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Jordan University
Mathematics Department
Mathematics for Engineering (II) . Second Exam. 22/04/2013

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Lecture time: 10:00-11:00

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1. Use the Divergence Theorem to evaluate $\iint_S \vec{F}\cdot\vec{n}\,dA$, where

$\vec{F}(x,y,z) = (xz\sin(yz)+x^3)i + (\cos(yz))j + (3zy^2)k$ and

S is the surface of the cone $z=\sqrt{x^2+y^2}$, $0 \le z \le 1$. [8 points]

$\iiint \operatorname{div} F\, dv = \iint F\cdot n\, dA$

$\operatorname{div} F = z\sin yz + 3x^2 + xz\cos(yz) - \frac{\sin yz}{z} + 6zy$

$\frac{xz\cos yz}{y} + x\sin yz - \frac{\sin yz}{y} + 3y^2$

$= \sin yz\left(z - \frac{1}{z} + x - \frac{1}{y}\right) + 3x^2 + \cos yz\left(\frac{xz}{y} + x\right) + 6zy + 3y^2$

$= \iint F\cdot n\, dA = \int_0^1\int_0^{2\pi}\int_0^1 \cdots\; r\,dr\,d\theta\,dz$

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

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$1 \ge r \ge 0$
$2\pi \ge \theta \ge 0$
$1 \ge z \ge 0$

$\int_0^{2\pi}\int_0^1 \sin(r\sin\theta)z\left(z - \frac{1}{z} + r\cos\theta\right.$

2. Use Stokes's theorem to evaluate $\iint_S (curl\vec{F})\cdot\vec{n}dA$, where

S is the surface of the hemisphere $x^2+y^2+z^2=9$ with $z\geq 0$, oriented outward;

$\vec{F}(x,y,z)=(2y\cos z)i+(e^x\sin z)j+(xe^y)k$ [7 points]

$$\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 2y\cos z & e^x\sin z & xe^y \end{vmatrix} = (xe^y - e^x\cos z)\hat{i} - (e^y - 2y\sin z)\hat{j} + (e^x\sin z + 2\cos z)\hat{k}$$

$$\bar{N} = grad\,(x^2+y^2+z^2-9) = 2xi + 2yj + 2zk$$

$$\text{curl } F \cdot N = [xe^y - e^x\cos z,\ -e^y + 2y\sin z,\ e^x\sin z + 2\cos z] \cdot [2x, 2y, 2z]$$

$$= 2x^2e^y - 2xe^x\cos z + -2ye^y + 4y^2\sin z + 2ze^x\sin z + 4z\cos z$$

$$\iint_S curl\ f\ ndA = \oint F.ndA$$

$C: x^2+y^2=9$

$N = k$

$F.N = xe^y$

$$\int_0^3\int_0^3 xe^y\, dA$$

$$= \int_0^3\int_0^3 xe^y\, dx\, dy$$

$$= \frac{9}{2}\int_0^3 e^y\, dy$$

$$\frac{9}{2}e^y\Big|_0^3 = \boxed{\frac{9}{3}e^3 - \frac{9}{2}}$$

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3. If $f(x)=\begin{cases} x+2, & -2\le x<0, \\ 2, & 0\le x<2, \end{cases}$, $f(x+4)=f(x)$ then

a) Find the Fourier series for $f(x)$,

b) Find $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. [8 points]

(a) $a_0 = \frac{1}{4}\int_{-2}^{0} x+2\, dx + \frac{1}{4}\int_{0}^{2} 2\, dx$

$= \frac{2}{4} + 1 = 1.5$

$a_n = \frac{1}{4}\int_{-2}^{0}(x+2)\cos\frac{\pi}{2}nx\, dx + \frac{1}{4}\int_{0}^{2} 2\cos\frac{\pi}{2}nx\, dx$

$= \frac{1}{4}\int_{-2}^{0} x\cos\frac{\pi}{2}nx\, dx + \frac{1}{4}\int_{-2}^{0} 2\cos\frac{\pi}{2}nx\, dx + \frac{1}{4}\int_{0}^{2} 2\cos\frac{\pi}{2}nx\, dx$

$= \frac{1}{4}\int_{-2}^{0} x\cos\frac{\pi}{2}nx\, dx + \frac{4}{4}\int_{0}^{2}\cos\frac{\pi}{2}nx\, dx$

$= \frac{x}{\frac{\pi}{2}n}\sin\frac{\pi}{2}nx + \frac{1}{\frac{\pi^2}{4}n^2}\cos\frac{\pi}{2}nx$ $\quad + \left.\frac{\sin\frac{\pi}{2}nx}{\frac{\pi}{2}n}\right|_0^2$

x $\cos\frac{\pi}{2}nx$

1 $\frac{1}{\frac{\pi}{2}n}\sin\frac{\pi}{2}nx$

0 $+\frac{1}{(\frac{\pi}{2}n)^2}\cos\frac{\pi}{2}nx$

$\frac{x}{\frac{\pi}{2}n}\sin\frac{\pi}{2}nx + \frac{1}{\frac{\pi^2}{4}n^2}\cos\frac{\pi}{2}nx$

4.5

$a_n = \left.\cancel{\frac{x}{\frac{\pi}{2}n}\sin\frac{\pi}{2}nx + \frac{\sin\frac{\pi}{2}nx}{\frac{\pi}{2}n}}\right|$

$a_n = \left.\frac{1}{\frac{\pi}{4}n^2}\cos\frac{\pi}{2}nx\right|_{-2}^{0}$

$= \frac{1}{\frac{\pi}{4}n^2}\left(1 - \cos\pi nx\right)$

$\boxed{a_n = \frac{(1-(-1)^n)}{\frac{\pi^2}{4}n^2}}$

$\boxed{f(x) = 1.5 + \sum_{n=-\infty}^{\infty}\left(\frac{1-(-1)^n}{\frac{\pi^2}{4}n^2}\right)\cos\frac{n\pi x}{2}}$

b_n

4. Find the Fourier transform of $f(x) = e^{-c|x|}$, $-\infty < x < \infty,\ c > 0$

[7 points]

$e^{-c\alpha}$

$e^{-c|\alpha|}$, $-\infty < \alpha < \infty$

$= e^{-c\alpha}$, $0 < \alpha < \infty$

$+ \int_{-\infty}^{0} \alpha$

$\frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-c\alpha} e^{-j\omega\alpha} d\alpha$

$= \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\alpha(c+j\omega)} d\alpha$

$= \frac{1}{\sqrt{2\pi}} \left. \frac{e^{-\alpha(c+j\omega)}}{-(c+j\omega)} \right|_0^\infty$

3.5

$= \frac{1}{\sqrt{2\pi}} \left(0 + \frac{1}{c+j\omega} \right)$

$\hat{f}(w) = \frac{1}{(c+j\omega)\sqrt{2\pi}}$ ✓

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