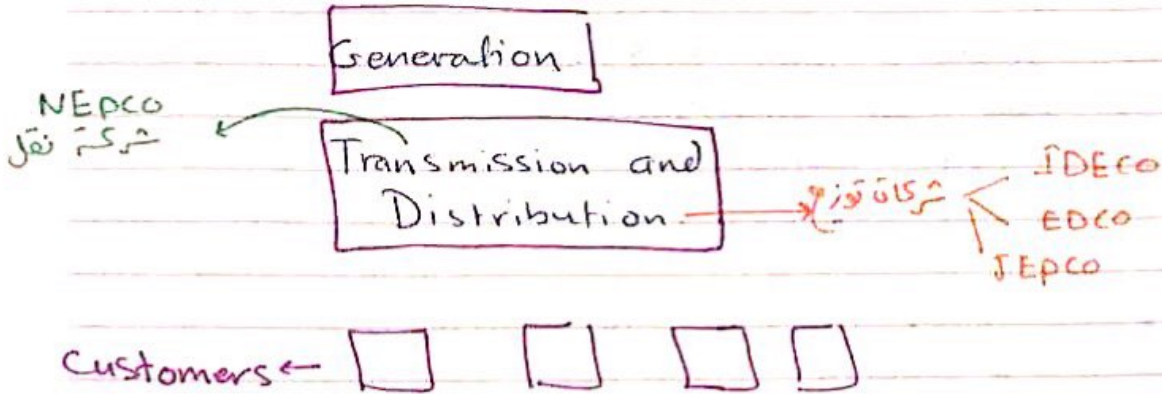


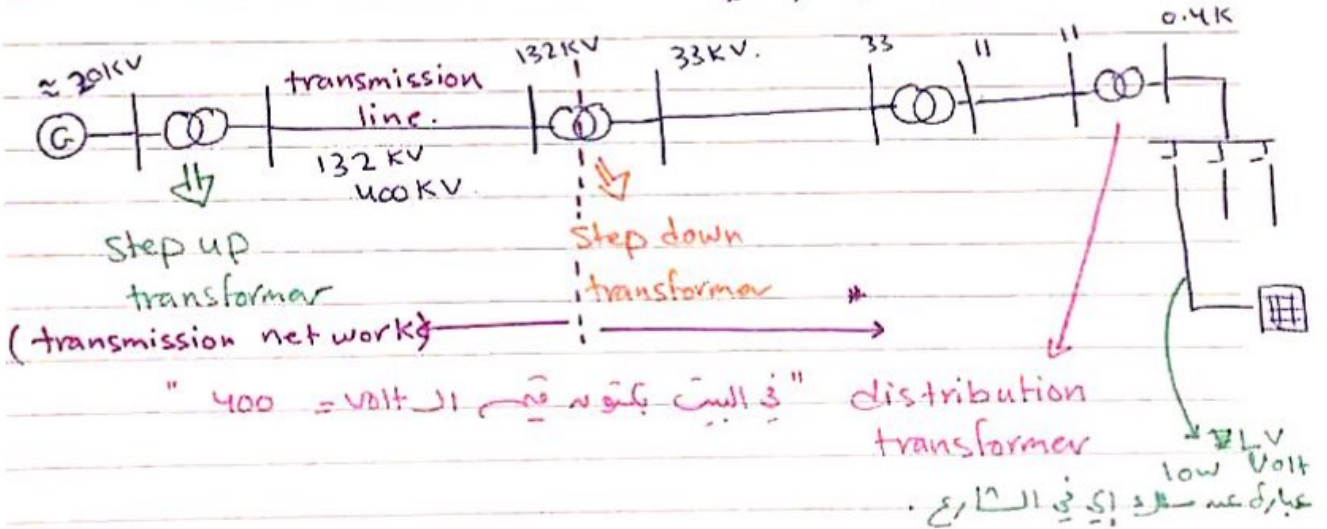
Power system :-



- customers
 - residential
 - Commercial
 - industrial

Fuel → (G) → electrical.

NEPCO → . Gen وبتبيع لـ



* distribution transformer (33 kV / 11 kV / 0.6 kV)

ERC Regulator → this performance standard

ERC = تنظيم الكهرباء

* Power system modeling \Rightarrow "transmission lines."

* Power system studies. 8-

- load flow studies
- Fault analysis.

* الكتب - power system analysis.

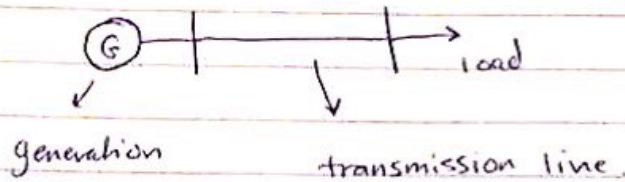
- Saddat
- Tom overbye.

* software \rightarrow power world.

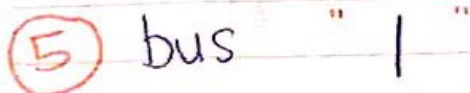
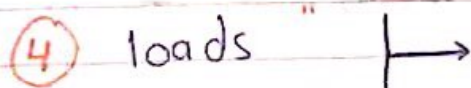
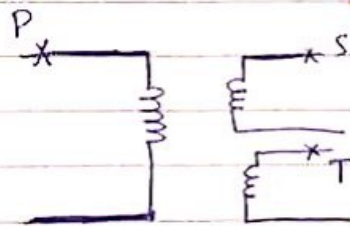
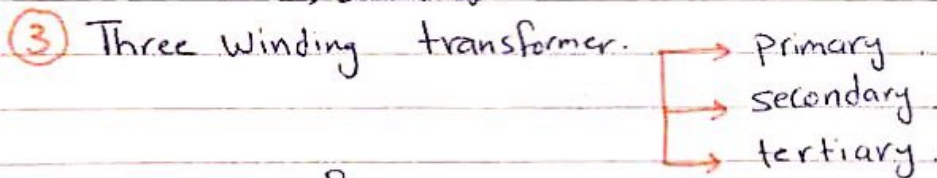
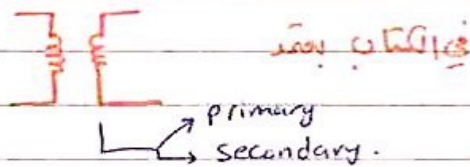
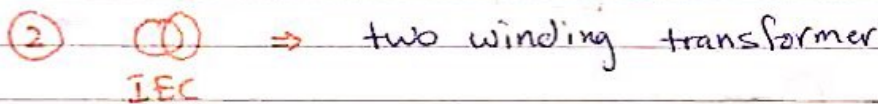
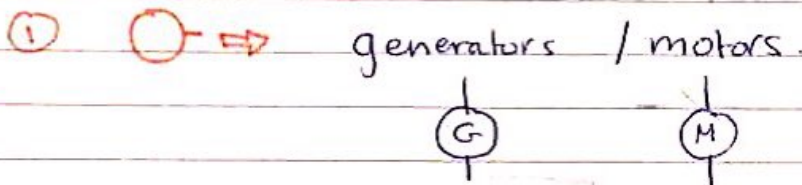
* Single line diagram:-

One-line diagram

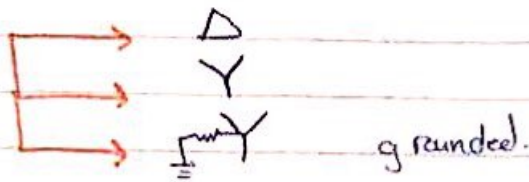
- relative interconnection between generators, transformer, transmission lines, loads.



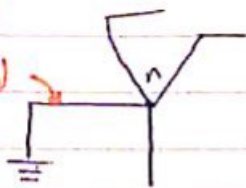
* Symbols:-



⑥ connections



solidly grounded



⑦ earth

* Switching element :-

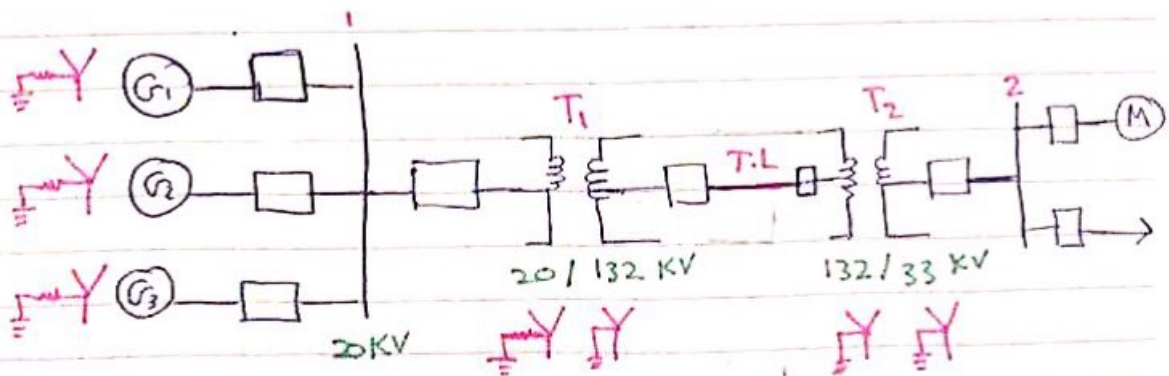
⑧ Isolator

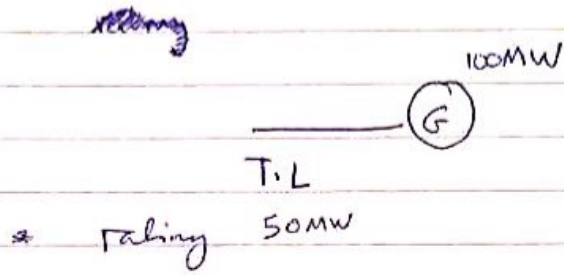
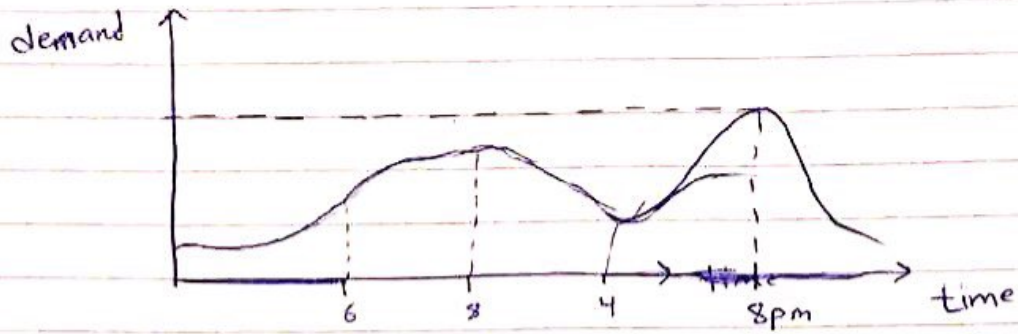
⑨ circuit breaker بقدر احتياج في fault

⑩ Fuse عوائل

- Load break switch
- Recloser
- sectionalizer.

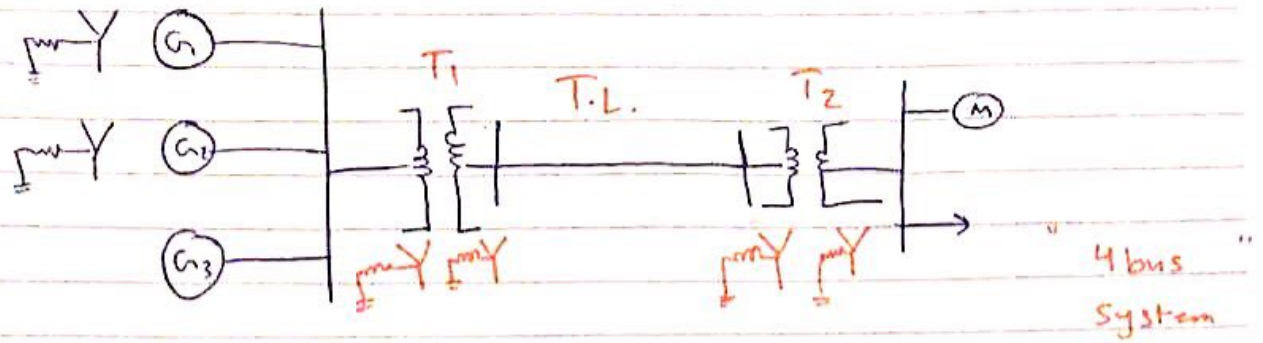
⑪ current transformer
- voltage transformer





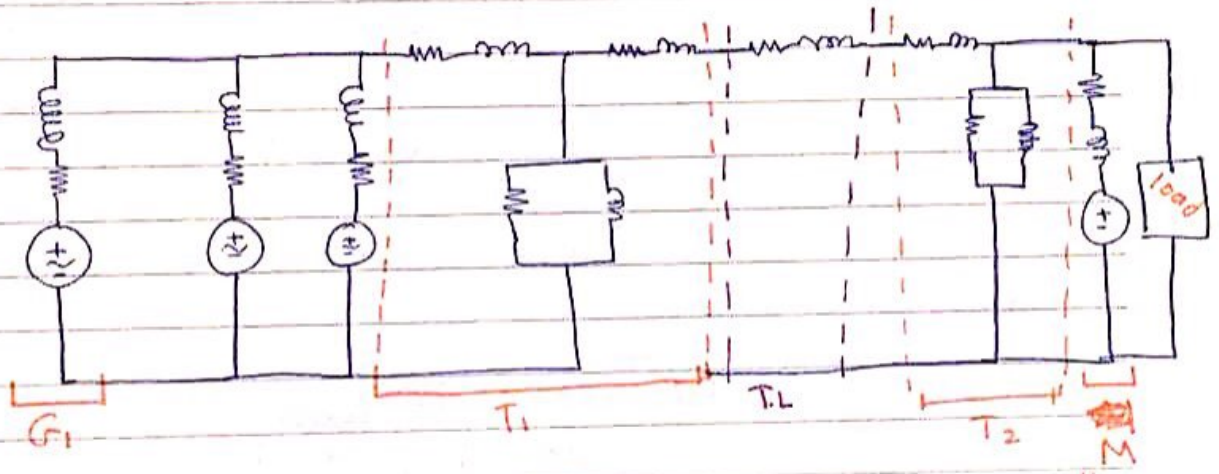
Impedance diagram / reactance diagram:-

9/12 Thur



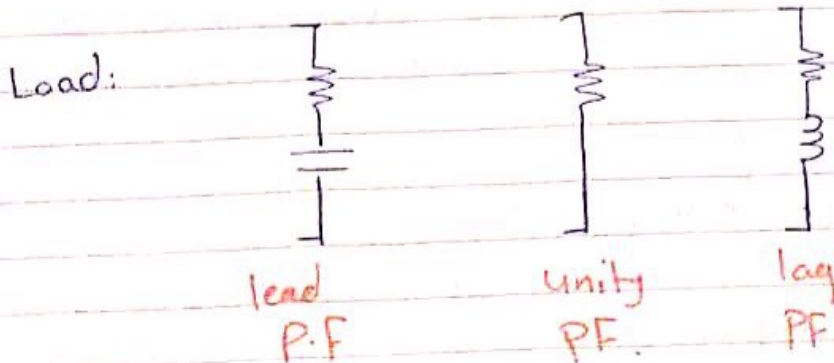
- power flow studies.
- short circuit studies.

* Impedance diagram / Reactance diagram :-



Reference → Neutral
 earth. Balance sys. vs 0 earth. / neutral 0 / 0
 earth. unbalance sys. vs 1 / 0 / 0
 " neutral/earth. / 0 / 0"

LV $R > X \rightarrow R \gg X$ (R=0) / 0 / 0
 HV $\rightarrow R = 0$ (R=0) / 0 / 0
 (Distribution 3) *
 Reactance 1 / 0 / 0



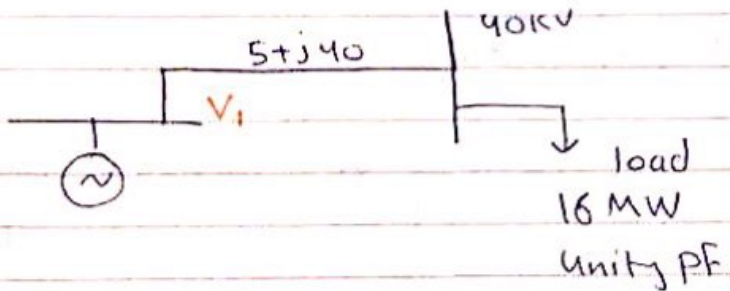
Z const.

$$Z = \frac{|V|}{|I|}$$

$$V \downarrow \rightarrow I \downarrow \rightarrow P \downarrow$$

Single - phase Power System

Ex:

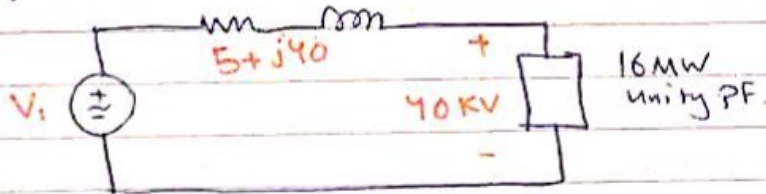


"شركه الـ 100"
3-ph
الـ 100 الـ 100
Single-phase.

Control generator

Sol:

neglecte Reactance.



unity PF
 $S = P$

$$S = V I^*$$

$$I = 400 \angle 0 \text{ A}$$

$$V_1 = 40 \text{ kV} \angle 0 + I (5 + j40)$$

$$V_1 = 45 \angle 24^\circ \text{ kV}$$

(45k = V) لازم بطلع الـ 45
V=40 الـ 40 الـ 45 الـ 45

$$S_{Gen} = V_1 I^*$$

$$S_{Gen} = 16.8 \text{ MW} + j 6.4 \text{ MVAR}$$

T.L losses neglected 0.3

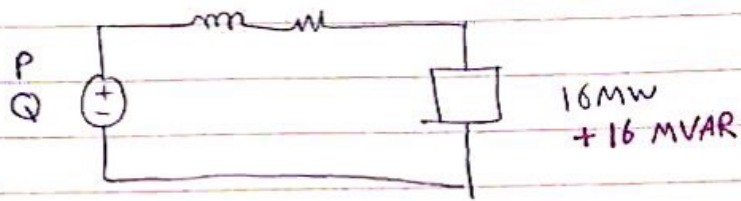
$$P = |V| |I| \cos \theta$$

$$= |I|^2 \left[\begin{array}{l} \angle 0 \rightarrow \text{unity} \\ + \cos \text{PF} \rightarrow \text{lead} \end{array} \right] \rightarrow - \cos \text{PF} \rightarrow \text{lag}$$

7

$S \uparrow \rightarrow I \uparrow \rightarrow \text{real power} \rightarrow P \uparrow$
 $\text{losses} \uparrow$

lossless line $\leftarrow R$ \leftarrow $\text{P}_{\text{const}} \text{ je } m \text{ Go } *$



$$S = V I^*$$

$$I = 565.7 \angle -45^\circ \text{ A} \quad \text{lag PF. (inductive) } Q$$

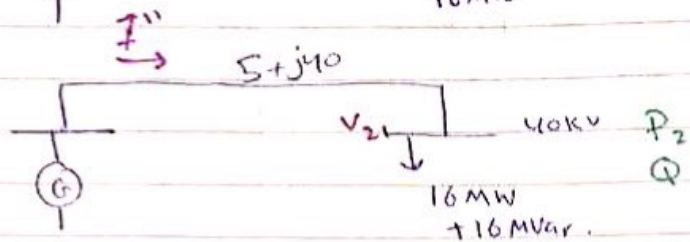
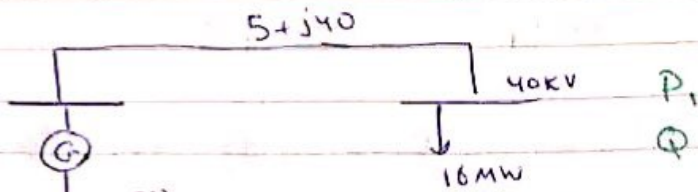
$$V_1 = \uparrow 59.6 \angle 13^\circ \text{ KV.}$$

زیادتر $S \uparrow$ \leftarrow زیادتر $I \uparrow$ \leftarrow $V = 40 \text{ KV}$ \leftarrow لازم از V_1

$$S_{\text{Gen}} = 17.06 + j 28.8 \text{ MVAR}$$

زیادتر Q \leftarrow losses.

$$\uparrow Q \text{ reactive power} \rightarrow \uparrow V_1$$

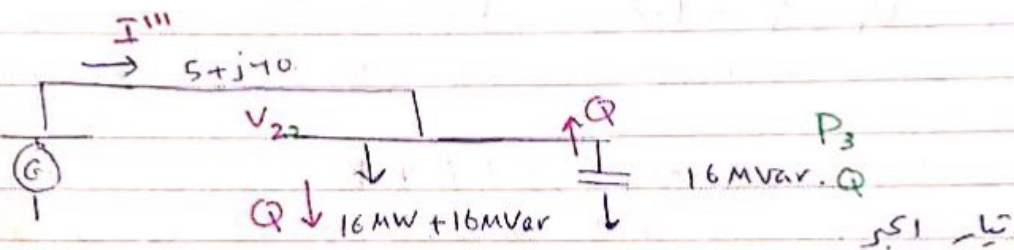


$$S = VI^*$$

$$S \uparrow \quad I \uparrow$$

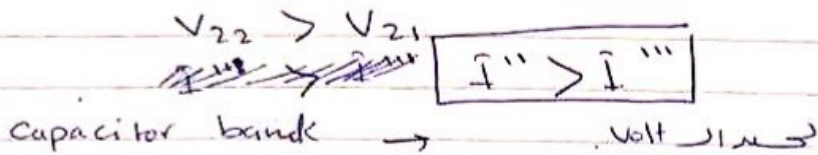
$$I \uparrow \rightarrow I^2 R \uparrow \rightarrow P = R + I^2 R \uparrow \rightarrow (\text{losses} \uparrow)$$

$$P_2 > P_1$$



Capacitor $C \rightarrow Q \uparrow$
 Load $\rightarrow Q \downarrow$

$$P_1 = P_3$$



\uparrow Voltage drop \leftarrow $\mu S, \omega$

Capacitors $X_c = -ve, Y_c = +ve$

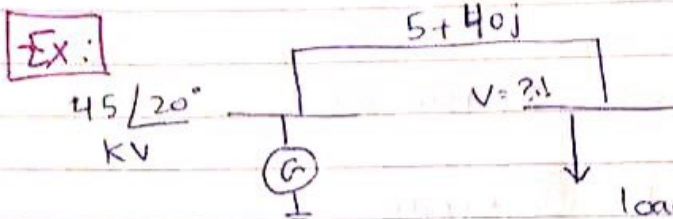
$$Y = g + jb$$

Conductance \swarrow \searrow Susceptance

loads: $\left\{ \begin{array}{l} \text{Constant power.} \\ \text{" impedance } [V \angle \theta, I \angle \phi] \\ \text{" current. } R \end{array} \right.$

$$Z = \frac{V}{I} \rightarrow V \downarrow \rightarrow I \downarrow \rightarrow P \downarrow$$

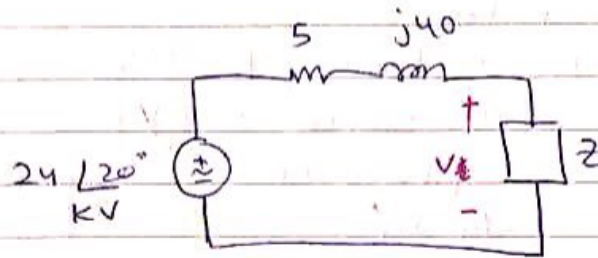
Z constant \Rightarrow volt \downarrow \leftarrow I



load can be modelled as a constant impedance.

single-phase power system
Find V ?

Sol:



load: consume 16 MW
with unity pf
when V is 40 KV
" عن V الارتفاع بـ 40KV "

$$Z = \frac{V^2}{S} = \frac{(40 \text{ K})^2}{16 \text{ M}} = 100 \Omega$$

$V \neq 40 \text{ KV}$ يعني
الارتفاع عن V الارتفاع

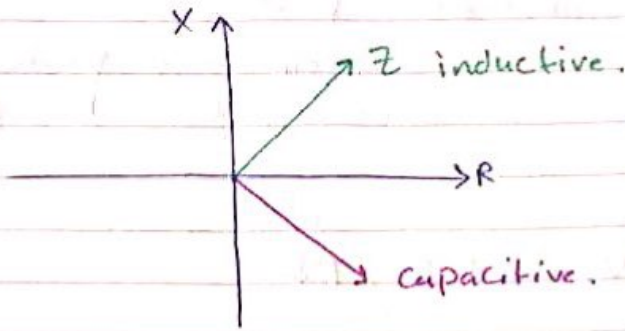
$$V = 45 \angle 20^\circ \times \frac{Z}{Z + 5 + j40}$$

$$V = 40 \angle -20^\circ \text{ KV}$$

if load \uparrow " 16 MW With 0.707 PF lagging).
when V is 40 KV.

$$Z = \frac{(40)^2}{16 / 0.707} = 70.7 \angle + \cos^{-1}(0.707) \text{ (S.P)}$$

10



\angle L -ve lag
 \angle C +ve lead.



* three-phase system. Δ

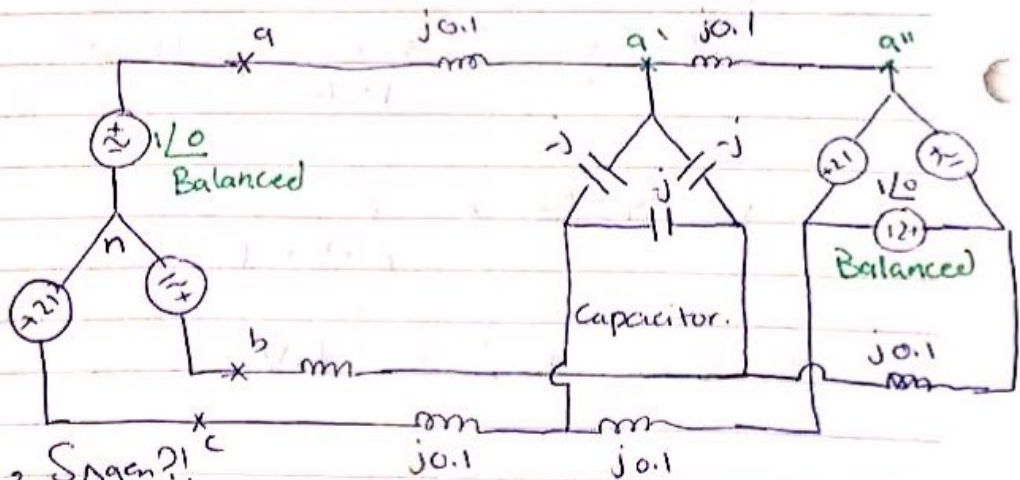
- Balanced System.
- per phase analysis

"load \rightarrow LV \Rightarrow unbalanced"

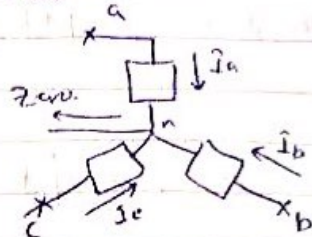
* per phase rules:

- 1 convert Δ loads to Y's
- 2 " " source " Y's
- 3 Solve phase a
- 4 $S = 3 V_a I_a^*$

Ex:



find S_{gen} , S_{load} ?



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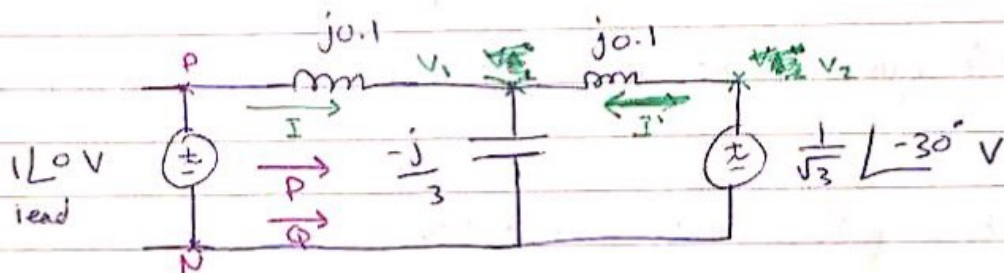
Sol: ① Convert Δ load to Y's

$$Z_Y = \frac{-j}{3}$$

② Convert Δ Source to Y's
 $V_{LL} = \sqrt{3} V_{LN} \angle 30^\circ$

$$V_{LN} = \frac{V_{LL}}{\sqrt{3}} \angle -30^\circ$$

$$V_{LN} = \frac{1}{\sqrt{3}} \angle -30^\circ$$



($Q \Rightarrow HV \rightarrow LV$)
 "Q reactive power flow moves from HV to LV" magnitude.

$P \propto \Delta \delta$ power angle.

$Q \propto \frac{\Delta V}{V}$
 \rightarrow magnitude.

$S_{gen} = 3 V_a I^*$
 Nodal analysis \Rightarrow

$$V_1 = 0.9 \angle -10.9^\circ \text{ V "lag"}$$

"Real power move from lead to lag"

$$S_{gen} = 3 * 1 \angle 0 \left(\frac{1 \angle 0 - 0.9 \angle -10.9}{j0.1} \right)$$

$$= \frac{5.1}{+ve} + j3.5 \text{ VA}$$

(inject real power)

\rightarrow +ve (inject reactive power). (12)

Power "Gen" consumed $\leftarrow (-)$ "Source" \leftarrow 2 sources

$$S_{\Delta gen} = 3 \times \frac{1}{\sqrt{3}} \angle -30^\circ \left(\left(\frac{1}{\sqrt{3}} \angle -30^\circ \right) - 0.9 \angle -10.9^\circ \right) / j0.1$$

$$= -5.1 - j4.7 \text{ VA}$$

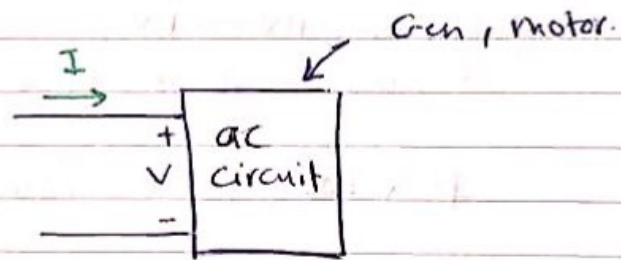
\downarrow -ve (consumed real power)
 \downarrow -ve (consumed reactive power)

* Capacitor \rightarrow Q (Source 2) Source 1 \leftarrow Q (Source 1) \leftarrow Q (Capacitor)

Q (Source 2) \leftarrow Q (Source 1) \leftarrow Q (Capacitor) \leftarrow Q (Source 1) \leftarrow Q (Capacitor)

* Direct of power flows

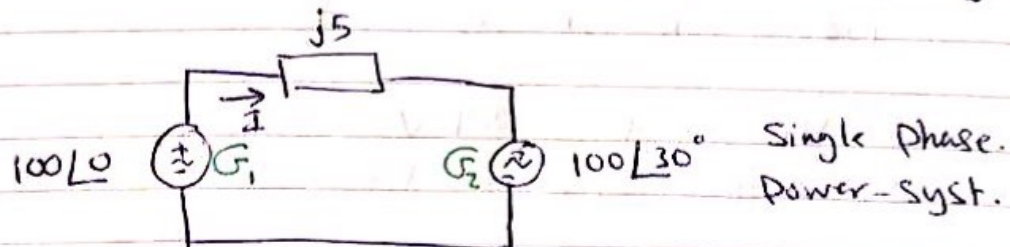
- Passive Sign Convention :-



* consume real power.
 ; reactive power.

"Supply \leftarrow Gen, motor"

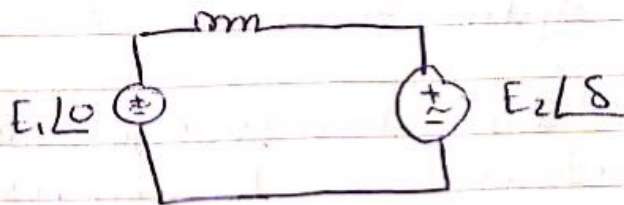
Ex:



which machines is Gen. or consuming real power?
 (real power Gen)
 (consumed \leftarrow Motor)

* H.W

Prove ?
 $P \propto IS$
 $Q \propto \Delta V$



Sol Ex: passive sign convention

$$I = \frac{100 \angle 0 - 100 \angle 30}{j5} = 10.35 \angle 95^\circ \text{ A}$$

$$S_{G_1} = V(-I)^* = 100 \angle 0 * (-I)^* = 1000 - j268 \text{ VA}$$

+ve consume real power. -ve inject reactive power.

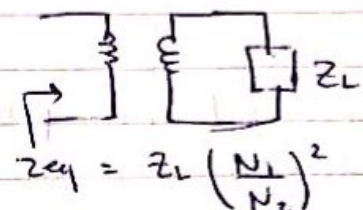
* لا يتم الا فرق الجهد +ve ، P, Q +ve

$$S_{G_2} = V(I)^* = (100 \angle 30) * (10.35 \angle 95)^* = -1000 - j268 \text{ VA}$$

-ve inject real power. -ve inject reactive power. $\angle 95 \rightarrow (\angle 95)^* = \angle -95$

* Perunit System & Δ

$$\text{Per unit} = \frac{\text{actual}}{\text{Base}}$$



$$Z_{L \text{ pu HV}} = Z_{L \text{ pu LV}}$$

* The Per unit system

Power System Quantities (Voltage, Current, power, impedance). can be expressed in per unit (% base value).

$V = 0.97 \text{ pu} \Rightarrow \text{voltage drop} = 3\%$

Calculations:

per unit = $\frac{\text{actual}}{\text{base}}$

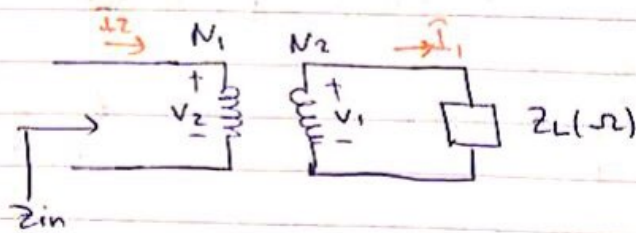
- 1] dimension less
- 2] angle is the same as the original quantity.

Ex: $V = 200 \text{ V} \angle 5^\circ$
 $V_{LN, \text{base}} = 230$

$V_{pu} = \frac{200 \angle 5^\circ}{230} \text{ pu}$

الزيادة ثابتة

* Calculation $\left\{ \begin{array}{l} \rightarrow \text{per unit (V pu, I pu)} \\ \rightarrow \text{actual value.} \end{array} \right.$



$Z_{in} = Z_L \left(\frac{N_1}{N_2} \right)^2$

الاسم \Rightarrow Name plate
 transformer $\left(\begin{array}{l} 33/11 \text{ KV} \\ 25 \text{ MVA} \\ Z = 15\% \end{array} \right)$
 الحثية

\rightarrow per unit (base \Rightarrow rated)

V, I, S, Z

two independent base value can be arbitrarily selected in one point in power system.

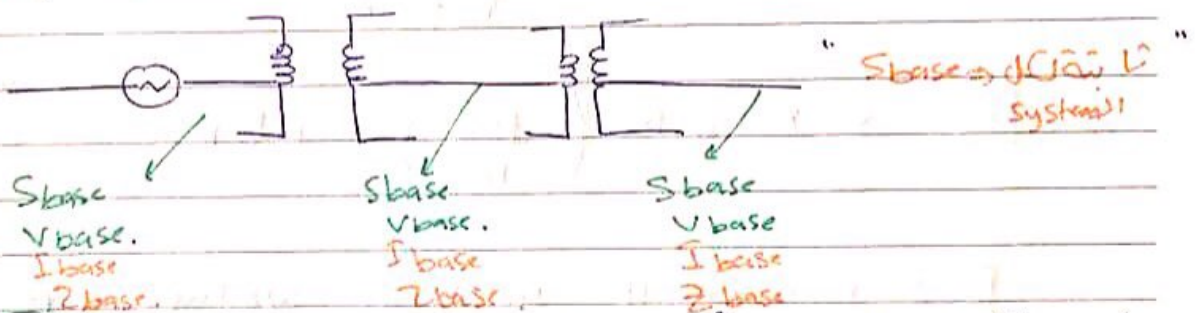
for $1\phi \Rightarrow V_{base}, S_{base}$ "Single phase"

$$I_{base} = \frac{S_{base, 1\phi}}{V_{base}}$$

$$Z_{base} = \frac{V_{base}^2}{S_{base}}$$

* 1ϕ power system

1] Pick $1\phi VA$ for the entire system



2] pick a voltage base for each different voltage level "transformer turns ratio"

to $Z_{pu, old}$ ($S_{base, old}, V_{base, old}$).

$Z_{pu, new}$ ($S_{base, new}, V_{base, new}$).

$$\rightarrow Z_{pu, new} = \frac{Z_{actual}}{Z_{base, new}}$$

$$Z_{pu, new} = \frac{Z_{pu, old} * Z_{base, old}}{Z_{base, new}}$$

$$Z_{pu\ new} = \frac{Z_{pu\ old} * (V_{old}^2 / S_{base\ old})}{(V_{new}^2 / S_{base\ new})}$$

$$Z_{pu\ new} = Z_{pu\ old} * \left(\frac{S_{base\ new}}{S_{base\ old}} \right) * \left(\frac{V_{old}}{V_{new}} \right)^2$$

- Ex:**
- (A) 25 MVA, 33/11 KV transformer, $X = 15\%$
 - (B) 10 MVA, 33/11 KV transformer, $X = 15\%$

Which one will result in higher voltage drop given a base 100 MVA?

Sol:

$$[A] \Rightarrow X = 0.15 * \left(\frac{100}{25} \right) = 0.6\ pu$$

$$[B] \rightarrow X = 0.15 * \left(\frac{100}{10} \right) = 1.5\ pu$$

B \rightarrow will result in large voltage drop.

(Voltage drop أكبر لأن المقاومة أكبر. (المقاومة بزيادة الجهد Voltage drop))

(($\uparrow R \rightarrow \uparrow$ Voltage drop.))

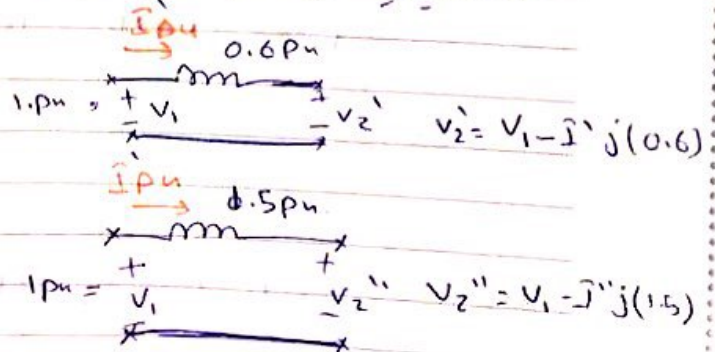
لكي نتأكد من أن الجهد عند الطرف الثاني أكبر من الجهد عند الطرف الأول (بافتراض أن الجهد عند الطرف الأول هو 1 pu) فنحن بحاجة إلى مقارنة الجهد عند الطرف الثاني في كلا الحالتين.

$$V_2' > V_2'' ?!$$

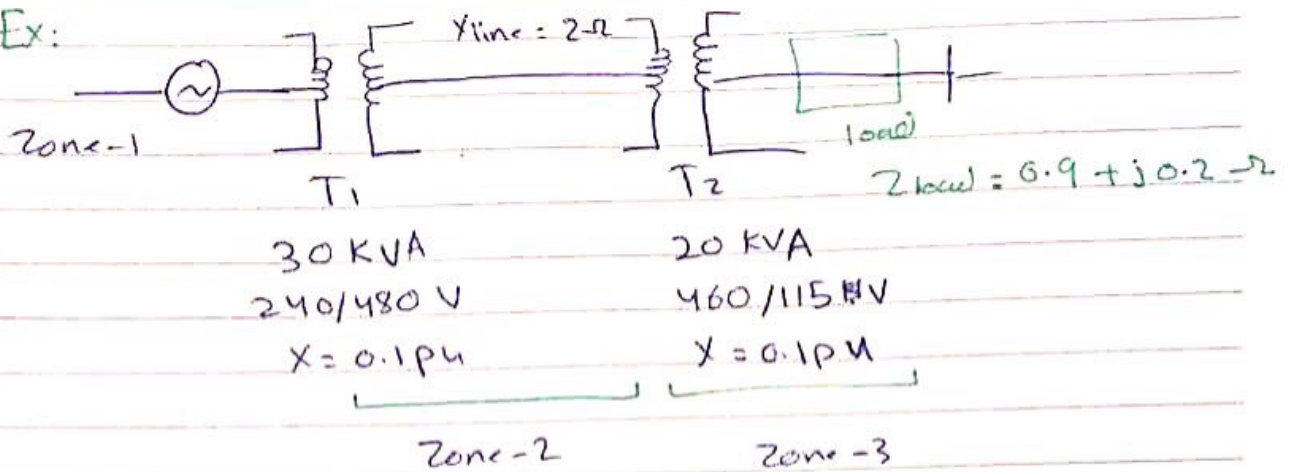
$$\text{Give } I'_{pu} = I''_{pu}$$

$$V_2' > V_2''$$

لأن الجهد عند الطرف الثاني أكبر من الجهد عند الطرف الأول (بافتراض أن الجهد عند الطرف الأول هو 1 pu) فنحن بحاجة إلى مقارنة الجهد عند الطرف الثاني في كلا الحالتين.



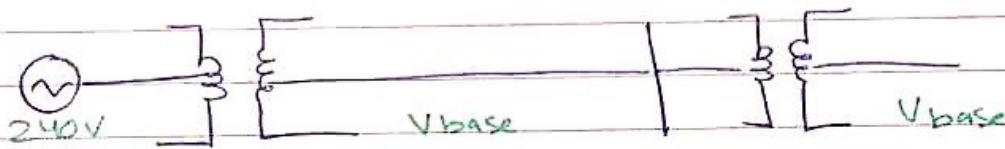
Ex:



"single phase"

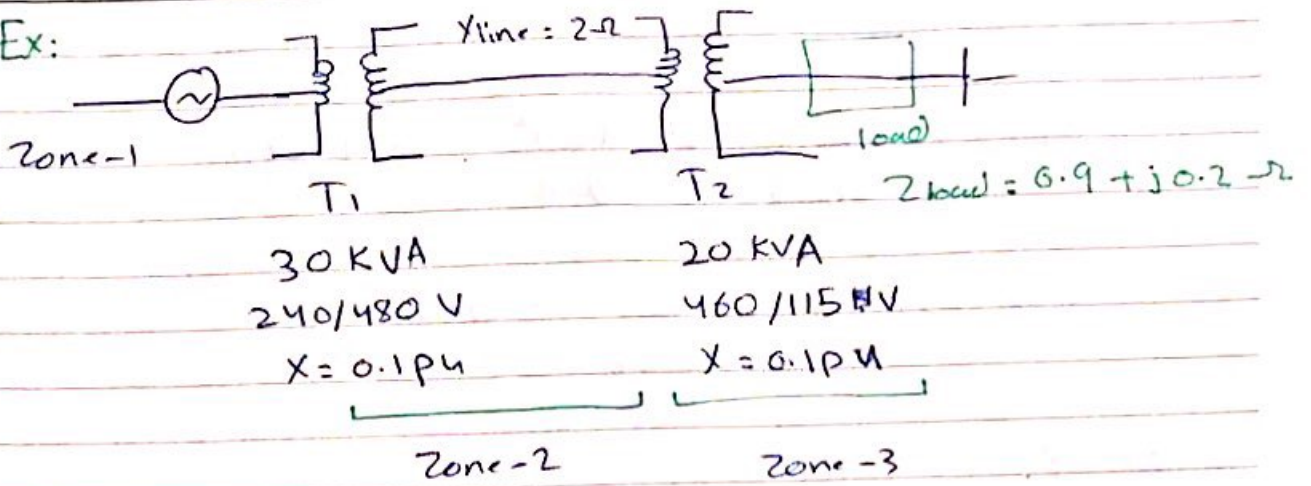
① Draw impedance diagram (all value in pu)
base in Zone-1 (30 KVA, 240 V)

Sol:



$$V_{base} = 30 \text{ KVA}$$

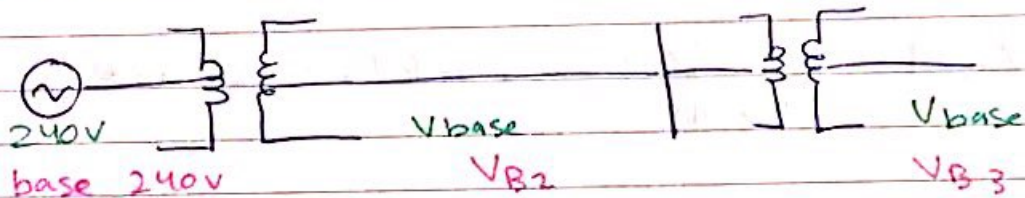
Ex:



'single phase'

- ① Draw impedance diagram (all value in pu)
base in Zone-1 (30 KVA, 240V)
- ② Find load current ?

Sol:



$$S_{base} = 30 \text{ KVA}$$

$$V_{B2} = 240 \times \left(\frac{480}{240} \right) = 480 \text{ V}$$

$$V_{B3} = 480 \times \left(\frac{115}{460} \right) = 120 \text{ V}$$

$$T_1 \Rightarrow X = 0.1 \text{ (30 KVA, 240/480V)}$$

$$X_{new} = 0.1 \left(\frac{30}{30} \right) \left(\frac{240}{240} \right)^2 = 0.1 \text{ pu.}$$

$$T_2 \Rightarrow X_{new} = 0.1 \times \left(\frac{30}{20} \right) \times \left(\frac{460}{480} \right)^2$$

$$= 0.1 \times \left(\frac{30}{20} \right) \times \left(\frac{115}{120} \right)^2$$

$$= 0.13789 \text{ pu}$$

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Zone 2:

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(480)^2}{30k} = 7.68 \Omega$$

$$\text{line} \Rightarrow X_{pu} = \frac{2}{7.68} = 0.2604 \text{ pu.}$$

Zone 3:

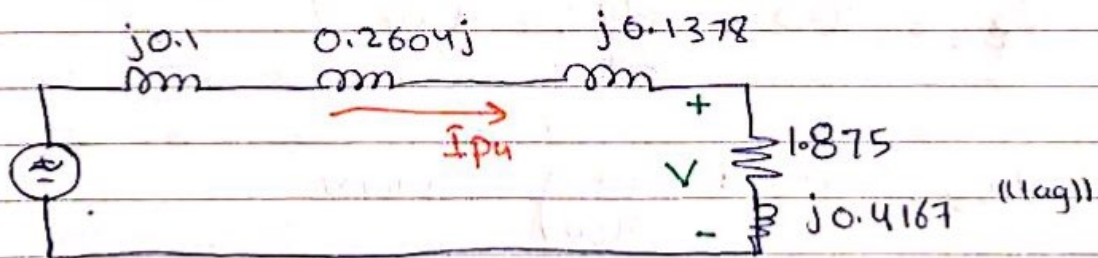
$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(120)^2}{30k} = 0.48 \Omega$$

$$X_{pu} = \text{-----}$$

$$Z_{load} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.4167 \text{ pu.}$$

$$S_{base} = P_{base} = Q_{base}$$

$$Z_{base} = R_{base} = X_{base}$$



2] if Voltage of the source is 220 V $\angle 0$
Find load current?!

$$\text{Sol: } V_{source \text{ pu}} = \frac{220V}{240} = 0.91 \text{ pu } \angle 0$$

$$I_{pu} = \frac{0.91 \angle 0}{j0.1 + 0.2604j + j0.1378 + 1.875 + j0.4167}$$

$$I_{pu \text{ base}} = \frac{S_{base, 1\phi}}{V_{base} \cdot LN} = \frac{30k}{120} = 250 \text{ A}$$

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$$I_{\text{actual, load}} = I_{\text{pu}} \cdot I_{\text{base}} \quad (\text{Zone 3}).$$

$$* V_{\text{sec}} = 220 \text{ V} \times \frac{480}{240} \text{ V}.$$

$$V_{\text{sec pu}} = \frac{220 \times \frac{480}{240}}{240 \times \frac{480}{240}} = \frac{220}{240}$$

$$V_{\text{pu primary}} = V_{\text{pu secondary}}.$$

* The per unit system - three phase ckt.

- In balanced system \Rightarrow per-phase analysis.
($\Delta \rightarrow Y$)

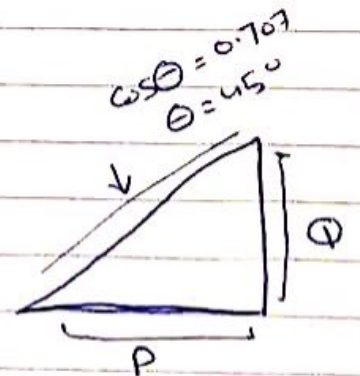
$$S_{\text{base, } 3\phi} = P_{\text{base, } 3\phi} = Q_{\text{base, } 3\phi}$$

Ex: load 100 MW, $\text{pF} = 0.707$ lag, $S_{\text{base, } 3\phi} = 100 \text{ MVA}$

Find $P_{\text{load pu}}$, $Q_{\text{load pu}}$?!

$$\text{Sol: } P_{\text{load pu}} = \frac{10}{100} = 0.1 \text{ pu}.$$

$$Q_{\text{load pu}} = \frac{10}{100} = 0.1 \text{ pu}.$$



$\theta = 45^\circ \rightarrow S = L_{\text{c}} \text{ ckt}$
($P = Q$) جهد

$S_{\text{base } 3\phi}$, V_{L-L} , Z_{base} , I_{base} .

$$S_{\text{base } 3\phi} \text{ و } V_{L-L \text{ base}} \Rightarrow$$

$$I_{\text{base } 3\phi} = \frac{S_{\text{base } 3\phi}}{\sqrt{3} V_{LL-\text{base}}}$$

Proof: $I_{\text{base } 3\phi} = \frac{3 S_{\text{base } 1\phi}}{\sqrt{3} (\sqrt{3} V_{L, \text{base}})}$

$$= \frac{S_{\text{base } 1\phi}}{V_{LN, \text{base}}}$$

$$= I_{\text{base } 1\phi}$$

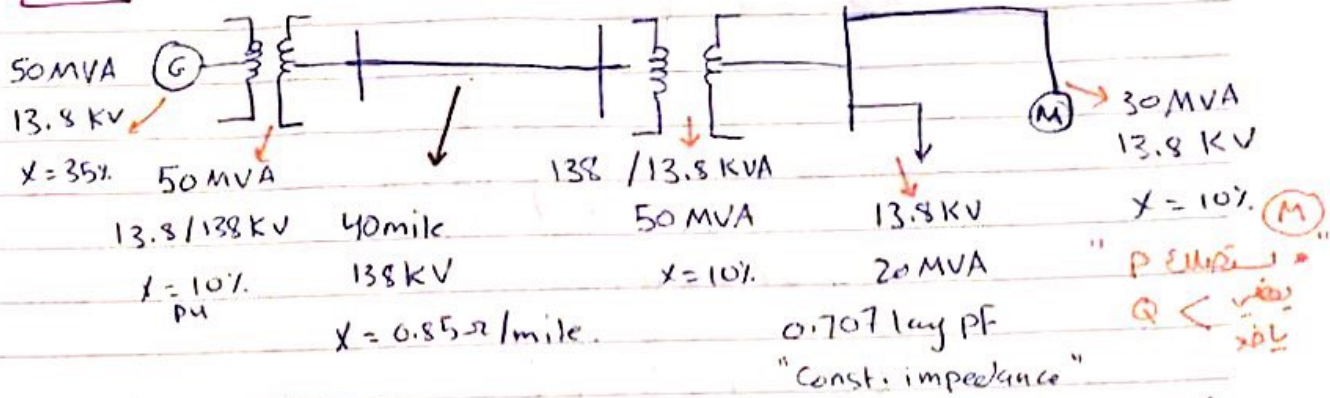
$$Z_{\text{base}} = \frac{V_{\text{base}, LN}}{I_{\text{base}, 1\phi}} = \frac{V_{\text{base}, LN}}{\frac{S_{\text{base}, 1\phi}}{V_{\text{base}, LN}}} = \frac{V_{\text{base}, LN}^2}{S_{\text{base}, 1\phi}}$$

$$= \frac{(V_{\text{base } LL} / \sqrt{3})^2}{S_{\text{base } 1\phi} / 3} = \frac{V_{\text{base}, LL}^2}{S_{\text{base}, 3\phi}}$$

$$Z_{pu, \text{new}} = Z_{pu, \text{old}} * \left(\frac{S_{\text{base } 3\phi, \text{new}}}{S_{\text{base } 3\phi, \text{old}}} \right) * \left(\frac{V_{LL, \text{old}}}{V_{LL, \text{new}}} \right)^2$$

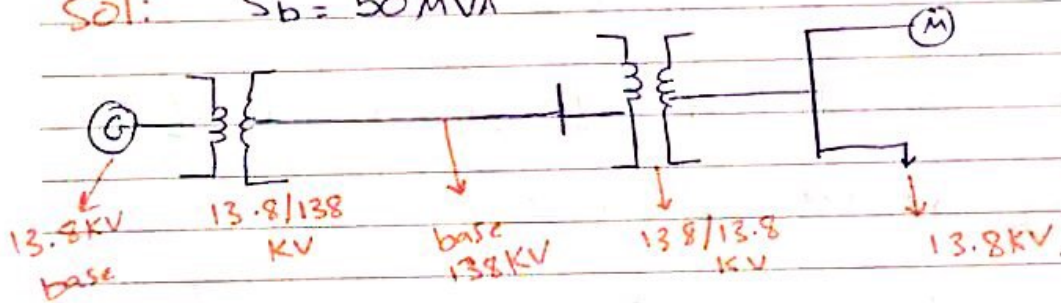
?? 21 *

Ex: 3 ϕ power system.



Draw impedance diagram (Mark all value in pu) base 13.8 kV, 50 MVA at the generation side!

Sol: $S_b = 50 \text{ MVA}$



T₁ \Rightarrow

$$X_{pu} = 0.1 * \left(\frac{50}{50} \right) * \left(\frac{13.8}{13.8} \right)^2 = 0.1 \text{ pu}$$

line \Rightarrow

$$X_{pu} = \frac{\text{actual}}{Z_{\text{base}}}$$

$$Z_{\text{base}} = \frac{(138)^2}{50 \text{ M}} = 380.88 \Omega$$

$$Z_{\text{base}} = \frac{V_{LL}^2}{S_{1\phi}}$$

$$= \frac{(V_{LL})^2}{S_{13\phi}}$$

$$X_{pu} = \frac{0.85 * 40}{380.88} = 0.089 \text{ pu}$$

$$T_2 \Rightarrow X_{pu} = 0.1 \text{ pu.}$$

$$G \Rightarrow X = 0.35 \times \left(\frac{50}{50} \right) \left(\frac{13.8}{13.8} \right)^2 \\ = 0.35 \text{ pu.}$$

$$M \Rightarrow X = 0.1 \left(\frac{50}{30} \right) \left(\frac{13.8}{13.8} \right) = 0.167 \text{ pu.}$$

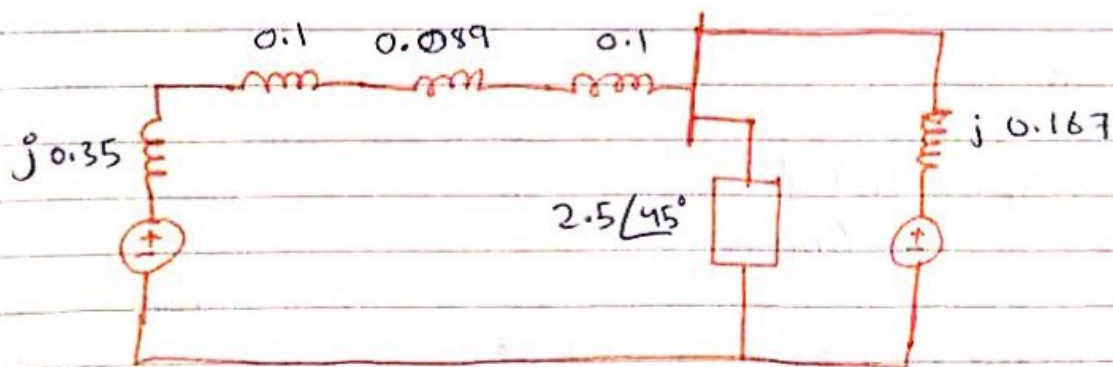
$$\text{load} \Rightarrow |Z_{\text{actual}}| = \frac{V^2}{S} = \left(\frac{13.8 \text{ kV}}{20 \text{ M}} \right)^2 \\ = 9.52 \Omega.$$

$$Z_{\text{actual}} = 9.52 \angle +\cos^{-1}(0) = 9.52 \angle 45^\circ$$

lag $\rightarrow Z \angle +ve \rightarrow I \angle -ve.$

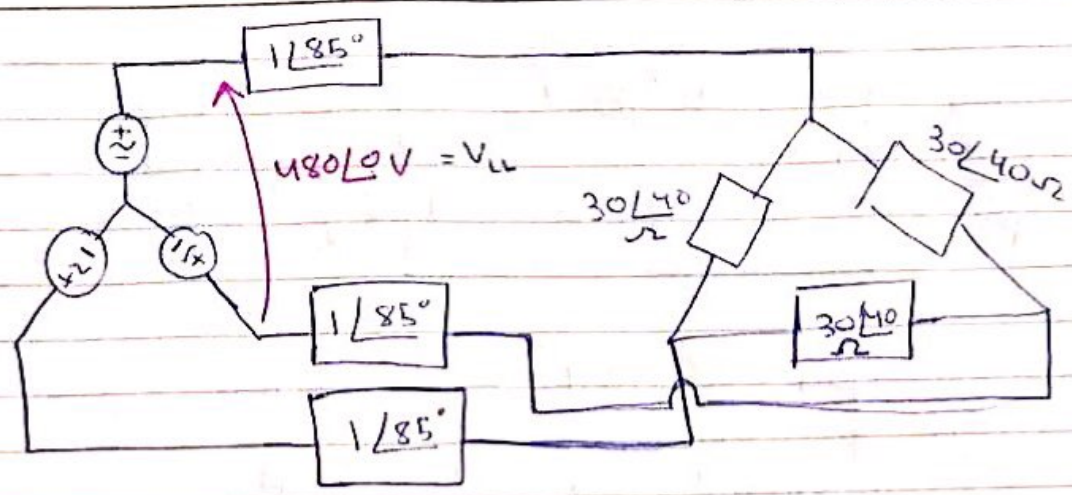
$$Z_{\text{base}} = \frac{V^2}{S} = \frac{(13.8)^2}{50} = 3.81 \Omega.$$

$$Z_{\text{load pu}} = \frac{9.52 \angle 45^\circ}{3.81} = 2.5 \angle 45^\circ$$



$$I_{\text{base (wuv)}} = \frac{S_{3\phi}}{\sqrt{3} V_{LL}} = \frac{S_{1\phi}}{V_{LN}}$$

Ex



Calculate pu and actual current in phase using $S_{1\phi} = 10 \text{ KVA}$, $V_{\text{base, L-L}} = 480 \text{ V}$.

Sol:

$$Z_{\Delta} \Rightarrow Z_Y$$

$$Z_Y = \frac{Z_{\Delta}}{3} = \frac{30 \angle 40^\circ}{3} = 10 \angle 40^\circ \Omega$$

$$Z_{Lpu} = \frac{Z_{\text{actual}}}{Z_{\text{base}}}$$

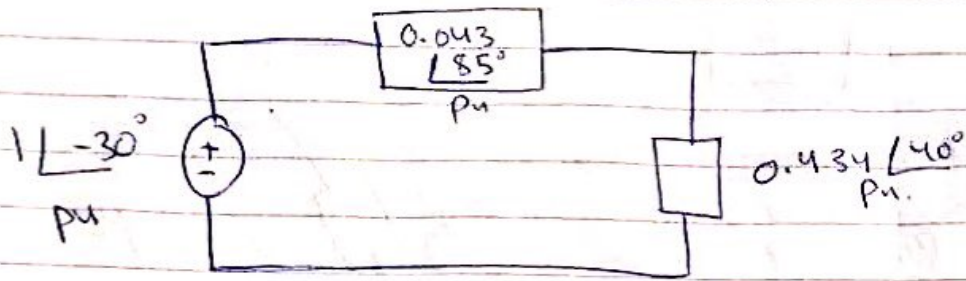
$$Z_{\text{base}} = \frac{(V_{\text{base, LL}})^2}{S_{\text{base, 3}\phi}} = \frac{(480 \text{ V})^2}{30 \text{ KVA}} = 23.04 \Omega$$

$$= \frac{(V_{\text{base LN}})^2}{S_{\text{base } 1\phi}} = \frac{(480/\sqrt{3})^2}{10}$$

$$Z_{Lpu} = \frac{10 \angle 40^\circ}{23.04} = 0.434 \angle 40^\circ \text{ pu}$$

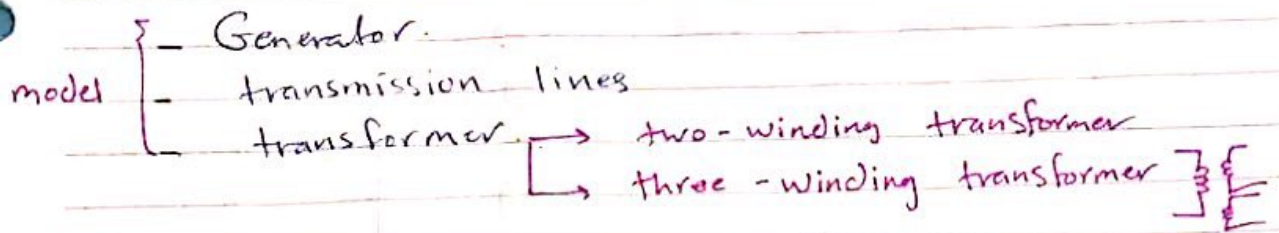
$$Z_{\text{line}} = \frac{1 \angle 85^\circ}{23.04} = 0.043 \angle 85^\circ \text{ pu}$$

$$\text{Source} = \begin{cases} V_{LN} = \frac{480}{\sqrt{3}} \angle -30^\circ \text{ V} \\ V_{pu} = \frac{480/\sqrt{3}}{480/\sqrt{3}} = 1 \text{ pu} \end{cases}$$



$$I_{pu} = 2.147 \angle -73.78^\circ \text{ pu}$$

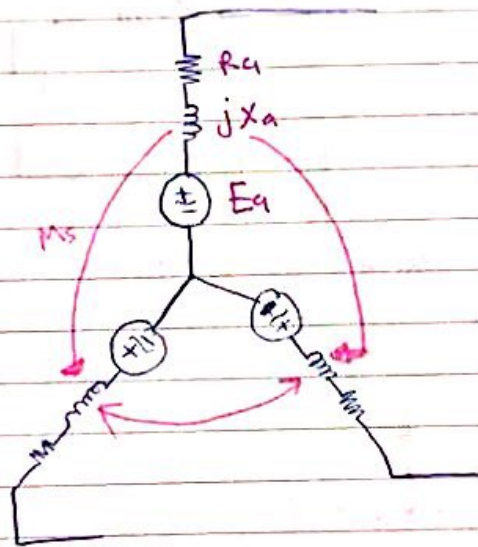
$$I_{actual} = 25.83 \angle -73.78^\circ \text{ A}$$



Power System

- ① models
- ② analysis
 - load flow
 - fault analysis

Three-phase synchronous Generator:-



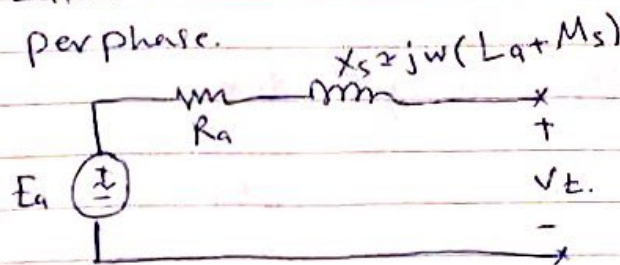
$E_a \equiv$ internal voltage.

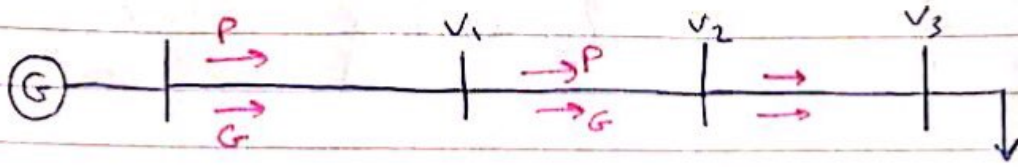
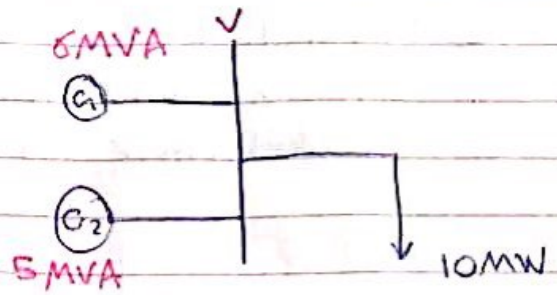
$R_a, jX_a \equiv$ armature winding.

$M_s \equiv$ mutual inductance.

equivalent ckt :

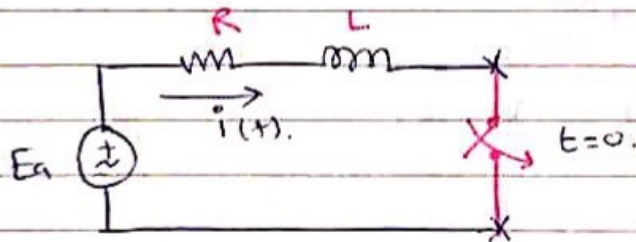
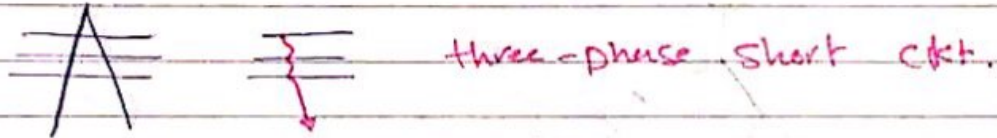
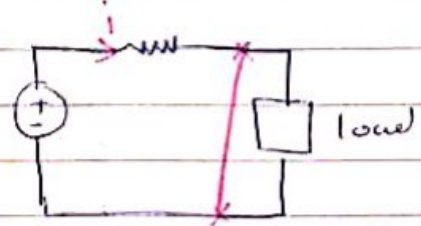
- balanced
- per phase.



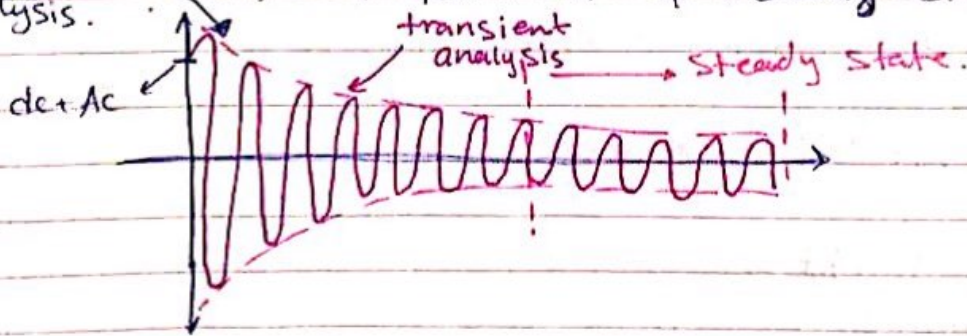


Fault analysis

↑ i_c (short ckt) fault side view



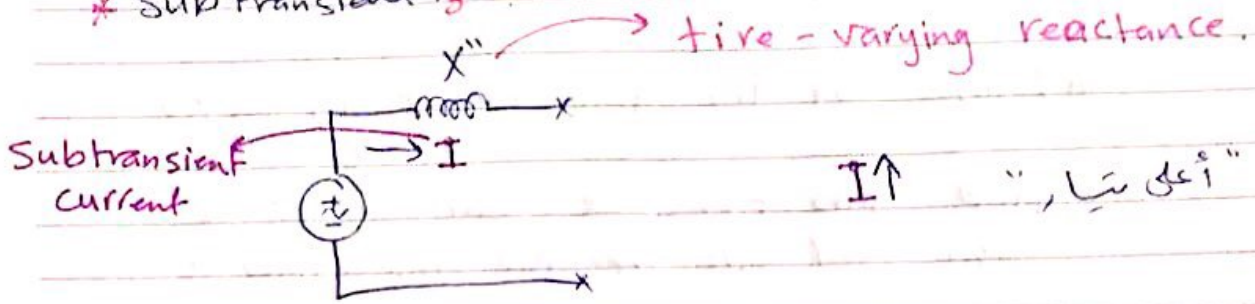
Subtransient analysis. $i(t) = i_{\text{transient}} + i_{\text{steady state}}$



Subtransient analysis SA

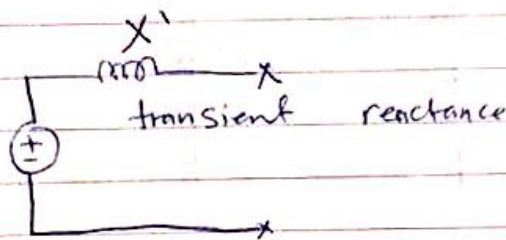
* Analysis 84

* Subtransient 8-

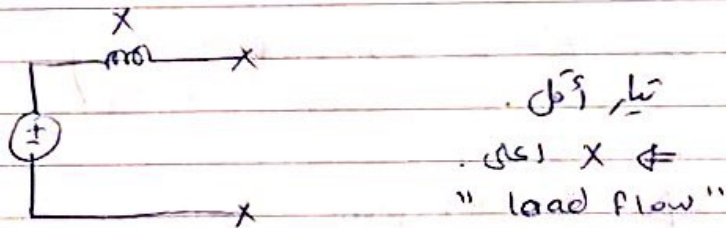


$x'' \equiv$ subtransient reactance.

* transient 8-

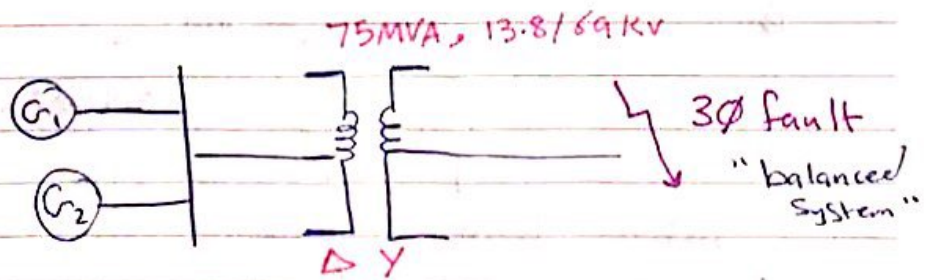


* Steady state 8-



$$x'' < x' < x$$

Ex:



G_1 : 50 MVA, 13.8 kV, $x'' = 25\%$

G_2 : 25 MVA, 13.8 kV, $x'' = 25\%$

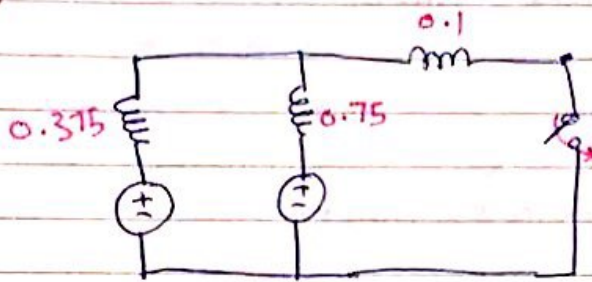
T_1 : 75 MVA, 13.8 Δ / 69 Y kV, $x'' = 10\%$

Unload condition, NO circulating current
[Before fault].

HV side of the transformer is 66 kV

Find sub-transient current at
the HV side of the transformer?

Sol:



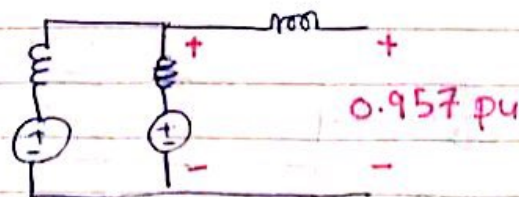
" $S_b = 75 \text{ KVA}$ "
," $k_b = 50 \text{ pu}$ "

$$G_1 \Rightarrow X'' = 25\% * \left(\frac{75}{50}\right) = 0.375 \text{ pu}$$

$$G_2 \Rightarrow X'' = 25\% * \left(\frac{75}{25}\right) = 0.75 \text{ pu.}$$

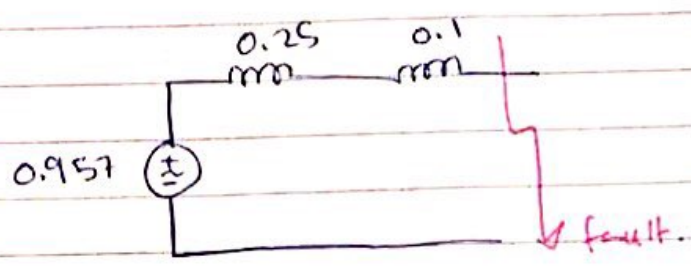
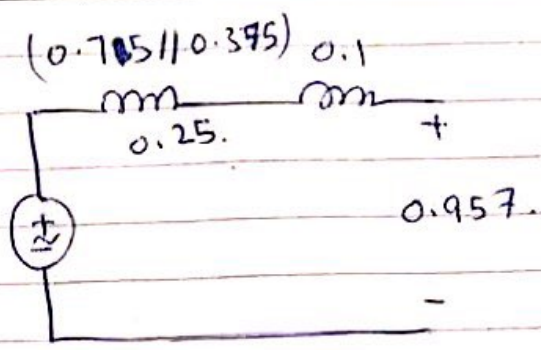
$$T_1 \Rightarrow X = 0.1 \text{ pu.}$$

⊕



before fault. "66 kV"

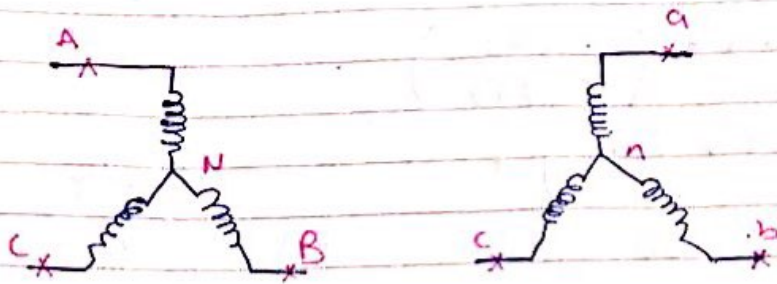
$$\frac{66}{69} = 0.957 \text{ pu.}$$



$$I_{pu}'' = \frac{0.95}{j0.25 + j0.1} = -j 2.734 \text{ pu}$$

$$I''_{\text{actual}} = -j 2.734 \times \left(\frac{75 \text{ M}}{\sqrt{3} \times 69 \text{ KV}} \right)$$

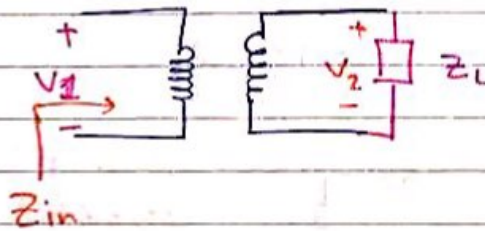
* Three-phase transformer Analysis $\Delta\Delta$



Turns ratio $\triangleq \frac{\text{Phase voltage } V_2}{\text{Phase voltage } V_1}$

Transformation ratio $\triangleq \frac{\text{L-to-L voltage secondary}}{\text{L-to-L voltage Primary}}$

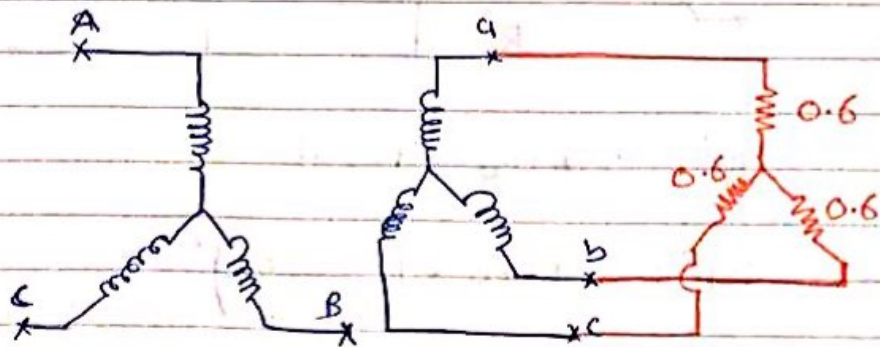
$Z_{in} = Z_L \left(\frac{N_1}{N_2} \right)^2$



$Z_{in} = \frac{V_2 (N_1 / N_2)}{i_2 (N_2 / N_1)}$

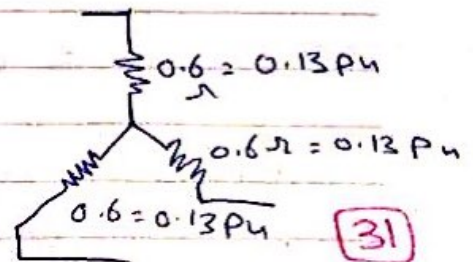
EX

10 MVA
66/6.6 KV



① referred to secondary $\Delta\Delta$

$R_{pu} = \frac{0.6}{(6.6)^2 / 10} = 0.13 \text{ pu}$



② referred to primary Δ

$$R_p = 0.6 \times \left(\frac{66/\sqrt{3}}{6.6/\sqrt{3}} \right)^2$$

$$= 0.6 \times \left(\frac{66}{6.6} \right)^2$$

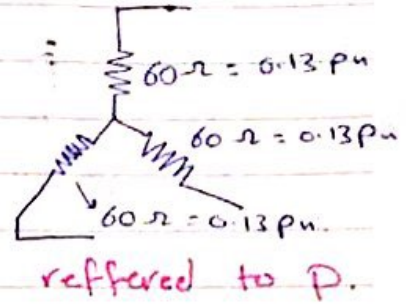
$$= 60 \Omega$$

$$R_p (\text{pu}) = \frac{0.6 \times \left(\frac{66}{6.6} \right)^2}{66^2 / 10}$$

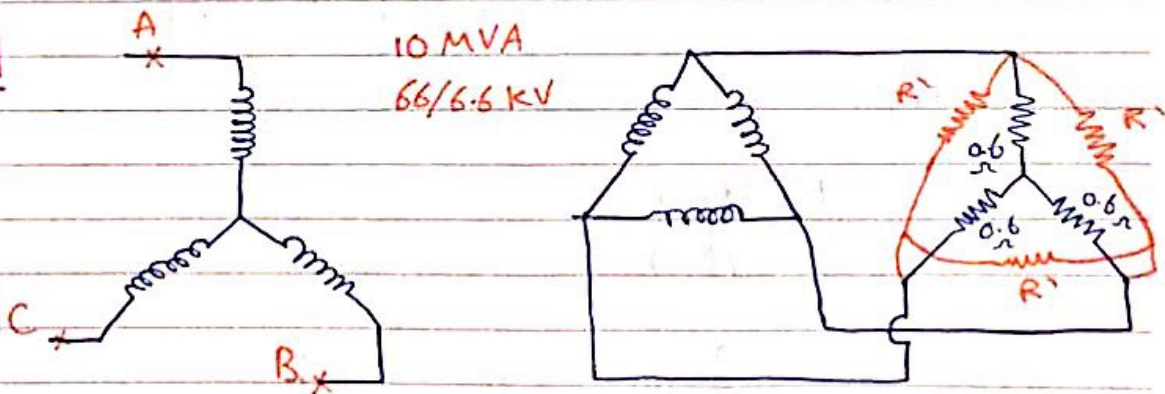
$$= \frac{0.6}{(6.6)^2 / 10}$$

$$= R_s \text{ pu}$$

$$= 0.13 \text{ pu.}$$



Ex



$$R' = 3 \times 0.6 = 1.8 \Omega$$

$$R_p' = 3 \times 0.6 \times \left(\frac{66/\sqrt{3}}{6.6} \right)^2$$

$$= 0.6 \times \left(\frac{66}{6.6} \right)^2$$

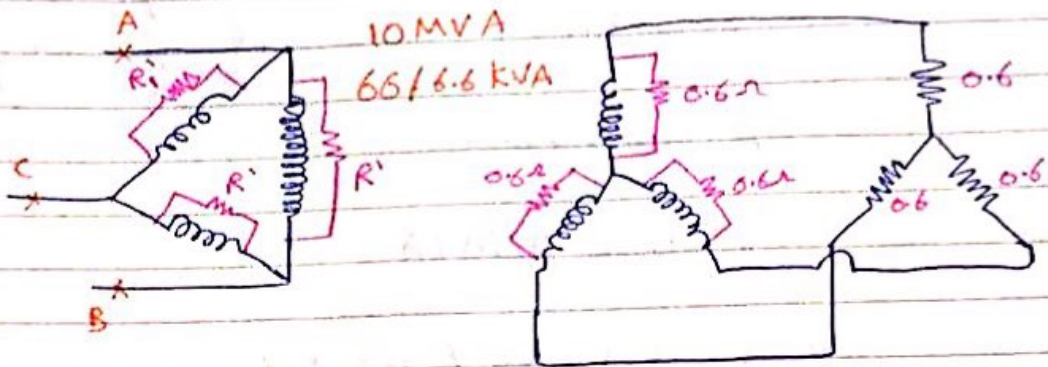
$$= 60 \Omega$$

$$R_p, \text{pu} = \frac{60}{(66)^2 / 10} = 0.13 \text{ pu}$$

32

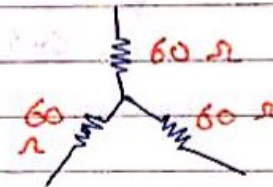
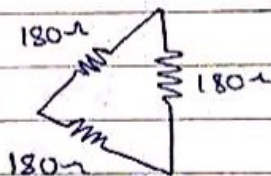
$$R_{s, pu} = \frac{0.6}{(6.6)^2/10} = 0.13 \text{ pu}$$

EX



$$R' = 0.6 \times \left(\frac{66}{6.6/\sqrt{3}} \right)^2$$

= 180 Ω \rightarrow Δ equivalent at primary



γ -equivalent at primary

$$Z_p = 0.6 \left(\frac{66}{6.6} \right)^2$$

$\therefore \gamma$ equivalent

transformation ratio

$$\left(\frac{V_{LL}}{V_{LL}} \right)^2$$

γ equivalent

$\therefore Z_{P(pu)} = Z_{S(pu)}$ irrespective of transformer connection.

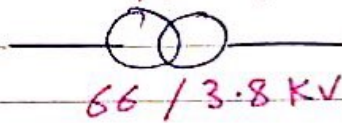
Ex 1 3 (1 ϕ transformer, each 25 MVA, 38.1/3.8 KV are connected Y- Δ with a balanced Y-connected load "resistor" R (Ω /phase) R = 0.6 Ω , Find the Y-equivalent resistors seen at primary side?!

Sol: 3 ϕ transformer.

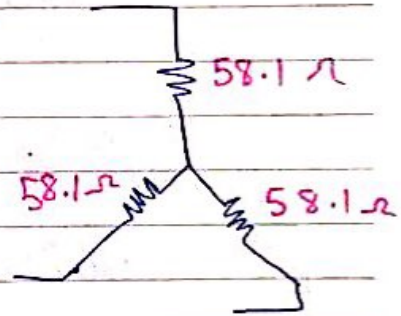
$$S \Rightarrow 3 \times 25 = 75 \text{ MVA}$$

$$66 = 38.1 \times \sqrt{3} / 3.8 \text{ KV}$$

75 MVA



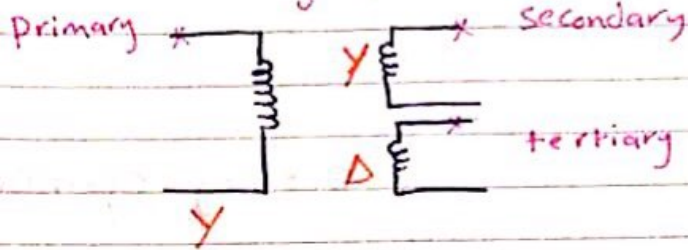
$$\Rightarrow 0.6 * \left(\frac{66}{3.8} \right)^2 = 58.1 \Omega.$$



$$Y_{\text{equivalent}} \xrightarrow[\text{ratio}]{\text{transformer}} Y_{\text{equivalent}}$$

$$\left(\frac{V_{LL}}{V_{LL}}\right)^2$$

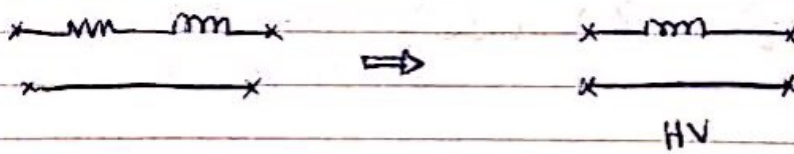
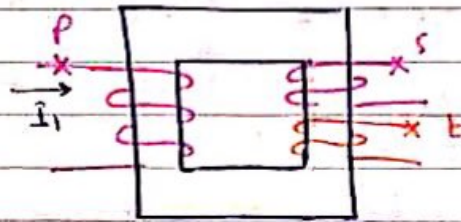
* three-winding transformer :-



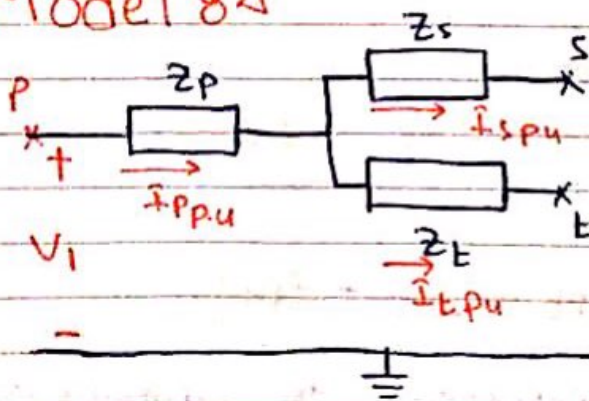
1 auxiliary voltage.

2 Reactive power compensation. "عشان انزبط ان volt"

3 harmonic Supereession.



* Model 8Δ



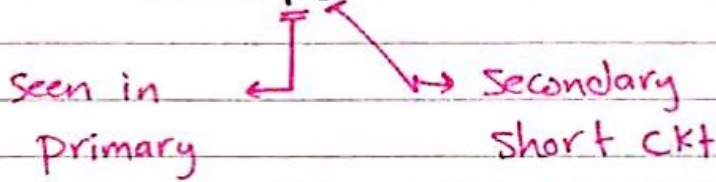
Short circuit tests

$$Z_{ps}$$

$$Z_{pt}$$

$$Z_{st}$$

Z_{ps} \Rightarrow tertiary open circuit.



Z_{ps} = Z_p و Z_s (actual value) و Z_{ps} = $Z_p + Z_s$ و Z_{ps} = $Z_p + Z_s$ و Z_{ps} = $Z_p + Z_s$

based on the model 84

$$Z_{ps} = Z_p + Z_s' \quad \dots \textcircled{1}$$

((actual value))

$$Z_{pt} = Z_p + Z_t' \quad \rightarrow \text{referred to primary} \quad \dots \textcircled{2}$$

$$Z_{st} = Z_s + Z_t' \quad \rightarrow \text{referred to secondary}$$

\rightarrow referred to primary.

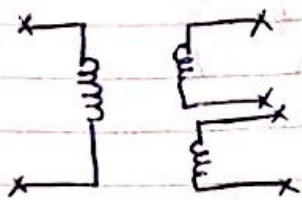
$$Z_{st}' = Z_s' + Z_t'' \quad \dots \textcircled{3}$$

$$Z_p = \frac{1}{2} (Z_{ps} + Z_{pt} - Z_{st})$$

$$Z_s = \frac{1}{2} (Z_{ps} + Z_{st} - Z_{pt})$$

$$Z_t = \frac{1}{2} (Z_{pt} + Z_{st} - Z_{ps})$$

all should referred to primary side



different rating.

Z_{ps} , Z_{pt} , Z_{st} [in per unit, rating winding]

all values are converted to common base.

Ex three winding transformer
 primary, Y connected, 66 KV, 15 MVA
 secondary, Y connected, 13.2 KV, 10 MVA
 tertiary, Δ connected, 2.3 KV, 5 MVA

short circuit \Rightarrow test $Z_{ps} = 7\%$ (15 MVA, 66 KV)

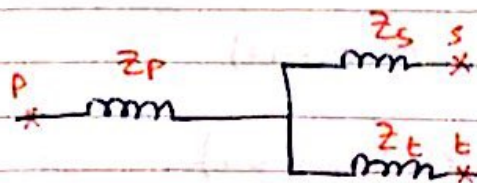
$Z_{pt} = 9\%$ (15 MVA, 66 KV)

$Z_{st} = 8\%$ (10 MVA, 13.2 KV)

Draw reactance diagram base [15 MVA, 66 KV] at primary?

"neglect resistance"

sol:



$$X_{pu, new} = X_{pu, old} * \left(\frac{S_{new}}{S_{old}} \right) * \left(\frac{V_{old}}{V_{new}} \right)^2$$

$$= 0.08 * \left(\frac{15}{10} \right) * \left(\frac{13.2}{13.2} \right)^2$$

$$= 0.12 \text{ pu}$$

8% (10MVA, 13.2KV)

pu (15MVA, 13.2KV)

OR $Z_{st} (\text{actual}) = 0.08 * \left(\frac{(13.2)^2}{10} \right) = 1.39 \Omega$

$$Z_{st}' = 1.39 * \left(\frac{66}{13.2} \right)^2 \Omega$$

$$Z_{st}' \text{ pu} = \frac{1.39 * \left(\frac{66}{13.2} \right)^2}{(66)^2 / 15} = 0.12 \text{ pu}$$

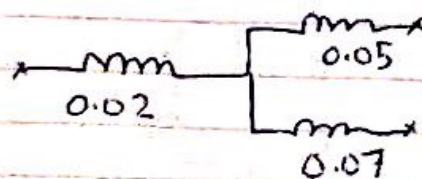
$$= \frac{0.08 * \left(\frac{13.2^2}{10} \right) * \left(\frac{66^2}{13.2^2} \right)}{66^2 / 15}$$

$$= 0.08 * \left(\frac{15}{10} \right) = 0.12 \text{ pu.}$$

$$Z_p = \frac{1}{2} (0.07 + 0.09 - 0.12)$$

$$Z_s = \frac{1}{2} (0.07 + 0.12 - 0.09)$$

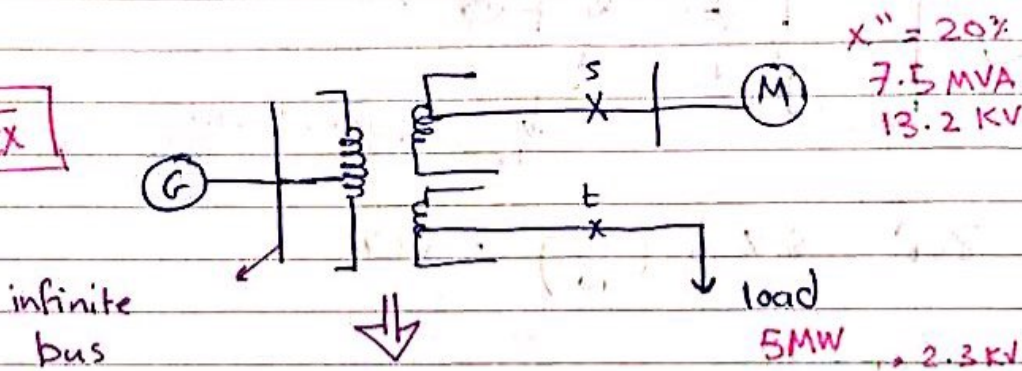
$$Z_t = \frac{1}{2} (0.09 + 0.12 - 0.07)$$



$Z_{PE}(pu) \neq Z_{TP}(pu)$

KVAR \neq KVAR

Ex



Primary 15 MVA, 66 KV

Secondary 10 MVA, 13.2 KV

tertiary 5 MVA, 2.3 KV

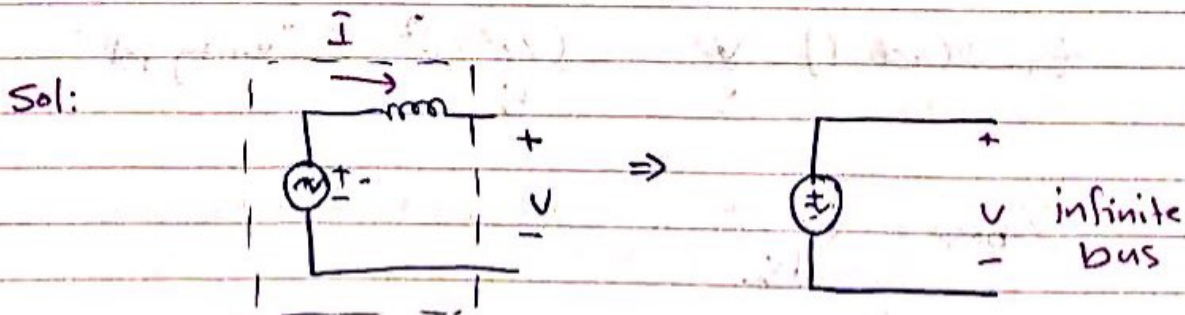
Unity PF

$Z_{ps} = 7\%$

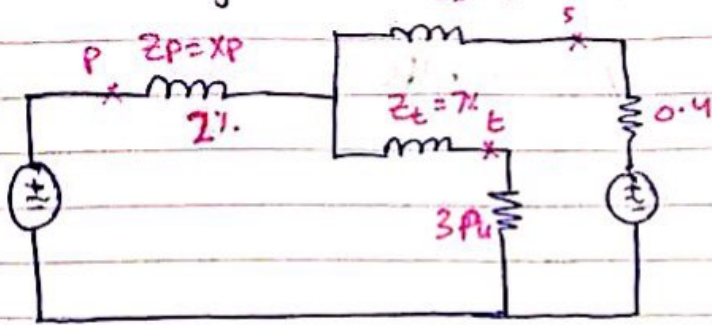
$Z_{pt} = 9\%$

$Z_{st} = 8\%$

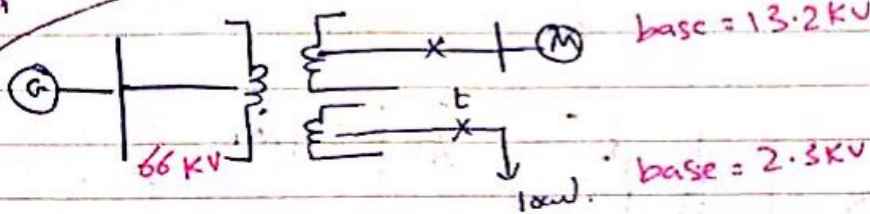
base 15 MVA, 66 KV at Generation side.



impedan diagram 8x $Z_s = 5\%$



15 MVA



$$Z_{st}' = 8\% \times \left(\frac{15}{10}\right) = 12\%$$

$$Z_p = 2\%$$

$$Z_s = 5\%$$

$$Z_t = 7\%$$

* Motor :-

$$X''_{\text{motor}} = 20\% \times \left(\frac{15}{7.5}\right) = 0.4 \text{ pu}$$

* load :-

$$Z_{\text{load (actual)}} = \frac{V^2}{S} = \frac{(2.3 \text{ kV})^2}{5} = \text{"unity PF"}$$

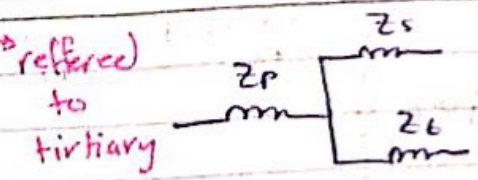
$$Z_{\text{base}} = \frac{(2.3)^2}{15}$$

$$Z_{\text{load}} \Big|_{\text{pu}} = \frac{15}{5} = 3 \text{ pu}$$

$$S = \frac{P}{\text{P.F}}$$

40

$$Z_{EP} (\text{pu}) = \frac{Z_E + Z_P}{(V_{LL E})^2 / S_E}$$



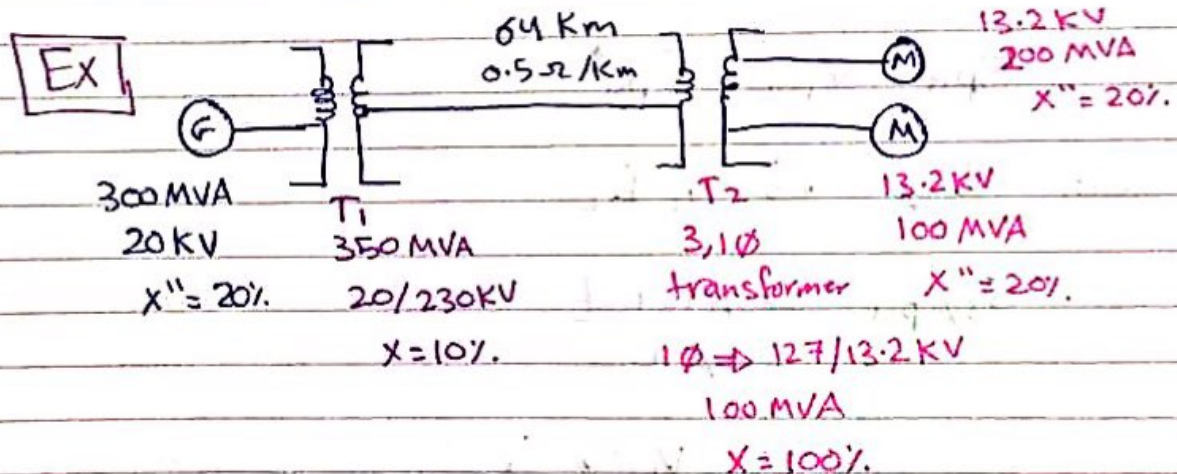
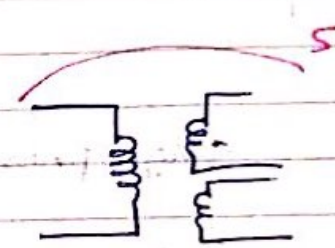
$$Z_{PE} (\text{pu}) = \frac{Z_P + Z_E}{(V_{LL P})^2 / S_{Primary}}$$

referred to primary.

$S_{EP} \neq S_{PE}$

referred to common base:-

$$Z_{EP} (\text{system}) = Z_{PE} (\text{system})$$



base 300 MVA, 20 KV at generation side.

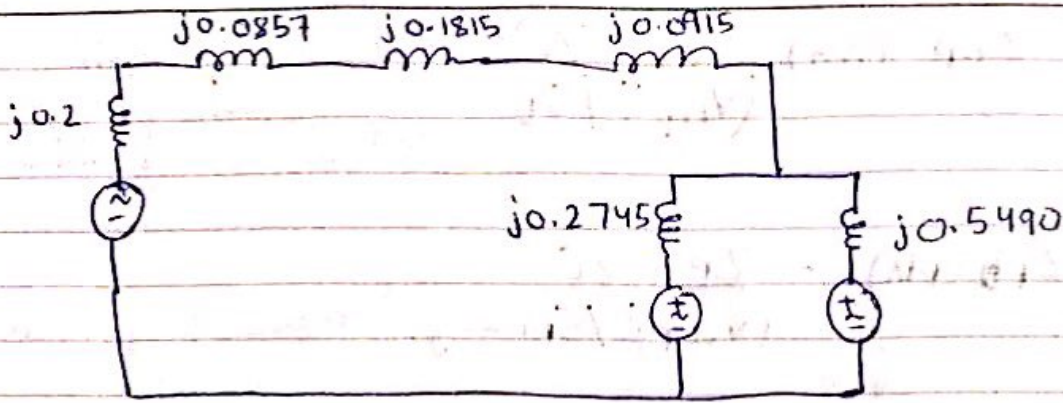
Sol:

TL (3 phase) 230 KV \Rightarrow Δ connection \Rightarrow $\sqrt{3}(127) = Y$

$$T_2 \Rightarrow 127 * \sqrt{3} \text{ KV} / 13.2 \text{ KV}$$

$$300 \text{ MVA} (3 * 100 \text{ MVA})$$

$$\Rightarrow 220 \text{ KV} / 13.2 \text{ KV} @ 300 \text{ MVA}$$



Prove: $S_{pu} = |V_{pu}| |I_{pu}|$ for 3ϕ system.

Single phase $\rightarrow S = V I^*$

3ϕ , balanced $\rightarrow S = 3 V I^*$

$S_{pu} = V_{pu} \cdot I_{pu}^*$

Sol: $S_{3\phi} = \sqrt{3} V_L I_L$

$S_{3\phi pu} = \frac{S_{3\phi actual}}{S_{3\phi base}}$

$= \frac{\sqrt{3} V_{LL} \cdot I_L}{\sqrt{3} \cdot V_{Lbase} \cdot I_{Lbase}}$

$S_{3\phi pu} = |V_{pu}| \cdot |I_{pu}|$ #

7/3 Tue

* اسأل الله العفو والعافية *

2. If the ~~Motor~~ Motor have input

$M_2 = 120 \text{ MW}$, [$M_2 = 120 \text{ MW}$, 13.2 kV, unity PF]

Find the Voltage terminal at generator ?!

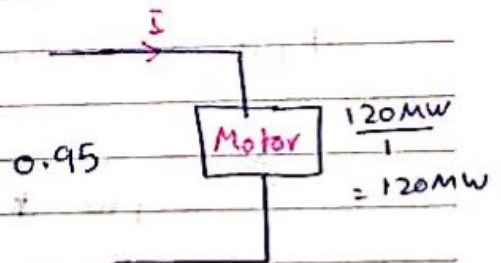
Sol:

$$V_{\text{motor}} \Big|_{\text{pu}} = \frac{13.2}{13.8} = 0.95 \text{ pu } \angle 0^\circ$$

$$S = V I^*$$

$$|S| = |V| |I|$$

$$\frac{120 \text{ MVA}}{3000 \text{ MVA}} = 0.95 I_1$$



$$I_1 = \frac{120 \angle 0}{300 \times 0.95} \text{ pu}$$

$$I_2 = \frac{60 \angle 0}{300 \times 0.95} \text{ pu}$$

$$I = I_1 + I_2 = 0.6273 \angle 0^\circ \text{ pu}$$

$$V = 0.6273 \angle 0 (j0.0857 + j0.185 + j1815 + j0.0915) + 0.95 \angle 0$$

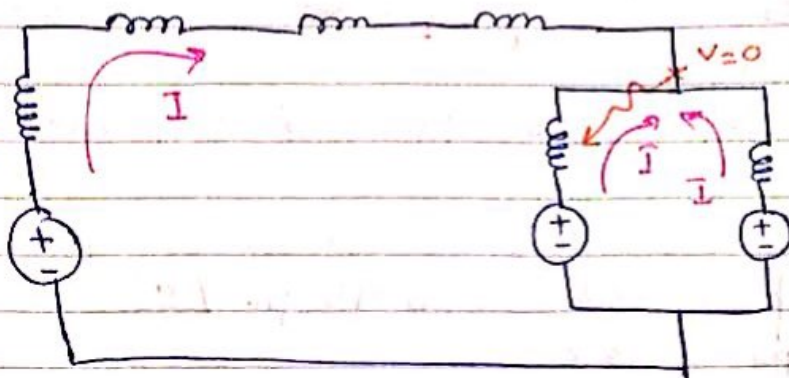
$$V = 0.9829 \angle 13.3^\circ$$

$$V_{LL}(G) = 0.9829 * 20 \angle 13.3 \text{ kV}$$

$$|V_{LL}(G)| = 0.9829 * 20 \text{ kV}$$

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3] if a 3 ϕ fault at the Motor bus
 pre-fault voltage 13.2 kV, find
 fault current?!



fault is at Motor bus
 Gen. will
 power into
 + current.

$$I_{f pu} = I_{G pu} + I_{M_1 pu} + I_{M_2 pu}$$

$$I_G = \frac{E_g}{j(0.2 + 0.0857 + 0.1815 + 0.915)} \text{ pu}$$

$$I_{M_1} = \frac{E_{m_1}}{j(0.5490)} \text{ pu}$$

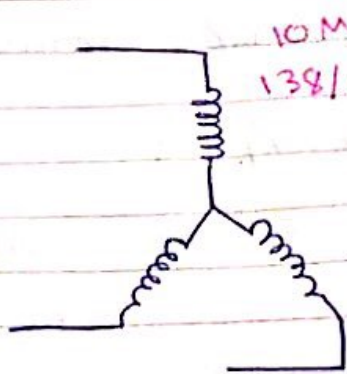
$$I_{M_2} = \frac{E_{m_2}}{j(0.2745)} \text{ pu}$$

$$I_{\text{actual}} = I_{f pu} \times \left(\frac{300 \text{ M}}{\sqrt{3} \times 13.8 \text{ k}} \right) \text{ I base}$$

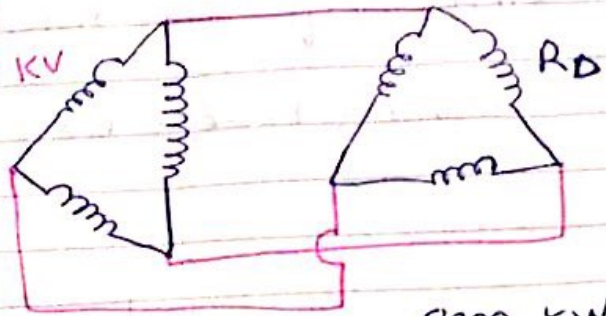
$$E_g = E_{m_1} = E_{m_2} = \frac{13.2 \text{ k}}{13.8 \text{ k}} = 0.95$$

Since the system unbalanced ($I \neq 0$)

Ex



10 MVA
138/13.8 KV



8000 kW
at rated volt
(balanced)

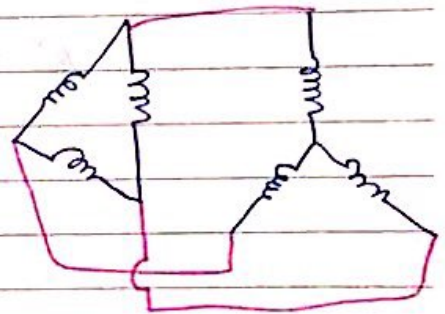
1) Find R_{y} resistance seen from HV-side ?!

$$\text{Sol: } P_{\text{load}} = 3 \times \frac{(13.8)^2}{R_{\Delta}} \Rightarrow$$

$$8000 \text{ k} = 3 \times \frac{V_L^2}{R_{\Delta}}$$

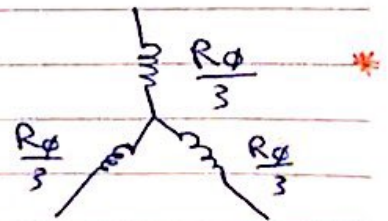
$$R_y = \frac{R_{\Delta}}{3} \times \left(\frac{138}{13.8} \right)^2$$

Seen HV side.

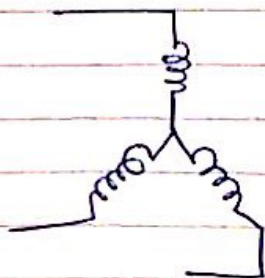


= (y) load \parallel $\frac{R_{\Delta}}{3}$ *

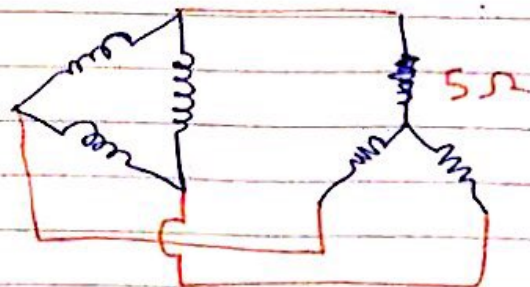
$$8000 \text{ k} = \frac{3 \times \left(\frac{138 \text{ k}}{\sqrt{3}} \right)^2}{R_{\Delta}/3}$$



Ex 1 ϕ transformer (1.2/0.12 KV) 7.2 KVA
 $X = 0.05$



Y

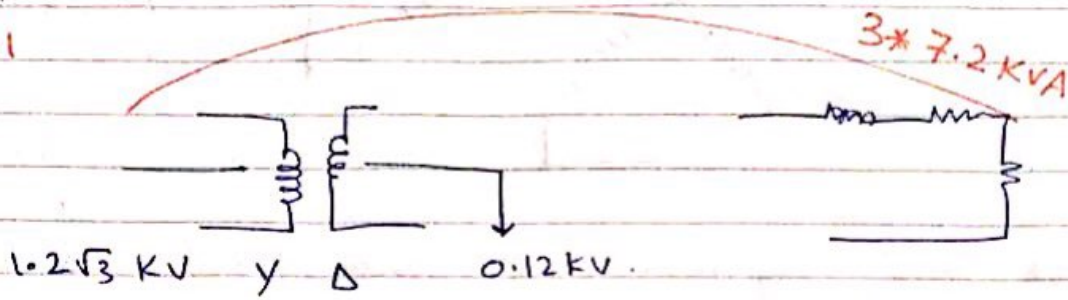


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Find Y_{eq} seen from primary transformation.

ratio $(1.2 \times \sqrt{3} \text{ KV} / 0.12 \text{ KV})$?

Sol



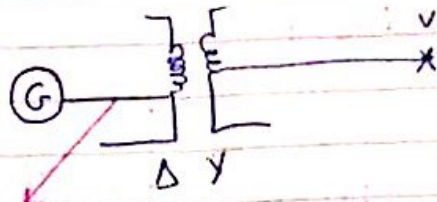
7.2 KV \rightarrow for 1 ϕ

R'_L (Y_{eq} load resistance seen for primary)

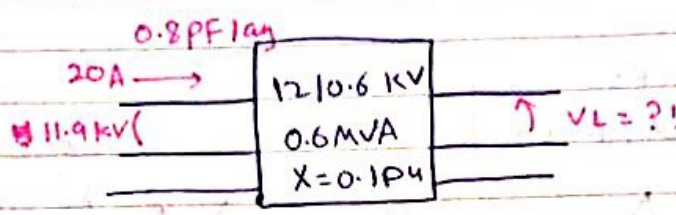
$$R'_L = 5 * \left(\frac{1.2 \sqrt{3}}{0.12} \right)^2$$

$$X' = 0.05 \left(\frac{(1.2 \sqrt{3} \text{ K})^2}{3 * 7.3 \text{ K}} \right)$$

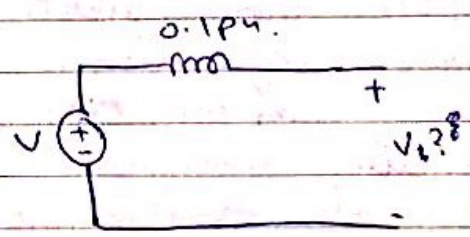
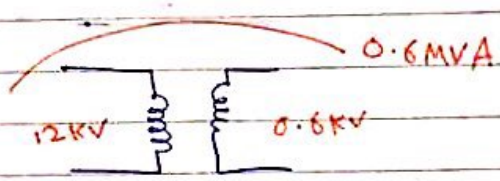
Ex :-



$V_{LL} = 11.9 \text{ kV}$
 $I_L = 20 \text{ A}$
 $\text{p.f.} = 0.8 \text{ lag}$
 0.6 MVA
 $X = 0.1 \text{ pu}$
 $12/0.6 \text{ kV}$



Sol:



$$V_s \text{ pu} = \frac{11.9}{12} \text{ pu} \angle 0^\circ$$

$$I_{\text{generator base}} = \frac{S_{\text{base}}}{\sqrt{3} V_{LL}} = \frac{0.6 \text{ M}}{\sqrt{3} * 12 \text{ k}} = 28.875 \text{ A}$$

$$I_{\text{pu}} = \frac{20}{28.875} \angle \cos^{-1}(0.8) = 0.693 \angle -36.87^\circ \text{ pu}$$

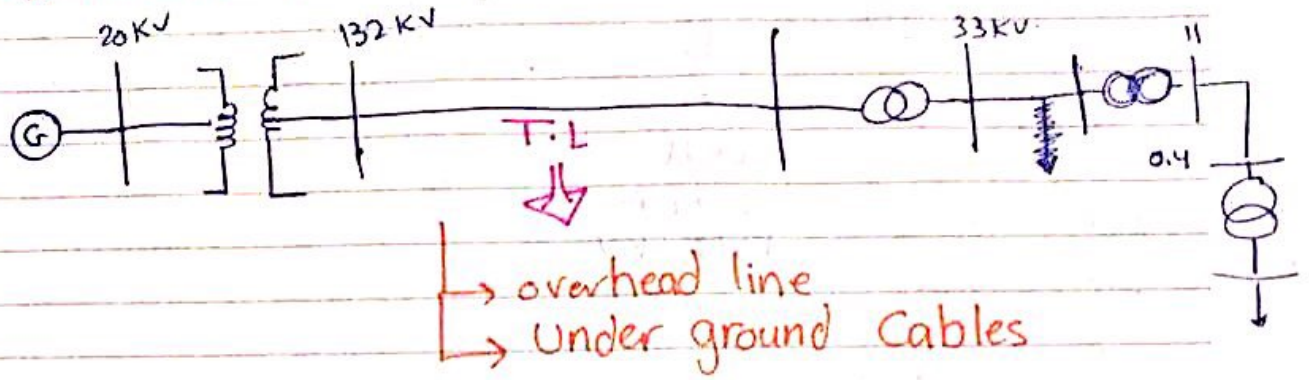
KVL:

$$V = V_s - jI(j0.1)$$

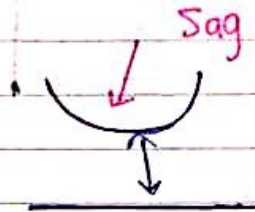
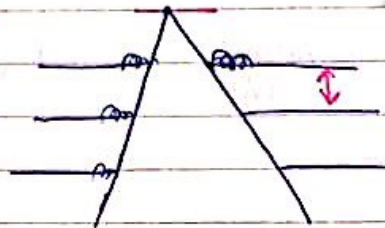
$$V_{\text{pu}} = \frac{11.9}{12} - (0.693 \angle -36.87^\circ)(j0.1) =$$

$$V_{L-L \text{ actual}} = V_{\text{pu}} * 0.6 \text{ k}$$

* Transmission Lines



[transmission networks
132 KV, 275 KV, 400KV]



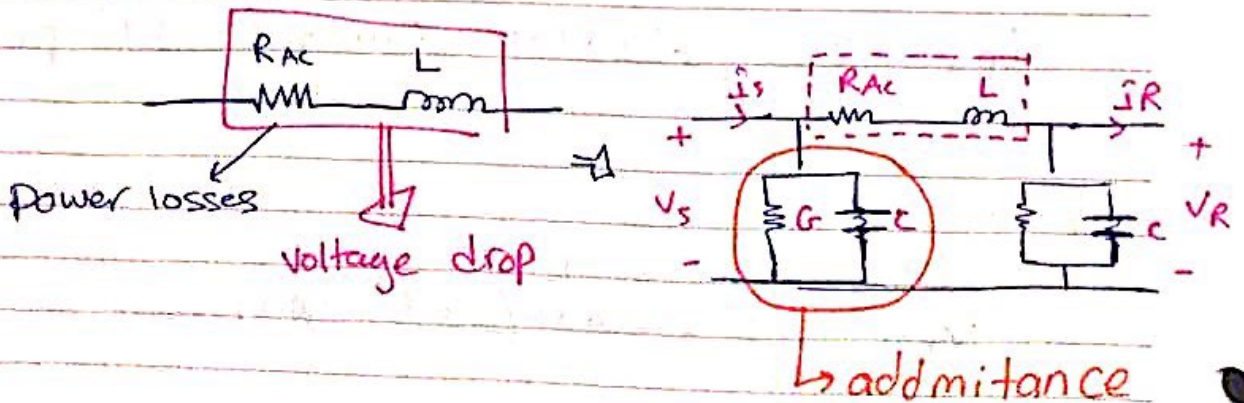
- * Power flow
- * Voltage and current along T.L.
- * Voltage regulation.
- * line compensation techniques.
- * Circle diagram.

⇒ Models of transmission lines

G, L, R, C

transmission line

- Short ($\leq 80 \text{ Km}$)
- medium ($80 \text{ Km} \leq \leq 240 \text{ Km}$)
- long ($\geq 240 \text{ Km}$)



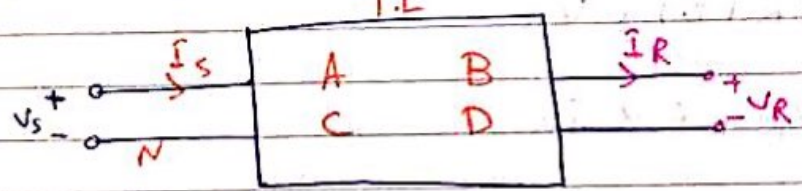
Tables $3 \times 11 \times 240 \text{ mm}^2, \text{Cu, XLPE}$

$R (\Omega/\text{km})$	$L (\text{H}/\text{km})$	$C (\text{F}/\text{m})$	$G (\text{S}/\text{m})$

G = conductance "loss between conductors, conductors and ground"

Corona \Rightarrow "type of loss between conductors and ground"

* Two-port network ((ABCD constant))



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Short
medium
long

$$V_s = A V_R + B I_R$$

$$I_s = C V_R + D I_R$$

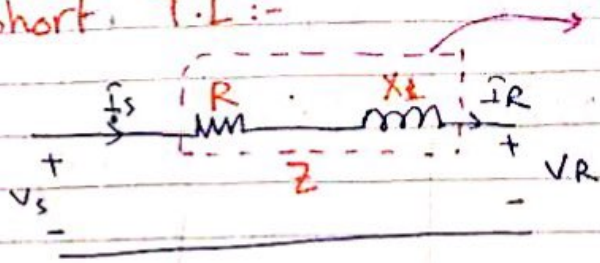
$$A = \frac{V_s}{V_R} \Big|_{I_R=0} \quad (\text{No-load})$$

B (Ω)

C (S)

D (dimensionless)

* Short T.L :-



$R' = 0.1 \Omega / \text{km}$, length = 50 km

$R = 5 \Omega$

$L = H / \text{km}$

$X_L = j \omega L = (j 2\pi f * L) * \text{length}$

$V_S = \hat{I}_R (Z) + V_R$

$V_S = V_R + Z \hat{I}_R$

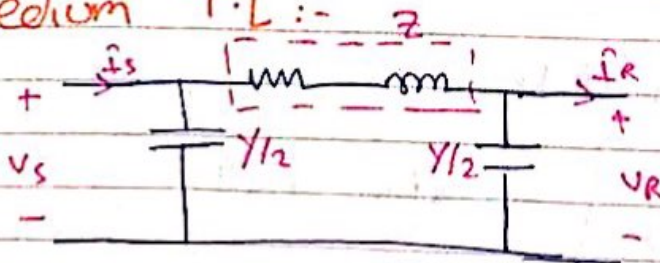
$A = 1$, $B = Z$

$\hat{I}_S = \hat{I}_R$

$C = 0$, $D = 1$

$AD - BC = 1$

* Medium T.L :-



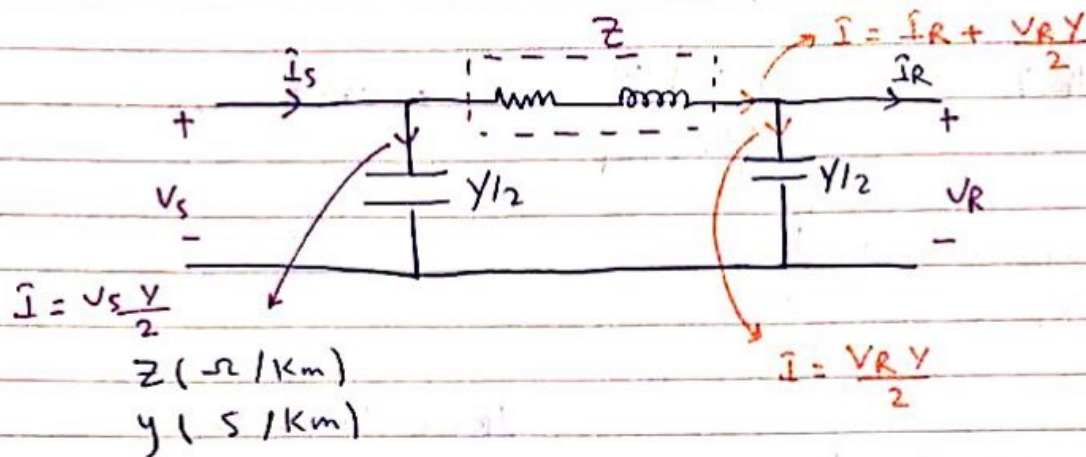
$C = X \text{ MF/km}$

$y = j \omega c \text{ S/km}$

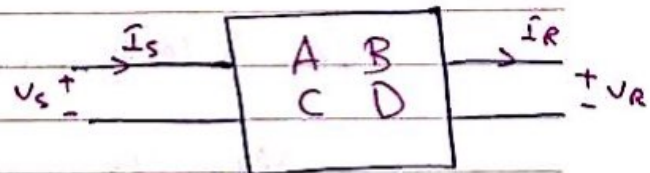
$Y = (j \omega c) * \text{length}$

Lump shunt capacitance located half at each end.

Medium transmission lines (80km - 240km)



$$\begin{bmatrix} V_S \\ \hat{I}_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ \hat{I}_R \end{bmatrix}$$



$$V_S = (\hat{I}_R + V_R \frac{Y}{2}) Z + V_R$$

$$V_S = V_R \left(1 + \frac{YZ}{2} \right) + \hat{I}_R Z$$

A (diminution loss) B (Ω)

$$\hat{I}_S = \frac{V_S Y}{2} + \hat{I}_R + \frac{V_R Y}{2}$$

$$\hat{I}_S = V_R \left[Y \left(1 + \frac{YZ}{4} \right) \right] + \hat{I}_R \left[1 + \frac{YZ}{2} \right]$$

$$\Rightarrow A = D = 1 + \frac{YZ}{2}$$

$$\Rightarrow B = Z$$

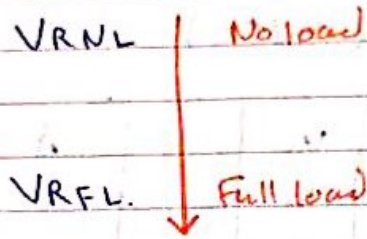
$$\Rightarrow C = Y \left(1 + \frac{YZ}{4} \right)$$

* Voltage Regulation :-

$$VR\% = \frac{|V_{RNL}| - |V_{RFL}|}{|V_{RFL}|} * 100\%$$

$V_{RNL} \equiv$ No load
 $V_{RFL} \equiv$ Full load

V_s fixed.



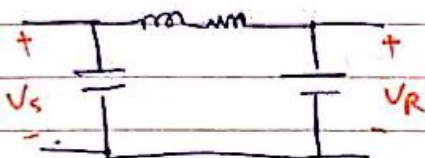
$V_s = A V_R + B I_R$ at No load $I_R = 0$.

$$V_R = \frac{V_s}{A}$$

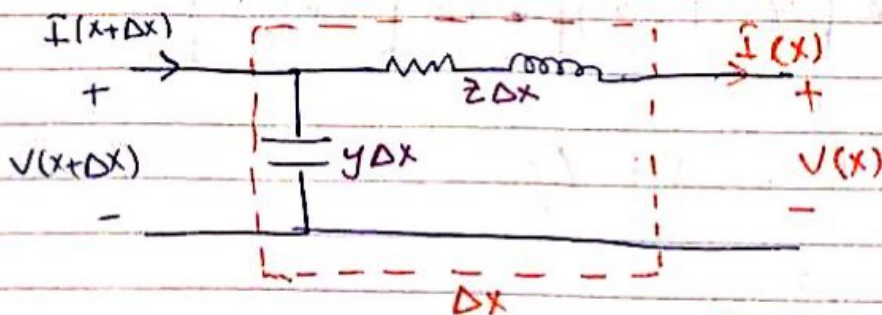
$A \approx 1$

$V_R > V_s$ under No load condition.

$(V_{RFL} < V_{RNL}) *$



* Long Transmission lines ($\geq 240\text{km}$)



(Port end ke direction se x)

$$\text{length} = L$$
$$V(x=L)$$

$$* V_{\text{nominal}} = 230V$$
$$V.D\% = 6\%$$
$$V = 94\% * 230$$

$$V_R = V(x=0)$$

$$V(x+\Delta x) = V(x) + (Z \Delta x) \hat{I}(x)$$

$$\frac{V(x+\Delta x) - V(x)}{\Delta x} = Z \hat{I}(x)$$

$$\Delta x \rightarrow 0$$

$$\frac{\partial V(x)}{\partial x} = Z \hat{I}(x) \rightarrow \textcircled{1}$$

$$\hat{I}(x+\Delta x) = \hat{I}(x) + V(x+\Delta x) y \Delta x$$

$$\frac{\hat{I}(x+\Delta x) - \hat{I}(x)}{\Delta x} = V(x+\Delta x) y$$

$$\Delta x \rightarrow 0$$

$$\frac{\partial \hat{I}(x)}{\partial x} = y V(x) \rightarrow \textcircled{2}$$

$$\frac{\partial^2 V(x)}{\partial x^2} = Z \frac{\partial \hat{I}(x)}{\partial x}$$

$$\frac{\partial^2 V(x)}{\partial x^2} = Z y V(x)$$

$$\frac{\partial^2 \hat{I}(x)}{\partial x^2} = Z y \hat{I}(x)$$

γ^2

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$$V(x) = A_1 e^{\delta x} + A_2 e^{-\delta x}$$

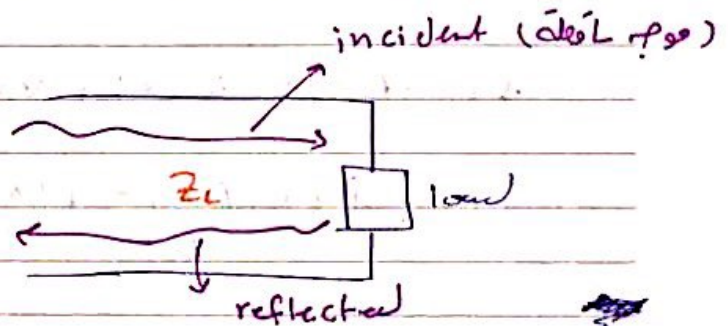
↳ distance from load

$\delta \triangleq$ propagation constant.

$$\delta = \sqrt{yz}$$

$$= \underbrace{\alpha}_{\text{attenuation constant}} + j \underbrace{\beta}_{\text{phase constant}}$$

$$\frac{\partial V(x)}{\partial x} = A_1 \delta e^{\delta x} - A_2 \delta e^{-\delta x}$$



$$\frac{\partial V(x)}{\partial x} = Z \hat{I}(x)$$

$$\hat{I}(x) = \frac{A_1 e^{\delta x} - A_2 e^{-\delta x}}{Z/\delta}$$

$$\frac{Z}{\delta} = \frac{Z}{\sqrt{yz}} = \sqrt{\frac{Z}{y}}$$

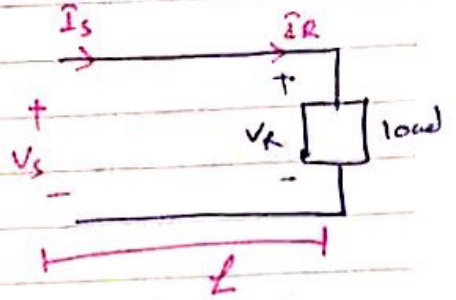
$$\hat{I}(x) = \frac{A_1 e^{\delta x} - A_2 e^{-\delta x}}{\sqrt{Z/y}}$$

Z_c (c/s impedance)

long transmission line :-

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

$$I(x) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{Z_c}$$



$$A_1, A_2 = ?!$$

Initial condition $x=0 \Rightarrow V(x=0) = V_R$
 $I(x=0) = I_R$

$$A_1 + A_2 = V_R \quad \dots \textcircled{1}$$

$$\frac{A_1 - A_2}{Z_c} = I_R \quad \dots \textcircled{2}$$

$$A_1 = \frac{V_R + Z_c I_R}{2}$$

$$A_2 = \frac{V_R - Z_c I_R}{2}$$

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$\Rightarrow Z (\Omega/\text{km}) = r + j\omega L \quad , \quad y (\text{S}/\text{km}) = j\omega C \rightarrow \frac{\text{MF}}{\text{km}}$$

$\frac{\Omega}{\text{km}} \quad \leftarrow \quad \frac{\text{H}}{\text{km}} \quad \leftarrow \quad \frac{\text{MF}}{\text{km}}$

$$\gamma = \sqrt{ZY} = \alpha + j\beta$$

$$Z_c = \sqrt{\frac{Z}{Y}}$$

$$I(x) = \frac{(V_R + Z_c I_R)/2}{Z_c} e^{\gamma x} - \frac{(V_R - Z_c I_R)/2}{Z_c} e^{-\gamma x}$$

$$V(x) = V_R \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2} \right) + I_R Z_c \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2} \right)$$

$$V(x) = V_R \cosh \gamma x + I_R Z_c \sinh \gamma x$$

$$I(x) = \frac{V_R}{Z_c} \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2} \right) + I_R \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2} \right)$$

$$I(x) = \frac{V_R}{Z_c} \sinh \gamma x + I_R \cosh \gamma x$$

((receiving end no $x=l$))

* at sending end ($x=0$)

$$V(x=0) = V_s = V_R \cosh \gamma l + I_R Z_c \sinh \gamma l$$

$$I(x=0) = I_s = \frac{V_R}{Z_c} \sinh \gamma l + I_R \cosh \gamma l$$

$$A = \cosh \gamma l$$

$$B = Z_c \sinh \gamma l$$

$$C = \frac{\sinh \gamma l}{Z_c}$$

$$D = \cosh \gamma l$$

$$AD - BC = 1$$

$$\cosh \delta L = \frac{1}{2} (e^{\delta L} + e^{-\delta L})$$

$$\delta = \alpha + j\beta$$

$$\cosh \delta L = \frac{1}{2} (e^{\alpha L} e^{j\beta L} + e^{-\alpha L} e^{-j\beta L})$$

$$* \cosh \delta L = \frac{1}{2} (e^{\alpha L} \angle \beta L + e^{-\alpha L} \angle -\beta L)$$

$$* \sinh \delta L = \frac{1}{2} (e^{\alpha L} \angle \beta L - e^{-\alpha L} \angle -\beta L)$$

$\angle \beta L \Rightarrow$ radian \Rightarrow degree \angle ω \neq
 calculator \angle ω \neq \angle

R8

$$\cosh (\alpha + j\beta L) = \cosh(\alpha L) \cos(\beta L) + j \sinh(\alpha L) \sin(\beta L)$$

$$\sinh (\alpha + j\beta L) = \sinh(\alpha L) \cos(\beta L) + j \cosh(\alpha L) \sin(\beta L)$$

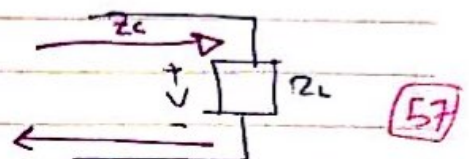
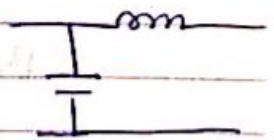
$$A = \cosh \delta L, \quad B = Z_c \sinh \delta L$$

$$C = \frac{\sinh \delta L}{Z_c}, \quad D = \cosh \delta L$$

* Lossless T.L Δ ($R = G = 0$)

$$Z = j\omega L, \quad y = j\omega C$$

$$Z_c = \sqrt{\frac{Z}{y}} = \sqrt{\frac{L}{C}} \quad \Omega \quad (\text{real})$$



Kashin Colors

$$\delta = \sqrt{ZY} = \alpha + j\beta$$

$$= \sqrt{j\omega L \cdot j\omega C} \Rightarrow$$

$$= j\omega\sqrt{LC}$$

$$\alpha = 0$$

β (Phase constant)

$$A = \cosh \delta l = \frac{1}{2} (e^{\delta l} / \beta L + e^{-\delta l} / -\beta L)$$

\Rightarrow lossless

$$\alpha = 0$$

$$A = \frac{1}{2} (\beta L + -\beta L) = \frac{1}{2} (e^{j\beta L} + e^{-j\beta L})$$

$$A = \cos \beta L = D$$

$$B = Z_c \sinh \delta L$$

$$\sinh \delta L = \frac{1}{2} (e^{\delta L} / \beta L - e^{-\delta L} / -\beta L) = \frac{1}{2} (j\beta L - -j\beta L)$$

$$= j \sin \beta L$$

$$B = Z_c (j \sin \beta L)$$

$$C = \frac{j \sin \beta L}{Z_c} = \frac{j \sin \beta L}{\sqrt{L/C}}$$

Ex

370 Km T.L, $z = 0.5240 \angle 79.04 \text{ } \Omega/\text{km}$

$y = 3.1728 \times 10^{-6} \text{ S/km} (\Rightarrow j\omega c \angle 90^\circ)$

load 125 MW, 215 KV, unity pf.
Find $V_s, \bar{I}_s, P_s, \text{V.R.}\%$?!

sol: $z = r + j\omega L \rightarrow \angle z \rightarrow \angle 60 \rightarrow 90^\circ$

$V_s = A V_R + B \bar{I}_R$

$A = \cosh \delta L$

$\delta L = \sqrt{zy} L = \sqrt{(zL)(yL)} = \sqrt{ZY}$

$\delta = \sqrt{zy} = \sqrt{0.5240 \angle 79.04 * 3.1728 \times 10^{-6} \angle 90^\circ}$

$\delta = \sqrt{0.5240 + 3.1728} \angle \frac{79.04 + 90}{2}$

مغز اوج δ
مغزب الكماتك
والزاوية δ \rightarrow $\frac{79.04 + 90}{2}$
ع δ ≈ 2 ≈ 1.2
الكز

$= \frac{0.0456}{\alpha L} + j \frac{0.4750}{BL}$

$\delta = \alpha + jB$

$\delta L = \alpha L + jBL$

$B \cong \text{phase constant} = \frac{0.4750}{370} = 0.001284 \text{ rad/km}$

$\alpha = \text{Nep/km}$

$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.5240 \angle 79.04}{3.1728 \times 10^{-6} \angle 90^\circ}}$

مغزب الكماتك Z_c \rightarrow $\frac{0.5240 \angle 79.04}{3.1728 \times 10^{-6} \angle 90^\circ}$
والزاوية δ \rightarrow $\frac{79.04 - 90}{2}$ ≈ -5.5 ≈ 1.2

$\angle z \rightarrow 60 \rightarrow 90^\circ$
 $\angle y \rightarrow 90^\circ$
 $Z_c \rightarrow -30$
 $(60 - 90)$
 $Z_c = \sqrt{-30}$
range $Z_c \rightarrow (-15 \rightarrow 0)$

$$Z_c = \sqrt{\frac{0.5240}{3.1728 \times 10^{-6}}} \angle \frac{79.04 - 90^\circ}{2}$$

$$= 406.4 \angle -5.48 \Omega$$

OHL \Rightarrow 400 Ω

$$V_s = A V_R + B I_R, \text{ per phase.}$$

$$V_{RLN} = \frac{215 \text{ K}}{\sqrt{3}} = 124.13 \text{ KV} \angle 0^\circ$$

$$= 124.13 \angle 0^\circ \text{ KV.}$$

To reference.

$$I_R ?! \rightarrow S = \sqrt{3} V_L I_R$$

$$\frac{25 \text{ MW}}{1} = \sqrt{3} \times 251 \text{ K} \times I_L$$

$$I_L = 335.7 \angle 0$$

$$\cosh \gamma L = \frac{1}{2} (e^{\alpha L} \angle BL + e^{-\alpha L} \angle -BL)$$

$$\alpha L = 0.0456, \beta L = 0.4750 \text{ rad.}$$

$$\cosh \gamma L = \frac{1}{2} (e^{0.0456} \angle 27.22^\circ + e^{-0.0456} \angle -27.22^\circ)$$

$$\cosh \gamma L = 0.8904 \angle 1.34^\circ$$

$$\sinh \gamma L = \frac{1}{2} (e^{\alpha L} \angle BL - e^{-\alpha L} \angle -BL)$$

$$\sinh \gamma L = 0.4597 \angle 84.93^\circ$$

$$V_s = \cosh \delta L V_R + Z_c \sinh \delta L I_R$$

$$A = \cosh \delta L$$

$$= 0.8904 / 1.34^\circ$$

$$0.8 \leq A \leq 1$$

$$0^\circ \leq \angle A \leq 5^\circ$$

$$B = Z_c \sinh \delta L =$$

$$= 406 \angle -5.4^\circ \times [0.45 \angle 84.93^\circ]$$

$$B \approx 190^\circ$$

$$V_s = 137 \angle 27.77^\circ \text{ KV}$$

$$I_s = C V_R + D I_R$$

$$= 332.31 \angle 26.33^\circ$$

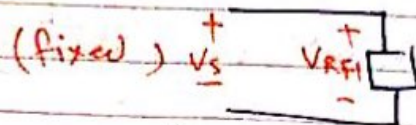
$$P_s = \operatorname{Re} \{ 3 V_s I_s^* \}$$

$$= \sqrt{3} I_L V_L \cos(\angle V - \angle I)$$

$$= \sqrt{3} \times [\sqrt{3} \times 137 \text{ K} \times 332 \times \cos(\angle 27.77 - 26.33^\circ)]$$

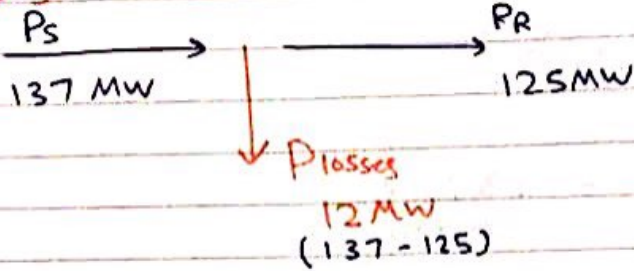
$$= 137.4 \text{ MW}$$

$$VR \% = \frac{|V_{RNL}| - |V_{RFL}|}{|V_{RFL}|}$$

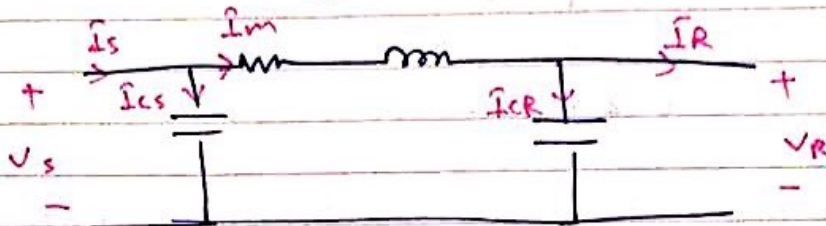


"VR" قدر في الـ Volt الـ تقريبا الـ load

losses :-

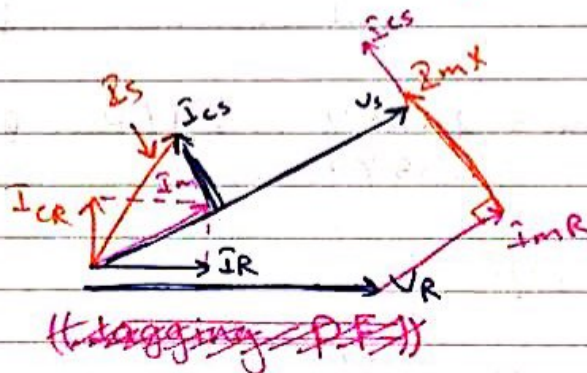


* Phasor diagram Δ



$I_{cs} \equiv$ current capacitor sending.
 $I_{CR} \equiv$ " " " receiving.

unity PF:



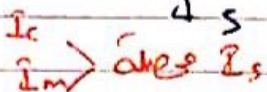
$$V_s = I_m R + j I_m X + V_R$$

$$V_s > V_R$$

$$\angle V_s > \angle V_R$$

$$I_s < I_R$$

$$I_s = I_m + I_{cs}$$



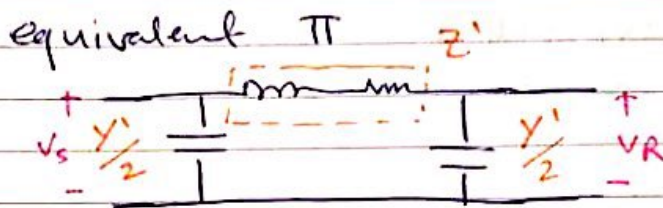
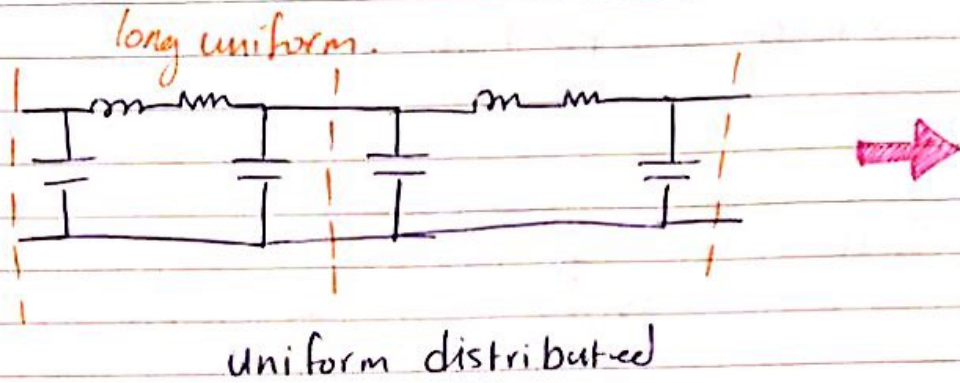
$$* i = C \frac{dV}{dt}$$

$$i = c \frac{d}{dt} (V e^{j\omega t})$$

$$i = \underbrace{j\omega C}_{90^\circ} \frac{dV}{dt}$$

$* I_c < I_R$
 لا يساوي
 model ال

π equivalent ckt of along line. 8Δ



A constant (π model) = $\frac{1 + Y'Z'}{2}$

B constant (π model) = Z'

$Z' = Z_c \sinh \gamma L$... ①

$Z' = \left(\sqrt{\frac{Z}{Y}} \sinh \gamma L \right) * \frac{ZL}{ZL}$

$Z' = ZL \sqrt{\frac{Z}{Y Z^2 L^2}} \sinh \gamma L$

$= Z \sqrt{\frac{1}{ZY L^2}} \sinh \gamma L$

$= Z * \frac{\sinh \gamma L}{\gamma L}$
 $\sqrt{ZY} = \gamma$

exact $Z' = Z * \frac{\sinh \gamma L}{\gamma L}$ correction factor ≈ 1

$$1 + \frac{Y'Z'}{2} = \cosh \delta L$$

$$\frac{Y'}{2} = \frac{\cosh \delta L - 1}{Z'}$$

$$\frac{Y'}{2} = \frac{\cosh \delta L - 1}{Z_c \sinh \delta L}$$

$$Z' = Z_c \sinh \delta L$$

$$\frac{Y'}{2} = \frac{Y}{2} \times \text{Correction factor}$$

$$\frac{\cosh \delta L - 1}{\sinh \delta L} = \tanh \delta L / 2$$

$$\frac{Y'}{2} = \frac{\tanh \delta L / 2}{Z_c} = \left(\frac{\tanh \delta L / 2}{\sqrt{ZY}} \right) \times \frac{YL/2}{YL/2}$$

$$\frac{Y'}{2} = \frac{YL}{2} \frac{\tanh \delta L / 2}{\sqrt{ZY} * YL/2}$$

$$\frac{Y'}{2} = \frac{Y}{2} \left[\frac{\tanh \delta L / 2}{\delta L / 2} \right]$$

→ Correction factor ≈ 1

- exact. $Y' \approx Y$ ← $Z' \approx Z_c$
- approximation $Y' \approx Y$ ← $Z' \approx Z_c$

* Surge impedance loading SA



line is terminated by its C/S impedance (Z_c)

$$Z_L = Z_c$$

$$Z_c = \sqrt{Z/Y}$$

lossless $\Rightarrow Z_c = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$

* overheadline. 8

132 KV	150 Ω
275 KV	315 Ω .

Cables :-

132 KV	37.1 Ω .
275 KV	50.4 Ω .

* Capacitance (Q $\mu\text{F}/\text{km}$)

Surge Impedance loading

$$Z_L = Z_c$$

* lossless T.L

$$Z_L = \sqrt{\frac{L}{C}}, \text{ real number.}$$

Voltage is constant $\Rightarrow V = V_R \underline{Bx}$

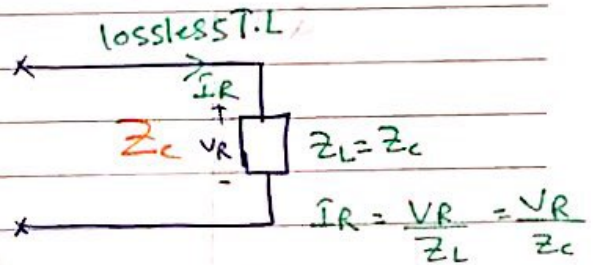
current is constant $\Rightarrow \hat{I} = \hat{I}_R \underline{Bx}$

$$SIL = \frac{(V_{rated})^2}{Z_c}$$

$$V_s = A V_R + B \hat{I}_R$$

$V(x)$

\hat{I} distance from load



$$V(x) = \cos \beta x V_R + j Z_c \sin \beta x \hat{I}_R$$

$x=L$

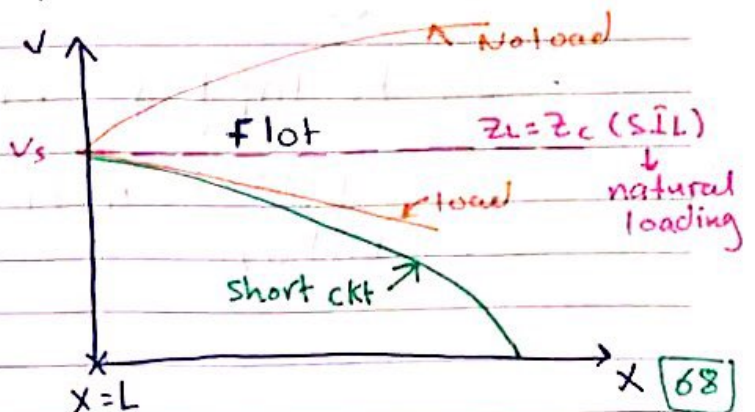
$$V(x) = \cos \beta x V_R + j Z_c \sin \beta x \frac{V_R}{Z_c}$$

$$V(x) = \cos \beta x V_R + j \sin \beta x V_R$$

$$V(x) = V_R (\cos \beta x + j \sin \beta x)$$

$$V(x) = V_R e^{j\beta x} = V_R \underline{Bx}$$

$$|V(x)| = V_R$$



$$\bar{I}(x) = \bar{I}_R \frac{L}{Bx}$$

$$V(x) = V_R \frac{L}{Bx}$$

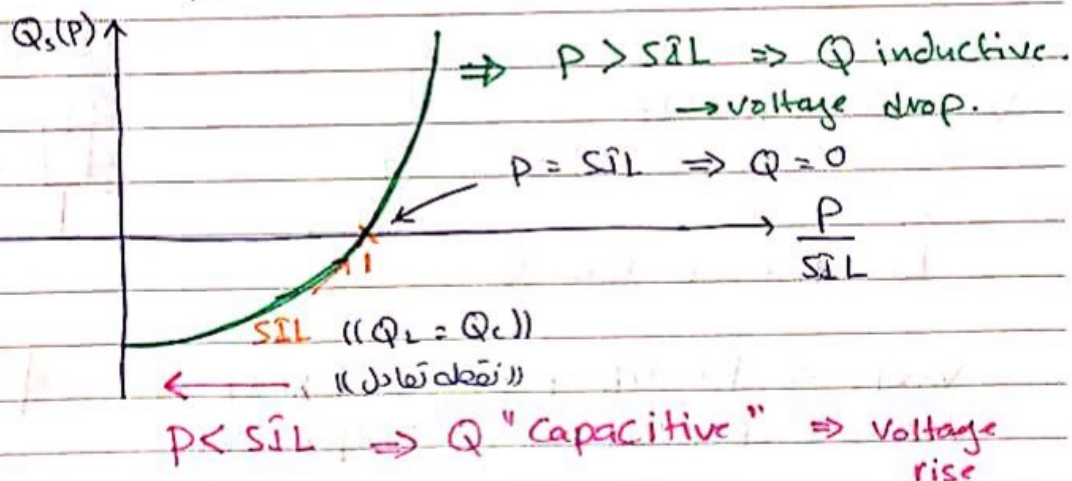
$$S_{IL} = 3 * \frac{|V_{RLN}|^2}{Z_c} \text{ MW}$$

$Z_c \rightarrow$ resistance "lossless T.L"

$$S_{IL} = 3 * \frac{\left(\frac{V_{LL}}{\sqrt{3}}\right)^2}{Z_c}$$

$$S_{IL} \triangleq \text{natural loading} = \frac{V_{LL}^2}{Z_c} = \frac{(V_{rated})^2}{Z_c} \text{ MW}$$

$$Z_c = \sqrt{\frac{L}{C}} \text{ in lossless.}$$



$$S_{IL} = \frac{(V_{rated})^2}{Z_c} = \frac{(V_{rated})^2}{\sqrt{L/C}} \text{ MW}$$

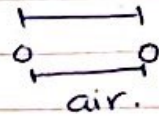
$$Z_c = \sqrt{\frac{L}{C}}$$

$$L = K \ln\left(\frac{D}{r}\right) \text{ , } D \equiv \text{spacing H/Km}$$

$$C = \frac{K}{\ln(D/r)} \text{ MF/Km. , } D \equiv \text{spacing between conductors.}$$

Over head line (OHL)

" Din cables \ll Din OHL "



" $L_{OHL} > L_{cables}$ "

" $C_{OHL} < C_{cables}$ "

" $Z_{cables} < Z_{OHL}$ "

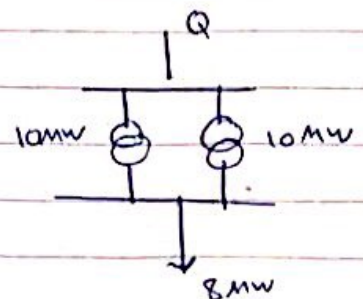
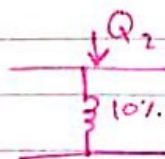
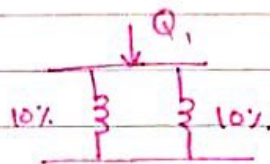
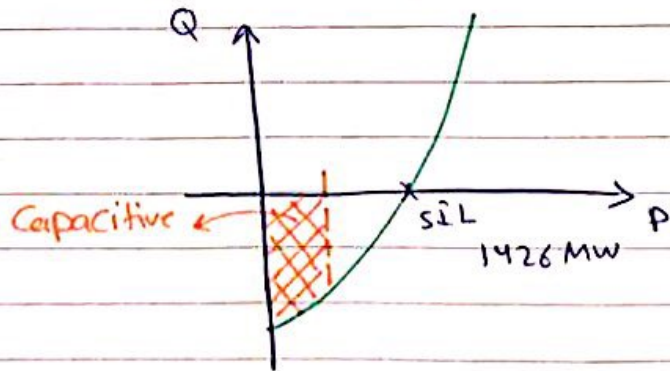
" $SIL_{cables} > SIL_{OHL}$ "

at 275 KV.

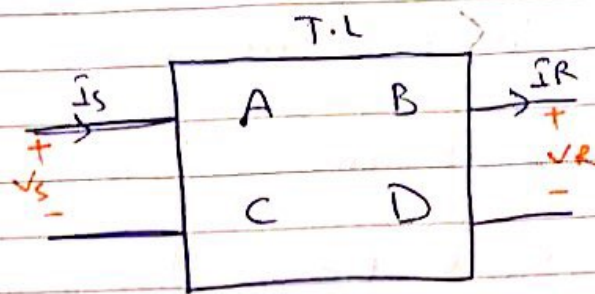
$$SIL_{OHL} = 240 \text{ MW}$$

$$SIL_{cables} = 1426 \text{ MW}$$

" injecting Q " \Leftrightarrow (Capacitive \leftarrow "تجاوب" cables)



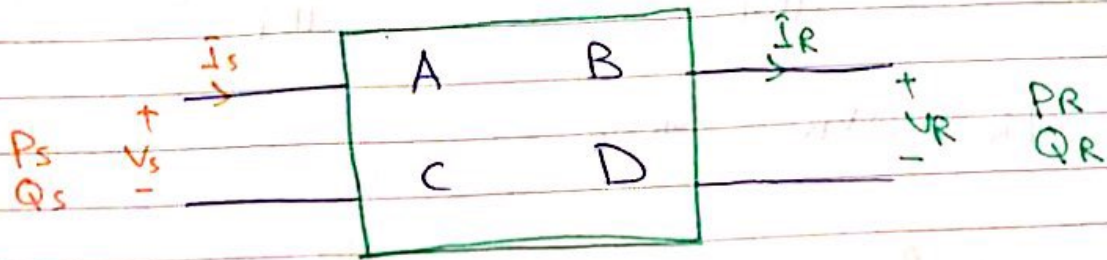
Complex power flow through T.L :-



$$\begin{bmatrix} P_R \\ Q_R \\ P_S \\ Q_S \end{bmatrix} = f \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

* Circle diagram [maximum power through T.L]

Complex power flow through transmission lines.



$$A = |A| \angle \theta_A$$

$$B = |B| \angle \theta_B$$

$$V_s = |V_s| \angle \delta$$

$$V_R = |V_R| \angle 0$$

$$V_s = A V_R + B I_R \quad , \quad S_R = V_R I_R^*$$

$$I_R = \frac{V_s - A V_R}{B}$$

$$I_R = \frac{|V_s| \angle \delta - |A| |V_R| \angle \theta_A}{|B| \angle \theta_B}$$

$$S_R = V_R \cdot I_R^*$$

$$= V_R \cdot \left(\frac{|V_s| \angle \delta - |A| |V_R| \angle \theta_A}{|B| \angle \theta_B} \right)^*$$

$$S_{R1\phi} = \frac{|V_{RLL}| |V_{sLL}|}{|B|} \angle \theta_B - \delta - \frac{|A| |V_R|^2}{|B|} \angle \theta_B - \theta_A$$

$$S_{R3\phi} = 3 \left(\frac{\frac{|V_{RLL}|}{\sqrt{3}} \cdot \frac{|V_{sLL}|}{\sqrt{3}}}{|B|} \angle \theta_B - \delta - \frac{|A| \frac{|V_{RLL}|^2}{\sqrt{3}}}{|B|} \angle \theta_B - \theta_A \right)$$

$$S_{3\phi} = \frac{|V_s| |V_R|}{|B|} \angle_{\theta_B - \delta} - \frac{|A| |V_R|^2}{|B|} \angle_{\theta_B - \theta_A}$$

$$P_R + jQ_R = S_{R3\phi}$$

$$P_R = \text{Re} \{ S_{R3\phi} \}$$

$$P_R = \frac{|V_s| |V_R|}{|B|} \cos(\theta_B - \delta) - \frac{|V_R|^2 |A|}{|B|} \cos(\theta_B - \theta_A)$$

$$Q_R = \text{Im} \{ S_{R3\phi} \}$$

$$Q_R = \frac{|V_s| |V_R|}{|B|} \sin(\theta_B - \delta) - \frac{|A| |V_R|^2}{|B|} \sin(\theta_B - \theta_A)$$

* θ_B و θ_A متغيران δ با نوع ال T.L

$$P_R = \frac{|V_s| |V_R|}{|B|} \cos(\theta_B - \delta) - \frac{|V_R|^2 |A|}{|B|} \cos(\theta_B - \theta_A)$$

* $|V_s|$ و $|V_R|$ ثابتين مع ال Power
 * "تابعية تغير في ال Power إلا اذا تغيرت δ "

* lossless \Rightarrow

$$\theta_B = 90^\circ$$

$$B = jZ_c \sin \beta L$$

$$\theta_A = 0$$

$$P_R = \frac{|V_s| |V_R|}{|B|} \cos(90 - \delta) - \frac{|A| |V_R|^2}{|B|} \cos(90)$$

$$P_R = \frac{|V_s| |V_R|}{|B|} \sin \delta$$

" $\cos(90 - \delta) = \sin \delta$ "

unit of $BL \equiv \frac{\text{rad}}{\text{km}} \cdot \text{length (km)} \hat{=} \text{electrical length.}$

$\lambda \equiv \text{wave length.}$

$\lambda f = v$

$v \equiv \text{velocity.}$

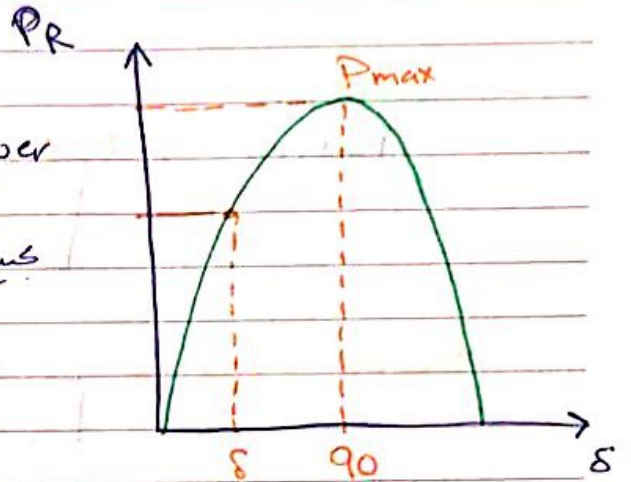
$$P_R = \frac{|V_s| |V_R|}{Z_c \sin\left(\frac{2\pi L}{\lambda}\right)} \sin \delta$$

\downarrow
length.

Stability \Rightarrow limitation power

$\delta \approx 30-45^\circ$

System \downarrow fault, \uparrow $|s| \uparrow$ \leftarrow μs
system stable, \downarrow μs

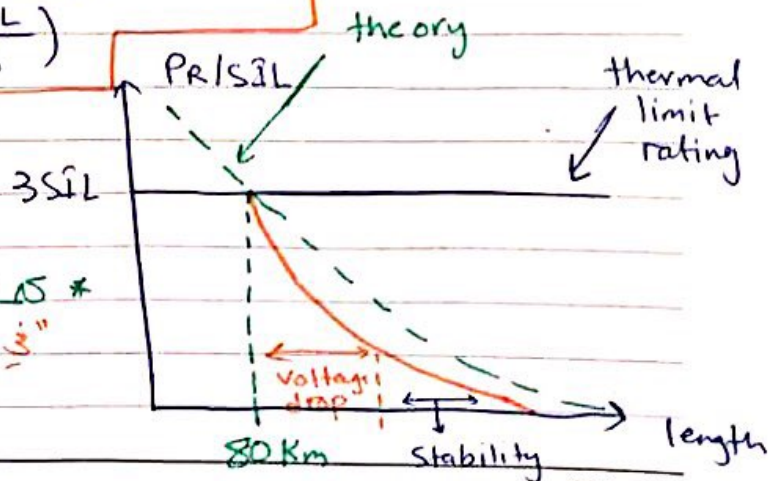


$$P_R = \frac{|V_s| |V_R|}{Z_c \sin\left(\frac{2\pi L}{\lambda}\right)} * \frac{|V_{rated}|^2}{|V_{rated}|^2} \sin \delta$$

$$P_R = |V_{spul}| |V_{Rpul}| \cdot \frac{SIL \sin \delta}{\sin \frac{2\pi L}{\lambda}}$$

$$\frac{|V_{rated}|^2}{Z_c} = SIL$$

$$\frac{P_R}{SIL} = \frac{|V_{spul}| |V_{Rpul}|}{\sin\left(\frac{2\pi L}{\lambda}\right)} \sin \delta$$



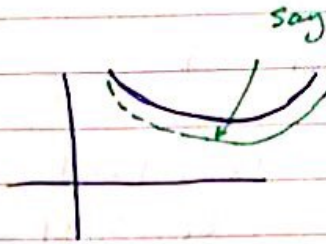
* $L \leq \lambda$ length ال طول، بقل
"في الكابلات لا يوجد مشكلة"

* Max power flow (limits).

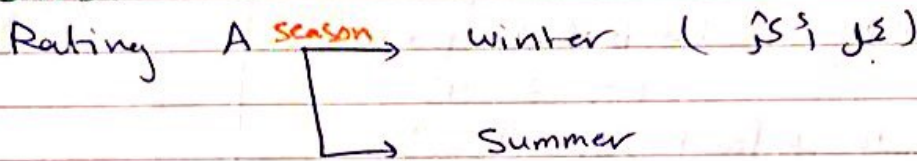
* thermal limit (heating)

- over head line (OHL)

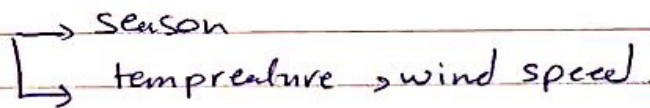
↑ heating → Sag
"نخس"



- cables.

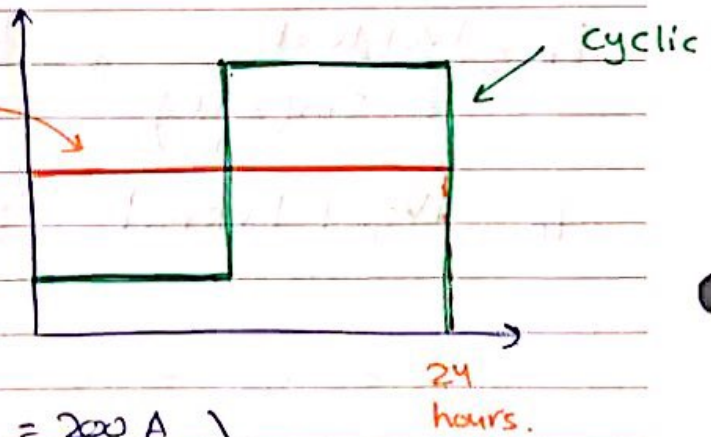


thermal rating



* Continuous rating

* Cyclic rating.



* line (Continuous rating = 200 A)
Cyclic " = 250 A)

* lossless T.L :-

$$P_R = \frac{|V_S| |V_R|}{|B|} \sin \delta$$

$\hookrightarrow Z_c \sin \beta L$

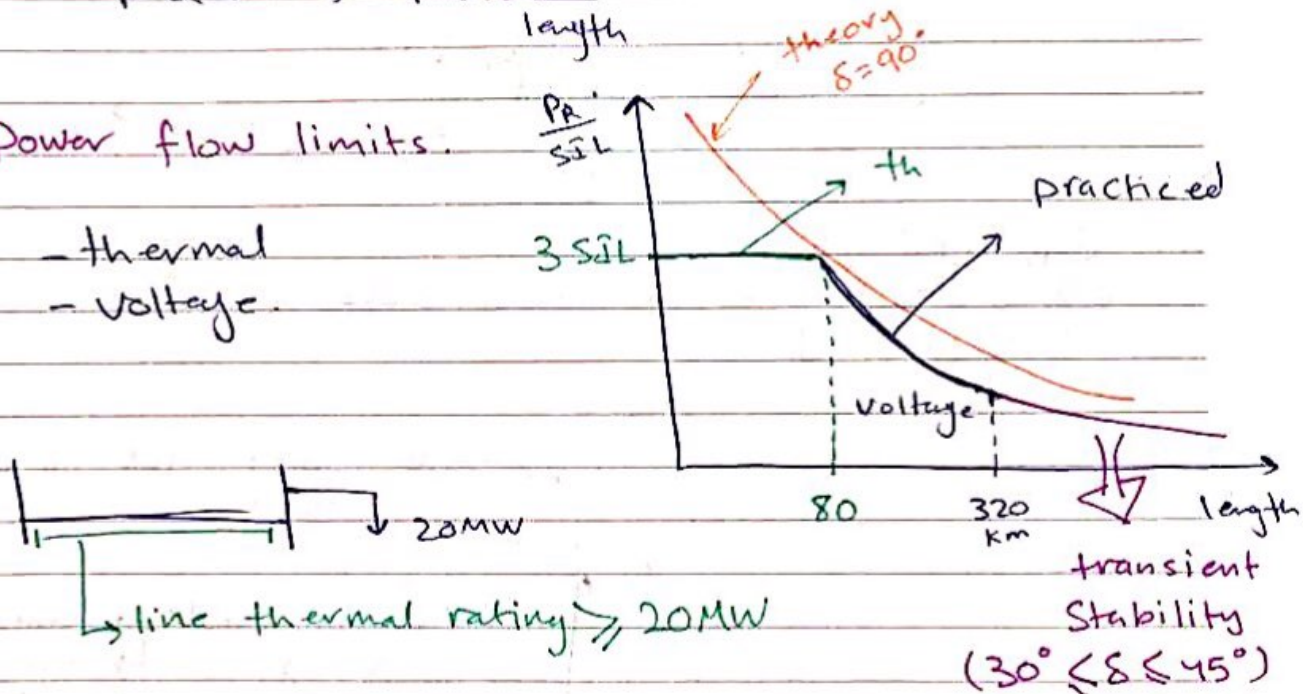
$$= \frac{V_S P_u \quad V_R P_u \quad S_{IL} \sin \delta}{\sin \left(\frac{2\pi L}{\lambda} \right)}$$

S_{IL} = Surge impedance loading.

$P \propto V^2$, $P \propto \frac{1}{\text{length}}$

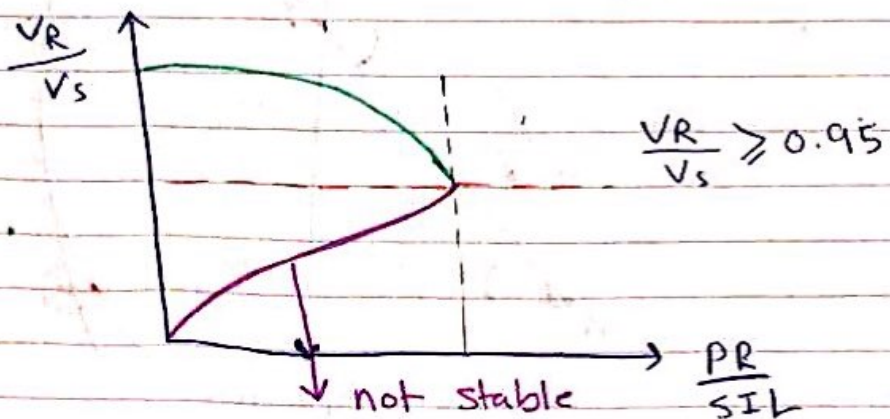
Power flow limits.

- thermal
- voltage.



* voltage.

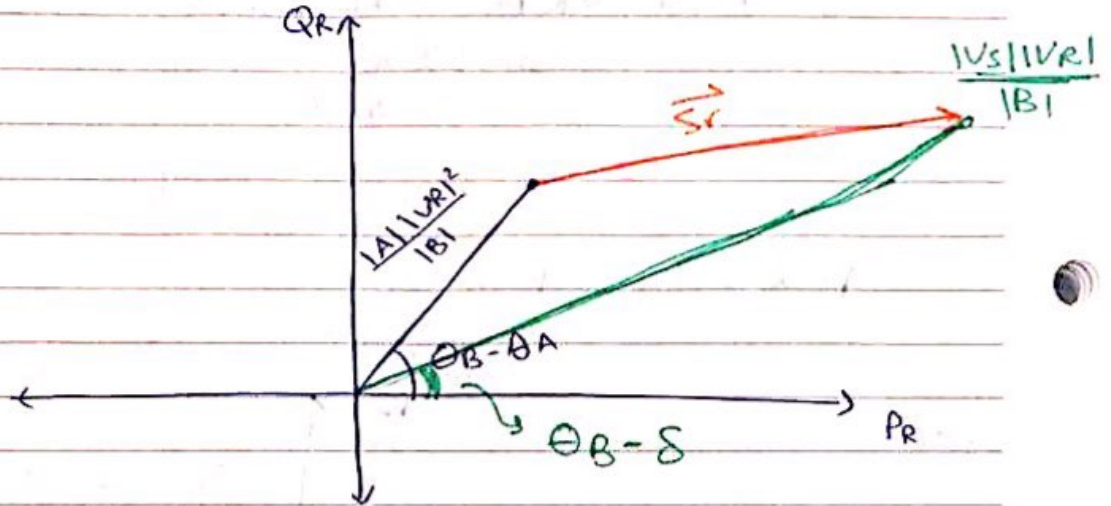
$$\uparrow P_R \rightarrow \uparrow Q_R \uparrow \rightarrow \Delta V. \quad (V \downarrow \text{ jeu } P \downarrow \text{ jai ;})$$



* Receiving-end power circle diagram

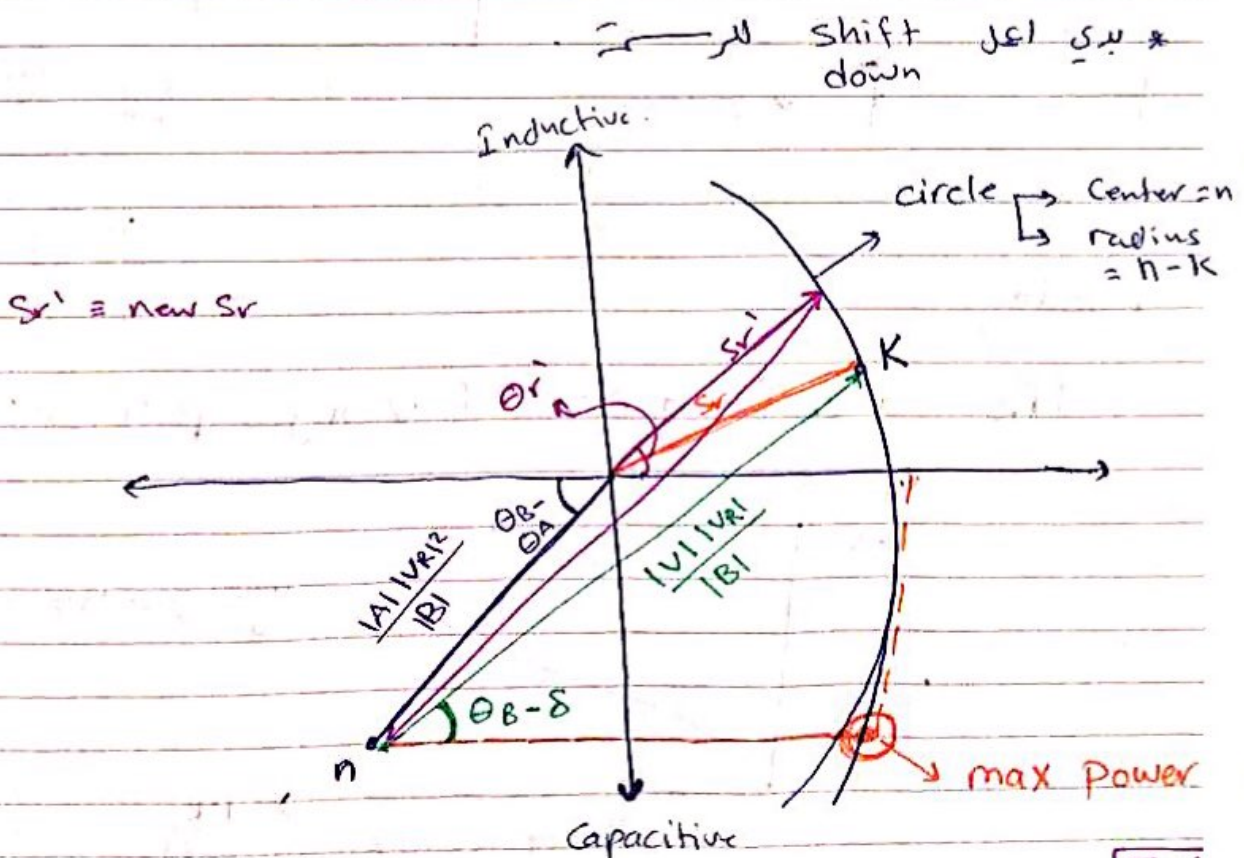
$$S_R = \frac{|V_S| |V_R|}{|B|} \angle \theta_B - \delta = \frac{|A| |V_R|^2}{|B|} \angle \theta_B - \theta_A$$

$$S_R = P_R + jQ_R$$



$\theta_B \approx 90^\circ$
 $\theta_A \approx 0^\circ$
 $\delta \approx 30^\circ$

$\Rightarrow \theta_B - \theta_A > \theta_B - \delta$



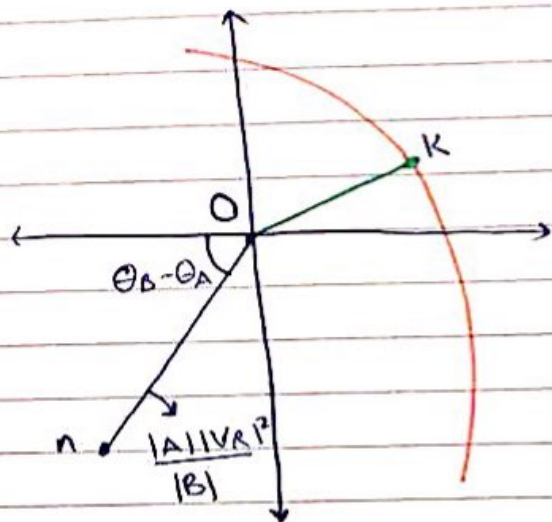
$|V_s| \triangleq \text{constant}$
 $|V_R| \triangleq \text{constant}$

max power. $\begin{cases} \rightarrow \text{lossless} \rightarrow \delta = 90^\circ \\ \rightarrow \text{lossy} \rightarrow \delta = \theta_B \end{cases}$

* Step to draw circle diagram $\delta \Delta$ (single phase LL) $\left(\begin{matrix} \leftarrow \text{برسم LL} \\ \text{phase} \end{matrix} \right)$

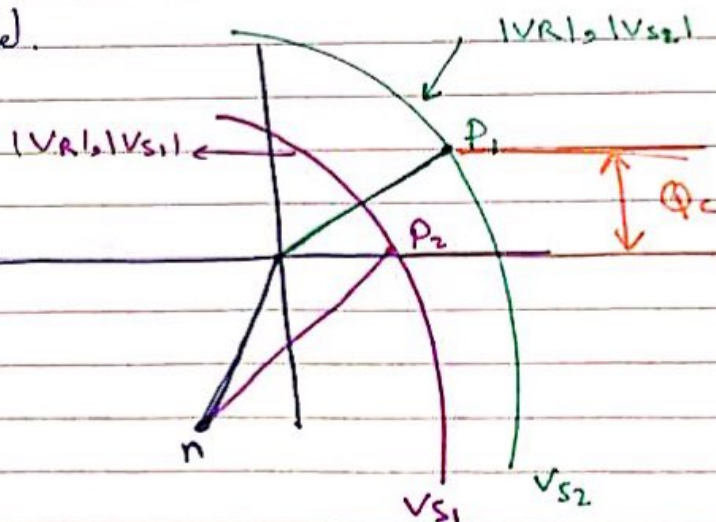
- center.
 - angle.
 - scale α (100M \rightarrow 1cm)

- Radius.
 - $\frac{|V_s| |V_R|}{|B|} \rightarrow$ برسم بالفرجا



if V_R constant \rightarrow
 (On) fixed

* V_s changed.



P_1 and P_2 have same V_R .

$V_s \rightarrow$ V_R $\left(\begin{matrix} \leftarrow \text{برسم LL} \\ \text{phase} \end{matrix} \right)$
 Q_c capacitor $\left(\begin{matrix} \leftarrow \text{برسم LL} \\ \text{phase} \end{matrix} \right)$

$V_{s2} > V_{s1}$
 $V_{s2} \text{ max power} > V_{s1} \text{ max power}$

Reactive power Compensation 8

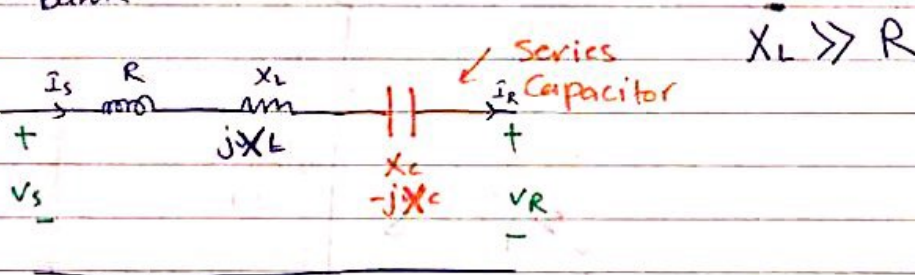
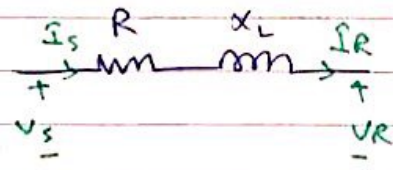
Series Compensation
"heavy load" (peak demand)

Shunt Compensation
light load.

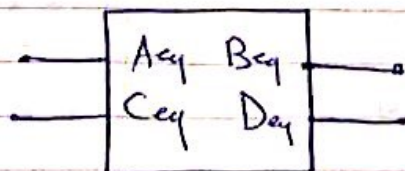
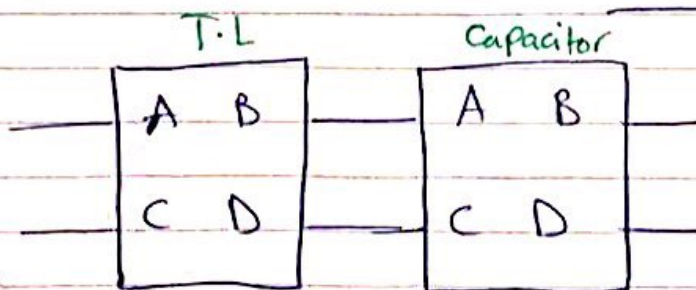
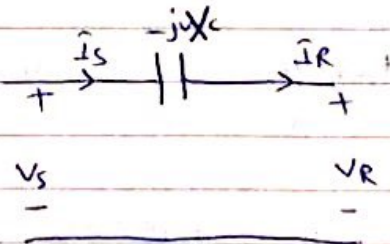
Series Compensation :-

- Short T.L

heavy load \Rightarrow voltage drop.
 \Rightarrow solution add capacitor bank



ABCD Constant series Capacitor



$$V_s = V_R - jX_c \hat{I}_R$$

$$\hat{I}_s = \hat{I}_R$$

$$A = 1, B = -j|X_c|, C = 0, D = 1$$

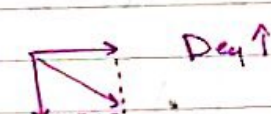
$$\begin{bmatrix} 1 & -j|X_c| \\ 0 & 1 \end{bmatrix}$$

$$\begin{matrix} \text{T.L} & & \text{Capacitor} \\ \begin{bmatrix} A & B \\ C & D \end{bmatrix} & \begin{bmatrix} 1 & -j|X_c| \\ 0 & 1 \end{bmatrix} \end{matrix}$$

* $P_{max} = \frac{|V_s| |V_R| \sin \delta}{|B|}$
 * P_{max} دى δ دى B دى كىلىدۇ

$$= \begin{bmatrix} A & -jA|X_c| + B \\ C & -jC|X_c| + D \end{bmatrix}$$

$|B_{eq}| < B$

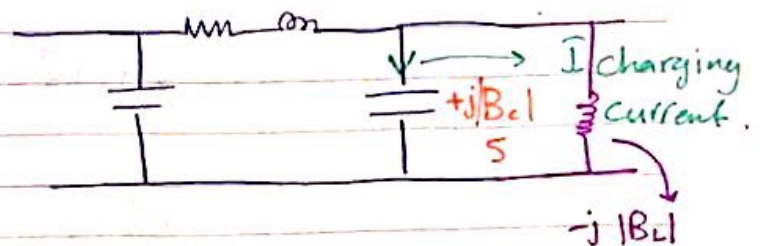


$$V_s = A_{eq} V_R + B_{eq} \hat{I}_R$$

$|V_R|$ constant \Rightarrow
 without series $\Rightarrow |V_{s1}|$
 with series $\Rightarrow |V_{s2}|$

Voltage drop. $\Rightarrow |V_{s2}| < |V_{s1}|$

* under light load: voltage rise.



81 Capacitor \rightarrow voltage rise

$$\hat{I}_{\text{charging current}} = |V_{LN}| B_c$$

$$X = j\omega L, \quad B_L = \frac{-j}{X_L}$$

$$\hat{I}_{\text{charging}} = |V_{LN}| (|B_c| - |B_L|)$$

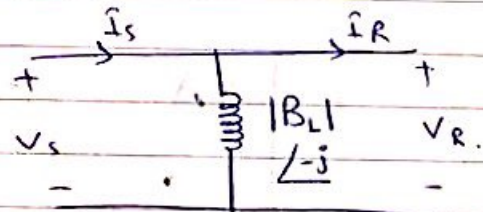
$$= |V_{LN}| |B_c| \left(1 - \frac{|B_L|}{|B_c|} \right)$$

compensation factor

80% compensation factor $\rightarrow B_L = 80\% B_c$

$$V_s = V_R$$

$$\hat{I}_s = \hat{I}_R + j|B_L| V_R$$



$$\begin{bmatrix} 1 & 0 \\ -j|B_L| & 1 \end{bmatrix}$$

(in light load)
All series react

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j|B_L| & 1 \end{bmatrix}$$

$$A_{eq} = A - j|B_L| B \Rightarrow \boxed{A_{eq} > A}$$

$\underbrace{0}_{\downarrow} \quad \underbrace{-90}_{\downarrow} \quad \underbrace{+90}_{\downarrow}$
 $-j \quad +j \quad -j * +j = +ve.$

$$V_s = A V_R + B \hat{I}_R$$

$$\downarrow V_R = \frac{V_s}{A \uparrow}$$

Ex AT.L serve a load (400 MVA, 0.8 PF lag
345 KV), receiving voltage 345 KV
T.L $A = D = 0.81 \angle 1.3^\circ$
 $B = 172.2 \angle 84.4^\circ \Omega$
 $C = 0.001933 \angle 90.4^\circ S$
 $-jX_s = -j 146.6$

① V.R.V. ?!

② Series capacitor ~~at~~ at mid of T.L
Find V.R.V. ?!

$$\text{mid T.L} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.9534 \angle 0.3^\circ & 90.33 \angle 84.1^\circ \\ 0.001014 \angle 90.1^\circ & 0.9534 \angle 0.3^\circ \end{bmatrix}$$

Sol. ①

$$V_R = \frac{|V_{RNL}| - |V_{RFL}|}{|V_{RFL}|} \times 100\%$$

V_s fixed.

$$V_{RFL} = \frac{345}{\sqrt{3}} = 199 \text{ KV}$$

$$V_s = A V_R + B \hat{I}_R$$

$$V_{RNL} = \frac{V_s}{A}$$

$$V_s = A V_R + B \hat{I}_R$$

at full load $V_R = 199 \text{ KV} \angle 0$

$$S = \sqrt{3} V_L \hat{I}_L$$

$$\hat{I}_L = \hat{I}_R = \frac{S}{\sqrt{3} V_L} = \frac{400}{\sqrt{3} (345)} = 669.4 \angle -36.87^\circ \text{ A}$$

$$\angle -\cos^{-1}(\text{PF}) = \angle -36.87$$

83

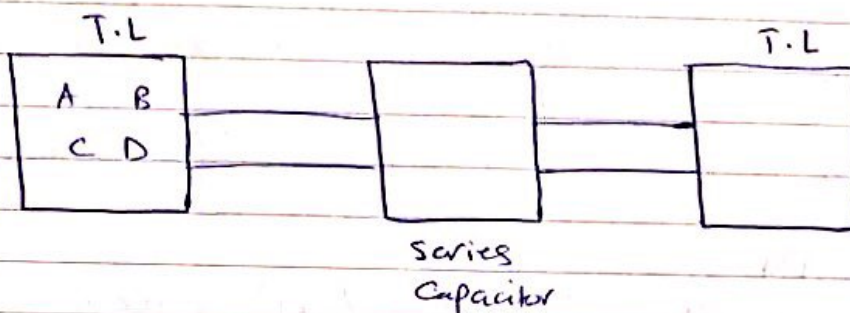
$$V_s = 256.8 \angle 20.1 \text{ KV}$$

$$V_{RNL} = \frac{V_s}{A} = \frac{256.8}{0.81}$$

$$V_R = \frac{\left| \frac{V_s}{A} \right| - |V_{RFL}|}{|V_{RFL}|} \times 100\%$$

$$V_R \% = 57.6 \%$$

Sol (2)



Series Capacitor \leftarrow ABCD equivalent

$$\text{Series Capacitor} = \begin{bmatrix} 1 & -j146.6 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.9534 \angle 0.3 & 90.33 \angle 84.1 \\ 0.001014 \angle 90.1 & 0.9534 \angle 0.3 \end{bmatrix} * \begin{bmatrix} 1 & -j146.6 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} & \\ & \end{bmatrix}$$

T.L → نقطة

$$A_{eq} = 0.96 \angle 1.2^\circ = D_{eq}$$

$$B_{eq} = 42 \angle 64.8^\circ$$

$$C_{eq} = 0.002 \angle 90^\circ$$

841

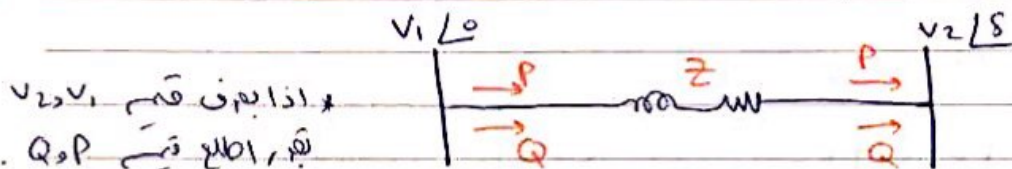
$$V_s = 216.7 \angle 4.5^\circ$$

$$VR\% = 14\%$$

* Load flow / Power flow 84

what?!

- Voltage (magnitude, angle)
- Complex power flow (P, Q) through T.L

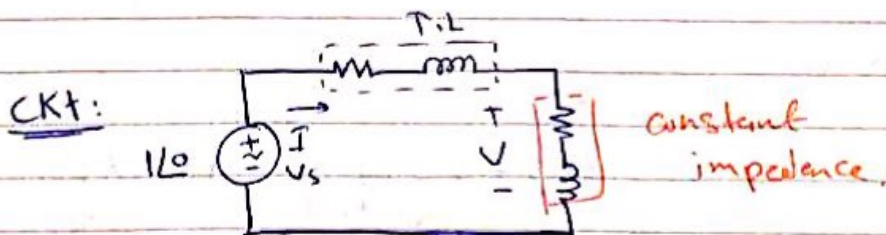


why?!

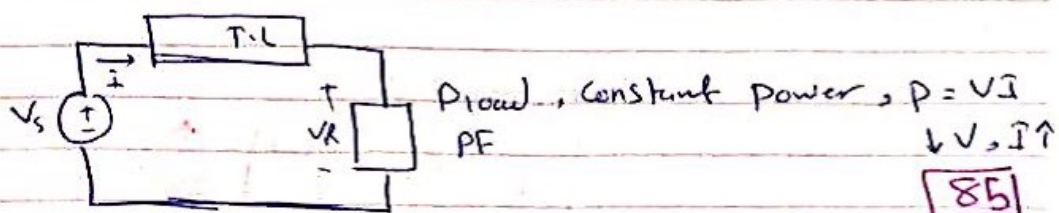
- planning
- operations

* check Voltage \leq limits \pm $\left\{ \begin{array}{l} \rightarrow \text{normal} \\ \rightarrow \text{contingency} \end{array} \right.$
 * " power flows \leq limits $\left\{ \begin{array}{l} \rightarrow \text{normal} \\ \rightarrow \text{contingency} \end{array} \right.$

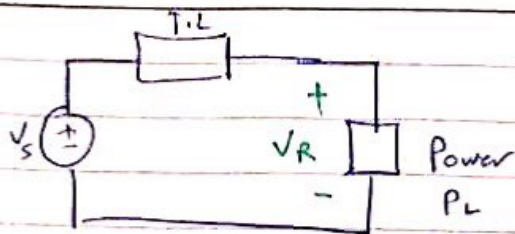
"loss of line"



$$I = V_s / Z, \quad P = I^2 R, \quad Q = I^2 X$$

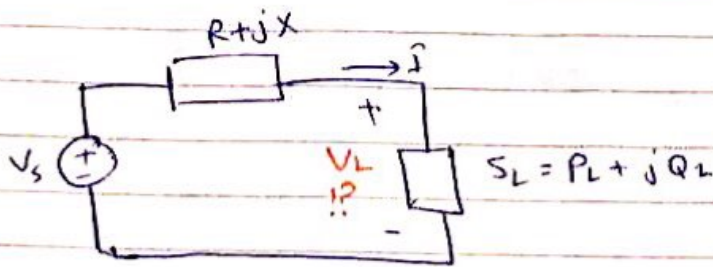


85



Find $V_R = ?!$ using

Iterations



$$S_L = V_L I^*$$

$$I = \frac{S_L^*}{V_L^*}$$

$$I = \frac{P_L - jQ_L}{V_L^*} \quad \dots \quad (1)$$

$$V_L = V_s - I(R + jX)$$

$$V_L = V_s - \frac{P_L - jQ_L}{V_L^*} (R + jX)$$

numerical steps

Numerical analysis

iteration:-

$$V_L^{(0)} \xRightarrow{\text{initial}} V_L^{(1)} = V_s - \frac{P_L - jQ_L}{V_L^{*(0)}} (R + jX)$$

$$V_L^{(1)} \Rightarrow V_L^{(2)} = V_s - \frac{P_L - jQ_L}{V_L^{*(1)}} (R + jX)$$

اگر ان الناتج سے $V_L^{(0)}$ بڑھتا ہے

tolerance.

$$|V_L^{(i+1)} - V_L^{(i)}| \leq \epsilon$$

86

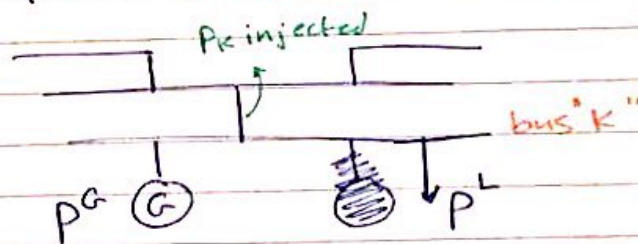
load flow \rightarrow Gauss - Sidel
 \rightarrow Newton - Raphson.

- * formulation power flow problem
- * Solutions.

* Formulation power flow Problem 84
 Inputs:-

* Single line diagram "data" \rightarrow T.L
 \rightarrow Generator
 \rightarrow Load.

- * bus. V_k
 δ_k
 P_k injected
 Q_k injected



$$KCI: P_k^{\text{injected}} = P_k^G - P_k^L$$

$$Q_k^{\text{injected}} = Q_k^G - Q_k^L$$

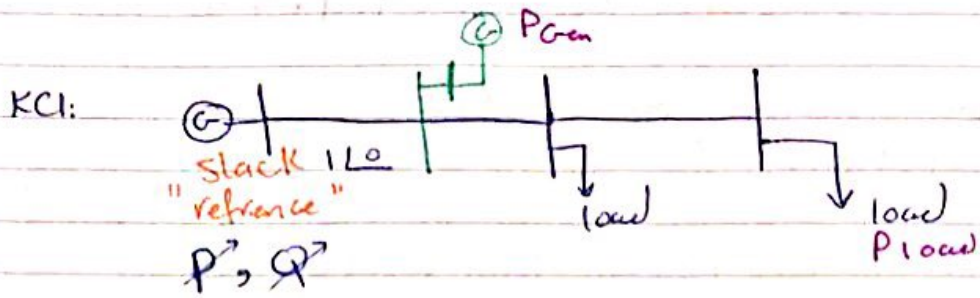
* classification / Bus types 85

* Load bus \rightarrow Known: P_k, Q_k
 PQ \rightarrow unknown: V_k, δ_k .

* PV bus \rightarrow Known: P, V_k (magnitude) "control volt"
 \rightarrow unknown: Q_k, δ_k

* Slack bus
"buffer"

Known: V_k, δ_k
UnKnown: P_k, Q_k



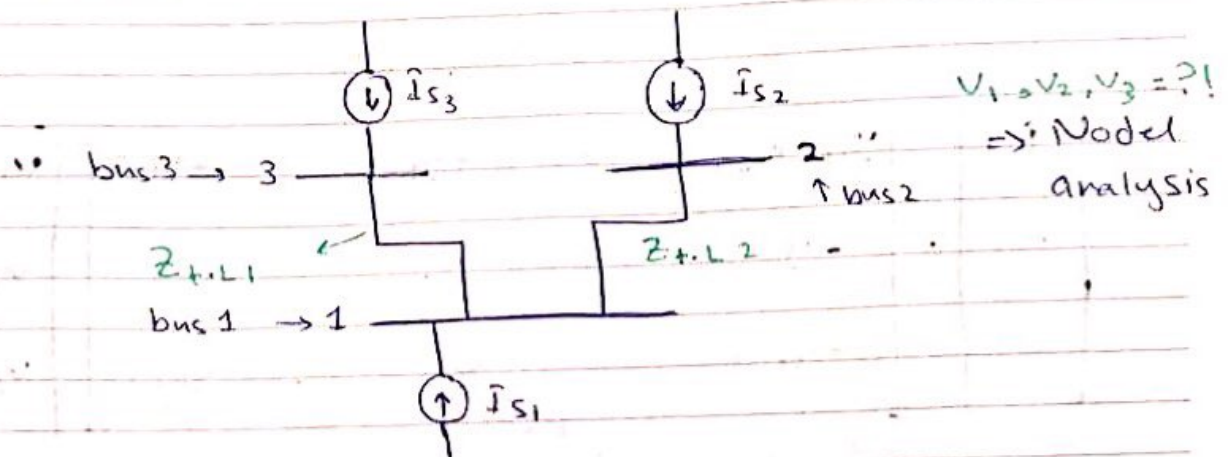
reference voltage, P, Q, δ, ω (Gen) $\omega, \delta, \omega, \delta, \omega$

$$P_{\text{slack}} + P_{\text{generation}} - P_{\text{load}} - P_{\text{losses}} = 0$$

balancing law

f (voltage)

↳ unknown

* Admittance matrix " Y_{bus} "

at bus 1

$$\sum \hat{I}_{in} = \sum \hat{I}_{out} \quad (KCL)$$

$$\hat{I}_{S1} = \frac{V_1 - V_2}{Z_{t.L2}} + \frac{V_1 - V_3}{Z_{t.L1}}$$

$$\hat{I}_{S1} = (V_1 - V_2) Y_{t.L2} + (V_1 - V_3) Y_{t.L1}$$

$$\hat{I}_{S1} = V_1 (Y_{t.L2} + Y_{t.L1}) - V_2 Y_{t.L2} - V_3 Y_{t.L1}$$

at bus 2

$$\begin{aligned} \hat{I}_{S2} &= (V_2 - V_1) Y_{t.L2} \\ &= V_2 Y_{t.L2} - V_1 Y_{t.L2} \end{aligned}$$

at bus 3

$$\hat{I}_{S3} = V_3 Y_{t.L1} - V_1 Y_{t.L1}$$

$$\vec{I} = Y \vec{V}$$

↳ injected

$$\begin{bmatrix} \vec{I}_{s1} \\ \vec{I}_{s2} \\ \vec{I}_{s3} \end{bmatrix} = \begin{bmatrix} y_{TL1} + y_{TL3} & -y_{TL2} & -y_{TL3} \\ -y_{TL2} & y_{TL2} & 0 \\ -y_{TL3} & 0 & y_{TL3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

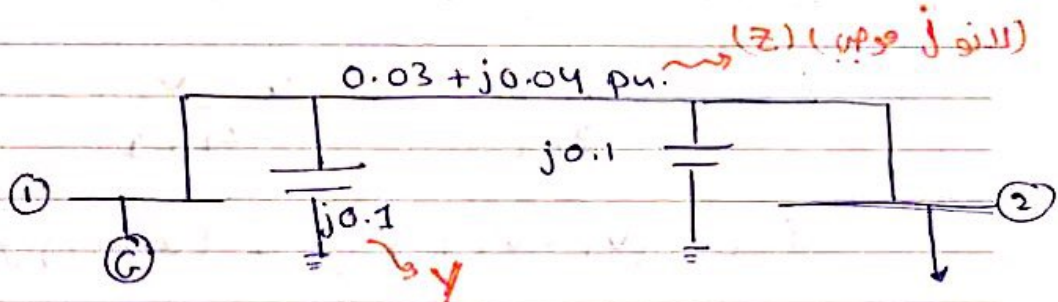
كل متبوع 1

لانه 2 متبوع مع 3

كل 1 متبوع مع 3

دائما diagonal موجب والباقي سالب

Ex



((التيار الجوز موجب))

Find Y matrix ?!

$$y_{TL} = \frac{1}{0.03 + j0.04} = 12 - j16$$

$$Y = \begin{bmatrix} 12 - j16 + j0.1 & -(12 - j16) \\ -(12 - j16) & -12 - j16 + j0.1 \end{bmatrix}$$

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Power Flow Solution :-

"iterative solution"

Gauss sidel Δ

$$f(x) = 0 \rightarrow \text{write it as } s$$

$$\boxed{X = F(x)} \quad \text{Fixed-point solution}$$

$$X^{(0)} \Rightarrow X^{(1)}$$

$$X^{(1)} \Rightarrow X^{(2)}$$

⋮

$$\text{until } \max |x^{i+1} - x^i| \leq \epsilon$$

Ex Solve $x - \sqrt{x} - 1 = 0$ by using fixed point method?

$$x^0 = 1 \Rightarrow \text{(initial condition)}$$

Sol: $x = \sqrt{x} + 1$

iteration \rightarrow	v	x^v
0		1 \rightarrow initial condition.
1		2
2		⋮
⋮		⋮
9		2.217 2.617

91

Power flow solution - Gauss Sidel

$$X = f(x)$$

$$KCI \quad \Delta I = 0$$

$$I_i = \sum_{j=1}^N y_{ij} V_j$$

$$S_i = V_i I_i^*$$

$$I_i = \frac{S_i^*}{V_i^*}$$

$$\frac{S_i^*}{V_i^*} = \sum_{j=1}^N y_{ij} V_j$$

$$\frac{S_i^*}{V_i^*} = y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^N y_{ij} V_j$$

$j \Rightarrow$ Bus 1, 2, ..., N
المحطات في النظام

$$V_i = \frac{1}{y_{ii}} \left[\frac{S_i^*}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^N y_{ij} V_j \right]$$

Complex voltage (المعقد الجهد) في Bus i (V_i^*)
Complex

$$X = f(x)$$

$$V_i = \frac{1}{y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^N y_{ij} V_j \right]$$

Let K iteration number.

$$V_i^{(K+1)} = \frac{1}{y_{ii}} \left[\frac{P_i - jQ_i}{V_i^{(K)*}} - \sum_{j=1}^{i-1} y_{ij} V_j^{(K+1)} - \sum_{j=i+1}^N y_{ij} V_j^{(K)} \right]$$

PQ bus $\Rightarrow P_i, Q_i \rightarrow$ Known
 $V_i \Rightarrow$ unknown

* 4 bus , 3 iteration.

iteration \leftarrow
 $V_2^{(4)} = f(V_1^{(4)}, V_2^{(3)}, V_3^{(3)}, V_4^{(3)})$
 bus \swarrow

$V_3^{(4)} = f(V_1^{(4)}, V_2^{(4)}, V_3^{(3)}, V_4^{(3)})$
 حساب باقی دو بار و محاسبه
 بی پی ای

PV bus $\Rightarrow P, Q \rightarrow$ unknown , $V \rightarrow$ Known
 $\delta \rightarrow$ unknown.
 Known

* باقی، فرض کنیم Q را نمی دانیم، V را

بی پی ای Q ① $S_i = V_i \cdot I_i^*$
 $S_i = V_i \left[\sum_{j=1}^N y_{ij} V_j \right]^*$

((بی پی ای Q را V محاسبه))
 $Q \rightarrow$ calculate V
 * Q را نمی دانیم، فرض کنیم V را
 PV bus \Rightarrow Volt

$S_i^* = V_i^* \left[\sum_{j=1}^N y_{ij} V_j \right]$

$P_i - jQ_i = V_i^* \left[\sum_{j=1}^N y_{ij} V_j \right]$

$Q_i = - \text{Im} \left\{ V_i^* \left[\sum_{j=1}^N y_{ij} V_j \right] \right\}$

Imaginary \mathcal{I}

② $V_i = \frac{1}{y_{ii}} \left[\frac{S_i^*}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^N y_{ij} V_j \right]$

③ Correction V.

$$V_i^{(k+1)} = |V_i^{\text{specified}}| * \frac{V_i^{(k+1)}}{|V_i^{(k+1)}|} \quad \left(\begin{array}{l} \text{V ثابت ال} \\ \text{S بطلع} \end{array} \right)$$

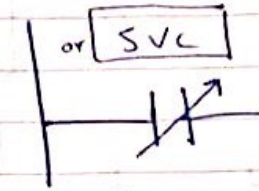
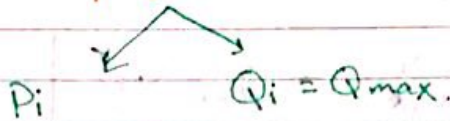
④ check Q:

$$Q > Q_{\max}$$

⇒ Voltage will not fixed

⇒ transform PV bus to

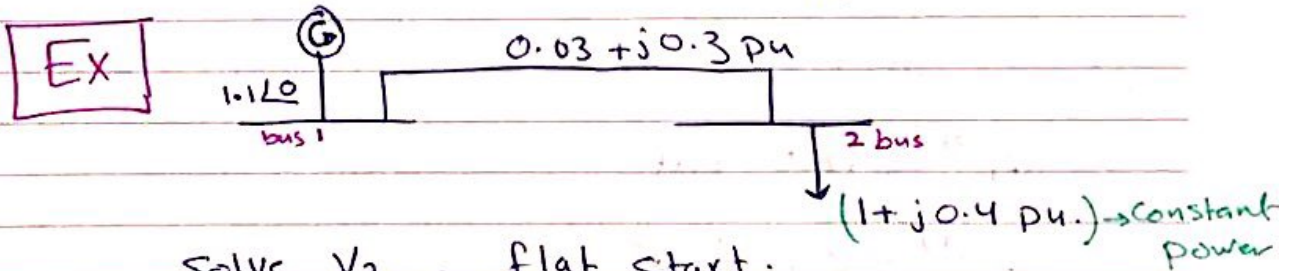
PQ bus



V fixed

$$Q_{\min} \leq Q \leq Q_{\max}$$

no voltage correction if $Q_{\max} < Q < Q_{\min}$ → PQ bus ← PV bus



Sol:

	Known	Unknown
bus 1 : slack	120	P_1, Q_1
bus 2 ⇒ PQ bus	$P_2 = 0 - 1 = -1$ $Q_2 = 0 - 0.4 = -0.4$	
	$S_2 = -1 - 0.4j$	

① Find Y matrix :-

$$y_{T.L} = \frac{1}{0.03 + j0.3} = 0.33 - j 3.3003 \text{ pu}$$

$$Y = \begin{bmatrix} 0.33 - j 3.3003 & -0.33 + j 3.3003 \\ -0.33 + j 3.3003 & 0.33 - j 3.3003 \end{bmatrix}$$

$$V_2 = \frac{1}{y_{22}} \left[\frac{S_2^*}{V_2^*} - \sum_{\substack{j=1 \\ j \neq 2}}^2 y_{ij} V_j \right]$$

$$V_2^{(k+1)} = \frac{1}{0.33 - j 3.3003} \left[\frac{-1 + j0.4}{(V_2^k)^*} - y_{21} \underbrace{V_1^k}_{1 \angle 0 \text{ slack}} \right]$$

((* Flat start \Rightarrow initial condition: $V_i^{(0)} = 1 \angle 0$))

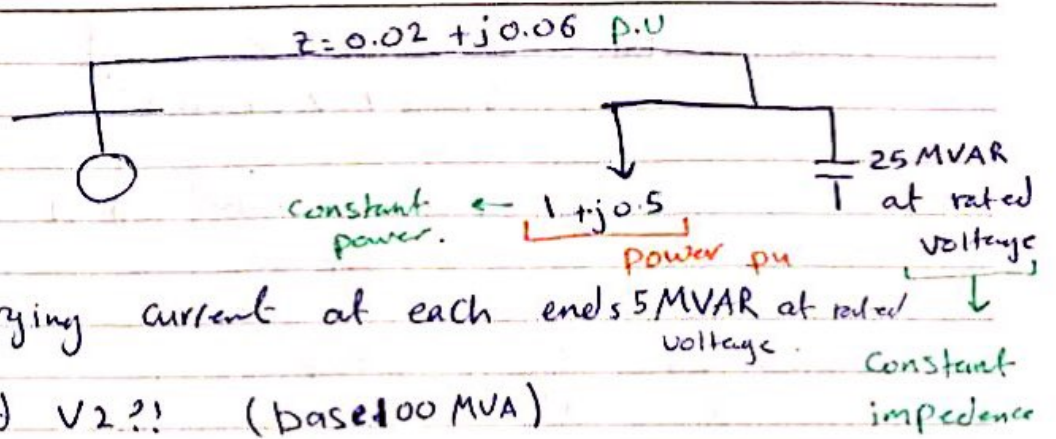
$$V_2^{(0)} = 1 \angle 0 \text{ flat.}$$

$$V_2^{(1)} = 0.99 \angle -16^\circ$$

$$V_2^{(2)} = 0.91 \angle -15^\circ$$

$$S_{10} = V_1 \hat{I}^* \\ \hat{I} = (V_1 - V_2) \cdot y_{T.L}$$

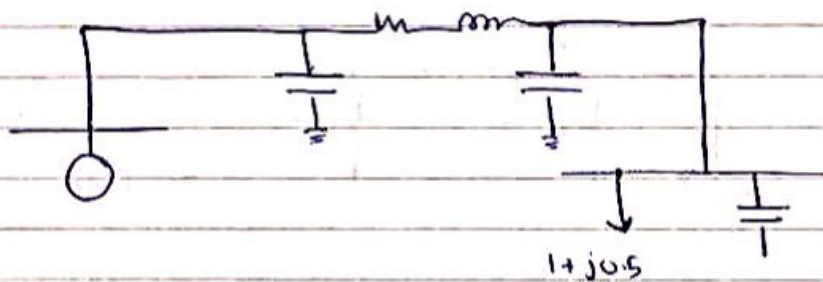
Ex



- ① Find V_2 ?! (base 100 MVA)
- ② I_1
- ③ $Q_{\text{capacitor}}$.

25 MV * قیومی بقیه کی باقیه ال Volt
 و load 1+j0.5 ← قیوم ال
 باقیه ال 25 م مع ال

Sol:



$$Y_{\text{capacitor pu}} = \frac{Y_{\text{actual}}}{Y_{\text{base}}}$$

$$= \frac{Y_{\text{actual}}}{S_{\text{base}} / (V_{LL \text{ base}})^2}$$

$$Q_{\text{capacitor}} = 3 * (V_{LL \text{ rated}})^2 * Y_{\text{actual}}$$

$$= (V_{LL \text{ rated}})^2 * Y_{\text{actual}}$$

$$= (V_{LL \text{ base}})^2 * Y_{\text{actual}}$$

$$Y_{\text{actual}} = \frac{Q_{\text{capacitor}}}{(V_{LL \text{ base}})^2}$$

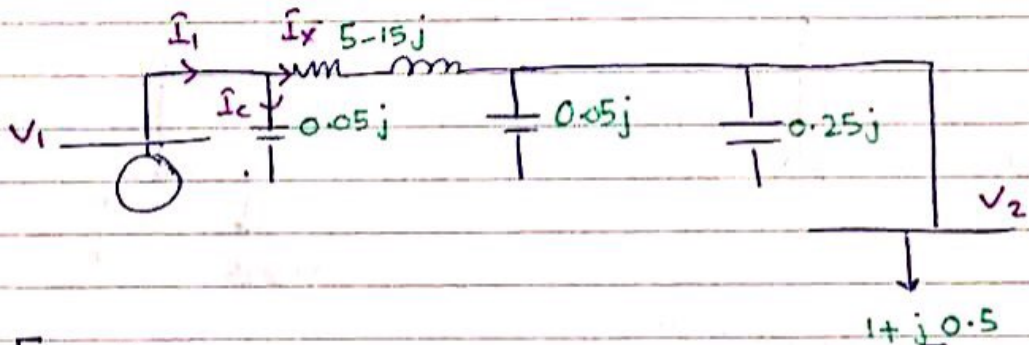
$$Y_{\text{capacitor pu}} = \frac{Q_{\text{capacitor}} / (V_{LL} \text{ base})^2}{S_{\text{base}} / (V_{LL} \text{ base})^2}$$

$$= \frac{Q_{\text{capacitor}} (\text{rated volts})}{S_{\text{base}}}$$

$$Y_{\text{capacitor pu}} = \frac{25}{100} = 0.25 \text{ pu.}$$

$$Y_{E.L} = \frac{5}{100} = 0.05 \text{ pu.}$$

$$y_{T.L} = \frac{1}{0.02 + j0.06} = 5 - j15$$



$$Y = \begin{bmatrix} 5 - j15 + j0.05 & -5 + j15 \\ -5 + j15 & 5 - j15 + j0.05 + j0.25 \end{bmatrix}$$

$$Y = \begin{bmatrix} 5 - j14.95 & -5 + j15 \\ -5 + j15 & 5 - j14.7 \end{bmatrix}$$

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Bus:-

① → slack bus $\left\{ \begin{array}{l} \text{Known } 1 \angle 0 \\ \text{unknown } P_1, Q_1 \end{array} \right.$

② → load bus PQ $\left\{ \begin{array}{l} \text{Known } \Rightarrow P_2 = -1, Q_2 = -0.5 \\ \text{unknown } \Rightarrow |V_2| \angle V_2 \\ \text{(initial } 1 \angle 0) \end{array} \right.$

$$V_2^{(1)} = \frac{1}{y_{22}} \left(\frac{S_2^*}{V_2^*} - \sum_{\substack{i=1 \\ i \neq 2}}^2 y_{2i} V_i \right) \quad (i \equiv \text{bus \#})$$

$$V_2^{(1)} = \frac{1}{5 + j14.7} \left(\frac{-1 + j0.5}{1 \angle 0} - y_{21} V_1 \right) \rightarrow 1 \angle 0$$

\downarrow
 $-5 + j15$

$$= 0.9687 \angle -3.36^\circ$$

② $\hat{I}_{\text{injected}} = YV$

$$\hat{I}_1 = y_{11} V_1 + y_{12} V_2 = 1.029 \angle -8.979$$

OR $\hat{I}_x = (V_1 - V_2)(5 - j15)$

$$\hat{I}_c = V_1(0.05j)$$

$$\hat{I}_1 = \hat{I}_x + \hat{I}_c$$

$$S_1 = V_1 \hat{I}_1^*$$

③ $Q_{\text{capacitor}} = ?!$

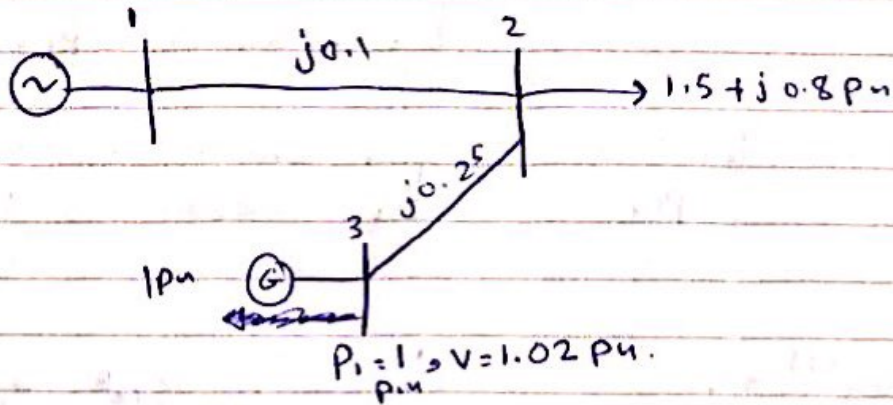
$$Q_{\text{capacitor}} = |V_2|^{pu^2} * y_{\text{capacitor}} \text{ pu.}$$

$$Q_{\text{capacitor actual}} = |V_2|^{pu^2} * y_{\text{capacitor}} \text{ pu} * \text{Spase.} \uparrow 100$$

98

Q capacitor = 23.2 MVAR.
actual

Ex:



find V_2, V_3 ?!

sol: Bus:

1 \rightarrow slack $\begin{cases} \text{Known } V_1 = 1 \angle 0 \\ \text{unknown } P_1, Q_1 \end{cases}$

2 \rightarrow PQ bus $\begin{cases} \text{Known } P_2 = -1.5, Q_2 = -0.8 \\ \text{unknown } V_2, \angle V_2 \end{cases}$

3 \rightarrow PV bus $\begin{cases} \text{Known } P_3 = 1, V_3 = 1.02 \\ \text{unknown } Q_3, \angle V_3 \end{cases}$

* $P_3^{\text{injected}} = 1 - 0 = 1$

$$Y = \begin{bmatrix} -j10 & j10 & 0 \\ j10 & -j14 & j4 \\ 0 & j4 & -j4 \end{bmatrix}$$

$$V_2 = \frac{1}{Y_{22}} \left(\frac{-1.5 + j0.8}{1 \angle 0} - (j10 * 1 \angle 0 + 4j * 1.02 \angle 0) \right)$$

$$V_2^{(1)} = 0.95744 \angle -6.4^\circ$$

99

$$Q_3 = -\text{Im} \left[V_3^* \sum_{i=1}^3 y_{3i} V_i \right]$$

$$Q_3 = -\text{Im} \left[V_3^* (y_{31} V_1 + y_{32} V_2 + y_{33} V_3) \right]$$

$\begin{matrix} \nearrow 1 \angle 0^\circ & \nearrow 0.957 \angle -6.4^\circ & \nearrow 1.02 \angle 0^\circ \end{matrix}$

$$Q_3 = -0.32038 \text{ pu.}$$

$$V_3 = \frac{1}{y_{33}} \left(\frac{1 - j0.32038}{1.02 \angle 0^\circ} - (0.957 \angle -6.4^\circ * j4) \right)$$

$$V_3^{(1)} = 1.0395 \angle 7.626^\circ \text{ pu.}$$

$V_3^{(1)} = 1.0395 > V_3 = 1.02 \Rightarrow$ *بقيمة الـ correct*

Correction:

$$V_3^{(1) \text{ correction}} = \frac{|V_3| * V_3^{(1)}}{|V_3^{(1)}|}$$

"Voltage (le voltage) ~ L¹ e local flow" *بجول*

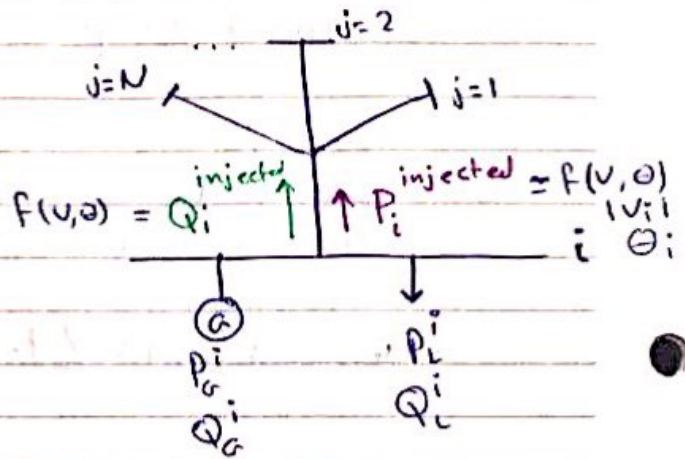
- Newton Raphson :-

- problem formulation \rightarrow (معادلات)

- solution \rightarrow (Find v_2, θ)

Power flow equations

• θ, V بجای P بدلائے اور θ, V



$$S_i = V_i (\bar{I}_i^{\text{injected}})^*$$

$$= V_i \left[\sum_{j=1}^N y_{ij} V_j \right]^*$$

$$V_i = |V_i| \angle \theta_i$$

$$V_j = |V_j| \angle \theta_j$$

$$y_{ij} = g_{ij} + j b_{ij}$$

$$S_i = |V_i| \angle \theta_i \left[\sum_{j=1}^N (g_{ij} + j b_{ij})^* |V_j| \angle -\theta_j \right]$$

$$S_i = \sum_{j=1}^N |V_i| |V_j| \left[(g_{ij} - j b_{ij}) (\cos \theta_{ij} + j \sin \theta_{ij}) \right]$$

$\angle \theta_i - \theta_j = \theta_{ij}$

$$S_i = P_i + j Q_i$$

$$P_i = \sum_{j=1}^N |V_i| |V_j| [g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}]$$

$$Q_i = \sum_{j=1}^N |V_i| |V_j| [g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}]$$

$$Y = G + jB$$

$$P_i^{\text{injected}} = P_G^i - P_L^i$$

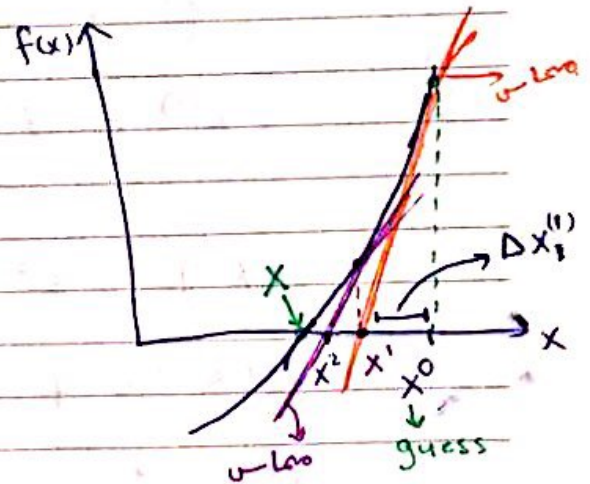
$$P_i^{\text{injected}} - P_G^i + P_L^i = 0$$

$f(v, \theta)$

Newton Raphson:

« بي اي اطلع اصلا، الاقران »

$$f(x) = 0 \rightarrow x^* (\text{roots})$$



* بلش نه x^0 بطلع قه ليه كثير فبانه لاه بطلع سو السيات
فبانه x^1 اقران x^0 نه x

$f(x^*) = 0$ Taylor series.

$$f(x^{k+1}) = f(x^k) + f'(x^k) (x^{k+1} - x^k) + \cancel{f''(x^k) (x^{k+1} - x^k)^2} + \cancel{f'''(x^k) (x^{k+1} - x^k)^3}$$

$$\frac{f''(x^k) (x^{k+1} - x^k)^2}{2!} + \frac{f'''(x^k) (x^{k+1} - x^k)^3}{3!}$$

$$0 = f(x^{k+1}) = f(x^k) + f'(x^k)(x^{k+1} - x^k)$$

input $x^k \rightarrow x^{k+1}$ next iteration

$$(x^{k+1} - x^k) = - [f'(x)]^{-1} f(x^k)$$

$$x^{k+1} = x^k - [f'(x^k)]^{-1} f(x^k)$$

Ex $f(x) = x^2 - 5x + 4$, $x^{(0)} = 6$

$$f(x^*) = 0 \text{ , } x^* = ?!$$

Sol: $f(x) = x^2 - 5x + 4$
 $f'(x) = 2x - 5$

$$x^{(1)} = x^{(0)} - \frac{f(x^{(0)})}{f'(x^{(0)})}$$

$$f(x^{(0)}) = 6^2 - 5 \times 6 + 4 = 10$$

$$f'(x^{(0)}) = 7$$

$$x^{(1)} = 6 - \frac{10}{7} = 4.57$$

$$x^{(2)} = 4.57 - \frac{f(4.57)}{f'(4.57)}$$

$$= 4.08$$

Sol using Gaussian:

$$x = f(x)$$

$$x = \frac{x^2 + 4}{5}$$

* Newton Raphson Δ
two function of 2 variable.

$$f(x, y) = 0$$

$$g(x, y) = 0$$

$$f(x^{k+1}, y^{k+1}) = f(x^k, y^k) + \frac{\partial f}{\partial x} f(x^k, y^k) \Delta x^k + \frac{\partial f}{\partial y} f(x^k, y^k) \Delta y^k = 0$$

$$g(x^{k+1}, y^{k+1}) = g(x^k, y^k) + \frac{\partial g}{\partial x} g(x^k, y^k) \Delta x^k + \frac{\partial g}{\partial y} g(x^k, y^k) \Delta y^k = 0$$

$\frac{\partial f(x^k, y^k)}{\partial x}$	$\frac{\partial f(x^k, y^k)}{\partial y}$	Δx^k	+	$f(x^k, y^k)$
$\frac{\partial g(x^k, y^k)}{\partial x}$	$\frac{\partial g(x^k, y^k)}{\partial y}$	Δy^k	+	$g(x^k, y^k)$

↓
Jacobian Δ

$\Delta x^k = x^{k+1} - x^k$
$\Delta y^k = y^{k+1} - y^k$

Newton Raphson:

$$f(x^*) = 0 \Rightarrow x^*$$

$$x^{k+1} = x^k - (f'(x^k))^{-1} f(x^k)$$

Generalization to multiple variables.

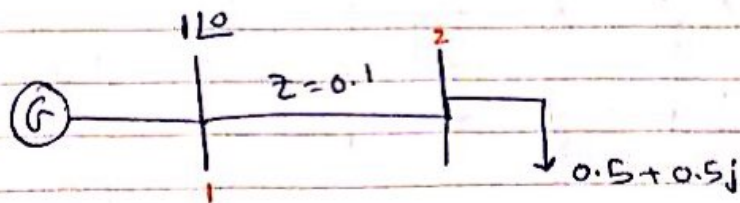
$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$f'(x) = \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_1(x)}{\partial x_2} & \dots & \frac{\partial f_1(x)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n(x)}{\partial x_1} & \frac{\partial f_n(x)}{\partial x_2} & \dots & \frac{\partial f_n(x)}{\partial x_n} \end{bmatrix}$$

↓
Jacobian.

$$x^{(k+1)} = x^k - J^{-1}(x^k) f(x^k)$$

Ex:



$V_2, \theta_2 = ?!$

Bus	Known	unknown
1	$V_1 = 1\angle 0$	P_1, Q_1
2	$P_2 = -0.5$ $Q_2 = -0.5$	V_2, θ_2

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$$P_i = \sum_j |V_i| |V_j| [G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}]$$

$$Q_i = \sum_j |V_i| |V_j| [G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}]$$

$$Y = \begin{bmatrix} -10j & j10 \\ j10 & -j10 \end{bmatrix} = G + jB$$

$$= j \begin{bmatrix} -10 & 10 \\ 10 & -10 \end{bmatrix}$$

$$P_1 = |V_1| |V_1| [-10 \sin \theta_{11}] + |V_1| |V_2| [10 \sin \theta_{12}]$$

$$P_1 = -10 |V_1| |V_2| \sin \theta_2$$

$$P_1 = -10 |V_2| \sin \theta_2$$

$$Q_1 = 10 - 10 |V_2| \cos \theta_2$$

* $\theta_{11} = \theta_1 - \theta_1 = 0$
 $\sin(0) = 0$
 explicit equation

$$\begin{cases} P_2 = 10 |V_2| \sin \theta_2 \\ Q_2 = -10 |V_2| \cos \theta_2 + 10 |V_2|^2 \end{cases}$$

implicit equation.

$\theta_{12} = \theta_{12} - \theta_2$
 (slack bus)
 $V_1 = 1, \theta_1 = 0$
 $\sin(\theta_{12}) = \sin(-\theta_2) = -\sin \theta_2$

Newton Raphson.

- Solve implicit equation.

$$f_1(x) = 10 |V_2| \sin \theta_2 + 0.5 = 0$$

$$f_1(x) = P_2 + 0.5$$

$$f_2(x) = -10|v_2| \cos \theta_2 + 10|v_2|^2 + 0.5 = 0$$

$$f_2(x) = Q_2 + 0.5$$

$$x = \begin{bmatrix} \theta_2 \\ |v_2| \end{bmatrix}$$

Flat start

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \text{rad}$$

$$x^{k+1} = x^k - J^{-1}(x^k) f(x^k)$$

$$J = \begin{bmatrix} \frac{\partial f_1(x)}{\partial \theta_2} & \frac{\partial f_1(x)}{\partial |v_2|} \\ \frac{\partial f_2(x)}{\partial \theta_2} & \frac{\partial f_2(x)}{\partial |v_2|} \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial P_2(x)}{\partial \theta_2} & \frac{\partial P_2(x)}{\partial |v_2|} \\ \frac{\partial Q_2(x)}{\partial \theta_2} & \frac{\partial Q_2(x)}{\partial |v_2|} \end{bmatrix}$$

$$J = \begin{bmatrix} 10|v_2| \cos \theta_2 & 10 \sin \theta_2 \\ 10|v_2| \sin \theta_2 & 20|v_2| - 10 \cos \theta_2 \end{bmatrix}$$

first iteration

$$\textcircled{1} \quad J(\theta^{(0)} = 0, v^{(0)} = 1) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$\begin{bmatrix} \theta^{(0)} \\ V^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\textcircled{2} \quad J^{-1}(\theta^{(0)}, V^{(0)}) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$f(x^{(0)})$$

$$f_1(\theta=0, |V|=1) = 0.5$$

$$f_2(\theta=0, |V|=1) = 0.5$$

Power and reactive
power "mis
match"

$$x^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \cdot \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$\begin{bmatrix} \theta^{(1)} \\ V^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.05 \\ -0.05 \end{bmatrix} = \begin{bmatrix} -0.05 \\ 0.95 \end{bmatrix}$$

$$\begin{bmatrix} \theta^{(2)} \\ V^{(2)} \end{bmatrix} = \begin{bmatrix} -0.5288 \\ 0.94574 \end{bmatrix}$$

after 2 iterations

$$P_1 = -10 |V_2| \sin \theta_2 = 0.4999 \text{ pu}$$

$$Q = 10 - 10 |V_2| \cos \theta_2 = 0.5557 \text{ pu}$$

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Kashin Colors

1 slack + NPQ

$$\bar{J} = \begin{array}{c|cccc}
 \frac{\partial P_2(x)}{\partial \theta_2} & \frac{\partial P_2(x)}{\partial \theta_3} & \dots & \frac{\partial P_2(x)}{\partial \theta_n} & \frac{\partial P_2(x)}{\partial V_2} & \frac{\partial P_2(x)}{\partial \theta_3} & \dots & \frac{\partial P_2(x)}{\partial V_n} \\
 \vdots & \frac{\partial P_i}{\partial \theta} & & & & \frac{\partial P}{\partial V} & & \\
 \frac{\partial P_n(x)}{\partial \theta_2} & \dots & \frac{\partial P_n(x)}{\partial \theta_n} & \frac{\partial P_n}{\partial V_2} & \dots & \frac{\partial P_n}{\partial V_n} & & \\
 \hline
 \frac{\partial Q_2(x)}{\partial \theta_2} & & \frac{\partial Q_2}{\partial \theta_n} & \frac{\partial Q_2}{\partial V_2} & \dots & \frac{\partial Q_{n-2}}{\partial V_n} & & \\
 \vdots & \frac{\partial Q}{\partial \theta} & & & & \frac{\partial Q}{\partial V} & & \\
 \frac{\partial Q_n}{\partial \theta_2} & & \frac{\partial Q_n}{\partial \theta_n} & \frac{\partial Q_n}{\partial V_2} & & \frac{\partial Q_n}{\partial V_n} & &
 \end{array}$$

at PV bus $Q=0 \rightarrow$

Slack ← will get θ_1 & V_1 & P_1 *

$$X = \begin{bmatrix} \theta_2 \\ \theta_3 \\ \vdots \\ \theta_n \\ |V_2| \\ \vdots \\ V_n \end{bmatrix}$$

* PV bus

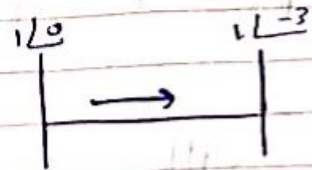
P known & Q unknown.

$Q=0$ if not

* DC load flow is "linear"

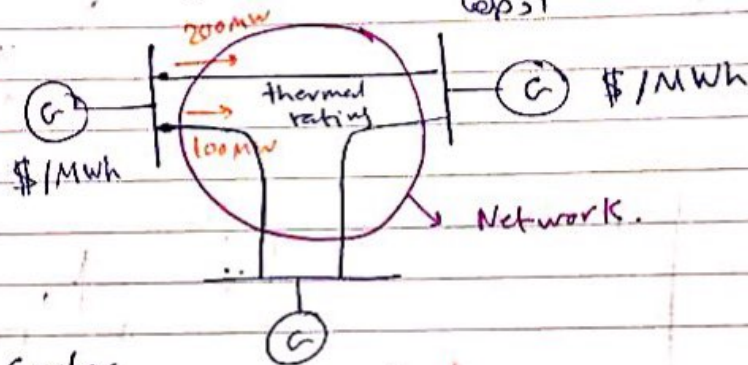
- Power.
- Angles unknown.
- Voltage fixed.

$\Delta P \propto (\Delta \delta)$
↳ angles.



* economic dispatch.

thermal rating نیو دی پاور لینے کے لیے
helps!



Control Center.

- ↳ ① Min-Cost
- ② Network.

* Fast Decoupled load flow :-

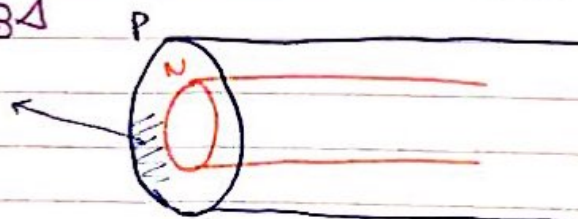
DC load no m-pi

$$\begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix}$$

transmission $\frac{X}{R} \gg 1$

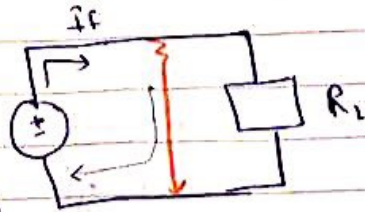
* Fault analysis is

die insulation breakdown.



① dangerous

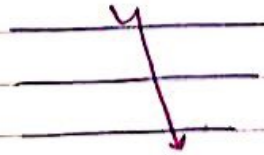
Personal Facilities stability



OHL

* Faults are in the line between the poles

الخط بين الأبراج



wind & ice.



open

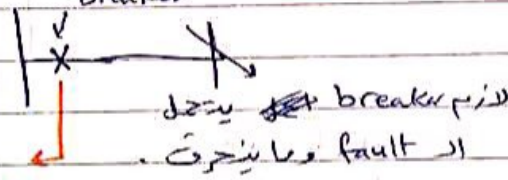
Fault Current Calculation:-

1) Design

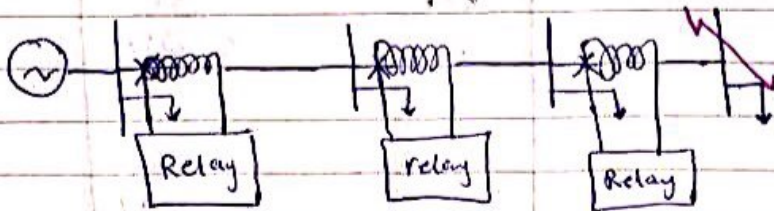
breaker

withstand fault current.

"worst-case" → max fault current



2) protection coordination studies



I setting
wait
open

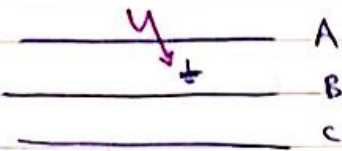
3) stability



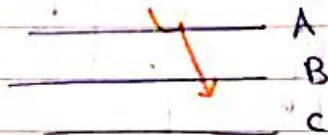
types of fault: - $\left\{ \begin{array}{l} \text{internal voltage method} \\ \text{thevenin.} \end{array} \right.$



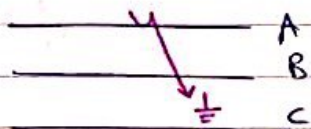
- three-phase short circuit
Symmetrical fault
balanced "per phase analysis"



line to current fault.



line to line fault.



line to line Ground
"LLG"

unbalanced \Rightarrow Symmetrical components

-ve seq

+ve seq

Zero seq

Symmetrical fault:

Three phase short circuit.



Objective.

- fault current
- contributions to fault current
- voltage
- effect of load current.

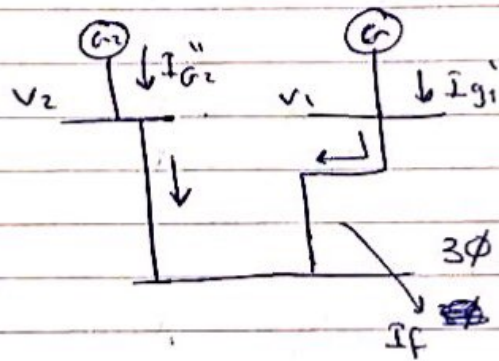
Methods:-

1) Internal voltage method

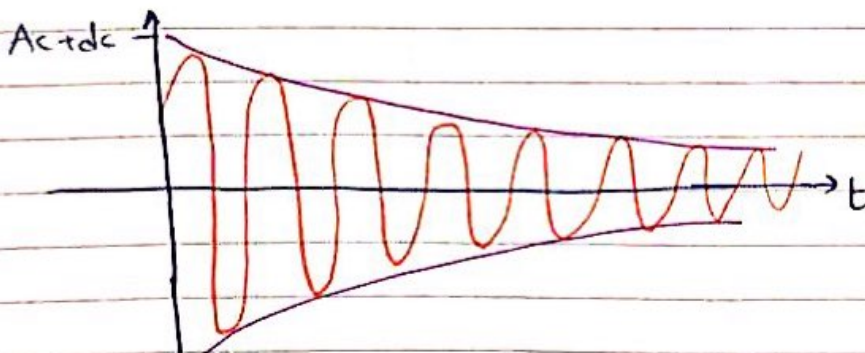
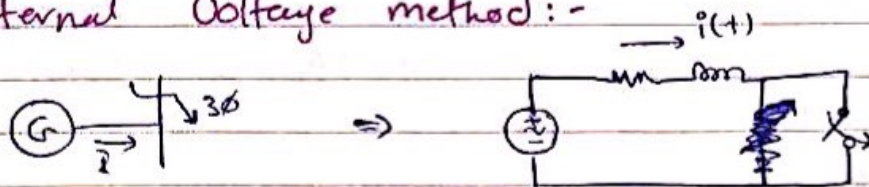
2) Thevenin.

3) Superposition

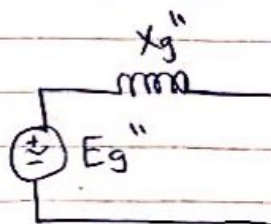
4) impedance matrix. ($Z_{bus} = Y^{-1}_{bus}$)
 ↳ complex system.



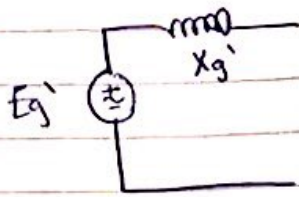
II Internal Voltage method:-



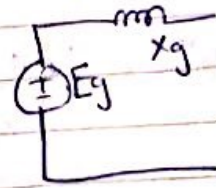
① Sub-transient



② transient



③ steady state



① Subtransient current :-

① E_g'' , E_m'' (internal voltage)

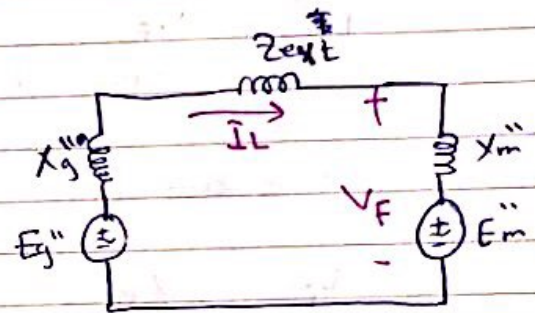
* Pre-fault analysis $\Rightarrow E_m'' \rightarrow E_g''$ $\frac{V_F}{jX_m''} \rightarrow \frac{V_F}{jX_g''}$



~~geometrical fault~~

$$E_g'' = V_F + \hat{I}_L (Z_{ext} + jX_g'')$$

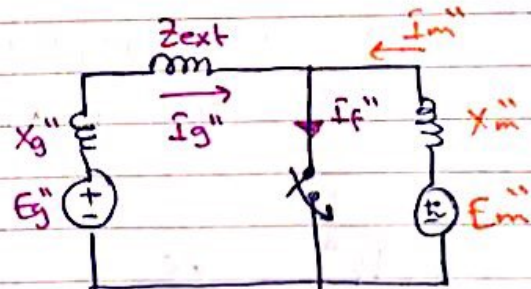
\downarrow pre-fault



$$E_m'' = V_F - \hat{I}_L (jX_m'')$$

Post fault :-

$$\hat{I}_g'' = \frac{E_g''}{jX_g'' + Z_{ext}}$$



$$= \frac{V_F + \hat{I}_L (jX_g'' + Z_{ext})}{jX_g'' + Z_{ext}}$$

$$\hat{I}_g'' = \frac{V_F}{jX_g'' + Z_{ext}} + \hat{I}_L \rightarrow \text{current (pre-fault)}$$

$$\hat{I}_m'' = \frac{E_m''}{jX_m''} = \frac{V_F - jX_m'' \hat{I}_L}{jX_m''} = \frac{V_F}{jX_m''} - \hat{I}_L$$

$$\hat{I}_F'' = \hat{I}_G'' + \hat{I}_M''$$

$$\hat{I}_F'' = \frac{V_F}{Z_{ext} + jX_G''} + \frac{V_F}{jX_M''}$$

$$\hat{I}_F'' = V_F \left(\frac{1}{Z_{ext} + jX_G''} + \frac{1}{jX_M''} \right)$$

$Z_{th} \Rightarrow$ seen from the fault

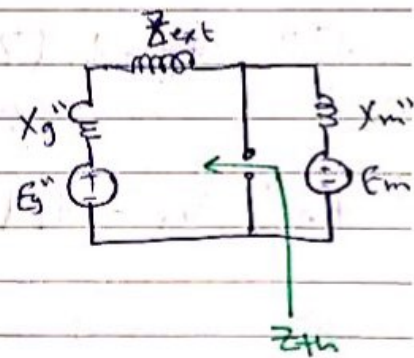
$$\hat{I}_F'' = \frac{V_F}{Z_{th}} \rightarrow \text{seen from the fault.}$$

2) Thevenin :-

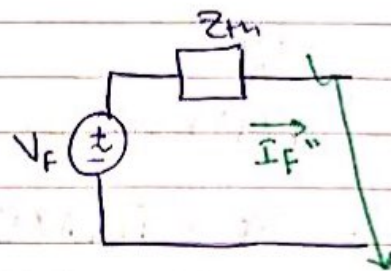
* V_F , $Z_{th} = ?$

$$Z_{th} = (Z_{ext} + jX_G'') \parallel jX_M''$$

$$Z_{th} = \frac{(Z_{ext} + jX_G'') * jX_M''}{Z_{ext} + jX_G'' + jX_M''}$$



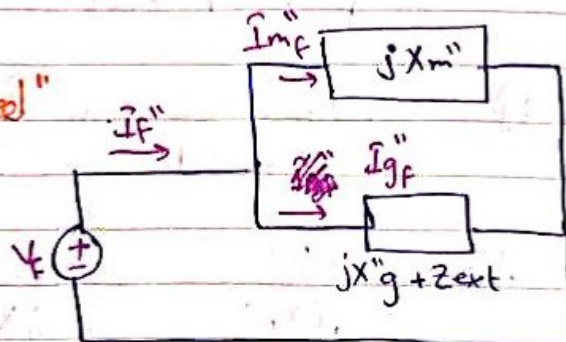
$$\hat{I}_F'' = \frac{V_F}{Z_{th}}$$



$$\hat{I}_{M_F}'' = \frac{V_F}{jX_M''}$$

$$\hat{I}_{G_F}'' = \frac{V_F}{jX_G'' + Z_{ext}}$$

"unloaded"

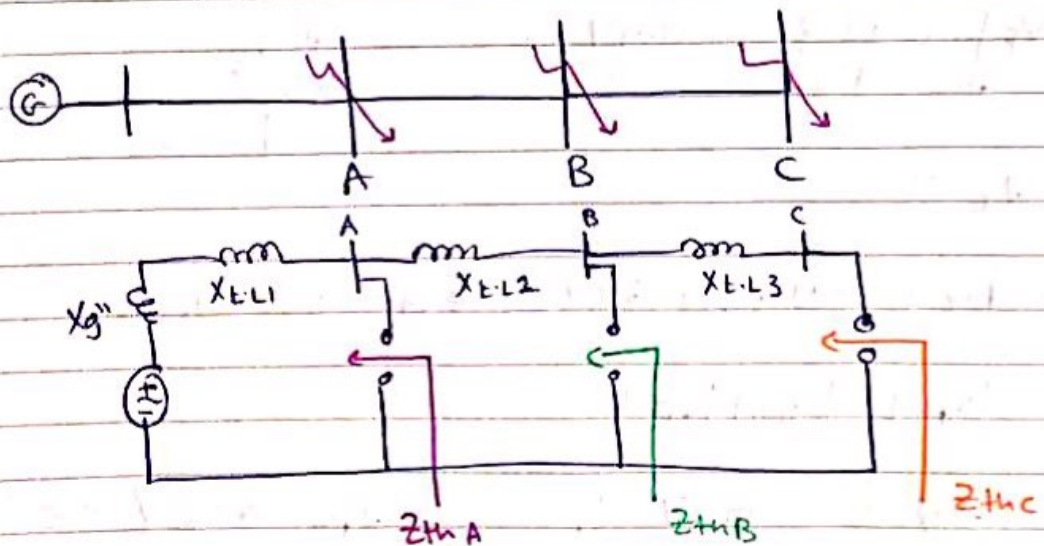


Correction:

$$I_m'' = \frac{V_F}{jX_m''} - I_L$$

$$I_g'' = \frac{V_F}{jX_g'' + Z_{ext}} + I_L$$

Internal	Thevenin
Pre fault (I_L, V_F)	V_F
E_g'', E_m''	Z_{th}
Contributions I_g'', I_m''	$I_F'' = \frac{V_F}{Z_{th}}$
Fault current: $I_F'' = I_g'' + I_m''$	Contribution "current division"
	Correction ($+ I_L$) $+ I_L$



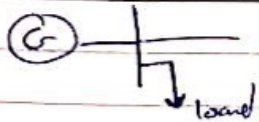
$$Z_{th}(A) = X_g'' + X_{E.L1}$$

$$Z_{th}(B) = X_g'' + X_{E.L1} + X_{E.L2}$$

↑ $Z_{th} \rightarrow \downarrow I_{fault}$.
 A (Source) \rightarrow A
 Fault: A (Source) \rightarrow A

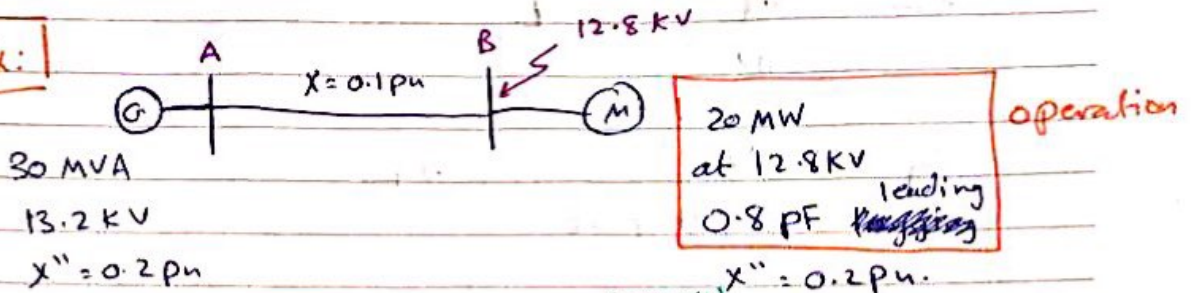
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$$I_F''(B) < I_F''(A)$$



impedance (ج) ← load $\sin \theta$

Ex:



all pu at Generator rating (30 MVA, 13.2 kV) calculate 3 ϕ subtransient fault current at Bus B contributions from motor and generator

- 1) internal voltage ?!
- 2) thevenin method. ?

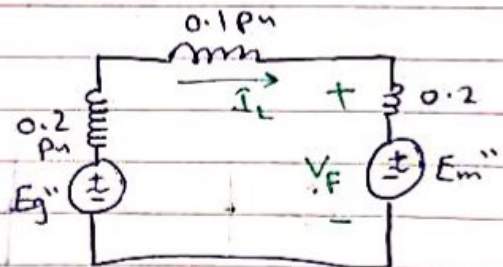
Sol: $V_F = 12.8 \text{ kV}$

1) internal voltage method.

$I_L = ?!$ V_F (before fault).
 E_g'' , E_m'' ?!

X'' , X' , X $\sin \theta$ $\cos \theta$
Subtransient X'' , L C
لا يوجد $\sin \theta$, $\cos \theta$

$$V_F |_{pu} = \frac{12.8}{13.2} = 0.97$$



I_L ?!

motor (20 MW, 12.8 kV, 0.8 PF leading).

$$|S_{pu}| = |V_{pu}| |I_{pu}|$$

$$\frac{20}{30} / 0.8 = 0.97 \text{ pu} \Rightarrow I_{pu} = 0.86 \angle +\cos^{-1}(0.8)$$

$$I_{pu} = 0.86 \angle 36.87 \text{ pu}$$

$$E_g'' = \hat{I}_L(j0.2 + j0.1) + 0.97 \angle 0$$

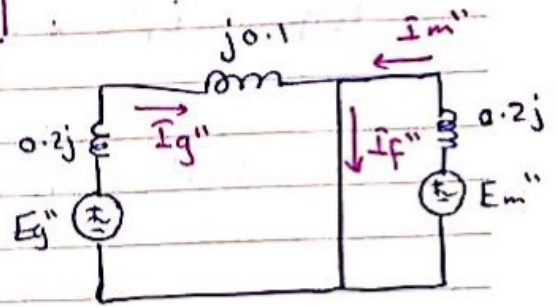
$$E_m'' = 0.97 \angle 0 - \hat{I}_L(j0.2)$$

$$E_g'' = 0.84 + j0.207 \text{ pu}$$

$$E_m'' = 1.074 - j0.138 \text{ pu}$$

$$\hat{I}_g'' = \frac{E_g''}{j0.2 + j0.1}$$

$$\hat{I}_m'' = \frac{E_m''}{j0.2}$$



$$\hat{I}_f'' = \hat{I}_g'' + \hat{I}_m'' = -j8.08 \text{ pu}$$

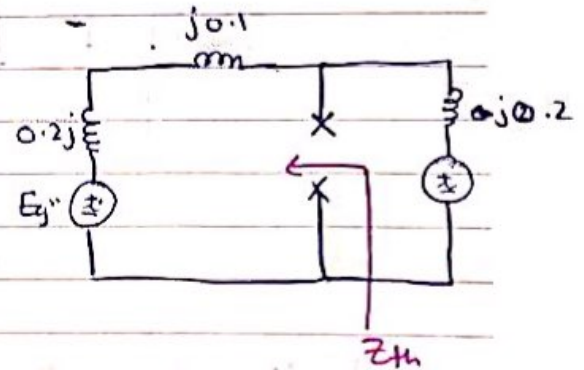
$$\hat{I}_f'' (\text{actual}) = -j8.08 \times \frac{30\text{M}}{\sqrt{3} \times 13.2\text{k}}$$

$$\hat{I}_f'' = -j10.6 \text{ kA}$$

② Thevenin s.d

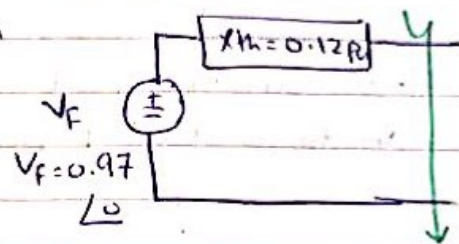
$$X_{th} = (j0.1 + 0.2j) \parallel (j0.2)$$

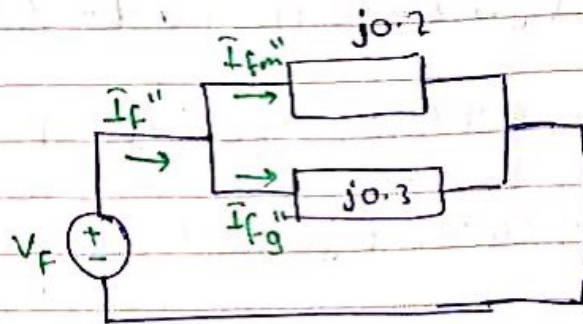
$$= 0.12 \text{ pu}$$



$$\hat{I}_f'' = \frac{V_f}{X_{th}} = \frac{0.97}{j0.12} = -j8.08 \text{ pu}$$

* نلاحظ انو طبعوننا الجواب ه لانو
* fault ال fault



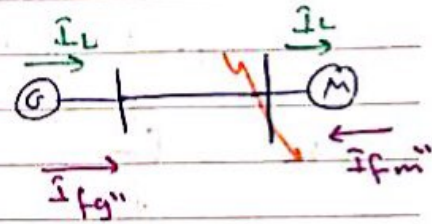


$$I_{fm}'' = I_f'' \times \frac{j0.3}{j0.5} = -j 3.23 \text{ pu}$$

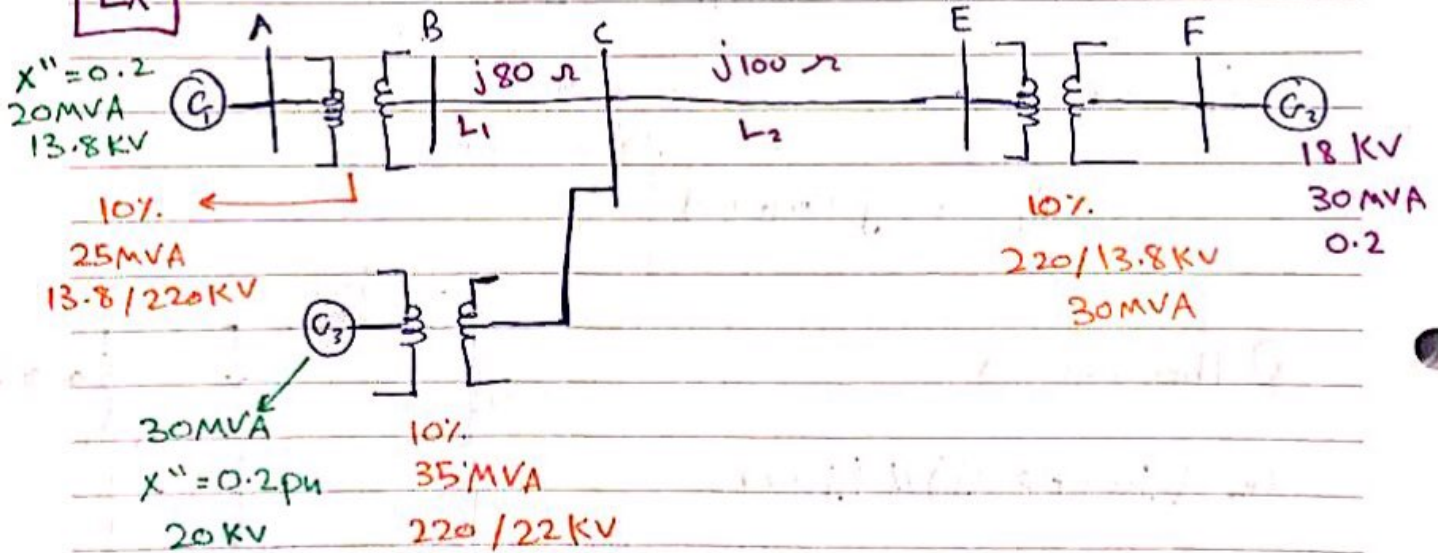
$$I_{fg}'' = I_f'' \times \frac{0.2j}{j0.5} = -j 4.85 \text{ pu}$$

Correction. $I_g = I_{fg}'' + I_L$

$$I_m'' = I_{fm}'' - I_L$$



Ex



all the perunit on a common base (50MVA, 13.8KV at generated (G1) side) fault.

Find subtransient current at bus C, pre fault voltage is 1 pu. Neglect load current. ?!

E_{g1}''		V_F	internal (ie up to) method.
E_{g2}''		$I_L = 0$	
E_{g3}''			

Sol: using thevenin

$$Z_{t.L}(pu) = \frac{80}{(220)^2/50}$$

$$= 0.083j$$

$$Z_{t.L} = \frac{100}{(220)^2/50} = 0.103j$$

$$Z_{th} = (j0.103 + j0.1 + j0.2) \parallel (j0.1 + j0.2) \parallel (j0.083 + j0.1 + 0.2) = j0.187$$

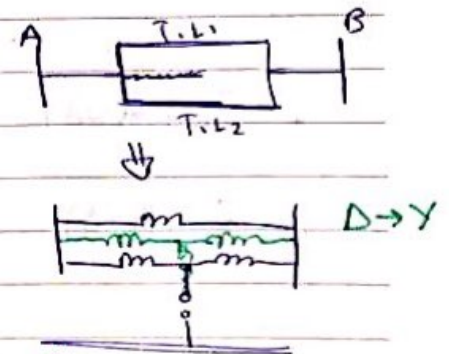
$$I_f'' = \frac{1}{Z_{th}}$$

$$= -j 5.348 \text{ pu}$$

$$I_{f \text{ actual}} = 5.348 \times \frac{50M}{\sqrt{3} \times 220k}$$

$$I_{f \text{ actual}} = 702 \text{ A}$$

Genar 11 se joi k fault 11 jai *



* Fault level & Short circuit level :-

- express fault in terms of MVA.

* - easier to compare fault at different voltage level.

16 KA	11 KV
16 KA	33 KV

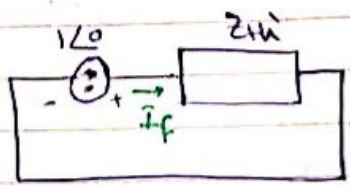
120

Kashin Colors

$$S_c^{MVA} = \sqrt{3} * V_{LL\ base} * \hat{I}_F$$

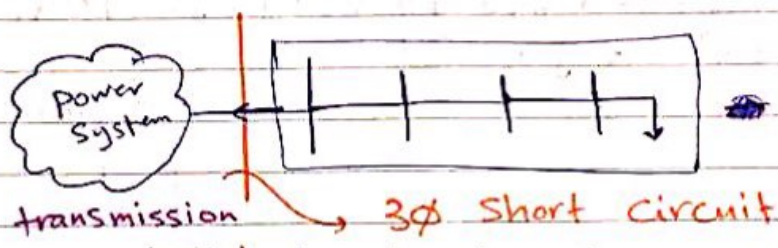
$$S_c(pu) = \frac{S_c(MVA)}{S_{base}} = \frac{\sqrt{3} * V_{LL\ base} * \hat{I}_F}{\sqrt{3} * V_{LL\ base} * \hat{I}_{base}} = \hat{I}_F(pu)$$

$$\hat{I}_F(pu) = \frac{1}{Z_{th}}$$



$$Z_{th} = \frac{1}{\hat{I}_F(pu)}$$

$V_F = 1 pu$



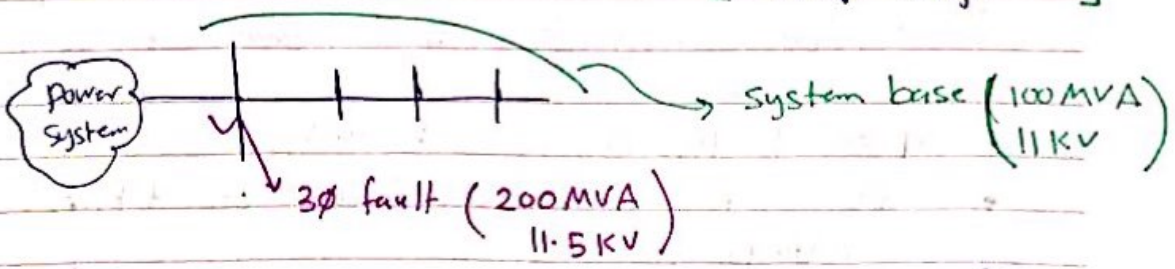
transmission, 3φ Short circuit.



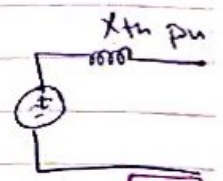
3φ short circuit
 Z_{th} value given, V_{th} value given, I_{sc} value
 $(0 = Z_{th})$ given
 value given

* $S_c(MVA)$ - Large \Rightarrow strong \Rightarrow [current $\uparrow \rightarrow Z_{th}$ small]
 \rightarrow voltage drop small

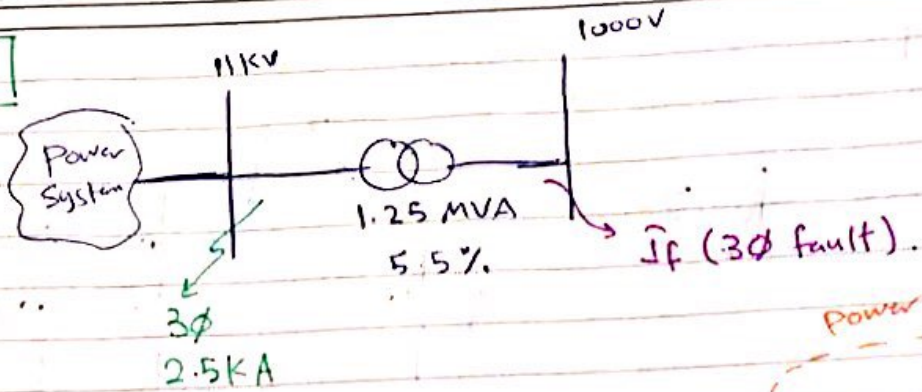
* $S_c(MVA)$: small \Rightarrow weak \rightarrow "fault current \uparrow "
 [Z_{th} large, voltage drop large]



$$X_{th\ pu} = j1 * \left(\frac{S_{new}}{S_{old}} \right) * \left(\frac{V_{old}}{V_{new}} \right)^2$$



Ex



Sol.

$$Z_{th} = \frac{11/\sqrt{3}}{2.5}$$

$$I_f (pu) = \frac{2.5 \text{ kA}}{1.25 \text{ MVA} / \sqrt{3} * 11 \text{ kV}}$$

$$I_f (pu) = \frac{1}{Z_{th} (pu)}$$

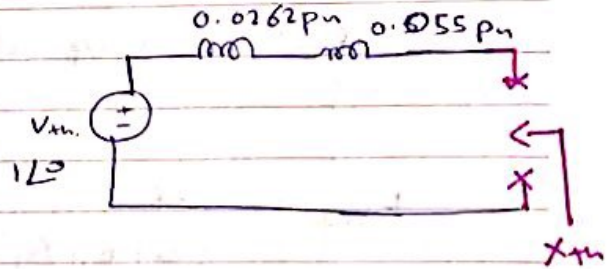
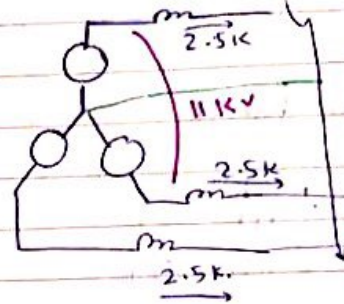
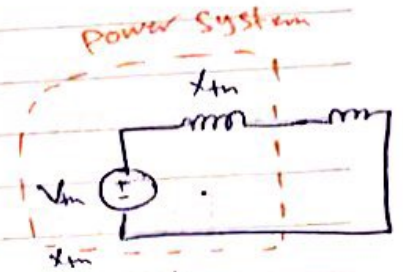
$$Z_{th} (pu) = 0.0262 \text{ pu}$$

thevenin

$$X_{th} = j0.055 + j0.0262$$

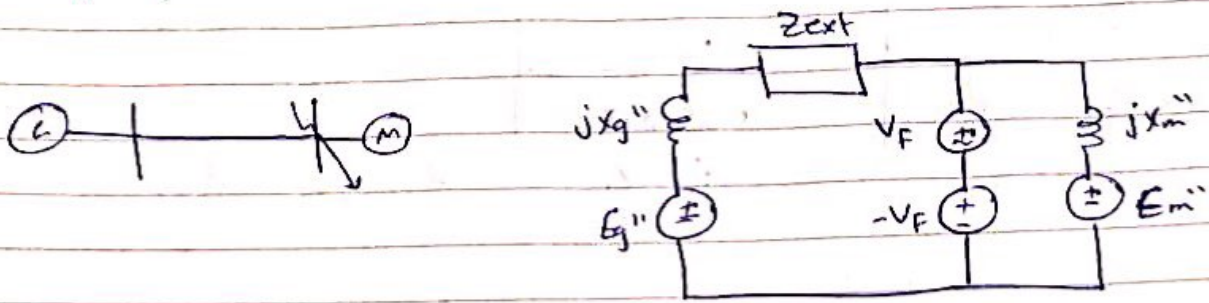
$$I_f |_{1000 \text{ V}} = 12.315 \text{ pu}$$

$$I_f (\text{actual}) = 12.315 * \frac{1.25 \text{ M}}{\sqrt{3} * 1000} = 8.88 \text{ kA}$$



$$\text{max SC Current} = \frac{1}{X} * \text{full load current}$$

③ Superposition:-



Solution (1):

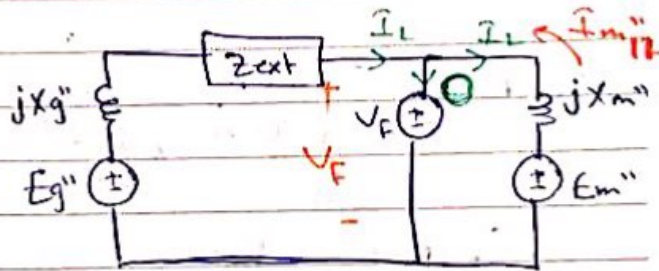
$$(Eg'', Em'', VF), -VF \Rightarrow \text{OFF}$$

ON

Pre fault "load flow"

$$\hat{I}_{g11} = \hat{I}_L$$

$$\hat{I}_{m11} = -\hat{I}_L$$



Solution (2)

$$(Eg'', Em'', VF) (-VF \text{ ON})$$

OFF

$$\hat{I}_{g12} = \frac{VF}{Z_{ext} + jXg''}$$

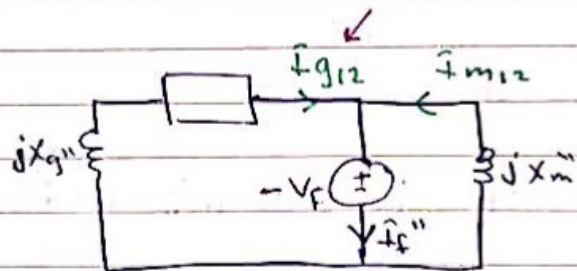
$$\hat{I}_g'' = \hat{I}_{g11} + \hat{I}_{g12}$$

$$\hat{I}_g'' = \hat{I}_L + \frac{VF}{Z_{ext} + jXg''}$$

$$\hat{I}_m'' = -\hat{I}_L + \frac{VF}{jXm''}$$

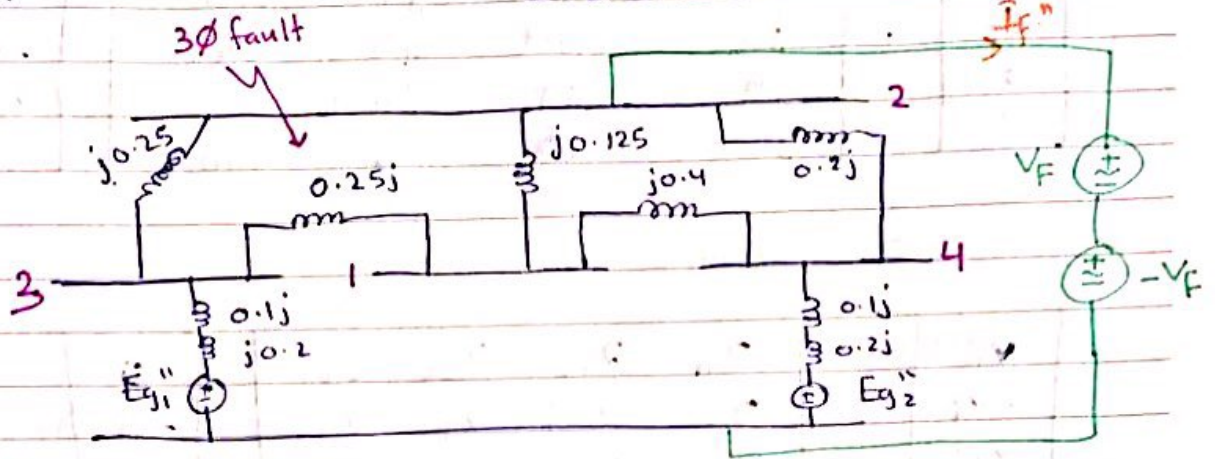
$$\hat{I}_f'' = \hat{I}_g'' + \hat{I}_m''$$

2nd solution.



"short -> source."
(-VF ↓ as by fault)

④ Z-matrix :- (Complex network)
 $Z\text{-bus} = Y^{-1}$



Neglect pre fault current. (V_F = pre fault voltage)

($-V_F$ off) \Rightarrow (V_F ON, E_{g1}'' , E_{g2}'' = ON)

Solution (1) :-

Pre-fault (Neglect load current)

I (branches) = 0

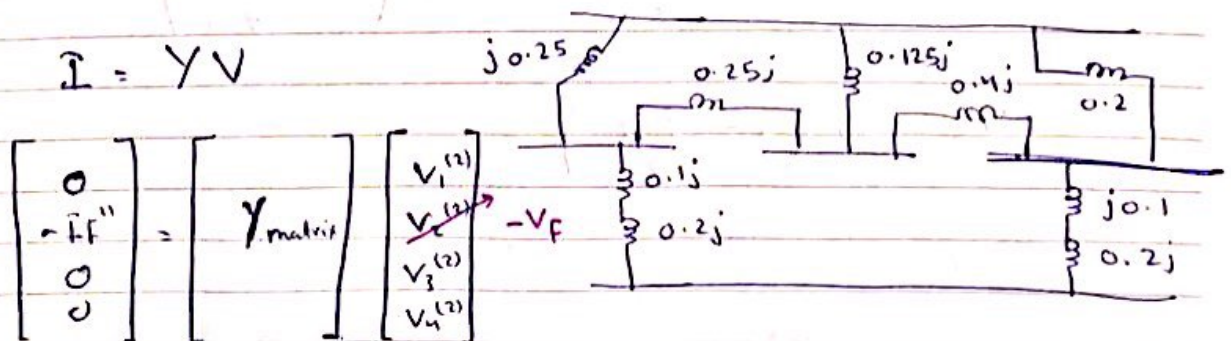
$$E_{g1}'' = E_{g2}'' = V_1^{(1)} = V_2^{(1)} = V_3^{(1)} = V_4^{(1)} \quad (\text{ماتریس، ولتاژ})$$

$$= V_F$$

Solution (2) :-

($-V_F$ ON / V_F , E_{g1}'' , E_{g2}'' \Rightarrow OFF)

$$I = YV$$



$$\begin{bmatrix} 0 \\ -\hat{I}_f'' \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j14.5 & j8 & j4 & j2.5 \\ j8 & -j17 & j4 & j5 \\ j4 & j4 & -j11.33 & 0 \\ j2.5 & j5 & 0 & j10.88 \end{bmatrix} \begin{bmatrix} V_1^{(2)} \\ -V_F \\ V_3^{(2)} \\ V_4^{(2)} \end{bmatrix}$$

$V_1^{(2)} \rightarrow$ Second solution.

$$\begin{bmatrix} V_1^{(2)} \\ -V_F \\ V_3^{(2)} \\ V_4^{(2)} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} 0 \\ -\hat{I}_f'' \\ 0 \\ 0 \end{bmatrix}$$

interest $Z_{\text{-bus}} = Y^{-1}$

is, up fault is at bus 2.

$$-V_F = Z_{21} (-\hat{I}_f'')$$

$$\hat{I}_f'' = \frac{V_F}{Z_{22}}$$

$$V_1^{(2)} = -Z_{12} \hat{I}_f'' = -\frac{Z_{12} V_F}{Z_{22}}$$

$$V_1 = V_1^{(1)} + V_1^{(2)}$$

$$V_1 = V_F - \frac{Z_{12} V_F}{Z_{22}} = V_F \left(1 - \frac{Z_{12}}{Z_{22}} \right)$$

$$V_3 = V_F \left(1 - \frac{Z_{32}}{Z_{22}} \right)$$

$$\hat{I}_f'' = \frac{V_F}{Z_{KK}}$$

$$V_i = V_F \left(1 - \frac{Z_{iK}}{Z_{KK}} \right)$$

\Rightarrow fault at bus K

$$Z = \begin{bmatrix} j0.2436 & j0.1938 & j0.1544 & j0.1456 \\ & j0.2295 & & \\ & & \cancel{j0.1494} & \\ & & j0.1494 & \\ & & & j0.1506 \end{bmatrix}$$

$$V_f = 1 \angle 0 \text{ at bus 2.}$$

$$I_f'' = \frac{1 \angle 0}{j0.2295} \rightarrow Z_{22} = -j4.3573 \text{ pu.}$$

1 pu fault voltage
for fault voltage

$$V_1 = V_f \left(1 - \frac{Z_{12}}{Z_{22}} \right) = 0.1556$$

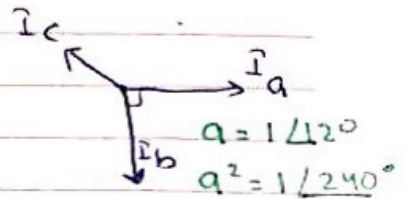
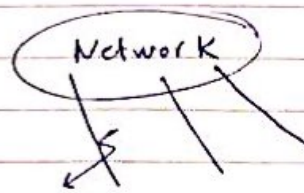
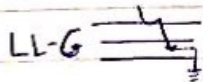
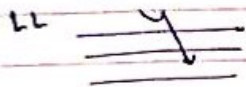
$$V_2 = 0 \quad (V_f - V_f = 0)$$

$$V_3 = V_f \left(1 - \frac{Z_{13}}{Z_{22}} \right) = 0.349j$$

$$V_4 = 0.3438 \text{ pu.}$$

$$I_{31} = \frac{V_3 - V_1}{j0.25} = -j0.7736 \text{ pu.}$$

* Symmetrical components and Asymmetrical fault Δ "unbalanced system"



unbalanced.

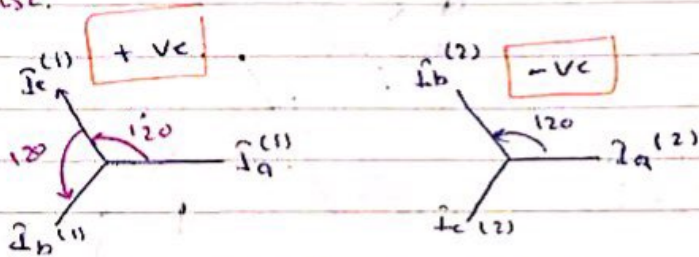
(operator) $a^3 = 1 \angle 0$

Symmetrical components :-

$$\begin{bmatrix} \hat{I}_a \\ \hat{I}_b \\ \hat{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \hat{I}_a^{(0)} \\ \hat{I}_a^{(1)} \\ \hat{I}_a^{(2)} \end{bmatrix}$$

Phase.

Symmetrical Component



$$a = 1 \angle 120^\circ$$

$$a^2 = 1 \angle 240^\circ$$

$$\hat{I}_b = a^2 \hat{I}_a^{(1)} + a \hat{I}_a^{(2)} + \hat{I}_a^{(0)}$$

$$\hat{I}_c = a \hat{I}_a^{(1)} + a^2 \hat{I}_a^{(2)} + \hat{I}_a^{(0)}$$

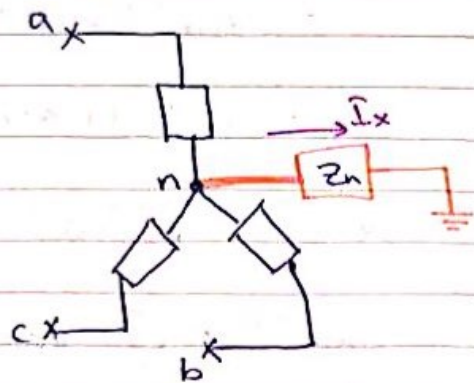
$$\begin{bmatrix} \hat{I}_a^{(0)} \\ \hat{I}_a^{(1)} \\ \hat{I}_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \hat{I}_a \\ \hat{I}_b \\ \hat{I}_c \end{bmatrix}$$

A^{-1}

① Balanced

$$\hat{I}_x = 0$$

$$V_n = V_E$$



② un balanced

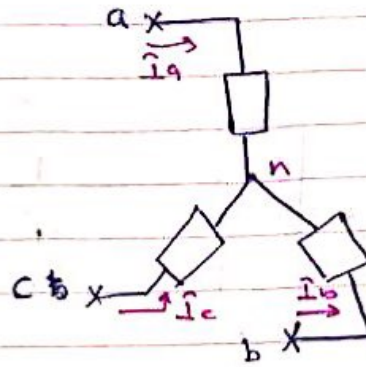
$$\hat{I}_x = \hat{I}_a + \hat{I}_b + \hat{I}_c = 3\hat{I}_a^{(0)}$$

$$* \hat{I}_a^{(0)} = \frac{1}{3} (\hat{I}_a + \hat{I}_b + \hat{I}_c) \Rightarrow 3\hat{I}_a^{(0)} = \hat{I}_a + \hat{I}_b + \hat{I}_c$$

$$V_n = 3\hat{I}_a^{(0)} \times Z_n$$

$$\hat{I}_a^{(0)} = 3(\hat{I}_a + \hat{I}_b + \hat{I}_c) = 0$$

• لا يوجد تيار في Ground في حالة التوازن

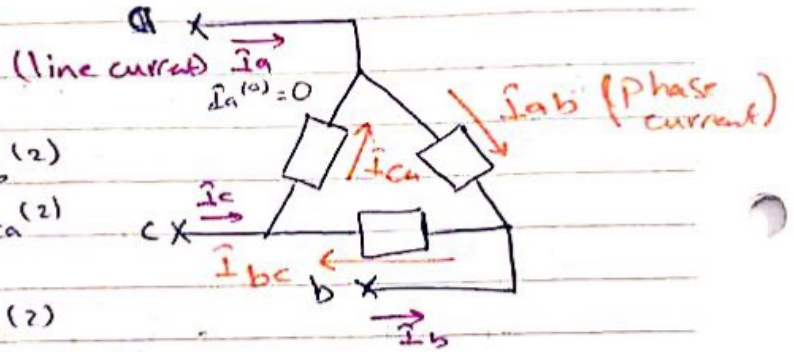


$$\hat{I}_a = \hat{I}_{ab} - \hat{I}_{ca}$$

$$= (\hat{I}_{ab}^{(0)} + \hat{I}_{ab}^{(1)} + \hat{I}_{ab}^{(2)}) - (\hat{I}_{ca}^{(0)} + \hat{I}_{ca}^{(1)} + \hat{I}_{ca}^{(2)})$$

$$= \hat{I}_{ab}^{(0)} + \hat{I}_{ab}^{(1)} + \hat{I}_{ab}^{(2)} - \hat{I}_{ca}^{(0)} - \hat{I}_{ca}^{(1)} - \hat{I}_{ca}^{(2)}$$

$$= (\hat{I}_{ab}^{(0)} + a \hat{I}_{ab}^{(1)} + a^2 \hat{I}_{ab}^{(2)})$$



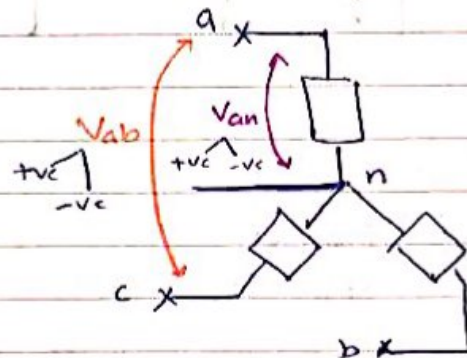
$$\hat{I}_a = \hat{I}_{ab}^{(1)} (1-a) + \hat{I}_{ab}^{(2)} (1-a^2)$$

$$\hat{I}_a = \hat{I}_{ab}^{(1)} (\sqrt{3}) \angle -30^\circ + \sqrt{3} \hat{I}_{ab}^{(2)} \angle +30^\circ$$

$$V_{ab}^{(1)} = V_{an}^{(1)} - V_{bn}^{(1)}$$

$$= V_{an}^{(1)} - a^2 V_{an}^{(1)}$$

$$= V_{an}^{(1)} (1-a^2)$$



$$V_{ab}^{(1)} = \sqrt{3} V_{an}^{(1)} \angle +30^\circ$$

$$V_{ab}^{(2)} = \sqrt{3} V_{an}^{(2)} \angle -30^\circ \quad \left(\frac{1-a}{1-1/\angle 120^\circ} = \sqrt{3} \right)$$

$$\Delta \text{ in delta} \Rightarrow \hat{I}_a^{(1)} = \sqrt{3} \hat{I}_{ab}^{(1)} \angle -30^\circ$$

$$\hat{I}_a^{(2)} = \sqrt{3} \hat{I}_{ab}^{(2)} \angle +30^\circ$$

3 ϕ Complex Power :-

$$S_{3\phi} = 3 V_a I_a^* \quad \text{"balanced"}$$

$$S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^* \quad \text{"unbalanced"}$$

$$S_{3\phi} = 3 V_a^{(0)} I_a^{*(0)} + 3 V_a^{(1)} I_a^{*(1)} + 3 V_a^{(2)} I_a^{*(2)}$$

↳ Symmetrical Component.

$$S_{3\phi} = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix}$$

Symmetrical

$$3 V_a^{(0)} I_a^{*(0)} = 0 \quad \text{when } \begin{cases} \rightarrow \Delta \text{ connect load} \\ \rightarrow Y \text{ load ungrounded} \end{cases}$$

Ex:

Find I_c ?!

$$\frac{1}{3} (10 \angle 0^\circ + 10 \angle 180^\circ + I_c) = 0$$

line

$$I_a^{(0)} = 0$$

$$I_a^{(0)}, I_a^{(1)}, I_a^{(2)} = ?!$$

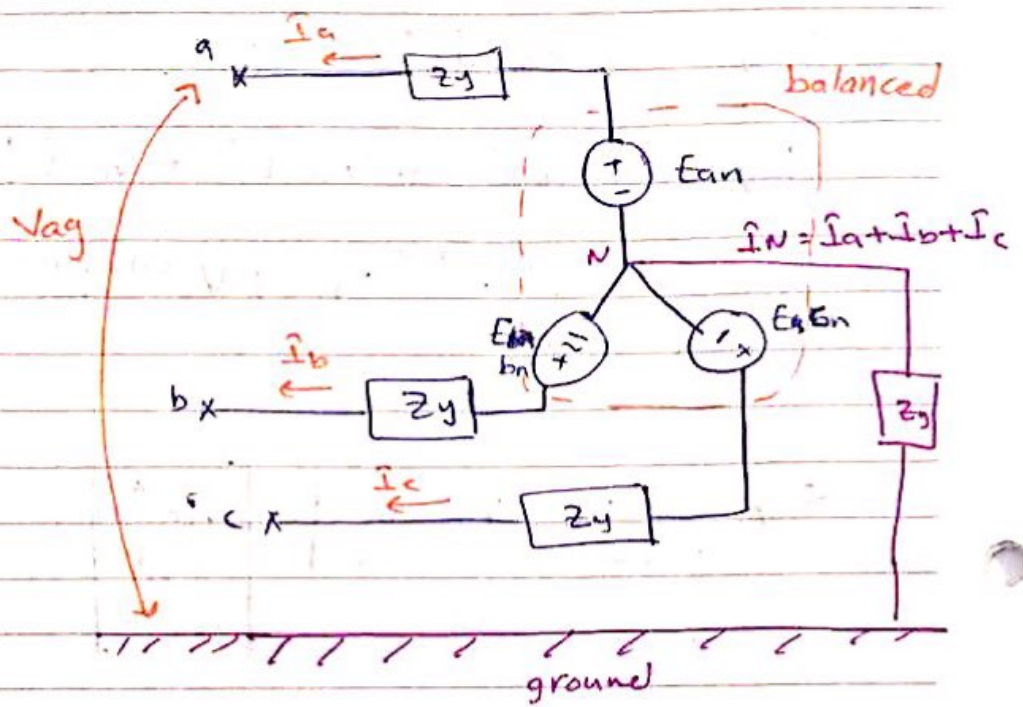
$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_a^{(0)} = 0$$

$$I_a^{(1)} = 5.78 \angle -30^\circ$$

$$I_a^{(2)} = 5.78 \angle +30^\circ$$

Sequence Network :-



$$-V_{ag} = \hat{i}_a Z_y + E_{an} - \hat{i}_n Z_n$$

$$-V_{ag} = \hat{i}_a Z_y - E_{an} - (\hat{i}_a + \hat{i}_b + \hat{i}_c) Z_n = 0$$

$$V_{ag} = E_{an} - \hat{i}_a (Z_y + Z_n) - \hat{i}_b Z_n - \hat{i}_c Z_n$$

$$V_{bg} = E_{bn} - \hat{i}_a Z_n - \hat{i}_b (Z_y + Z_n) - \hat{i}_c Z_n$$

$$V_{cg} = E_{cn} - \hat{i}_a Z_n - \hat{i}_b (Z_n + Z_y)$$

coupling in phase.

Symmetrical components :-

$$V_p = E_p - Z_p \hat{I}_p$$

$p \triangleq$ phase.
 $s \triangleq$ symmetrical.

$$AV_s = E_p - Z_p A \hat{I}_s$$

$$V_s = A^{-1} E_p - A^{-1} Z_p A \hat{I}_s$$

$$A^{-1} E_p = \begin{bmatrix} E_a^{(0)} \\ E_a^{(1)} \\ E_a^{(2)} \end{bmatrix}$$

generator balanced.
→ current unbalanced

generator balanced. $\begin{bmatrix} 0 \\ E_a^{(1)} \\ 0 \end{bmatrix}$

$A^{-1} Z_P A$

$$\frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_y + 3Z_n & Z_n & Z_n \\ Z_n & Z_y + 2Z_n & Z_n \\ Z_n & Z_n & Z_y + 2Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_y + 3Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix}$$

$V_s = A^{-1} E_P - A^{-1} Z_P A \hat{I}_s$

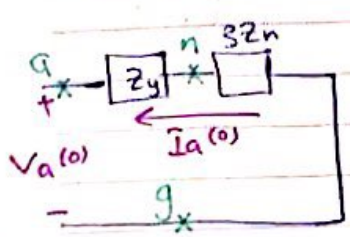
$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a^{(1)} \\ 0 \end{bmatrix} - \begin{bmatrix} Z_y + 3Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix} \begin{bmatrix} \hat{I}_a^{(0)} \\ \hat{I}_a^{(1)} \\ \hat{I}_a^{(2)} \end{bmatrix}$$

Coupling is 0. $\hat{I}_a^{(0)}$ (is not zero) is all real symmetric (is js x

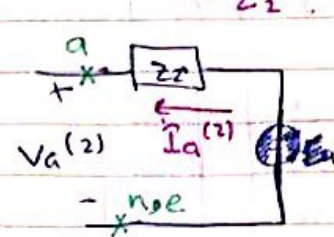
$V_a^{(0)} = -\hat{I}_a^{(0)} (Z_y + 3Z_n)$ Z_0

$V_a^{(1)} = E_a^{(1)} - \hat{I}_a^{(1)} Z_y$ Z_1

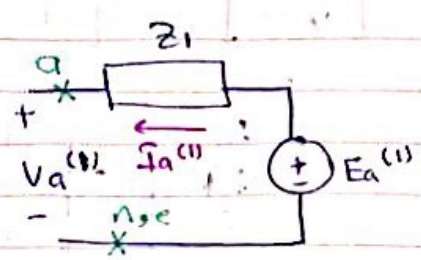
$V_a^{(2)} = -\hat{I}_a^{(2)} Z_y$ Z_2



Zero seq
g = ground



-ve seq

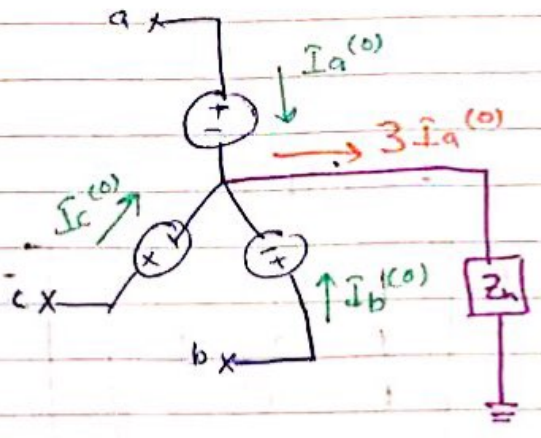


+ve seq
n = neutral
e = earth

O.C ← zero seq. n. b.s up, W, L, s.c. n. s.b. d.s x

فولتاج و كورنت = e, n في فولتاج

$$V_n = 3Z_n I_a^{(0)}$$



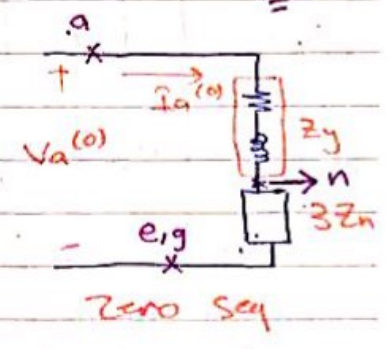
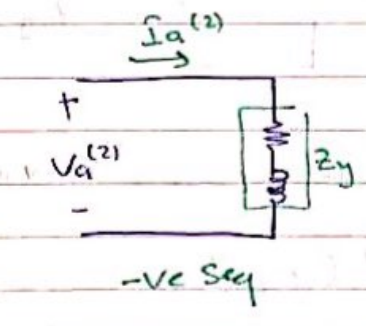
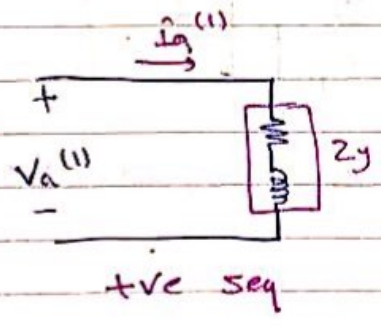
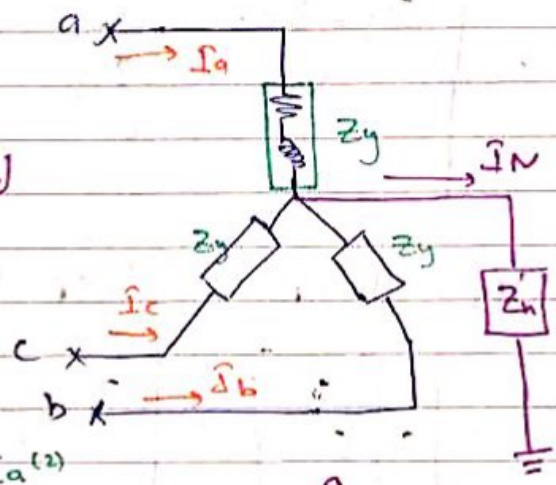
Sequence network with loads Z_y

input $\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$, Z_y

$I_a, I_b, I_c = ?$

unbalanced

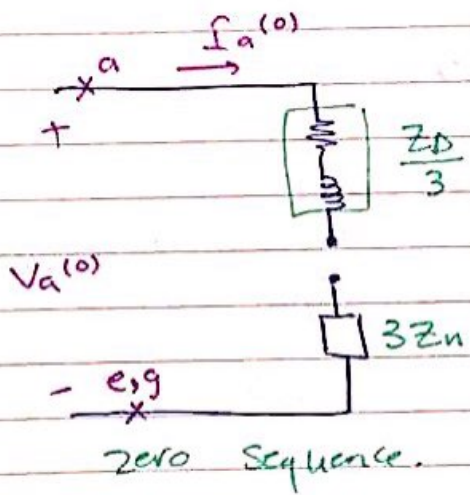
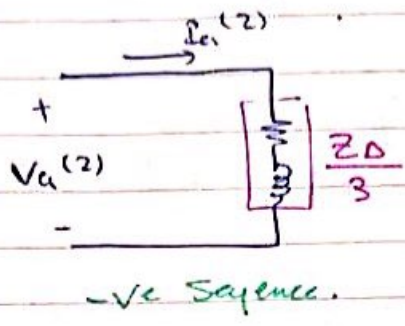
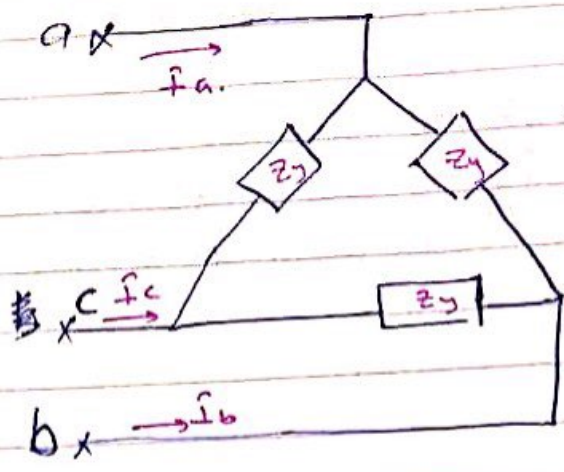
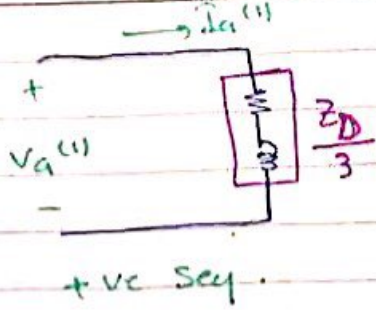
$$V_a \equiv V_{ag}$$



$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

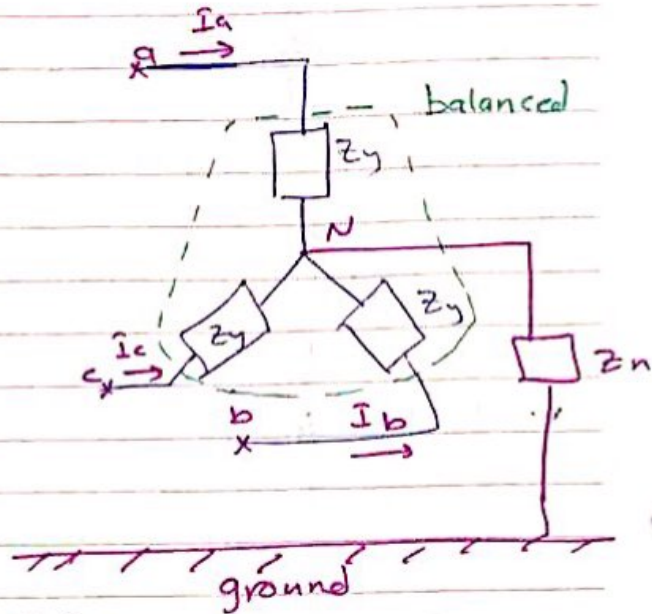
Circuit $\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$

$$\begin{bmatrix} I_a^{(0)} \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$



Sequence Networks of impedance Δ

load \Rightarrow balanced
 $I_a, I_b, I_c \rightarrow$ unbalanced.
 \rightarrow voltage unbalanced.



$$V_a = I_a Z_y + (I_a + I_b + I_c) Z_n$$

$$V_a = I_a (Z_y + Z_n) + I_b Z_n + I_c Z_n$$

$$V_b = I_a Z_n + I_b (Z_y + Z_n) + I_c Z_n$$

$$V_c = I_a Z_n + I_b Z_n + I_c (Z_y + Z_n)$$

$$Z_p = \begin{bmatrix} Z_y + Z_n & Z_n & Z_n \\ Z_n & Z_y + Z_n & Z_n \\ Z_n & Z_n & Z_y + Z_n \end{bmatrix}$$

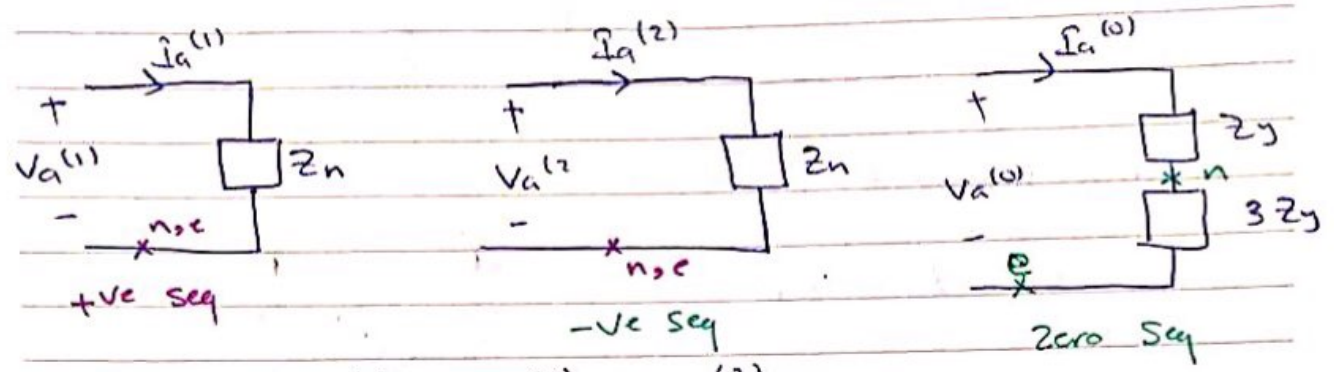
$$V_p = Z_p I_p$$

$$A V_s = Z_p A I_s$$

$$V_s = A^{-1} Z_p A I_s$$

$$\begin{aligned} a + a^2 &= -1 \\ 1 + a + a^2 &= 0 \end{aligned}$$

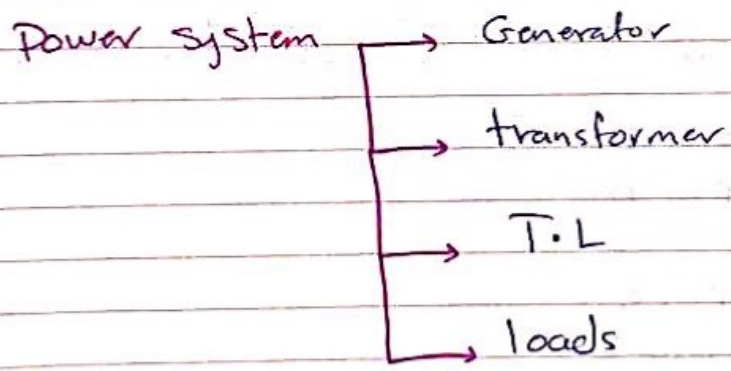
$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \begin{bmatrix} Z_y + 3Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$



$$V_{an} = V_{an}^{(0)} + V_{an}^{(1)} + V_{an}^{(2)}$$

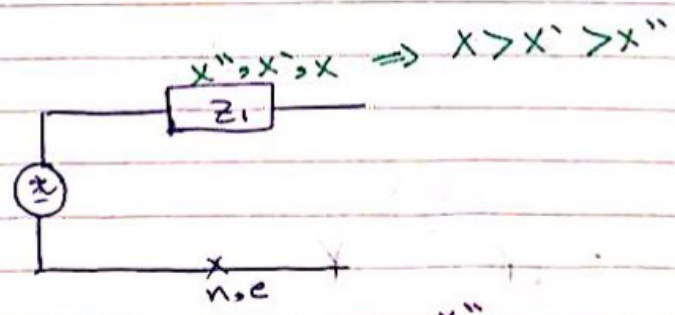
$$V_{ae} = V_{ae}^{(0)} + V_{ae}^{(1)} + V_{ae}^{(2)}$$

* Sequence diagram Δ

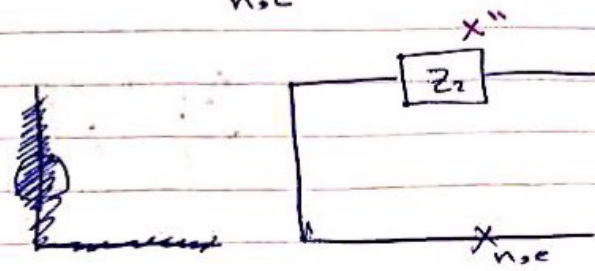


\Rightarrow Generator :-

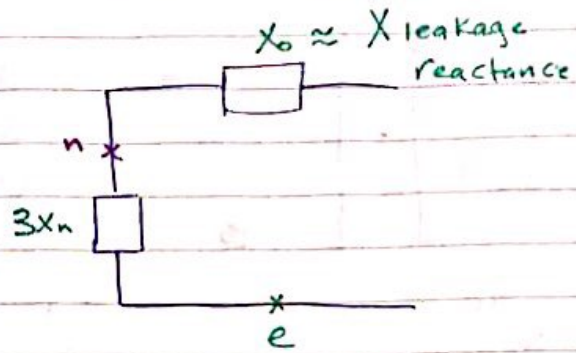
+ve Sequence :-



-ve Sequence :

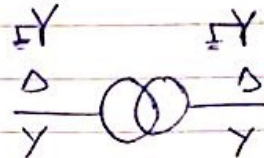


Zero Sequence:

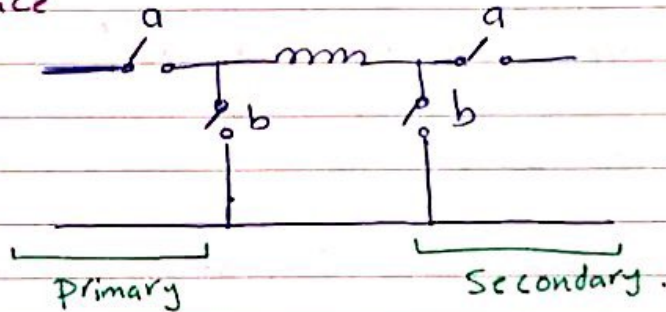


* transformer δ/Δ

$$X_1 = X_2 = X_0$$



Zero Sequence



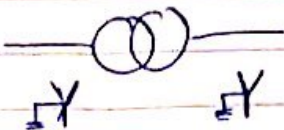
close a
open b



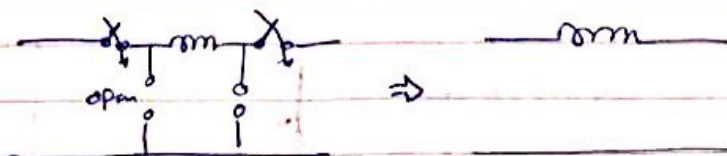
open a
close b

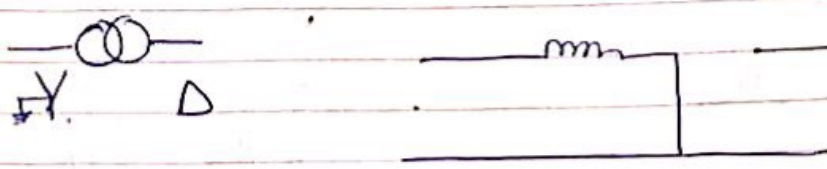
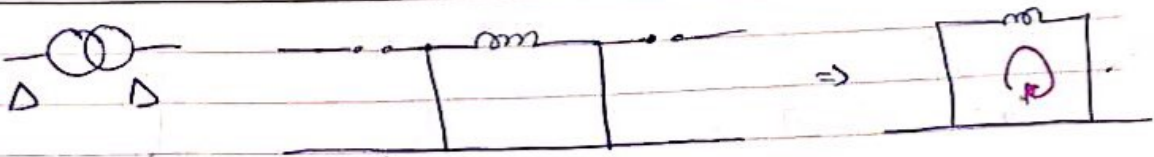


open a
open b

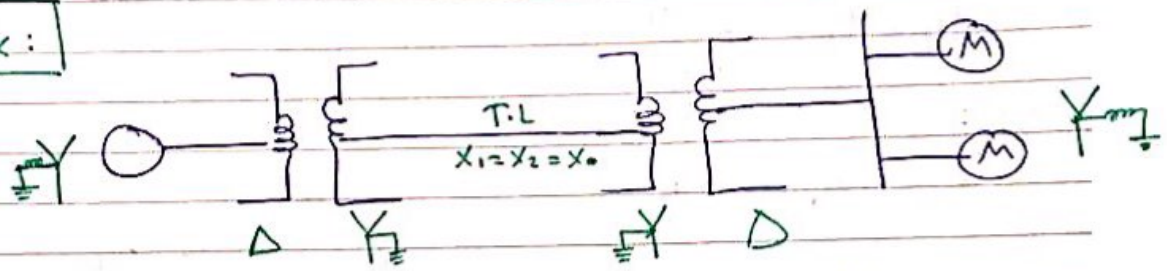


Zero Sequence

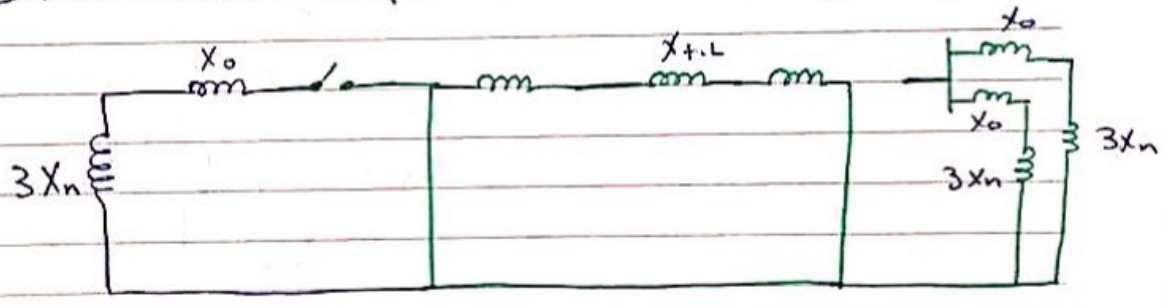




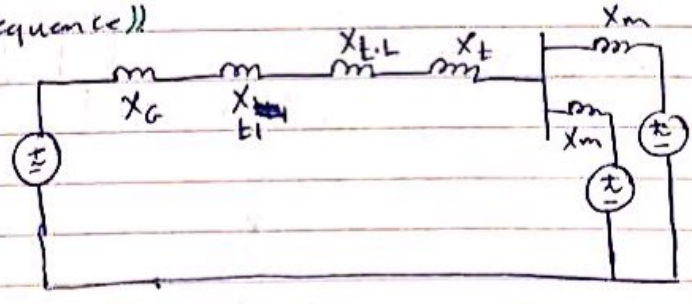
Ex:



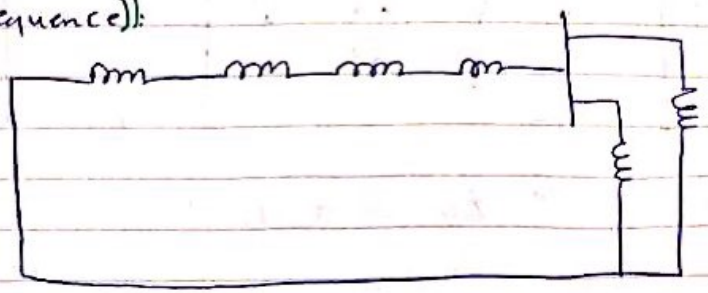
Draw zero sequence networks?



(+ve sequence):



(-ve sequence):

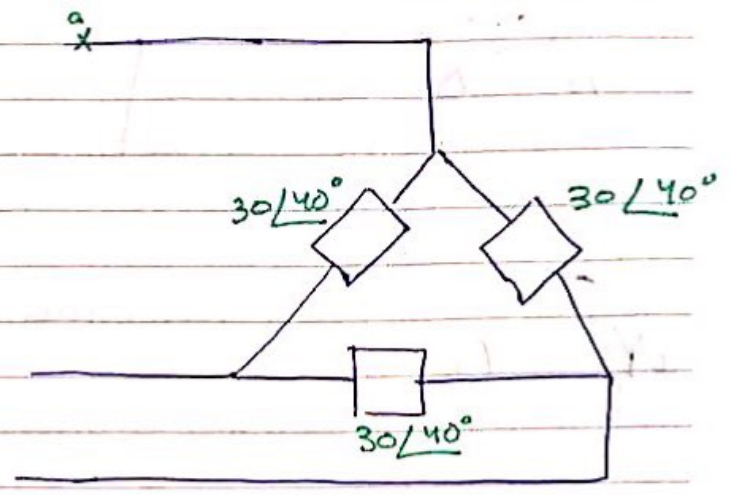


Ex

$V_{ag} = 277 \angle 0$

$V_{bg} = 260 \angle -120^\circ$

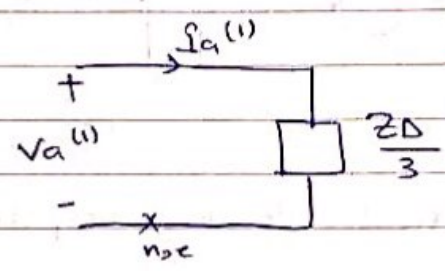
$V_{cg} = 295 \angle 115^\circ$



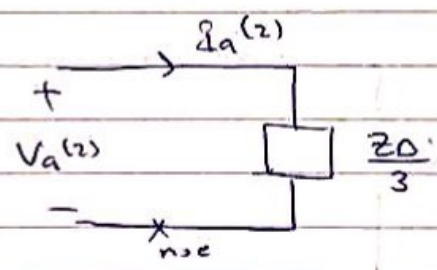
I_a, I_b, I_c ?!

Sol:

+ve sequence :-

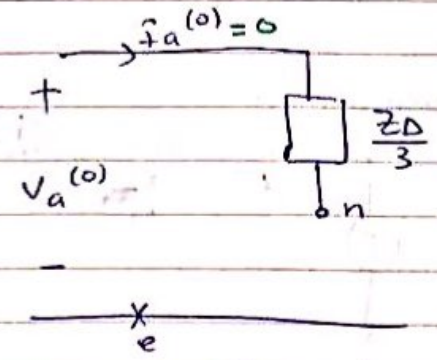


-ve sequence:



Zero sequence:

(الفولت بالنسبة لـ g) $V_a \rightarrow$



$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$V_a^{(0)} = I_a^{(0)} Z_0 \rightarrow I_a^{(0)}$

$$V_a^{(1)} = \hat{I}_a^{(1)} \frac{Z_D}{3} \Rightarrow \hat{I}_a^{(1)}$$

$$V_a^{(2)} = \hat{I}_a^{(2)} \frac{Z_D}{3} \Rightarrow \hat{I}_a^{(2)}$$

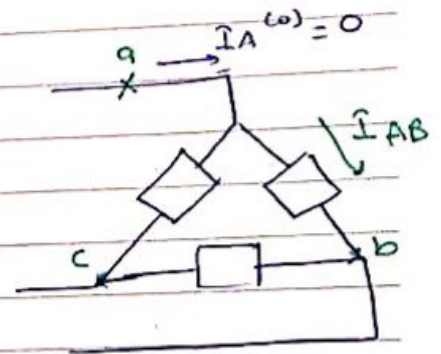
$$\begin{bmatrix} \hat{I}_a \\ \hat{I}_b \\ \hat{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \hat{I}_a^{(0)} \\ \hat{I}_a^{(1)} \\ \hat{I}_a^{(2)} \end{bmatrix}$$

$$\hat{I}_{AB} = \hat{I}_{AB}^{(0)} + \hat{I}_{AB}^{(1)} + \hat{I}_{AB}^{(2)}$$

$$\hat{I}_A^{(1)} = \hat{I}_{AB}^{(1)} \sqrt{3} \angle -30^\circ$$

$$\hat{I}_A^{(2)} = \hat{I}_{AB}^{(2)} \sqrt{3} \angle +30^\circ$$

line phase



$$\hat{I}_{AB}^{(0)} = ?!$$

$$V_{ab}^{(1)} = \sqrt{3} V_i^{(1)} \angle +30^\circ$$

$$V_{ab} = V_a - V_b$$

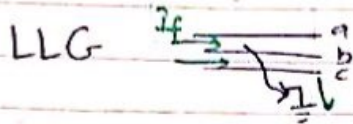
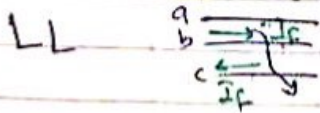
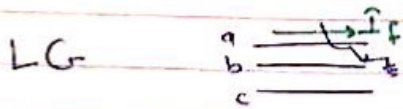
$$\hat{I}_{ab} = \frac{V_{ab}}{Z_D}$$



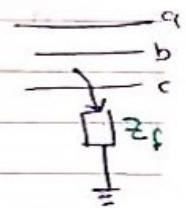
$$V_a, V_b, V_c.$$

$$S_{3\phi} = 3 V_a^{(0)} \times \hat{I}_a^{(0)*} + \dots + \dots$$

* Unsymmetrical fault analysis :-



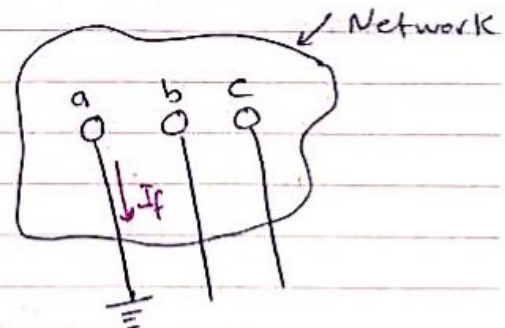
- * Analysis Current
- * Analysis Voltage
- * Sequence network
(+ve, -ve, Zero)
- * I_f
- * impact of Z_f (impedance of fault)



Single line to Ground :-

$$I_f = I_a$$

$$I_b = I_c = 0 \text{ (unloaded)}$$



$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_f \\ 0 \\ 0 \end{bmatrix}$$

$$I_a^{(0)} = \frac{1}{3} I_f = \frac{1}{3} I_f$$

$$I_a^{(1)} = \frac{1}{3} I_f = \frac{1}{3} I_f$$

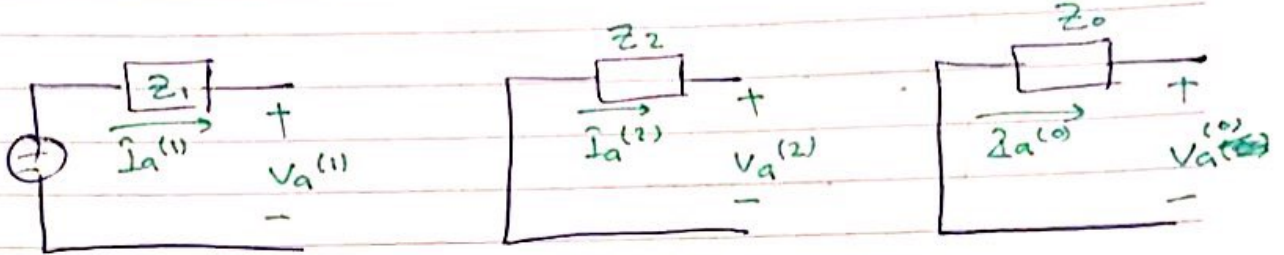
$$I_a^{(2)} = \frac{1}{3} I_f = \frac{1}{3} I_f$$

$$\hat{I}_a^{(0)} = \hat{I}_a^{(1)} = \hat{I}_a^{(2)}$$

$$V_a = 0$$

$$V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = 0$$

$$\hat{I}_a^{(0)} = \hat{I}_a^{(1)} = \hat{I}_a^{(2)}$$

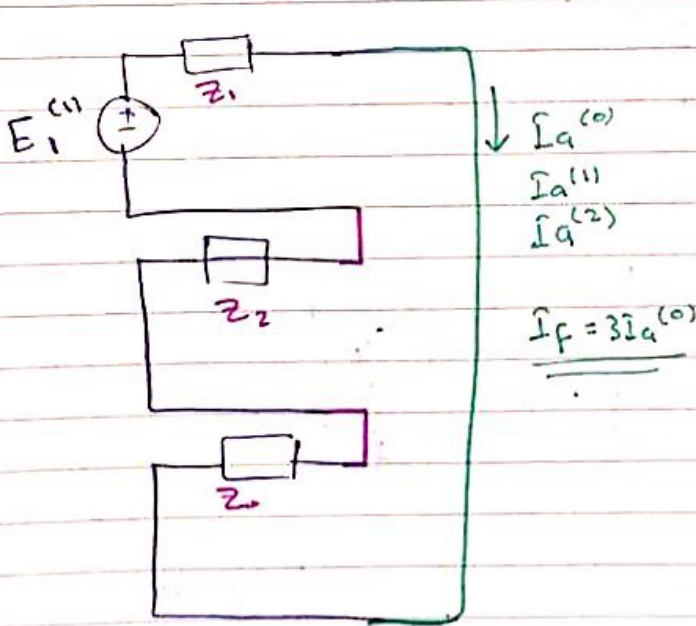


+ve seq

-ve seq

Zero seq

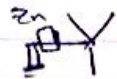
$Z_0, Z_1, Z_2 \Rightarrow Z_{th}$ thevenin from fault.



برفوقه ز0
in series ←

$$\hat{I}_f = 3 \hat{I}_a^{(0)} = \frac{3 E_1^{(1)}}{Z_1 + Z_2 + Z_0}$$

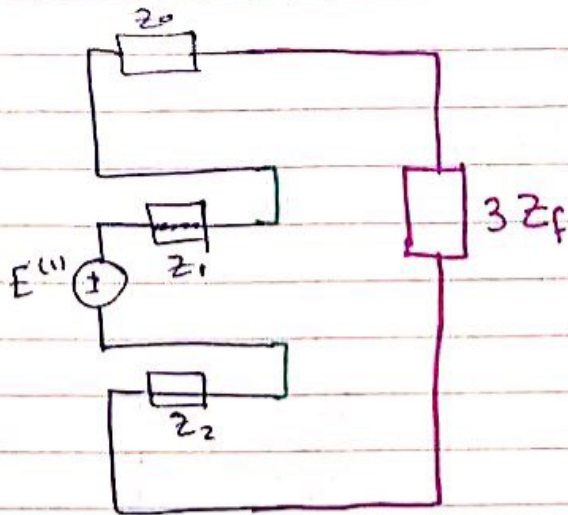
$Z_0 > Z_1, Z_2$
لا يتركب في الاصل



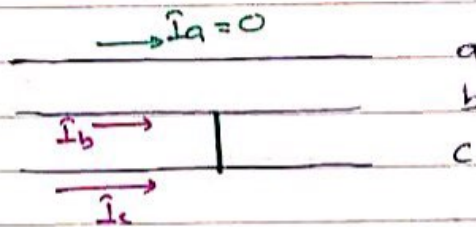
Impact of Z_f 84

$$V_a = \hat{I}_f Z_f = 3 \hat{I}_a^{(0)} Z_f = \hat{I}_a^{(0)} 3 Z_f$$

$$V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = \hat{I}_a^{(0)} (3Z_f)$$



Line to line fault :-



$$\hat{I}_a = 0 \quad \hat{I}_b = \hat{I}_f \quad , \quad \hat{I}_c = -\hat{I}_b = -\hat{I}_f$$

$$\hat{I}_b = -\hat{I}_c$$

$$\begin{bmatrix} \hat{I}_a^{(0)} \\ \hat{I}_a^{(1)} \\ \hat{I}_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \hat{I}_a \\ \hat{I}_b \\ \hat{I}_c \end{bmatrix}$$

$$\hat{I}_a^{(0)} = 0$$

$$\hat{I}_a^{(1)} = \frac{1}{3} (a \hat{I}_b - a^2 \hat{I}_b)$$

$$\hat{I}_a^{(2)} = \frac{1}{3} (a^2 \hat{I}_b - a \hat{I}_b)$$

$$\hat{I}_a^{(2)} = 0$$

$$\hat{I}_a^{(1)} = -\hat{I}_a^{(2)}$$

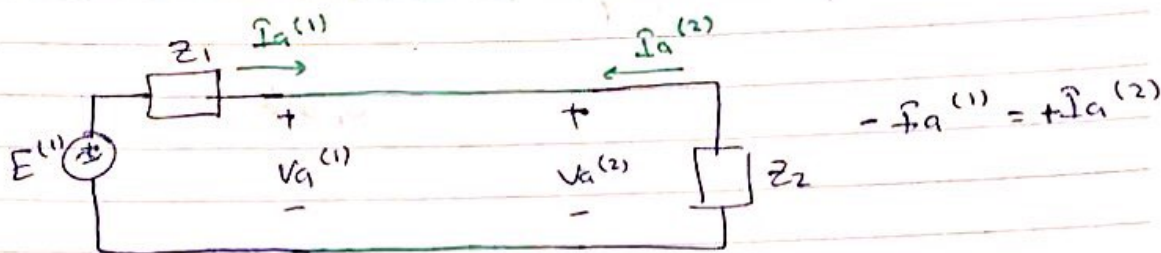
$$V_b = V_c$$

$$V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(2)} = V_a^{(0)} + a V_a^{(1)} + a^2 V_a^{(2)}$$

$$V_a^{(1)} (a^2 - a) = V_a^{(2)} (a^2 - a)$$

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$$V_a^{(1)} = V_a^{(2)} \Rightarrow \text{parallel.}$$



$$\hat{I}_a^{(1)} = \frac{E}{Z_1 + Z_2}$$

$$\hat{I}_f = \hat{I}_b$$

$$\begin{bmatrix} Z_a \\ \hat{I}_b \\ \hat{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \hat{I}_a^{(1)} \\ \hat{I}_a^{(1)} \\ \hat{I}_a^{(1)} \end{bmatrix}$$

$$\hat{I}_b = a^2 \hat{I}_a^{(1)} - a \hat{I}_a^{(1)}$$

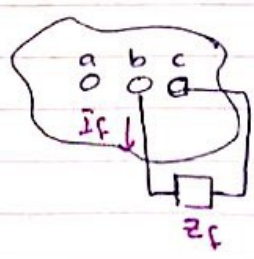
$$\hat{I}_f = \hat{I}_b = \hat{I}_a^{(1)} (a^2 - a)$$

$$|\hat{I}_f| = \sqrt{3} \left| \frac{E^{(1)}}{Z_1 + Z_2} \right|$$

impedanz Z_f :-

$$V_b - V_c = \hat{I}_f Z_f$$

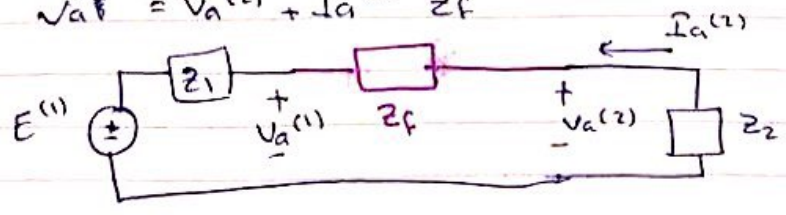
$$V_b = V_c + \hat{I}_f Z_f$$



$$V_a^{(1)} (a^2 - a) = V_a^{(2)} (a^2 - a) + (\hat{I}_a^{(1)} + a^2 \hat{I}_a^{(1)} + a \hat{I}_a^{(1)}) Z_f$$

$$V_a^{(1)} (a^2 - a) = V_a^{(2)} (a^2 - a) + \hat{I}_a^{(1)} (a^2 - a) Z_f$$

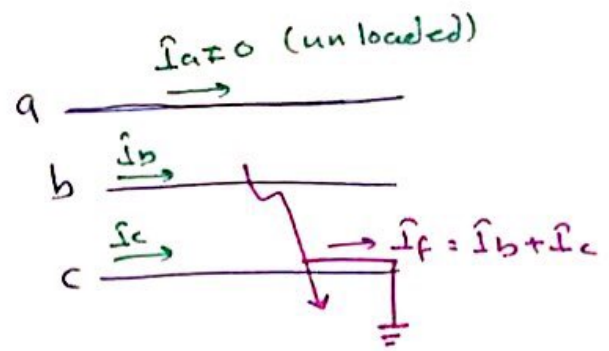
$$V_a^{(1)} = V_a^{(2)} + \hat{I}_a^{(1)} Z_f$$



4/5 Thur

Double line to ground fault:-

- 1) Analysis of current
- 2) " Voltage
- 3) Sequence networks.
- 4) impact of Z_f .



$$I_a = 0$$

$$I_a^{(0)} + I_a^{(1)} + I_a^{(2)} = 0, \text{ KCL}$$

$$V_b = V_c$$

$$V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(2)} = V_a^{(0)} + a V_a^{(1)} + a^2 V_a^{(2)}$$

$$V_a^{(1)} (a^2 - a) = V_a^{(2)} (a^2 - a)$$

$$\boxed{V_a^{(1)} = V_a^{(2)}}$$

$$V_b = 0$$

$$V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(2)} = 0$$

$$V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(1)} = 0$$

$$V_a^{(0)} + V_a^{(1)} (a^2 + a) = 0$$

$$V_a^{(0)} + V_a^{(1)} (-1) = 0$$

$$\boxed{V_a^{(0)} = V_a^{(1)}}$$

$$a^2 + a = ?!!$$

$$1 + a^2 + a = 0$$

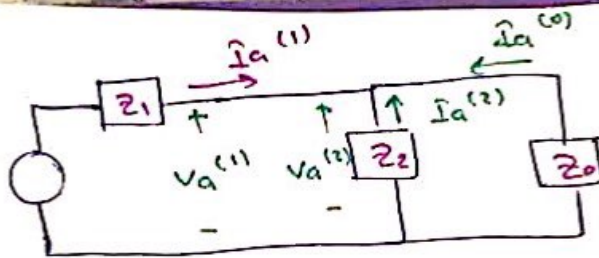
$$a^2 + a = -1$$

$$I_a^{(0)} + I_a^{(1)} + I_a^{(2)} = 0$$

$$V_a^{(1)} = V_a^{(2)}$$

$$V_a^{(1)} = V_a^{(0)}$$

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$$\begin{matrix} \hat{I}_a^{(0)} \\ \hat{I}_a^{(1)} \\ \hat{I}_a^{(2)} \end{matrix} \begin{matrix} \left. \begin{matrix} \rightarrow \hat{I}_b \\ \rightarrow \hat{I}_f \end{matrix} \right\} \rightarrow \hat{I}_f \end{matrix}$$

* impact \$Z_f\$:-

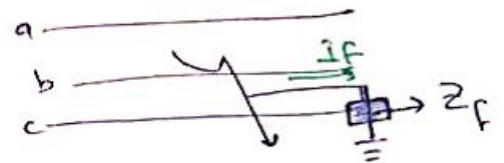
$$V_b = V_a^{(0)} - V_a^{(1)}$$

$$V_b = \hat{I}_f Z_f$$

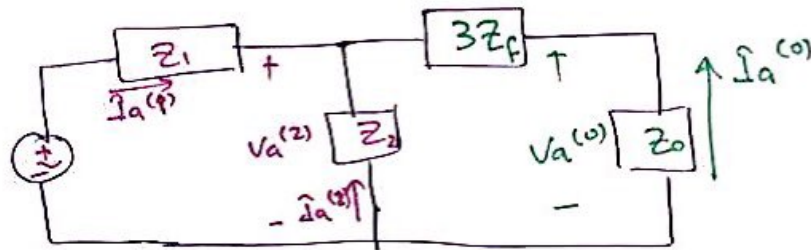
$$V_b = 3 (\hat{I}_a^{(0)}) Z_f$$

$$V_b = \hat{I}_a^{(0)} 3Z_f$$

$$V_a^{(0)} - V_a^{(1)} = \hat{I}_a^{(0)} (3Z_f)$$

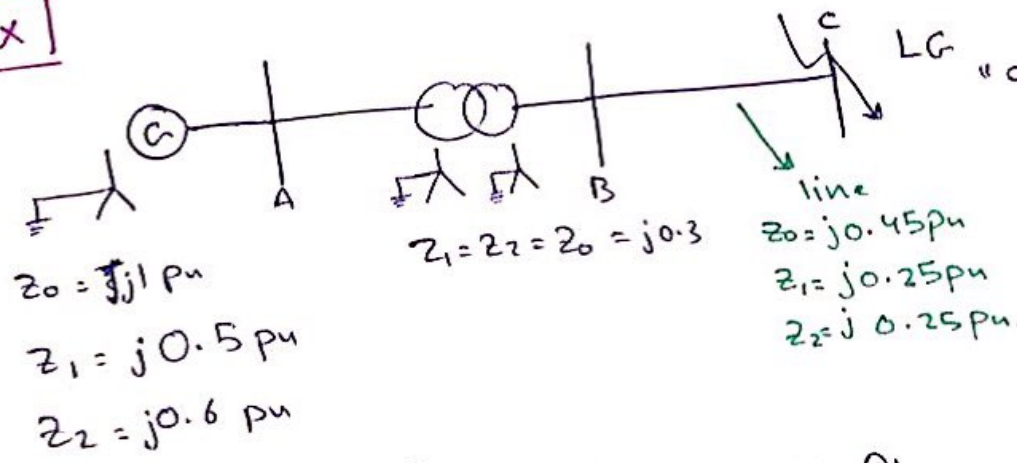


\$V_b \neq 0\$
 due to fault \$Z_f\$, \$V_b\$ is not zero.



$$\begin{aligned} \hat{I}_f &= \hat{I}_b + \hat{I}_c \\ &= (\hat{I}_a^{(0)} + a^2 \hat{I}_a^{(1)} + a \hat{I}_a^{(2)}) + (\hat{I}_a^{(0)} + a \hat{I}_a^{(1)} + a^2 \hat{I}_a^{(2)}) \\ &= \hat{I}_a^{(0)} + \hat{I}_a^{(1)} + \hat{I}_a^{(2)} = 0 \end{aligned}$$

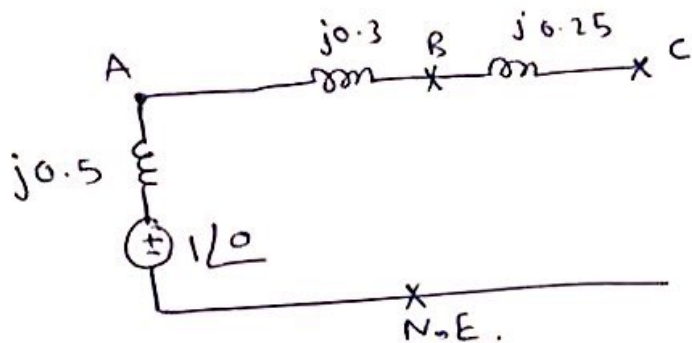
Ex



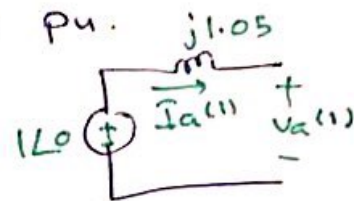
"on a common base"
 Neglect load current,
 $V_f = 1 pu$

- ① find LG fault at bus C ?!
- ② Line to fault at bus B ?!

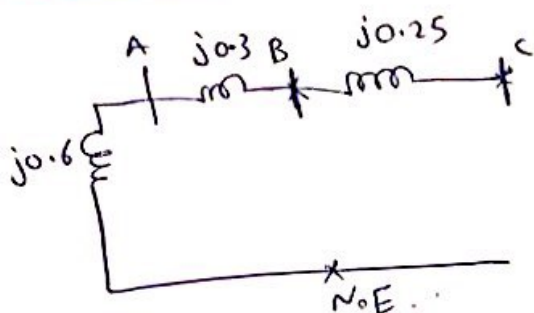
+ve Sequence:



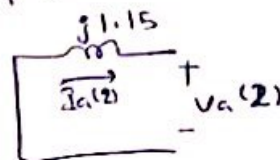
$Z_1 = j1.05 \text{ pu}$



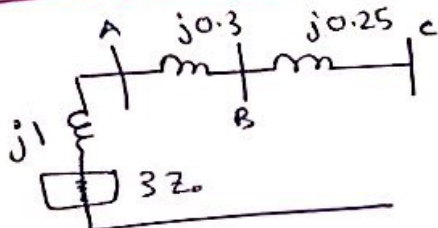
-ve Sequence:



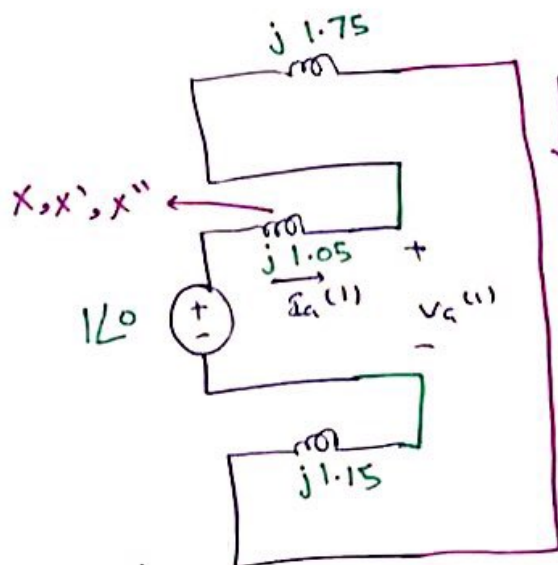
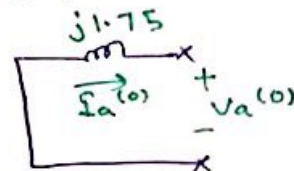
$Z_2 = j4.15 \text{ pu}$



Zero Sequence:



$Z_0 = j1.75 \text{ pu}$



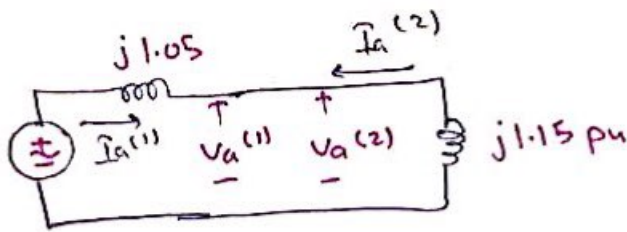
$I_a^{(0)} = I_a^{(1)} = I_a^{(2)}$

$I_{LG} = 3 I_a^{(0)}$

$I_a^{(0)} = \frac{1}{j1.75 + j1.05 + j1.15}$

$I_f = 3 I_a^{(0)} = 0.759 \text{ pu}$

② line to line fault. at bus C



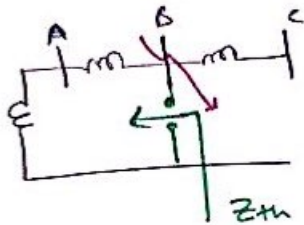
$$\hat{I}_a^{(1)} = -\hat{I}_a^{(2)}$$

$$\hat{I}_a^{(1)} = \frac{1 \angle 0}{j1.05 + j1.15}$$

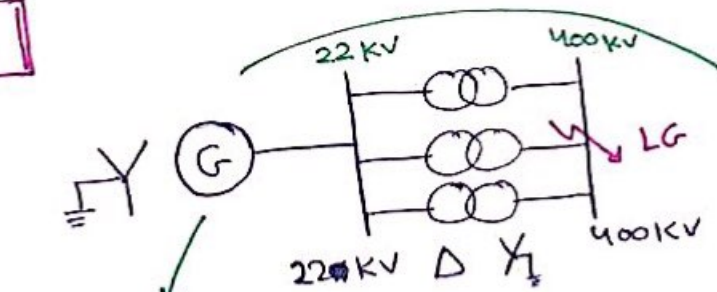
$$\hat{I}_a^{(2)} = -\hat{I}_a^{(1)}$$

$$\hat{I}_f = \sqrt{3} \hat{I}_a^{(1)}$$

2th ال Z_{th} من Z_{th} bus B (fault site, Z_{th} *)



Ex



$S_b = 1000 \text{ MVA}$

باقي V_b * ratio for transformer

22kV
825 MVA.
 $X_1 = 0.14 \text{ pu}$
 $X_2 = 0.13 \text{ pu}$
 $X_0 = 0.15 \text{ pu}$

each ---
300MVA
 $X_1 = X_2 = X_0 = 0.14 \text{ pu}$

LG fault
at bus 2?!
(base 1000 MVA))

$$V_f = 1 \angle 0 \text{ pu}$$

Sol:

Generator \Rightarrow

$$X_1 = 0.14 * \left(\frac{1000}{825} \right) = 0.17 \text{ pu}$$

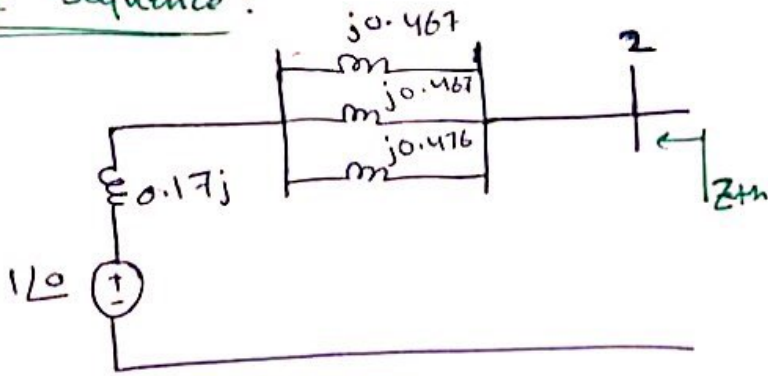
$$X_2 = 0.158 \text{ pu}$$

$$X_0 = 0.182 \text{ pu}$$

148

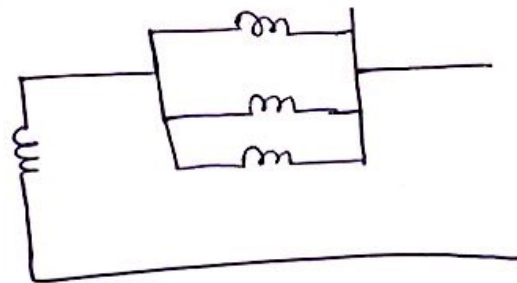
transformer: $X_1 = X_2 = X_0 = 0.14 * \left(\frac{1000}{300}\right) = 0.467 \mu$

+ve Sequence:



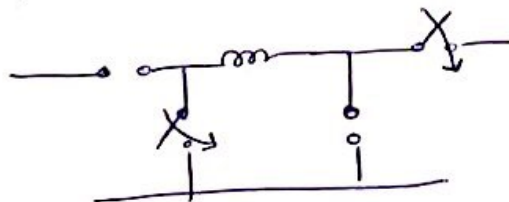
$$Z_{th} = j \left(\frac{j0.467}{3} + 0.17 \right)$$

-ve Sequence:

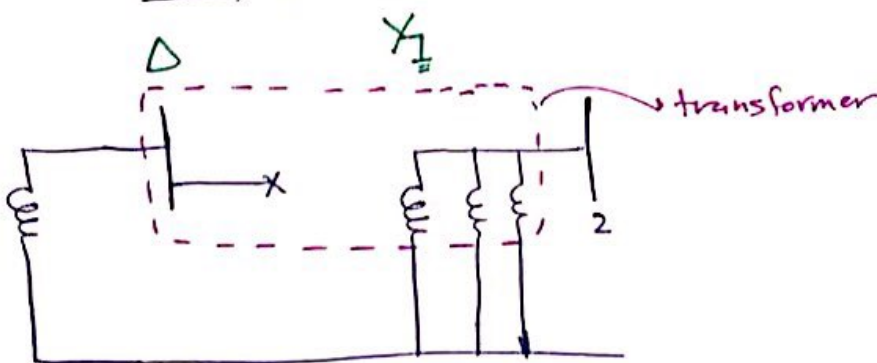
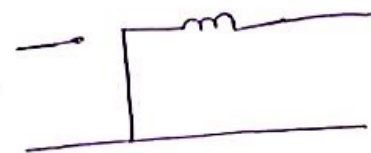


$$Z_2 = j \left(\frac{0.467}{3} + j0.158 \right)$$

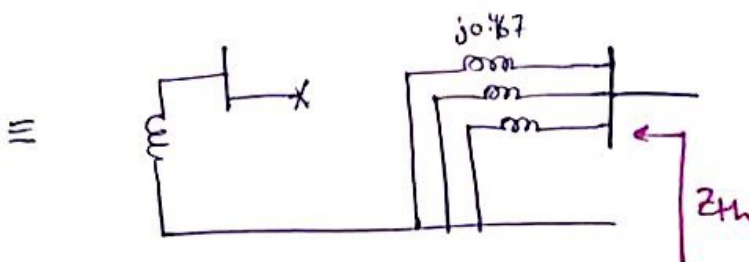
Zero sequence :-



=>

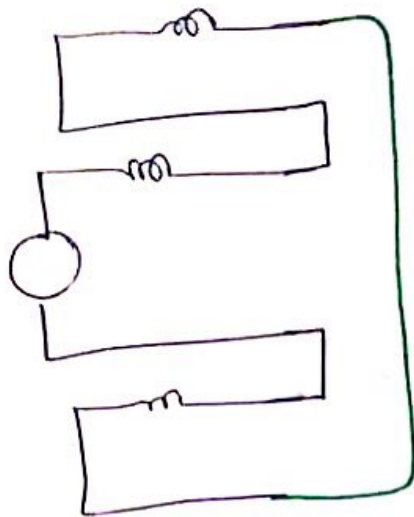


\sim $X_1 = X_2 = X_0 \sim 15 \mu$
 $\cdot Z_{th} \sim$



$$Z_0 = \frac{j0.467}{3}$$

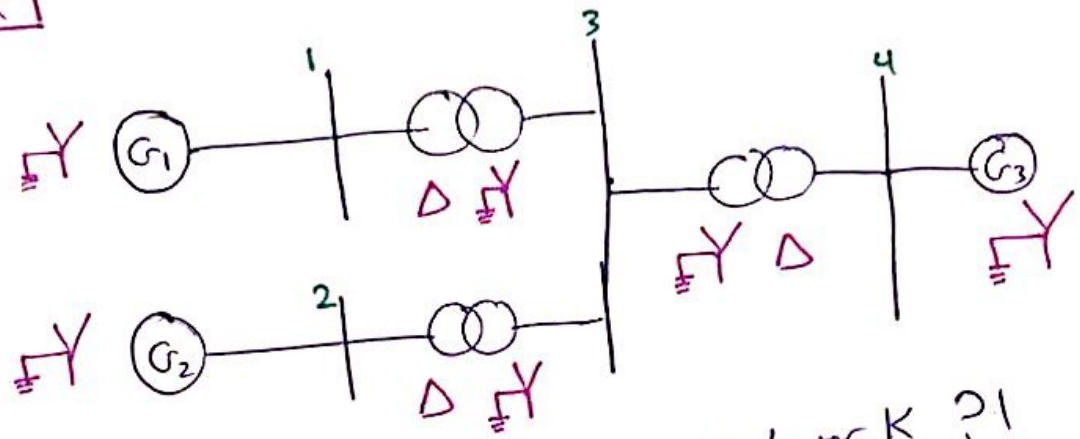
LG



$I_a^{(0)}$

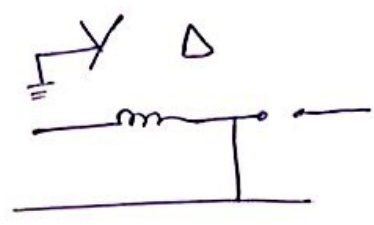
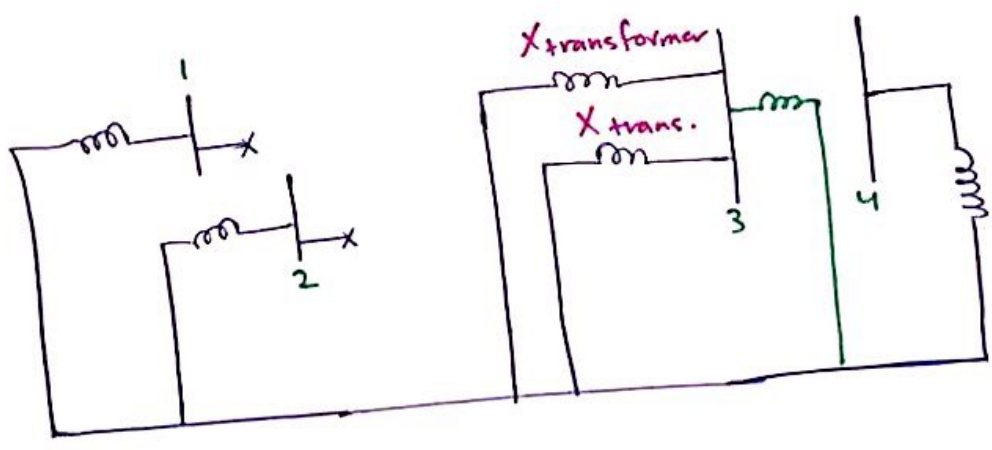
$I_f = 3 I_a^{(0)}$

EX

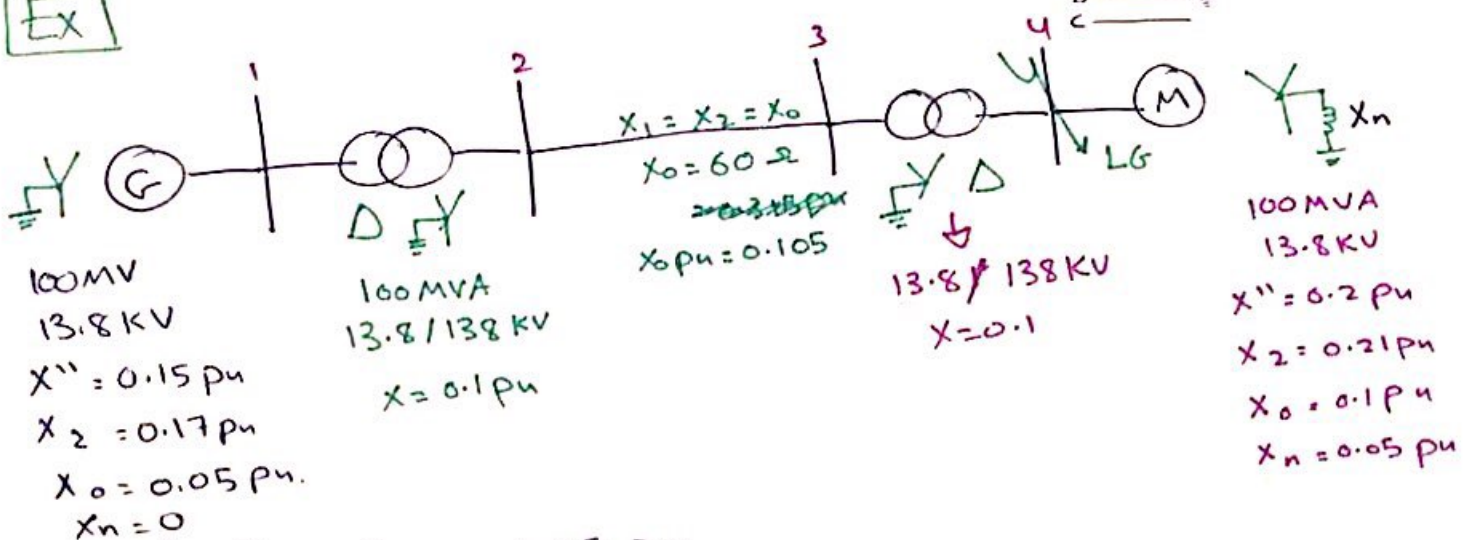


Draw zero sequence network K ?!

Sol:

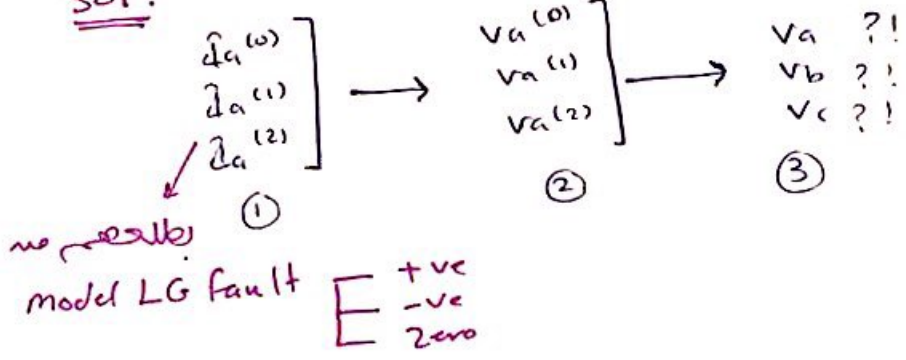


Ex 1

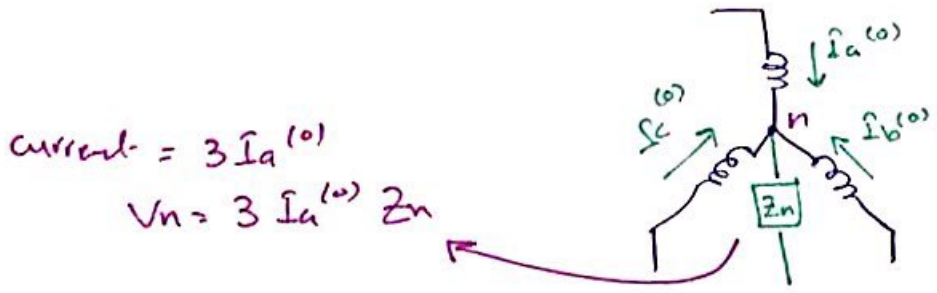
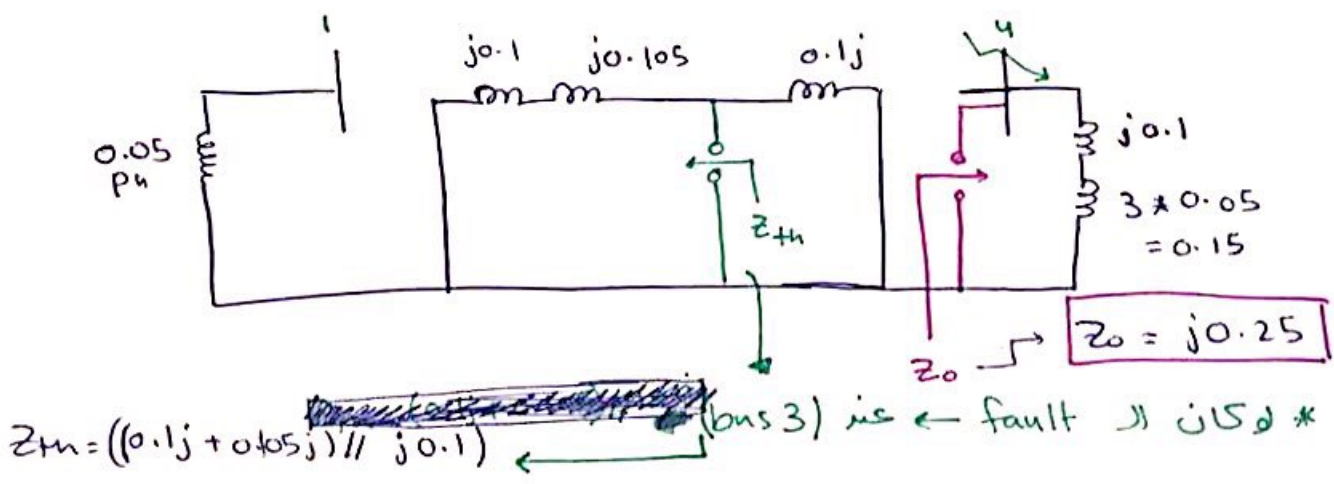


Determine phase voltage at bus 4 LG fault ?!

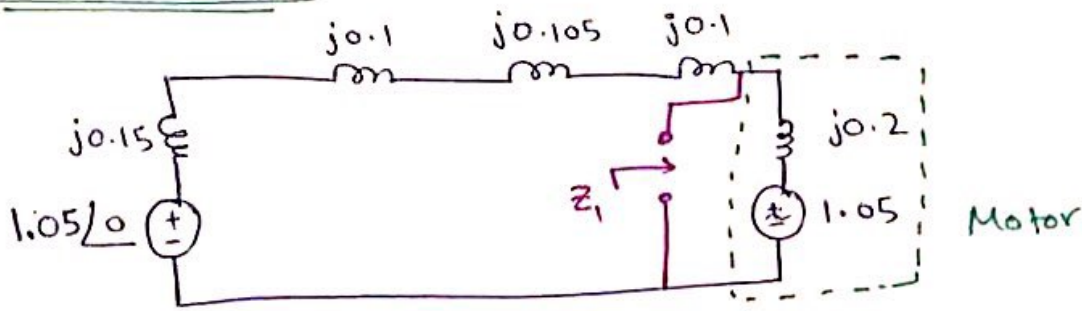
Sol:



* Zero sequence:



+ve Sequence :-

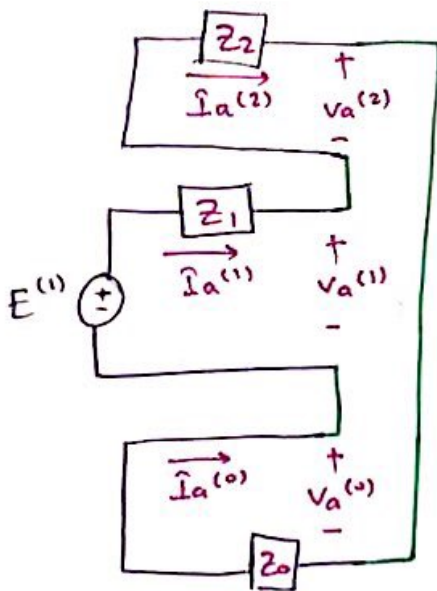


$$Z_1 = (j0.15 + j0.1 + j0.105 + j0.1) \parallel (j0.2) = j0.13893$$

$$Z_2 = j0.14562$$

$$E^{(1)} = 1.05 \text{ pu}$$

LG fault:



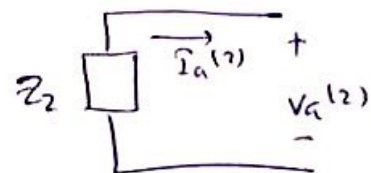
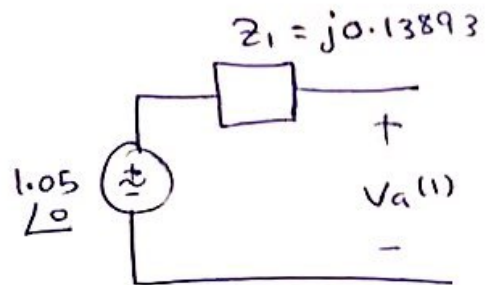
$$\begin{aligned} I_a^{(0)} &= I_a^{(1)} = I_a^{(2)} \\ &= \frac{1.05 \angle 0}{Z_1 + Z_2 + Z_0} \end{aligned}$$

$$I_a^{(0)} = -j1.96427 \text{ pu}$$

$$\begin{aligned} V_a^{(1)} &= 1.05 \angle 0 - I_a^{(1)} (Z_1) \\ &= 1.05 \angle 0 - I_a^{(1)} (0.13893) \end{aligned}$$

$$V_a^{(2)} = -I_a^{(2)} Z_2$$

$$V_a^{(0)} = -I_a^{(0)} Z_0$$



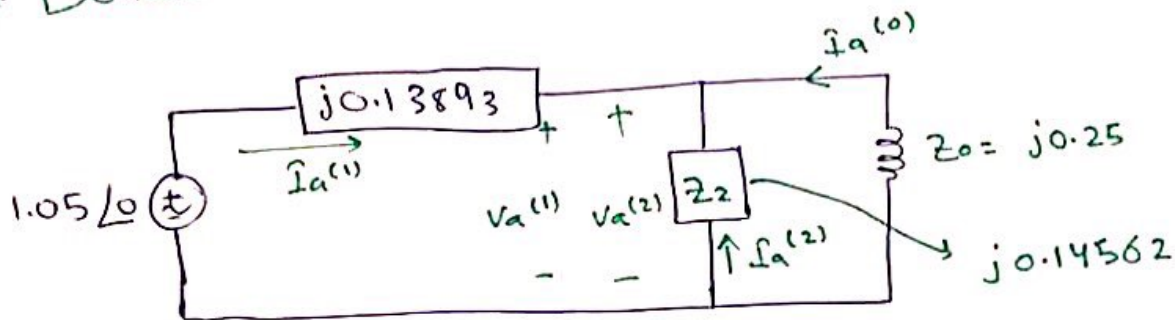
$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ & a^2 & a \\ & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix}$$

$$V_a = 0 \text{ pu}$$

$$V_b = 1.179 \angle 231.3^\circ \text{ pu}$$

$$V_c = 1.179 \angle 128.7^\circ \text{ pu}$$

* Double line to Ground fault :



- 1) Fault current (LLG)
- 2) neutral fault current
- 3) Contribution to the fault from the motor ?!

$$\textcircled{1} \quad \hat{I}_a^{(1)} = \frac{1.05 \angle 0}{Z_1 + (Z_2 \parallel Z_0)}$$

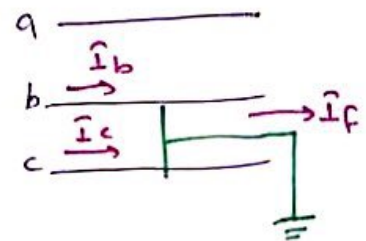
$$\hat{I}_a^{(1)} = -j 4.5464 \text{ pu}$$

$$\hat{I}_a^{(2)} = -\hat{I}_a^{(1)} * \frac{Z_0}{Z_0 + Z_2}$$

$$\hat{I}_a^{(2)} = j 2.873 \text{ pu}$$

$$\hat{I}_a^{(0)} = -\hat{I}_a^{(1)} * \frac{Z_2}{Z_0 + Z_2}$$

$$\hat{I}_a^{(0)} = j 1.6734 \text{ pu}$$



② Neutral current ?!

$$\hat{I}_N = \hat{I}_A + \hat{I}_B + \hat{I}_C$$

$$\begin{bmatrix} \hat{I}_A \\ \hat{I}_B \\ \hat{I}_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \hat{I}_a^{(0)} \\ \hat{I}_a^{(1)} \\ \hat{I}_a^{(2)} \end{bmatrix}$$

$$\hat{I}_A = 0$$

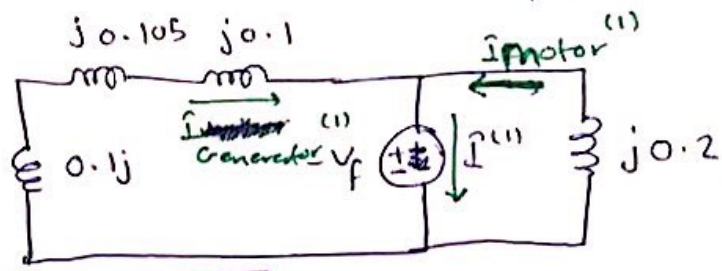
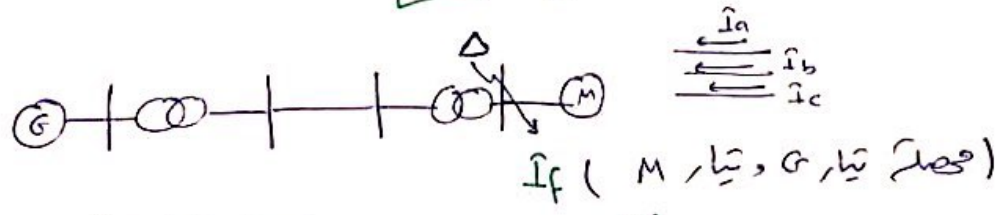
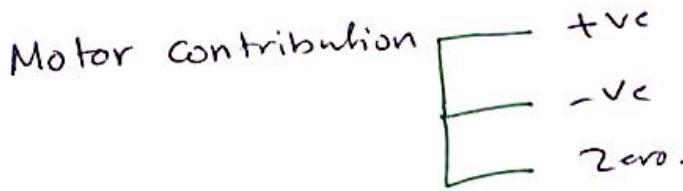
$$\hat{I}_B = 6.89 \angle 158^\circ \text{ pu}$$

$$\hat{I}_C = 6.89 \angle 21^\circ \text{ pu}$$

$$\hat{I}_N = 21 \angle 90^\circ \text{ KA}$$

→ base = 4.18 KA

③



current division
Zero, -ve, +ve

$$\hat{I}_{\text{motor}}^{(0)} = 0$$

$$\hat{I}_{\text{motor}}^{(1)} = 1.9813 \angle 172.643^\circ \text{ pu}$$

$$\hat{I}_{\text{motor}}^{(2)} = 1.9813 \angle 7.357^\circ \text{ pu}$$

$$\begin{bmatrix} \hat{I}_{\text{motor } a} \\ \hat{I}_{\text{motor } b} \\ \hat{I}_{\text{motor } c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \hat{I}_{\text{motor}}^{(0)} \\ \hat{I}_{\text{motor}}^{(1)} \\ \hat{I}_{\text{motor}}^{(2)} \end{bmatrix}$$

Z_{bus} - matrix

$$\begin{cases} \rightarrow Z^{(1)} = Y^{(1)-1} \\ \rightarrow Z^{(2)} = Y^{(2)-1} \\ \rightarrow Z^{(0)} = Y^{(0)-1} \end{cases}$$

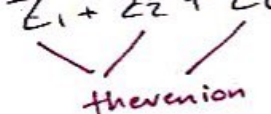
$$\hat{I}_n^{(1)} = \frac{V_F}{Z_{nn}^{(1)}} \rightarrow \text{location fault at bus } n$$

LG:

$$\hat{I}_n^{(0)} = \hat{I}_n^{(1)} = \hat{I}_n^{(2)}$$

$$= \frac{V_F}{Z_{nn}^{(0)} + Z_{nn}^{(1)} + Z_{nn}^{(2)} + 3Z_f}$$

$$* \text{LG} \Rightarrow \hat{I}_a^{(0)} = \frac{E^{(1)}}{Z_1 + Z_2 + Z_0}$$



LL:

$$\hat{I}_n^{(1)} = -\hat{I}_n^{(2)} = \frac{V_F}{Z_1 + Z_2} = \frac{V_F}{Z_{nn}^{(1)} + Z_{nn}^{(2)}}$$

LLG:

$$\hat{I}_n^{(1)} = \frac{V_F}{Z_{nn}^{(1)} + [Z_{nn}^{(2)} \parallel Z_{nn}^{(0)}]}$$

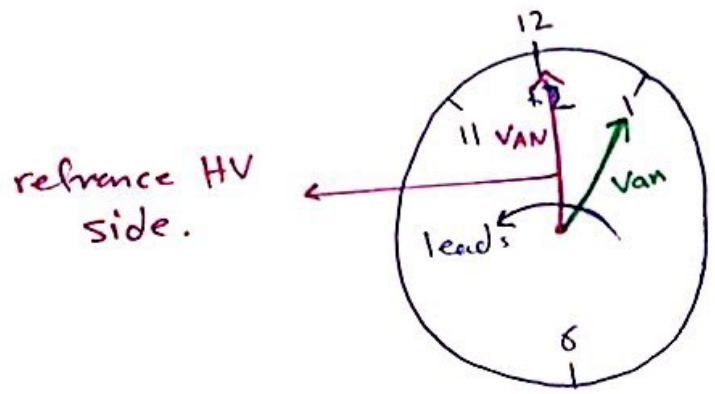
$$\hat{I}_n^{(2)} = -\hat{I}_n^{(1)} * \frac{Z_{nn}^{(0)}}{Z_{nn}^{(0)} + Z_{nn}^{(2)}}$$

$$\hat{I}_n^{(0)} = -\hat{I}_n^{(1)} * \frac{Z_{nn}^{(2)}}{Z_{nn}^{(0)} + Z_{nn}^{(2)}}$$

Impact of vector group on fault current :-

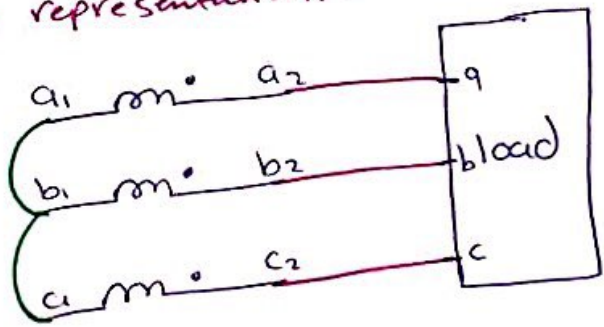
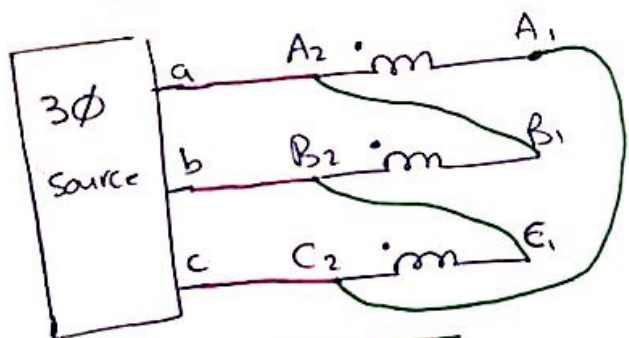
* vector group of transformers :-

Ex: \underline{HV} \underline{LV} $X \rightarrow$ clock number
 D y \downarrow angle between equivalent Y at HV side and LV side.

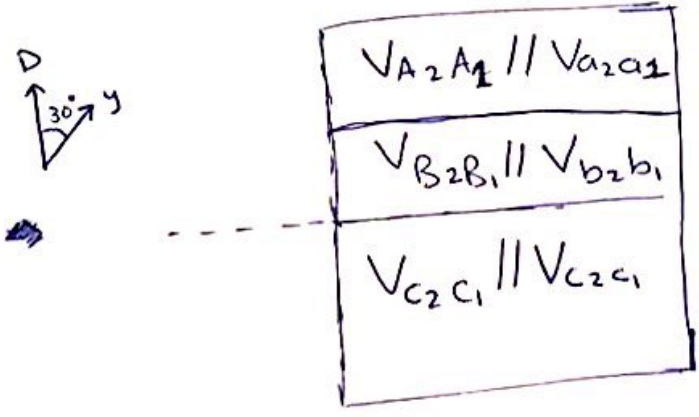
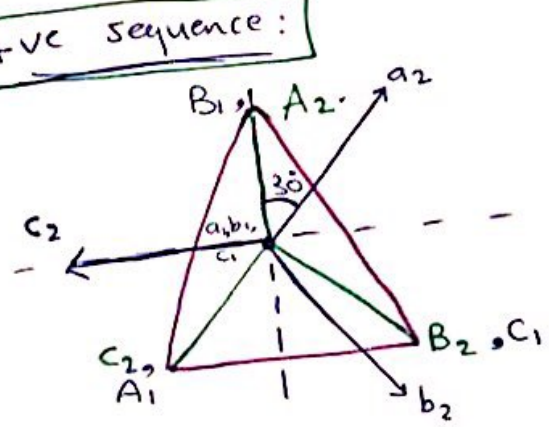


V_{AN} leads V_{an} by 30°
 I_A leads I_c by 30°
 $S = \sqrt{3} I^*$

Ex Dy1 schematic representation :-



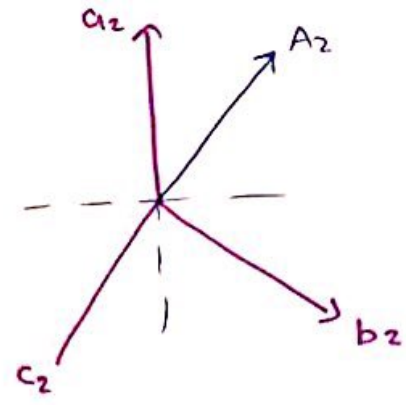
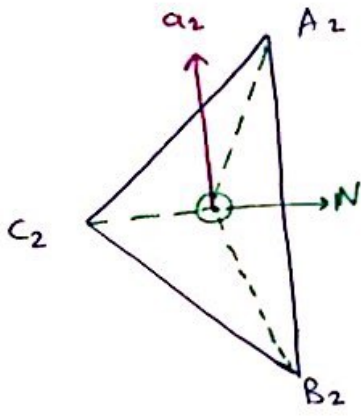
+ve sequence:



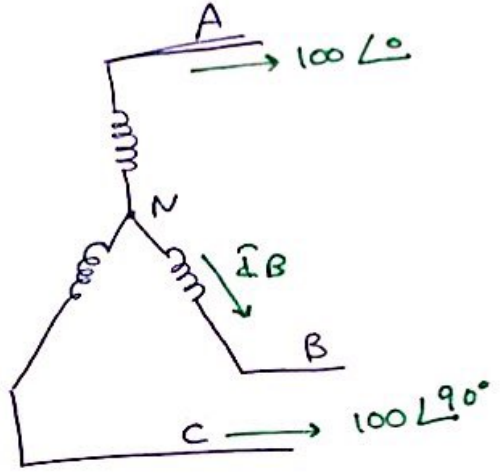
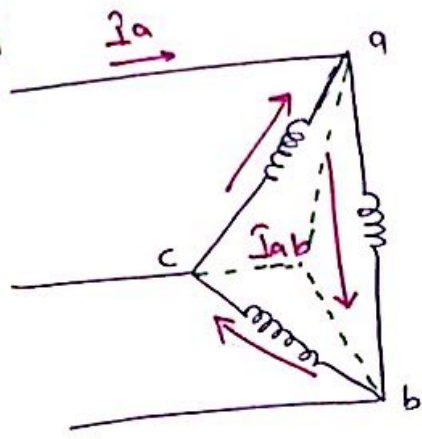
- ① HV side.
 - ② ~~clock~~ clock a2
 - ③ b2, c2
- A1, B1, C1 ?!

$$V_A^{(1)} = V_{a_2 a_1} / +30^\circ$$

$$I_A^{(1)} = I_{a_2 a_1} / +30^\circ$$



Ex



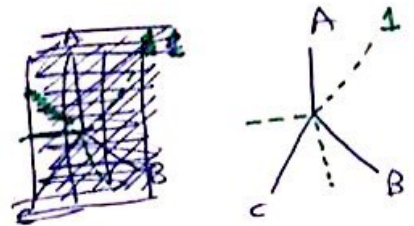
13.2/66 KV
10 MVA

Find

- ① \hat{I}_B ?!
- ② vector group ?!
- ③ \hat{I}_{ab}
- ④ \hat{I}_a .

Sol: ① $\hat{I}_A + \hat{I}_B + \hat{I}_C = 0$
 $\hat{I}_B = 141 \angle -135^\circ \text{ A}$

② vector group = Yd 1

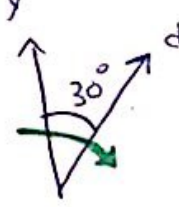


③ $\hat{I}_{ab} = \hat{I}_A * \left(\frac{66/\sqrt{3}}{13.2} \right)$

$\hat{I}_{ab} = 289 \angle 0^\circ \text{ A}$

$\hat{I}_{ca} = \hat{I}_C * \left(\frac{66/\sqrt{3}}{13.2} \right) = 289 \angle 90^\circ$

$$\hat{I}_a = 408.71 \angle -45^\circ$$



$$\hat{I}_a = \hat{I}_a^{(1)} + \hat{I}_a^{(2)}$$

د، لیا بی ی لیا بی ی ی
" 30 زاویه"

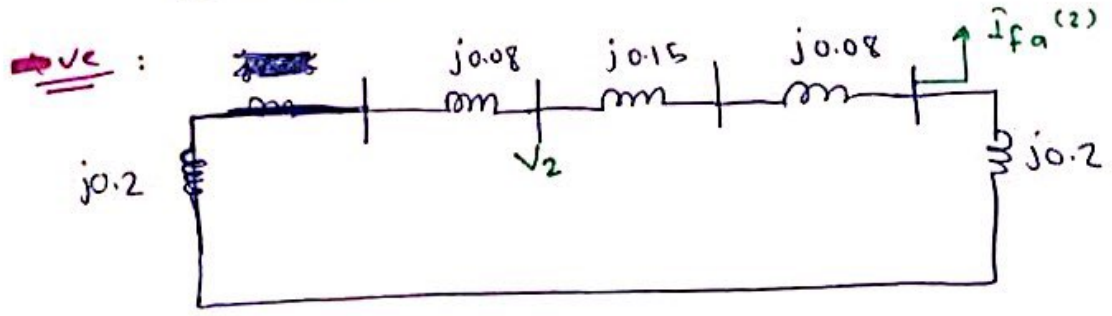
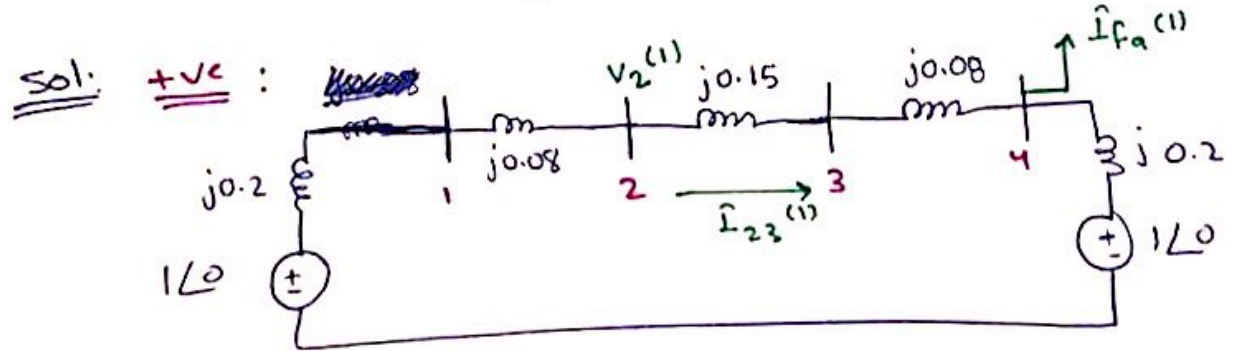
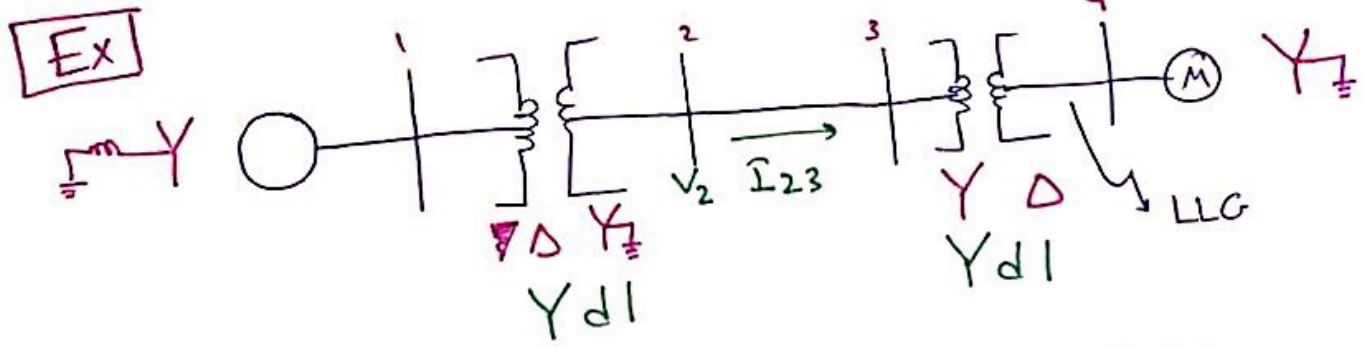
$$\hat{I}_a^{(1)}_{pu} = \hat{I}_A^{(1)}_{pu} \angle -30^\circ pu$$

و Col -ve فی

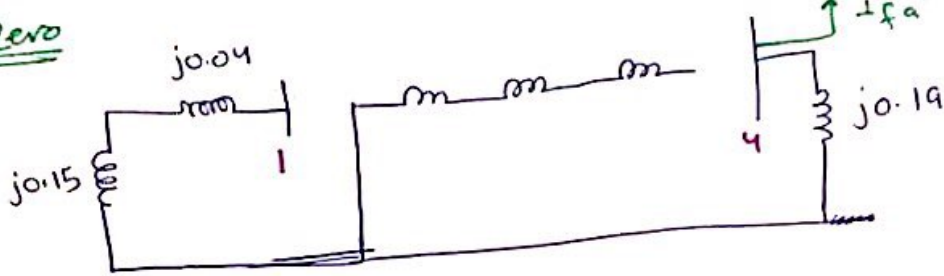
$$\hat{I}_a^{(2)}_{pu} = \hat{I}_A^{(2)}_{pu} \angle +30^\circ pu$$

$$\begin{bmatrix} \hat{I}_A^{(1)} \\ \hat{I}_A^{(2)} \\ \hat{I}_A^{(0)} \end{bmatrix} \rightarrow A \begin{bmatrix} \hat{I}_A \\ \hat{I}_B \\ \hat{I}_C \end{bmatrix}$$

$$\frac{\hat{I}_A^{(1)}}{\hat{I}_{base}} \rightarrow \frac{S_{base}}{\sqrt{3} V_L}$$



Zero



fault current :

$$Z_1 \quad Z_{44}^{(1)} = 0.1437$$

$$Z_2 \quad Z_{44}^{(2)} = 0.1437$$

$$Z_0 \quad Z_{44}^{(0)} = 0.19$$

$$\hat{I}_a^{(1)} = -j 4.437 \text{ pu.}$$

$$\hat{I}_a^{(2)} = j 2.547 \text{ pu.}$$

$$\hat{I}_a^{(0)} = j 1.9$$

$$\hat{I}_a = 0$$

$$\hat{I}_b = 6.67 \angle 154^\circ \text{ pu.}$$

$$\hat{I}_c = 6.67 \angle 24.4^\circ \text{ pu.}$$

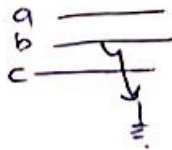
$$V_y = V_{4a}^{(1)} + V_{4a}^{(2)} + V_{4a}^{(0)}$$

$$V_{4a}^{(1)} = V_{4a}^{(2)} = V_{4a}^{(0)} = 0.30 \text{ pu.}$$

$$V_{42a}$$

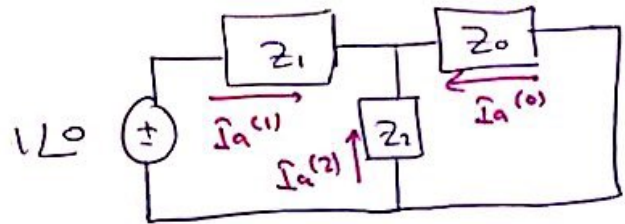
$$V_{42b} = 0$$

$$V_{42c} = 0.$$



$$\hat{I}_{23} = ?!$$

$$\begin{bmatrix} \hat{I}_{23 \rightarrow a} \\ \hat{I}_{23 \rightarrow b} \\ \hat{I}_{23 \rightarrow c} \end{bmatrix} \longrightarrow \begin{matrix} \hat{I}_{23a}^{(1)} \\ \hat{I}_{23a}^{(2)} \\ \hat{I}_{23a}^{(0)} \end{matrix}$$



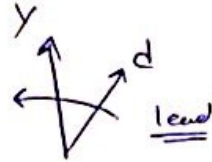
without phase shift:

$$\hat{I}_{23a}^{(1)} = -j 4.437 * \frac{0.2}{0.71} = -j 1.249$$

$$\hat{I}_{23a}^{(2)} = j 2.547 * \frac{0.2}{0.71} = +j 0.718$$

$$\hat{I}_{23a}^{(0)} = 0$$

with phase shift:



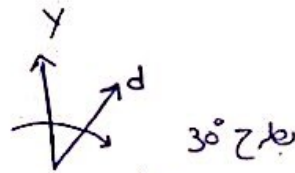
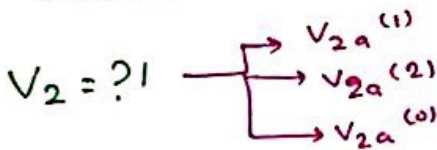
عند تيار، د وبي
تأخر، ا، ي

$$\hat{I}_{23,a}^{(1)} = -j 1.249 \angle +30^\circ$$

$$\hat{I}_{23,a}^{(2)} = j 0.7118 \angle -30^\circ$$

$$\hat{I}_{23,a}^{(0)} = 0$$

$\hat{I}_{23,a}, \hat{I}_{23,b}, \hat{I}_{23,c} \dots$



$$V_{2a}^{(1)} = \left(1 \angle 0 - \hat{I}_{23}^{(1)} \angle -30 * j 0.28 \right) * \angle 30^\circ$$

$$V_{2a}^{(1)} = 1 \angle 30 - \hat{I}_{23}^{(1)} * j 0.28$$

$$V_{2a}^{(2)} = 0 - \hat{I}_{23}^{(2)} * j 0.28$$

$$V_{2a}^{(0)} = 0$$

$$V_{21,a}$$

$$V_{21,b} \quad !?$$

$$V_{21,c}$$