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University of Jordan

College of Engineering & Technology

Department of Electrical Engineering

Fall Term – A.Y. 2015-2016

Digital Signal Processing , second exam,

sec . 1

Number of questions :5

Name: [redacted] Student Number: [redacted]

Q1. A system is described by the difference equation

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$$y[n] = x[n] - 2x[n-6]$$

a) Compute $H(e^{j\omega})$ of the system.

b) Determine the output of the system for the following input:

$$x[n] = 3 + 6\cos\left(\frac{\pi}{3}n\right) + 5\sin\left(\frac{2\pi}{5}n + \frac{\pi}{2}\right) \qquad -\infty < n < \infty$$

a) $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$

$Y(e^{j\omega}) = X(e^{j\omega}) - 2e^{-j\omega 6}X(e^{j\omega})$

$Y(e^{j\omega}) = (1 - 2e^{-j6\omega})X(e^{j\omega})$

$\therefore \boxed{H(e^{j\omega}) = 1 - 2e^{-j6\omega}}$

$e^{-j\frac{12\pi}{5}} = \cos\frac{12\pi}{5} - j\sin(\frac{\pi}{5})$

$= 1\angle -\frac{2}{5}\pi$

$= 1\angle -72$

$e^{-j2\pi} = \cos 2\pi - j\sin 2\pi$

$= +1$

b) $H(0) = 1 - 2 = -1$

$H(\frac{\pi}{3}) = 1 - 2e^{-j\frac{6\pi}{3}} = 1 - 2e^{-j2\pi} = 1 - 2(+1)$ ~~2.236∠63.4~~ $= -1$

$H(\frac{2\pi}{5}) = 1 - 2e^{-j\frac{6(2\pi)}{5}} = 1 - 2e^{-j\frac{12\pi}{5}} = 1 - 2(1\angle -72) = 1.94\angle 78.65$

$$\boxed{y[n] = -3 - 6\cos(\frac{\pi}{3}n) + 9.7\sin(\frac{2\pi}{5}n + \frac{\pi}{2} + 78.65°)}$$

168.65

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Q2. For the sequence $x[n] = \{1 \quad 2\}$,

a) Find the D_2 matrix.
b) Find X[k], k= 0, 1. (The DFT of x[n]).
c) Find the D_2^{-1} matrix.
d) Find the IDFT of X(k).

a) $D_2 = \begin{bmatrix} 1 & 1 \\ 1 & W_2 \end{bmatrix}$ $\quad W_2 = e^{-j\frac{2\pi}{N}} = e^{-j\pi} = \cos\pi - j\sin\pi = -1$

$\cancel{= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}$

b) $X[k] = D_2\, x[n]$

$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1+2 \\ 1-2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ $\quad \therefore X[k] = \cancel{\{3} \quad -1\}.$

c) $D_2^{-1} = \frac{1}{N} D^* = \cancel{\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}$

d) $x[n] = D_2^{-1} X[k]$

$= \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \cancel{\begin{bmatrix} \frac{3}{2} - \frac{1}{2} \\ \frac{3}{2} + \frac{1}{2} \end{bmatrix}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad \therefore x[n] = \{1 \quad 2\}$ #.

$$X[K] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N} nK}$$

$$x[n] = \frac{1}{N} \sum_{n=0}^{N-1} X[K]\, e^{j\frac{2\pi}{N} nK}$$

$N=4$

Q3. Let X[k], $0 \le k \le 3$, be a 4-point DFT of a 4-length real sequence x[n], given by:

$$X[k] = \{ \overset{0}{6} \quad \overset{1}{-1-j} \quad \overset{2}{0} \quad \overset{3}{-1+j} \}$$

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Evaluate the following functions of x[n] without computing the IDFT of X[k]:

a) x[0] b) x[2] c) $\sum_{n=0}^{3} x[n]$ d) $\sum_{n=0}^{3} e^{j\pi n} x[n]$ e) $\sum_{n=0}^{3} |x[n]|^2$

a) $x[0] = \frac{1}{N}\sum_{K=0}^{3} X[K]\, W_n^{0K}$

$= \frac{1}{4}\sum_{K=0}^{3} X[K] = \frac{1}{4}\{6 - 1 - \cancel{j} + 0 - 1 + \cancel{j}\} = \boxed{\frac{4}{4}} = 1$

$W_n = e^{-j\frac{2\pi}{N}}$

$W_N^{-2K} = e^{-j\frac{2\pi}{4}\cdot -2K}$

$= e^{j\pi K}$

$= \cos(K\pi) + j\sin K\pi$

$= (-1)^K$

b) $x[2] = \frac{1}{N}\sum_{K=0}^{3} X[K]\, W_n^{-2K}$

$= \frac{1}{4}\sum_{K=0}^{3} X[K](-1)^K$

$= \frac{1}{4}\{6 + -1\times(-1-j) + 0 + -1(-1+j)\}$

$= \frac{1}{4}\{6 + 1 + \cancel{j} + 1 - \cancel{j}\} = \boxed{\frac{8}{4}} = 2$

$e^{j2\frac{\pi}{4}(2)K} = e^{j\pi K}$ ✓

c) $X[0] = \sum_{n=0}^{3} x[n]\, W_n^{0K}$

$\therefore \sum_{n=0}^{3} x[n] = X[0] = \boxed{6}$

$-\frac{\pi}{2}K = \pi$

$\therefore K = -2$

d) $X[K] = \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi}{4}nK}$

$X[K] = \sum_{n=0}^{3} x[n]\, e^{-j\frac{\pi}{2}nK}$

not defined

$0 < K < 4$

$\therefore \sum_{n=0}^{3} e^{j\pi n} x[n] = X[-2] = 0!$

$-1-j = \sqrt{2}\angle -135$

$-1+j = \sqrt{2}\angle 135$

e) $\sum_{n=0}^{3} |x[n]|^2 = \frac{1}{4}\sum_{K=0}^{3} |X[K]|^2$

$= \frac{1}{4}\left(6^2 + (\sqrt{2})^2 + 0 + (\sqrt{2})^2\right)$

$= \boxed{10}$

Q5. The transfer function of a causal LTI discrete-time system is given by

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$$H(z) = \frac{0.5 - 0.3z^{-1}}{1 + 0.3z^{-1}}$$

a) Determine the impulse response h[n] of the system.

b) Determine the output of the system y[n] if the input $x[n] = (0.2)^n\mu[n]$.

a)

$$\begin{array}{r|l} & -1 \\ 1+0.3z^{-1} & 0.5 - 0.3z^{-1} \\ & -1 - 0.3z^{-1} \\ \hline & 1.5 \quad 0 \end{array}$$

$$\Rightarrow \cancel{ZT}\ H(z) = -1 + \frac{1.5}{1+0.3z^{-1}}$$

$$\boxed{h[n] = -\delta[n] + 1.5(-0.3)^n \mu[n]}$$

b) $$x[z] = \frac{1}{1-0.2z^{-1}}$$

$$y[z] = H[z] \cdot X(z)$$

$$= \left(-1 + \frac{1.5}{1+0.3z^{-1}}\right)\frac{1}{1-0.2z^{-1}}$$

$$= \frac{-1}{1-0.2z^{-1}} + \frac{1.5}{(1+0.3z^{-1})(1-0.2z^{-1})} = \frac{-1}{1-0.2z^{-1}} + \frac{A_1}{(1+0.3z^{-1})} + \frac{A_2}{(1-0.2z^{-1})}$$

$$A_1 \Big|_{z=-\frac{10}{3}} = \frac{1.5}{1-0.2\left(-\frac{10}{3}\right)} = \frac{9}{10} = 0.9$$

$$A_2 \Big|_{z=5} = \frac{1.5}{1+0.3(5)} = \frac{3}{5} = 0.6$$

$$\therefore\ Y(z) = \frac{-1}{1-0.2z^{-1}} + \frac{0.9}{1+0.3z^{-1}} + \frac{0.6}{1-0.2z^{-1}}$$

$$y[n] = -(0.2)^n \mu[n] + 0.9(-0.3)^n \mu[n] + 0.6(0.2)^n \mu[n] \ \#.$$

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$$\Rightarrow H(z) = -1 + \frac{1.5}{1+0.3z^{-1}}$$

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b) $X(z) = \dfrac{1}{1-0.2z^{-1}}$

$$Y[z] = H[z] \cdot X(z)$$

$$= \left(-1 + \frac{1.5}{1+0.3z^{-1}}\right)\frac{1}{1-0.2z^{-1}}$$

$$= \frac{-1}{1-0.2z^{-1}} + \frac{1.5}{(1+0.3z^{-1})(1-0.2z^{-1})} = \frac{-1}{1-0.2z^{-1}} + \frac{A_1}{(1+0.3z^{-1})} + \frac{A_2}{(1-0.2z^{-1})}$$

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