22.5

Jordan University
Mathematics Department
Mathematics for Engineering II, First Exam, 30/10/2014

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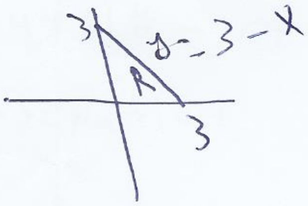
1) (7 points) Evaluate $\iint_S \vec{F}\cdot\hat{n}\,dS$ where $\vec{F} = z\,k$ and S is the part of the plane $x + y + z = 3$ in the first octant.



~~$x = u$~~
~~$y = v$~~
~~$z = 3 - u - v$~~
~~$\vec{r}(t) = \langle u, v$~~
~~$\vec{r}(x,y) = z$~~

$x = x$
$y = y$
$z = 3 - x - y$

$\vec{r}(x,y) = \langle x, y, 3 - x - y\rangle$

$\vec{N} = \langle 1, 1, 1\rangle$

7

$$\iint_S \vec{F}\cdot\hat{n}\,dS = \iint_R f(r(x))\cdot\vec{N}$$

$$= \iint_R \langle 0, 0, 3 - x - y\rangle\cdot\langle 1,1,1\rangle\,dy\,dx$$

$$= \int_0^3\int_0^{3-x} 3 - x - y\;dy\,dx$$

$$= \int_0^3 3y - xy - \frac{y^2}{2}\Big|_0^{3-x}dx$$

$$= \int_0^3 3(3-x) - x(3-x) - \frac{(3-x)^2}{2}\,dx$$

continuos.

$\Rightarrow$

~~$\int_0^3 9 - 3x - 3x + x^2$~~

$$= \int_0^3 9 - 3x - 3x + x^2 - \frac{9}{2} - \frac{x^2}{2} + 3x\,dx$$

$$= \int_0^3 -3x + \frac{x^2}{2} + \frac{9}{2}\,dx$$

$$= -\frac{3x^2}{2} + \frac{x^3}{6} + \frac{9}{2}x\Big|_0^3$$

$$= -\frac{27}{2} + \frac{27}{6} + \frac{27}{2} = \boxed{\frac{27}{6}}$$

2) Consider the vector field $\vec{F} = 2xy^2\, i + (2x^2y + e^z)\, j + ye^z\, k$

a) (2 points) Show that $\vec{F}$ is conservative.

$$\text{curl}\,\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy^2 & 2x^2y+e^z & ye^z \end{vmatrix}$$

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$$= \left(\frac{\partial ye^z}{\partial y} - \frac{\partial(2x^2y+e^z)}{\partial z}\right) i - \left(\frac{\partial ye^z}{\partial x} - \frac{\partial 2xy^2}{\partial z}\right) j + \left(\frac{\partial (2x^2y+e^z)}{\partial x} - \frac{\partial 2xy^2}{\partial y}\right) k$$

$$= (e^z - e^z)\, i - (0-0)\, j + (4xy - 4xy)\, \hat{k} = \vec{0}$$

$\therefore$ consevative

b) (3 points) Find the potential of $\vec{F}$.

~~potential = div$(\vec{F}) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y}$~~ $+ \frac{\partial F_3}{\partial z}$

$\vec{F} = \nabla f$

$\int 2xy^2\, dx = x^2y^2 + g(y,z)$ ~~$= 2y^2 + 2x^2$~~ $+ ye^z$

$2x^2y + g'(y,z) = 2x^2y + e^z$

$g(y,z) = e^z y + f(z)$

$f(x,y,z) = x^2y^2 + e^z y + f(z)$

$e^z y + f'(z) = ye^z \Rightarrow f(z) = K$

$\therefore$ its potential $x^2y^2 + e^z y + K$

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c) (3 points) Find $\int_C \vec{F} \cdot dr$, where C is the part of the helix $\vec{r}(t) = \cos(t)\, i + t\, j + \sin(t)\, k, \quad 0 \le t \le 2\pi$.

$$\int_C \vec{F} \cdot dr = \int_0^{2\pi} f(r(t)) \cdot r'(t)$$

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$$\vec{r}'(t) = \langle -\sin t, 1, \cos t \rangle$$

$$\int_C \vec{F} \cdot dr = \int_0^{2\pi} 2\cos \ldots \cancel{\cdot \langle -\sin t, 1, \cos t}$$

$$= \int_0^{2\pi} \langle 2t^2\cos t,\ 2t\cos^2 t + e^{\sin t},\ t e^{\sin t} \rangle \cdot \langle -\sin t, 1, \cos t \rangle$$

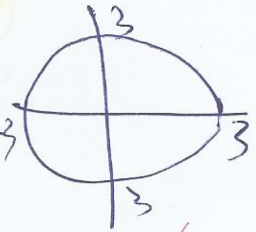
$$= \int_0^{2\pi} -2t^2 \cos t \sin t + 2t\cos^2 t + e^{\sin t} + t\cos t\, e^{\sin t}\, dt$$

$$= \int_0^{2\pi} -t^2 \sin 2t + \cancel{t + t\cos 2t} + e^{\sin t} + t\cos t\, e^{\sin t}\, dt$$

~~$=$ Area $= \frac{1}{2}$~~

3) (5 points) Find the work done by the force $\vec{F} = < \ln(x^3+7) - y,\ \frac{1}{1+y^6} - 5x - 1 >$ moving along the circle $x^2 + y^2 = 9$ where motion is counter clockwise.

$\cancel{W = \int_C \vec{F} \cdot dr}$



$\cancel{r = < 3\cos t, 3\sin t >}$

$W = \oint_C \vec{F} \cdot dr = \iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dx\,dy$

$= \iint_R -5 + 1\, dxy = \iint_R -4\, r\,dr\,d\theta$

$= \int_0^{2\pi}\int_0^3 -4r\,dr\,d\theta$

$= \int_0^{2\pi} -\frac{4r^2}{2}\Big|_0^3\, d\theta$

$= \int_0^{\pi} -18\, d\theta = -\cancel{3}6\pi$

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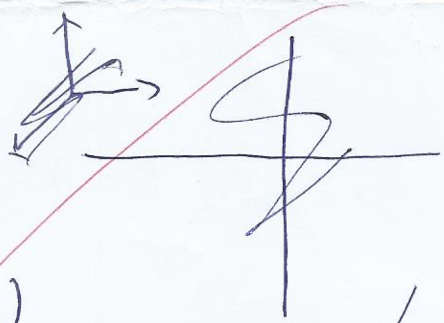
4) (5 points) Find $\int_C \vec{F} \cdot dr$ where $\vec{F} = xz\,i + y\,j + yz\,k$ and C is the curve $z = 1 - x^2$, $-1 \le x \le 3$ with $y = 3$.



$\cancel{(3,3,-8)}$

$\cancel{\int \vec{F} \cdot dr}$

$\cancel{(-1,3,0)}$

$\cancel{\vec{F} = \int}$

$\vec{F}$ is from $a = (-1, 3, 0)$ to $b = (3, 3, -8)$

$\int_{(-1,3,0)}^{(3,3,-8)} \vec{F} \cdot dr = \int \frac{yz^2}{2} + \frac{x^2 z}{2} + K \Big|_{(-1,3,0)}^{(3,3,-8)}$

$= (96 - 36) - (0)$

$= \boxed{60}$

$\int x z\, dx = \frac{x^2}{2} z + g(y, z)$

$g'(y, z) = y$

$g(y, z) = \frac{y^2}{2} + f(z)$

$f(x, y, z) = \frac{x^2}{2} z + \frac{y^2}{2} + f(z)$

$\frac{x^2}{2} + f'(z) = yz$

$f(z) = \frac{yz^2}{2} + \frac{x^2 z}{2} + K$

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