15/30

The University of Jordan

School of Engineering

Department of Electrical Engineering

Fall Term – A.Y. 2016-2017

Probability and Random Variables, EE321, Second Exam

Name: [illegible] Student Number: 0140754 section: 1

Q1 Given that random variable X is continuous type, we form the random variable $y = g(x)$.

$Y = 2F_x(x)+4$

a) Find $f_Y(y)$ if $g(x) = 2F_X(x)+4$. $Y = 2F_x(x)+4$

b) Find $y = g(x)$ such that Y is uniform in the interval (6, 10).

2

a) $F_x(x) = P\{X \le x\}$

$F_x(x)$ $\quad \cancel{T^{-1}(y)} = \dfrac{y-4}{2}$

$\cancel{f_y(y) = f_x(T^{-1}(y))\dfrac{dT^{-1}(y)}{dy}}$

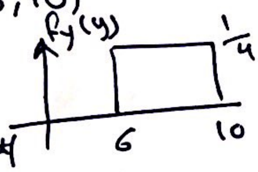
$\cancel{f_y(y) = \frac{1}{2} f_x\left(\frac{y-4}{2}\right)}$

$F_x(x) = \dfrac{y-4}{2}$

$f_y(y) = f_x(T^{-1}(y))\dfrac{dT^{-1}(y)}{dy}$

$f_y(y) = f_x\left(\dfrac{y-4}{2}\right) * \dfrac{1}{2}$

b) $Y \sim u(6,10)$



$\cancel{g(x) = 2F_x(x)+4}$

$f_y(y) = \begin{cases} \frac{1}{4}, & 6 < Y < 10 \\ 0, & o.w \end{cases}$

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~~$E[X] = \sum X_i P\{X_i\}$~~

2

Q2. The Poisson density function is given by

$$f_X(x) = e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k)$$

a) Find the characteristic function $\emptyset_X(\omega)$.
b) Use the result in part (a) to find the mean and variance of the random variable X.

a) $\Phi_x(\omega) = E[e^{j\omega x}] = \int_{-\infty}^{\infty} e^{j\omega x} * e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k) \, dx$

$= e^{-b} \sum_{k=0}^{\infty} \frac{e^{j\omega k}}{j\omega} \frac{b^k}{k!} u(x-k)$

b) $\bar{X} = E[X] = m_1 = -j \left. \frac{d\Phi_x(\omega)}{d\omega} \right|_{\omega=0}$

$= -j \left(e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} * \frac{jk e^{j\omega k}\delta(x-k) - j e^{j\omega k} u(x-k)}{(j\omega)^2} \right.$

$E[X^2] = m_2 = (-j)^2 \left. \frac{d^2\Phi_x(\omega)}{d\omega^2} \right|_{\omega=0}$

$\Rightarrow \sigma_x^2 = m_2 - m_1^2$

$C_{XY} = R_{XY} - \bar{X}\bar{Y}$

$= E[XY] - \bar{X}\bar{Y}$

$R_{XY} = C_{XY} + \bar{X}\bar{Y}$

$\sigma_x = 1.9$

$\theta = \pi/6$

$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\rho\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}\right)$

$\rho = \frac{C_{XY}}{\sigma_x\sigma_y}$

3

Q3. Two Gaussian random variables X and Y have covariance $C_{XY}= 0.8$. The standard deviation of X is 1.9. A coordinate rotation of $\frac{\pi}{6}$ is known to transform X and Y to new random variables Y1 and Y2 that are statistically independent.

$\frac{\pi}{6} \rightarrow 30^\circ$

a) Find the new random variables Y1 and Y2 in terms of X and Y.

b) Find σ_Y^2. $\rightarrow m_2 - m_1^2$

a)

$E[Y_1]E[Y_2] = E[Y_1 Y_2]$

$C_{Y_1 Y_2} = 0$

$\rho\sigma_x\sigma_y = 0.8$

$\rho\sigma_{y_1}\sigma_{y_2} = 0$

$\theta = \tan^{-1} 0$

$\theta = 0$

3 b) $\sigma_y^2 = (\sigma_y)^2$

$\rho = \frac{C_{XY}}{\sigma_x\sigma_y}$, $\tan 2\theta = \frac{2\rho\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}$

$\rho = \frac{0.8}{1.9\,\sigma_y}$

$\sqrt{3} = \frac{2\rho(1.9)\frac{8}{19\rho}}{(1.9)^2 - \sigma_y^2}$

$\frac{19}{8}\rho = \frac{1}{\sigma_y}$

$\boxed{\sigma_y = \frac{8}{19\rho}}$

~~$6.25 - \sqrt{3}\sigma_y^2$~~

$6.25 - \sqrt{3}\sigma_y^2 = 1.6$

$\rightarrow \boxed{\sigma_y^2 = 2.686}$

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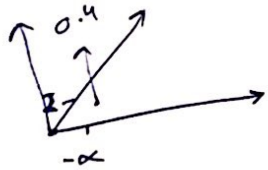

Q4. Discrete random variables X and Y have the joint density

$$f_{XY}(x,y) = 0.4\delta(x+\alpha)\delta(y-2) + 0.3\delta(x-\alpha)\delta(y-2) + 0.1\delta(x-\alpha)\delta(y-\alpha) + 0.2\delta(x-1)\delta(y-1)$$

Determine the value of α, if any, such that :

a) X and Y are uncorrelated. $\rightarrow C_{XY}=0$

b) X and Y are orthogonal. $\rightarrow R_{XY}=0$

6

a) $E[X]E[Y] = E[XY]$

$\bar{X}\ \bar{Y} = E[XY]$

$C_{XY}=0$

$E[XY] - \bar{X}\bar{Y} = 0$

$E[X]=\bar{X} = 0.4(-\alpha) + 0.3(\alpha) + 0.1(\alpha) + 0.2(1)$

$\bar{X} = 0.2$

$E[Y] = \bar{Y} = 0.4(2) + 0.3(2) + 0.1\alpha + 0.2(1)$

$= 0.8 + 0.6 + 0.1\alpha + 0.2$

$\bar{Y} = 1.6 + 0.1\alpha$

$E[XY] = 0.4(-\alpha)(2) + 0.3(\alpha)(2)$

$+ 0.1(\cancel{\alpha})(\alpha) + 0.2(1)(1)$

$= -0.8\alpha + 0.6\alpha + 0.1\alpha^2 + 0.2$

$= -0.2\alpha + 0.1\alpha^2 + 0.2$

$\underline{C_{XY}=0}$

$\cancel{0.2(1.6+0.1\alpha) = 0}$

$-0.2\alpha + 0.1\alpha^2 + 0.2 - (0.2(1.6) + 0.2(0.1)\alpha) = 0$

$\boxed{\cancel{\alpha = 0.8}}$

$\boxed{\cancel{\alpha_1 = -0.45}}$

$\boxed{\alpha_1 = -0.45}$

$\boxed{\alpha_2 = 2.65}$

b) $R_{xy} = 0$

$C_{xy} = \bar{X}\bar{Y}$

$\cancel{E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} XY f_{XY}(x,y)\,dx\,dy = 0}$

$E[XY] = 0$

$E[XY] = 0.4(-\alpha)(2) + 0.3(\alpha)(2) + 0.1\alpha^2 + 0.2 = 0$

$0.2 + -0.8\alpha + 0.6\alpha + 0.1\alpha^2$

-0.2α

$\alpha^2 - 2\alpha + 2 = 0$

$\boxed{\alpha_1 = 1+i}$

$\boxed{\alpha_2 = 1-i}$

Q5. Three statistically independent random variables X1,X2, and X3 are defined by:

$$\overline{X1} = -1 \quad \sigma_{X1}^2 = 2.0$$
$$\overline{X2} = 0.6 \quad \sigma_{X2}^2 = 1.5$$
$$\overline{X3} = 1.8 \quad \sigma_{X3}^2 = 0.8$$

2

Use the central limit theorem to write an approximate density function for the sum Y=X1+X2+X3.

$\cancel{f_Y(y) = \int_{-\infty}^{y} \frac{d F_Y(y)}{dy}}$

$E[Y] = -1 + 6.6 + 1.8$

$=$