15a_y \quad m/s^2}$

$\boxed{2}$ $\vec{a} = \frac{\partial \vec{u}}{\partial t} = \frac{\partial u_x}{\partial t} a_x + \frac{\partial u_y}{\partial t} a_y + \frac{\partial u_z}{\partial t} a_z$

$= 18a_x + 15a_y$

$\frac{\partial u_x}{\partial t} = 18 \Rightarrow u_x = 18t + A$

$\frac{\partial u_y}{\partial t} = 15 \Rightarrow u_y = 15t + B$

$\frac{\partial u_z}{\partial t} = 0 \Rightarrow u_z = C$

$u(t=0) = 4a_x + 3a_z$

$18\times 0 + A = 4 \Rightarrow A = 4$, $15\times 0 + B = 0 \Rightarrow B = 0$

$C = 3$

$u(t) = (18t + 4)a_x + 15t a_y + 3a_z$

$u(t=1) = 22a_x + 15a_y + 3a_z \quad m/s$

$\boxed{3}$ $K.E = \frac{1}{2} m|\vec{u}|^2 \qquad |\vec{u}|^2 = 484 + 225 + 9$

$= 718 \text{ J}$

Five Apple

11th Aug. 2014

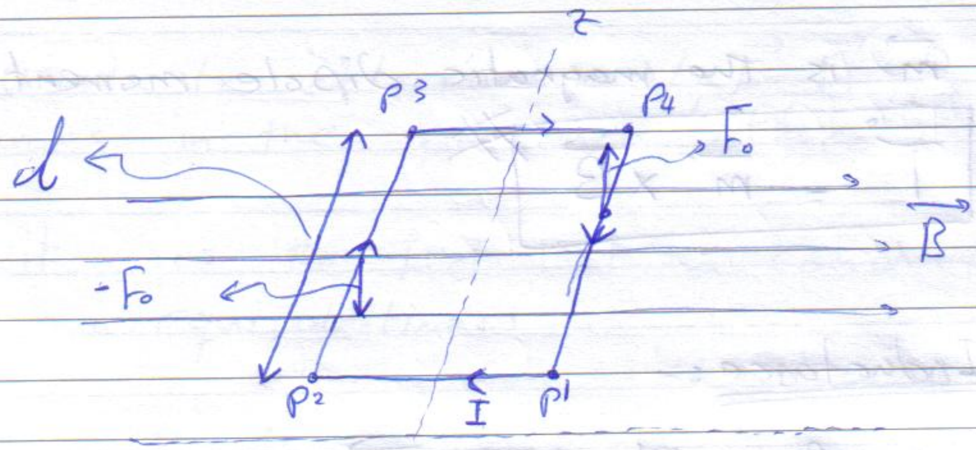
$\boxed{4}$ Integrate the velocity and substitute the initiale conditions we obtaine:-

$$(x, y, z) = (9t^2 + 4t + 1, 7.5t^2 - 2, 3t)$$

at $t = 1$ $(x, y, z) = (14, 5.5, 3)$

Example 8.2 :- study this example.

Magnetic Tourque :-



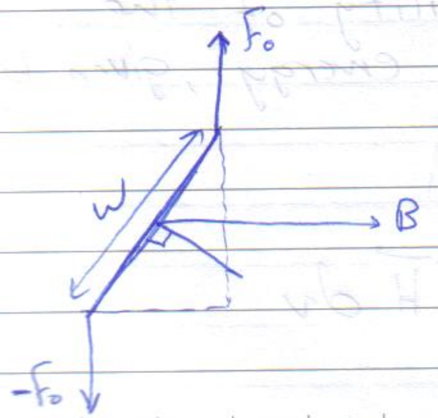
$$d\vec{F} = I\,d\vec{l} \times \vec{B}$$

$$\vec{F} = I\int_{P_2}^{P_3} dz\,a_z \times \vec{B} + I\int_{P_4}^{P_1} dz\,a_z \times \vec{B}$$

$$= I\int_{P_2}^{P_3} dz\,a_z \times \vec{B} - I\int_{P_1}^{P_4} dz\,a_z \times \vec{B} = 0$$

$$= \vec{F_o} - F_o$$

where $|F_o| = IBl$ when $\vec{B}$ is uniform.



$$|T| = |F_o|\,w \sin\alpha$$

$$= IBlw\sin\alpha$$

$$|T| = IBS\sin\alpha$$

13th. Aug. 2014

Define $\vec{m} = I S \, a_n$

where $\vec{m}$ is the magnetic dipole moment.

hence $\boxed{\vec{T} = \vec{m} \times \vec{B}}$ ※

※ <u>The Inductance:-</u>

Magnatic flux $\Psi = \int \vec{B} \cdot \vec{ds}$

If the circuit has N identical turns, then the flux Linkage is

$$\lambda = N \Psi$$

It turns out that $\lambda \propto I$

$$\lambda = L I \Rightarrow L = \frac{\lambda}{I} = \frac{N\Psi}{I}$$

L is Known as the inductance.

* it is a measure of the ability of the conductor to store magnatic energy, given by

$$W_m = \frac{1}{2} L I^2$$

or generally $W_m = \frac{1}{2} \int_v \vec{B} \cdot \vec{H} \, dv$

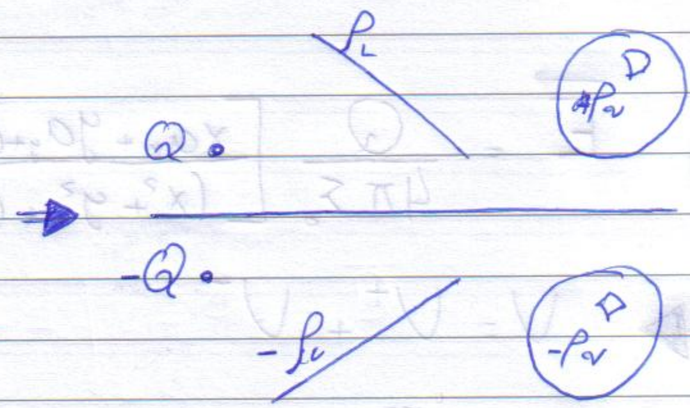
14th. Aug. 2014

※ The Method of Images :- (CH#6)

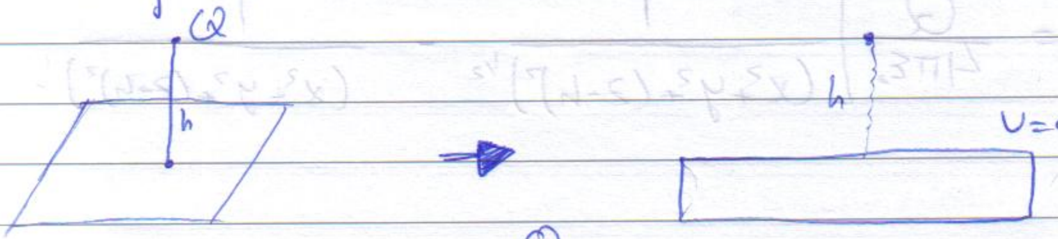
* used to determine $V, \vec{E}, \vec{D}, \rho_s$ due to charges in the presence of conductors.

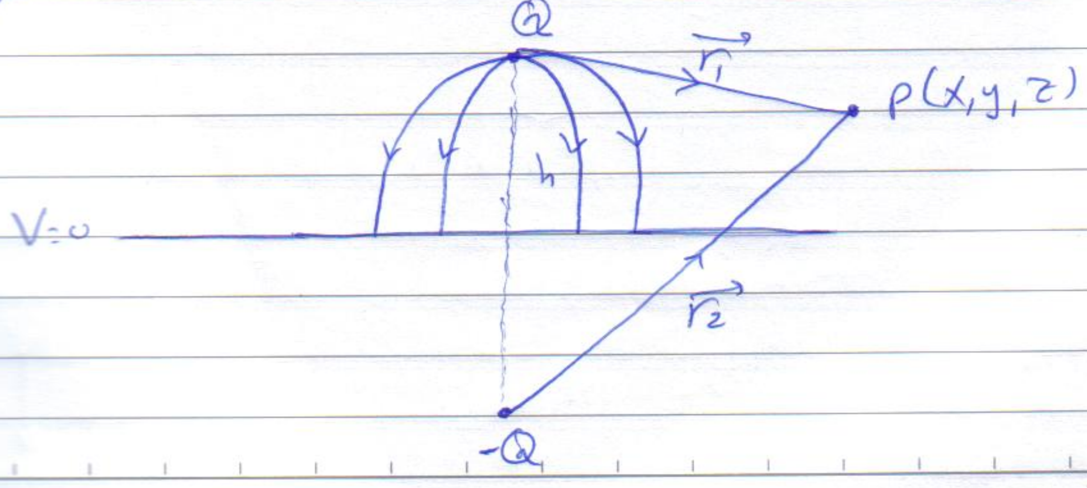
* it uses the fact that a conducting surface is an equipotentianal.

$\boxed{\text{perfect conductor} \equiv \text{equipotentianal}}$



Example :- a point charge above a grounded conductiong plane.





$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$= \frac{Q\vec{r_1}}{4\pi\epsilon_0|\vec{r_1}|^3} - \frac{Q\vec{r_2}}{4\pi\epsilon_0|\vec{r_2}|^3}$$

$$\vec{r_1} = (x, y, z) - (0, 0, h) = (x, y, z-h)$$

$$\vec{r_2} = (x, y, z) - (0, 0, -h) = (x, y, z+h)$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\left[\frac{xa_x + ya_y + (z-h)a_z}{(x^2+y^2+(z-h)^2)^{3/2}} - \frac{xa_x + ya_y + (z+h)a_z}{(x^2+y^2+(z+h)^2)^{3/2}}\right]$$

$$V = V^+ + V^-$$

$$V = \frac{Q}{4\pi\epsilon_0|\vec{r_1}|} - \frac{Q}{4\pi\epsilon_0|\vec{r_2}|}$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{(x^2+y^2+(z-h)^2)^{1/2}} - \frac{1}{(x^2+y^2+(z+h)^2)^{1/2}}\right]$$

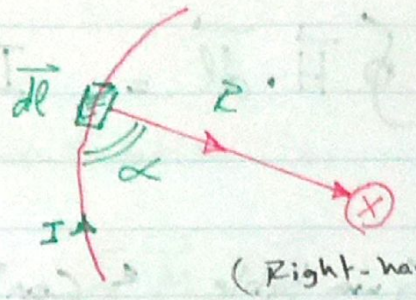
14th. Aug. 2017

* Example :- a line charge above a conducting plane
(See book)

EM المحاضرة الإضافية

CH7 : Magnetostatics :

→ Biot - Swart's Law :



(Right-hand-Rule)
thamb in
chreaction of
Current and
Magnatic field
with other
finger

→ $dH \propto \dfrac{I\,\vec{dl}\ \sin\alpha}{R^2}$

$\therefore\ \vec{dH} = \dfrac{k \cdot I\,\vec{dl}\ \sin\alpha}{R^2} = \dfrac{I\,dl \cdot 1 \cdot \sin\alpha}{4\pi R^2}$

Cross Product

$\vec{dH} = \dfrac{I\,\vec{dl} \times a_R}{4\pi R^2} \cdot \dfrac{R}{R} = \dfrac{I\,\vec{dl} \times \vec{R}}{4\pi\,|\vec{R}|^3}$

→ $\vec{H} = \displaystyle\int_L \dfrac{I\,\vec{dl} \times \vec{R}}{4\pi\,|\vec{R}|^3} \overset{oR}{=} \int_S \dfrac{k\,ds \times \vec{R}}{4\pi\,|\vec{R}|^3}$

(Double Integral)

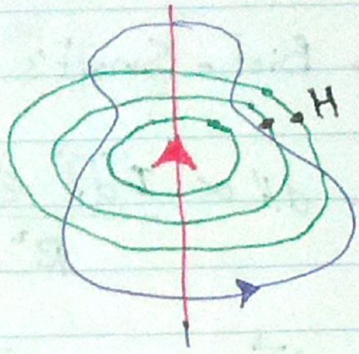
oR $= \displaystyle\int_V \dfrac{J\,dv \times \vec{R}}{4\pi\,|\vec{R}|^3}$

(triple Integral)

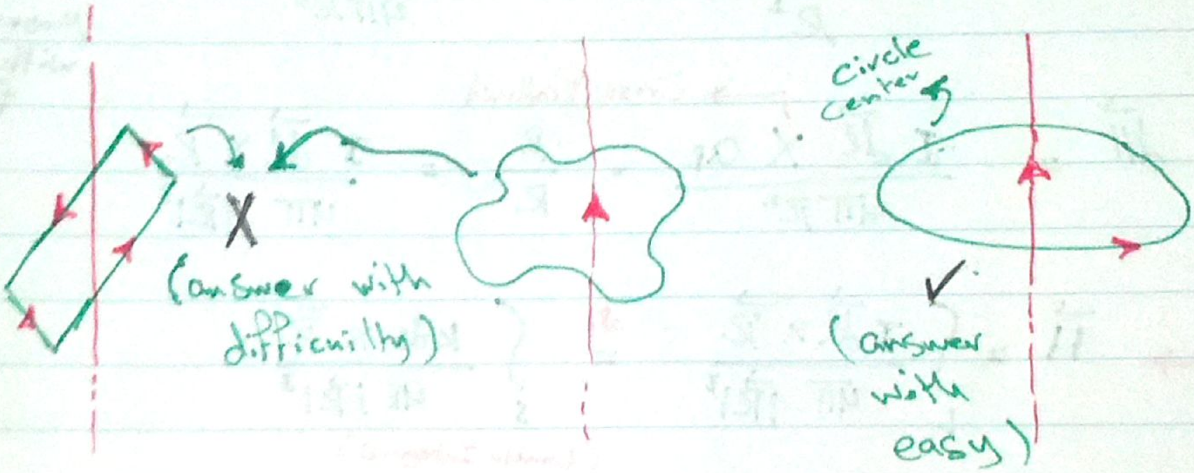
* ~~For $\rho = a$, $I_{enc} = I$~~

* Ampere's Circuit Law :

$$\oint \vec{H} \cdot \vec{dl} = I_{enc}$$:

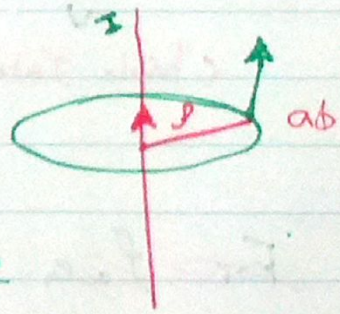


* Suitable & Centers together with atilization of Symmetry, Simplity the Calculation Considirectly



* Example : Consider the infinite Filmantry Straight Conductor

→ In this Case :



$$\vec{H} = H_\phi \, a_\phi$$

a Suitable Contour (Path) is a Circle

→ $\vec{dl} = \rho \, d\phi \, a_\phi$

$$I_{enc} = \oint_{circle} H_\phi \cdot a_\phi \cdot dl = \oint_{circle} H_\phi \, a_\phi \cdot \rho \, d\phi \, a_\phi$$

$\therefore I_{enc} = \int_{\phi=0}^{2\pi} H_\phi \, \rho \, d\phi = 2\pi H_\phi \rho$

$$\boxed{H_\phi = \frac{I}{2\pi\rho}}$$

$\therefore$ a generalization is

$$\sum I_i = \oint \vec{M} \cdot \vec{dl}$$

$$\boxed{\vec{H} = \frac{I}{2\pi\rho} a_\phi}$$

Special Cases :

① Semi-infinite Conductor :

$\alpha_1 = 90°$ $\alpha_2 = \text{Zero}$

$$\boxed{H = \frac{I}{4\pi\rho} a_\phi}$$

② infinite Conductor :

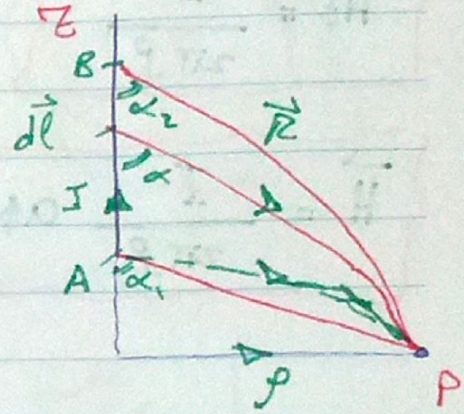
$\alpha_1 = 180°$ $\alpha_2 = \text{Zero}$

$$\boxed{\vec{H} = \frac{I}{2\pi\rho} a_\phi}$$

* See the two example in Book (Determine H for a Current loop)

* **Example** : Straight Filemntery Conductor (P264)

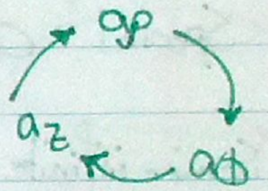
$$d\vec{H} = \frac{I}{4\pi} \frac{dz\, a_z \overset{\text{Cross Product}}{\times} (+\rho a_\rho - z a_z)}{[\rho^2 + z^2]^{3/2}}$$



$$= \frac{I}{4\pi} \frac{\rho dz\, a_\phi - \text{zero}}{[\rho^2 + z^2]^{3/2}}$$

$$\cancel{d\vec{H} = \frac{I}{4\pi} \frac{dz\, a_z \times (+\rho a_\rho - z a_z)}{[\rho^2 + z^2]^{3/2}}}$$

$$\vec{H} = \frac{I}{4\pi} \rho a_\phi \int_A^B \frac{1}{[\rho^2+z^2]^{3/2}} dz$$



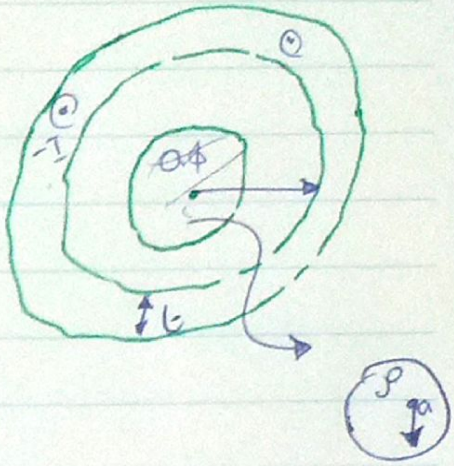
let $z = \rho \cos\alpha$ (see Book) $dz = \rho - \csc^2 x$

$$\boxed{H = \frac{I\rho}{4\pi} [\cos\alpha_2 - \cos\alpha_1]\, a_\phi}$$ Final answer

H is given By Concentric circles where the centre is the conctrre

* Example : Consider an infinity long coaxial tranmesion line , (see Book) for full solution

$$I = \oint \vec{J} \cdot \vec{ds}$$



$$\vec{H} = H_\phi \, a_\phi$$

$$\vec{dl} = \rho \, d\phi \, a_\phi$$

$$|J| = \frac{I}{\pi a^2}$$

$$\boxed{I_{enc} = \frac{I \rho^2 \pi}{\pi a^2} = \frac{I \rho^2}{a^2}}$$

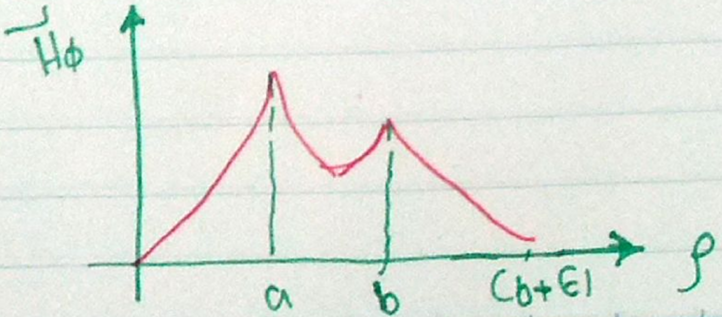
$$\frac{I\rho^2}{a^2} = \int_{\phi=0}^{2\pi} H_\phi \, a_\phi \cdot \rho d\phi \, a_\phi = H_\phi \rho \int_0^{2\pi} d\phi$$

$$= 2\pi H_\phi \rho = \frac{I\rho^2}{a^2}$$

$$\therefore \boxed{H = \frac{I\rho}{2\pi a^2} a_\phi} \qquad 0 \le \rho < a$$

$[\rho \uparrow \rightarrow H \uparrow]$

For $\rho = a$, $I_{enc} = I$



{CH 7}

Maxwell's Equations

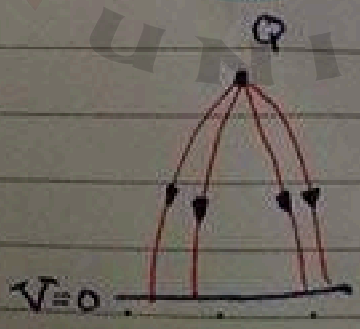
10-8-2014
Sunday

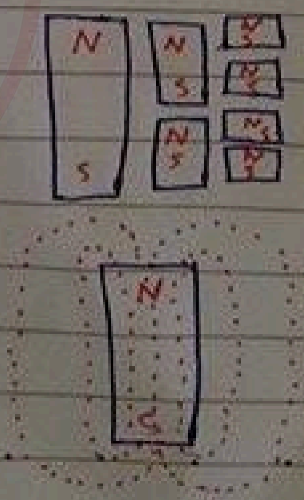
No:

→ A compact formulations for the "EM"

So far, They are:

differential or Point form	Integral Form	
1-) $\nabla \cdot \bar{D} = \rho_v$	$\oint \bar{D} \cdot \bar{ds} = \int \rho_v \, dv$	Gauss Law
2-) $\nabla \cdot \bar{B} = 0$	$\oint B \cdot \bar{ds} = 0$	- non-existans of magnetic monopole - magnetic fields are closed
3-) $\nabla \times \vec{E} = 0$	$\oint \bar{E} \cdot \bar{dL} = 0$	conservation of the electrostatic
4-) $\nabla \times \bar{H} = \bar{J}$	$\int \bar{H} \cdot dL = \oint J \cdot ds$	Ampere's Law





(CH8)

* Magnetic Forces *

10-8-2014
Sunday

No.

Ex) what are the diffrences betwen electrostatic and the magnetostatic fields?!

((See Book))

$F_m = Q\,\bar{u} \times \bar{B}$ m: magnetostatic

* Force is Perpendicular to "$\bar{u}$"

$F_e = Q\,\bar{E}$ e: electrostatic

* Force is the same direction of "$\bar{E}$"

* Due to both fields:

$$\bar{F} = Q\,\bar{E} + Q\,\bar{u} \times \bar{B}$$
$$= Q(\bar{E} + \bar{u} \times \bar{B})$$

Force on a Current Element

10-8-2014
Sunday

No.

$$I d\bar{l} = K d\bar{s} = \bar{J} dv$$

also,,,

$$\bar{J} = \rho_v \bar{u}$$

$$\Rightarrow I d\bar{l} = \rho_v \bar{u} d\bar{v} = \rho_v d\bar{v} \bar{u} = d\bar{q} \bar{u}$$

Hence,,,

$$d\bar{F}_m = dq\, \bar{u} \times \bar{B}$$
$$= I d\bar{l} \times \bar{B}$$

$$\Rightarrow \bar{F}_m = \oint_L I d\bar{l} \times \bar{B}$$

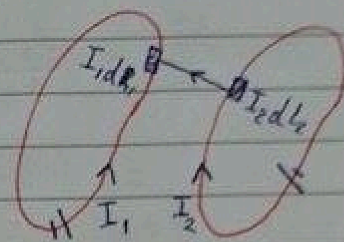
→ Giving the force imposed on a conducter due to an external magnetic field.

Force Betwen Two Current Elemenes

11-8-2014

monday

No.



$$d(dF_{m_1}) = \oint\oint \frac{\mu_0 I_1 I_2 \times d\bar{L}_1 \times d\bar{L}_2 \times \bar{a}_{R_1}}{4\pi R_{21}^2}$$

$$d(dF_1) = I_1 d\bar{L} \times d\bar{B}_1 \quad \text{--- (1)}$$

see Examples in Book

$$dB_1 = \frac{\mu_0 I_2 d\bar{L}_2 \times a_{R_1}}{4\pi R_{21}^2} \quad \text{--- (2)}$$

$d(dF_m) =$ (2) into (1)

EX 8.1

a charge partical of mass "2 kg" having a "3 c" charge at "(1,-2,0)" with velocity "4ax + 3 az" m/s in an electric faild "12ax + 10ay" V/m. at tim = "1 s" Find?!

1-) The accelaration
2-) Velocity
3)
4-) The Position

Sol,,,

1) $F_e = Q\bar{E} = m\bar{a}$

$$\bar{a} = \frac{QE}{m} = \frac{3}{2}(12a_x + 10a_y)$$

$$= \boxed{18a_x + 15a_y \ m/s^2}$$

11-8-2014
monday

No.

2)

$$\bar{a} = \frac{d\bar{u}}{dt} = \frac{du_x}{dt} a_x + \frac{du_y}{dt} a_y + \frac{du_z}{dt} a_z$$

$$= 18 a_x + 15 a_y$$

$\Rightarrow \frac{du_x}{dt} = 18 \Rightarrow u_x = 18t + A$

$u_y = 15t + B$

$u_z = C$

3)

$u(t=0) = 4a_x + 3a_z = 4a_x + 0 \cdot a_y + 3a_z$

$8 \times 0 + A = 4 \Rightarrow \boxed{A = 4}$

$15 \times 0 + B = 0 \Rightarrow \boxed{B = 0}$

$\boxed{C = 3}$

$u(t) = (18t + 4)a_x + 15t\, a_y + 3a_z$

$u(t=1) = (18+4)a_x + 15a_y + 3a_z$

AE93

$K.E = \frac{1}{2} m |\bar{u}|^2$

$= \frac{1}{2} \times 2 \times (22^2 + 15^2 + 3^2)$

$= 718 J$

4) Integrating the Velocity we obtain (x, y, z)

$(x, y, z) = 9t^2 + 4t, \; 7.5t^2 - 2, \; 3t$

at $(t=1) \Rightarrow (14, 5.5, 3)\, m$

11-8-214

No.

EX 4.2 Study the example " see books"

end of second materail