University of Jordan
Elect. Eng. Dept.

Numerical Methods 903301 First Exam Sun. 24/10/2016
Student Name: عمر عبدالرحمن أحمد عبدربه Reg. No. 0144364 Mark out of 20 **18**
Answers should be written in ink **Exam Duration: 50 min**

Q1 a) A **decimal** computer uses **7** digits for number representation. For floating point number representation, it allocates **4** digits for the significand , **1** digits for the exponent ,**1** digit for the number sign, and **1** digit for the exponent sign . Furthermore, this computer uses **number normalization** and does number **chopping.** Use this computer to answer the following questions .

4 digits

What **floating point number** this computer gives for **2/700** 2.857×10^{-3} 14 ...... 2.857×10^{-3} ✓

What **floating point number** this computer gives for **(2/700)** ^{2} ...... 8.162×10^{-6} ✓

What result this computer gives for $(10^{-7}x\ 10^{-8})\ /\ 10^9$ 10^{15} ...... ~~over~~ under flow ✗

10^{-15} → 2 digits exponent answer will not be show

If **2222** and **4444** were **declared as natural numbers**, their **product** will be ...9874568 ✓

3-

b) Using a **base 3 computer** , with **four** digits allocated for the significand and no restriction on the remaining allocations . What normalized floating point answer will this computer give for **(0.3)**$_{10}$. What is the **percent absolute error** involved.

$(0.3)_{10} \overset{?}{=} (\ )_3$

$0.3\times3 = 0.9$, $0.9\times3 = 2.7$, $0.7\times3 = 2.1$, $0.1\times3 = 0.3$

$(0.0220022002 2\ldots)_3$

$= (2.200)_3 \times 3^{-2}$

$(2.2)_3 \times 3^{-2} = \left(2+\frac{2}{3}\right)\times 3^{-2}$

$= 0.29629$

(true value − approximate) × 100%

$= 0.37037\%$

2

c) Write a MATLAB m.file to implement the following iterative process $x_{i+1} = x_i\,(2 - N\,x_i)$ where **N = 0.125**. Your program should stop when fulfilling a relative error of **10^{-8} %.**

```
X1 = 0 ; X0 = 10;
N=0.125, K=0 , error = ( X1 - X0 ) / X1 x100;
while (error) > 10^-8 10
for k=1:10
X1 = X0 * (2 - N * X0);
X0 = X1 ;
end
end
fprintf X1 ,
```

1

Confirm the convergence of your iterative process by calculating to your calculator maximum accuracy. Use an initial estimate of **10** for x_i . **Write down** results of the first **5** iterations. What answer do you get?.

1st = 7.5
2nd = 7.96875
3rd = 7.9987793
4th = 7.99999998
5th = 8

converges to 8

1

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Q2 Use the **false position** method **twice** to determine a zero of $f(x) = x^4 - x - 10 = 0$; in the interval [1 2].

$$x_r = x_u - \frac{f(x_u)(x_u - x_l)}{f(x_u) - f(x_l)}$$

$f(1) = -10$

$f(2) = 4$

$f(1.714) = -3.077$

if $f(x_l) \cdot f(x_u) > 0$

$x_u = x_r$

else

$x_l = x_r$

end

$$x_{r1} = 2 - \frac{4(2-1)}{4+10}$$

$x_{r1} = 1.7142857...$

$x_l = x_{r1} = 1.714$

$x_l = x_{r2} = 1.8383$

$$x_r = 2 - \frac{4(2-1.714)}{4+3.077}$$

$x_{r2} = 1.8383$

$f(1.8383) = -0.4171$

x_l

$$x_{r3} = 2 - \frac{4(2-1.8383)}{4+0.4171}$$

$x_{r3} = 1.8535$

$f(1.8535) = -0.0494$

3

In order to confirm your answer use the fixed point method. Why $g(x) = \frac{10}{x^3 - 1}$ with initial **x=2** is a bad choice. Demonstrate by performing **six** iterations.

it is a very bad choice

1st estimate $= 1.428571429$

2nd " $= 5.220700152$

3rd " $= 0.07077446885$

4th " $= -10.00354637$

5th " $= -9.979399646 \times 10^{-3}$

6th " $= -9.999990062$

2

Never try $g(x) = \frac{\sqrt{x+10}}{x}$ as it is very slow. Think of a better choice for ***g(x)*** which results in fast convergence. Perform **six** iterations starting with **x=2**.

$x = \frac{\sqrt{x+10}}{x}$

$x^2 = \sqrt{x+10}$

$\cancel{x^4 = x+10}$

$\cancel{x = x^4 - 10}$

$\boxed{x = \sqrt[4]{x+10}}$

$x = (10+x)^{\frac{1}{4}}$ converges to 1.85

1st $= 1.861209718$

2nd $= 1.855804597$

3rd $= 1.85559314$

4th $= 1.85558 4866$

5th $= 1.855584542$

6th $= 1.855584529$

3

8

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Numerical Methods 903301 — First Exam — Sun. 24/10/2016

Student Name: حمزة عبدالرحمن عبدربه — Reg. No. 0140754 — Mark out of 20: 10

Answers should be written in ink — **Exam Duration: 50 min**

Q1 a) A **decimal** computer uses **7 digits** for number representation. For floating point number representation, it allocates **4** digits for the significand , **1** digits for the exponent ,**1** digit for the number sign, and **1** digit for the exponent sign . Furthermore, this computer uses **number normalization** and does number **chopping.** Use this computer to answer the following questions .

1,1,1,4

What **floating point number** this computer gives for **2/700** .... 2.857×10^{-3} ✓

What **floating point number** this computer gives for **(2/700)** ^{2} .... 8.162×10^{-6} ✓

What result this computer gives for $\left(10^{-7} x\ 10^{-8}\right) / 10^{9}$ .... $10^{\cancel{-6}\,-15}$ but this will not be shown because (underflow) ✓

If **2222** and **4444** were **declared as natural numbers**, their **product** will be .... ~~9.874×10⁶~~ 9.874×10^{6} ✗

2.5

b) Using a **base 3 computer** , with **four** digits allocated for the significand and no restriction on the remaining allocations . What normalized floating point answer will this computer give for **(0.3)**$_{10}$. What is the **percent absolute error** involved.

$\boxed{\text{Normalized floating point answer} = 2.962 \times 10^{-1}}$ ~~2.962×10⁻¹~~

$(0.3)_{10} \rightarrow (.0220)_3$ 2.2×3^{-2} ✓

$\epsilon_t = (\text{True value} - \text{approximate value}) * 100\%$

$= (.3 - .2962) \times 100$

~~$\epsilon_t = 3.704 \times 10^{-2}\%$~~

$\boxed{\epsilon_t = 3.704 \times 10^{-3}\ \%}$ ×100

1

c) Write a MATLAB m.file to implement the following iterative process $x_{i+1} = x_i\,(2 - N x_i)$ where **N = 0.125**. Your program should stop when fulfilling a relative error of **10^{-8} %**.

```
N= 0.125 ;
e= (X-Y)/X ;
while abs(e) >10^-8 10
 Y=X
 X= X* (2-N*X) ;
 e= (X-Y) /X ;
end
```

5

1

Confirm the convergence of your iterative process by calculating to your calculator maximum accuracy. Use an initial estimate of **10** for x_i . **Write down** results of the first **5** iterations. What answer do you get?.

1st = 3.5

2nd = 5.46875

3rd = 7.19909668

4th = 7.919819234

~~4th = 7.486096699 × 10¹⁶~~

~~5th = 7.005205473 × 10³³~~

5th = 7.999196381

it converges to 8

0.5

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Q2 Use the **false position** method **twice** to determine a zero of $f(x) = x^4 - x - 10 = 0$; in the interval [1 2].

$$xr = xu - \frac{f(xu)(xu - xL)}{f(xu) - f(xL)}$$

1) $xu_1 = 2 \rightarrow f(xu) = 4$ +
$xL_1 = 1 \rightarrow f(xL) = -10$

$xr_1 = 12/7$
$f(xr_1) = -3.077$

*if $f(xu) \cdot f(xr) > 0$
$xu = xr$
else
$xL = xr$

2) $xL_2 = xr_1 = 12/7 \rightarrow f(xL_2) = -3.077$
$xu_2 = 2 \rightarrow f(xu_2) = 4$
$xr_2 = 1.8385$
$xL_3 = xr_2 = 1.8385$

$\boxed{xr_1 = 1.714}$ 28
$\boxed{xr_2 = 1.838}$

3

In order to confirm your answer use the fixed point method. Why $g(x) = \frac{10}{x^3 - 1}$ with initial **x=2** is a bad choice. Demonstrate by performing **six** iterations.

1st = 1.428571429
2nd = 5.220700152
3rd = 0.07077446885
4th = −10.00354637
5th = $-9.79399646 \times 10^{-3}$
6th = −9.999990062

As shown from the six iteration ~~this~~ g(x) is a divergance not a convergence. which is bad.

2

Never try $g(x) = \frac{\sqrt{x+10}}{x}$ as it is very slow. Think of a better choice for *g(x)* which results in fast convergence. Perform **six** iterations starting with **x=2**.

*the better choice

$x = \frac{\sqrt{x+10}}{x}$

$x^2 = \sqrt{x+10}$

$x^4 - x - 10 = 0$

$g_2(x) = x^4 - 10$ better Choice (convergence)

1st = 6
2nd = 1286
3rd = $2.735041121 \times 10^{12}$
4th = $5.595713846 \times 10^{49}$

*the six iterations $g(x) = \frac{\sqrt{x+10}}{x} \Rightarrow$ divergence
1st = 1.732050808
2nd = 1.977544842
3rd = 1.750078633
4th = 1.958677002
5th = 1.765544808
~~6th~~

6th = 1.942796342

you can see $g_1(x)$ divergence which is bad

~~because less calculations~~

0