Probability
Fall 16
*power_unit
Dr. Deya' Abdnadi

Q1 $R_{xx}(\tau)$ is given, WSS process, ~~Atott~~ Gaussian

~~(1) mean, variance P{X<~~ $R_{xx}(\tau) = 4e^{-2|\tau|}$

1) $P\{X(t) \leq 3\}$

2) $E[(X(t+1) - X(t-1))^2]$

Q2 $R_{xx}(\tau) = 36 + 25e^{-\tau}$, WSS process

find
1) mean
2) Variance
3) $C_{xx}(\tau)$
4) AC power
5) DC power

Q3

$$F_X(x) = \begin{cases} 0 & , x < 0 \\ 0.1x & , 0 \leq x < 3 \\ 0.4x - 0.6 & , 3 \leq x < 5 \\ 1 & , 5 \leq x \end{cases}$$

هي 5 مش مهمة

find
1) $f_X(x)$
2) $P\{1 < X < 3\}$
3) $P\{0 < X \leq 3\}$

Q4 $I \sim U(-5, 10)$
$R = 1000\,\Omega$
find $f_P(p)$, $\bar{P}$

Q5

$$Y = T(X) = \begin{cases} -2 & , \leq X \leq -2 \\ 0 & , -2 < X < 1 \\ 1 & , 1 < X < 4 \\ 6 & , 4 < X \end{cases}$$

$f_X(x) \sim U(-4, 12)$
find 1) $f_Y(y)$ 2) $F_Y(y)$
3) mean 4) Variance

Q_1 $R_{xx}(\tau) = 4e^{-2|\tau|}$

1) $P\{X(t) \leq 3\} = F_X(3) = F\left(\frac{3-\bar{X}}{\sigma_x}\right) = F\left(\frac{3-0}{2}\right) = F(1.5)$ from table

$f_X(x) \sim N(\bar{X}, \sigma_x^2)$

$\lim_{|\tau|\to\infty} R_{xx}(\tau) = \bar{X}^2$

$4e^{-2(\infty)} = \bar{X}^2$

$\boxed{\bar{X} = 0}$

$\sigma_x^2 = m_2 - \cancel{m_1^2}^{\,0}$

$m_2 = E[X^2(t)] = R_{xx}(0)$

$\boxed{m_2 = 4}$

$\boxed{\sigma_x^2 = 4}$

2) $E[(X(t+1) - X(t-1))^2]$

$= E[X^2(t+1) + X^2(t-1) - 2X(t+1)X(t-1)]$

$= R_{xx}(0) + R_{xx}(0) - 2R_{xx}(2)$

$= 2(4) - 2(4e^{-4})$

$= 8 - 8e^{-4}$

Q_2 $R_{xx}(\tau) = 36 + 25e^{-\tau}$, WSS

1) $\bar{X}$

$\lim_{|\tau|\to\infty} R_{xx}(\tau) = \bar{X}^2$

$\bar{X}^2 = 36$

$\boxed{\bar{X} = \pm 6}$

2) $\sigma_x^2 = m_2 - m_1^2$

$m_2 = E[X^2(t)] = R_{xx}(0)$

$m_2 = 61$

$\sigma_x^2 = 61 - 36$

$\boxed{\sigma_x^2 = 25}$

3) $C_{xx}(\tau) = R_{xx}(\tau) - \bar{X}^2$

$\boxed{C_{xx}(\tau) = 25e^{-\tau}}$

4) AC Power = 25 watt

5) DC power = 36 watt

$\sigma_x^2 = m_2 - m_1^2$

AC power

total power

DC power

Q_3

1) $f_X(x) = \begin{cases} 0 & , \ x<0 \\ 0.1 & , \ 0 \leq x < 3 \\ 0.4 & , \ 3 \leq x < 5 \\ 0 & , \ 5 \leq x \end{cases}$

2) $P\{1 \leq x < 3\} = F_X(3) - F_X(1)$

$= 0.1(3) - 0.1(1)$

$= 0.2$

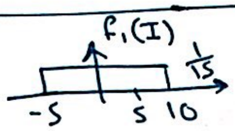
3) $P\{0 < x \leq 3\} = F_X(3) - F_X(0)$

$= 0.4(3) - 0.6$

$= 0.6$

Q_4

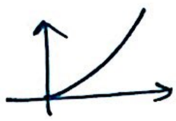
$I \sim U(-5, 10)$



, $R = 1000$

$P = I^2 R$

$P = 1000 I^2$



monotonically increasing

$f_P(P) = f_I(T^{-1}(P)) * \left| \frac{dT^{-1}(P)}{dp} \right|$

$T^{-1}(P) = \frac{P^{\frac{1}{2}}}{10\sqrt{10}}$

$= \frac{1}{15} * \frac{1}{2} \frac{P^{-\frac{1}{2}}}{10\sqrt{10}}$

هاي عشان
من $-5 \to 5$

$$\boxed{f_P(P) = \begin{cases} \frac{*2P^{-\frac{1}{2}}}{300\sqrt{10}} & , \ 0 < P < 25k \\ \frac{P^{-\frac{1}{2}}}{300\sqrt{10}} & , \ 25k < P < 100k \end{cases}}$$

$I^2 R$

$1000(\pm 5)^2$

$1000(10)^2$

$\bar{P} = E[1000 I^2] = \int_{-5}^{10} 1000 I^2 * \frac{1}{15} \, dI$

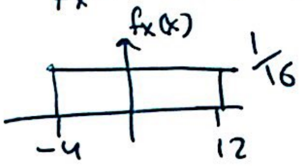
$= \frac{1000}{3(15)} I^3 \Big|_{-5}^{10}$

$\boxed{\bar{P} = 25 kW}$

Q5

$$Y = T(x) = \begin{cases} -2, & x \le -2 \\ 0, & -2 < x \le 1 \\ \cancel{2}1, & 1 < x < 4 \\ 6, & 4 < x \end{cases}$$

$f_x(x) \sim U(-4, 12)$



$$f_x(x) = \begin{cases} 1/16, & -4 < x < 16 \\ 0, & o.w \end{cases}$$

1) $f_y(y) = \sum P\{Y_n\}\,\delta(y - y_n) \quad \cancel{P\{Y_n\}\delta(y-y_n)} \rightarrow P\{Y_n\} = P\{X_n\}$

$$f_y(y) = \frac{2}{16}\delta(y+2) + \frac{3}{16}\delta(y) + \frac{3}{16}\delta(y-1) + \frac{8}{16}\delta(y-6)$$

$Y = -2$

$P\{X \le -2\} = F_X(-2)$

$\cancel{F_X(-2)} = \frac{2}{16}$

$Y = 0$

$P\{-2 < X < 1\}$

$= F_X(1) - F_X(-2)$

$= \frac{5}{16} - 2/16 = 3/16$

$Y = 1 \rightarrow P\{1 < X < 4\} = F_X(4) - F_X(1)$

$= \frac{8}{16} - \frac{5}{16} = 3/16$

$Y = 6 \rightarrow P\{4 < X\} = 1 - F_X(4)$

$= 1 - 8/16 = 8/16$

* to ~~check~~ Check add them up = 1

$$F_X(x) = \begin{cases} 0, & x < -4 \\ \frac{x+4}{16}, & -4 < x < 16 \\ 1, & 16 < x \end{cases}$$

~~Y=2~~
~~P{~~

2) $F_y(y) = \sum P(y_n)\, u(y - y_n)$

$$F_y(y) = \frac{2}{16}u(y+2) + \frac{3}{16}u(y) + \frac{3}{16}u(y-1) + \frac{8}{16}u(y-6)$$

3) $\bar{Y} = E[Y] = \sum Y_n\, P\{y_n\}$

$$\bar{Y} = (-2)\left(\frac{2}{16}\right) + 0\left(\frac{3}{16}\right) + 1\left(\frac{3}{16}\right) + 6\left(\frac{8}{16}\right)$$

$$\boxed{\bar{Y} = \frac{47}{16}}$$

4) $\sigma_y^2 = m_2 - m_1^2$

$$= \frac{299}{16} - \left(\frac{47}{16}\right)^2$$

$$\boxed{\sigma_y^2 = \frac{2575}{256} = 10.05}$$

$$E[Y^2] = (-2)^2\left(\frac{2}{16}\right) + (0)^2\left(\frac{3}{16}\right) + (1)^2\left(\frac{3}{16}\right) + (6)^2\left(\frac{8}{16}\right)$$

$$= 299/16$$

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