26/40 13/20

The University of Jordan

School of Engineering

Department of Electrical Engineering

Fall Term – A.Y. 2016-2017

Probability and Random Variables, EE321, First Exam

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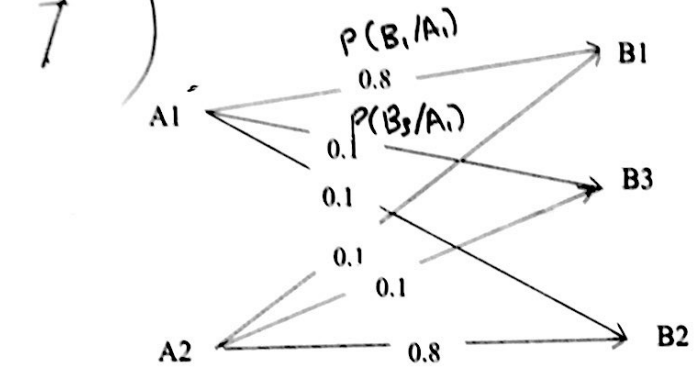
Q1. For the communication channel shown. A1 and A2 are sent are sent at the transmitter , the received values are B1 B2 and B3. P(A1)=0.4 and P(A2)=0.6.

7



$P(B_1/A_1) = .8$
$P(B_2/A_2) = .1$
$P(B_3/A_1) = .1$
$P(B_3/A_2) = .1$
$P(B_2/A_1) = .1$
$P(B_2/A_2) = .8$

Find:

a) The P(B1), P(B2) and P(B3).
b) Given that the received value is B3 find the probability that it came from A1.
c) Knowing that B1 and B2 are the correct reception of A1 and A2, respectively. Find the probability of error for this channel.

a) $P(B_1) = P(B_1/A_1)\,P(A_1) + P(B_1/A_2)\,P(A_2)$
$= 0.38$

$P(B_2) = P(B_2/A_1)\,P(A_1) + P(B_2/A_2)\,P(A_2)$
$= 0.52$

$P(B_3) = P(B_3/A_1)\,P(A_1) + P(B_3/A_2)\,P(A_2)$
$= 0.1$

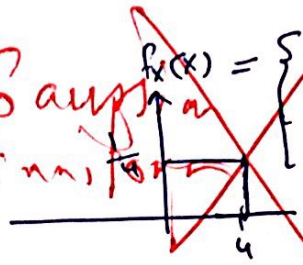
b) $P(A_1/B_3) = \frac{P(B_3/A_1)\,P(A_1)}{P(B_3)}$

$= \frac{.1 * .4}{.1} = 0.4$

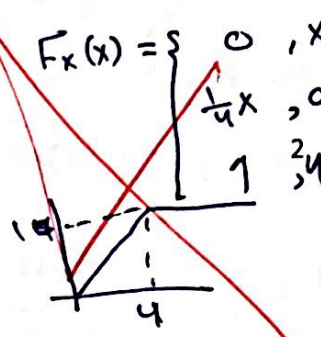
c) $P_{error} = P(B_3) + P(B_3/A_2)$

This Gaussian is not uniform

$f_X(x) = \begin{cases} 0, & x<0 \\ \frac{1}{4}, & 0<x<4 \\ 0, & 4<x \end{cases}$



$F_X(x) = \begin{cases} 0, & x<0 \\ \frac{1}{4}x, & 0<x<4 \\ 1, & 4<x \end{cases}$



Q2. If X is N(0,4); find,

3

a) 1) $P\{1 \le X < 7 / (x > 1)\}$ 2) $P\{0.9 < \sqrt{x} < 6\}$

b) $f_X(x/\{1 \le x \le 2\})$

a) 1) $P\{(1 \le x < 7) / (x > 1)\} = \dfrac{P\{(1 \le x < 7) \cap (x > 1)\}}{P\{(x > 1)\}}$

$= \dfrac{F_X(7) - F_X(1)}{1 - F_X(1)}$

$= \dfrac{1 - \frac{1}{4}}{1 - \frac{1}{4}}$

$= 1$

2) $P\{0.9 < \sqrt{x} < 6\}$

$= P\{0.81 < x < 36\}$

$= F_X(36) - F_X(0.81)$

$= 0.7975$

b) $f_X(x / 1 \le x \le 2) = \begin{cases} \frac{f_X(x)}{F_X(2) - F_X(1)}, & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$

$\dfrac{f_X(x)}{F_X(2) - F_X(1)}$

$\dfrac{\frac{1}{4}}{}$

$f_X(x / 1 \le x \le 2) = \begin{cases} 0, & x<0 \\ \frac{\frac{1}{4}x}{\frac{1}{4}}, & 1 \le x \le 2 \\ 0, & 4<x \end{cases}$

$f_X(x / 1 \le x \le 2) = \begin{cases} 0, & x<1 \\ x, & 1 \le x \le 2 \\ 0, & 2<x \end{cases}$

Q2

$X \sim N(0,4)$ (a_x, σ_x^2)

a)

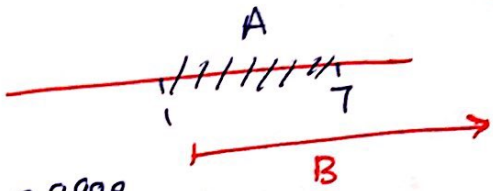
1. $P\{(1 \le X \le 7) / (X > 1)\} = \dfrac{P\{(1 \le X \le 7) \cap (X > 1)\}}{P\{X > 1\}}$

$= \dfrac{P\{1 \le X \le 7\}}{P\{X > 1\}}$

$= \dfrac{F_x(7) - F_x(1)}{1 - F_x(1)}$

$F_x(7) = F\left(\frac{7-0}{2}\right) = 0.9998$

$F_x(1) = F\left(\frac{1-0}{2}\right) = 0.6915$



$A \cap B = A$

$= 0.9993$

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2. $P\{0.9 < \sqrt{X} < 6\}$

$= P\{0.81 < X < 36\}$

$= F_x(36) - F_x(0.81)$

$= F(18) - F(0.405)$

$= 1 - 0.6591$

$= 0.3409$

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b) $f_x(X / (1 \le X \le 2))$

$f_x(X) = \dfrac{1}{\sqrt{2\pi\sigma_x^2}} e^{-(X-a_x)^2/2\sigma_x^2}$

$= \dfrac{1}{\sqrt{8\pi}} e^{-X^2/8}$, $-\infty < X < \infty$

$F_x(2) = F(1) = 0.8413$

$f_x(X / (1 \le X \le 2)) = \begin{cases} \dfrac{f_x(x)}{\int_1^2 f_x(x)\,dx} & , 1 \le X \le 2 \\ 0 & , o.w \end{cases}$

$= \begin{cases} \dfrac{f_x(X)}{F_x(2) - F_x(1)} \\ 0 \quad , o.w \end{cases}$

$= \begin{cases} \dfrac{1}{\sqrt{8\pi}(0.8413 - 0.6915)} e^{-X^2/8} & , 1 \le X \le 2 \\ 0 & , o.w \end{cases}$

b) ~~$P_{error} = P(B_3) + P(B_3/A_2) + P(B_2/A_1)$~~

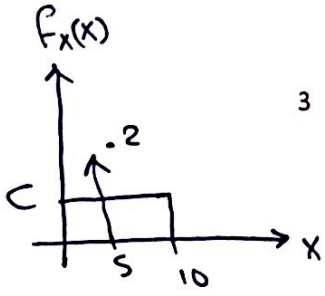
c) $P_{error} = \frac{.4}{2} = .2$

(correct)
$A_1 \Longrightarrow B_1$
$A_2 \longrightarrow B_2$

c)
$$P_e = P(B_2/A_1)\,P(A_1) + P(B_1/A_2)\,P(A_2) + P(B_3/A_1)\,P(A_1) + P(B_3/A_2)\,P(A_2)$$

$P_e = 0.2$

$P_{correct} = 1 - P_e = 0.8 = P(B_1/A_1)\,P(A_1) + P(B_2/A_2)\,P(A_2)$



Q3. A random variable X has the following density function:

$$f_X(x) = C[u(x) - u(x-10)] + 0.2\delta(x-5)$$

a) Find the constant C such that $f_X(x)$ is a valid density function.
b) Find and sketch $F_X(x)$.
c) Evaluate $P\{1 < x < 5\}$.
d) Evaluate $P\{x \leq 5\}$.

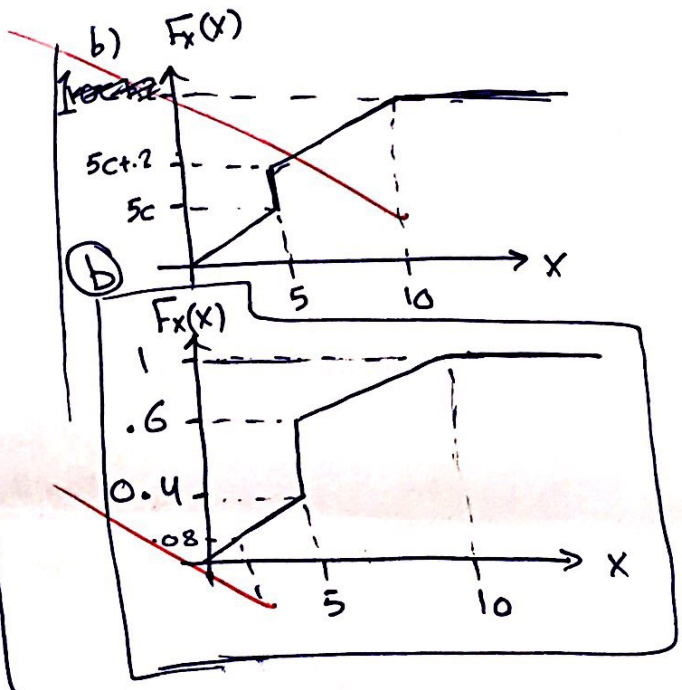
a) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$= \int_{0}^{10} C\,dx + .2\,u(x-5) = 1$

$10C + .2 = 1$

a) $C = 0.08$

b) $F_X(X)$



c) $P\{1 < x < 5\}$

$= F_X(5) - F_X(1)$

$= .08 * 5 - .08$

$= 0.32$

d) $P\{x \leq 5\}$

$= F_X(5)$

$= 0.6$

$$F_X(x) = \begin{cases} .08x & , 0 < x < 5 \\ .08x + .2 & , 5 \leq x < 10 \\ 1 & , 10 < x \end{cases}$$

6x6

1
2

N = n

6, 1 1, 6
5, 2 2, 5
4, 3 3, 4

$P(7) = \frac{6}{36}$

Q4. A pair of dice is rolled n times.

a) Find the probability that "seven" will not show at all.
b) Find the probability of obtaining double six at least once.
c) For part b), find the minimum number n of throws required to assure a 50% success of obtaining double six at least ones.

a) P(7 will show from n=1) = 6/36

P(7 will not show from n=1) = 30/36

P = 30/36 → Probability a "7" will not be shown

$P(n=0) = \binom{n}{0} p^n (1-p)^{N-n}$

$= N$

$P(k=0) = \binom{n}{0} p^0 (1-p)^n$

$P(k=0) = \left(\frac{6}{36}\right)^n$

n: # of all occurances

k: # of 7's

b) a) $P = \frac{30}{36}$ (Probability 7 will not)

A: event that 7 will not be shown

$P(A) = \binom{n}{0} p^0 (1-p)^n$

$P(A) = \left(\frac{6}{36}\right)^n$

$\left(\frac{30}{36}\right)^n$

B: obtaining double six

b) $P = \frac{1}{36}$

$P(B) = \binom{n}{1} p^1 (1-p)^{n-1}$

$P(B) = \frac{n!}{(n-1)!}\left(\frac{1}{36}\right)^1\left(\frac{35}{36}\right)^{n-1}$

c) $P(B) = .5$

$\left(\frac{35}{36}\right)^{n-1} =$

$\frac{1}{2} = \frac{n!}{(n-1)!}\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{n-1}$

$18\left(\frac{36}{35}\right)^{-n} = \frac{n!}{(n-1)!}$ ⇒

c) $\frac{1}{2} = \frac{n}{36}\left(\frac{35}{36}\right)^{n-1}$

$\frac{1}{2} = \frac{n}{36} * \frac{36}{35} * \left(\frac{35}{36}\right)^{n}$

$17.5 = n\left(\frac{35}{36}\right)^{n}$

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a) A ≡ Seven occurs

$f_7 \rightarrow f_{16}, f_{25}, f_{34}$
$f_{61}\ \ f_{52}\ \ f_{43}$

$P(A) = \frac{1}{6}$

bernoulli trials.

$P\{A \text{ happens o times of n trials}\} = \binom{n}{0} P^{0}(A)\, P^{n}(\bar{A})$

$= \left(\frac{5}{6}\right)^{n}$

b) B ≡ double six

$P(B) = \frac{1}{36}$, $P(\bar{B}) = \frac{35}{36}$

$P\{B \text{ never happens o times of n trials}\} = \binom{n}{0} P^{0}(B)\, P^{n}(\bar{B})$

$= \left(\frac{35}{36}\right)^{n}$

$P\{B \text{ happens at least once}\} = 1 - \left(\frac{35}{36}\right)^{n}$

c)

$1 - \left(\frac{35}{36}\right)^{n} > 1/2$

$\left(\frac{35}{36}\right)^{n} > \frac{1}{2}$

$n = \frac{\log 2}{\log 36 - \log 35}$

$= 24.6$

$n = 25$

Table B-1 Values of $F(x)$ for $0 \leq x \leq 3.89$ in steps of 0.01

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000