University of Jordan
Elect. Eng. Dept.
Second Exam

Numerical Methods 903301 | Monday 28/11/2016
Student Name: ? | Reg. No. 0140754 | Mark out of 30
Answers should be written in ink | **Exam Duration: 70 min**

13

Q1) From numerical methods point of view , in not more than one line , briefly answer

i) The advantage of **direct methods** for solving systems of equations over **iterative methods.**

*less errors

~~Accurate~~ can solve multiple equations by using matrices and Gauss elimination

ii) The advantage of the **fixed point** method over the **modified secant** method.

Simple, has convergence *

one initial pt | 2 initial pts

iii) The advantage of the **secant method** over the **Newton Raphson method.**

N-R method can't solve not differantiable equations ~~(linear)~~

iv) A **disadvantage** of the Newton Raphson method when solving for zeros of polynomials of **repeated (identical) zeros** .

It is slow

4

v) The number of mathematical operations involved when using naïve Gauss elimination to calculate the **inverse of an nxn matrix**.

$\frac{2}{3}n^3 + O(n^2)$

order of the equation

vi) An advantage of the LU factorization method .

to solve a group of linear equation that have A fixed and b can change ~~Ax=b~~ Ax=b

vii) Write down the error formula involved with the **Newton-Raphson method** relating E_{i+1} to E_i .

$$E_{t,i+1} = \frac{-f''(x_r)}{2f'(x_r)} E_{t,i}^2$$

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Q2 When determining the reciprocal of a number **N** i.e. **x = 1/N** using the **fixed point method** , a possible choice is **g(x) = (1+N) x - 1.** Prove by **demonstration** and **analytically** that this is a **bad choice**. Use an initial estimate of **8** to calculate **1/0.2**. Perform **5** iterations.

$x_i = 8$ $g(x) = (1+N)x - 1$ $x = \frac{1}{N}$ ~~N=0.2~~

$g(x) = x$

$N = \frac{1}{x}$

$g'(x) < 1$ bad ~~good choice~~

$g'(x) = 1$

1

Stick to the use of the **fixed point method** , look for a better choice for **g(x)** which leads to convergence and **confirm** such convergence by iterating your choice to the calculation of **1/0.2** using an initial estimate of **8.**

0

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Q3) a) Formulate the solution of $x^2 = \sin(x)$ as a zero finding problem, hence use the **secant method** to find the solution starting with ***initial estimates 0.8* and 1**. Write down the first ***5*** iterations to ***10*** significant digits.

$x_{i-1} = .8, 1 \quad x_i = 1$

$$x_{i+1} = x_i - \frac{f(x_i)\,[x_i - x_{i-1}]}{f(x_i) - f(x_{i-1})}$$

~~$x_1 = 0.8995$~~

~~$x_1 = 0.8945738773$~~

$x_1 = 0.5117968546$

$x_2 = 0.2580049300$

$x_3 = 0.0183223939$

$x_4 = 2.218081078 \times 10^{-4}$

$x_5 = 1.256196447 \times 10^{-8}$

$f(x) = \sin^{-1} x^2$

2

b) i) Use Taylor's series to **derive** the Newton Raphson method .

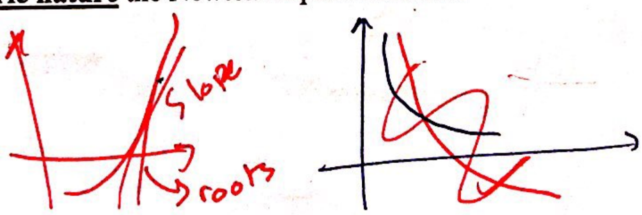
$f(x) = 0$

$f(x_{i+1}) = f(x_i) + (x_{i+1} - x_i) f'(x_i) + \cdots$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad ; \quad f(x_i) =$$

0

ii) Sketch the **geometric nature** the Newton Raphson method.



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Q4) Given the system of equations $A x = b$; *i.e.* $\begin{bmatrix} 0 & -1 & 4 \\ 8 & -2 & -1 \\ -1 & 5 & -2 \end{bmatrix} x = \begin{bmatrix} 10 \\ 1 \\ 3 \end{bmatrix}$

Do not use the LU factorization method.

Instead, Use any method based on Gauss elimination to determine

i) **the solution x.** → G.E (Partial

ii) **the determinant of A**

iii) **and the inverse of A.**

6

I) $\begin{bmatrix} 8 & -2 & -1 \\ -1 & 5 & -2 \\ 0 & -1 & 4 \end{bmatrix} x = \begin{bmatrix} 10 \\ 1 \\ 3 \end{bmatrix}$

$R_2 + \frac{1}{8} R_1$

$R_3 + \frac{1}{4.75} R_2$

$\cancel{\begin{bmatrix} 8 & -2 & -1 \\ 0 & 4.75 & 1.875 \\ 0 & 0 & 4.39 \end{bmatrix} x = \begin{bmatrix} 10 \\ 2.25 \\ 3.23 \end{bmatrix}}$

$\cancel{4.39 x_3 = 3.23 \Rightarrow x_3 = 0.737}$

$\cancel{x_2 = 2.12 67}$

$\begin{bmatrix} 8 & -2 & -1 \\ 0 & 4.75 & -2.125 \\ 0 & 0 & 3.55 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_1 \end{bmatrix} = \begin{bmatrix} 10 \\ 2.25 \\ 3.47 \end{bmatrix} \Rightarrow 3.55\, x_1 = 3.47$

$x_1 = 0.977$

$x_2 = 0.91$

$x_3 = 1.599$

$\cancel{X = \begin{bmatrix} 1.38 \\ 0.16 \\ 0.74 \end{bmatrix}}$ $\cancel{X = \begin{bmatrix} 1.599 \\ 0.910 \\ 0.977 \end{bmatrix}}$ $\cancel{X = \begin{bmatrix} 0.977 \\ 0.91 \\ 1.599 \end{bmatrix}}$

$\cancel{X = \begin{bmatrix} 0.977 \\ 1.599 \\ 0.91 \end{bmatrix}}$

II) $|A| = 8 * 4.75 * 3.55 * -1 * -1 = 134.9$ (Pivots)

III) A^{-1}

$$\left[\begin{array}{ccc|ccc} 8 & 0 & 0 & \frac{1.89}{3.55} & 1.06 & 0.53 \\ 0 & 4.75 & 0 & \frac{2.125}{3.55} & 0.14 & 1.126 \\ 0 & 0 & 3.55 & 1 & \frac{1}{38} & \frac{1}{4.75} \end{array}\right]$$

?

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Numerical Methods 903301 | Second Exam | Monday 28/11/2016

Student Name: عمر عبدالمحسن عبد ٢ | Reg. No. | Mark out of 30

Answers should be written in ink 0144364 | Exam Duration: 70 min

18

Q1) From numerical methods point of view , in not more than one line , briefly answer

i) The advantage of **direct methods** for solving systems of equations over **iterative methods.**

~~most of the times they are faster~~

less error , No errors building up

ii) The advantage of the **fixed point** method over the **modified secant** method.

one initial point and modified secant uses two

iii) The advantage of the **secant method** over the **Newton Raphson method.**

you don't have to evaluate the derivation

iv) A **disadvantage** of the Newton Raphson method when solving for zeros of polynomials of **repeated (identical) zeros** .

f'(x) will equal zero and then we can not evaluate

it will be slow

v) The number of mathematical operations involved when using naïve Gauss elimination to calculate the <u>**inverse of an nxn matrix**</u>.

$\frac{2}{3}n^3 + n^2$

vi) An advantage of the LU factorization method .

faster and easier and better usage on the matlab

2

vii) Write down the error formula involved with the **Newton-Raphson method** relating E_{i+1} to E_i .

$$|E_{i+1}| = \frac{-f''(x_r)}{2f'(x_r)}|E_i|^2$$

$$E_{t,i+1} = \frac{-f''(x_r)}{2f'(x_r)} E_{t,i}$$

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Q2 When determining the reciprocal of a number N i.e. x = 1/N using the **fixed point method** , a possible choice is **g(x) = (1+N) x - 1**. Prove by **demonstration** and **analytically** that this is a **bad choice**. Use an initial estimate of **8** to calculate **1/0.2**. Perform **5** iterations.

$g'(x) = (1+N)$

$= 1 + \frac{1}{8} > 1$

which means it will diverge

$X = (1+0.2)8 - 1$

1st = 8.6

2nd = 9.32

3rd = 10.184

4th = 11.2208

5th = 12.46496 diverges

3

Stick to the use of the **fixed point method** , look for a better choice for **g(x)** which leads to convergence and **confirm** such convergence by iterating your choice to the calculation of **1/0.2** using an initial estimate of **8**.

$x = (1+N)x - 1$

$\boxed{\frac{x+1}{1+N} = x}$ now

1st = 7.5

2nd = 7.083333333

3rd = 6.736111111

4th = 6.446759259

5th = 6.205632716

it will converge to 5

4

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$f(x) = x^2 - \sin(x)$

Q3) a) Formulate the solution of $x^2 = \sin(x)$ as a zero finding problem, hence use the **secant method** to find the solution starting with ***initial estimates 0.8*** **and 1**. Write down the first *5* iterations to *10* significant digits.

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

$x = \sqrt{\sin(x)}$

$x = \sin^{-1}(x^2)$

$$= 0.8 - \frac{f(0.8)(0.8-1)}{f(0.8) - f(1)}$$

$f(0.8) = -0.0773560909$

$f(1) = 0.1585290152$

$f(0.965\ldots) = 0.1105165092$

$$x_{i+1} = 1 - \frac{f(1)(1-0.8)}{f(1) - f(0.8)}$$

Which one?!

$f(0.916\cdots) = 0.04602596[illegible]$

$x_{i+1} = 0.96598418$

$f(0.88\ldots) = 4.251995417 \times 10^{-3}$

$x_{i+2} = 0.9161160478$

$f(0.876\ldots) =$

$x_{i+3} = 0.8805258866$

$x_{i+4} = 0.8769033148$

2

b) i) Use Taylor's series to **derive** the Newton Raphson method .

$$f(x_{i+1}) = f(x_i) + (x_{i+1} - x_i) f'(x_i) + \frac{(x_{i+1}-x_i)^2}{2!} f''(x_i) + \cdots$$

$= 0$

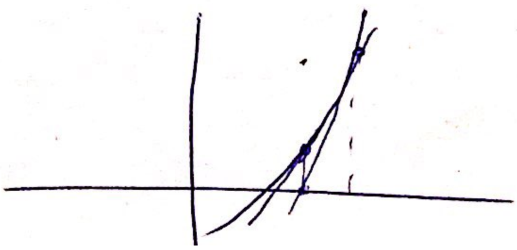
using first two terms

$$\frac{\cancel{f(x_{i+1})} - f(x_i)}{f'(x_i)} = x_{i+1} - x_i$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

0

ii) Sketch the **geometric nature** the Newton Raphson method.



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Q4) Given the system of equations $A\,x = b$; *i.e.* $\begin{bmatrix} 0 & -1 & 4 \\ 8 & -2 & -1 \\ -1 & 5 & -2 \end{bmatrix} x = \begin{bmatrix} 10 \\ 1 \\ 3 \end{bmatrix}$

Do not use the LU factorization method.

Instead, Use any method based on Gauss elimination to determine

i) **the solution x.**

ii) **the determinant of A**

iii) **and the inverse of A.**

i) $\left[\begin{array}{ccc|c} -1 & 5 & -2 & 3 \\ 8 & -2 & -1 & 1 \\ 0 & -1 & 4 & 10 \end{array}\right]$

$R_2 = R_2 - (-8R_1)$

$\left[\begin{array}{ccc|c} -1 & 5 & -2 & 3 \\ 0 & 38 & -17 & 25 \\ 0 & -1 & 4 & 10 \end{array}\right]$

$R_3 = R_3 - \left(\frac{-1}{38} R_2\right)$

$\left[\begin{array}{ccc|c} -1 & 5 & -2 & 3 \\ 0 & 38 & -17 & 25 \\ 0 & 0 & \frac{135}{38} & \frac{405}{38} \end{array}\right]$

$x_3 = 3$

$38x_2 + 3(-17) = 25$

$x_2 = 2$

$-x_1 + 5(2) - 2(3) = 3$

$x_1 = 1$

$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

ii) determinant of A

$= -1 \times \cancel{38} \times \frac{135}{\cancel{38}} = -135$

iii)

$A^{-1} = \frac{1}{-135} \begin{bmatrix} \frac{9}{135} & \frac{18}{135} & \frac{9}{135} \\ \frac{32}{135} & \frac{4}{135} & \frac{17}{135} \\ \frac{8}{135} & \frac{-1}{135} & \frac{38}{135} \end{bmatrix}$

6

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