

# Measurements

## NoteBook

By: Asma Hakouz



- Note that physical quantities should be defined with both kind & magnitude.
- The Standard measure of each kind of a physical quantity is its unit, where the unit is defined as a sample of a quantity
- The # of times the unit occurs in any given amount of the same quantity, is the number of measure.

→ for example:-

$$\text{distance} = \underbrace{100}_{\substack{\leftarrow \\ \# \text{ of measure}}} \underbrace{\text{meter}}_{\rightarrow \text{unit}}$$

so, the physical quantity is the length & is defined by the unit (meter), without the unit # of measure has no physical meaning.

- Note that there are 2 types of units:-

#### 1. Fundamental units:

- Unit of length
- ~ ~ time.
- ~ ~ mass

#### 2. Auxiliary Fundamental Units:-

measure of quantities in electrical, thermal & illumination fields.

- In general, all other physical quantities are derived from the fundamental quantities & known as derived units.
- In the actual physical world, there are 6 fundamental quantities:-
  1. Mechanics: <sup>Length</sup> L, <sup>Mass</sup> M, <sup>Time</sup> T.
  2. Thermodynamics: Temperature difference.
  3. Electrical field: current (Ampere).
  4. Optics: light intensity (Candela).

SI Units:

- Note that the SI units system gives the definition of unit symbols & the dimension of the units. & it's known as the International system.

<u>Fundamental quan.</u>	<u>SI unit</u>	<u>unit symbol</u>	<u>Dimension</u>
Mass	Kilogram	kg	M
Time	Second	s	T
Length	Meter	m	L
Temp.	Kelvin	K	Θ
current	Ampere	A	I
Light intensity	Candela	Cd	-

Electrical units:

<u>Unit</u>	<u>Symbol</u>	<u>Dimension</u>	
Hertz	Hz	$1/T$	frequency
Watt	W	$L^2 M / T^3$	Power
Volt	V	$L^2 M / (T^3 I)$	Voltage
Ohm	$\Omega$	$L^2 M / (T^3 I^2)$	resistance
Ampere	A	I	current
Farad	F	$T^4 I^2 / (L^2 M)$	capacitance
Henry	H	$L^2 M / (T^2 I^2)$	Inductance

• Note that in SI system, multiples or submultiples of a unit can be used, which are related to multiples or submultiples of 10, by adding a multiplier prefix to the name of the base unit.

<u>Prefix</u>	<u>Symbol</u>	<u>multiplier</u>			
atto	a	$10^{-18}$	Giga	G	$10^9$
femto	f	$10^{-15}$	Tera	T	$10^{12}$
Pico	p	$10^{-12}$	Peta	P	$10^{15}$
nano	n	$10^{-9}$	exa	E	$10^{18}$
micro	$\mu$	$10^{-6}$			
milli	m	$10^{-3}$			
kilo	k	$10^3$			
Mega	M	$10^6$			

• Some Common electrical units:-

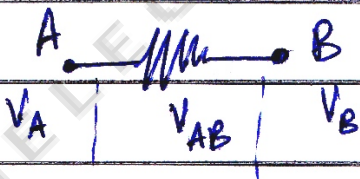
1. Volt: SI unit for potential difference, which is the force that causes the charge to move through an electric CKT.

On the basis of moving charges, a difference of (1) volt exists between two points in a CKT if a (1) joule is required to move a (1) coulomb charge from a point of lower potential to the point of higher potential.

$$(1)V = (1)J / (1)\text{coulomb}$$

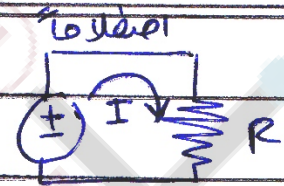
$$= \frac{\text{watt} \cdot \text{second}}{\text{Ampere} \cdot \text{second}}$$

(1)V = 1 watt / 1 Ampere ← the potential difference across an element dissipating 1W when a (1)A current is flowing through it.



2. Ampere: SI unit for current, and it's a measure of the rate of electric charge <sup>that</sup> passes a given point in a CKT, so a current of (1) Ampere will cause a charge of (1) coulomb to pass a given point in a CKT in (1) second.

$$1 \text{ Ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}$$



3. Ohm: SI unit for electrical resistance which is a measure opposition to a steady current flow due to molecular properties of the conductor. also, it can be defined in terms of voltage across the element divided by the current flowing through it.

$$(1) \Omega = \frac{(1)V}{(1)A}$$

\* R is passive element

(consumes power)

\* Active element: produces power.

4. farad: SI unit for capacitance. Where capacitance is a measure of the charge that's stored in the capacitor as a function of the applied voltage across the capacitor & it's equal to the stored charge divided by the voltage across the capacitor

$$(1) F = \frac{(1) \text{ coulomb}}{(1) \text{ Volt}}$$

5. Henry: SI unit for inductance, which is a proportionality factor that relates induced voltage in a coil of wire to the rate of change of current with time

$$(1) \text{ Henry} = \frac{(1) \text{ Volt}}{(1) \text{ Ampere}}$$

6. Hertz: SI unit of frequency, which is defined as a number of cycles of the waveform per unit second.

$$1 \text{ (Hz)} = \frac{(1) \text{ cycle}}{(1) \text{ sec}}$$

• Logarithmic Response Unit:

Note that when measurements are made on individual electrical components, measured values are usually used to evaluate the performance of the system. So, when such components are connected, the performance of the system is expressed as the ratio of the output signal to the input signal.

if  $> 1$  Gain,  $< 1$  loss/attenuation.

So, to simplify the calculations, it's convenient to express this ratio in terms of logarithmic Response Units, such as Decibels (dB), & Neper.

$$A_p = 10 \log \left( \frac{P_{out}}{P_{in}} \right)$$

if  $P_{out} > P_{in}$  (Gain) (dB +ve)

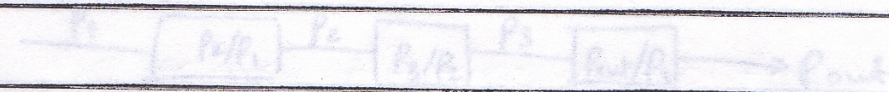
if  $P_{out} < P_{in}$  (Loss) (dB -ve)

if  $P_{out} = P_{in}$  (No change)

Advantages of dB:

1. Use of (log<sub>10</sub>) gives 2 advantages:

1. When several systems are connected in cascade, the gain of the overall system is the product of the power gain of the individual systems.



$$A_p = \frac{P_2}{P_1} \cdot \frac{P_3}{P_2} \cdot \frac{P_{out}}{P_3}$$



### • Decibel (dB):

- dB is a unit that simplifies calculations involving power & voltage gain with cascade <sup>stage</sup> systems.
- It can be defined in terms of the input & output power levels of the system

$$A_{dB} = 10 \log_{10} \left( \frac{P_{out}}{P_{in}} \right)$$

if  $P_{out} > P_{in} \rightarrow A_{dB} (+ve)$  **Gain**

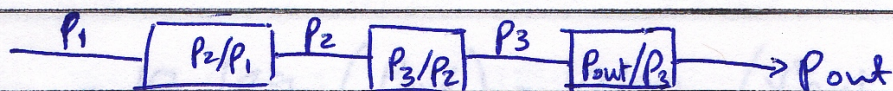
if  $P_{in} > P_{out} \rightarrow A_{dB} (-ve)$  **Loss / Attenuation**

if  $P_{in} = P_{out} \rightarrow A_{dB} = 0$  **No Change**

### → Advantages of dB:-

the use of  $(\log_{10})$  gives 2 advantages:-

1. when several systems are connected in cascade, the gain of the overall system is the product of the power gain of the individual systems.



$$A_p = \frac{P_2}{P_1} \cdot \frac{P_3}{P_2} \cdot \frac{P_{out}}{P_3} = \frac{P_{out}}{P_1}$$

However, if the individual system power gains are in dB, then the overall gain or loss is obtained by simple addition of the dB Gain or Loss of each system.

$$A_{dB} = 10 \log_{10} \left( \frac{P_{out}}{P_{in}} \right)$$

$$\therefore A_p = 10 \log_{10} \left( \frac{P_2}{P_1} \right) + 10 \log_{10} \left( \frac{P_3}{P_2} \right) + 10 \log_{10} \left( \frac{P_{out}}{P_3} \right)$$

2. In General, System gains or losses may vary from very small to very large numbers, but by using a logarithmic scale, this wide range can be comprised to more convenient range.

In some applications, we might need a current or voltage gain, which can be calculated as follows:-

$$P = \frac{V^2}{R} \quad \therefore \frac{P_{out}}{P_{in}} = \frac{V_{out}^2}{V_{in}^2} \cdot \frac{R_{in}}{R_L}$$

$$A_{dB} = 10 \log_{10} \left( \frac{P_{out}}{P_{in}} \right) = 10 \log_{10} \left( \frac{V_{out}^2}{V_{in}^2} \cdot \frac{R_{in}}{R_L} \right)$$

Voltage gain (dB)

$$A_{dB} = 20 \log_{10} \left( \frac{V_{out}}{V_{in}} \right) + 10 \log_{10} \left( \frac{R_{in}}{R_L} \right)$$

if  $R_L = R_{in}$ , then  $A_{dB} = 20 \log_{10} \left( \frac{V_{out}}{V_{in}} \right)$

current gain (dB)

current gain  $A_{dB} = 20 \log_{10} \left( \frac{I_{out}^2}{I_{in}^2} \times \frac{R_{out}}{R_{in}} \right)$

$A_{dB} = 20 \log_{10} \left( \frac{I_{out}}{I_{in}} \right)$  if  $R_{in} = R_{out}$

ex: An Amplifier has a voltage gain  $\left( \frac{V_{out}}{V_{in}} = 10000 \right)$   
find the gain in dB if  $R_L = R_{in}$ .

gain<sub>dB</sub> =  $20 \log_{10} (10^4) = 80 \text{ dB}$

ex2: An amplifier has a V. gain of 100, if  $R_{in} = 1.6 \text{ k}\Omega$   
&  $R_L = 400 \Omega$ , find the V. gain in dB.

V. gain =  $20 \log_{10} (100) + 10 \log_{10} \left( \frac{1600}{400} \right)$   
=  $40 + 6 = 46 \text{ dB}$

ex3:- a signal path has the following elements  
in series: a coupler with 2dB loss,  
a Transmission line with 5dB loss & an  
amplifier with 10dB gain. find the total  
path gain/loss in dB.

$A_{dB \text{ total}} = -2 - 5 + 10 = +3 \text{ dB (gain)}$

ex4:- if 100 mV is applied to an amplifier having an input resistance of 75  $\Omega$  & whose output is connected to a 300  $\Omega$  load, determine the output voltage if the overall gain is 12 dB.

$$12 \text{ dB} = 20 \log_{10} \left( \frac{V_{out}}{100 \text{ m}} \right) + 10 \log_{10} \left( \frac{75}{300} \right)$$

$$V_{out} = 794.3 \text{ mV}$$

● Neper (Np):-

Power  $\Rightarrow$   $A_{np} = 0.5 \ln \left( \frac{P_{out}}{P_{in}} \right) \dots (1)$        $\log_e = \ln$   
 $e = 2.718281$

voltage  $\Rightarrow$   $A_{np} = \ln \left( \frac{V_{out}}{V_{in}} \right) \dots (2)$

## • Measurement & errors:-

→ Note that, the measurement involves using an instrument as a physical means of determining a quantity or variable.

• Instrument:- a device for determining the value or magnitude of a quantity or variable.

• Note that there are number of terms that should be employed in measurements work:-

→ Accuracy:- a measurement of closeness with which an instrument reading approaches the actual value.

→ Precision:- a measure of repeatability of series of measurements.

→ Note that, accuracy implies precision while precision doesn't necessarily imply accuracy

so, a precise instrument can be very inaccurate.

$$\text{Precision} = 1 - \left| \frac{x_i - \bar{x}}{x_i} \right|$$

$(x_i)$  is the value of the  $i$ -th measurement  
 $(\bar{x})$  is the avg. value of  $n$  measurements.

→ Sensitivity:- it's a measure of change in reading of an instrument for a given change in the measured value

→ Resolution:- the smallest change in the measured quantity that will produce a detectable change in the instrument reading.

→ Error: It's the deviation from the true value of the measured quantity, & it could be defined as an absolute quantity or a percentage

a) Absolute error:- difference between expected value & measured value.

$$= X_e - X_m$$

b) Percent error =  $\left| \frac{X_e - X_m}{X_e} \right|$

• Range: it describes the limits of magnitude over which a quantity may be measured.

it's normally specified by stating its lower & upper limits.

• Span: algebraic difference between the lower & upper limits of instrument's range.

## ● Errors:-

there are 3 major types of errors:-

1) Systematic error: the error that remains constant with repeated measurement, it arises from the inaccuracies in the manufacture of an instrument or from improper adjustment of an instrument, and it doesn't change with time. thus it can be measured & compensated.

### 1-a) Zero error:

all readings have the same amount of error.

### 1-b) Scale error:-

all readings are multiplied with the same factor.

It depends on the magnitude of the reading, such as a voltmeter that reads 1V as 10V & 2V as 20V and so on.

### 1-c) Response time error:

this is due to the instrument's inability to follow dynamic changes in the measured quantity.

### 1-d) Loading Error:

the instrument extracts sufficient energy from the system under measurement. So that the value of the measured parameter is changed

\* systematic errors can be removed easily by regular adjustments of the instrument or by using a correction factor.

2) Random error:- the errors that happen due to unknown causes & are observed when a magnitude of measurement fluctuates in an unpredictable manner.

### 2-a) Rounding error:-

this occurs when readings are between 2 scale graduations! & are rounded up or down.

### 2-b) Periodic error:-

this occurs when an analog meter readings swing/fluctate about the correct value.



### 2-c) Noise:-

the sensitivity of the instrument is changed or the reading is altered due to outside interference.

### 2-d) Backlash:-

the reading either lags or leads the correct value, because of mechanical friction or damping.

### 2-e) Ambient Influences:-

error due to conditions external to the measuring system. such as, temperature, atmospheric variations.

\* to minimize these errors by a skilled observer.

## 3) Gross error:

### 3-a) Human error:

this occurs when the operator makes mistakes, such as reading the wrong scale or value.

### 3-b) Equipment faults:

this error source can be large & sometimes erratic.

gross error can be min. by careful operator attention & cross checking & frequent equipment calibration.

errors called limiting errors

Limiting errors: Accuracy of full scale value.

Percentage error =  $\frac{\text{Max. limiting error}}{\text{Scale reading}} \times 100\%$

Ex: An analog Ammeter with a 0-100 A range & stated accuracy of 1% of full scale i.e. for the reading 30 A determine the magnitude of the limiting error & the percentage error of the reading.

Sol: limiting error = Acc. x full scale value

$= 0.01 \times 100 = 1 \text{ A}$

Percentage error =  $\frac{\text{Limiting error}}{\text{reading}} \times 100\%$

$= \frac{1}{30} \times 100\% = 3.3\%$

∴ State that the perc. errors are larger at lower portion of an analog scale & it's better to take the measurement at the higher portion of the scale.

## • Accuracies & Tolerances:

→ Note that instrument meters are usually guaranteed to be accurate with a certain percentage errors called limiting errors.

→ Limiting errors = Accuracy  $\times$  full scale value.

→ Percentage error =  $\frac{\text{Maximum error} \times 100\%}{\text{Scale reading}}$

→ ex: An analog Ameter with a 0-100 A range & stated accuracy of 3% of full scale is presently reading 30 A, Determine the magnitude of the limiting error & the percentage error of the reading.

Sol: limiting error = Acc.  $\times$  full scale value  
 $= 0.03 \times 100 = 3A$

Percentage error =  $\frac{\text{limiting error} \times 100\%}{\text{reading}}$

$= \frac{3A}{30} \times 100\% = 1\%$

• Note that the perc. errors are larger at lower portions of an analog scale, & it's better to take the measurement at the higher portion of the scale.

ex2:- An oscillator is guaranteed to have a max. frequency drift of  $\pm 100$  pulse/million at a freq. of 5MHz, determine the max. freq. drift and the percentage drift.

Sol:- 
$$\text{drift} = \frac{\pm 100}{10^6} \times 5000000 = \pm 500 \text{ Hz}$$

freq. range  $\rightarrow (4.995 \text{ MHz} - 5.005 \text{ MHz})$

Percentage drift =  $\frac{500}{5M} \times 100\% = 0.01\%$

### ● Component tolerances:-

- $\rightarrow$  Note that electric components such as R, C, ... have tolerance specified as a percentage of the nominal value (true value).
- $\rightarrow$  Note that Capacitor may have different values for (+ve) tolerance than for (-ve) tolerance.
- $\rightarrow$  Note that in division & multiplication, tolerances will be added.

ex:- The D.C. voltage across a Resistor with tolerance of  $\pm 10\%$  is measured to an accuracy of  $\pm 3\%$ , what would be the max. percentage error in determining the power dissipated by the resistor.



ex:- The measurement of 5 resistors all marked  $100\ \Omega$ , result in the following readings  $99.2\ \Omega$ ,  $98.1\ \Omega$ ,  $100.3\ \Omega$ ,  $100.4\ \Omega$ ,  $100.1\ \Omega$  determine the mean value.

Solution:-

$$\bar{R} = \frac{\sum R_i}{5} = 99.6\ \Omega$$

2) Deviation from the avg. value:-

$$d_i = X_i - \bar{X}$$

ex: determine  $d_i$  for the data of previous ex:

Solution:-  $\bar{X} = 99.6$

$$d_1 = 99.2 - 99.6 = -0.4$$

$$d_2 = -1.5$$

$$d_3 = 0.7 \quad d_5 = 0.5$$

$$d_4 = 0.8$$

3) Average Deviation ( $\bar{D}$ )

$$\bar{D} = \left( \frac{\sum |d_i|}{n} \right)$$

it must be as small as possible.

ex:- for previous ex:  $\bar{D} = \frac{3.9}{5} = 0.8$

→  $(\bar{D})$  is a measure of how much the data is varied from avg value, & it's always +ve. & it gives an indication <sup>about</sup> the precision of the measurement.

4) Standard deviation:- (s)

$$S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^n d_i^2}{n-1}}$$

→ note that (s) / root mean <sup>square</sup> is mathematically & statistically more convenient & meaning full for analyzing set of readings.  
ex:- S for previous ex:

$$S = \sqrt{\frac{0.4^2 + 0.5^2 + 0.7^2 + 0.8^2 + 1.5^2}{4}} = 0.97 \Omega$$

Monday

10/3

التاريخ

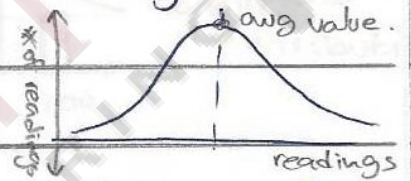
اليوم

Lecture #6

الموضوع

## • Normal (Gaussian) distribution of errors-

Note that it's useful to graph a large # of readings vs the # of times each reading occurs. (graph is called Histogram).



- if all readings are taken with equal care, & errors are random, then the graph. will have a gaussian (normal) distribution (bell-shaped).
- the narrower the bell, the more probable that the center of the bell represents the true value.

→ example: The following list of measured values of 20 resistors having the same marked value. Draw the histogram & determine the avg. value & the standard deviation for this data.

Readings (measured) values of resistance	# of times reading occurs
---	------------------------------

92  $\Omega$

1

93  $\Omega$

1

94  $\Omega$

2

95  $\Omega$

3

96  $\Omega$

4

97  $\Omega$

3

98  $\Omega$

3

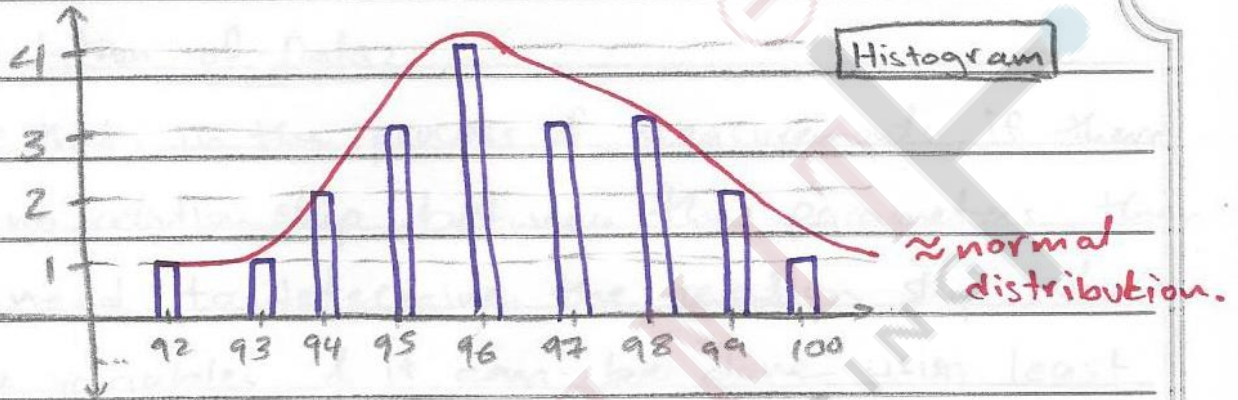
99  $\Omega$

2

100  $\Omega$

1





$$\bar{x} = \frac{(92 + 93 + 94 \times 2 + 3 \times 95 + 4 \times 96 + 3 \times 97 + 3 \times 98 + 99 \times 2 + 100)}{20}$$

$$\bar{x} = 96.35$$

→ Standard deviation:-

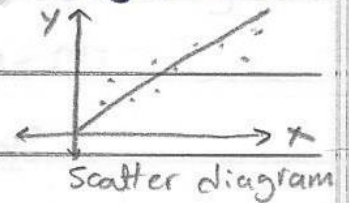
Reading	$d_i$	$d_i^2$
92	-4.3	18.49x1
93	-3.3	10.89x1
94	-2.3	5.29x2
95	-1.3	1.69x3
96	-0.3	0.09x4
97	0.7	0.49x3
98	1.7	2.89x3
99	2.7	7.29x2
100	3.7	13.69x1

$$\sum d_i^2 = 83.9$$

$$S = \sqrt{\frac{\sum d_i^2}{n-1}} = 2.15$$

## • Correlation of Data:

Note that, in the process of measurements, if there is no relationship between the parameters, then we need to determine the relationship between the variables, & it can be done using least square regression line.



### → Least Square Regression Line:-

↳ a systematic method to find the best line to represent the relation between data.

\* assume that, the relationship is linear.

∴ the straight line equation can be obtained from :-

$$Y = mX + b$$

↳  $Y$  : dependent variable

↳  $X$  : independent

↳  $m$  : slope of the line

↳  $b$  :  $y$ -intercept point ( $x=0$ )

where :-

$$m = \frac{n \sum (xy) - \sum x \sum y}{n \sum (x^2) - (\sum x)^2} \quad \dots (1)$$

$$b = \frac{\sum y - m \sum x}{n} \quad \dots (2)$$

example: An experiment measured the following voltages across a given thermocouple at various temperatures assuming that there's a linear relationship, determine the relationship between the thermocouple voltage and temp. in the form of a straight line.

Temp. (F)	Voltage (V)	$XY$	$X^2$
-100	-2.58	258	10000
-20	-1.11	22.2	400
2	-0.64	-1.28	4
13	-0.44	-5.72	169
25	-0.16	-4	625
30	-0.04	-1.2	900
32	0	0	1024
80	1.09	87	6400
100	1.54	154	10000
190	3.71	704.9	36100

$$\Sigma X = 352 \quad \Sigma Y = 1.37 \quad \Sigma XY = 1214.1 \quad \Sigma X^2 = 65622$$

$$m = \frac{10(1214.1) - (352)(1.37)}{10(65622) - (352)^2} = (0.0219) \text{ V/F}$$

$$b = \frac{\Sigma Y - m \Sigma X}{n} = \frac{1.37 - (0.0219)(352)}{10} = -0.6339 \text{ V}$$

$$\therefore Y = 0.0219 X - 0.6339$$

## • DC meters:

1. The D'Arsonval Movement meter

2. The Hot wire movement meter

## → The D'Arsonval Movement:

Note that the D'Arsonval movement is the most common electromechanical meter & it's also called "permanent Magnet Moving Coil (PMMC)"

The meter movement is driven by current & uses the force arising from the interaction of a magnetic field & a current carrying conductor to rotate a moving coil against restraining force of spiral spring as shown in the figure (next page)

\* Note that the resulting force is perpendicular to both the magnetic field & direction of current flow.

$$F = B L I$$

• F: force

• B: flux density

• I: current

• L: length of conductor

Immersed in a magnetic field.

- Note that the torque is produced on a single-turn <sup>N=1</sup> current carrying coil when it's immersed in a magnetic field.

$$T = 2Fr$$

$$= \underline{2BLI}r \quad 2rL = \text{Area (A)}$$

$$\therefore T = BIA \text{ for single-turn coil}$$

if we increase the # of turns (n), then

$$T = nBIA$$

### Construction of PMMC:

Basic parts of the D'Arsonval meter movement:

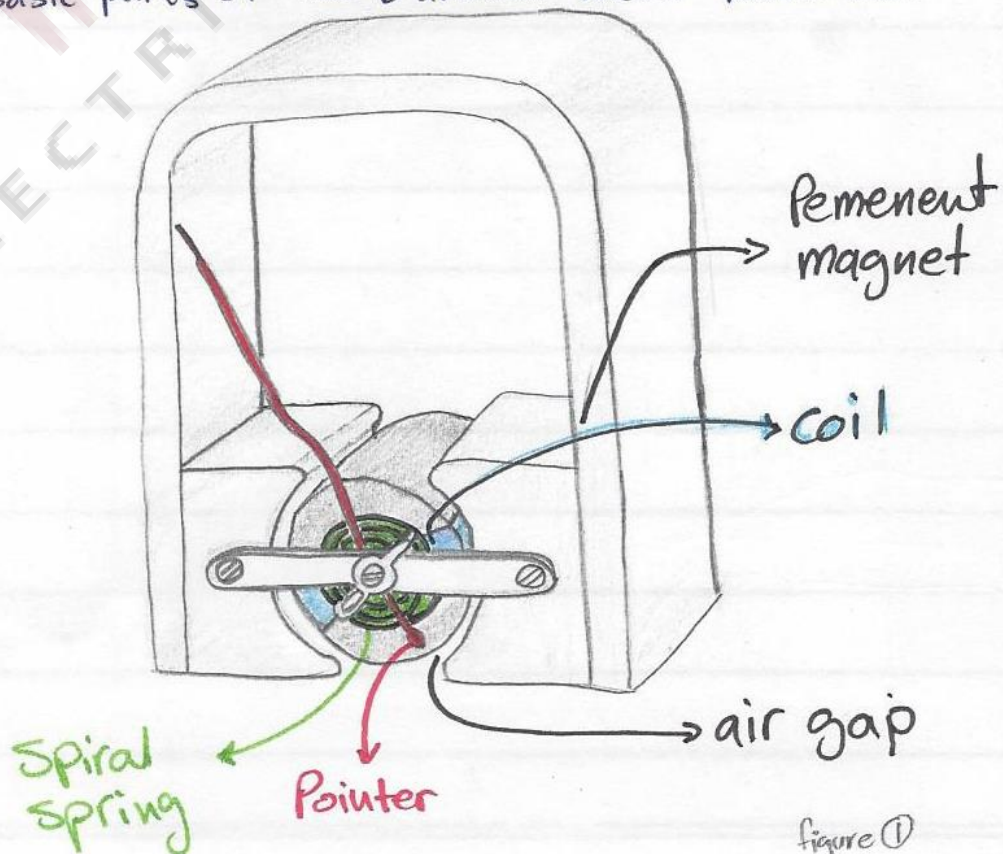


Figure ①

• Note that, the torque may be increased by adding more turns, or by increasing the area of the coil but there are limitations for them; <sup>①</sup> where increasing the dimension of the coil means greater inertia, bearing loading, physical size.

& <sup>②</sup> additional turns increase the electrical resistance of the meter.

• PMMC characteristics:

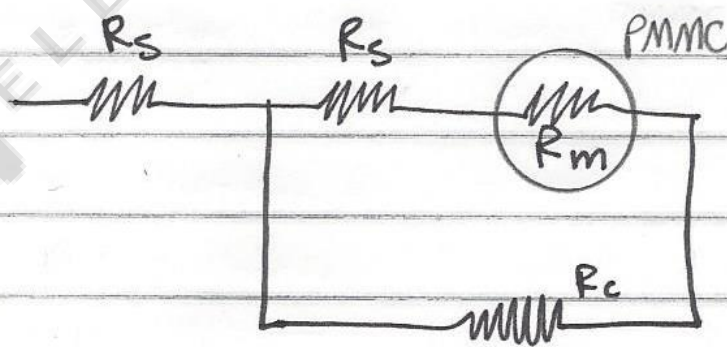
1) the meter indicates the average value of the current in the coil.

2) the meter's response is relatively slow due to mechanical inertia & control damping.

3) if a signal applied to the meter to be measured with DC component & AC component the meter will read only the avg value of the D.C component. Since <sup>the</sup> avg value of the A.C. component is zero!

4) PMMC meter is relatively sensitive, which depends on the coil resistance and the range of the meter.

5) the magnetic field strength & spring tension tend to decrease with an increment in temperature. also the resistance of the coil increases with temperature. Thus the net result of these changes is that the meter tends to read slightly low at high Temp. , for this reason meters that must operate over a wide range of Temp. must have some form of compensation, which can be done by connecting a resistance wire with <sup>(-ve)</sup> low temp. coefficient in series with the moving coil to swap out the effect of any change in the coil resistance.



$R_s$ : swapping resistance

$R_m$ : low temp. resistor wire

$R_c$ : copper resistance ~

• The Hot Wire Movement meter:-

This type of meters is used with low accuracy applications.

It depends on the expansion of a heated wire to move the pointer, so it's cheap & simple. but it suffers from a non-linear scale & lack of sensitivity & error due to change in temperature.

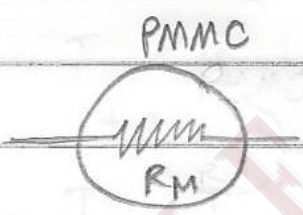


17/3

- The Galvanometer: (for detection of current) it doesn't provide accurate values.

Note that: a galvanometer is an application of the PMMC movement, & it's used to detect the presence of extremely low level current, it indicates only the direction & relative magnitude of the current.

- DC Ameter



to reduce  $R_m$ , we connect it with shunt resistance (to increase the range of readings) & increase accuracy.

Note that the ameter is the fundamental building block of all analog meters, it's then possible to create other types of meters to measure voltage & resistance.

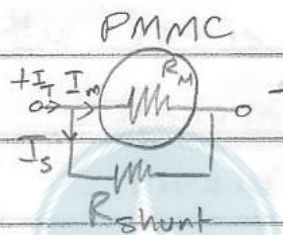
also, the PMMC movement is an ameter. the amount of current required to deflect the meter to full-scale is called the "full-scale deflection"  $I_{FSD}$ .

& the resistance of the coil windings is

$R_m$

usually,  $I_{FSD} = 100 \mu A$

to measure larger current, we have to connect the PMMC with shunt resistance as shown below:



$$V_{shunt} = V_{meter}$$

$$I_s R_s = I_m R_M$$

$$I_s = \frac{I_m R_M}{R_s}$$

$$I_T = I_s + I_m$$

$$\therefore R_s = \frac{I_m R_M}{I_T - I_m}$$

When  $I_m = I_{FSD}$ , then

$$R_s = \frac{I_{FSD} R_M}{I_T - I_{FSD}}$$

$$= \frac{R_M}{\frac{I_T}{I_{FSD}} - 1}$$

$$\text{let } N = \frac{I_T}{I_{FSD}}$$

$$\therefore R_s = \frac{R_M}{N - 1}$$

this is used to adjust the range of the ammeter.

ex:- determine the size of the required shunt  $R_s$ , & its power rating to use a (0-100)  $\mu$ A PMMC movement meter having an internal resistance  $R_M = 1 \text{ k}\Omega$  as the basis for 0-10 mA ammeter.

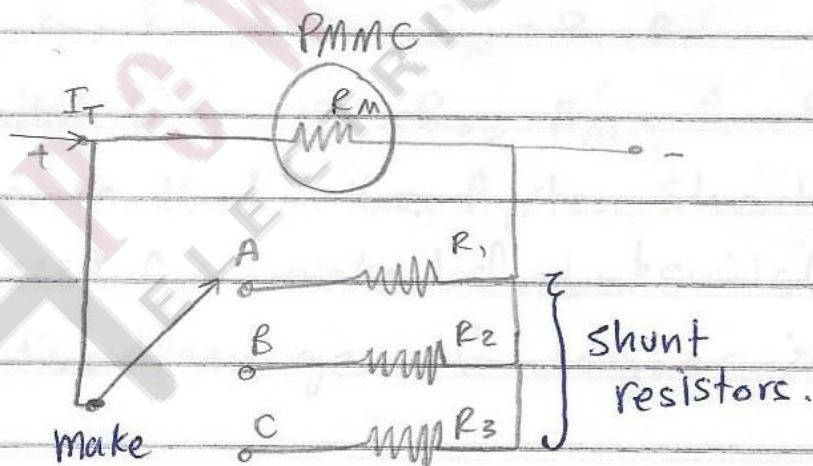
$$R_s = \frac{I_{FSD} R_M}{I_T - I_{FSD}} = \frac{(100 \times 10^{-6})(1 \times 10^3)}{(10 \times 10^{-3}) - (100 \times 10^{-6})} = 10.1 \Omega$$

$$I_s = I_T - I_{FSD} = 10 \text{ mA} - 100 \mu\text{A} = 9.9 \text{ mA}$$

$$V_{shunt} = (9.9 \text{ mA})(10.1) = 99.99 \text{ mV}$$

$$P_{shunt} = I_s V_s = (9.99 \text{ mA})(99.99) \text{ mV} = 0.999 \text{ mW}$$

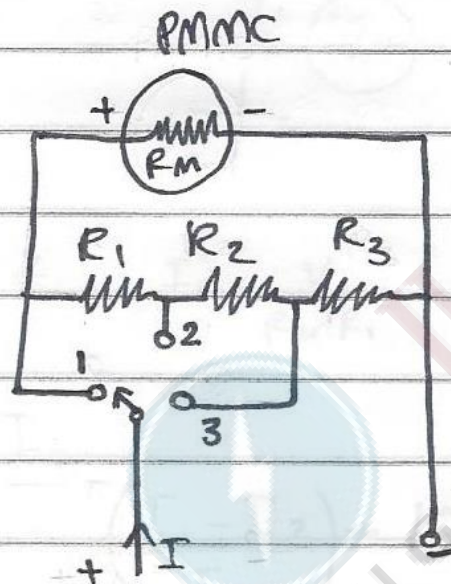
### • Multiple range DC ammeter



before  
break  
switch

\* Note that multiple ranges can be added by using "make before break" switch to choose particular shunt  $R$  (range). so we have to make sure the PMMC meter is never without  $R_{shunt}$

another type of switches: (Ayrton Shunt ammeter)



Switch @ 1:

$$R_s = R_1 + R_2 + R_3, \quad R_M' = R_M$$

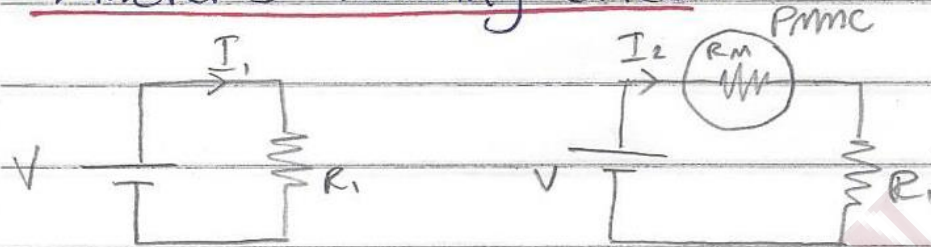
Switch @ 2:

$$R_s = R_2 + R_3, \quad R_M' = R_1 + R_M$$

Switch @ 3:  $R_s = R_3, \quad R_M' = R_1 + R_2 + R_M$

\*Note that, the Ayrton shunt eliminates the need for make before break switch, but has the disadvantage of higher insertion resistance.

• Ammeter's loading effect:



$$I_1 = \frac{V}{R_1} \quad \text{--- (1)} \quad \neq \quad I_2 = \frac{V}{R_M + R_1} \quad \text{--- (2)}$$

$$I_{\text{error}} = I_1 - I_2$$

$$\text{percentage error} = \left( \frac{I_1 - I_2}{I_1} \right) \times 100\%$$

$$= \left( 1 - \frac{I_2}{I_1} \right) \times 100\%$$

$$\text{from (1) \& (2)} \quad \frac{I_2}{I_1} = \frac{R_1}{R_1 + R_M}$$

$$\therefore \text{percen. error} = \left( 1 - \frac{R_1}{R_1 + R_M} \right) \times 100\%$$

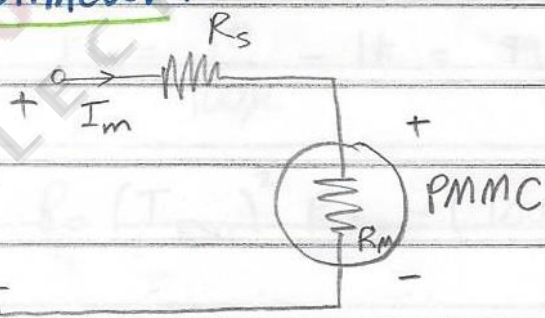
Note that the loading error occurs due to the existance of the internal resistance of the coil, since the coil in PMMC movement is not ideal.

24/3

### ● Precautions in using DC Ammeter:

1. never connect an Ammeter directly with a voltage source.
2. Observe the polarity when connecting the Ammeter, since a large current in the reverse direction causes deflection against the stop at the lower end of the scale, thus it could be damaged.
3. Initially set the multi-range Ammeter to its highest range setting before inserting it in the CKT.

### ● DC Voltmeter:



$R_s$ : multiplier resistance.

Basic voltmeter CKT.

Note that the DC voltmeter consists of a PMMC meter in series with suitable multiplier resistor.

at full-scale deflection, the voltage will be the peak value.

$$V_p = V_{FSD} \quad \text{at } I_{FSD}$$

$$R_{total} = R_M + R_s = \frac{V_p}{I_{FSD}} = \frac{V_{FSD}}{I_{FSD}}$$

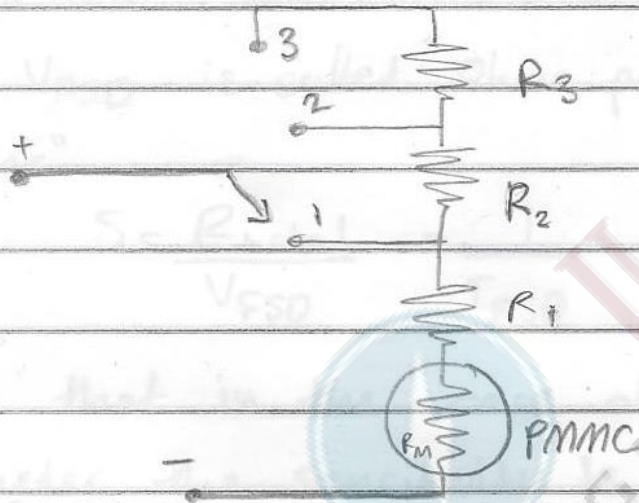
$$R_s = \frac{V_{FSD}}{I_{FSD}} - R_M$$

ex: It's required to construct a voltmeter of 0-10 Volt using PMMC meter with  $I_{FSD} = 100 \mu A$  & a coil resistance of  $1k \Omega$ , determine the required value & power rating of  $R_s$ .

$$R_s = \frac{10}{100 \mu} - 1k = 99k \Omega$$

$$P_{R_s} = (I_{FSD})^2 R_s = (100 \times 10^{-6})^2 \times 99 \times 10^3 = 0.99 \text{ mW}$$

● Multiple Range DC voltmeter:



ex: the required full-scale voltage ranges are (1, 5, 10) V respectively, determine the values of the 3 resistances ( $R_1, R_2, R_3$ ) to construct a Voltmeter using PMMC with  $100\mu A = I_{FSD}$ ,  $R_m = 1k\Omega$ .

$$R_s = \frac{V_{FSD}}{I_{FSD}} - R_m$$

● for (0-10) v range

$$V_{FSD} = 10V$$

$$R_1 + R_2 + R_3 = \frac{10}{100\mu} - 1k = 99k\Omega$$

● for (0-1) v range:  $V_{FSD} = 1V$

$$\therefore R_1 = \frac{1}{100\mu} - 1k\Omega = 9k\Omega$$

● for (0-5) v range:  $V_{FSD} = 5V$

$$R_3 = 50k\Omega$$

$$R_1 + R_2 = \frac{5}{100\mu} - 1k\Omega = 49k\Omega$$

$$\therefore R_2 = 40k\Omega$$



• Voltemeter's Ohms per volt: (Sensitivity factor)

\* Note that the ratio of the total  $R$  divided by  $V_{FSO}$  is called "Ohms-per-Volt" or "Sensitivity factor"

$$S = \frac{R_{total}}{V_{FSO}} = \frac{1}{I_{FSO}}$$

\* Note that in the case of multiple range Voltmeter, the sensitivity is constant.

ex: determine the Ohms-per-Volt & rating of the volt meter in the previous example.

range # 1.

$$S = 10k = 10k\Omega/V$$

range # 2:

$$S = \frac{49 + 1}{5} = 10 k\Omega/V$$

range # 3:-

$$S = \frac{99 + 1}{10} k = 10 k\Omega/V$$

check:-

$$S = \frac{1}{I_{FSO}} = \frac{1}{100\mu} = 10 k\Omega/V \quad \#$$

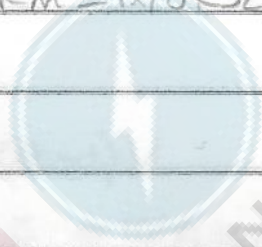
\* Note that, the Ohms/Volt of a voltmeter is also useful in calculating the size of the  $R_s$

$$R_s = V_{FSD} \left( \frac{S}{I} \right) - R_M$$

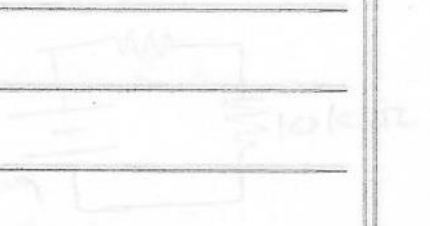
for example (the previous ex):

$$R_s = V_{FSD} S - R_M = 1 \times 10^3 \times 2 - 1k = 9k \Omega$$

and so on.

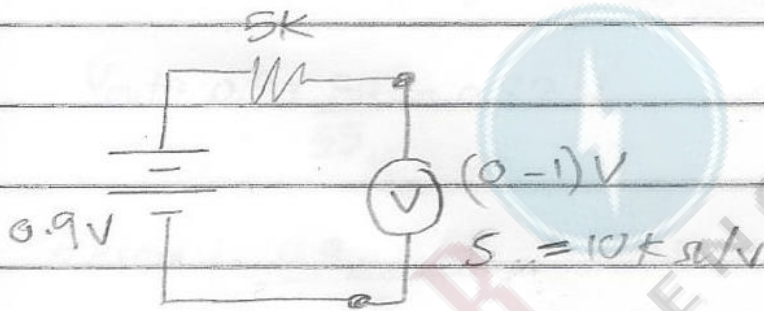


POWER ELECTRICAL ENGINEERING



### • Voltmeter's loading effect:

ex: for the DC voltmeter shown below, it's connected to the output of a signal source that has an equivalent  $R$  of  $5k\Omega$ , & open ckt terminal voltage ( $0.9V$ )



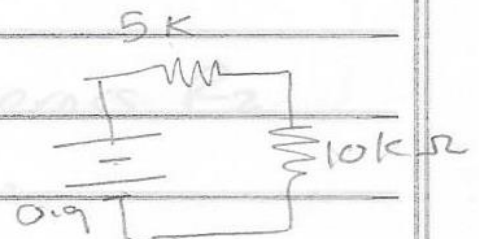
a. determine the actual voltage read by the voltmeter

b. determine the Percent error introduced by the voltmeter's loading

c. if the voltmeter's range is changed to  $5V$ , determine the reading & the percent error.

a)  $S = 10k\Omega/V$ ,  $V_{FS} = 1$

$$R_{total} = S \times V_{FS} = 10k\Omega$$



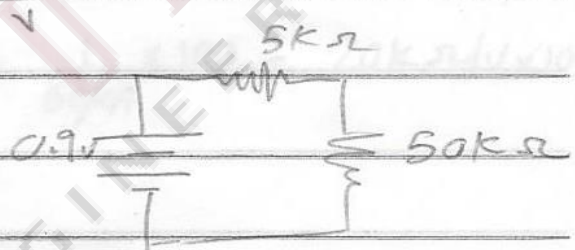
$$V_{out} = 0.9 \left[ \frac{10k}{15k} \right] = 0.6V$$

b) % error =  $\frac{0.9 - 0.6}{0.9} \times 100\% = 33.33\%$

c) S is the same even if the range is changed.

$R_{total} = S \times V_{FS} = 10k\Omega \times 5V = 50k\Omega$

$V_{out} = 0.9 \times \frac{50}{55} = 0.82V$



% error =  $\frac{0.9 - 0.82}{0.9} \times 100\% = 8.9\%$

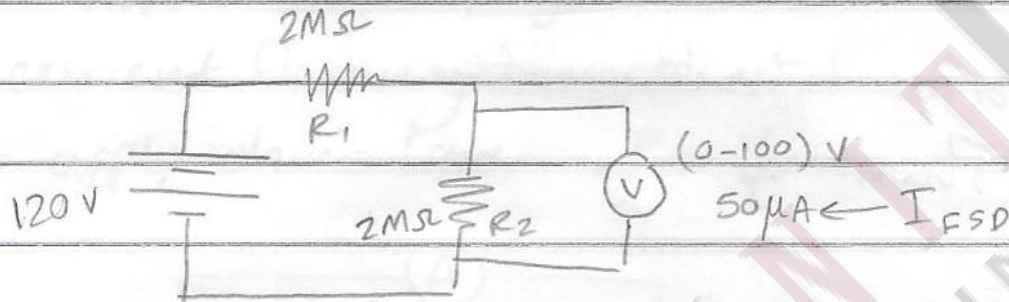
↑ range → ↓ loading error.

but the resolution decreases.

ex: a voltmeter that uses a 50μA PMMC meter is set to the 100V range to measure the voltage across  $R_2$ .

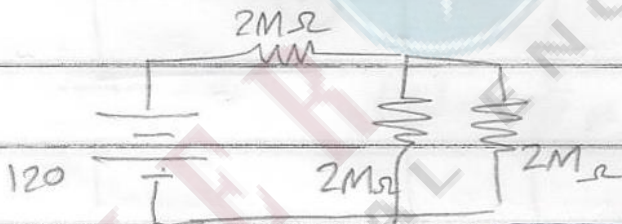
a) determine the voltage read by the meter.

b) determine the voltage across  $R_2$  without the meter loading.



$$a) R_{total} = \frac{S V_{FSD}}{I_{FSD}} = \frac{1}{I_{FSD}} V_{FSD} = \frac{1}{50 \mu A} \times 100 = 20 K \Omega / V \times 100$$

$$= 2 M \Omega$$



$$V_{R2} = 120 \times \frac{1M}{(1+2)M} = \boxed{40V}$$

$$b) V_{R2(\text{without meter})} = 120 \times \frac{2}{2+2} = \boxed{60V}$$

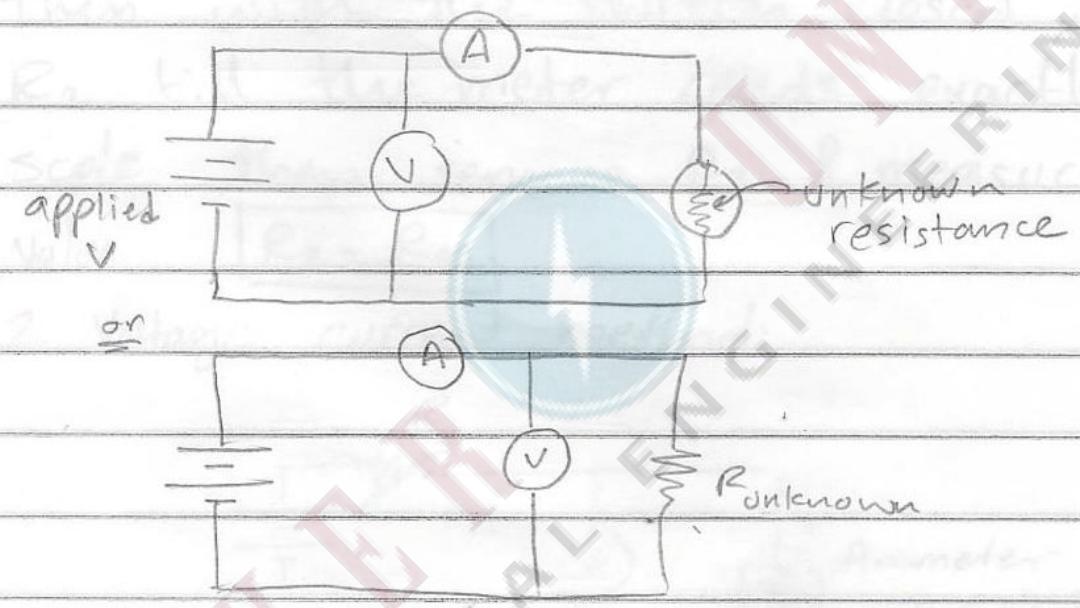
### DC Resistance measurement using Ammeter & Voltmeter:

to measure a DC R using Voltmeter Ammeter method:

1. apply a voltage across the resistance to be measured, ~~measure~~

2. measure the voltage across the R & the current flowing through it.

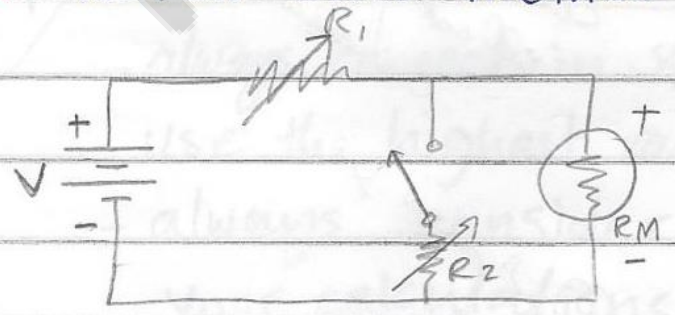
3. apply ohm's law.  $R = \frac{V}{I}$



check the "series type ohmmeter" & "shunt type ohmmeter"

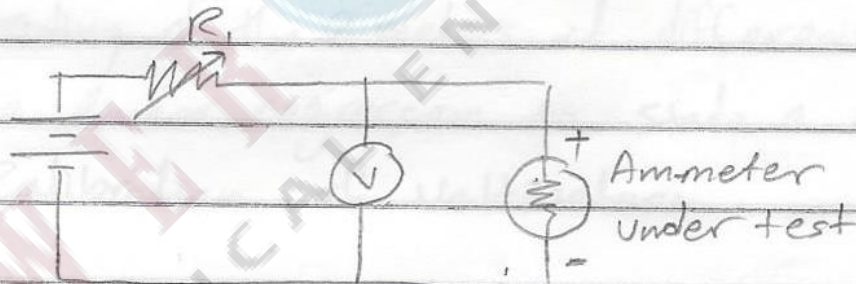
Determination of the internal resistance of a meter:

1. The half scale method:



→ connect the meter as shown in the ckt, with the switch opened,  $R_1$  is adjusted so that the meter reads full scale, then with the switch closed, adjust  $R_2$  till the meter reads exactly half scale. then remove  $R_2$  & measure its value,  $R_2 = R_m$

2. Voltage current method:



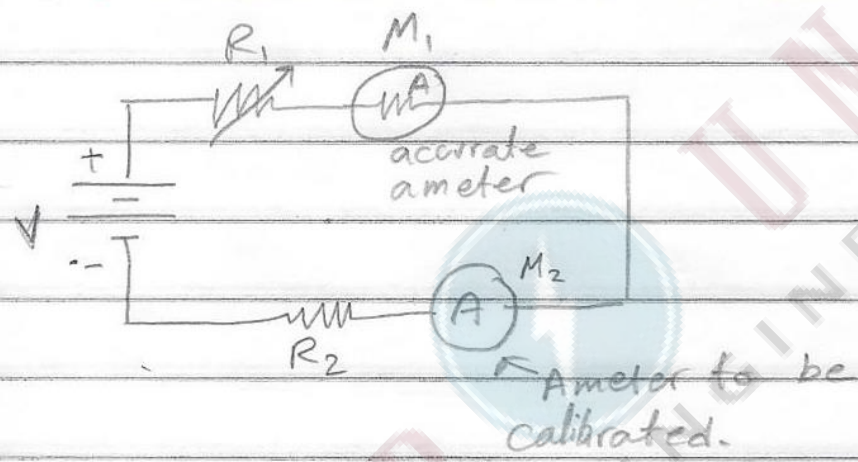
adjust  $R_1$  until the Ammeter reads  $I_{FSD}$  then  $R_m = \frac{V}{I_{FSD}}$  from the voltmeter.

• Precautions before using DC Voltmeter:

- always connect in //.
- use the highest range first then adjust.
- always consider loading effect in your calculations.

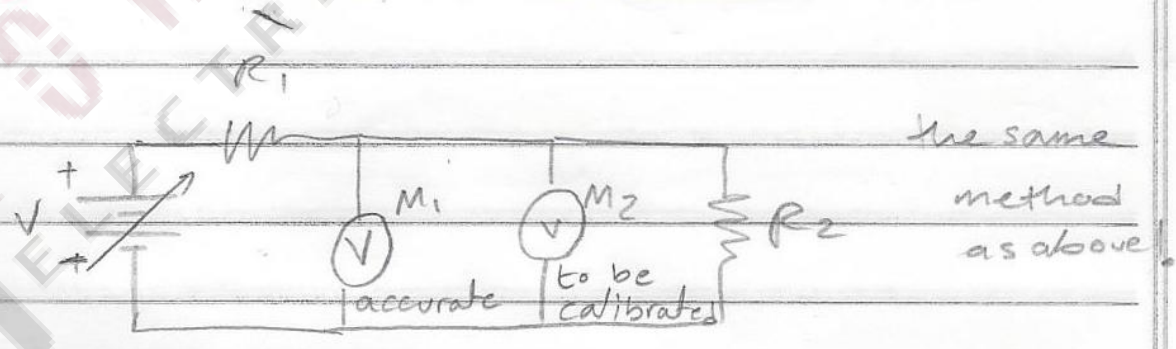
• meter calibration :

→ Calibration of an ammeter:



record reading of the 2 meters at different ( $R_1$ ) values then use linear regression to state a relation between them.

→ Calibration of voltmeter:



→ end of 1<sup>st</sup> exam material.



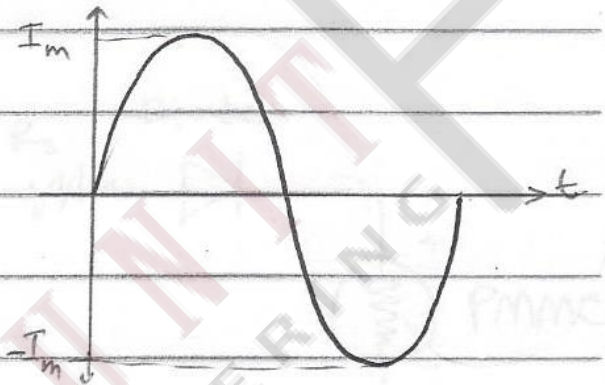
• AC indicating meters:

Revision:

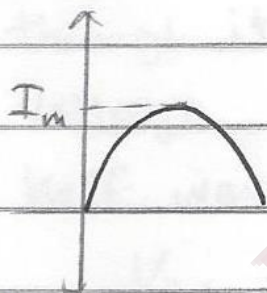
$$i = I_m \sin(\omega t)$$

$$I_{avg} = 0$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

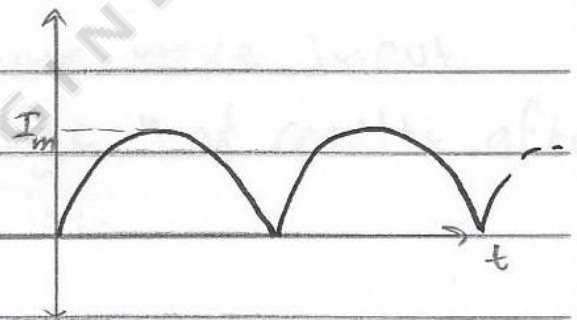


$I_m$  (peak value),  $I_{rms}$  (effective value)



↳ Half-wave rectifier

$$I_{avg} = \frac{I_m}{\pi} = 0.3183 I_m$$



↳ Full-wave rectifier

$$I_{avg} = \frac{2I_m}{\pi} = 0.8366 I_m$$

→ Rectification Instruments:

Note that the PMMC meter reads the average value of the current that passes through it.

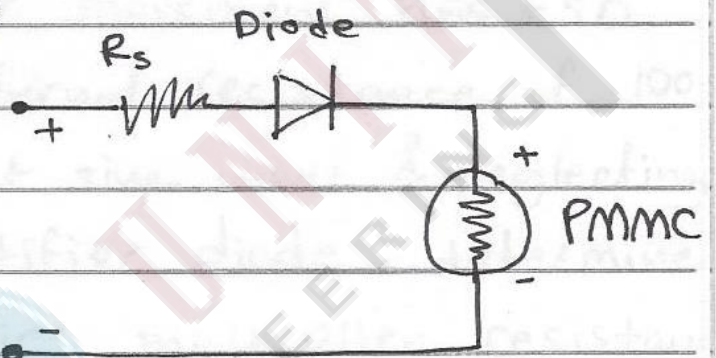
Hence, if an AC sine wave current passes through a PMMC, the meter will read zero.

Thus, in order to use PMMC meter in AC CRT, the AC quantity must be made unidirectional (+ve only or -ve only). which can

be done by using rectifier CKTs.

1) Half-wave rectifier:

Note that in this CKT the meter will read the avg. of the current passing



through it, so for a sine wave input voltage, the avg voltage that results after half wave rectification is:-

$$V_{avg} = \frac{V_m}{\pi}$$

consequently, the AC voltmeter is not as sensitive as the DC voltmeter.

ex:- let's measure  $50V_{rms}$  using the CKT above.

$$V_{avg} = \frac{50\sqrt{2}}{\pi} = 22.5 \text{ Volt} \rightarrow \text{"read by AC Voltmeter"}$$

while the D.C. meter will read 50V

so, the sensitivity of the A.C meter using half-wave rectifier is (45%) of the sensitivity of the DC meter.

$$\uparrow \frac{22.5}{50} \times 100\% = 45\%$$

Example:- using the previous ckt. make an RMS reading AC voltmeter having an FSD of 100 V, the meter movement has FSD of 1 mA & an internal resistance of 100  $\Omega$  assuming a perfect sine wave & neglecting the drop of the rectifier diode, determine the value of the series multiplier resistance

Solution:-

$$R_T = \frac{V_p}{I_p} = R_s + R_m$$

$$V_p = \sqrt{2} V_{rms} = \sqrt{2} (100) = 141.4 \text{ V}$$

$$I_p = \pi \times 1 \text{ mA} = 3.142 \text{ mA}$$

$$\frac{V_p}{I_p} = R_s + R_m$$

$$\frac{141.4}{3.142 \text{ mA}} = R_s + 100 \rightarrow R_s = 44.9 \text{ k}\Omega$$

2) Full-wave rectifier:-

the full-wave rectifier ensures that the current in PMMC meter is in one direction.

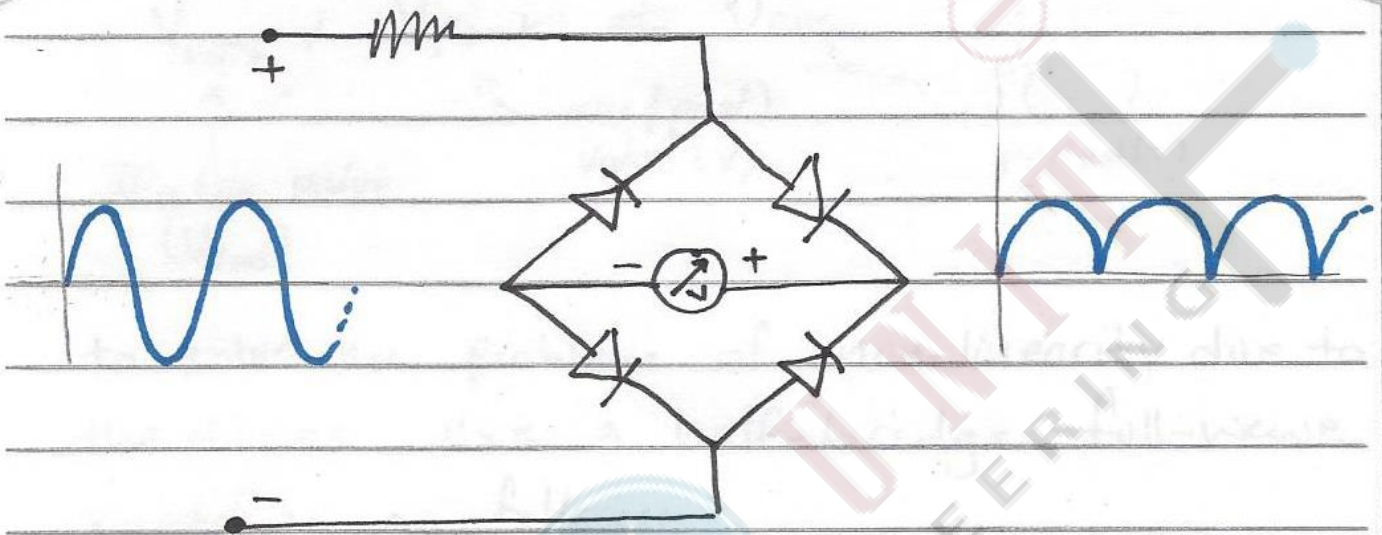
The avg. current in the meter is twice as much as the avg. current in the half-wave rectifier. &  $V_{avg} = \frac{2V_p}{\pi}$

8/4/2014

التاريخ

اليوم

الموضوع



\* The disadvantage of this CKT is the existance of 2 diodes in series with the meter at all time. which will increase the non-linearity of the low level of the applied voltage.

Ex:- An AC voltmeter using full-wave bridge rectifier CKT. is to measure the rms value of a sine wave. the CKT uses an FSD of  $100\mu A$  & an internal meter resistance of  $500\Omega$ .

Assume ideal diodes, determine the value of the series multiplier resistor if the meter is to read  $100 V_{rms}$  full-scale.

Solution:-  $R_T = R_s + R_M = V_p / I_p$

$$I_p = \frac{\pi}{2} I_{avg} = \frac{\pi}{2} (100 \times 10^{-6}) = 157 \mu A$$

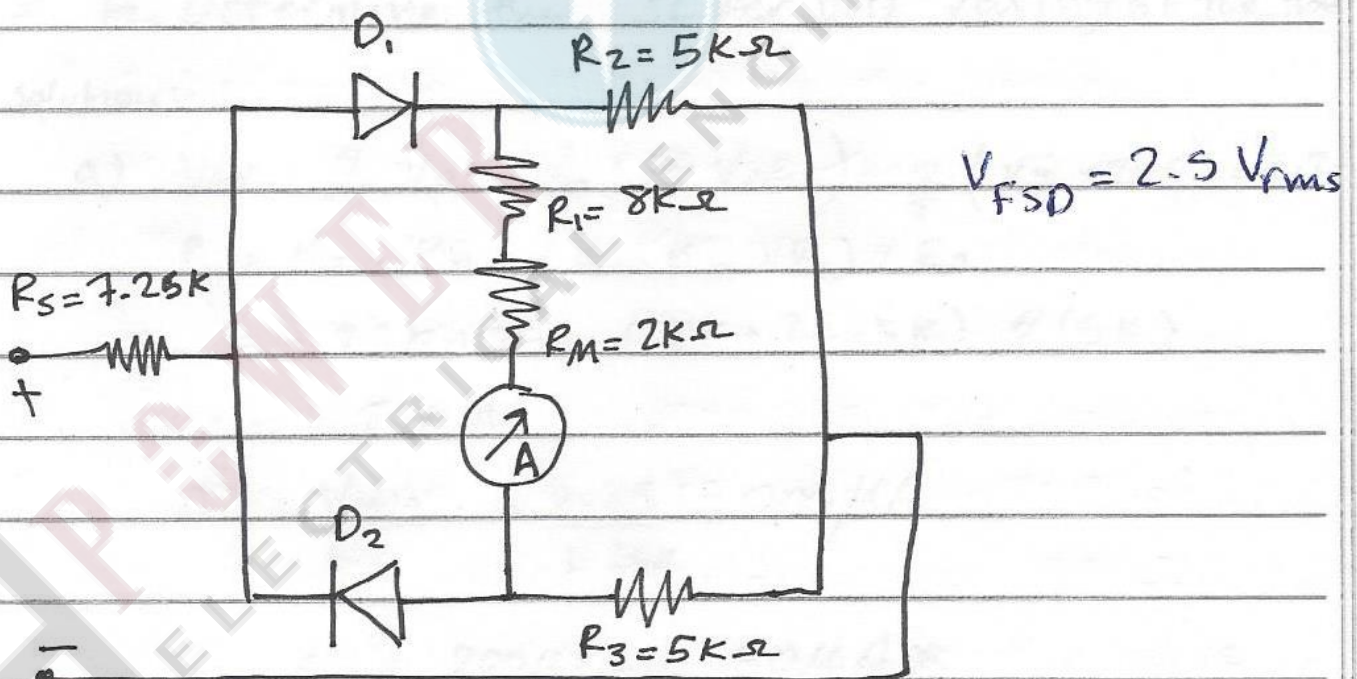
$$R_s + R_M = V_p / I_p$$

$$R_s + 500 = \frac{141.4}{157 \mu} \Rightarrow R_s = 900.1 K\Omega$$

$$V_{RMS} \neq V_{peak} \neq V_{avg}$$

$V_{RMS}$  → effective value ( $V_{rms}$ )  
 $V_{peak}$  → max (peak) value (V)  
 $V_{avg}$  →  $\frac{\int ( )}{\text{Period}(T)}$

to solve the problem of non-linearity due to the diodes use a half-bridge full-wave rectifier as follows:



Note that this type of meters is used in AC multimeter & it's designed in this ckt to have an FSD reading of  $2.5 V_{rms}$ . The advantage of this ckt is that it has only one diode in the path of the current at any time.

in this CKT ( $\frac{3}{4}$ ) the current passes through  $R_2$ , while the remaining ( $\frac{1}{4}$ ) passes through the meter.

ex:- assuming the forward diode resistance is  $250 \Omega$ .

a- verify that the meter measurement must have an FSD of  $50 \mu A$ .

b- Determine the  $\Omega$  per Volt rating of the meter.

Solution:-

$$a) V_{avg} = \frac{2}{\pi} V_p = \frac{2}{\pi} (\sqrt{2} V_{rms}) = \frac{2}{\pi} (\sqrt{2} \times 2.5) = 2.25 V$$

$$R_T = R_s + R_D + (R_1 + R_M + R_3) \parallel R_2$$

$$= 7.25 K + 250 + (8 K + 2 K + 5 K) \parallel (5 K)$$

$$= 11.25 K \Omega$$

$$I_T = \frac{V_{avg}}{R_T} = \frac{2.25}{11.25 K} = 200 \mu A$$

$$I_{meter} = 200 \mu \times \frac{1}{4} = 50 \mu A$$

$$b) S = \frac{R_T}{V_{FSD}} = \frac{11.25 K}{2.5 V} = 4.5 K \Omega / V$$

9/4/2014

التاريخ

اليوم

الموضوع

• The Iron-vane meter:- (depends on magnetic repulsion)

\* Note that the Iron-vane meter is used for the measurement of both AC voltage & current at low freq. (it can be also used for D.C measurements).

The meter's moving vane & pointer are attached to a shaft mounted within a bearing & they're free to rotate over a limited range, while the restoring torque is provided by a spiral spring.

\* The basic iron-vane movement depends on magnetic repulsion.

\* The construction of the meter produces corresponding north & south poles on opposite ends of the plates when the current flows in the coil. Since the polarity of both plates is the same, they repel each other. This repelling action always produces torque in the same direction.

\* Note that this type of meters can be used for both AC & D.C measurements.

↑ low freq.

It's inexpensive & used where accuracies of (5-10)% are satisfactory.

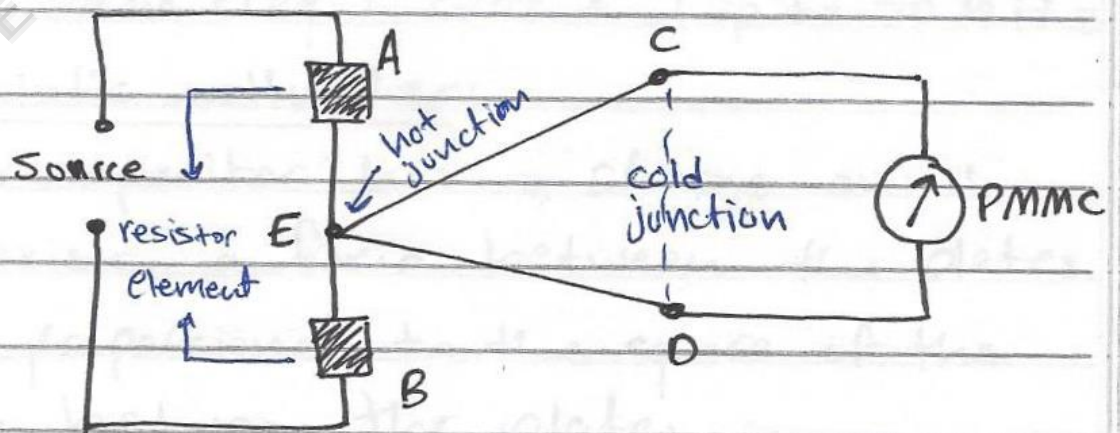
See Figure 1.

• The Thermo Couple meter:

\* Note that when 2 dissimilar metals are joined, a voltage is generated at the junction. where the voltage is proportional to junction's temperature.

So, as junction's temp. increases, so does the voltage across the junction.

This is called the "Seebeck effect" which forms the basis for the thermo-couple meter.



ED → metal #1

EC → ~ #2



the current flowing through the resistor (AB) causes the resistance wire to heat, this heat is transferred to the hot junction of the thermocouple at (E) then the voltage generated by the thermocouple forces a small D.C current through the PMMC meter. Where the heat produced at the thermocouple's junction is proportional to the square of the (rms) current value.

So, the meter has a non-linear square law response, thus, the scale crowds at the lower portion & spreads out at the upper end.

\*The meter is mainly used for measurements at RF (radio freq.) current (up to 50 MHz)

### • Electro Static voltmeter:

When a capacitor has a charge on it, there exists a force between the plates. & it's proportional to the square of the voltage between the plates.

This principle can be used to construct electrostatic voltmeters.

these meters come in two forms:-

1. Attracted Disc.
2. Rotating Plate.

### 1) Attracted Disc Electrometer:-

\* It's simply a horizontal circular plate which is suspended a small distance above a fixed plate.

this causes the electrostatic lines of force to be perpendicular to the plates & uniformly distributed over the area

$$\text{force of attraction} \equiv F = 8.85 \times 10^{-12} \times \frac{V^2 A}{2d^2}$$

A:- area of the moving plate ( $m^2$ )

d:- plate's spacing (m)

V:- voltage between plates (v)

## 2) Rotating plate electrostatic voltmeter.

(see figure 2):

this meter consists of 2 or more semicircular plates similar to a variable tuning capacitor, one plate is fixed to the frame, while the other is mounted to a shaft that supports a pointer & is restrained by a spiral spring.

Torque  $\propto$  square of the  $V_{rms}$  value.

\* this meter can be used for both AC & D.C over a wide range of freq.

## ● Electro dynamo meter:

(see figure 3).

\* Note that the electro dynamo meter's principle is similar to the PMMC's, except that the permanent magnet is replaced with electromagnet (coils).

The polarity of both <sup>the</sup> electromagnet & the moving coil changes with the direction of the coil. Hence, the resultant torque is always unidirectional (upscale).

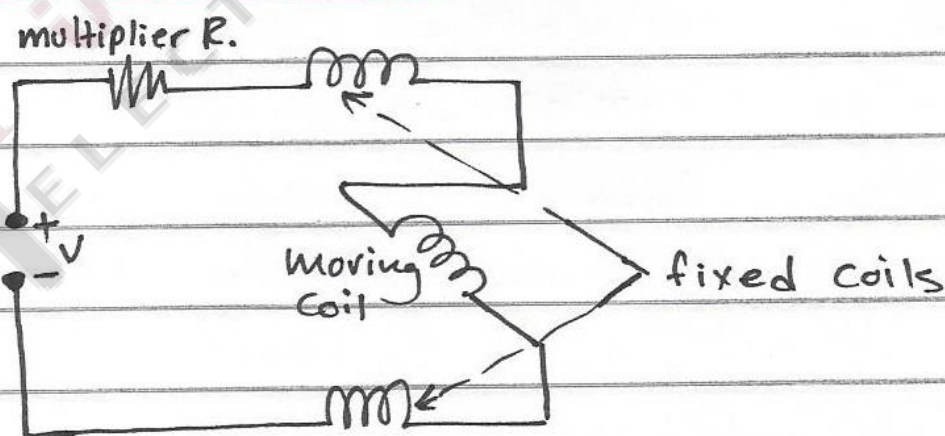
\* It's used for A.C & D.C measurements & it gives the RMS value of the waveform without the need of rectification.

\* The torque developed by the moving coil is proportional to the product of the magnetic field strength & the current flowing through the moving coil.

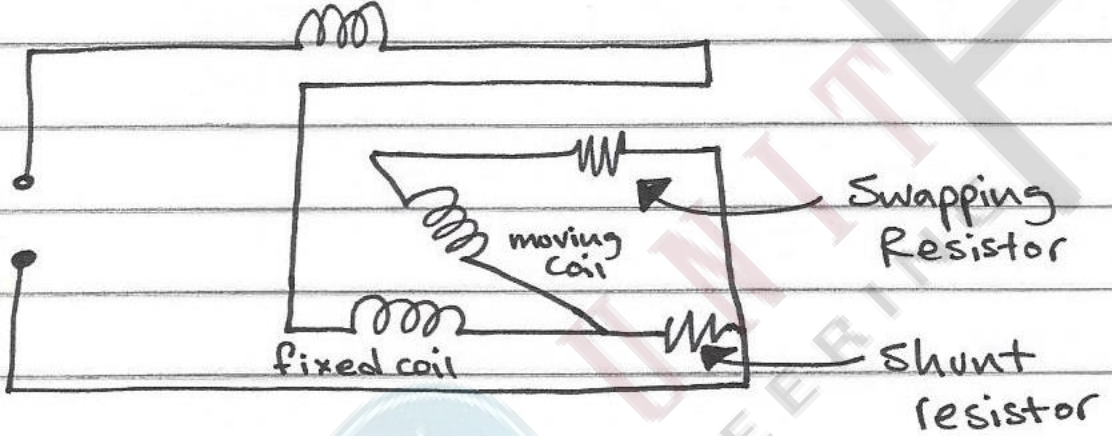
\* The freq. rate of operation is up to 200 Hz so it can be used in power lines.

\* this meter can be used for Voltage & current measurements

\* Voltmeter's connection:-



\* Ameter's connection:



Iron-vane movement that depends on magnetic repulsion.

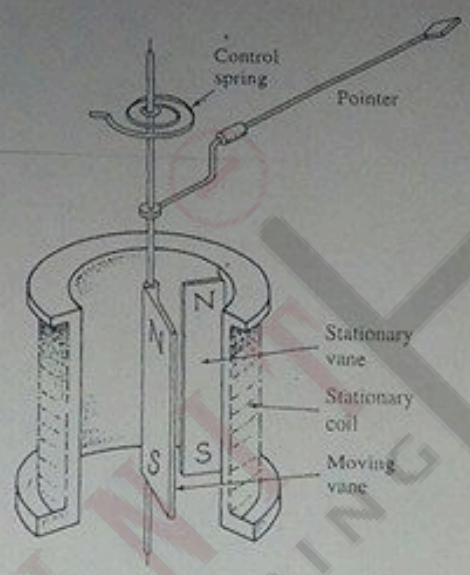
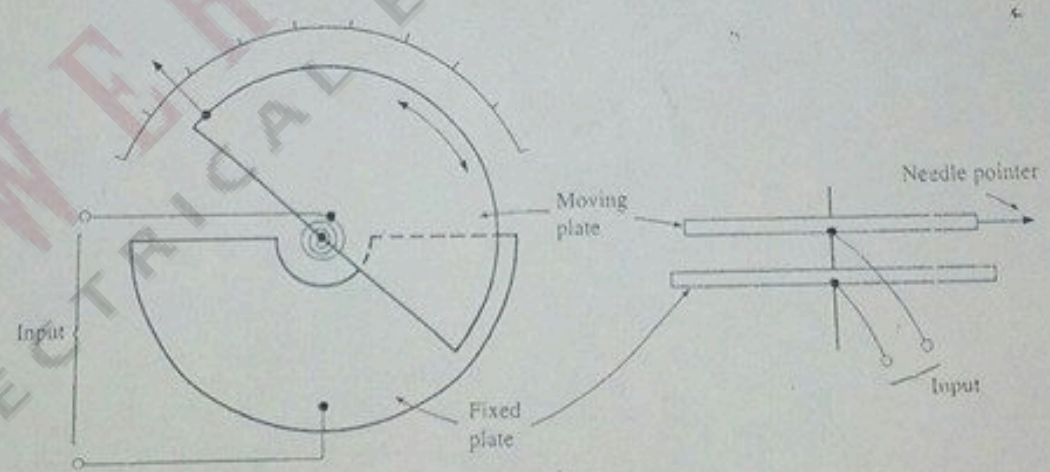


Figure 1

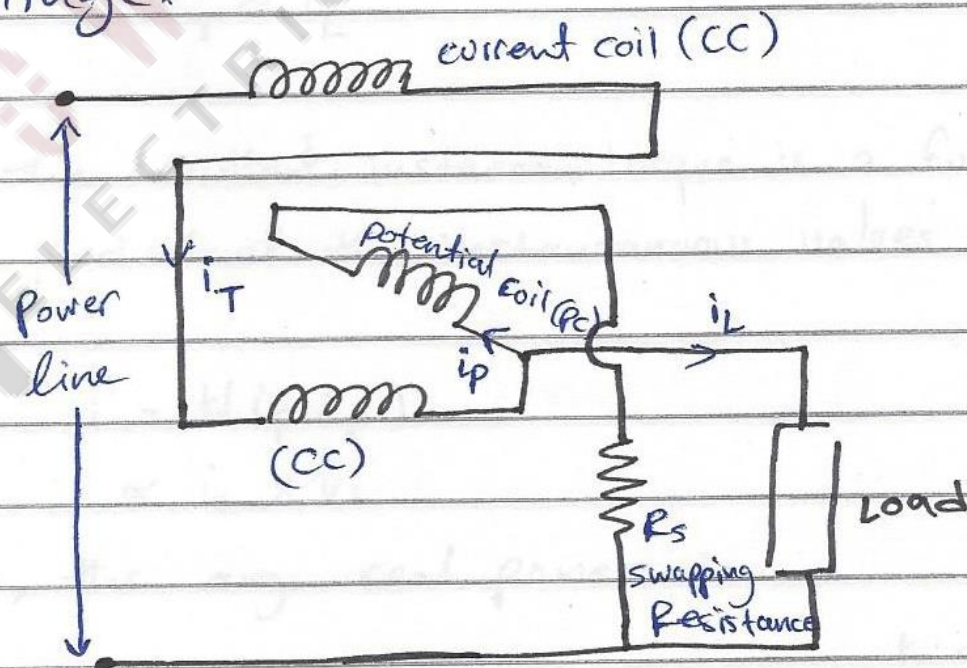


The rotating plate electrostatic voltmeter.

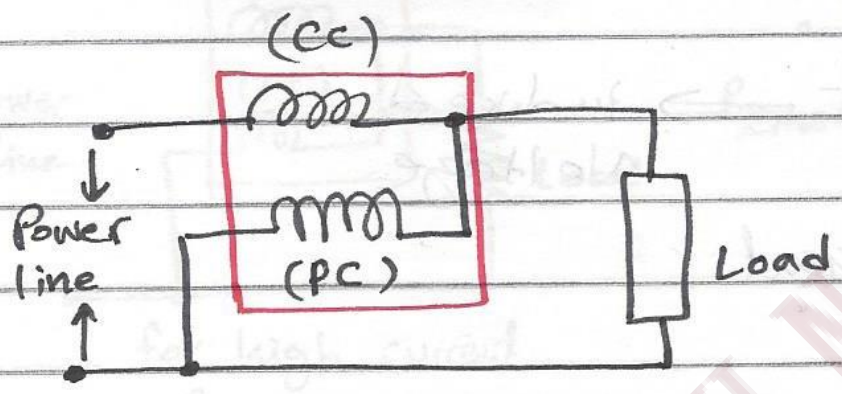
Figure 2

## • Single Phase Wattmeter

\* a single phase low freq. Watt meters ( $< 400\text{Hz}$ ) for Power measurements are constructed with electrodynamic meter. Where the fixed current coil (CC) is connected in series with load & it carries current approximately equal to the load current. & the moving potential coil (PC) is in series with fixed resistor is connected across the Power line & carries a small current proportional to the line voltage.



Single - phase Wattmeter.



Simplified CKT.

\*Note that the moving coil current ( $i_p$ ) is usually in the range (10-50)mA

$$i_T = i_p + i_L$$

$i_L$  usually is  $\gg i_p$  ,  $i_p = \frac{V_L}{R_s}$  ← very large

$$\therefore i_T \approx i_L$$

So, the resultant instantaneous torque is a function of the product of the instantaneous values  $i_p, i_T$

$$P = k(i_T \times i_p)$$

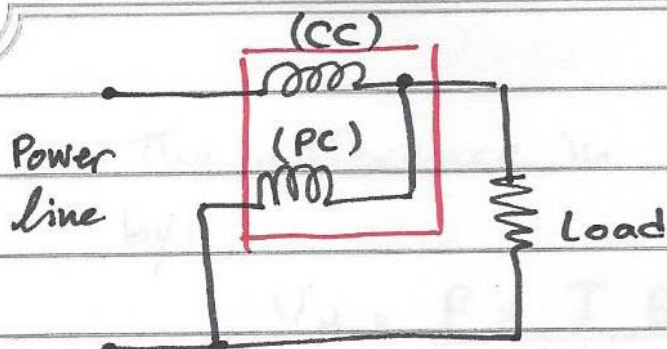
$$\propto i_L \times V_L$$

Then, the avg. real power is:

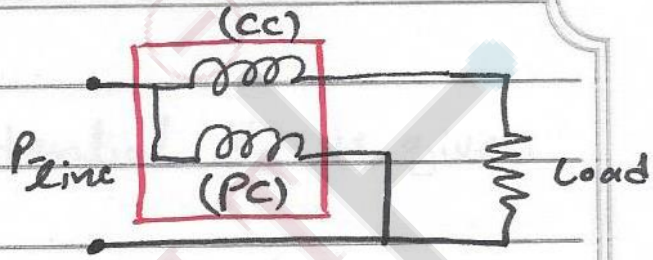
$$P_{avg} = \frac{k}{T} \int_0^T (V_L i_L) dt$$

k: constant  
T: period of one cycle





for high current  
& high voltage



low current  
& high voltage  
(I need current's reading to be more accurate)

• The hall effect wattmeter: see figure (1)

\* Note that when a strip of conducting material that carries current in the presence of transfer magnetic field, as shown in the figure, then a potential difference will be produced between the two plates of the conductor which are perpendicular to the field & the current

Semiconductor materials usually gives sufficient output to use full applications. This phenomenon is called "the Hall Effect"

→ The difference in potential  $V_H$  is given by:

$$V_H = \frac{R_H I B}{d}$$

$R_H$ : the Hall coefficient of the material.

→ the "Hall element" is a device that produces an output voltage that is proportional to the product of the current & flux density.

\* Note that the magnetic field is a function of load current, & the current through the hall element is a function of the load voltage. So the hall output voltage is a function the product of load voltage & load current. & is proportional to the instantaneous power dissipated in the load.

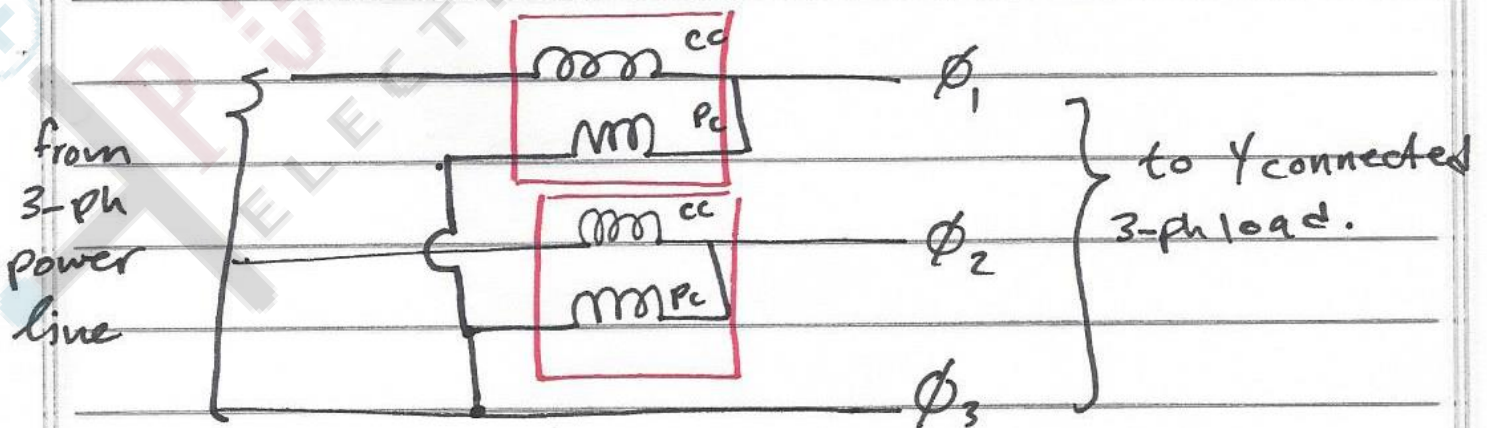
\* Note that, a PMMC meter may be used to average the Hall output voltage & produce a reading that represents the avg. power.

• Poly-phase Power measurements:

1. Poly phase measurements using single-phase wattmeter: (used for balanced systems)

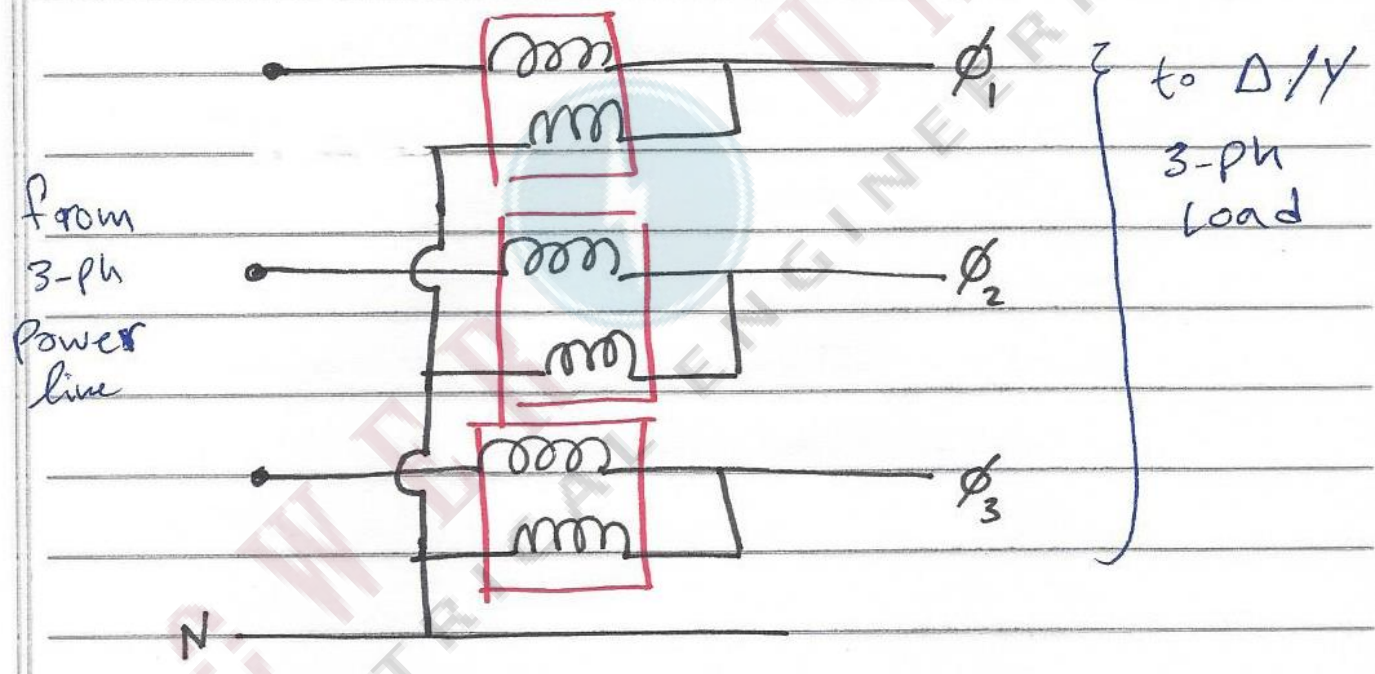
1-1 In this case we can use only 1 wattmeter & measure the power consumed in 1 leg of the load. then multiply it by # of phases to obtain total power.

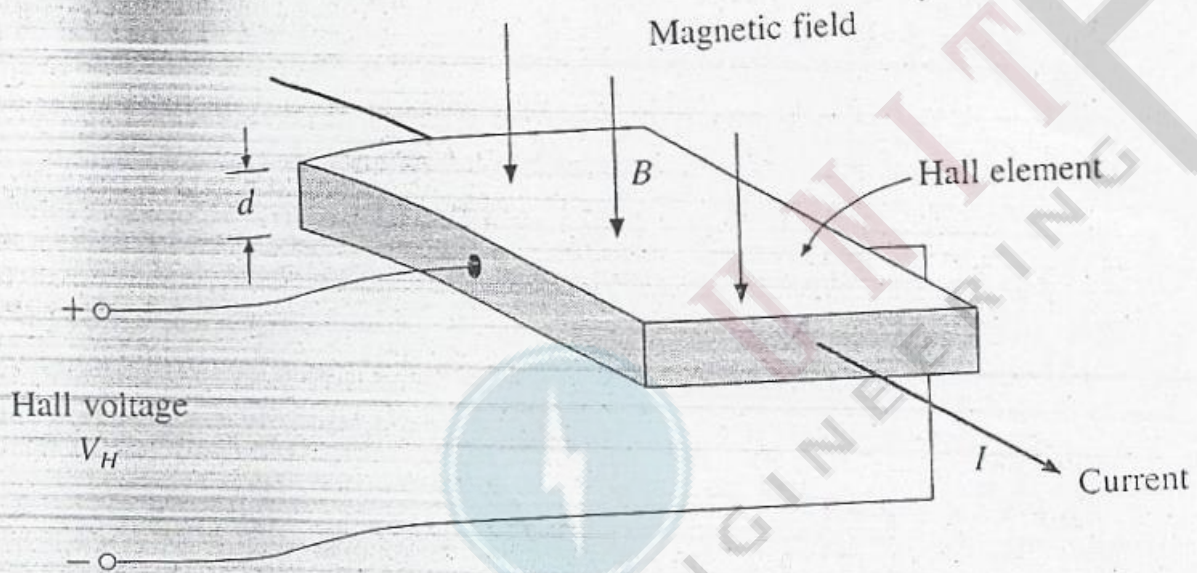
1-2 In three phase system its possible to measure the total power using only two single phase wattmeters as shown below.



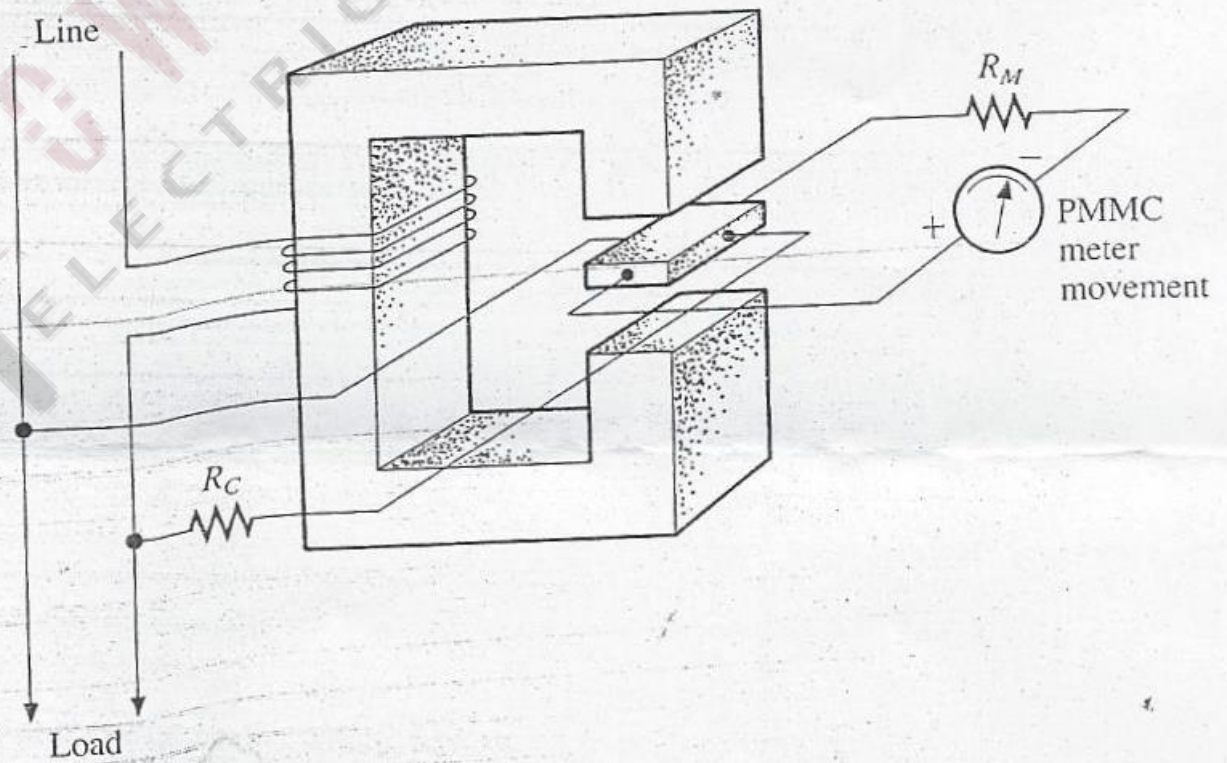
to calculate  $P_{total}$  in  $n$ -phase system we need  $(n-1)$  wattmeters.

\* in 3-ph. 4-wires system, 3 wattmeters are connected as shown below, so the total 3-phase power is the sum of the 3 meters readings.





The Hall effect.



Hall watt transducer.

16/4/2014

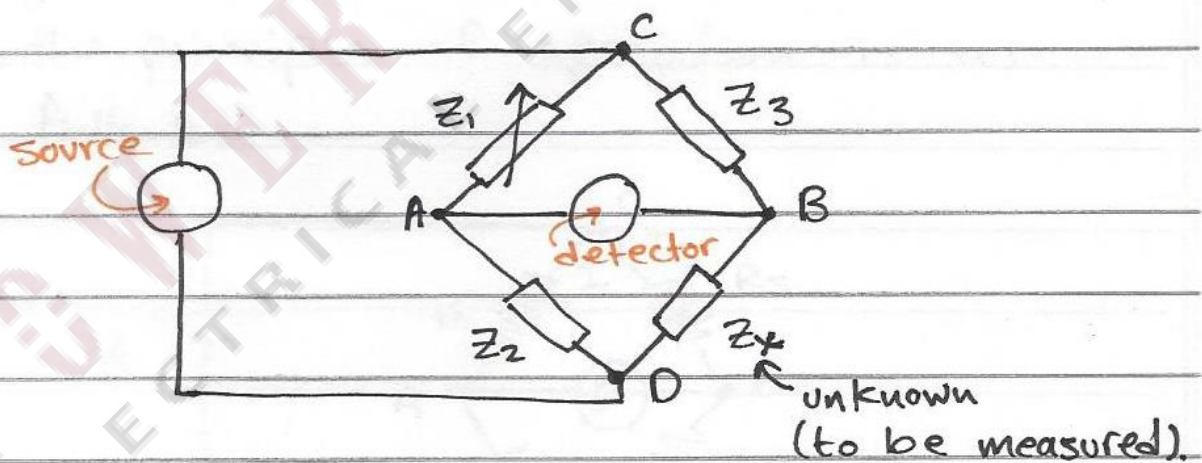
التاريخ

اليوم

الموضوع

### • AC & DC bridges:

- Bridge CKTs are used to measure many quantities such as resistance, capacitance, inductance, Impedance, Admittance & frequency.
- In general, a bridge CKT consists of 4 branch networks with two ports, one is connected to excitation source (input) & the other is connected to the output detector as in the figure.



Note that if  $Z_1$  is adjusted until the detector reads zero, in this case the voltage difference between A & B is zero, so the bridge is said to be balanced, then the unknown  $Z_x$  can be determined in terms of the unknown impedance so, the bridges are considered

## Comparison instruments.

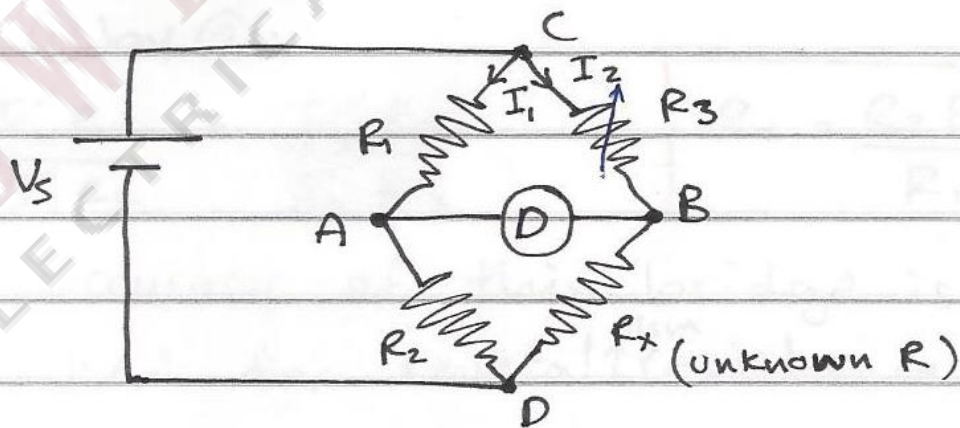
### • Resistance bridge:

based on the magnitude of quantity to be measured several resistance bridges are available such as:-

#### 1. Wheatston bridge:

which can be used for resistances greater than several ohms.

the principle of operation is as follows:



$R_3$  is adjusted until the detector reads zero volt, this means that  $V_{AB} = 0$

$\therefore V_A - V_B = 0 \Rightarrow V_A = V_B$ , so there's no current flow in the detector branch.  
hence, the current through  $R_1$  will

also flow through  $R_2$ , and using the same argument,  $I_2$  passes through  $R_3$  &  $R_x$ .

Consider branches AC & BC, then:

$$V_{CA} = V_C - V_A$$

$$V_{CB} = V_C - V_B$$

Since C is common point, then

$$V_A = V_B \Rightarrow I_1 R_1 = I_2 R_3 \dots (1)$$

also,

$$V_{AD} = V_{BD}$$

$$I_1 R_2 = I_2 R_x \dots (2)$$

Divide (1) by (2):

$$\frac{I_1 R_1}{I_1 R_2} = \frac{I_2 R_3}{I_2 R_x} \rightarrow R_x = \frac{R_2 R_3}{R_1}$$

the accuracy of this bridge is 0.1% & is used for several <sup>ohm</sup> resistance.

example:-

A Wheatstone bridge has the following resistance at balance:

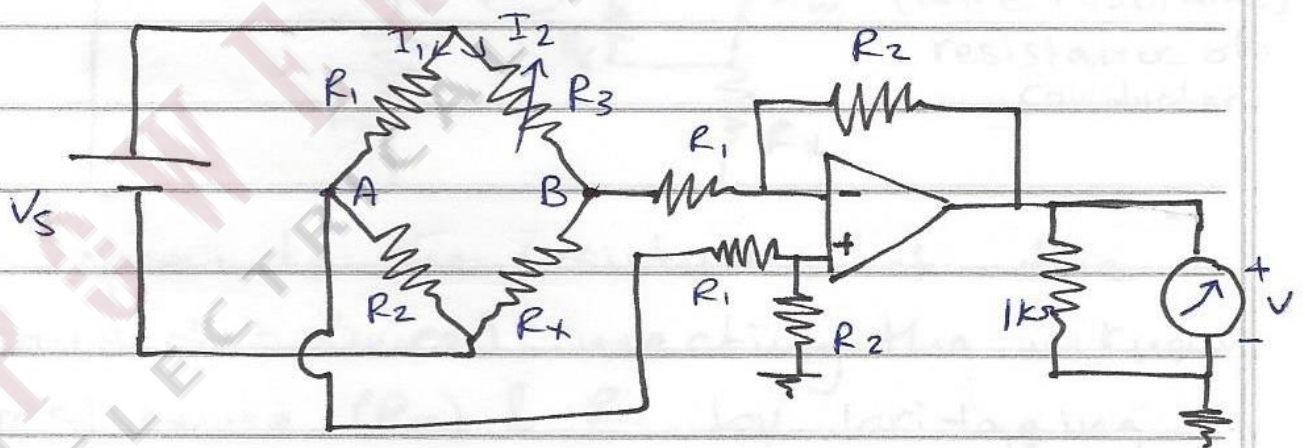
$$R_1 = 10K\Omega, \quad R_2 = 5K\Omega, \quad R_3 = 2786\Omega$$

find  $R_x$ ?



Sol:-  $R_x = \frac{R_2 R_3}{R_1} = \frac{(5K)(2786)}{10K} = 1393\Omega$

- to increase the sensitivity of wheatstone bridge, the galvanometer can be replaced by a voltmeter with an operational Amplifier connected as differential Amp. This amplifier expands the voltage difference between A & B for more accurate balance, as shown in the figure.

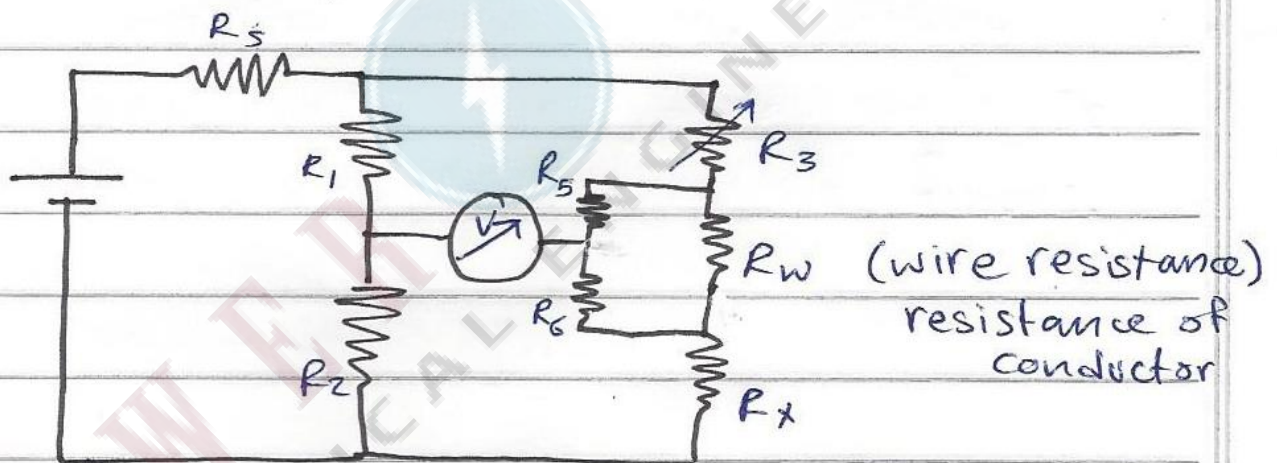


$$V_o = \frac{R_2}{R_1} (V_A - V_B)$$

## 2. Kelvin bridge:

When measuring values of small resistors below  $1\Omega$  the small resistance of leads or wires & electrical conductance

can cause a significant error when using a wheatstone bridge. So, modified wheatstone bridge is used to allow accurate resistance measurements ( $1\mu\Omega - 1\Omega$ ) this bridge is called "kelvin bridge".



$R_w$ : represents the resistance of the conductor (wire) connecting the unknown resistance ( $R_x$ ) &  $R_3$  by bridging  $R_w$  with 2 resistances  $R_5$  &  $R_6$  that have the same ratio as  $R_1$  &  $R_2$  then the effect of the voltage drop across  $R_w$  is eliminated.

At balance, voltage across  $R_2$  equals the voltage across the series combination of  $R_5$  &  $R_x$ , so:

$$R_x = \frac{R_2 R_3}{R_1} + \frac{R_5 R_w}{R_5 + R_6 + R_w} \left( \frac{R_2}{R_1} \cdot \frac{R_6}{R_5} \right)$$

- AC bridges:

- ↳ Capacitance bridge:

at balance:

$$Z_1 Z_x = Z_2 Z_3 \quad \text{--- (1)}$$

$$Z_1: R_1, \quad Z_2: R_2$$

$$Z_3 = \frac{1}{j\omega C_1}, \quad Z_x = \frac{1}{j\omega C_x}$$

Put into (1)

$$R_1 \frac{1}{j\omega C_x} = R_2 \frac{1}{j\omega C_1}$$

$$\boxed{C_x = \frac{R_1}{R_2} C_1} \quad [\text{Farad}]$$

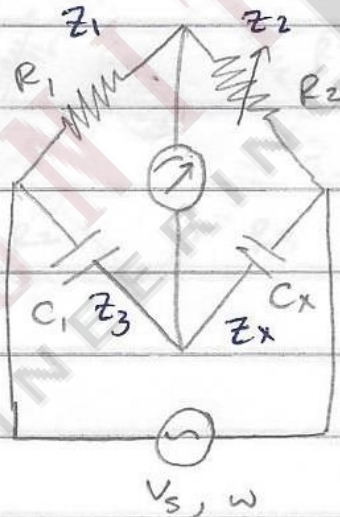
ex: in the capacitance bridge  $C_1$  is a standard capacitor whose value =  $0.01 \mu\text{F}$ ,  $R_1 = 100 \Omega$ , if  $R_2$  can be adjusted from  $[50-500] \Omega$  determine the range of the unknown capacitor.

$$C_x = \frac{R_1}{R_2} C_1 \quad \rightarrow \text{for } R_2 = 500 \Omega$$

$$\rightarrow \text{for } R_2 = 50 \Omega \quad C_x = 0.002 \mu\text{F}$$

$$C_x = 0.02 \mu\text{F}$$

$$C_x \in [0.002 \mu\text{F}, 0.02 \mu\text{F}]$$



a) Series Resistance Capacitance comparison bridge:

at balance:-

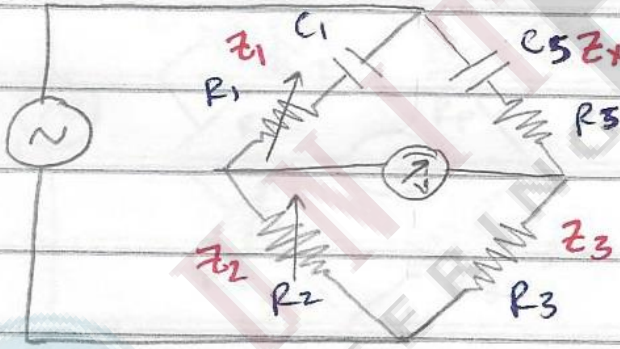
$$Z_1 Z_3 = Z_2 Z_x \dots (1)$$

$$Z_1 = R_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_x = R_3 + \frac{1}{j\omega C_s} = R_3 - \frac{j}{\omega C_s}$$



plug into (1)

$$\left( R_1 - \frac{j}{\omega C_1} \right) R_3 = R_2 \left( R_3 - \frac{j}{\omega C_s} \right)$$

$$\frac{R_1 R_3 - j R_3}{\omega C_1} = \frac{R_2 R_3 - j R_2}{\omega C_s}$$

$$\rightarrow R_1 R_3 = R_2 R_3 \rightarrow \boxed{R_s = \frac{R_1 R_3}{R_2}}$$

also

$$\rightarrow \frac{R_3}{\omega C_1} = \frac{R_2}{\omega C_s} \rightarrow \boxed{C_s = \frac{R_2 C_1}{R_3}}$$

these values are independent on freq. thus it can be applied at any freq.

### b. Parallel RC bridge comparison bridge:

at balance:-

$$Z_1 Z_3 = Z_2 Z_x \quad \dots (1)$$

$$Z_1 = \left( \frac{R_1 \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} \right)$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_x = \left( \frac{R_p \frac{1}{j\omega C_p}}{R_p + \frac{1}{j\omega C_p}} \right)$$

Plug into (1):

$$\frac{\frac{R_1 R_3}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_p R_3}{R_p + \frac{1}{j\omega C_p}}$$

$$R_1 R_3 R_p C_p \omega - j R_1 R_3 = R_1 R_2 R_p \omega C_1 - j R_p R_2$$

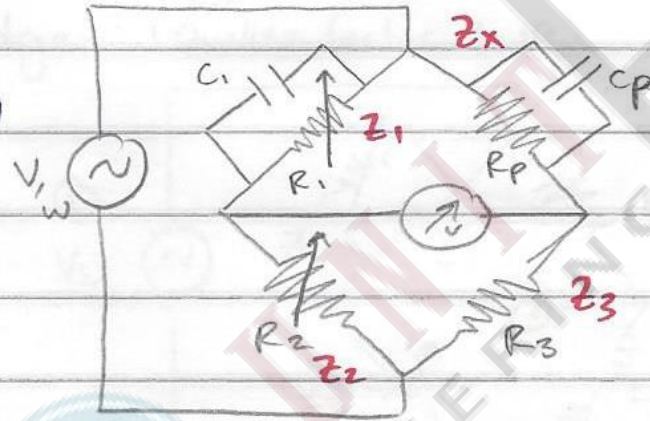
$$\rightarrow C_p R_3 = C_1 R_2$$

$$\boxed{C_p = C_1 \frac{R_2}{R_3}}$$

$$\rightarrow R_1 R_3 = R_p R_2$$

$$\boxed{R_p = \frac{R_1 R_3}{R_2}}$$

these values are independent on freq. thus it can be applied at any freq.



↳ Inductance bridges:

a) Maxwell Bridge: (Quality factor 1-10)

at balance:

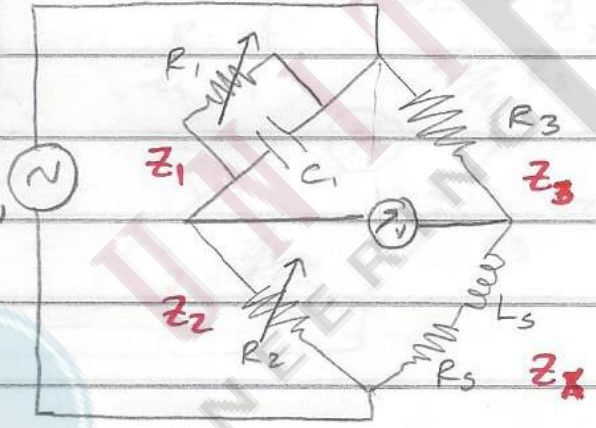
Z<sub>1</sub>Z<sub>x</sub> = Z<sub>2</sub>Z<sub>3</sub> ... ①

Z<sub>1</sub> =  $\frac{R_1 \times \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}}$

Z<sub>2</sub> = R<sub>2</sub>

Z<sub>3</sub> = R<sub>3</sub>

Z<sub>x</sub> = R<sub>s</sub> + jωL<sub>s</sub>



plug in ①:

$\frac{\frac{R_1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} (R_s + j\omega L_s) = R_2 R_3$

→  $L_s = R_2 R_3 C_1$

→  $R_s = \frac{R_2 R_3}{R_1}$

Z<sub>x</sub> = R<sub>s</sub> + jωL<sub>s</sub>

plug in ① and simplify:

$(\frac{R_1}{j\omega C_1}) (R_s + j\omega L_s) = (R_1 + \frac{1}{j\omega C_1}) R_3$

### b) Hay Bridge (Quality factor > 10)

at balance:- ( $V = 0$ )

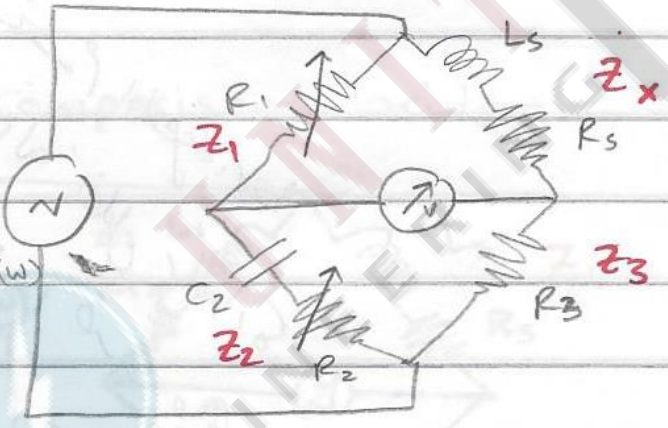
check its derivation.

$$R_s = \frac{(wC_2)^2 R_1 R_2 R_3}{1 + (wR_2 C_2)^2}$$

↓ depends on (w)

$$L_s = \frac{R_s}{R_2 w^2 C_2}$$

$$L_s = \frac{R_1 R_3 C_2}{1 + (wR_2 C_2)^2}$$



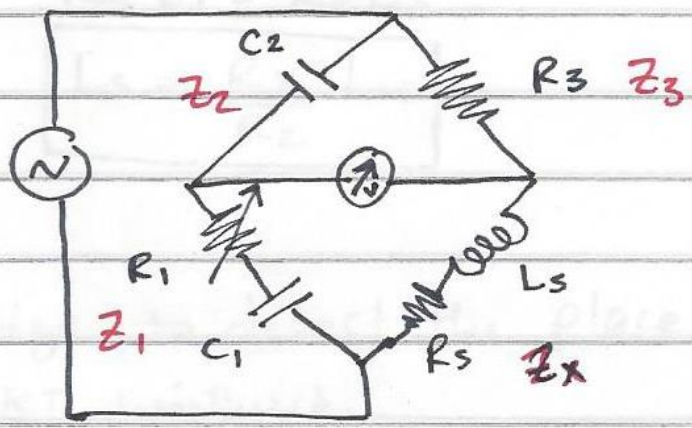
### c) Owen bridge: (for wide range of inductance).

at balance:

$$Z_2 Z_x = Z_1 Z_3 \quad \text{--- (1)}$$

$$Z_1 = R_1 - \frac{j}{wC_1}$$

$$Z_2 = \frac{1}{jwC_2}, \quad Z_3 = R_3$$



$$Z_x = R_s + jwL_s$$

plug in (1) then simplify:



23/4/2014 التاريخ

اليوم

الموضوع

$$\rightarrow R_s = \frac{R_2 R_3}{C_1}, \quad L_s = R_1 R_3 C_2$$

d) Inductance comparison bridge:

at balance:-

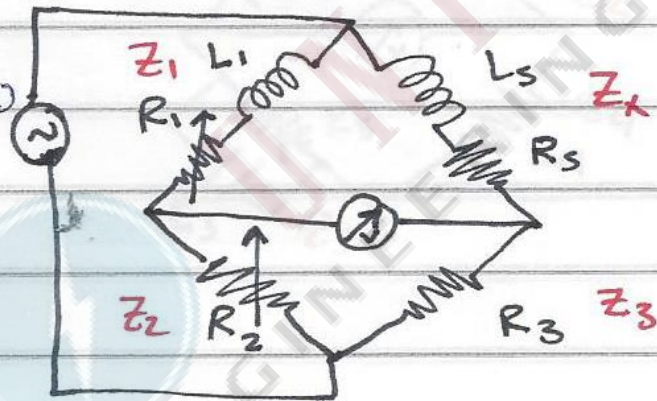
$$Z_1 Z_3 = Z_2 Z_x \quad \text{--- (1)}$$

$$Z_1 = R_1 + j\omega L_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_x = R_s + j\omega L_s$$



into (1)  $(R_1 + j\omega L_1) R_3 = R_2 (R_s + j\omega L_s)$

$$R_1 R_3 = R_2 R_s, \quad R_3 L_1 = R_2 L_s$$

$$R_s = \frac{R_1 R_3}{R_2}$$

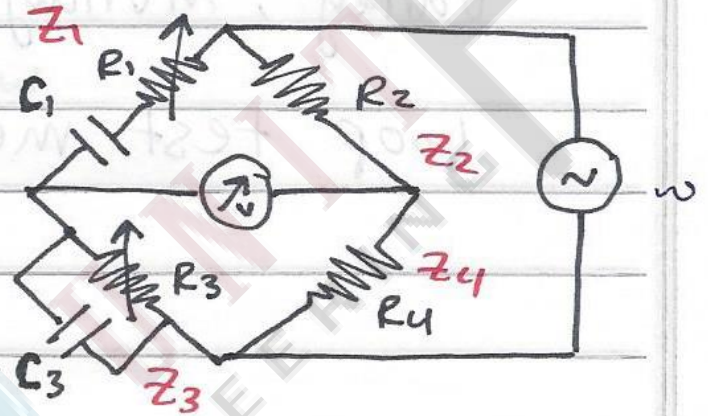
$$L_s = \frac{R_3 L_1}{R_2}$$

H.W: how to use a bridge to detect the place of errors in a CKT. Introduce

$\rightarrow R_s = \frac{R_1 R_3}{R_2} = \frac{R_1 R_3}{R_2}$   
 $L_s = \frac{R_3 L_1}{R_2} = \frac{R_3 L_1}{R_2}$   
 $R_1 R_3 = R_2 R_s, \quad R_3 L_1 = R_2 L_s$

## • Wien bridge:

Note that this bridge is used to measure the freq. of an AC supply.



$$Z_1 = R_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = R_2$$

$$Z_3 = \frac{R_3 \left( \frac{1}{j\omega C_3} \right)}{R_3 + \frac{1}{j\omega C_3}}$$

$$R_3 + \frac{1}{j\omega C_3}$$

$$Z_4 = R_4$$

at balance:-

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left( R_1 + \frac{1}{j\omega C_1} \right) R_4 = R_2 \left( \frac{R_3 / j\omega C_3}{R_3 + \frac{1}{j\omega C_3}} \right)$$

$$\rightarrow \omega^2 = \frac{1}{R_1 R_3 C_1 C_3}$$

$$R_1 R_4 + \frac{R_4}{j\omega C_1} = \frac{R_2 R_3}{1 + j\omega R_3 C_3}$$

$$R_1 R_4 + \frac{R_4}{j\omega C_1} + j R_1 R_4 R_3 C_3 \omega + \frac{R_4 R_3 C_3}{C_1} = R_2 R_3$$

imaginary parts are equal:-

$$\frac{R_4}{\omega C_1} = R_1 R_3 C_3 \omega$$

$$\boxed{\omega^2 = \frac{1}{R_1 R_3 C_1 C_3}} \quad \#$$

$$f = \frac{1}{2\pi \sqrt{R_1 R_3 C_1 C_3}}$$

## • Transducers:

- \* Transducer is any device that converts energy from one form to another. & in electrical engineering measuring systems, a transducer develops a usable electrical output signal in response to a specific physical phenomenon, such as: mechanical force, acceleration, pressure, Temp., physical position, light intensity, .... etc.
- \* Transducers may be classified into:
  1. Self-generating transducers:  
e.g: solar cells.  
↳ which develop their own voltage or current as an output when subjected to a specific physical input.
  2. Externally-powered transducers:  
these devices require power from an external source.

## ● Selection considerations & specifications:

1) Sensitivity: it's defined as the ratio of the output per unit input & it must be good enough for the resolution of the system.

2) Range: the transducer must be able to respond over an appropriate range values of the parameters under measurement.

3) Physical properties: <sup>the</sup> transducer must fit properly within the measuring system from mounting, protection, shielding & electrical connection.

4) Loading effect & distortion: all transducers absorb some energy from the physical phenomenon being measured, then it's essential that the transducer must not significantly distort the measured quantity.

5) Frequency response: the transducer must be able to accurately respond to the max. rate of change of the phenomenon being considered.

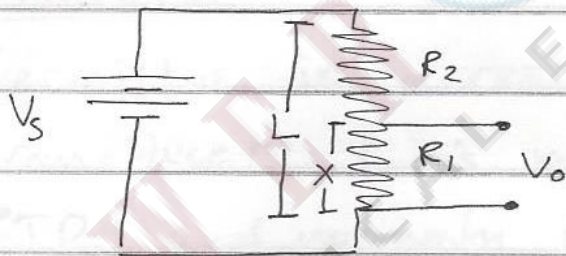
- 6) Electrical output format: the form of the output signal must be compatible with the rest of the measuring system, otherwise there should be some conversion.
- 7) Output Impedance: the output impedance of the transducer must be compatible to the system to avoid loading effects.
- 8) Power requirements.
- 9) Noise: noise is any unwanted signal present in a signal so the output signal of the transducer should be free of noise.
- 10) Accuracy & error
- 11) Calibration: Note that the properties of many transducers can drift with time & aging, this must be compensated by periodic recalibration.
- 12) Environment: transducer's performance is affected by environmental factors such as (temp., humidity & dust, ...)

### 13) Cost.

- Resistance changing transducers: (simple & cheap).

this type of transducers has very wide range of applications, the change of Resistance is achieved by:

1. mechanical linkage
2. direct change of physical parameters



$$V_o = V_s \left( \frac{R_1}{R_1 + R_2} \right)$$

$$\equiv V_s \left( \frac{x}{l} \right)$$

$V_s$  is constant.

$\therefore V_o \propto x$

- Resistance Temperature Detector (RTD):

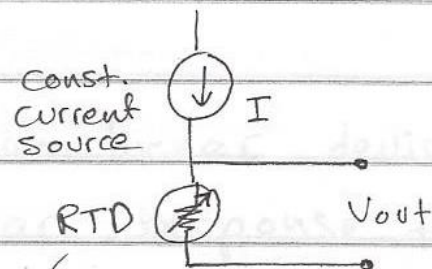
Note that in this

OKT:

$$V_o = R_{RTD} \times I$$

hence, any change of

$R_{RTD}$ , which is proportional to the temp, will produce proportional change in  $V_o$ .



it's linear over a wide range of Temp.

\* it has a (+ve) Temp. coefficient. that is virtually linear over a certain range of temp.

The RTD resistance is given by:-

$$R_T = R_0 [1 + \alpha(T - T_0)]$$

↳  $R_0$ :  $R_{RTD}$  at a standard temp. reference ( $T_0$ )

↳  $T_0$ : standard Temp. reference.

↳  $\alpha$ : temp. coefficient

↳  $T$ : RTD's temp.

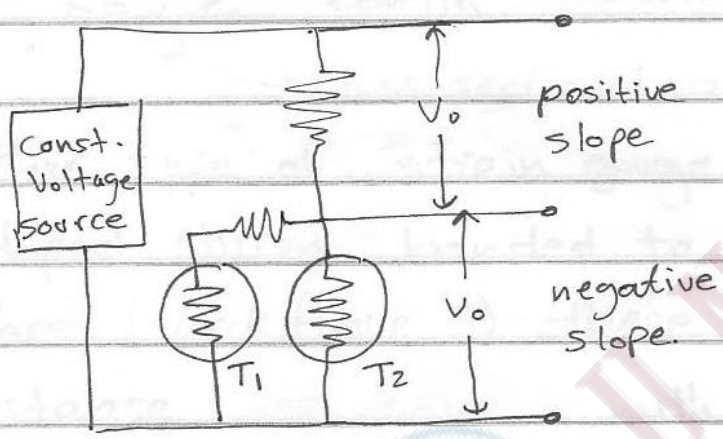
### • Thermistors:

Thermistor is a resistance changing transducer, & it's more sensitive than RTD, and normally it has (-ve) temp. coefficient, but thermistors with (+ve) temp coefficients can be found.

These devices have high temp. coefficients which allows the measurement of small change in temp.

Thermistor is a non-linear device & it can give a linear response to temp. by combining of (2) or more thermistors.





$$V_o = \mp m T + b$$

$m$ : slope.

$b$ : value of  $V_o$  at  $T=0$

$T$ : temp.

• Strain gauge: (check the figure)

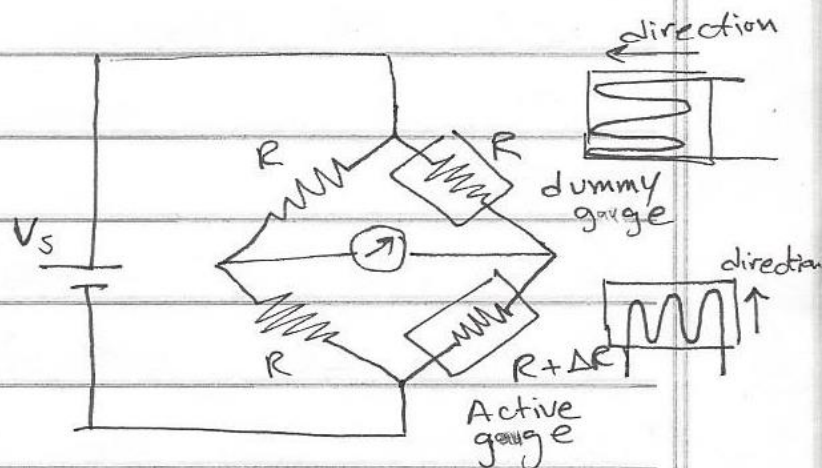
Note that the strain gauge is a resistance changing transducer, it consists of a fine wire (diameter  $\approx 0.0025$  cm) bonded to the face of mounting plates. normally, the wire is looped back & forth many times to allow more length.

by applying a stress along the proper axis, the length of the wire will increase, & the cross sectional area will decrease, so the value of  $R$  will change.

$$R = \frac{\rho l}{A}$$

$\rho$  → resistivity.  
 $l$  → length  
 $A$  → cross-sectional area

- another type of strain gauge uses lengths of doped silicon bonded to a slightly elastic surface. (check figure 3) these silicon-based resistance will change with stress.
- One of the short comings of strain gauge is that the resistance varies with temp. to compensate for this <sup>temp.</sup> effect a second identical strain gauge can be used, placed in a perpendicular direction to the active gauge. The second (dummy) gauge because of its orientation is affected by temp. only. Since temp. changes are equal on each gauge, temp. variations do not affect the output.



wheatstone bridge.

## • Oscilloscope:

→ Note that the Cathode-Ray Oscilloscope (CRO) is a universal instrument that displays waveforms on the phosphor-coated screen of a cathode-ray tube (CRT).

→ The oscilloscope presents a two-dimensional graph of a signal's Amplitude versus time.

horizontal axis.

vertical axis

## - Oscilloscope Basics:

→ the basic oscilloscope has six subsystems:

1. Cathode-ray tube (CRT)

2. Vertical Amplifier.

3. Horizontal Amplifier.

4. Time base

5. Trigger Circuit.

6. Calibrated attenuator.

→ Note that the actual display is created by moving a focused beam of high velocity electrons across the phosphor-coated screen of the CRT.

→ A small spot of light forms where the electron beam strikes the screen; this leaves a glowing trail as the beam moves across the screen. (which is known as trace).

→ Before coming to the screen, the electron beam passes first between a pair of vertical & horizontal deflection plates. A voltage applied to the vertical deflection plates produces an electric field between them, which deflects the electron beam either up or down in the vertical plane.

The direction in which the beam is deflected depends on the relative polarity of the plates (vertical plates)

also, a voltage applied to the horizontal plates deflects the electron beam either right or left in the horizontal plane.

So, by simultaneously applying the proper horizontal & vertical deflection voltages, the electron beam can be directed anywhere on the CRT

- Normally, the observed signal is displayed on the oscilloscope's vertical input.
- this signal must be calibrated to be within the working range of the oscilloscope. So, it first passes through a calibrated input attenuator that permits adjustment of the vertical gain.
- This attenuator usually has front-panel multiposition switch calibrated in terms of (Volts/division)
- Then, the signal is magnified by the vertical amplifier, which has a fixed gain & a <sup>two transistors working simultaneously.</sup> push-pull output stage that drives the vertical deflection plates with the required deflection voltage.
- the horizontal Amplifier also has a push-pull output stage, which drives the horizontal deflection plates. The input to the horizontal Amplifier can be switched between two possible types of input signal:

1) An external signal.

2) An internal time-base generator to provide a horizontal sweep signal.

→ The oscilloscope's internal time-base generator (sweep generator) provides a sawtooth waveform for the horizontal deflection plates to determine (time/division).

→ The trigger CKT initiates the sweep at a particular point in the waveform, and it has two principal controls:-

1) Variable trigger level, which selects the voltage at which the input signal initiates a sweep.

2) Slope; it determines whether the sweep begins on the +ve or -ve-going slope on the waveform.

## ● Cathode Ray Tube: (check the figure)

\* it's a vacuum tube.

\* Note that the tube uses electrostatic focus & electrostatic deflection.

a directly heated cathode releases free electrons when heated by ↑ <sup>enclosed filament</sup> heater

\* the cathode is surrounded by a control grid, which is a cylinder with a small hole on its end for the passage of electrons.

\* the purpose of the control grid is to adjust the magnitude of the electron stream that passes through it on the way to the screen.

→ the more (-ve) the control grid's voltage with respect to the cathode the fewer electrons get through. & the less intense the trace appears on the screen.

→ next, the electrons pass by the 1<sup>st</sup> accelerator anode (H), which is a disc

or cylinder with a small hole at the center.

→ this electrode is kept at a high (+ve) voltage with respect to the cathode in order to accelerate the electrons.

then, electrons pass through focusing electrode (F) that adjusts the voltage on the focus electrode with respect to accelerating anode.

→ then, the electrons pass through a second accelerating anode (A), that is biased at high (+ve) voltage with respect to the cathode.

\* these elements are called "electron gun"

→ at the end, the electron beam strikes the phosphor-coated screen. where the kinetic energy of the electrons is converted to light & heat.

→ Usually the internal part of the tube <sup>(CRT)</sup> is coated with a conducting material called "Aquaday" which provides shielding from stray electromagnetic fields, & prevents



light from striking the back of the screen.

### • Oscilloscope's Probes:

- the signal to be observed is normally connected to vertical input connector of the oscilloscope via a probe & a coaxial cable.
- the probe should not interfere the normal function of the CRT under test & should not distort the signal.
- types of probes:-

#### 1) 1x Probe (times-1 probe)

it's the simplest probe in use & it consists of a length of shielded coaxial cable with a convenient probe tip at one end & a connector compatible with the oscilloscope's input. (usually used for D.C. & low freq. AC measurements) it's not suitable for high freq. applications.

## 2) 10x probe (times-10 probe):

- \* (for high freq. applications)
- \* this probe attenuates the signal by a factor of 10 but there's freq. compensation.
- \* Usually the input impedance of this probe is 10 times <sup>higher than</sup> the impedance of the oscilloscope itself.

## 3) High Voltage probes:

- \* this probe is used for high voltage applications (about 500V), which needs a special attenuator probe.

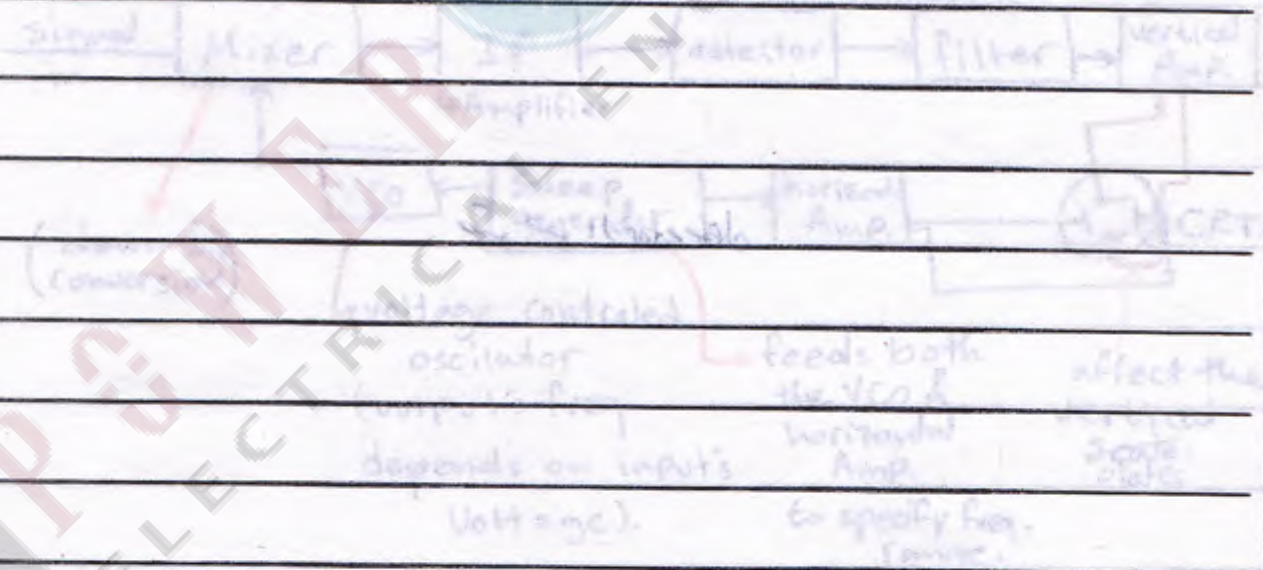
## • The graticule:

(Check the figure)

- \* A rectangular grid, called a "graticule", is placed on or over the screen of a CRT to facilitate accurate measurements of signal voltages & time periods.
- \* In general, it's arranged into a pattern: 8 squares high & 10 wide. Each of them represents a fixed # of units (V/division) & (time/div)

also, these squares have 5 minor divisions to each major division, each minor div represents 0.2x value of the major div.

### 1. Spectrum Analyzer:

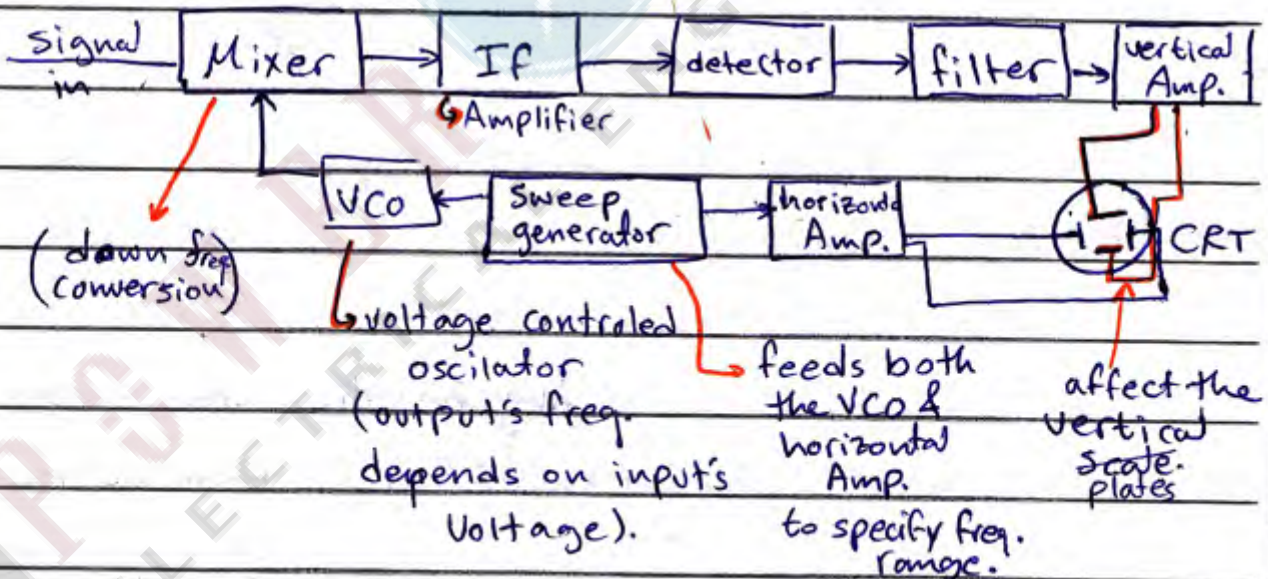


Spectrum analyzer is a narrow BW filter that is swept across a fixed range of freq. while providing a visual display of output amplitude vs. freq.

- Waveform analyzer:

- Note that the oscilloscope is used to analyze the signal in the time domain & the analyzer is used to analyze the signal in the freq. domain.

### 1. Spectrum Analyzer: (it's like the superheterodyne R)



Spectrum analyzer is a narrow BW filter that is swept across a fixed range of freq. while providing a visual display of output Amplitude Vs freq.

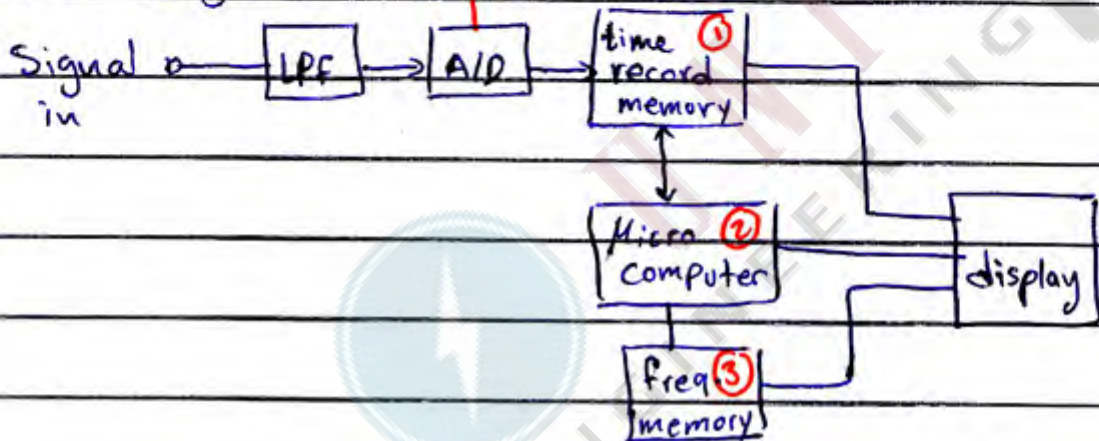
## 2. Fourier Analyzer.

nyquist rate.

$$f_s \geq 2f_m$$

Block diagram:

sampling & quantization & coding.



**Adv** \* it can display both amplitude & phase.

\* better resolution. \* measures the direct transfer func.

① Stores signal sample until its has enough samples to analyzed.

② does an (fft) algorithm. (Fast Fourier Transform)

→ \* the Fourier spectrum of any periodic signal is displayed on a Fourier analyzer based on Fourier series expansion to obtain a Fourier spectrum.

Note that the input signal is filtered using LPF to remove any out-of-band

freq. components & then it's sampled & digitized at a regular intervals until enough samples have been accumulated. the computer computes (FFT) then the results are stored in memory for which it can be displayed in a (CRT)

→ The Fourier analyzer has several advantages:-

1) low freq. coverage.

2) high freq resolution.

3) direct transfer function measurements.

3. Wave analyzer (freq. selective Voltmeter):

The wave analyzer is referred to as a freq. selective voltmeter & it's basically a tuneable finite BW BPF driving a meter CRT. It's tuned over a given range of freq. & the amplitude of the signal's component are displayed on the meter as they come within filter's range.

#### 4. Distortion Analyzer:

→ Note that due to the non-linear elements in a system — harmonic freq. are created from a pure sine wave.

this is known as harmonic distortion.

→ Percent  $N$ th harmonic distortion =  $\frac{V_N}{V_1} \times 100\%$

$V_N$ : rms voltage in the  $n$ th harmonic.

$V_1$ : rms ~ ~ ~ ~ ~ fundamental harmonic.

→ rather than specifying the amount of distortion at a particular harmonic a more generalized measure called "Percent total harmonic distortion %THD" is used.

$$\%THD = \sqrt{(2^{nd}\%)^2 + (3^{rd}\%)^2 + \dots + (N^{th}\%)^2}$$

So, a distortion analyzer is used to determine (THD). & it's a narrow Band reject filter, followed by a broadband detector & a meter ckt. so, it measures the amplitude of the fundamental & harmonics components: as well as any noise that might exist.

also can be written as:

$$\% \text{THD} = \frac{\sqrt{(\text{harmonics})^2 + (\text{noise})^2}}{\sqrt{(\text{fundamental})^2 + (\text{harmonics})^2 + (\text{noise})^2}}$$

### 5. Audio Analyzer:

- its a general purpose audio test instrument that performs several low freq. measurements as well as measuring distortion.

5. Trigger Circuit.

6. Calibrated attenuator.

Note that the actual display is created by moving a focused beam of high velocity electrons across the phosphor-coated screen of the CRT.



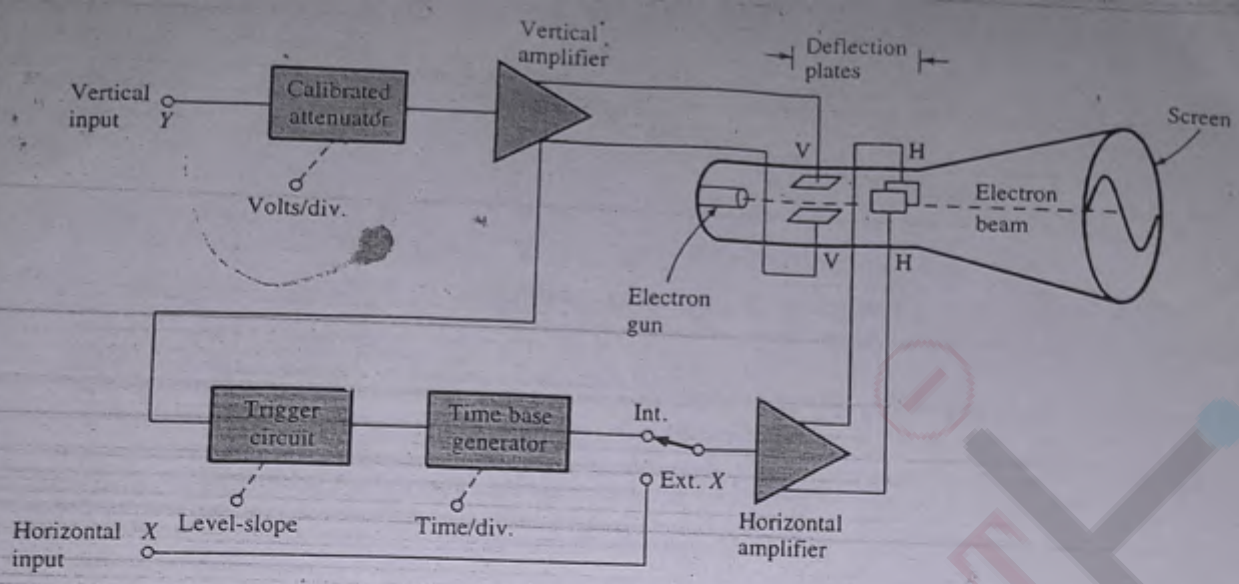


Figure Cathode-ray oscilloscope block diagram.

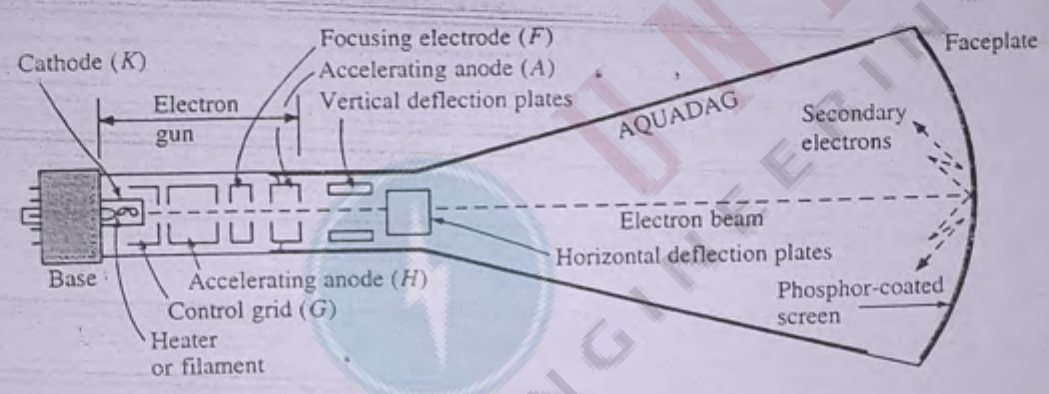
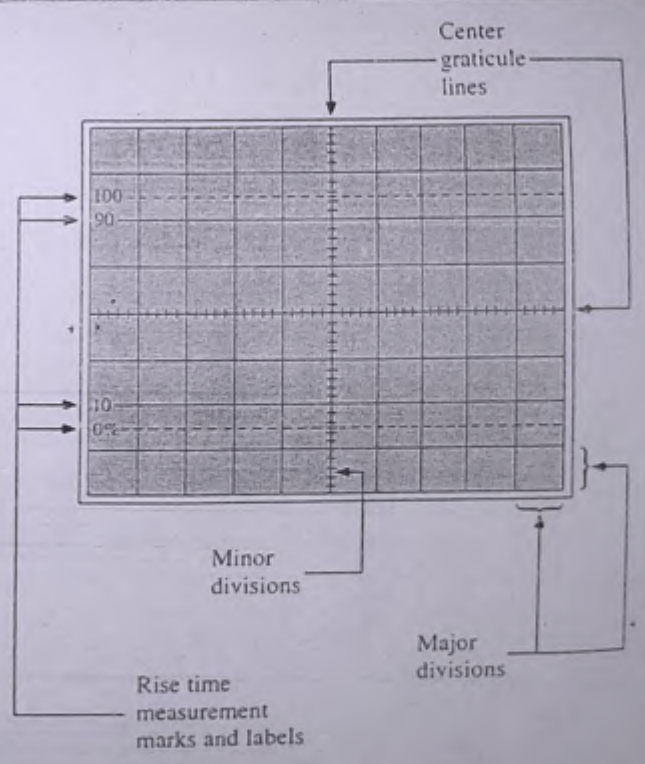


Figure Cathode-ray tube construction.

Figure The graticule.



- Grounds & grounding:

the electrical ground is a low resistance connection between a given CKT & the earth, & it's usually a long copper rod (ground rod) driven at least (4feets) into the earth, & the ground connection made to it!

& it's usually a zero voltage reference.

- AC power line: (check the figure)

single phase power line comprises 3 lines (hot, neutral, ground).

\* ground faults refer to the return of current to ground by any path other than neutral wire.

Note that there are 2 possible paths for current flow in the AC power line:

1. (Hot) to (neutral): is the correct & safe path

2. (Hot) to (ground): where the current returns directly to ground via a

ground conductor rather than a neutral conductor, this situation is called "ground fault".

The current flowing in the gnd. wire as a result of gnd. fault is called the fault current.

this situation is remedied by the installation of gnd. fault interrupter (GFI), which compares current flowing in the hot & neutral lines & disconnects the supply line if the currents aren't equal.

a (GFI) may be actuated for fault currents as low as (5mA)

GFI is a transformer connected to a sensitive AC relay.

As long as the hot & neutral line currents are equal, the two oppositely connected transformer windings cancel each other's magnetic fields & ~~the~~ <sup>no</sup> ~~low~~ voltage is across the winding connected to the relay coil.

1- Potential difference between 2 or more points of a ground plane to which the external ground is connected.

2- inductive coupling.

3- capacitive  $\approx$  between the system & ground.

4- Common-mode noise.

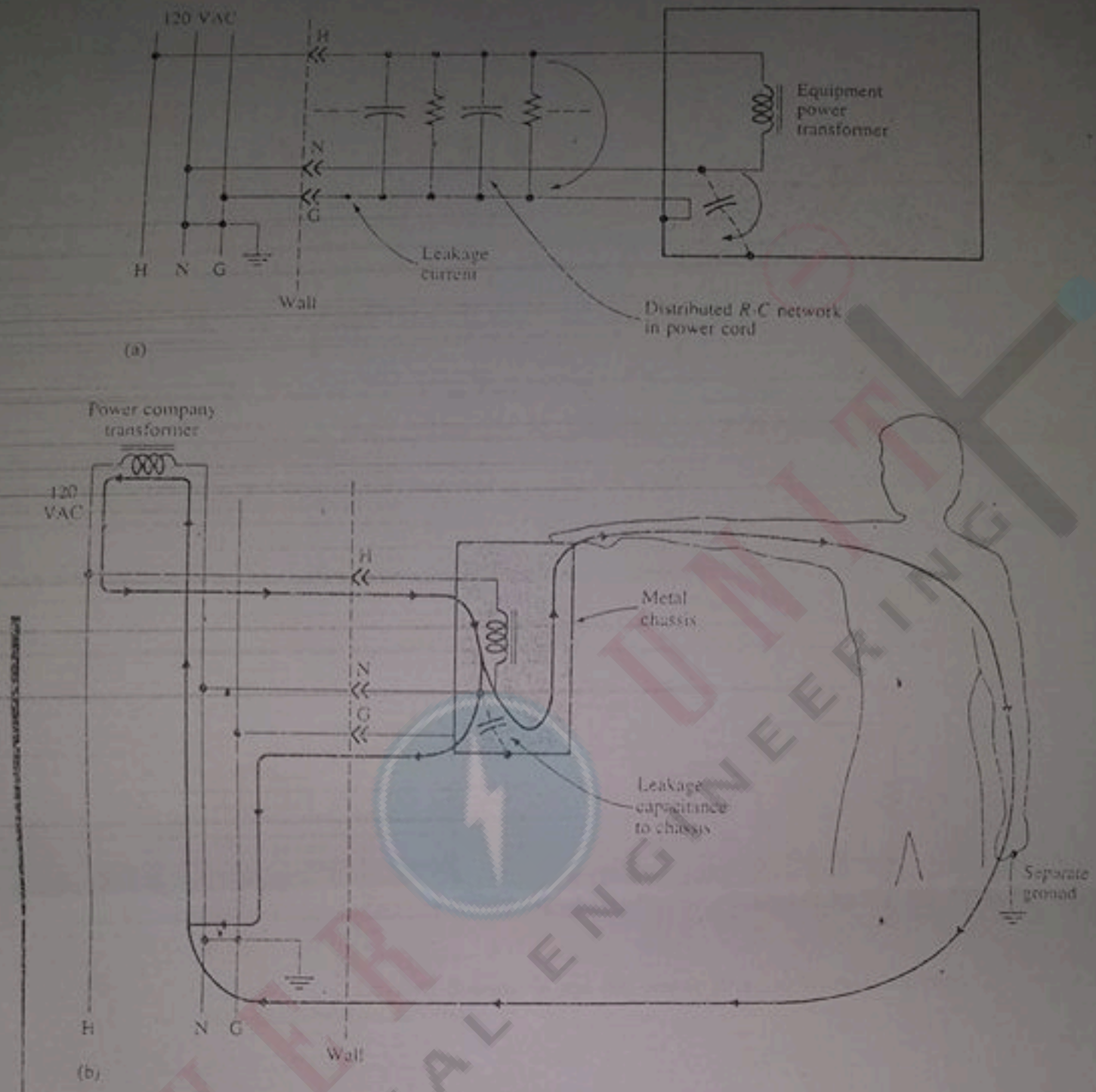


Figure Leakage current paths. (a) Path created by distributed R-C network in AC power cord and leakage capacitance to equipment case. (b) Path created by leakage capacitance to equipment case through the human body and to another ground point.

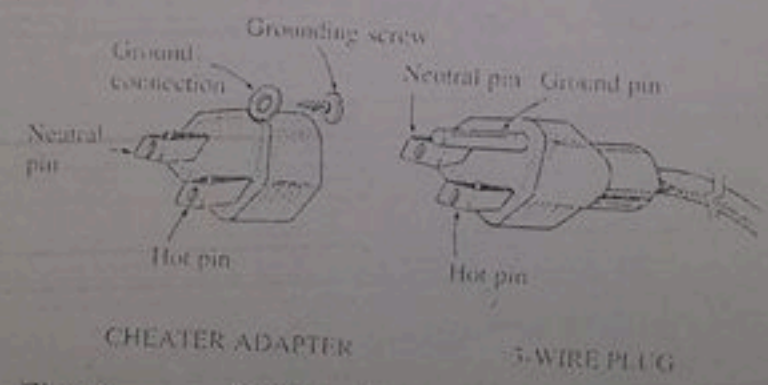


Figure Cheater adapter.

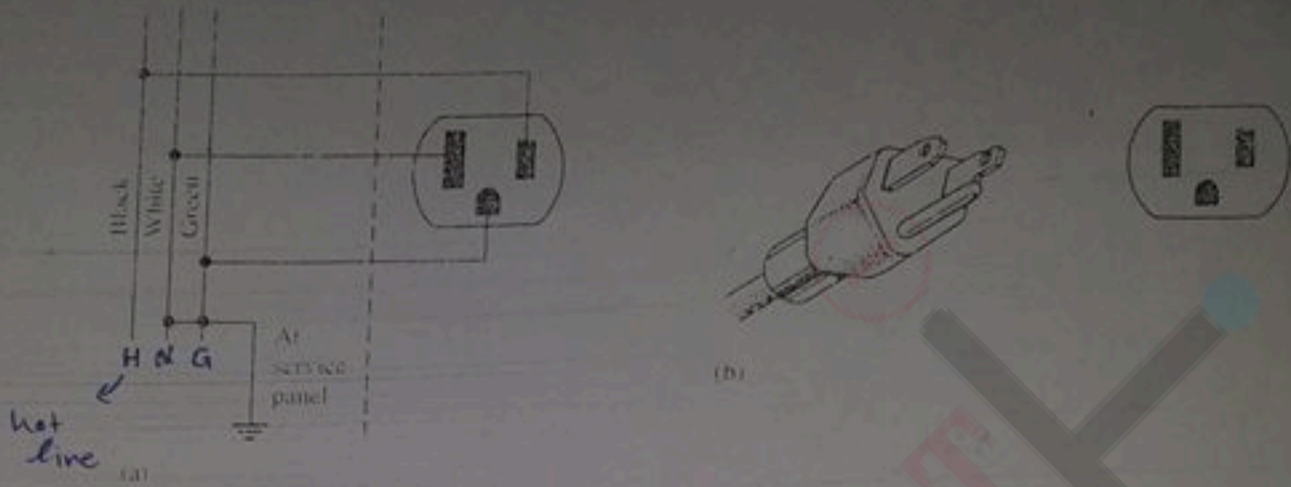


Figure The 3-wire power AC power line. (a) Wire connections. (b) Polarized plug and receptacle.

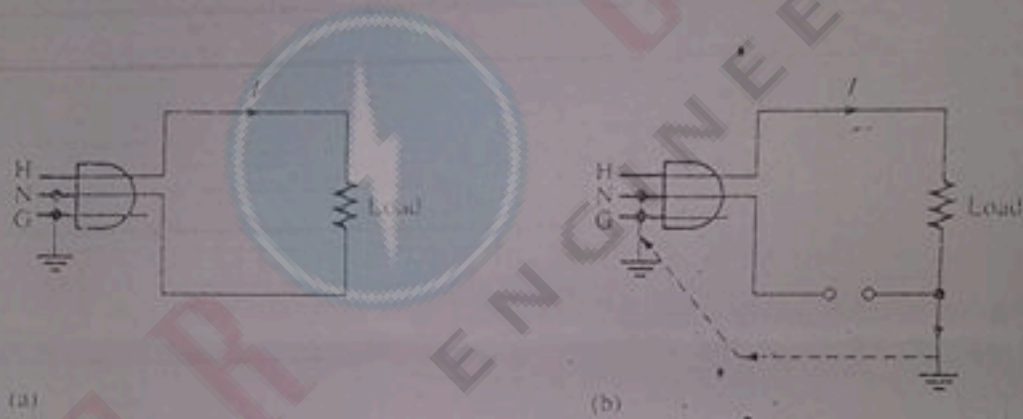


Figure Current flowing in the AC power line. (a) Normal current path. (b) Ground fault path.

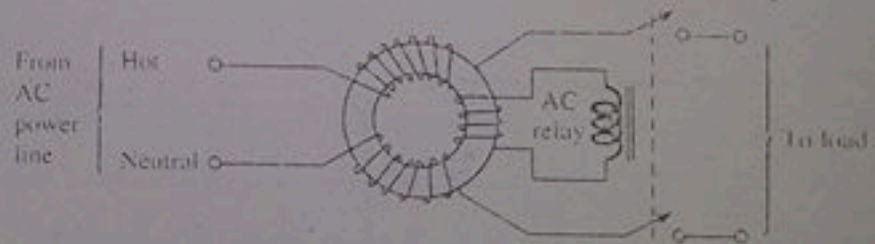


Figure Ground-fault interrupter.