Probability

NoteBook

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ELECTRICAL ENGINEERING

الأحد

Lec #1 2019 / 6 / 22

* Random Variable (R.V)

↳ a function that maps inputs into outputs

(input) Domain — Function — (output) Range
- observation
- outcomes (values of X)

(varies) غير معروفة inputs ← متغيرات

- we have to have certain Sequences to give very exact Probabilty.

$X \in [-10, 60]$

X is not a variable it is a random variable.

* Descrite Random variable

حجر النرد → has 6 values only (finite descrite)

Random Variable has at least 2 values

_ like binary (Coin رمي قطعة)

_ $\bar{e}^{n}$ is describle & infinite
↳ countable (we can know value of n)

$n \in \{0, 1, 2, 5\}$
↳ it is a discrete & Finite
so it is digital domain

/ /

- if we have an infinte Random variable but it's is avarage = 7 So, we expect most values are 7 or around 7

- Random process (funtion of time)
↳ change with time process.

* Stationary

↳ values doesn't change with time

* non - Stationary

↳ values change with time.

cos & sin. the smoothest function we can use it in one point
← ليس له علاقة لا يمين ولا شمال.

* Binary Random variable

$X \in \{0, 1\}$
$X \in \{R, G\}$
$X \in \{5, -16\}$

R, G are lables we can represent them as a function

→ function → deterministic

if $X \in \{1, 2, 3\}$
$1+2=3$ → out comes
$1+3=4$ → measurment value
كـ ليس من ضمن المجموعة

* if outcomes of this function it's limits exist from right & Left so it is differentiable.

* if it is infinite it is integrable

Lec # 2

الاثنين
2014 / 6 / 23

→ The Random variable RV

RV X

X → statistics احصائيات
→ corrolation
→ Higher order statistics

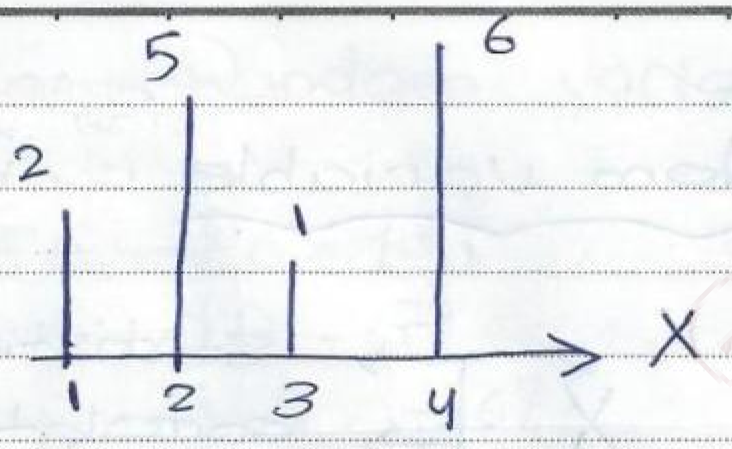
→ statistics
→ corrolation → Auto corrolation
⇓ F.T → Forier / Forian transformer
PSD we applied it to random Var.
عشان نعرف خصائصه

→ Higher order statistics
هي في الحقيقة جزء من ال statistics بس كنا فصلناها لان نظرة عمومية
{Higher order statistics}

* Probability density function (Pdf)

↳ measure the outcomes of the random variable & approximate the outcome frequency of each element in the Range.

→

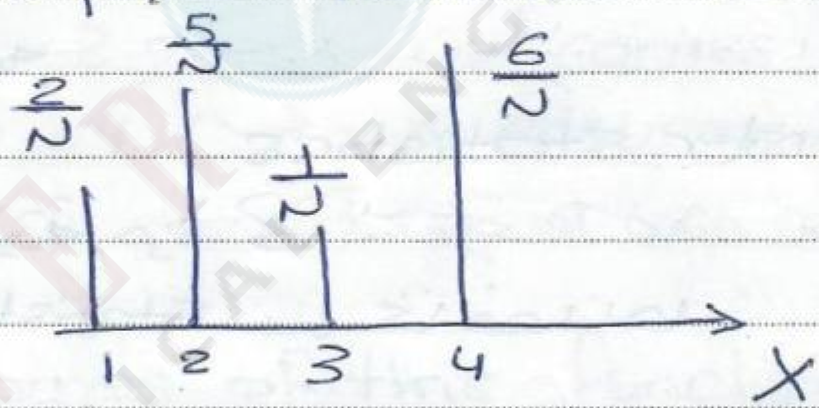


عندي 14 ⇐ outcomes

← كلما زاد عدد الـ experiment
كلما كان stable أكثر.

Normalize → become such that the area under the curve equal $\boxed{1}$

حتى اعمله Normalize بقسم عدد المرات على N



* if I have discrete R.V → The curve will be discrete
* if I have contin. RV → The curve will be conti.

(مثلا لو قلنا كم نتوقع انه يظهر رقم معين ؟ بيعطيه Percentage معينة ، حيث مجموع الـ Percentage يساوي 100%)

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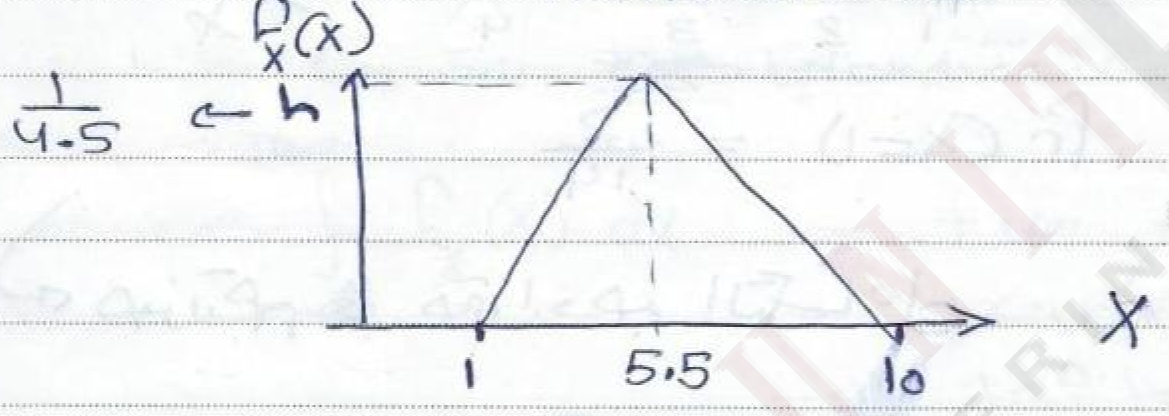
Example :-

free variable

outcome

X pdf $f_X(X)$ of X

X ↓ name of R.V



to Find h ?

$\boxed{1}$ $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$$4.5 * h = 1 \rightarrow h = \frac{1}{4.5}$$

← إذا كنا عند نقطة واحدة ، كم ال value عند X ؟

← بهمني اعرف كشان أوجد ال Prob.

($f_X(x)\,dx$ → لو كان ال f_X صفر ال Prob رح تساوي صفر.)

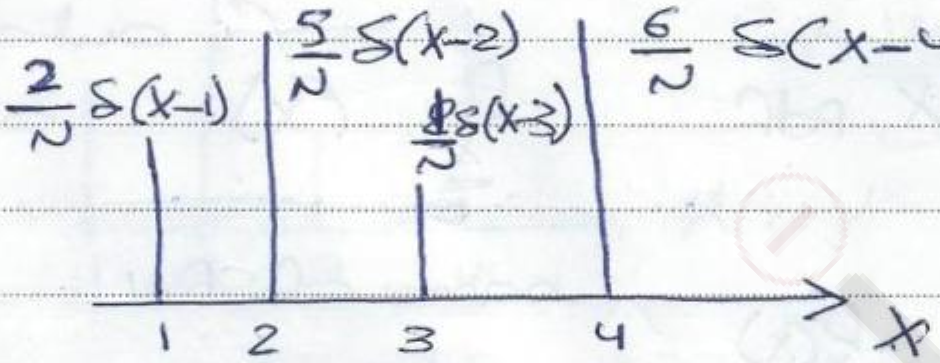
$\boxed{2}$ $Pr(X = x_0) = \int_{x_0^-}^{x_0^+} f_X(x)\,dx$

$Pr\ X = X_0 = 0$ for cont. RV

لانه تقسمها على رقم كبير من اجل norm. تحكس discrete لا يساوي صفر.

For discrete $Pr \; X = X_0 \neq 0$

Impulse function ⟹



$$Pr(x=1) = \frac{2}{14}$$

منتكم منه اوجد التكامل.

التكامل لل discrete حولناه لـ Summation

$$f_X(x) = \frac{2}{14}\delta(x-1) + \frac{5}{14}\delta(x-2) + \frac{1}{14}\delta(x-3) - - - + \frac{6}{14}\delta(x-4)$$

* Probability distrubtion function PDF

$$\boxed{3} \quad F_X(x) = \int_{-\infty}^{x} f_X(\alpha)\, d\alpha$$

حطيت α بدل X
لاني عوضت X
كحدود التكامل

$$Pr(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f_X(x)\, dx$$

$$= \int_{-\infty}^{x_2} f_X(x)\, dx - \int_{-\infty}^{x_1} f_X(x)\, dx$$

$$= F_X(x_2) - F_X(x_1)$$

$\boxed{4}$ $F_X(X_0) \geq 0$ $\quad 0 \leq F_X(X) \leq 1$

$F_X(x)$ is a non ~~discreting~~ decreasing function.

so +ve مشتقة دايما

$\int_{-\infty}^{\infty} f_X(X)\, dX \rightarrow$ +ve أنا بجمع quantities infite على range.

[Always ~ منحنى التكرار Positive]

هل بقدر احكي إنه

$f_X(X) = \frac{dF_X(X)}{dX}$?

هاد غير دقيق (غير صحيح) إلا تحت certain conditions ح نوخذهم بعدين

* متى يكون ال function pdf ?

لازم يكون → +ve

→ Finite range

→ bounded

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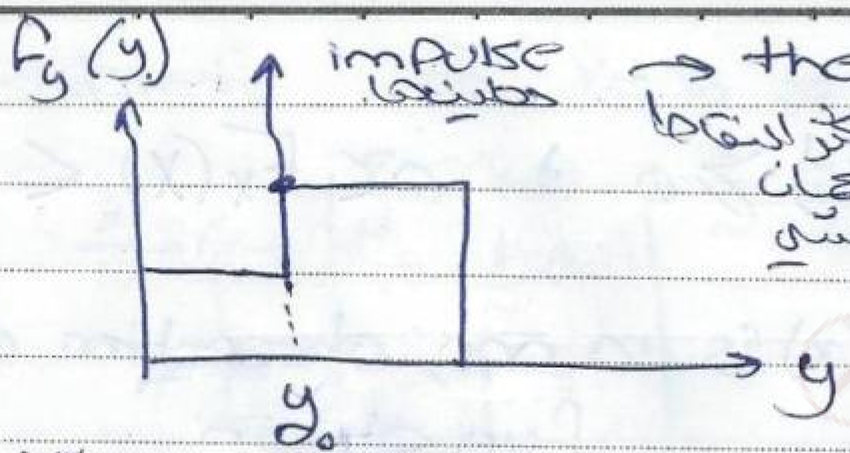


→ the y_0
مش بس اكثر احتمالا
بتكون كمان
اعلى شي

Pure هون استخدم ماله → it's continious
mathmatics
حتى اوجدهم الصين   every where & not
والبس
في واحد فرنسي حط   differentiable
اشي كشان نحلها function

الثلاثاء

* Lec # 3

2014/ 6/ 24

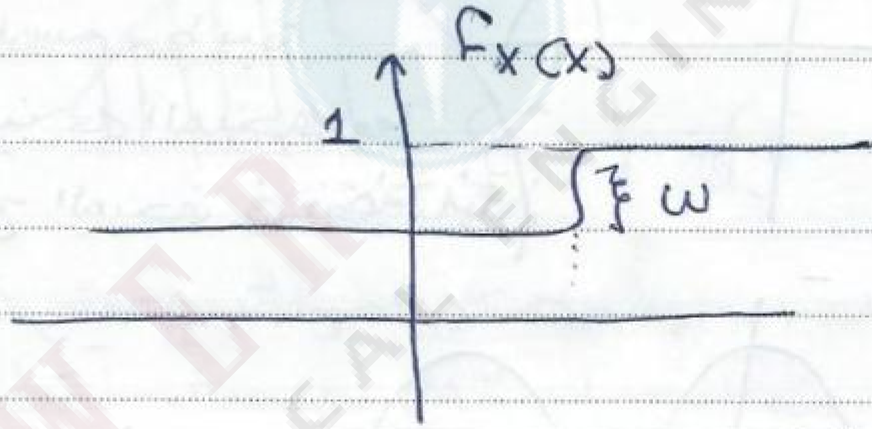
1- $Pr(x_0 \le X \le x_1) = F_X(x_1) - F_X(x_0)$

$$F_X(-\infty) = 0$$

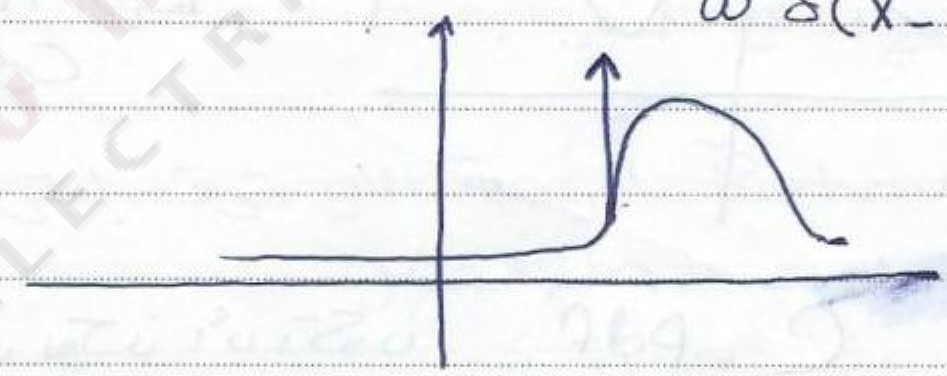
$$F_X(+\infty) = 1 = \int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

2- $F_X(X)$ is a non decreasing function

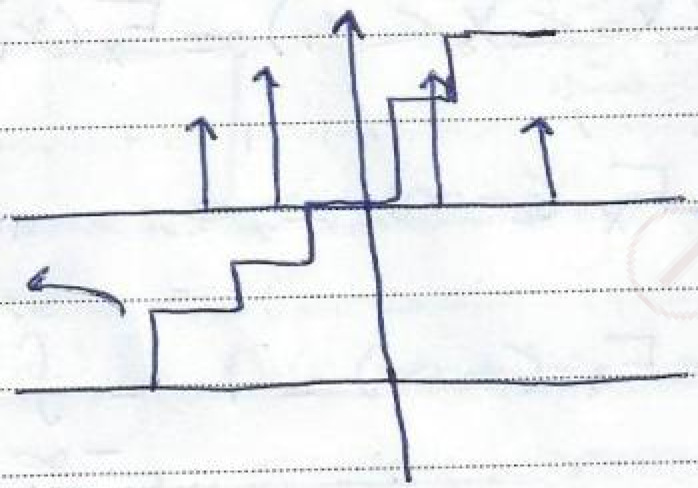
Zero or undefined ← يعني المشتقة



$w\,\delta(X - X_0)$ → المشتقة الثانية تعطي impulse



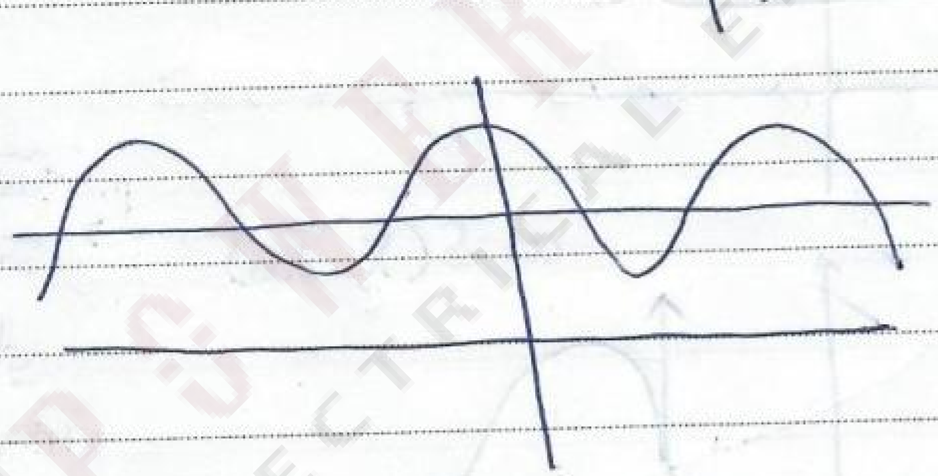
الرقم pdf كان يمثل ب impulse هي عبارة عن
prob ، احتمال حدوثه يساوي الـ weight
للـ impuls



مشتقة ال impulse

convex

اللي ارسم خط بين نقطتين على المنحنى وما يقطع المنحنى في نقطة ثالثة



non Convex

هل يمكن أن تكون pdf ؟ لجزء في حالة non - convex

إذا كان non convex

معناها

لتكون

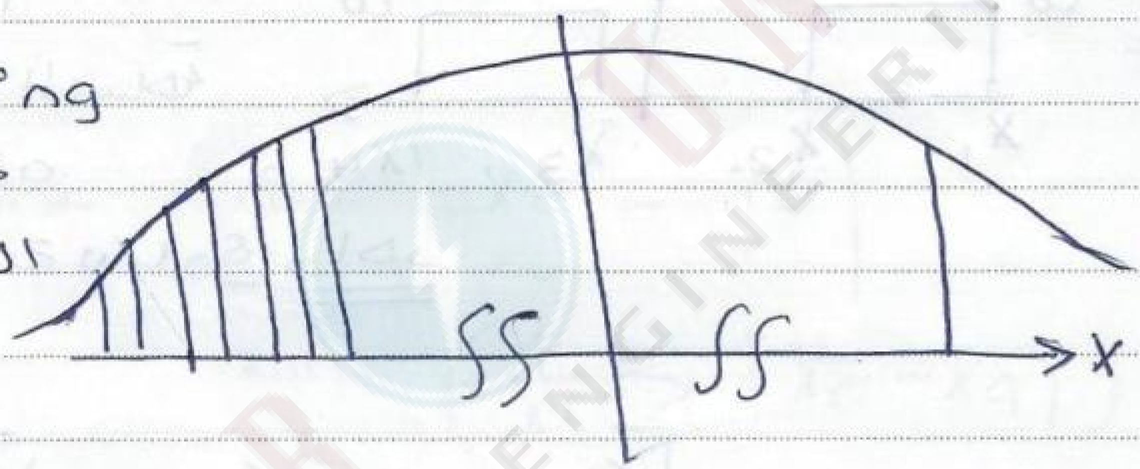
$$f_X(x) = \frac{dF_X(x)}{dx} \Rightarrow$$

دالة صحيحة

← بنادرس الـ dencity
لانه الـ Variables اللي متعامل معهم
الـ behaviour تبعهم مش رح يكون Smooth
في الـ prob. مش رح تكون homogenius
فبالتالي مش رح تكون العلاقة خطية .

(الـ prob هي تكامل الـ dencity)

Sampling
على
الـ X_axis



← نقسم الـ cont لفترات صغيرة
لانه بنعرف انه احتمال حدوث نقطة
بيساوي صفر .

← الـ Behaviour للرسمة اللي فوق اقرب
اشي الها guassian
رح نشوف بعدين كيف .

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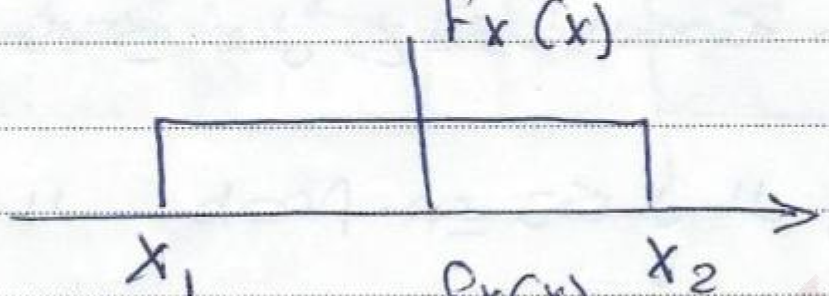
* Examples (هيه بالاصل ex بس لأهميتهم شرحوا كل شي كال.)

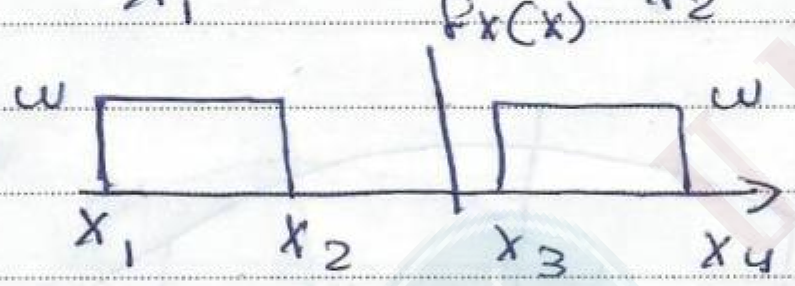
[1] Uniform distrbuted RV

Fig ①



يجب أن تكون Bounded

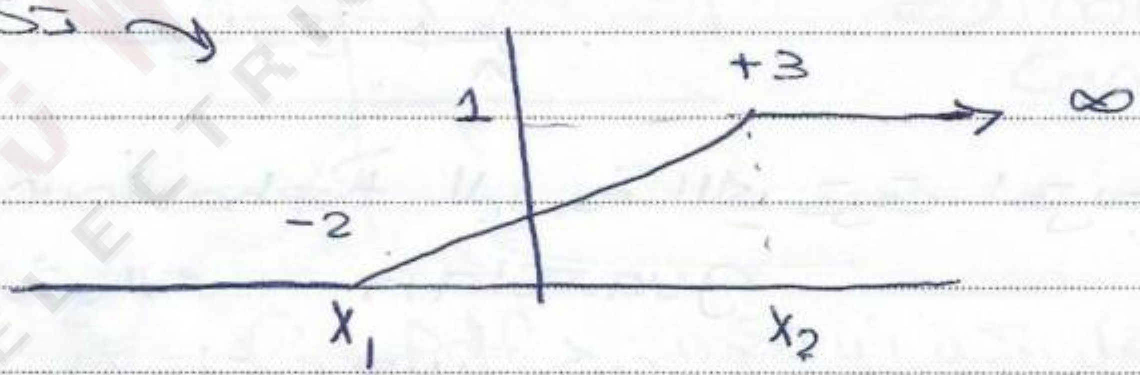
كادي لو كانت مقطوعة



اللي يهمنا أنه تكون المساحة تحت هذه المنحنيات بتساوي واحد

$$\text{area} = w\left[(X_2 - X_1) + (X_4 - X_3)\right]$$

Fig ① تكامل



شو احتمالية اقل من 5 + ؟ 100%

على الرسمة الاولى pdf

$$Pr(X \le 1) = \int_{-\infty}^{1} f_X(x)\,dx = \int_{-2}^{1} \frac{1}{5}\,dx$$

$$= \frac{3}{5}$$

حل

الرسمة الثانية ← $Pr(X \le 1) = F_X(1) = 0.6$

PDF $Pr(-1 \le X \le 1.5)$

$= F_X(1.5) - F_X(-1)$

$= \frac{3.5}{5} - \frac{1}{5} = \frac{2.5}{5} = 0.5$

الفرق كن الصح ← $\min |x_e - x_a|$

$\min \left(\frac{1}{N} \sum |x_e - x_a| \right)$

look like avarage

we called it L_∞ Norm

mean square error

$$\boxed{mse = \frac{1}{N} \sum (x_e - x_a)^2}$$

L_2 Norm

(*) الأكثر measure سيستخدم في الهندسة (*)

* Review

normalized

Probability density function

(frequency) لـ يمثل التكرار

outputs لكل نقطة على ال

وإذا كان Continous فكل نقطة

حقيقة ال Prob ← تعتبرها Zero

So in practical → we need to find prob to the point in a range

ex:-

range between x_1 & x_2

Lec #4

الأربعاء
٢٥ / ٦ / ٢٠١٤

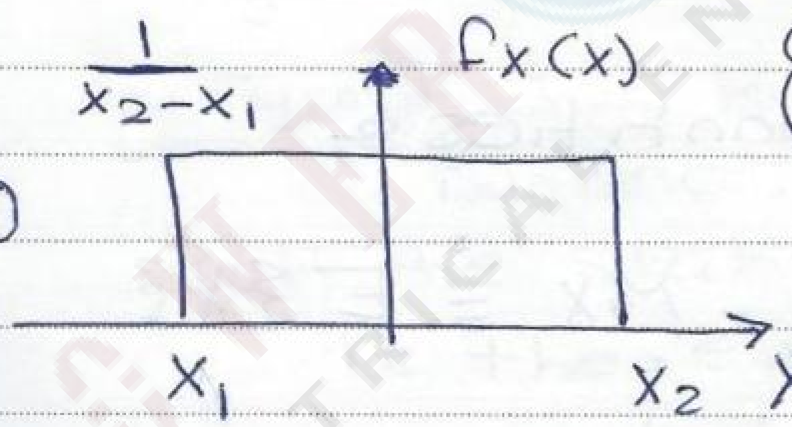
1 * Uniform random variable ← مناسبه نصف
- a- Uniform RV
- b- discrete Uniform

probability density function

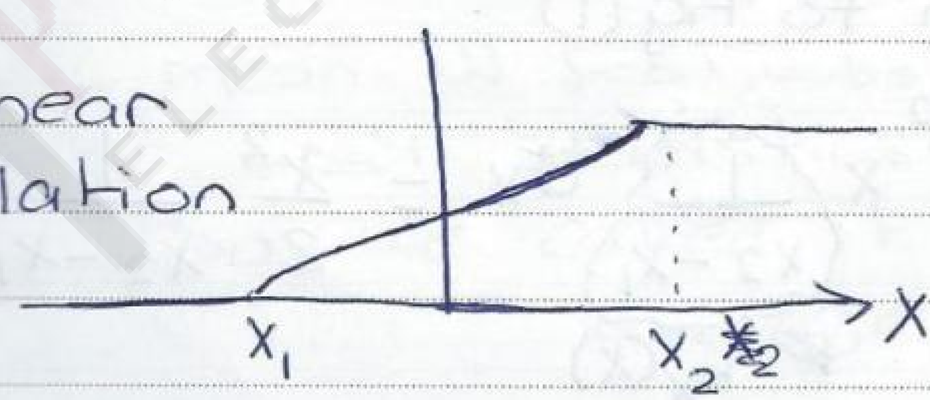
R. V X → دائما على ال Horizantal axis ← يمثل ال outcomes

Horizantal axis → out comes (R.V)
vertical axis → يمثل خاصية من خواص هاد ال R.V
{خاصية من خصائصه الاحصائية}
اسمها ↓ probability density function

Fig ①



linear relation



$$Pr(x_i \leq X \leq x_j) = \int_{x_i}^{x_j} f_X(x)\, dx$$

← The 1st operator to Find Pr of R.V

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← other operator

Expectation :-

$$E\{\alpha\} \triangleq \int_{-\infty}^{\infty} \alpha \cdot f_x(x)\, dx$$

$(-\infty \rightarrow \infty)$ range نضيف كل ال

α related to that Random variable

RV سبب بنا نكون كارحين بأنهانا إله

$F_X(X)$ ← X سمينا اسمه

$F_y(y)$ إذا كان y بكون كنا

~~* exam~~

* physical quantities :-

① mean $\mu_X = E\{X\}$

mean to Fig ①

$$\mu_X = \int_{x_1}^{x_2} x \frac{1}{x_2 - x_1}\, dx = \frac{x^2}{2} \frac{1}{x_2 - x_1}$$

$f_x(x)$

$$= \frac{x_2^2 - x_1^2}{2(x_2 - x_1)} = \frac{x_2 + x_2}{2} \quad \text{---} \boxed{1}$$

ALADIB.net

/ /

$$E\{\underbrace{X^2}_{\text{Total Power}}\} = \int_{x_2}^{x_1} x^2 f_x(\overset{\text{Free variable}}{x})\, dx$$

$$= \int_{x_1}^{x_2} \frac{x^2}{x_2 - x_1}\, dx = \left. \frac{x^3}{3(x_2 - x_1)} \right|_{x_1}^{x_2}$$

$$= \frac{x_2^3 - x_1^3}{3(x_2 - x_1)} \quad \text{--- } \boxed{2}$$

from $\boxed{1}$ ∴ This is the statistical mean

لوكانوا هذه الارقام not uniform

رح يتغير مش رح يطلع الجواب حاصل جمعهم / 2

عادة إحنا متعامل بحياتنا اليومية مع numerical avarage

But the statistical avarage is the expectation of X (μ_X)

∴ mean أيهما بيعطي وصف أفضل لل statistics ال أفضل

So, we called it the true avarage

في حالة ال uniform رح يطلعوا متطابقات ما في فرق

From [2] certain x موعليانقش
outcome
of the R.V

واحنا حكينا بالبداية ال R.V اللي رح نتعامل
معها هي mathmatical {numerical Random var.}
عشان هيك صفتدر نجمعها أو نطرحها
ونطبق عليها قوانين الرياضيات اللي صفروضا -
(التربيع ، تكعيب ، ...)

* The expectation usually give us a number

* what is the expectation of something square ?
Something $\xrightarrow{\text{عبارة عن}}$ number $\xrightarrow{\text{وهاد}}$ Voltage
الرقم ممكن يمثل

So, the expectation of voltage square is Power

② σ_x^2 Varience → قديه تتمركز الـ signal حول المركز تبعها

$$\sigma_x^2 = E\{(x - \mu_x)\}$$

$$mse = E\{(x - x_e)^2\}$$

الـ mean square error شبيه بالـVarience

إذا افترضت σ_x^2 = صفر

then التشتت = صفر

linear operator ← $E\{\alpha\} = \int \alpha f_x(x)\,dx$

$$\alpha \underbrace{\int f_x(x)\,dx}_{1}$$

$$E\{\alpha\} = \alpha$$

$$E\{(x-\mu_x)^2\} = E\{x^2 - 2\mu_x x + \mu_x^2\}$$

$$= E\{x^2\} - 2\mu_x E\{x\} + E\{\mu_x^2\}$$

$$= E\{x^2\} - 2\mu_x^2 + \mu_x^2$$

$$\boxed{\sigma^2 = E\{x^2\} - \mu_x^2}$$

$$E\{x^2\} = \underset{\substack{\downarrow \\ \text{AC power} \\ V_{RMS}^2}}{\sigma_x^2} + \underset{\substack{\downarrow \\ \text{d.c power} \\ V_{dc}^2}}{\mu_x^2}$$

/ /

* notes

أنا بشوف هل ال R.V

اللي معي ال range تبعه قليل بحيث

معناها ال varience تبعه رح يكون قليل

اذا كان ال range كبير ال variene كبير

from this concept

↳ the deterministic number

كم ال Varience تبعها ؟؟

zero

فبقدر احاطهم وكأنهم Random variables

∞ كلما قل ال varience

كلما صارت هذه ال quantity

أقرب لل deterministic كلما كانت ال error أقل

R.V ← لما يكون signal بقدر أعرف من

ال statistics تبعته ← True RMS values

&

True dc value

[3] n^{th} moment → subscript or super script

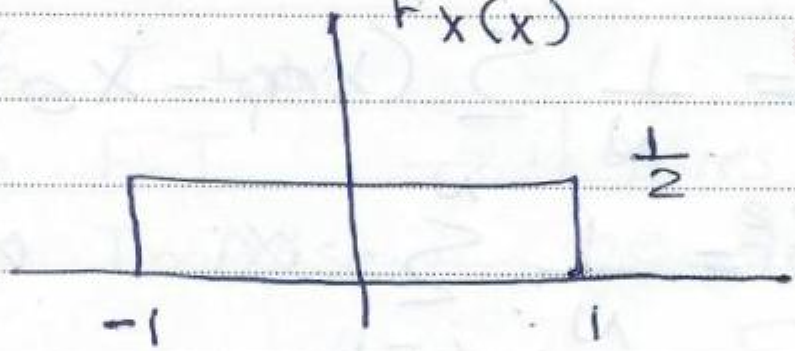
$$m_n \triangleq E\{X^n\}$$

$$= \int_{-\infty}^{\infty} x^n f_x(x)\, dx$$

for uniform R.V $X \in (-1, 1)$

لـ من هاي الجملة

نكتفي بكل المعلومات



→ special case symmetrical around y-axis

find the n^{th} momment

$$m_n = \int_{-1}^{1} \frac{x^n}{2} dx = \left.\frac{x^{n+1}}{2(n+1)}\right|_{-1}^{1}$$

$$= \frac{1}{2(n+1)} - \frac{(-1)^{n+1}}{2(n+1)} = \begin{cases} 0 & n \text{ odd} \\ \frac{1}{n+1} & n \text{ even} \end{cases}$$

second moment → $m_2 = \sigma_X^2 + \mu_X^2$

لو غيرت الرسمة مش بيطلع الجواب اللي طلع معنا صح

* لو كانت $f_X(x)$ not uniform ← مابنعرف شي بس التكامل بصير أصعب .

في هاد المثال $\sigma_X^2 = \frac{1}{3}$

The 3rd momment is zero

هادا ال pdf ← هل له معنى لو أخذنة

ال F.T ؟؟

Yes, we called it spectrum

$$mse = \frac{1}{N} \sum_{N} (x_{a_i} - x_e)^2$$

$$\epsilon^2 = \frac{1}{N} \sum_{i=1}^{N} \alpha_i \qquad \alpha_i \geq 0$$

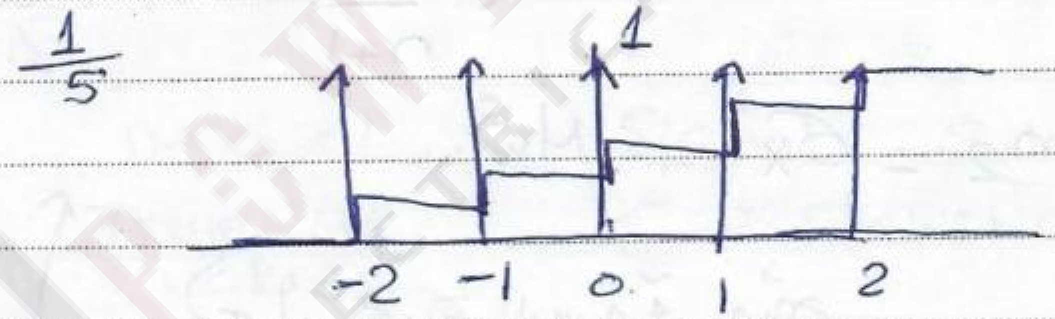
find the optimal α_i's?

الأحسن

Solution:

$$\alpha_i = \alpha_j \quad \forall i, j$$

b- discrete Uniform



$$\frac{1}{5}\delta(X+2)$$

$$\mu_X = E\{X\} = \sum_{-2}^{+2} \frac{1}{5} X = 0$$

* varience is the same
* the moment has changed

/ /

$$\int_{-\infty}^{\infty} \delta(x - x_0) \cdot g(x)\, dx = \begin{cases} g(x_0) \\ 0 \rightarrow \text{لوكانت الفترة كلها خارج ال } x_0 \end{cases}$$

when I take the F.T → it gives
to Impulse me describe
F.T

/ /

From [2] certain وهلأ بنقش X outcome of the R.V

واحنا حكينا بالبداية ال R.V اللي رح نتعامل معها هي mathmatical (numerical Random var.)

كشان هيك منقدر نجمعها أو نطرحها ونطبق عليها قوانين الرياضيات اللي منعرفها - (الترتيع ، تكعيب ، ...)

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σ_x^2 → AC power V_{RMS}^2

μ_x^2 → d.c Power V_{dc}^2

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أقل

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& True dc value

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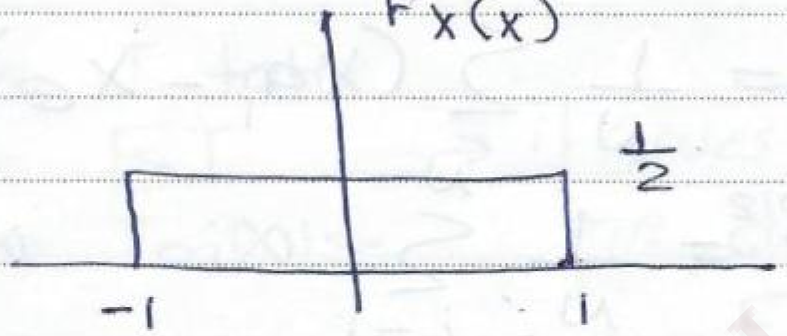
له من هاي الجملة

نكتفي بكل المعلومات



→ Special case symmtrical around Y-axis

find the n^{th} momment

$$m_n = \int_{-1}^{1} \frac{x^n}{2} dx = \left. \frac{x^{n+1}}{2(n+1)} \right|_{-1}^{1}$$

$$= \frac{1}{2(n+1)} - \frac{(-1)^{n+1}}{2(n+1)} = \begin{cases} 0 & n \text{ odd} \\ \frac{1}{n+1} & n \text{ even} \end{cases}$$

second moment → $m_2 = \sigma_X^2 + \mu_X^2$

لو غيرت الرسمة منه

بطلع الجواب اللي طلع

معنا صح

* لو كانت $f_X(x)$ not uniform ← ما بتغير شي بس التكامل بصير أصعب.

في هاد المثال $\sigma_X^2 = \frac{1}{3}$

The 3rd momment is zero

هادا ال pdf ← هل له معنى لو أخذنا ال F.T ؟؟

Yes, we called it spectrum

$$mse = \frac{1}{N} \sum (x_{a_1} - x_e)^2$$

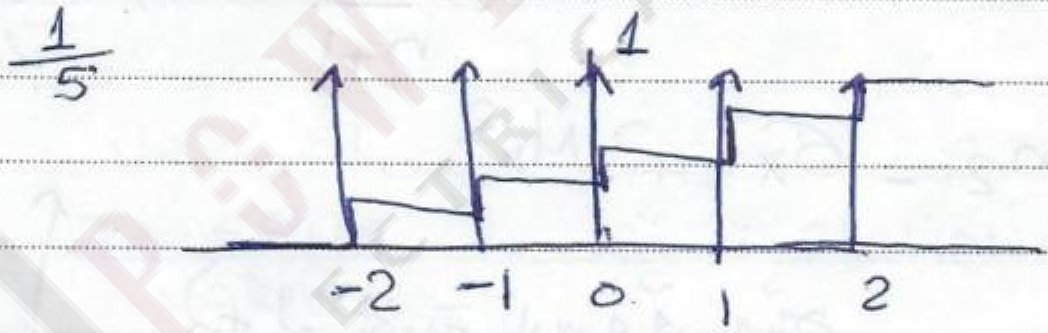
$$\epsilon^2 = \frac{1}{N} \sum_{i=1}^{N} \alpha_i \qquad \alpha_i \geq 0$$

find the optimal α_i's?
الأحسن

Solution:

$$\alpha_i = \alpha_j \quad \forall i, j$$

b- discrete Uniform



$$\frac{1}{5} \delta(X+2)$$

$$M_X = E\{X\} = \sum_{-2}^{+2} \frac{1}{5} X = 0$$

* Varience is the same
* the momment has changed

$$\int_{-\infty}^{\infty} \delta(x-x_0)\cdot g(x)\,dx = \begin{cases} g(x_0) \\ 0 \rightarrow \text{لوكانت الفترة كلها خارج ال } x_0 \end{cases}$$

when I take the F.T to Impulse $\rightarrow$ it gives me describe F.T

فرع اساسي من ال data compression

الحفنيس

Lec 5 　　　　　　2014/ 6 / 26

1 _ Random Variable

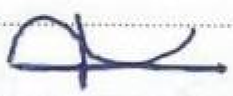
2 _ Exponential R.V

uniform → سهل → application on data compression

more uniform

الأفضل

A/D convertor



$V(t)$ — $\otimes$ — $\boxed{Q}$ — X 2^m output

- analog ال
- to ∞ levels
- cont. in time & Almpt.

$\uparrow$

$\sum \delta(t - t_i)$

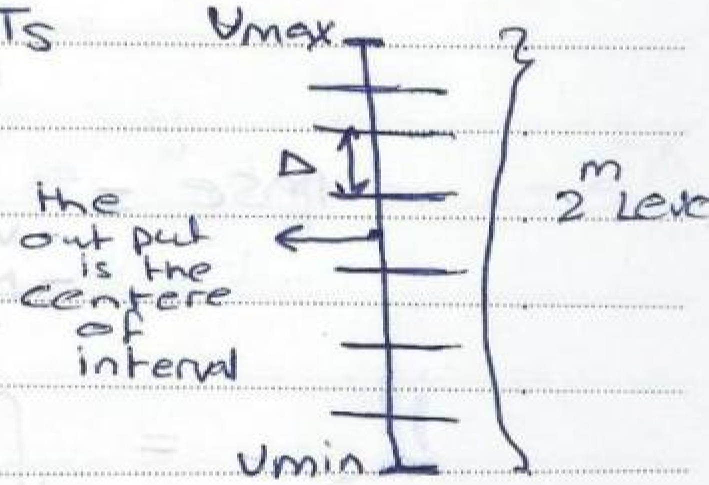
$= nT_s$

↳ descrite uniform

time axis ال بحول (*)

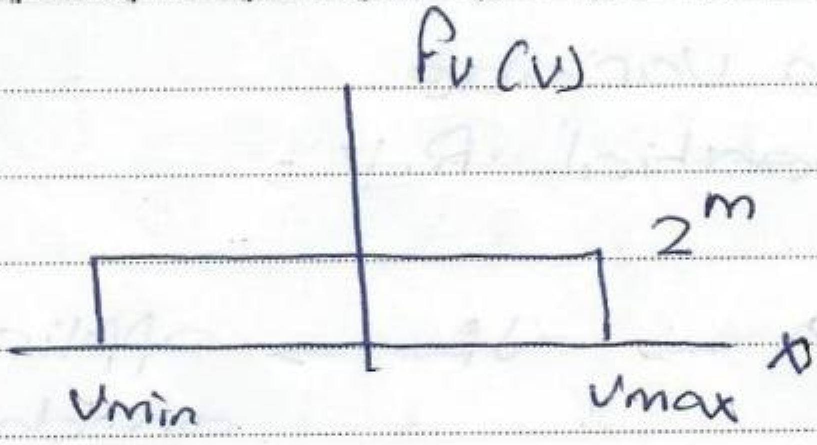
descrite ال axis

← بوخذ ال Signal وبقسمها equal intervals



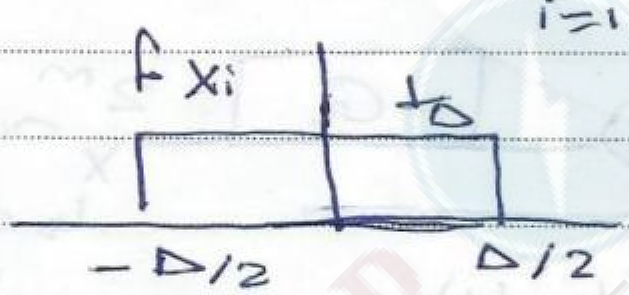
if signal random → out put random

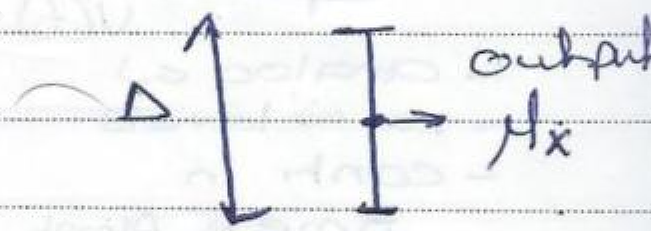
$f_X(x)$



$$Pr\,(f_X(x)) = \frac{1}{V_{max} - V_{min}}$$

$$f_X(x) = \sum_{i=1}^{2^m} \frac{1}{2m}\,\delta(x - x_i)$$





$$mse = \int_{-\Delta/2}^{\Delta/2} x^2 f_X(x)\,dx$$

$$= \int_{-\Delta/2}^{\Delta/2} \frac{x^2}{\Delta}\,dx = \left.\frac{x^3}{3\Delta}\right|_{-\Delta/2}^{\Delta/2} = \frac{\Delta^2}{12}$$

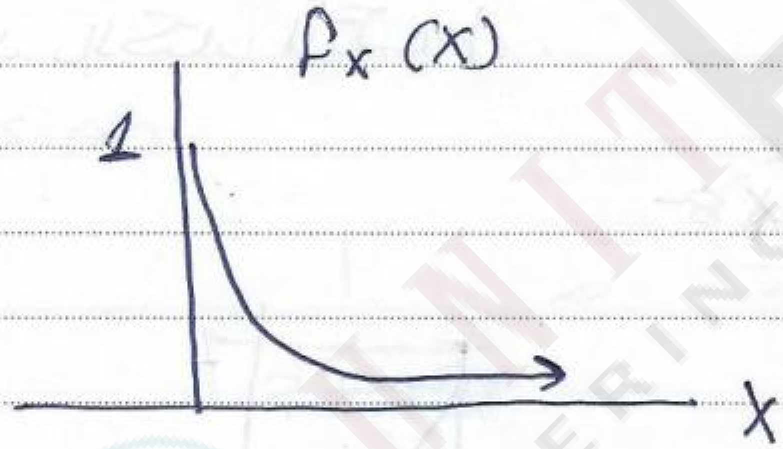
هاي النتيجة ما تنسوها للمستقبل

$$f_X(x) = \alpha e^{-x} u(x)$$

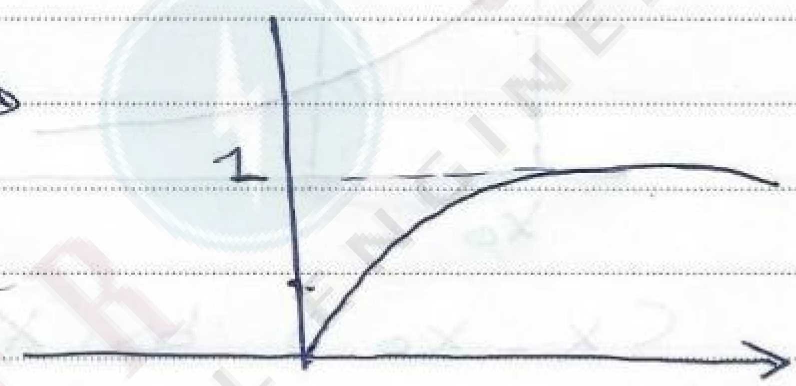
$$\int_0^{\infty} f_X(x)\,dx = 1$$

$$1 = -\alpha e^{-x} \Big|_0^{\infty} = \alpha$$

$$f_X(x) = e^{-x} u(x)$$



non decreasing function →



$$F_X(x) = \int_0^x e^{-y} \, dy = 1 - e^{-x}$$

(*) Applications:

→ Internal usage
→ Arrival rate of packet Transmission
ترسل وتستقبل

I can find Mean & varience

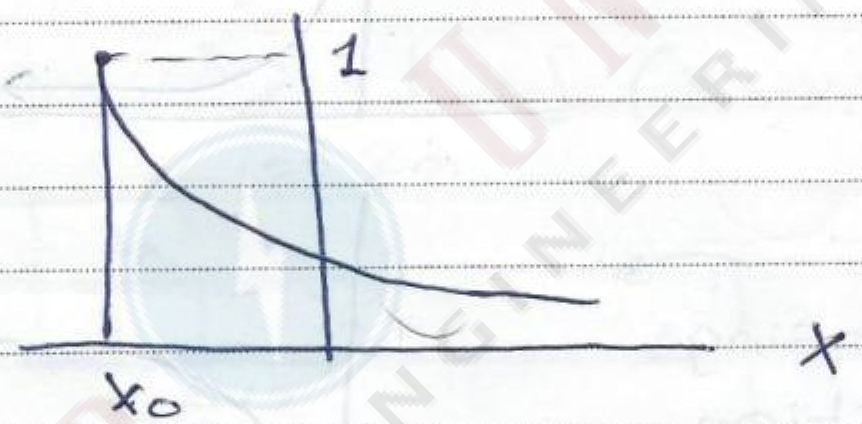
$$M_x = \int_0^{\infty} x e^{-x} \, dx$$

$$\sigma^2 = E\{(X-\mu_x)^2\} = m_2 - \mu_x^2$$

$$m_2 = \int_0^\infty x^2 e^{-x}\, dx$$

موجود بالكتاب التكامل.

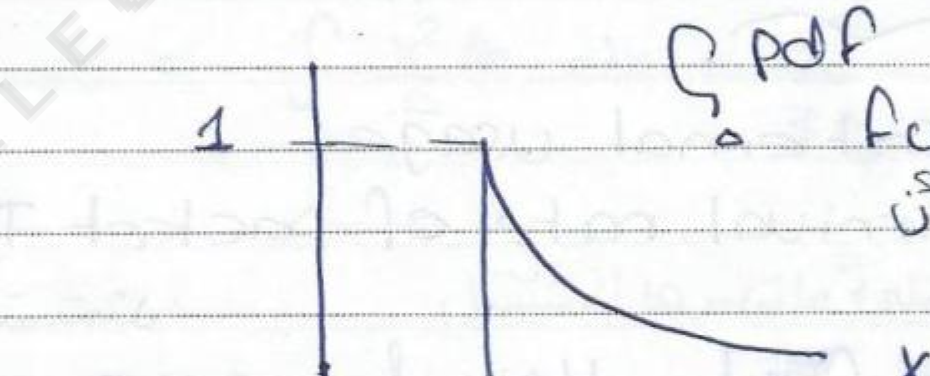
Ex8-



محل كل x بحط $(x - x_0)$

هي سالبة متغير موجبة

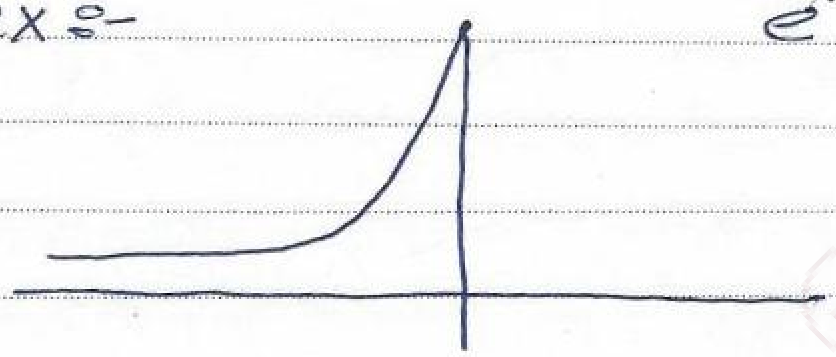
$$f_X(x) = e^{-(x+x_0)}\, u(x-x_0)$$



هل هذا pdf ؟
function.
نعم ، لأن
المساحة
تحت
المنحنى = 1

ex:-

$e^{x}\, u(-x)$



الفكرة انه
المساحة =1

3 _ Gaussian

$$f_X(x) = \frac{1}{\sqrt{2\pi\, \sigma_x}}\, e^{-\frac{(x-\mu_x)^2}{2\,\sigma_x^2}}$$

$\Downarrow$

حفظ

POWER UNIT

Prob Notebook
Dr.Jamal Rahal

By . Farah Abussamin

بأفكارنا_نُبدع

second week

الاثنين

Lec # 6 2014 / 6 / 30

cont.

$\boxed{3}$ Gaussian R.V → pdf f_n

مشهور ← يحدد نوع ال R.V

← كل شي في الطبيعة إذا في إله Random behavior
هو gaussian

Noise → The Random movement of electronics in the material (as air).

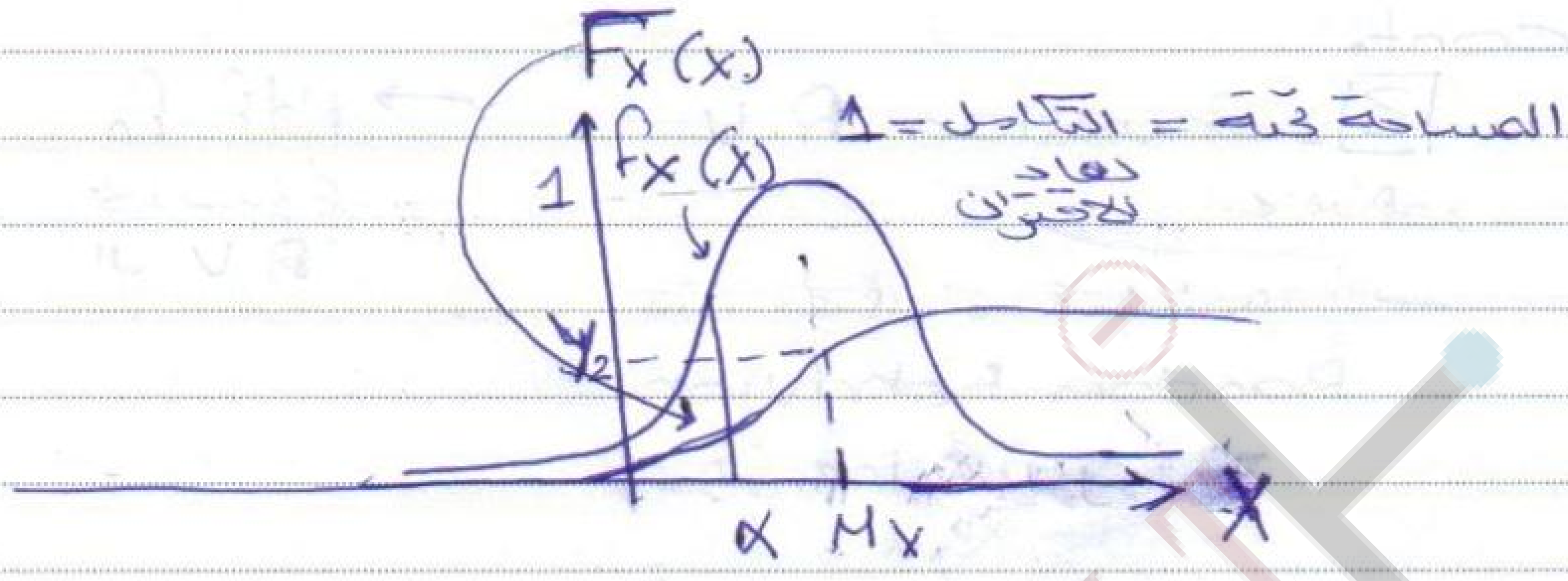
Noise (Thermal Noise)
Vm(t) = noise process
amplitude is gaussian R.V .

Gaussian R.V ≡ Normal distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{\frac{-(x-\mu_X)^2}{2\sigma_X^2}}$$

$N(\sigma_X^2, \mu_X)$ ← تأكدي من الكتاب

$N(3, -7)$ مثال ←



$$F_X(\alpha) = Pr(X \le \alpha) = \int_{-\infty}^{\alpha} f_X(x)\,dx$$

طريقة لحل التكامل من الجدول

$$Q(\alpha) \triangleq \int_{-\infty}^{\alpha} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\,dt = N(1,0)$$

special case

→ normal distribution when $M_X = 0$ & $\sigma = 1$

$$F_X(\alpha) = \int_{-\infty}^{\alpha} \frac{1}{\sqrt{2\pi}\,\sigma_X} e^{-\frac{(x-M_X)^2}{2\sigma_X^2}}\,dx$$

let $u = \frac{X - M_X}{\sigma_X}$

$$du = \frac{dx}{\sigma_X}$$

$$F_X(\alpha) = \int_{-\infty}^{\frac{\alpha - M_X}{\sigma_X}} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}$$

Usually nois is zero mean noise

Ex: Find $F_x(0.3)$?

given $\sigma_x^2 = 3$, $M_x = 0$

$\underset{\text{Standard deviation}}{\sigma}$

$$F_x(0.3) = Q\left(\frac{0.3-0}{3}\right)$$

$$= Q(0.1)$$

من الجدول $= 0.46017$

approximation

$$Q(\alpha) \cong \left[\frac{1}{(1-a)\alpha + a\sqrt{\alpha^2+b^2}}\right]\frac{e^{-\frac{\alpha^2}{2}}}{\sqrt{2\pi}}$$

↑ حفظ *

$a = \frac{1}{\pi}$, $b = 2\pi$

*يكون أدق كلما كانت ال arguments (α)

تحت ال Q اكبر

$Q(0) = \frac{1}{2}$ → $-\infty \to \infty$

نصف المساحة

$$Q(-\beta) = 1 - Q(\beta)$$

* Remember

σ_x related to ac power

M_x related to dc power

$$m_n = E\{x^n\}$$

ما يقدر أحد ← $$= \int_{-\infty}^{\infty} x^n \cdot f_x(x)\, dx$$
Integral الـ

So

في كتير Function
تس الطلوع
مو مطلوب → moment generating function $\phi(x)$

$$3\text{rd moment} = 0$$ من أهم خصائص
gaussian
كل احصائياته
to the 2nd order

$\boxed{HW\#1}$

$$g(t) = \frac{1}{\sigma_t\sqrt{2\pi}}\, e^{-\frac{(t - M_t)^2}{2\sigma_x^2}}$$

رح يطلع
exactly
the same
لكن بعض شغلات
صغيرة .

Find F.T $\{g(t)\} = G(f)$

4- Laplacian

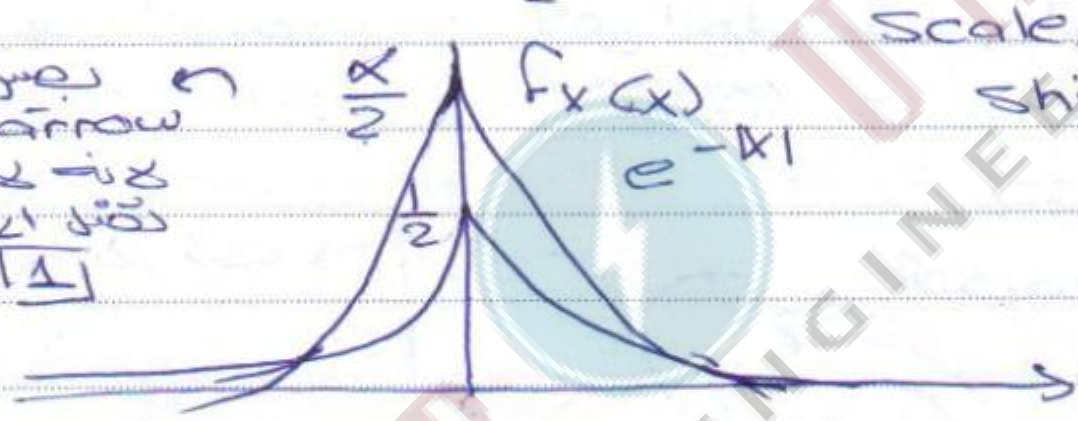
$$f_X(x) = \frac{1}{2} e^{-|x|}$$

$$f_X(x) = \frac{\alpha}{2} e^{-\alpha|x|}$$ → this is general

$$f_X(x) = \frac{\alpha}{2} e^{-\alpha|x-\mu_x|}$$ → more general

بس كثرة الـ scale وكمان shift

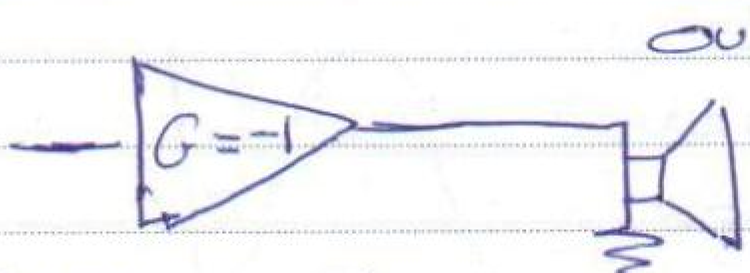
بس narrow لانه لازم تمثل المساحة $\boxed{1}$



* Video signals & Audio signals → guassian ليس لكن شبيه فيه

double sided ← exponantial إنه بس مش square

Audio



مسمع نفس الكلام لانه اشارة الصوت symmtrical around zero

الـ phase ليس له أهمية باشارة الصوت لكنه مهم باشارة الصورة

Phase الاشارة الي فيها
3rd order منهتم بال
$\alpha =$ مقلوب
ال x اس

الثلاثاء
Lec # 7

2014 / 7 / 1

Band width ← لما يزيد الـ Correlation ← يقل الـ

Laplacian :-

$$f_X(x) = \frac{\alpha}{2} e^{-\alpha |X - M_X|}$$

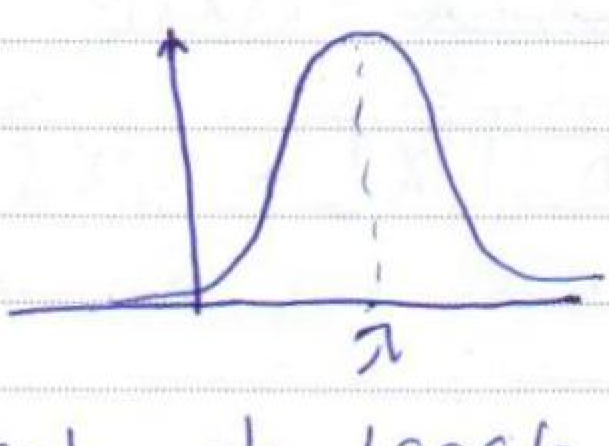
→ Audio signals
→ Video signals

For outdoor
↳ * Rician , Rayligh , log normal
الأشهر شيء , gaussian

mean = SNR
Signal noise ratio

For indoor wirless
↳ * Nakag— , ~~Erlang~~ weibul

* Poisson , Er Lang
descrite يعني , traffic
→ voice communication
خلوي أو أرضي أو سير السيارات



حزمة من الأوامر →

Packet at least zero

↳ exp

Lognormal $\quad y = \log |x|$

Random variables :-

صوت الضوضاء

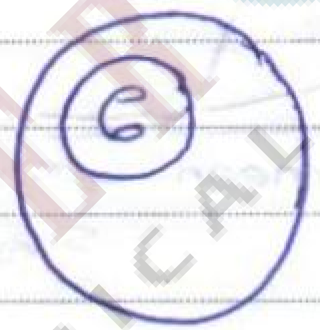
$X \rightarrow f_X(x) \;,\; F_X(x)$

$\rightarrow \underbrace{P_r(x_0 \le X \le x_1)} = F_X(x_1) - F_X(x_0)$

This ~~is~~ represent subset of the total outcome set

C event

Pr هي الـ
تمثل الكثافة
أو المساحة
تحت هنا
الـevents



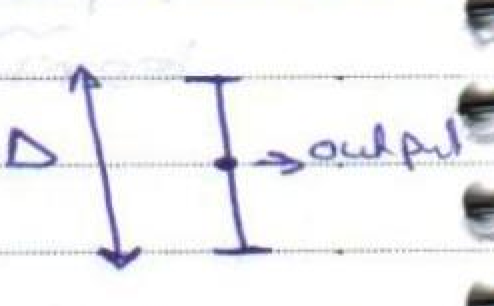
Range

$$P_r(C) = \int_{x_0}^{x_1} f_X(x)\, dx$$

Local mean :-

$$M_x^{l} = \int_{x_0}^{x_1} x\, f_X(x)\, dx$$

أبسط مثال
A to D convertor



$$m_1 = E\{x\} = \mu_x$$

Linear operator $\leftarrow$ $E\{g(x)\} \overset{?}{=} g(\mu_x)$ $\rightarrow$ yes, if $g(x)$ is linear

$$\int \frac{dF(x)}{dx}\,dx \overset{?}{=} \frac{d}{dx}\int F(x)\,dx$$ $\rightarrow$ yes

$$E\{g(x)\} = \int_{-\infty}^{\infty} g(x)\, f_x(x)\,dx$$

* Ex:-

$$g(x) = a + bx$$

$$E\{g(x)\} = \int (a+bx) f_x\,dx$$

$$= a + b\mu_x$$

if $g(x)$ is linear

$$E\{g(x)\} = g(\mu_x)$$

* Ex:-

$$g(x) = x^2$$

$$E\{x^2\} = \int x^2 f_x\,dx = m_2 \neq \mu_x^2$$

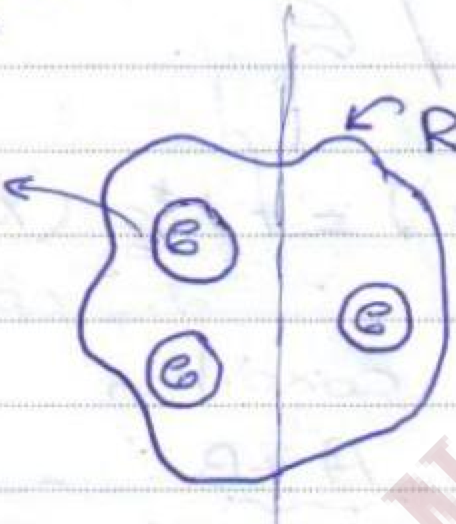
non linear.

الاربعاء

Lec # 8 2014 / 7 / 2

* events :-

Single R.V event



Mathmatical & Logical applied to the outcome Random variable

$x > 3$

$x_1 \leq x \leq x_2$

$x_1 \leq x$ & $x \geq x_2$ overlap

$x_1 \leq x$ or $x \geq x_3$ unioun

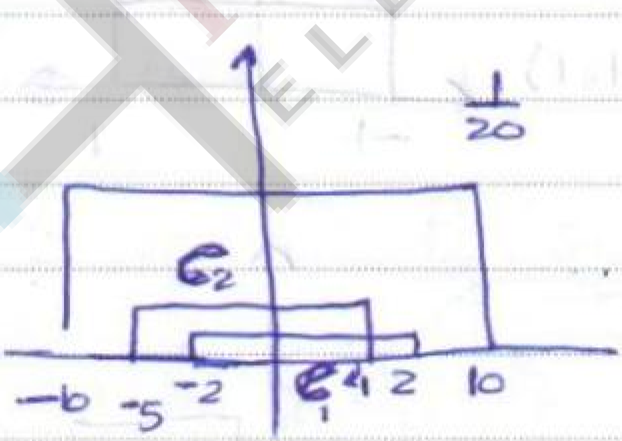
مش شرط at same point

two Random var x, y

$$P_c(C) = \int_C f_x(x)\,dx$$

independent
↳ no intersection
dependent
↳ intersection

Ex :- Uniform R.V $x \in (-10, 10)$ find the prob. of $-2 \leq x \leq +2$ OR $-3 \leq x \leq 1$



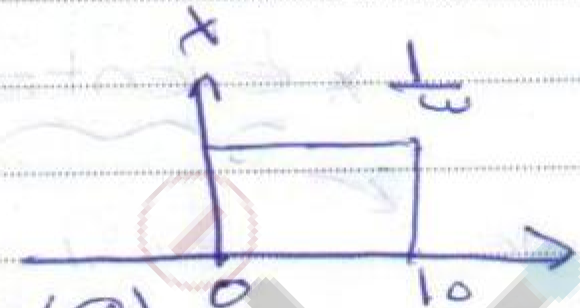
$= Pr(\epsilon_1 \text{ or } \epsilon_2) = \frac{5}{20}$

$Pr(x \leq -5)$ or $Pr(x \geq 5)$

$$\int_{-10}^{5} f + \int_{5}^{10}$$

بحرف R.V جديد

Conditioned R.V



$$C_1 / C_0$$

$$f_{x'}(x') = f_{x^*C_2}(x/C_2)$$

condition

$$A \cup B = \underbrace{A+B} - A \cap B$$

$$P_r(A \cup B) = P_r(A+B) - P_r(A \cap B)$$

$$P_r(A \cup B) = P_r(A+B) - P_r(A,B) \rightarrow \text{joint event}$$

$$A/B = \frac{A \cap B}{B} \quad \text{normalized to } B$$

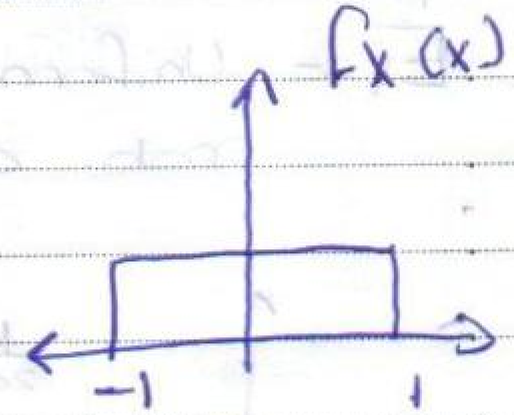
$$P_r(A/B) = \frac{P_r(A \cap B)}{P_r(B)} \rightarrow \text{Joint event}$$

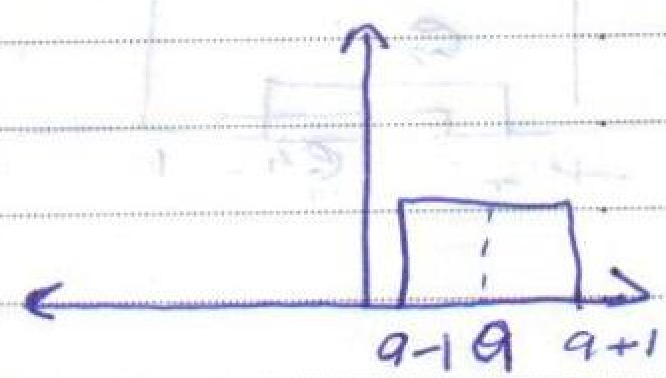
$$f_{A,B}(A,B) = f_1(A/B) \cdot f(B)$$

Ex :- $Y = \underset{\text{R.V}}{X} + \underset{\text{constant}}{a}$

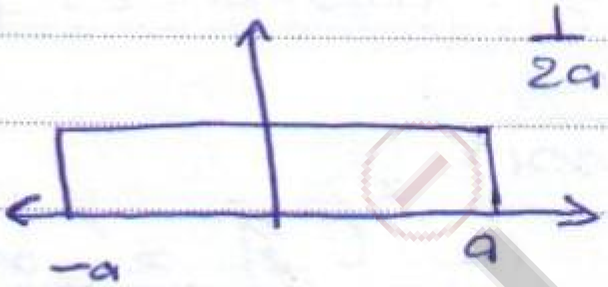
$$f_y(y) = f_x(x/a)$$

x :- uniform $\in (-1,1)$

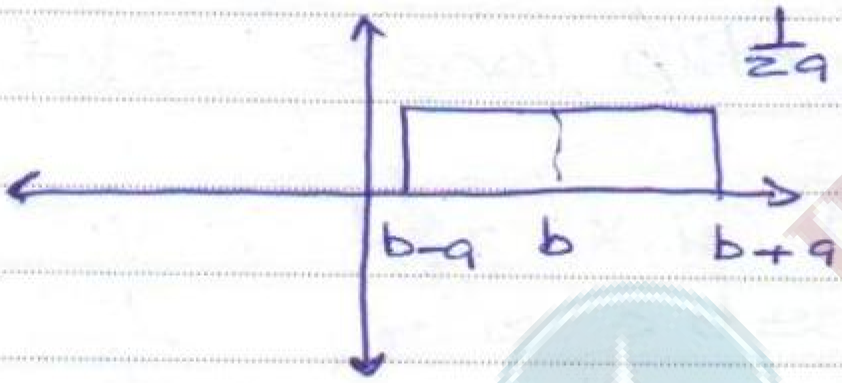




Ex:- $y = a.X$



$Z = ax + b$



$$P_r(X(A)) = \frac{P_r(X, A)}{P_r(A)}$$

$$P_r(X, A) = P_r(X(A)) \cdot P_r(A)$$

$A_1, A_2, A_3, \ldots A_N$

total Area $= \sum_{i=1}^{N} A_i$

$$\sum_{i=1}^{N} P_r(X, A_i) = P_r(X) \Rightarrow \text{total } P_r \text{ low}$$

$$P_r(t) = \sum_{i=1}^{N} P_r(X/A_i) \cdot P_r(A_i)$$

↳ Total probability low

الكفيس

Lec # 9 2014/7/3

- noise + constant → constant deteministic

معلوم 100%

لكن ال R.V غير معلوم 100%

* Ex: Signal with noise

Let X is $N(\sigma_x^2, 0)$

Y is a decistion variable

Find the error rate:

$\underbrace{\text{error event}}$

↓

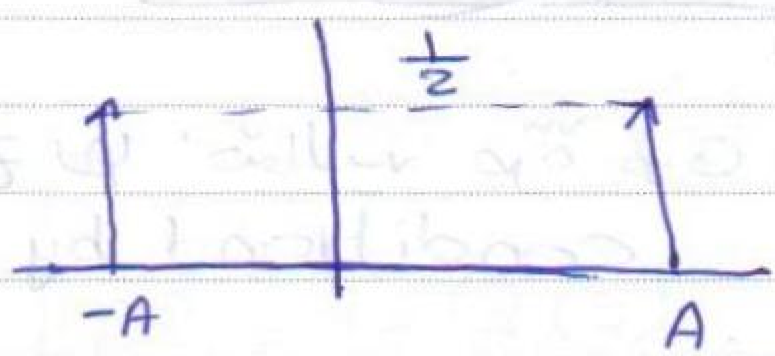
prob. of error

variable $\xrightarrow{\text{هل هو}}$ له قيمة محددة -

أي discrete -

أو cont. -

Let $y \in \{-A, +A\}$



we estimate y from z such that

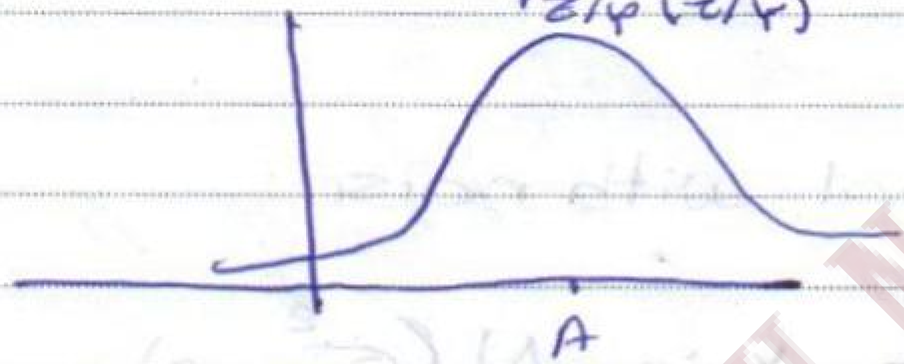
$z = y + x$

y → decistion, x → noise

→ if $y = +A$ what is the prob?

$f_{Z/Y}(Z/Y)\Big|_{Y=+A}$ → this will be normally distrebuted (gaussian) shifted around $+A$

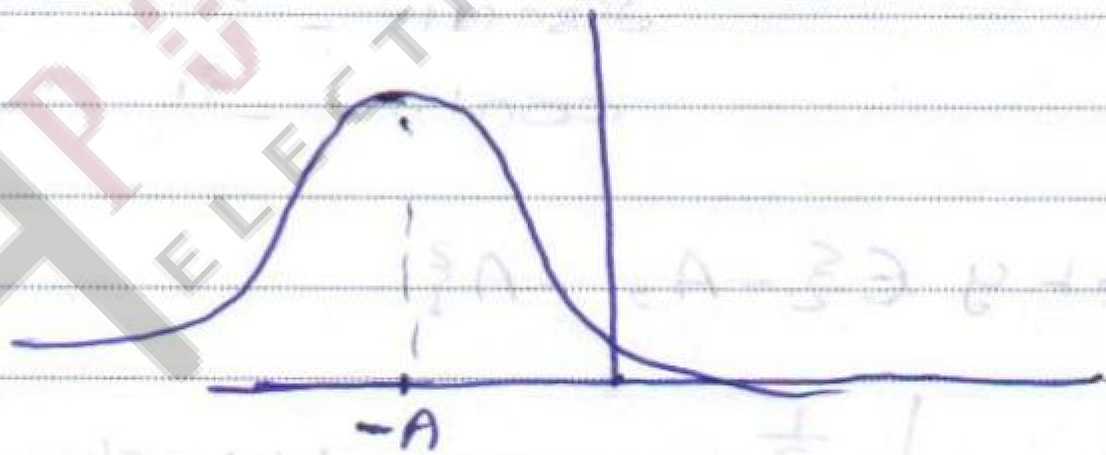
$f_{Z/Y}(Z/Y)$



$$f_{Z/Y}(Z/Y)\Big|_{Y=+A} = \frac{1}{\sigma_X\sqrt{2\pi}} e^{-\frac{(X-A)^2}{2\sigma_X^2}}$$

→ if $Y = -A$

$$f_{Z/Y}(Z/Y)\Big|_{Y=-A} = \frac{1}{\sigma_X^2\sqrt{2\pi}} e^{-\frac{(X+A)^2}{2\sigma_X^2}}$$



(أصبح Z لها احتمالين مرة فوق ومرة تحت
conditiond by y)

/ /

→ error event when $y = +A$
and x from $(-\infty$ to $-A)$

OR

when $y = -A$
and x from $(+A$ to $\infty)$

$$\cancel{P_{re} = \int_{-\infty}^{-A} f_X(x)\,dx = P_r(y=A) + \int_{+A}^{\infty}}$$

$$P_{re} = \int_{-\infty}^{-A} f_X(x)\,dx \cdot Pr(y=A) + \int_{+A}^{\infty} f_X(x)\,dx \cdot P_r(y=-A)$$

$$= \int_{-\infty}^{-A} f_X(x)\,dx = Q(-A/\sigma_X)$$

$$= 1 - Q(A/\sigma_X)$$

BER نفسها ↑

Bit error rate

* when $A = 5v$, $\sigma_X^2 = 1$

$a = \frac{1}{\pi}$
$b = 2\pi$
$\alpha = 5$

$BER = 1 - Q(5)$

$$= \left[\frac{1}{\alpha(1-a) + a\sqrt{\alpha^2 + b^2}}\right] \frac{e^{-\frac{\alpha^2}{2}}}{\sqrt{2\pi}} = 2.49 \times 10^{-7}$$

~~2.86~~ *

$$f(x/y) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

$f_Y(y)$ → constant
given لانه

* $f_{XY}(x,y)$ joint pdf
يعني 2R.V

Temprcture مثلا
مع عمر الاسنان

Probability

NoteBook

Dr. Jmal Rahal

By: Farah Abu Alssamin

بأفكارنا_نُبدع

3rd week

Lec # 10 2014 / 7 / 6

Two Random variables

Joint pdf's

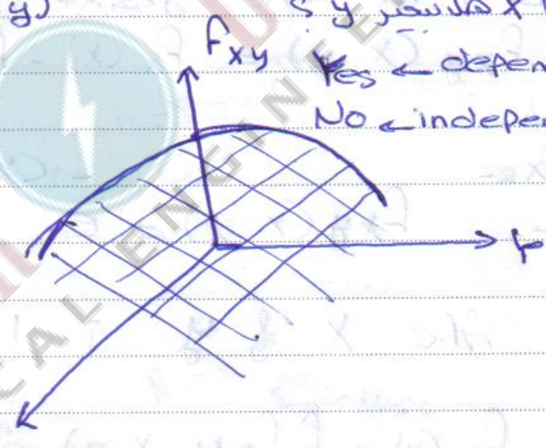
Let X & y are two random variables

$f_{xy}(x, y)$ إذا غيرنا X هل ستتغير y ؟

Yes ← dependent if

No ← independent

كأنّا منثبّت واحد ونغير الثاني → Joint pdf can be written as condition pdf function



$$f_{xy}(x, y) = f(x/y) \cdot f_y(y)$$
$$= f(y/x) \cdot f_x(x)$$

dependency → محليا إحنا ما منعرفها

Volume under the surface XY $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1$

~~Pr(X≤X₀≤X~~

$$Pr(X \le X_o , y \le y_o) = \int_{-\infty}^{y_o}\int_{-\infty}^{X_o} f(x,y)\,dx\,dy$$

$$\hookrightarrow \triangleq F_{XY}(X_o, Y_o)$$

ع‍ وأ

distrubited function

Independent RV's

$\hookrightarrow$ have the Joint pdf

$$f_{xy}(x,y) = f_x(x) \cdot f_y(y)$$

* Ex:-

$$f_{xy}(x,y) = e^{-(x+y)}\, u(x,y)$$

ARe X & y independent?

$$f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x,y)\,dy$$

$$f_y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

Marginal pdf's

$$f_x(x) = \int_0^{\infty} e^{-x} \cdot e^{-y}\,dy = e^{-x} u(x)$$

$$f_y(y) = \int_0^{\infty} e^{-x} \cdot e^{-y}\,dx = e^{-y} u(y)$$

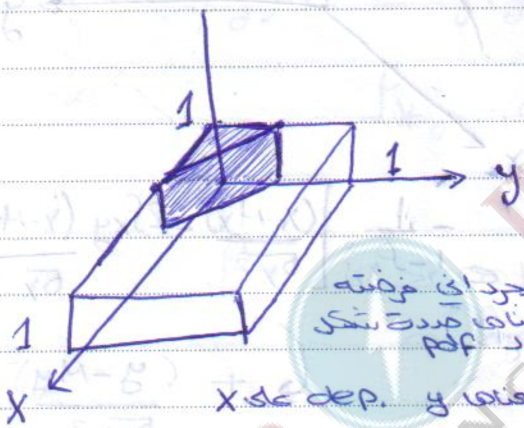
$$f_x \cdot f_y = e^{-x} e^{-y} u(x) \cdot u(y) = e^{-(x+y)} u(x,y)$$

X & y are independent

/ /

* إذا كان حاصل الضرب لا يساويها أصلاً
معناها independent .

* Ex : X, y are independent uniform
$x, y \in (0, 1)$



← بمجرد إني فرضته معناها محددة شكل الـ pdf

$$\boxed{y = ax + b}$$

← معناها y dep. على X

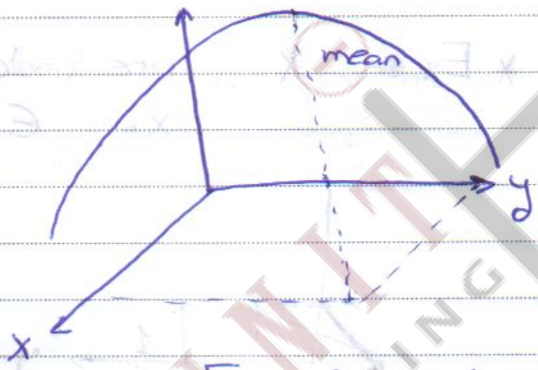
← كإني حصرتهم بـ Region معناها فرضتهم dependent

$$Pr(\underbrace{0.1 \le X \le 0.3}_{R},\ \underbrace{0.5 \le y}_{})$$

$$Pr = \iint_R f_{xy}(x, y)\, dx\, dy$$

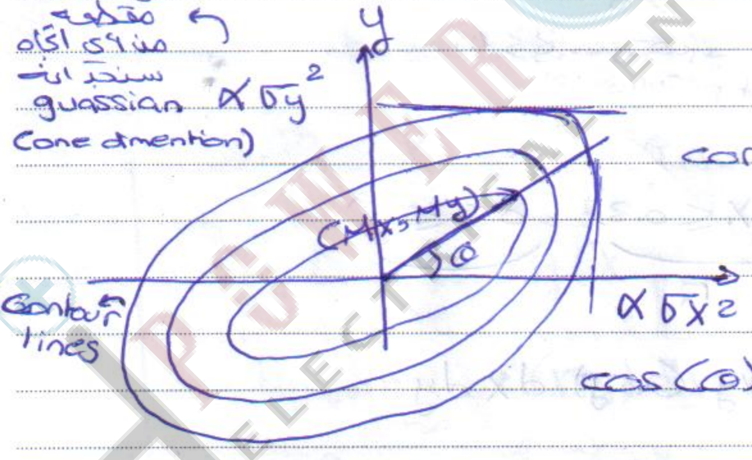
$R \in x\text{-}y$ plan

2-D Gaussian R.V



$$f_{xy}(x,y) = \frac{1}{\sqrt{\pi^2 \sigma_x^2 \sigma_y^2 (1-\rho_{xy}^2)}} e^{-\frac{1}{1-\rho_{xy}^2}\left[\frac{(x-\mu_x)^2}{\sigma_x^2} - 2\rho_{xy}\frac{(x-\mu_x)(y-\mu_y)}{\sigma_x \sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}\right]}$$

لو أخذنا مقطوعه مناوى أكاه سنجد انه guassian (one dimention)



الميل بيمثل corrolation

$\cos(\theta) \propto \rho_{xy}$

→ if the relation between x & y non_linear → usually they are not guassian

← إذا التشتت باتجاه x اكبر معناها σ_x أكبر
وكذلك بالنسبة لـ y

← الخط المستقيم اللي رسمته يمثل
new Random variables
منحنى Z.

$$\int_{-\infty}^{\infty} f(x,y)\,dx = f_y(y) \quad N(\sigma_y^2, M_y)$$

$\rho_{xy} = 0$ ⇒ Two gaussian are independent.

$\rho \equiv$ Corrolation coeffecient
معامل الارتباط

Lec # 11 2014 / 7 / 7

Random variable prediction

[1] Modling → we expect a certain Model
∴ this model should expressed in close form.

* Ex :

$$\underbrace{X = t^2 - 5t + \log(t)}_{\text{deteremonistic}}$$

$$\hat{X} = \alpha t^2 - \beta / \ln(t-1)$$

$\underbrace{X}_{\text{actual Data}}$ $\underbrace{\hat{X}}_{\text{model}}$

[2] Fit the model

How to measure the goodness of this model ? Find mse

$$mse = E\{|X - \hat{X}|^2\}$$

$$E\{|X - \underbrace{t^2 + 5t - \log(t)}_{\text{model}}|^2\}$$

→ minimizing the error

له بنشتق ونساوي

بالصفر فنوجد قيمة $\bar{E}$ * لما كان Linear

بنعوضها في mse لـ   بصير أقل minimiz

→ Find normalized mse

$$\text{normalized error} = \frac{\bar{\epsilon}^2}{\sigma_x^2}$$

$$\boxed{y(x) = ax + b}$$

الـ mean الـ First order moment

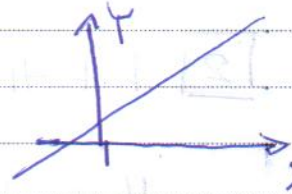
والـ Varience الـ 2nd order moment

↳ from gaussian distributed we cannot predict any higher

يمثل الـ Slope ولا يساويه

ρ = corrolation coeffecient

$$\rho_{xy} = \frac{E\{xy\}}{\sigma_x \sigma_y}$$



normalized

عشان مايزيد سرعته عن ال 1 وماينقص عن -1

$$|\rho_{xy}| \leq 1$$

/ /

if $\rho_{xy} = +1$ or -1 → They are exactly correlated Linearly dependent $y = x$ or $y = -x$

if $\rho_{xy} = 0$ → Joint pdf يصبح حاصل ضرب ال Marginals pdf
X & y are uncorrelated
it means that they are independent

if $\mu_x = 0$ ⇩

if $\mu_x \neq 0$
دحتطلع معي معادلة ثانية

$$\rho_{xy} = \frac{E\{x[ax+b]\}}{\sigma_x \sigma_y}$$

$$E\{ax^2 + bx\} = \rho_{xy}\, \sigma_x \sigma_y$$

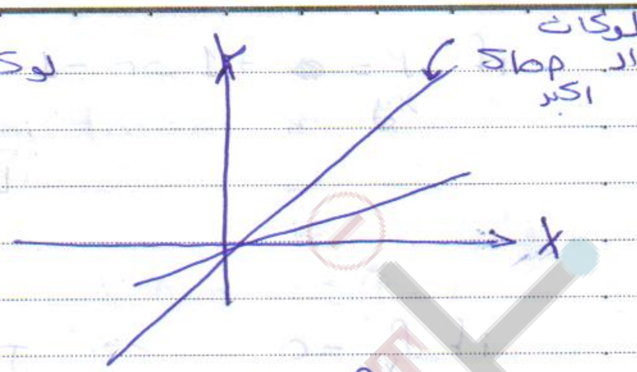
$$aE\{x^2\} + bE\{x\} = \rho_{xy}\, \sigma_x \sigma_y$$

$$a\sigma_x^2 = \rho_{xy}\, \sigma_x \sigma_y$$

$$\boxed{a = \rho_{xy}\left(\frac{\sigma_y}{\sigma_x}\right)}$$

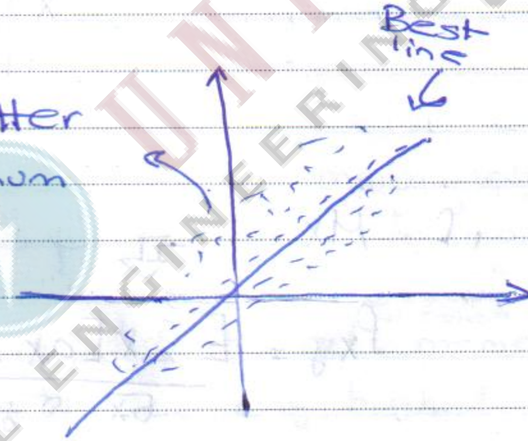
$$\boxed{\rho_{xy} = a\frac{\sigma_x}{\sigma_y}}$$

لو كان ال Slope كبير
معناها y كبيرة
معناها a كبيرة
يعني ممكن تكونه



* ρ فقط في حالة ال 1st order

we can expect in scatter a curve with minimum error



* Two R.V's

X, y

$f_{xy}(x, y)$

$f_x(x) = \int_y f_{xy}\, dy$

$f_y(y) = \int_x f_{xy}\, dx$

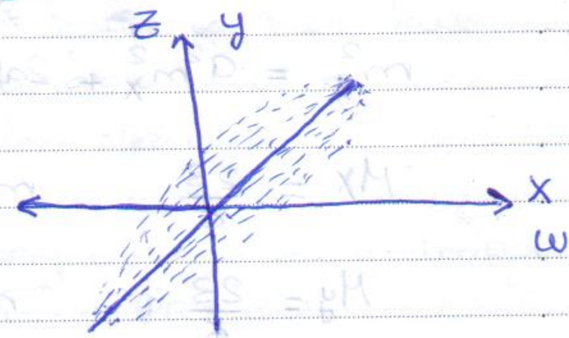
correlation just measure the 1st order

& assume

$y = ax + b$

$a \propto \rho_{xy}$

$\rho_{xy} = \dfrac{E\{x \cdot y\}}{\sigma_x \, \sigma_y}$ Normlization



هل الترابط بين x & y ← نفسه بين w & z is

No!

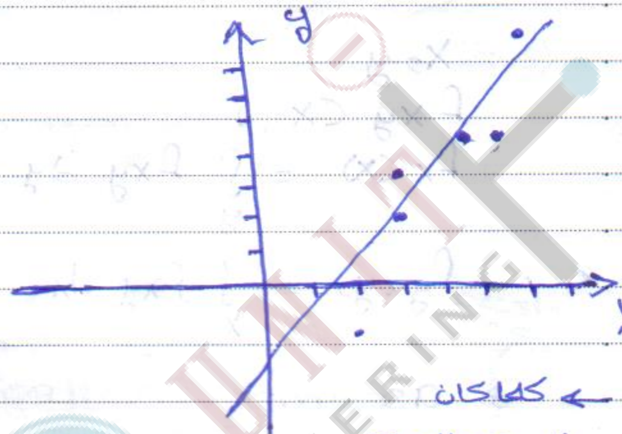
مع انه نفس ال correlation

$y = g(x) \quad P(x)$

$\downarrow$ exp $\quad \downarrow$ Poly set of points in the space

* Ex 8- given the scatter, find the relation between x & y.

x	y
2	-1
3	2
5	5
6	9
4	5
3	3



كلما كان ال Scatter ← يغطي مساحة أكبر كلما كان ال m أحق

$y = ax + b$

$M_y = E\{y\} = a M_x + b$ --- ①

$m_y^2 = E\{y^2\} = E\{(ax+b)(ax+b)\}$
$= a^2 E\{x^2\} + 2ab E\{x\} + b^2$

$m_y^2 = a^2 m_x^2 + 2ab M_x + b^2$ --- ②

$M_x = \frac{23}{6}$ $\quad m_x^2 = \frac{99}{6}$

$M_y = \frac{23}{6}$ $\quad m_y^2 = \frac{145}{6}$

Sub into ① $\quad \frac{23}{6} = a \times \frac{23}{6} + b$

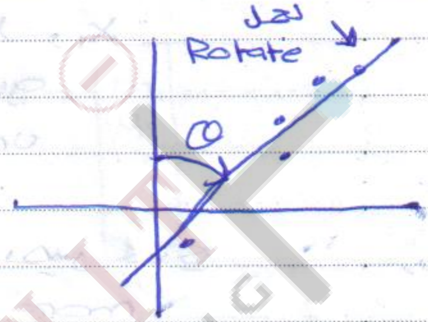
Sub into ② $\quad \frac{145}{6} = \frac{99}{6} a^2 + \frac{46}{6} \times a \left(\frac{23}{6} - \frac{23}{6}a\right) + \cancel{\left(\frac{23}{6}\right)^2}$

$+ \left(\frac{23}{6} - \frac{23}{6}a\right)^2$

take the true → $\boxed{a = 1.29}$ $\quad a = -2.29$ ✗

* Vectors of Random variables

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$



N_Dimentional R.V

$$\boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} R \end{bmatrix}}_{\text{Rotation matrix}} \begin{bmatrix} X \\ Y \end{bmatrix}}$$

* We can decorrolate ~~s~~ two Random variables using Rotation operator.

$\boxed{\text{decorrolation}}$ is a very important process

أفضل الأشياء الي ما إلها علاقة ببعض

so I can remove the depedency withen two Random variables with rotating, ecpecially if the RV's were guassian R.V

بمعنى
الي بيحصل

Identically distributed R.V's

$x_1, x_2, x_3, \dots x_n$

مثلا كلهم guassian

uniform " "

كلما زاد عدد ال R.V بيصير عندي more accurate to the prediction

$$E\{\vec{X}\} = \vec{\mu}_X = \begin{bmatrix} \mu_{X_1} \\ \mu_{X_2} \\ \vdots \\ \mu_{X_N} \end{bmatrix}$$

$$m_X^n = E\{X^n\}$$

$$m_X^2 = E\{X X^H\}$$

→ Hermite
واحد
مش زماني

* Vector R.V's (continue)

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} \qquad M_X = \begin{bmatrix} M_{X_1} \\ M_{X_2} \\ \vdots \\ M_{X_N} \end{bmatrix}$$

transpose complex conjegate

$$C_{XX} = E\{X X^H\} \quad \text{covariance Matrix}$$

$$= \begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{bmatrix} & & \end{bmatrix} = \begin{bmatrix} X_1X_1 & X_1X_2 & \cdots & X_1X_N \\ X_2X_1 & & & \\ \vdots & & & \\ X_NX_1 & & & X_NX_N \end{bmatrix}_{N \times N}$$

$$= E\left\{ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \right\}_{N \times N}$$

$$\Downarrow$$

$$= \begin{bmatrix} \sigma_X^2 & \sigma_X^2 \rho_{12} & \cdots & \sigma_X^2 \rho_{1N} \\ & \ddots & & \\ & & \ddots & \\ & & & \sigma_X^2 \end{bmatrix}$$

assume $M_X = \vec{0}$

$$\rho_{12} = \frac{E\{X_1 X_2\}}{\sigma_X^2}$$

مشترك بطلع σ_x^2 كعامل

$$\sigma_x^2 \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & \ddots & & \\ \vdots & & \ddots & \\ \rho_{N1} & & & 1 \end{bmatrix}$$

$$\boxed{\rho_{ij} = \rho_{ji}}$$

→ If we applied certain transformation of R.V's (function of R.V's)

Special case

$$\vec{Y} = A\vec{X} + \vec{b} \quad b=0$$

$$y = ax + b \quad \text{linear Transformation}$$

(أهم ترانزفورم)

For DFT

$$A = \begin{array}{c c} & \xrightarrow{m} \\ \downarrow n & \begin{bmatrix} e^{\frac{-j2\pi nm}{N.M}} \\ \\ \end{bmatrix} \end{array}$$

If $N = M$ square transfomation

$$n = 0, 1, 2, 3, \ldots N-1$$

$$m = 0, 1, 2, \ldots M-1$$

Generally $M = N$

$$A_{4\times4} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-\frac{2\pi}{16}} & -j\frac{4\pi}{16} & \\ 1 & & & \\ 1 & & & \end{bmatrix}$$

مهمة
تعوّار

application
ecpecially
in images

$$W\text{-}H = H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix}$$

$$H_8 = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix}$$

$\vec{Y} = A\vec{X}$ is a projection transform
project $\vec{X}$ into the axis of A

→ to define the space, I need to define unit vector.

$$\vec{\phi} = \text{Eig}[A]$$

→ to decorralate $\vec{X}$ we project X into $A = \text{Eig}[C_{xx}]$

$$C_{yy} = \sigma_y \, I_{N\times N}$$

Identity Matrix

مضروب بـ σ_y to presente the energy

/ /

eigen vector $\Phi = [\Phi_1 \ \Phi_2 \ \cdots \ \Phi_N]$

كلهم متعامدين ← $a_x \quad a_y \quad a_z \quad a_4$

$\det(\Phi) = 1$

↳ orthonormal space

يعني أن كل ال vector متعامدين على بعض ← $\langle \phi_i , \phi_j \rangle = \begin{cases} 1 & i=j \\ 0 & \text{otherwise} \end{cases}$

$\phi_i^H \phi_j$ يمثل space هنا ال

Norm of the vector

مع نفسه ← (inner product)

Lec # 14

الخميس 2014/7/10

Quiz

Find the correlation coefficient between x, y where

$$f_{xy}(x,y) = \alpha e^{-(x+y-2xy)} u(x,y)$$

$$\rho_{x,y} = \frac{E\{x,y\}}{\sigma_x \sigma_y}$$

$\Downarrow$

$$E\{x,y\} = \iint_0^\infty E$$

$\rightarrow y = ax + b$

$$\rho_{xy} = a \frac{\sigma_x}{\sigma_y}$$

$$x + y - 2xy = 0 \Rightarrow y = 2xy - x$$
$$= x(2y-1)$$

$$\boxed{\frac{x}{2x-1} = y} \qquad \boxed{\frac{y}{2y-1} = x}$$

$$y = \frac{x}{2x-1} \qquad \simeq$$

$$y = ax + b$$

$x = 1$

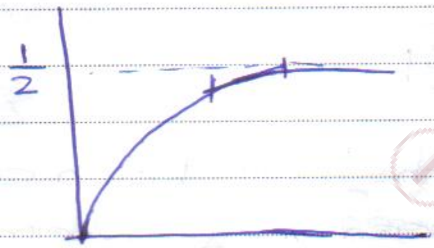
$1 = a + b$

$x = 2$

$$\frac{2}{3} = 2a + b = \cancel{2a}+1$$
$$= 2a + 1 - a$$

$$\boxed{a = -1/3}$$

we can Linearize



$$1 - e^{-\beta x} = 1 - ax$$

* Special case of function of Random variables

$$\vec{y} = A\vec{x} + \cancel{\vec{b}}$$ ما بهمني كثير

1. Forier transform
2. Cosine transform
3. W-H transform
4. K-L transform

هدول مجهود خارج تدريس عرفهم ؟

$$E\{yy^H\} = C_{yy} = \underset{\sim}{I}$$ identity matrix

K-L Transform

$$= E\{Ax \cdot (Ax)^H\}$$
$$= E\{A x x^T A^T\}$$
$$= A E\{x x^T\} A^T$$

$$\boxed{I = A C_{xx} A^T}$$

General Eigen matrix equation

$$\boxed{A C_{xx} A^T - I = 0} \quad \text{---} \; (*)$$

$$a C_{xx} a^T - \lambda = 0$$

$(B - \lambda I) = 0 \rightsquigarrow$ is a special case of equation (*)

$$\boxed{A = Eig\{C_{xx}\}}$$

H.W assume

① $C_{xx} = \begin{bmatrix} 2 & 0.9 & 0.1 \\ 0.1 & 2 & 0.1 \\ 0.1 & 0.9 & 2 \end{bmatrix}$

Find A? ~~why~~ Using matlab.

② $C_{xx} = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$

Find A?

POWER UNIT

Probability

NoteBook

Dr. Jamal Rahal

By: Farah AbuAlssamin

بأفكارنا_نُبدع

4th week

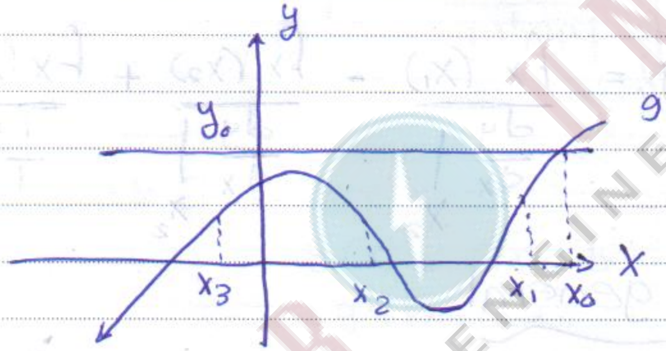
* Functions of Random variables

→ Decoltokration :

1) $y = g(x)$

$$X \; - \boxed{g(x)} - \; y$$

$f_X(x)$? $f_y(y)$



$$P_r(y \le y_0) = P_r(X \le X_0) - P_r(X \le X_2) + P_r(X \le X_3)$$

$$P_r(X \le X_0) \longleftrightarrow P_r(y \le y_0)$$

$$P_r(y \le y_0) = P_r(X \le X_0)$$

$$F_y(y) \Big|_{y = y_0 = g(x)} = F_X(x) \Big|_{X = X_0}$$

$$y_0 = g(x_0)$$

$$f_y(y)\,dy = f_x(x)\,dx$$

$$f_y(y) = f_x(x) \cdot \frac{dx}{dy}$$

$$f_y(y) = \frac{f_x(x)}{\left(\frac{dy}{dx}\right)}\Bigg|_{x=g^{-1}(y)}$$

$$f_y(y)\,dy = \frac{f_x(x_1)}{\frac{dy}{dx}\Big|_{x_2}} - \frac{f_x(x_2)}{\frac{dy}{dx}\Big|_{x_2}} + \frac{f_x(x_3)}{\frac{dy}{dx}\Big|_{x_3}}$$

In general

$$\left\{ f_y(y) = \sum \frac{f_x(x_i)}{\left|\frac{dy}{dx}\right|\Big|_{x_i}} \right\}$$

1

* Ex:- $y = g(x) = ax$, $y = ax$

$g' = \frac{dy}{dx} = a$ $\qquad x = \frac{y}{a}$

So, $f_y(y) = \frac{f_x(x)}{|a|}\Bigg|_{x=g^{-1}(y)}$

$$= \frac{f_x\left(\frac{y}{a}\right)}{|a|}$$

Cont. → **Functions of Random variables**

$y = g(x)$

$$X \longrightarrow \boxed{\overset{?}{g(\,)}} \longrightarrow y$$

full wave rectifier → absolut value

$$f_Y(y) = \sum_{i=1}^{?} \frac{f_X(g^{-1}(y))}{|g'(y)|} \Bigg|_{x_i = g^{-1}(y)}$$

* Ex :-

(2)

$g(x) = ax + b$

$g'(x) = a$

$y = ax + b \Rightarrow x = \frac{y-b}{a}$

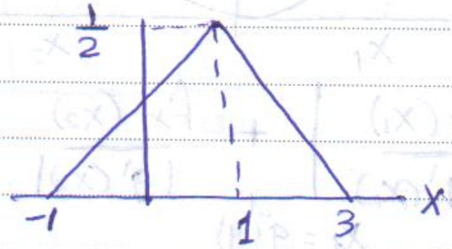
$$f_Y(y) = \frac{f_X\left(\frac{y-b}{a}\right)}{|a|}$$

b → shift

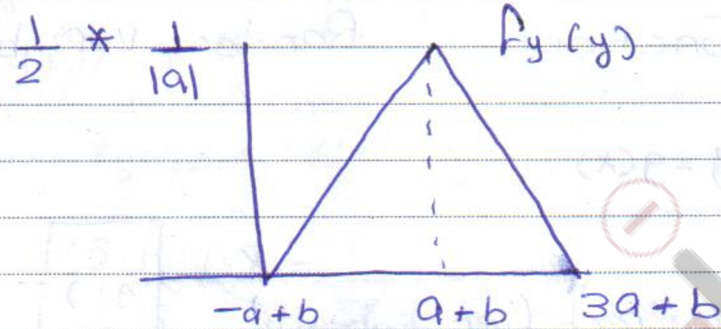
a → scale

لو كان ال f_X guassian ← بيطلع ال f_Y gaussian

→ Let $f_X(x)$

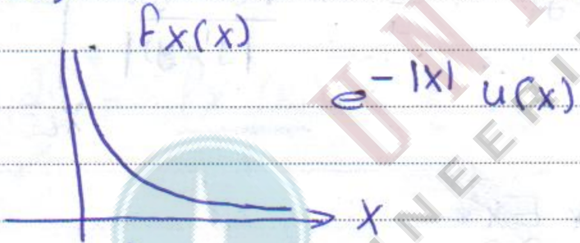


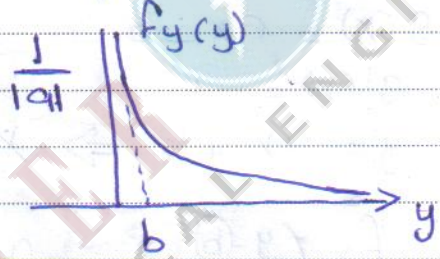
⇒



→ Let $f_x(x)$

③





$$f_y(y) = \frac{1}{|a|} e^{-\frac{(y-b)}{a}} \, u(y-b)$$

④ $y = g(x)$

$y = ax^2$

$$f_y(y) = \left| \frac{f_x(x_1)}{g'(x_1)} \right| + \left| \frac{f_x(x_2)}{|g'(x_2)|} \right|$$

$x_1 = g^{-1}(y)$ $\quad$ $x_2 = g^{-1}(y)$

$$y = ax^2 \qquad y' = 2ax$$

$$x = \pm \sqrt{\frac{y}{a}}$$

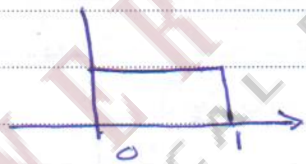
$$f_Y(y) = \frac{f_X(+\sqrt{y/a})}{2|a|\sqrt{y/a}} + \frac{f_X(-\sqrt{y/a})}{2|a|\sqrt{y/a}} \quad *$$

تحديها zero

$y \cdot a > 0$

هل يمكن يساوي صفر ؟!

Let $f_X(x)$



$$f_Y(y) = \frac{1}{|2a\sqrt{y/a}|}$$

$0 < y \le a$

$y = 0$ عند

undefined

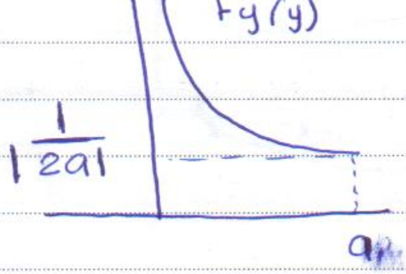
(Pole)

$y = ax^2$

$x = 0 \rightarrow y = 0$

$x = 1 \rightarrow y = a$

*



Let :

الدكتور فارض a +ve

$$f_X(x) = \frac{1}{2} e^{-|x|}$$

$$f_Y(y) = \frac{\frac{1}{2} e^{-|\sqrt{y/a}|}}{|2a\sqrt{y/a}|} + \frac{\frac{1}{2} e^{-|\sqrt{y/a}|}}{|2a\sqrt{y/a}|}$$

(لو a -ve كان يتحسب y سالبة)

$$= \frac{e^{-\sqrt{y/a}}}{|2a\sqrt{y/a}|} \begin{cases} y>0 & \text{و } a>0 \\ y<0 & \text{for } a<0 \end{cases}$$

cont. → Functions of Random variables

(5)

$$y = ax^2 + bx + c$$

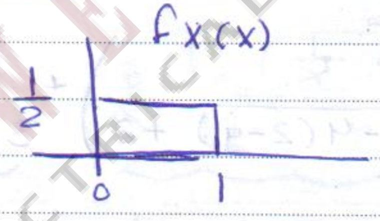
$$0 = ax^2 + bx + \underbrace{(c - y)}_{c'}$$

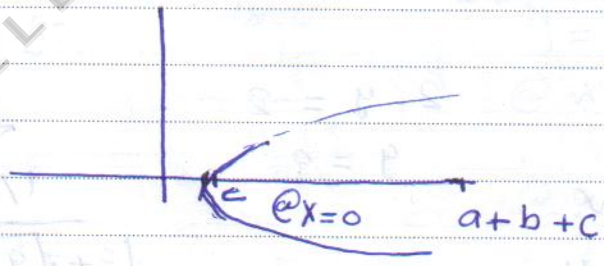
$$y' = 2ax + b$$

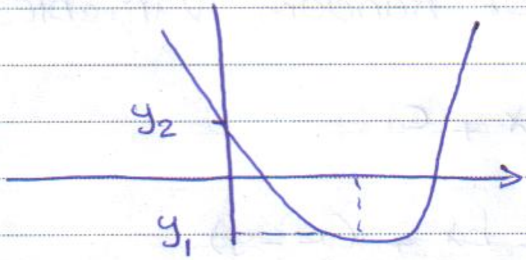
$$x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4a(c-y)}}{2a}$$

$$f_y(y) = \frac{f_X(x_1)}{|2ax_1 + b|} + \frac{f_X(x_2)}{|2ax_2 + b|}$$

Let :-







ال Range المسموح به

$y_1 \to y_2$

y_2 مش شرط تكون عند ال x_2

$$y = x^2 - 3x + 2$$
$$= (x-1)(x-2)$$

$$x_{1,2} = \frac{3 \pm \sqrt{9-4(2-y)}}{2}$$

$$y_1 = (1.5)^2 - 3*1.5 + 2 = -\frac{1}{4}$$

$y_2 = 2$ لما $x=0$

$$f_y(y) = \left(\frac{1/2}{2(3-\sqrt{9-4(2-y)}+2)}\right) + \frac{f_x(x_2)}{3+\sqrt{9-4(2-y)}+2}$$

صفر في حالة $y = +2$ (طلعت برا الفترة)

ال Term الاول $0 < y < 2$

$2 - y = 2$

$y = 0$

ال Term التاني $-\frac{1}{4} \leq y \leq 0$

$$\frac{1/2}{|3+\sqrt{9-4(2-y)}+2|}$$

$-\frac{1}{4} \geq y \geq 0$ مع ال Two terms موجودين

هل يمكن أن تكون y is $\boxed{-ve}$!

No، بمجرد ما صارت exp y كلها صارت $\boxed{+ve}$

$$y = e^{\alpha x}$$

$$y' = \alpha e^{\alpha x}$$

$$0 = y - e^{\alpha x}$$

$$x = \frac{1}{\alpha} \ln(y)$$

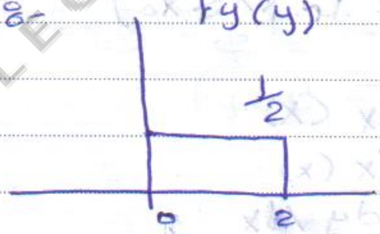
هل x حط $\frac{1}{\alpha}\ln(y)$

$$f_y(y) = \frac{f\left(\frac{1}{\alpha}\ln(y)\right)}{|\alpha| e^{\alpha x}}$$

$$= \frac{f_x\left(\frac{1}{\alpha}\ln(y)\right)}{|\alpha| \underbrace{e^{\alpha \frac{1}{\alpha}\ln(y)}}_{y}}$$

$$= \frac{f_x\left(\frac{1}{\alpha}\ln(y)\right)}{|\alpha|\, y} \qquad y > 0$$

* Ex :-



$$y = e^{3x}$$

@ $x = 0$

$y = 1$

@ $x = 2$

$y = e^6$

$$1 \le y \le e^6$$

$$f_y(y) = \frac{1/2}{3y} = \frac{1}{6y}$$

لو كانت

$$f_X(x) = e^{-x}\, u(x)$$ ~ داخ موجبة x معناها

$$f_y(y) = \frac{f_X(\ln y)}{y} \qquad y > 0$$

!! ما ضربتها

معنا $u(y)$

$$= \frac{-y}{y} = -1 \;? \quad u(y) \;??$$

!! هاي مشكلة ما بجيب سالب !!

$x=0$ at $x=\infty$

$y=1$ $y=\infty$

$1 \le y \le \infty$

table جدول

X_Y مش

Function

$$P_r(y \le y_0) = P_r(X \le x_0)$$

$$F_y(y) = F_X(x)$$

$$f_y(y) = \frac{f_X(x)}{dy/dx}$$

$$e^{-x} = \frac{1}{e^x} = \frac{1}{e^{\ln y}} = \frac{1}{y}$$

Lec # 18 الاربعاء . 2014 / 7 / 16

→ cont.

then : $f_y(y) = \frac{f_x(x)}{|y'|} = \frac{1}{y^2} \quad y \geq 1$

$x = \ln y$

⑥

$y = \ln(x) \qquad x > 0$

$x = e^y$

$|y'| = \left|\frac{1}{x}\right|$

$f_y(y) = e^y f_x(e^y) \quad -\infty < y < \infty$

* Ex 8-

$y = \ln(x)$

$f_x(x) = e^{-x} u(x)$

$f_y(y) = e^y e^{-x} \, 1$

$x = e^y$

$= e^y e^{-e^y} \quad -\infty < y < \infty$

let $e^y = z$

$f_y(y) = z e^{-z}$

z مستحيل -ve ومستحيل zero لكن يمكن $z \to \infty$

لوكانت

$f_X(x) = e^{-x}\, u(x)$ ~ دائمًا موجبة معناها x

$$f_y(y) = \frac{f_X(\ln y)}{y} \quad y > 0$$

نضيف $u(y)$

!! ماحطيتها

$$= \frac{-y}{y} = -1 \; ؟ \; u(y) \; ??$$

!! هاي مشكلة ما بصير سالب

$x=0$, $y=1$ at $x=\infty$, $y=\infty$

$1 \leq y \leq \infty$

هدول Table
مش XY
Function

$$P_r(y \leq y_0) = P_r(X \leq x_0)$$

$$F_y(y) = F_X(x)$$

$$f_y(y) = \frac{f_X(x)}{dy/dx}$$

$$e^{-x} = \frac{1}{e^x} = \frac{1}{e^{\ln y}} = \frac{1}{y}$$

Lec # 18 الاربعاء . 2014 / 7 / 16

→ cont.

then: $f_Y(y) = \frac{f_X(x)}{|y'|} = \frac{1}{y^2} \quad y \geq 1$

$x = \ln y$

⑥

$y = \ln(x) \qquad x > 0$

$x = e^y$

$|y'| = \left|\frac{1}{x}\right|$

$f_Y(y) = e^y f_X(e^y) \quad -\infty < y < \infty$

* Ex 8-

$y = \ln(x)$

$f_X(x) = e^{-x} u(x)$

$f_Y(y) = e^y e^{-x} \Big|_{x = e^y}$

$= e^y e^{-e^y} \quad -\infty < y < \infty$

let $e^y = z$

$f_Y(y) = z e^{-z}$

z مستحيل -ve ومستحيل zero لكن يمكن $z \to \infty$

ALADIB.net

$$f_y(y) = \sum_{i=1}^{?} \frac{f_x(g^{-1}(x))}{|g'(x)|} \quad \therefore \begin{bmatrix} \text{general} \\ \text{form} \end{bmatrix}$$

* Ex:- $y = \ln(a x^2)$

find $f_y(y)$?

المرحلة الأولى
$$\begin{cases} z = a x^2 \\ f_z(z) = f_x(?) \end{cases}$$

المرحلة الثانية
$$\begin{cases} f_y(y) = \frac{f_z(z_i)}{|g'(z_i)|} \\ y = \ln(z) \end{cases}$$

$$f_x(x) = e^{-x} u(x)$$

$$z = a x^2$$

$$f_z(z) = \frac{f_x(\sqrt{z/a})}{|2a\sqrt{z/a}|} + \cancel{\frac{f_x(-\sqrt{z/a})}{|2a\sqrt{z/a}|}}$$

zero because of $u(x)$ 0

let $a = 1$

$$f_z(z) = \frac{f_x(\sqrt{z})}{2z} = \frac{e^{-\sqrt{z}}}{2\sqrt{z}} \quad z > 0$$

$$\rightsquigarrow \quad y = \ln(z)$$

$$f_y(y) = e^{y} f_z(e^{y})$$

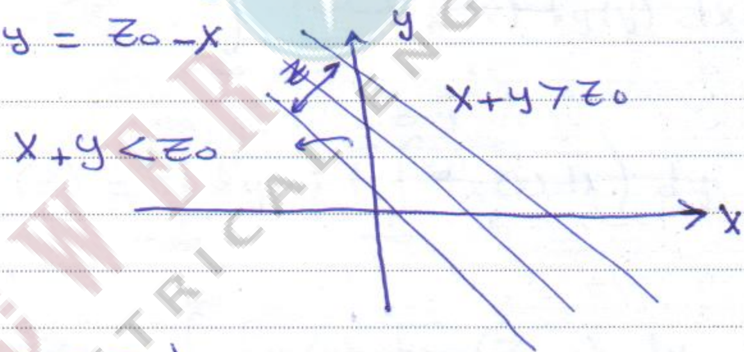
$$f_y(y) = \frac{e^{y/2} e^{-e^{-y/2}}}{2}$$

* Functions of Two R.V's

$Z = X + Y$ (only)

$z_0 = x + y$

$y = z_0 - x$



$$P_r(Z \le z_0) = $$

$$= \iint_{x\,y} f_{xy}(x,y)\, dx\, dy$$

Left half plane

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{z-y} f_{xy}\, dx\, dy = F_Z(z)$$

$\rightsquigarrow$

$$\boxed{f_Z(Z) = \frac{d}{dx} F_Z(Z)}$$

$~~Z~~ \; X$

* Ex:

$$Z = X + Y$$

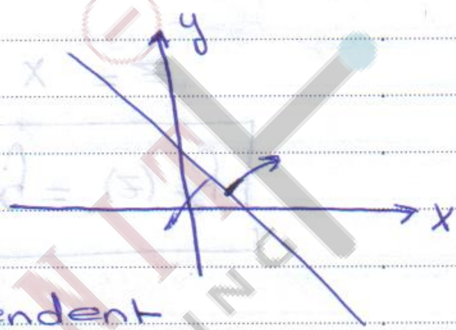
$$f_X(x,y) = e^{-(x+y)} u(x,y)$$

الحفيس

Lec # 19 2014 / 7 / 17

→ Functions of two R.v's

$$Z = X + Y$$



$$F_Z(z) = \int_{-\infty}^{\infty}\int_{-\infty}^{z-y} f_{x,y}(x,y)\,dx\,dy$$

if x & y are independent

$$f_{xy} = f_x \cdot f_y$$

$$F_Z(z) = \int_{-\infty}^{\infty}\int_{-\infty}^{z-y} f_x(x)\cdot f_y(y)\,dx\,dy$$

$$F_Z(z) = \int_{-\infty}^{\infty} f_y(y)\left(\int_{-\infty}^{z-y} f_x(x)\,dx\right)dy$$

$$F_Z(z) = \int_{-\infty}^{\infty} f_y(y)\,F_x(z-y)\,dy$$

$$\int f_Z(z)\,dz = \int_{-\infty}^{\infty} f_y(y)\cdot F_x(z-y)\,dy$$

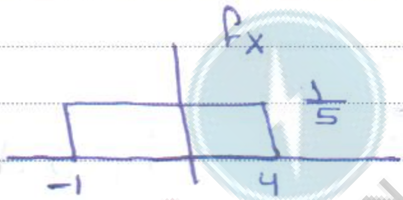
$$= f_y * F_x$$

* For independent R.V's X & Y → the pdf of thier sum is the convolution

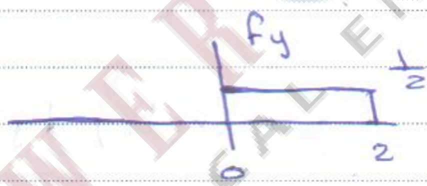
$Z = X + Y$

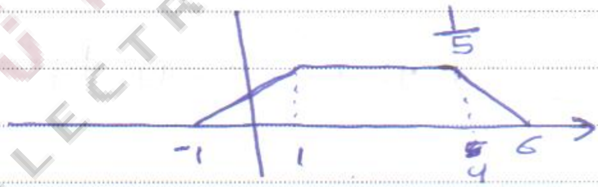
$$\boxed{f_Z(z) = f_X(x) * f_Y(y)}$$

* **Ex:-**



$Z = X + Y$
find $f_Z(z)$?

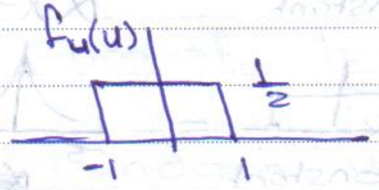




/ /

*Ex:-

$W = Z + U$ $\qquad$ $Z = X + Y$ نفسها في المثال السابق

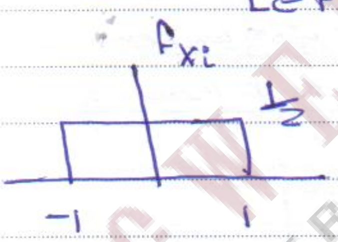


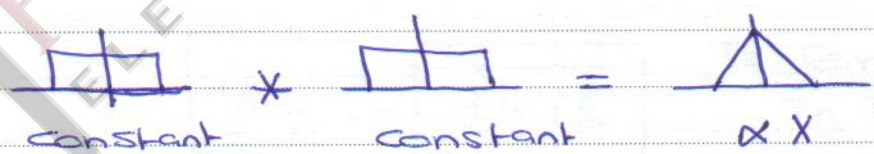
$$W = X + Y + U$$

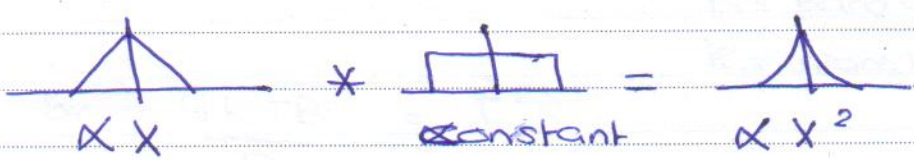
$$\therefore \; f_W(w) = f_{X_1} * f_{X_2} * f_{X_3} \cdots * f_{X_N}$$

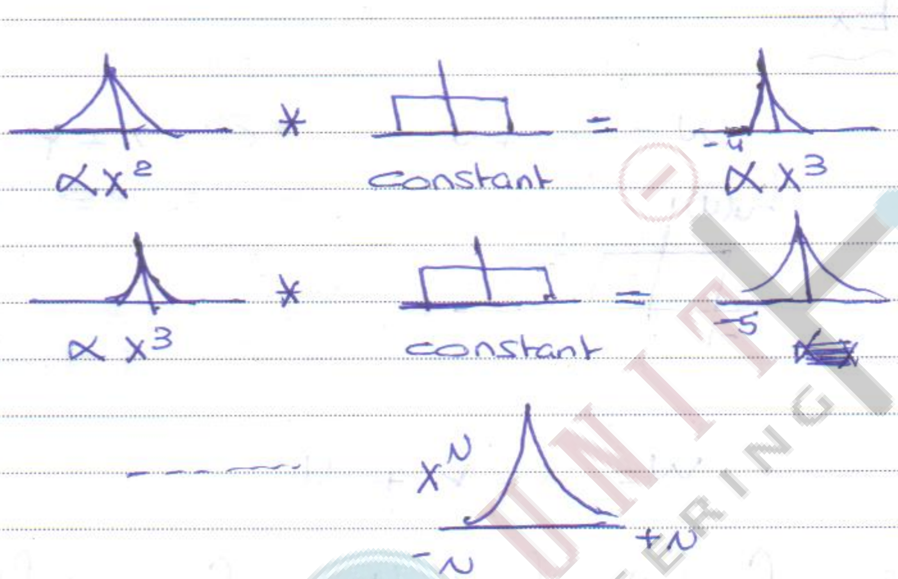
*Ex:

Let $N \to \infty$ (iid)
↓
independent
identically
distributed








∝x²
*
constant
=
-4
∝x³
∝x³
*
constant
=
-5
x^N
-N
+N

* Central Limit Theorem :-

(*) sum of too many (infinit) independent Random variables is a Gaussian Random variable.

Thermal Noise

$$\sum_{i=1}^{\infty} e_i \rightarrow \text{guassian RV}$$

الكترونات بتوزع بعين وشمال فالـ average الهم صفر

$$M_n = \sum M_{e_i} = 0$$

$$\sigma_n^2 = \sum \sigma_{e_i}^2 = \text{power}$$

$$\boxed{\sigma_n^2 = 4KTB}$$

$$\boxed{V_{rms} = \sqrt{4KTB}}$$

T = Temp. in kelfen

B = Band width

K = constant

$$\cancel{P} = \frac{4KTB}{R} = I^2R$$

POWER UNIT

Probability
NoteBook

By: Farah Abu Alssamin
Dr. Jamal Rahal

بأفكارنا_نُبدع

5th Week

(7) $y = \sqrt{x}$ $x = y^2$

$x, y > 0$

$y' = \frac{+1}{2\sqrt{x}}$

$$f_y(y) = f_x(y^2) \cdot 2y \quad y > 0$$

(8)

$y = \frac{1}{x}$ $y' = \frac{-1}{x^2}$

$x = \frac{1}{y}$

$$f_y(y) = \frac{f_x(\frac{1}{y})}{y^2} \cdot \quad y \neq 0$$

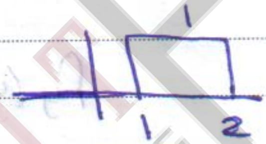
كله abs عشان
Pdf value

Quiz

given X uniform $\in (0,1)$

$$y = (x+1)^2$$

find $f_y(y)$?



$$w = x+1$$

$$f_w(w) = f_x(w-1)$$

$$y = w^2$$

$$f_y(y) = \frac{1}{2\sqrt{5}} \qquad 1 \le y \le 4$$

^^ ∪

* مصيبة ال $\boxed{Pr}$ تطلع سالب *

هيك نوعها الأسئلة في الامتحان

case $\boxed{1}$

$$y = F_X^{-1}(x)$$

X is RV with

$f_X(x) \longrightarrow F_X(x)$

$$X \longrightarrow \boxed{F_X^{-1}(x) = g(x)} \longrightarrow$$

$$X = F_X(y)$$

$$y' = g'(x)$$

$$f_Y(y) = \frac{f_X(F_X(y))}{|g'(F_X(y))|}$$

another case

case $\boxed{2}$

$$y = F_X(x)$$

$$y' = f_X(x)$$

$$X = F_X^{-1}(y)$$

$$f_Y(y) = \frac{f_X(F_X^{-1}(y))}{f_X(x)|}$$

$x = F_X^{-1}(y)$

Case $\boxed{3}$

$$y = F_y(x)$$

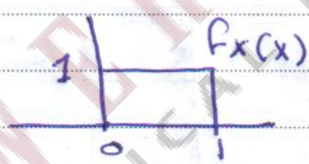
$$y' = f_y(x)$$

$$x = F_y^{-1}(y)$$

$$f_y(y) = \frac{f_x(F_y^{-1}(y))}{f_y(F_y^{-1}(y))}$$

For case $\boxed{1}$

if $x \in (0,1)$



$F_x(x) = x$

$f_y(y) = 1 \qquad 0 \le y \le 1$

∴ So, In uniform the input is equal to output

/ /

For case $\boxed{2}$

$$f_y(y) = 1 \qquad 0 \leq y \leq 1$$

case ① نفس النتيجة ل :

For case $\boxed{3}$

$$f_y(y) = \frac{1}{f_y(F_y^{-1}(y))}$$

(*) From uniform we can find another pdf if we ~~the~~ take $y = F_y(x)$

Case $\boxed{4}$

$$y = F_y^{-1}(x) \Rightarrow x = F_y(y)$$

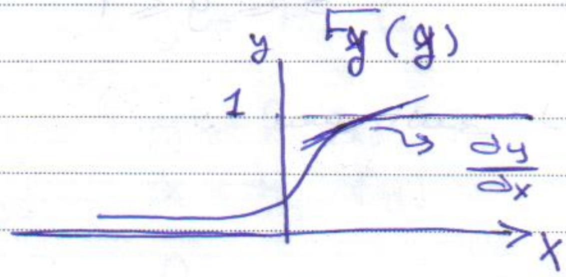
$$g' = \frac{d F_{\{y\}}^{-1}(x)}{dx}$$

$$f_y(y) = \frac{f_x(F_y(y))}{\frac{d}{dx}(y)}$$

For uniform :-

$$f_y(y) = \frac{1}{|g'(F_y(y)|}$$

→ Now, Can we generate R.V for Uniform ?

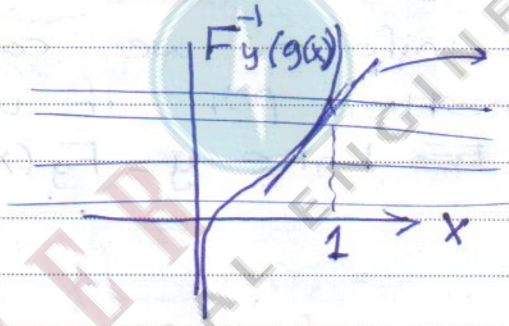


أي شي capital تقريباً هيك بكون شكله (Non dec.)

$$Y = F_y^{-1}(x) \Rightarrow x = F_y(y)$$

$y = g(x)$

حل واحد فقط



ميل المماس (مشتقة)

$\frac{dx}{dy}$

$$Y' = \frac{1}{f_y(y)}$$

$$f_y(y) = \frac{1}{g'(x)} = \frac{1}{\frac{1}{f_y(y)}} = f_y(y)$$

Result →

/ /

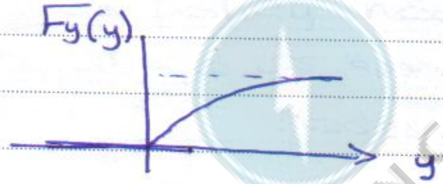
⊛ If we implement a device to perform the function $F_Y^{-1}(x)$
→ we can generate an arbitrary Random variable with $f_Y(y)$ using uniform $x \in (0,1)$.

* Ex:

$$f_Y(y) = e^{-y}\, u(y)$$

$$x \rightarrow \boxed{g = F_Y^{-1}(\cdot)} \rightarrow y$$

$$F_Y(y) = 1 - e^{-y}$$



$$g(x) = F_Y^{-1}(x) = -\ln(1-x)$$

$$1 - e^{-y} = x$$

$$1 - x = e^{-y}$$

$$\boxed{y = \ln(1-x)}$$

* الامتحان إلى هنا *

Lec ※ 21 الاثنين 2014 / 7 / 21

* Simulation

case I

*Ex: computer network simulation :-

→ channels of data, each with certain capacity

→ we have nodes connected with channels.

→ policy nodes. → لتقرر أي يتفتح

(أي device موجودة كل هذه ال network nodes تختبر)

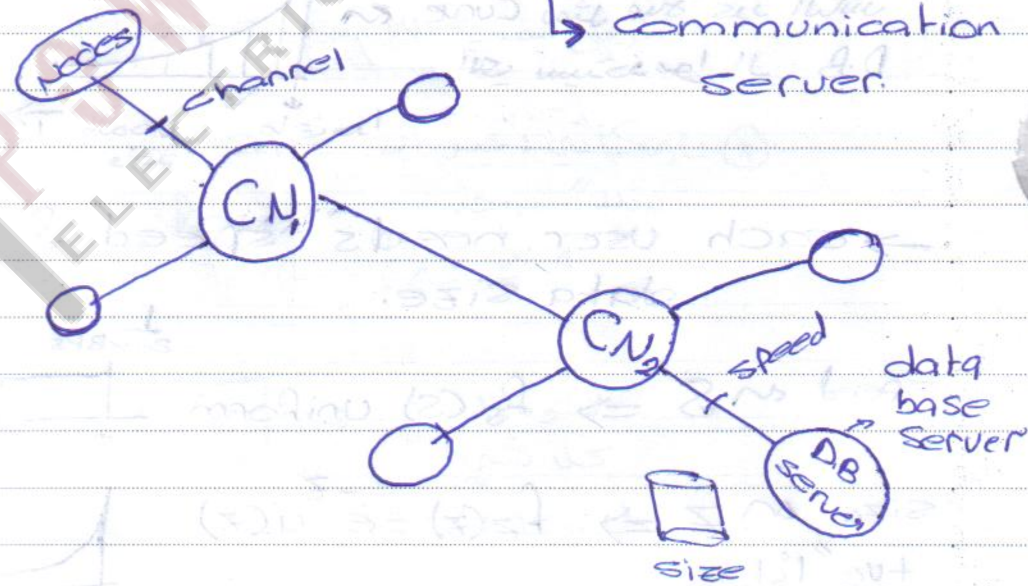
* Policy nodes could be :-

(IF statement) (control node) CN

→ servers

→ Router

→ communication server



* we need to simulate the network Shown (as example) to operate a D.B server.

time out → يعني تفصل الاتصال بينك وبين ال server

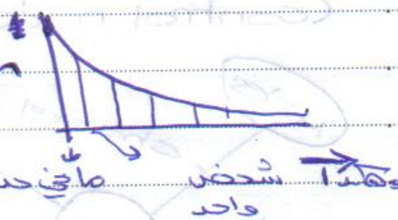
* we need to find the size and link speed as a function of network users.

* every user uses the D.B (email) randomly with pdf

demand $f_X(x) = e^{-x} u(x)$

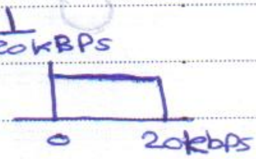
(أسوأ حالة إنه ال pdf يكون Uniform)

~ curve بيمثل منه "عدد الناس الي سيستخدموا ال D.B"

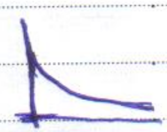


→ each user needs speed & data size.

speed ~ S ⇒ $f_S(s)$ uniform



size ~ Z ⇒ $f_Z(z) = e^{-z} u(z)$



دائماً "+ve"

Single sided

/ /

* Simulation :-

1- each loop will represent a Δt.

So, start.

⇓

(Initilize variables)

Loop:- Generate X out put

$V = Rand$

$X = -\ln(1-V) \rightarrow$ represent demand (عدد الناس)

$f_X = e^{-X}$

$F_X = 1 - e^{-X}$

$F_X^{-1} = -\ln(1-y)$

$S = Rand * 20k$

$u = Rand$

$Z = -\ln(1-u)$

Result Array :-

Speed (itteration) = X.S

size (itteration) = X.Z

Loop:- برمج للفسه اللي فوق

* بإمكاني أغير الـ varience

مثلاً بوحد

$f_Z = \beta e^{-\beta Z}$ $\quad f_X = \alpha e^{-\alpha X}$

$\boxed{6\square} \rightarrow$ لبتوخذ كل شي

لانه كل القياسات ضمنها.

/ /

$\overline{\text{speed}}$ = 120 kbps ~ total speed for server

$\overline{\text{speed}}$ = 10 kbps

→ ~~Link~~ Link speed 6Σ
= 120 + 6 * 10 = 180 kbps

الثلاثاء

Lec # 22 2014 / 7 / 22

Case II

Common channel simulation :

Tx

deteminstic signal (Number = A)

→ ch → Add R.V x where x is $N(\sigma_x^2, M_x)$

multiply by y where y uniform $\in (0, 10)$

@ Rx we recive

$$Z = yA + X$$

$$Z = y_i A_i + X_i + \Sigma$$

$$mse = E\{|Z - A|^2\} = E\{|yA + X - A|^2\}$$

$$= E\{|A(y-1) + X|^2$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |A(y-1) + x|^2 \cdot f_{xy}(x, y)\, dx\, dy$$

→

* Initial condition : Δt

Generate A arbitrary

fix $A = 100$

Loop :- $N =$ Number of loops

N times

Generate y

Generate X

$Z = 100\,y - X$

$mse = |Z - A|^2$

goto Loop

$$mse = \frac{\sum mse_i}{N}$$

Simulation accuracy :-

10 Readings

↳ each has an error of 6

$$\text{Avarage error} = \frac{NE}{N} = N \rightarrow ??$$

Reading A True

error X

$Z = A + \underset{\to \text{error}}{X}$

→

* After N readings :-

$$Total = \sum_{A=1}^{N} A + X_n$$

$$Avg = \frac{1}{N} \sum_{n=1}^{N} A + X_n$$

$$\boxed{Avarage = A + \bar{M}_X}$$

⊛ By default Uniform ⊛

$$\tilde{M}_X = \frac{1}{N} \sum_{n=1}^{N} X_n$$ ⇝ Estimate of the mean

if $\tilde{M}_X$ as $N \to \infty$

↳ then the estimate is onBiased.

* Varience of the estimate

$$\sigma_X^2 = E\{ |\tilde{M}_X - M_X|^2$$

Best linear unbiased estimate

↳ BLUE

Minimum varience Estimate

MVE

(Must algerithem) DSP ??

POWER UNIT

Probability

NoteBook

Dr. Jamal Rahal

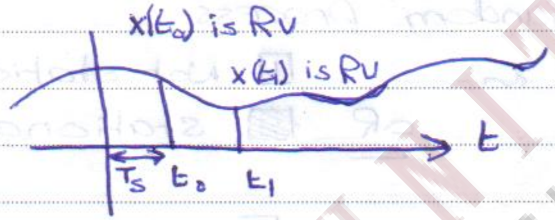
By: Farah Abu Alssamin

بأفكارنا_نُبدع

Lec 23 الأربعاء 2014 / 7 / 23

* Random Process :-

$X(t)$



$X(t_0) \rightarrow f_{X(t_0)}(X(t_0))$

$X(t_1) \rightarrow f_{X(t_1)}(X(t_1))$

إشارة الصوت
مثال على
Random
Process ← شكلها كل مرة
بتغير

$f_s \geq 2 f_m$ ← Sampling

$T_s \leq \frac{T_m}{2}$

$\boxed{\text{data rate} = \underline{\underline{m}} \cdot f_s}$

m: A/D

ex:- 4-bits A/D

(*) Standards :

$F_s = 8$ ksps

rate $= 8 \times 8 = 64$ kbps

→ Random process

احتمالية انه تطلع نفس الاشارة ← [1] Not stationary

OR [2] stationary

[2] Ergodic

OR Not Ergodic

→ Stationary → Strict sense
→ wide sense

Strict → independent with time.

هاد يسمى Ergodic

$$\underbrace{\sin(\omega t + \underset{\substack{\uparrow \\ R.V}}{\theta})}$$

يصبح Function ال Random

any change in R.V will be reflected to the same shape (sin بضل)

$$M = \int x\, f_x(x)\, dx$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, dt$$

Ergodic $\rightsquigarrow$ Time Avarage = statistical Avarage

$$\int dt = \int dx$$

/ /

Quiz ※ 4

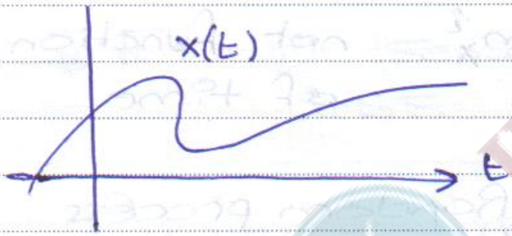
$$f_{xy}(xy) = e^{-(x+y)} u(x,y)$$

are x & y ~~are~~ independent?

"أخذنا مثله سابقاً"

* Random processes :-

$X(t)$ → wide sence staionary
→ ident. distrbuted



$X(t) = \underline{A}\ \text{rect}(t)$

1- $M_X(t) = E\{X(t)\} = \int_{-\infty}^{\infty} X(t)\, f_X(x)\, dx$

if Ergodic

$$M_X(t) = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, dt$$

$$m_X^2 = E\{X^2\} = \int X^2 f_X\, dx$$

if Ergodic

$= \frac{1}{T}\int |x(t)|^2 dt$: Total Avarge Power →

$$m_X^2 = \underbrace{\sigma_X^2}_{\substack{\text{A.C} \\ \text{Power}}} + \underbrace{\mu_X^2}_{\substack{\text{d.c} \\ \text{Power}}}$$

For stationary random process

$m_X^i(t) = m_X^i$ not function of time

Stationary Random process
$f_X(x)$ is not function of time
→ strict sense stationary
SSS

→ For stationary → not a function of time but relative to time

$\mu_X(t) = \mu_X$

$m_X^i(t) = m_X^i$

→ Only the Auto correlation function.

$$\boxed{R_{XX}(t_1, t_2) \triangleq E\{X(t_1) \cdot X^*(t_2)\}}$$

↳ In general For zero mean

→

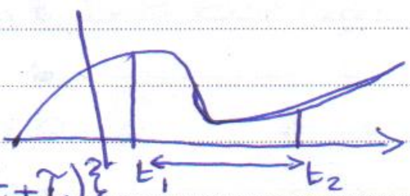
$$R_{XX}(t_1, t_2) \triangleq E\{(X(t_1) - \mu_X)(X(t_2) - \mu_X)\}$$

لو كانت $\boxed{t_2 = t_1}$ ← بيتحول بتمثل ال Total Power

→ For wide sense stationary WSS

$$R_{XX}(t_1, t_2) = R_{XX}(t_2 - t_1) = R_{XX}(\tau)$$

لو كانت $\boxed{t_2 = t_1}$ ← تكون ال correlation coefficient أعلى ما يمكن



$$R_{XX}(\tau) = E\{X(t) \cdot X^*(t+\tau)\}$$

Charactristics of $R_{XX}(\tau)$

① $R_{XX}(\tau) = R_{XX}(-\tau)$

② $R_{XX}(0) = \sigma_X^2 + \mu_X^2$

③ $R_{XX}(\tau_1) \leq R_{XX}(\tau_2)$ $\quad \tau_2 < \tau_1$

اللاتين

Lec # 25 2014 / 8 / 4

cont. Random process :-

* Auto corrolation function :-

* Ex :-

if $y = a\cos(wt)$
where a is a uniform $a \in (0,1)$
is y WSS ?

$$R_{YY}(t_1, t_2) = E\{y(t_1) \cdot y^*(t_2)\}$$

$$= \int_0^1 a\cos(wt_1) \cdot a\cos(wt_2)\, da$$

$$E\{g(x)\} = \int g(x) f_x\, dx$$

$$= \int_0^1 a^2 \frac{1}{2}[\cos(w(t_1+t_2)) + \cos(w(t_1-t_2))]\, da$$

$$R_{YY}(t_1, t_2) = \frac{1}{6}[\cos(w(t_1+t_2)) + \cos(w\tau)]$$

$$\tau = t_2 - t_1$$

so y is not WSS

* Ex 2:

if $y = 5\cos(\omega t + \theta)$

θ is uniform $\in (-\pi, \pi)$

is y WSS ?

$$= \cancel{\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{25}{2}[\cos}$$

$$R_{yy}(t_1, t_2) = \int_{-\pi}^{\pi} \frac{1}{2}\, 5\cos(\omega_1 t + \theta) \cdot 5\cos(\omega t_2 + \theta)\, d\theta$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{25}{2}\left[\cos(\omega(t_1+t_2)+2\theta) + \cos(\omega(t_2-t_1))\right] d\theta$$

$$= \frac{25}{2}\cos(\omega(t_2-t_1))$$

$$= \frac{25}{2}\cos(\omega \tau_1) = R_{yy}(\tau)$$

$\therefore$ y is WSS

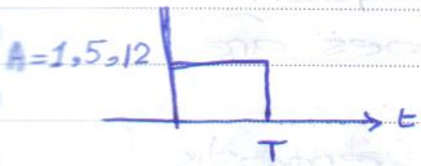
* Ex 38-

Binary signal

R.V $X \in \{-1, 1\}$ → uniformly distributed

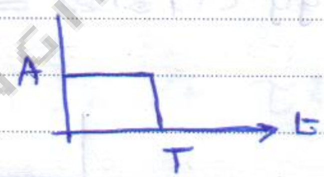
signal $w(t)$

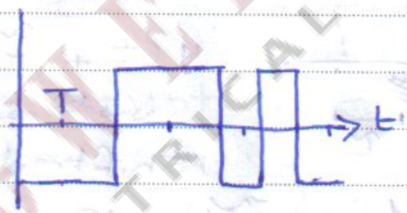
له يعني احتمال ال -1 & 1
لكل واحد منهما $\frac{1}{2}$



$$y = X \cdot w(t)$$

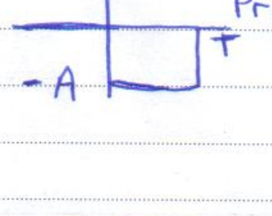
In general





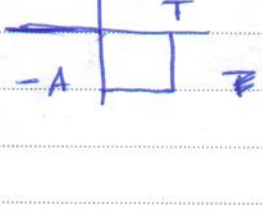
(مهم جدا")

find $R_{yy}(t_1, t_2)$



when $X = 1$
$A = \frac{1}{2}$

$Pr = \frac{1}{2}$



$R_{yy}(t_1, t_2) = E\{y(t_1) \cdot y^*(t_2)\}$

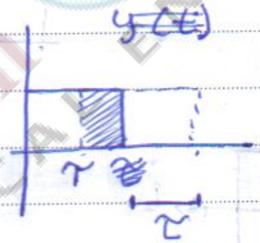
$= \iint y(t_1)\, y(t_2) \cdot f_{y(t_1) y(t_2)}(\quad)\, dy(t_1)\, dy(t_2)$

$\boxed{1}$ assume the sequence of one's & zeroes are independent

$\boxed{2}$ assume y is ergodic R.P

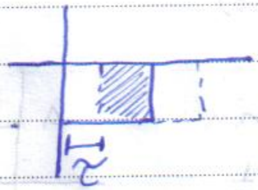
$R_{yy}(t_1, t_2) = R_{yy}(t, t+\tau)$

$\tau = 0$ قطعة

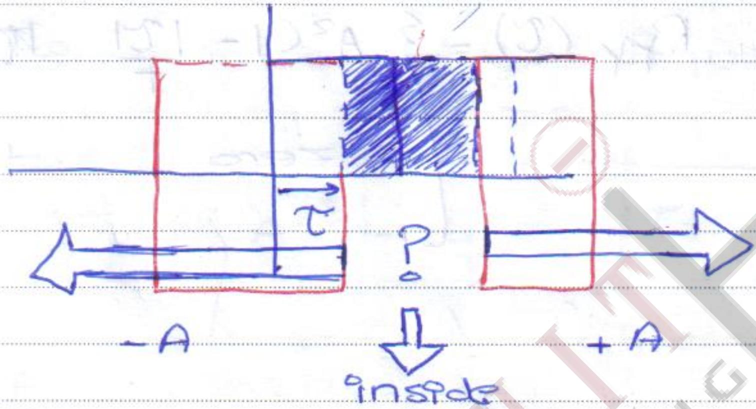


قطعة $y(t+\tau)$ نفسها بس shifted بمقدار τ

$X = +1$



$X = -1$



$$R_{YY}(t_1, t_2) = R_{YY}(t, t+\tau)$$

$$E\{y(t) \cdot y(t+\tau)\}$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} y(t) \cdot y(t+\tau)\, dt$$

Expectation outside the over lapped

$R_{yy}(t, t+\tau)$

$$\Rightarrow = A * A * \frac{1}{4} + {-A} * A * \frac{1}{4} + A * {-A} * \frac{1}{4} + {-A} * {-A} * \frac{1}{4} = 0$$

Inside Area $\quad \frac{1}{T} A^2 (T - \tau) \quad \tau < T$

$$A^2\left(1 - \frac{\tau}{T}\right) \quad \tau < T \quad \tau > 0$$

$$A^2\left(1 + \frac{\tau}{T}\right) \quad \tau \le 0$$

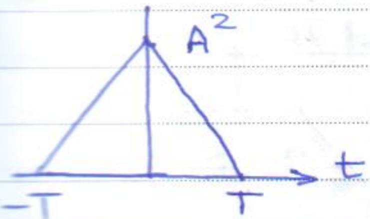
$$R_{YY}(\tau) = \begin{cases} A^2\left(1 - \frac{|\tau|}{T}\right) & |\tau| \le T \\ \text{zero} & \text{otherwise} \end{cases}$$

الثلاثاء

Lec ※ 26 2014 / 8 / 5

→ * Binary independent sequence

$$R_{yy}(\tau) = \begin{cases} A^2\left(1 - \frac{|\tau|}{T}\right) & |\tau| \le T \\ 0 & \text{otherwise} \end{cases}$$



$R_{yy}(\tau) = R_{yy}(-\tau)$
$R_{yy}(0) = \underbrace{\sigma_y^2}_{\text{Ac power}} + \underbrace{M_y^2}_{\text{dc power}}$

characterstics of auto corrolation

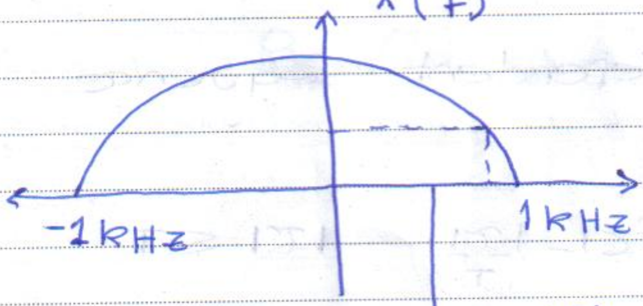
$$\lim_{\tau \to \infty} |R_{yy}(\tau)| = M_y^2$$

if $M_y^2 \ne 0$ $A^2 = \sigma_y$

$R_{YY} =$

$S_{YY}(F) = F.T\{R_{yy}(\tau)\}$

Power spectral density



→ Band limited signal

→ if it pass through filter it will be the same

any point here in f it represent $e^{j2\pi f}$

$$X(f) = \int_{\text{all time}} x(t)\, e^{j2\pi f t}\, dt$$

$\cancel{X(f) = \int_{\text{all t}}}$

$$X(f) \longrightarrow \boxed{H(f)} \longrightarrow Y(f) = X(f) \cdot H(f)$$

convolution in time domain = multiply in frequency domain

filter

High pass filter

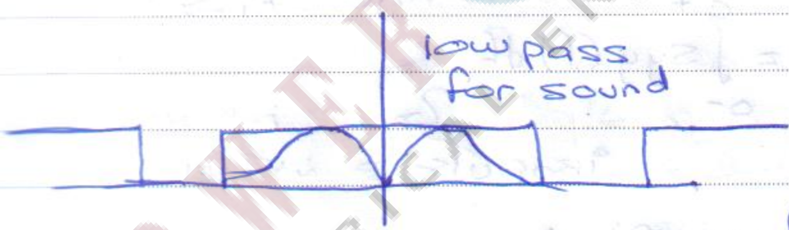
Low pass filter

Band pass filter

من خلال ال Filter باجز ال Signal من دون noise

to insolute signals

→ in mobile (signal)



high pass
for DSL
Signal

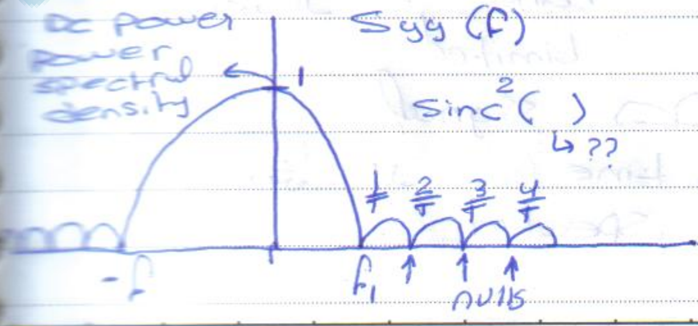
these 2 signals arrive together
I can insolute them by filters.

at f=0
DC power

power spectral density

$S_{yy}(f)$

$\text{sinc}^2(\;)$ ↳ ?? $= \dfrac{\sin(\pi t)}{\pi t}$



→

null to null bandwidth $\int_{-f_1}^{f} S_{yy}(f)\,df \simeq 95\%$ from σ_x^2

بحكي 95% من ال Power

* إذا بدي أكثر باخذ 2nd null ، 3rd null

$R_{yy}(0) = \sigma_y^2 + M_y^2$ = total power

$$= \int_{-\infty}^{\infty} S_{yy}(f)\,df$$

$\int_{f_1}^{f_2} S_{yy}(f)\,df$ = power inside the range $f_1 \to f_2$

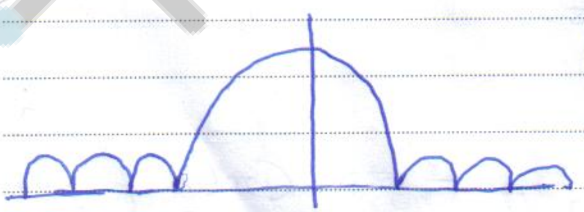
$$M_y^2 = \int_{0^-}^{0^+} S_{yy}(f)\,df$$

بكون صفر ← إلا إذا كان في impulse

if $M_y \neq 0$ → بصير عندي time spectral impulse

عند ال nulls وعند ال zero

وعند ال peak



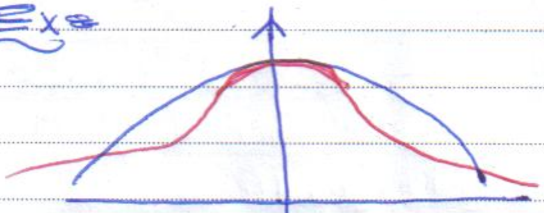
منجيب ال Band Limited signal

منحسش ال Line Spectra ← لانها بتعمل DC Power

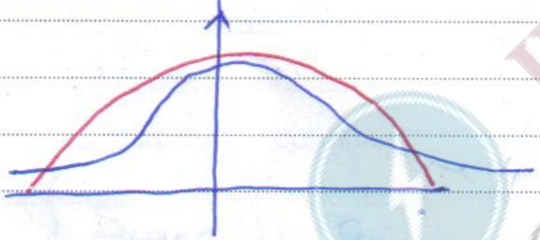
هنا بحط أكثر noise

* we study the Random process in frequency to know the characteric of the signal

* Ex:



High correlation
Low band width



high Correlated → less bandwidth

Encoding
~ Enception

$\boxed{\text{Pre-process}} \rightarrow \boxed{Tx}$

* Information measure

Entropy

$$H_i = -P_i \log_2 P_i$$

symbol x_i with probabilty P_i carries H_i information.

الاربعاء

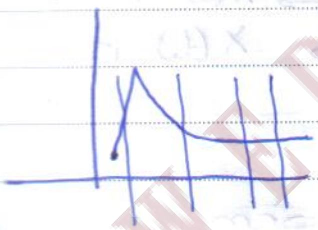
Lec 27 2014/8/6

* Auto correlation & power spectral ~~pow~~ density

Signal يمثل ال
in time domain

not stationary in small time
short terms
Stationary بكون
وكأنه

we do auto correlation and WSS
aply mathmatics to find optimal
bandwidth for example



devide them

for example the signal for yes

this for No

أغلب شي رح نتعامل
مع WSS

$$R_{xx}(\tau) = E\{x(t) \cdot x^*(t+\tau)\}$$ in cont. domain

$F.T[R_{xx}(\tau)] \to S_{xx}(f)$ power spectral density

$R_{xx}[m] = E\{x[n] \cdot X^*[n+m]\}$

$FT\ [R_{xx}[m]] \rightarrow S_{xx}[k]$ energy spectral density

If binary signal $p(t)$ if it multiply by $g(t)$

$x(t) = \underbrace{p(t)}_{\text{Random}} \cdot \underbrace{g(t)}_{\text{deterministic signal}}$ → constant (not random)

$p(t)$ is random → $x(t)$ random

if $p(t)$ is wss → $x(t)$ is wss

F.T is linear transform

$X(t) \xrightarrow{F.T} x(f)$ also wss and R.P
R.P

$R_{xx}(f, \Delta f) = E\{x(f) \cdot x^*(f+\Delta f)\}$

$S_{xx}(f) = F.T(R_{xx}(\tau))$

$= F.T(E\{x(t) \cdot x(t+\tau)\})$

معلومات عن كل ال signal

$= E\{\{x(f) \cdot x(f)\} e^{j\alpha\tau}\}$

إذا بدي أوجد psd لوترآت

Freq domain ⟵ signal الـ in time

↳ F.T

deteminstic

$X(t) \rightarrow \boxed{H(f)} \rightarrow Y(f)$

$X(t) \rightarrow \boxed{H(t)} \rightarrow y(t)$

$S_{xx}(x(f))$ $H(f)$ $S_{yy}(f)$

we must have the power spectral density

filtering

$$X(t) = \frac{P(t)}{R.P} * h(t) \rightarrow det$$

/ /

DFT $R_{xx}[m] = E\{x[n]\cdot x^*[n-m]\}$

$$x[n] = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$m\begin{cases} 0 \\ 0 \\ 0 \end{cases} \begin{bmatrix} x_m \\ \\ x_{n+m} \end{bmatrix}$$

$\vec{x}\cdot\vec{x}_m$ to find the expectation above

coloum vector = ناتج الضرب

$$R_{xx}[m] \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline x \\ x \\ x \\ \hline 0 \\ 0 \end{bmatrix}$$ values

$w = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$

$\vec{x} = x_T \cdot \vec{w}$

$\vec{x}_m = x_T \cdot \vec{w}_m$

نفس ال wx كدا

بس shifted

as window length decrease mesurment reselution ∂ increase

Lec 28 الخميس 2014/8/7

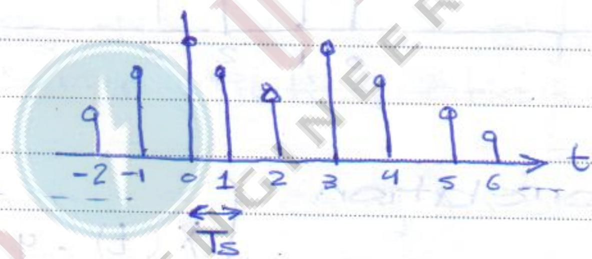
$$R_{xx}[m] = E\{x[n]\, x^*[n+m]\}$$

Exe-

$x[0] = x(t=0)$

$x[1] = x(t=1T_s)$

$x[n] = x(t=nT_s)$



$x[n]$

plus

$X[z] = \sum x[n]\, z^{-n}$ → delay one T_s

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{bmatrix}$$

Exe-

$x[0] = 1$

$x[1] = 5$

$x[2] = 3$

$X[z] = x[0]\, z^0 + x[1].\, z + x[2]\, z$

$X[z] = 1 + 5z + 3z^2$

$\rightarrow \quad y[z] = 1 + 2z$

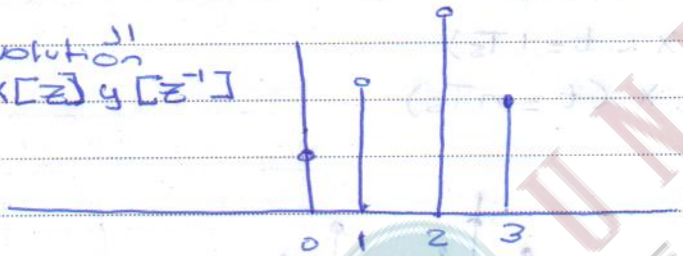
$$X[z]\,y[z] = (1 + 5z + 3z^2) + 2z + 10z^2 + 6z^3$$

ال correlation $\nearrow$

$$= 1 + 7z + 13z^2 + 6z^3$$

ال Convolution

$X[z]\,y[z^{-1}]$



correlation $\int X(t)\cdot y(t+\tau)\,dt$

convolution $\int X(t)\cdot y(\tau - t)\,dt$

* Ex:

Let $R_{xx}[m] = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} \rightarrow$ $R_{xx}[0]$ تمثل Power

not practical هنا الرقم $\leftarrow$

$\begin{bmatrix} 1.5 \\ 0.3 \\ 0.1 \end{bmatrix} \rightarrow$ more practical

$$\boxed{R_{xx}(0) \geq |R_{xx}(\tau)|}$$

لا يمكن أن تكون $R_{xx}(0)$ سالبة لأنها Power

/ /

* Practical signals

$$R_{XX}(\tau_1) \geq R_{XX}(\tau_2)$$

$$\boxed{\tau_1 < \tau_2}$$

* For periodic signal the auto corrolation is also periodic

* Periodic signal is a power signal can't carry information

* energy signal can carry information

$$\lim_{\tau \to \infty} |R_{XX}(\tau)| = M_X^2$$

(Autocorrolation function is a real & even function)

* No phase information can be obtained from 2nd order statistics.

* Audio signal has No phase information

Vedio signal has phase information

/ /

$$R_{xx}[m] = \begin{bmatrix} 1.5 \\ 0.3 \\ 0.1 \\ 0.08 \\ 0.05 \end{bmatrix}$$

size

N

لأنه قيمهم } 0.08

صغيرة قريبة الله } 0.05

3points بس كاينة زي

Zero وباقي

منعتبرهم Zero

$$\begin{bmatrix} 1.5 \\ 0.3 \\ 0.1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

* Q8- find $S_{xx}[k]$?

$$S_{xx}[k] = \frac{1}{N} \sum_{n=0}^{2} X[n] e^{-jnk \cdot \frac{2\pi}{N}}$$

resolution

$$\Delta f = \frac{f_s}{N}$$

frequency resolution

(as N goes to zero $\Delta f \to \infty$
بصير لـ continous.)

$$S_{xx}[0] = \frac{1.9}{3}$$

$$S_{xx}[1] = \frac{1.5}{3} + \frac{0.3}{3} e^{\pm j\frac{2\pi}{3}} + \frac{0.1}{3} e^{\pm j\frac{4\pi}{3}}$$

$$S_{xx}[2] = \frac{1.5}{3} + \frac{0.3}{3} e^{\pm j\frac{4\pi}{3}} + \frac{0.1}{3} e^{\pm j\frac{8\pi}{3}}$$

// دائماً

$$R_{xx}[m] = R_{xx}[-m]$$

(even)

/ /

$$\frac{0.3}{3} e^{\pm j\frac{2\pi}{3}} \Rightarrow \frac{0.3}{3}\underbrace{\left(e^{+j\frac{2\pi}{3}} + e^{-j\frac{2\pi}{3}}\right)}_{2\cos(2\pi/3)}$$

$$= 0.5 + 0.2\cos(2\pi/3) + \frac{0.2}{3}\cos(4\pi/3)$$

$$= 0.5 + 0.2\cos(4\pi/3) + \frac{0.2}{3}\cos(8\pi/3)$$

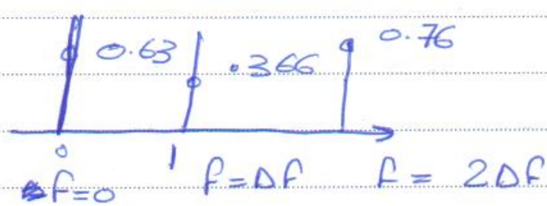
$$\cancel{S_{xx}[k] = 0.5 + \sum_{k=1}^{2} \cos\left(k \cdot \frac{2\pi}{3}\right)}$$

$$S_{xx}[k] = \begin{bmatrix} 0.633 \\ 0.366 \\ 0.76 \end{bmatrix} \approx>$$ تأكدما من الأرقام ?

Total power هو ال power spectral density لـ مجموع ال

1.5 تقريبا

$S_{xx}[k]$



resolution بزيد ال

Probability

NoteBook

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الثلاثاء

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* System Response to R.P :-

R.P $x(t)$ → $\boxed{\begin{matrix} h(t) \\ H(f) \end{matrix}}$ → $y(t)$? $R_{yy}(\tau)$? $S_{yy}(f)$

$R_{xx}(\tau)$
$S_{xx}(f)$

$y(t) = x(t) * h(t)$
x & y are dependent
y(t) could be Uncorrelated

* using a certain system , we can decorrelate X

$R_{yy}(\tau) = A\,\delta(\tau)$
$S_{yy}(f) = A \quad -\infty \le f \le \infty$

This system is called Whitening Filter

white → يحتوي كل الترددات من $-\infty \leftarrow \infty$

X could be colored process & we generate a white process by whitening process.

if $R_{xx}(\tau) = B\,\delta(\tau)$

$S_{xx}(f) = B \quad -\infty \le f \le \infty$

& y has

$R_{yy}(\tau)$?
$S_{yy}(f)$

This Filter we called it Innovation Filter

$$R_{yy}(\tau) = E\{y(t) \cdot y^*(t+\tau)\}$$

$$= E\{(x(t) * \underbrace{h(t)}_{\text{constant}}) \cdot (x(t+\tau) * h(t+\tau))\}$$

(الذي يكامل بالنسبة لـ X مو لـ t)

$$= E\{(\int x(v)\,h(v+\tau)\,dv)(\int x(u+\tau)\,h(u-\tau-v)\,du)\}$$

E Linear operator → ندخله على التكامل

$$R_{yy}(\tau) = h(t) * R_{xx}(\tau) * h^*(t) \quad --- \quad (*) \text{ معادلة}$$

(*) In digital Form :

$$R_{xx}[m] = E\{\vec{X} \cdot \vec{X}_m\}$$

$$\vec{Y} = H \cdot \vec{X}$$

$$R_{yy} = E\{H\vec{x} \cdot (H\vec{x})^*\}$$
$$= HE\{x\, \vec{x}_m\} H^*$$

$$\Rightarrow R_{yy} = H R_{xx} H^*$$

لونرجع لمعادلة (*) بتصير

هاي اللي بتهمك

$$\boxed{S_{yy}(f) = |H(f)|^2 S_{xx}(f)}$$

$$R_{yy} = H R_{xx} H^*$$

$$h = [h_1 \; h_2 \; \cdots \; h_N]$$

$$H = \begin{bmatrix} h_0 & h_1 & \cdots & h_N & 0 & 0 & 0 \\ & h_0 & h_1 & \cdots & & & 0 \\ & \varnothing & & \ddots & & & \\ 0 & 0 & 0 & & h_0 & h_1 & \cdots\; h_N \end{bmatrix}$$

* Whiting filter

✓ $S_{yy}(f) = \text{constant} = |H(f)|^2 \cdot S_{xx}(f)$

$$\Rightarrow |H(f)|^2 = \frac{k}{S_{xx}(f)}$$

✓ $R_{yy}(\tau) = k \cdot \delta(\tau)$ Uncorrelated R.P

white noise

Thermal noise in electronic ckts is a white noise with pdf $N(\sigma^2, 0)$

$$|H(f)|^2 = H(f) \cdot H^*(f)$$

* Innovation Filter

white noise $\begin{cases} S_{xx}(f) = k \\ R_{xx}(\tau) = k\,\delta(\tau) \end{cases}$

$$S_{yy}(f) = |H(f)|^2 \cdot k$$

$$R_{yy}(\tau) = k\, FT^{-1}(|H(f)|^2)$$

$$= k\, h(\tau) * h^*(\tau)$$

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الأربعاء :)

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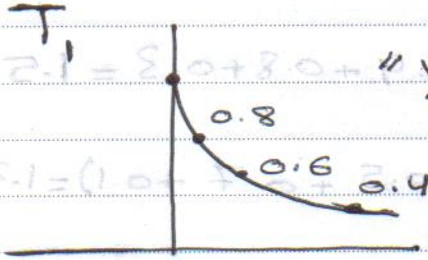
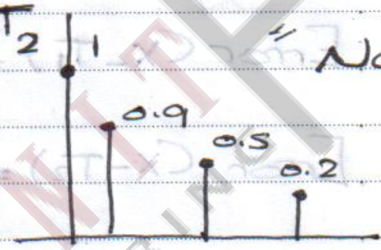
* Ex :-

$$X = [1\ 2\ 3\ -1\ -5\ 3\ 7\ 2\ 1\ 2\ 3\ 4]$$

T_1 "yes"



T_2 "No"



$$R_{xx}[m] = E\{X[n] \cdot X[n+m]\}$$

$$R_{xx}[0] = \frac{1}{12}\sum_{n=1}^{12} X[n]\,[n+0] = \frac{132}{12} = 11 \Rightarrow \underbrace{\frac{11}{11}}_{\text{Normalized}}$$

$$R_{xx}[1] = \frac{+52}{12} = \frac{4.33}{11}$$

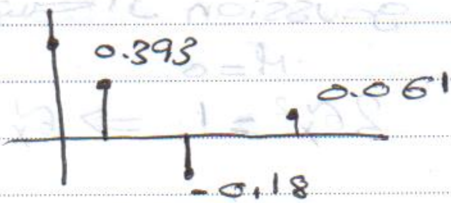
$$\Rightarrow [0\ 0\ 1\ 2\ 3\ -1\ -5\ 3\ 7\ 2\ 1\ 2\ 3\ 4]$$

$R_{xx}[2]$

$$R_{xx}[2] = \frac{-24}{12} = \frac{-2}{11}$$

$$R_{xx}[3] = \frac{0.67}{11} \Rightarrow [0\ 0\ 0\ 1\ 2\ 3\ -1\ -5\ 3\ 7\ 2\ 1\ 2\ 3\ 4]$$

X



* Fiitness ?!

$$Error = \sum | R_{template}[m] - R_{xx}[m] |^2$$

$$Error(X\_T_1) = \sum 0 + 0.4 + 0.8 + 0.3 = 1.5$$

$$Error(X - T_2) = \sum (0 + 0.5 + 0.7 + 0.1) = 1.3$$

Threshold ex = 0.5 ~ both values have to eq 0.5 or if we have to choose between them choose less ex

~ better to devide by $(N-M)$ Not N

⊛ for small data & small m :

$$R_{xx}[m] = \frac{1}{N-m} \sum_{i=1}^{N} x[n]\, x[n+m]$$

↑

biased

if N ⇒ un biased (for large data)

* $5 \& e^{-x^2}$ ⇒ شكله

يشبه ال gaussian

$\mu = 0$

$2\sigma_x^2 = 1 \Rightarrow \sigma_x^2 = \frac{1}{2}$

$$\sum \frac{3^n}{n!} = ?$$

e^3