Lecture 1: What is MATLAB?

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EE201: Computer Applications. See Textbook Chapter 1.

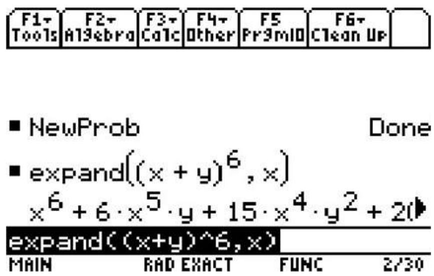
MATLAB

- MATLAB (MATrix LABoratory) is a numerical computing environment and programming language.
- Developed by MathWorks.
- MATLAB is widely used to solve engineering and science problems in academic and research institutions as well as the industry.
- In MATLAB, problems are expressed in familiar mathematical notation.
- MATLAB is an interactive system whose basic data element is a matrix (remember C/C++ arrays!).
- Open-source alternative is: GNU Octave.
- Paid alternative: LabVIEW MathScript

MATLAB can be used for:

- Matrix manipulations (math computations).
- Data analysis, exploration, and plotting.
- Implementation of algorithms.
- Creation of user interfaces.
- Data acquisition.
- Interfacing with programs written in other languages, (e.g., C, C++, Java, and Fortran).
- An optional toolbox (with MuPAD symbolic engine) allows accessing symbolic computing.
- An additional package, Simulink®, adds graphical simulation and model-based design.

Like a *VERY* advanced calculator



Would you go to an engineering exam without a calculator?

Solving Simultaneous Equations

- Find the values of x and y that satisfy the following equations simultaneously :
- Can be solved by hand to get: $x = 1, y = 2$
- Remember how?

$$2x + y = 4$$
$$x - y = -1$$

Simultaneous Equations

- Solving simultaneous equations:
- Can be solved by hand to get:

  $x = 1.2, y = 2.8, z = 0.6$
- How?

$$\begin{aligned} 2x + y + 2z &= 4 \\ x - y - z &= -1 \\ y - 2z &= 4 \end{aligned}$$

Solving Simultaneous Equations

- Many variables:

$$
\begin{array}{rrrrrrrrrrcr}
2x_1 & -x_2 & & +3x_4 & & -x_6 & +2x_7 & & +3x_9 & +x_{10} & = & 1 \\
x_1 & & +x_3 & +3x_4 & +2x_5 & +x_6 & & & +3x_9 & -x_{10} & = & 2 \\
3x_1 & +3x_2 & -x_3 & -x_4 & +2x_5 & +3x_6 & -x_7 & +2x_8 & +3x_9 & +x_{10} & = & 1 \\
2x_1 & +3x_2 & +3x_3 & +2x_4 & +x_5 & +2x_6 & +x_7 & & & +x_{10} & = & 3 \\
3x_1 & -x_2 & -x_3 & & +2x_5 & -x_6 & +x_7 & +3x_8 & +x_9 & +2x_{10} & = & 2 \\
x_1 & & -x_3 & +x_4 & +2x_5 & & -x_7 & +3x_8 & -x_9 & +2x_{10} & = & 3 \\
x_1 & +x_2 & & +x_4 & -x_5 & +x_6 & +x_7 & +2x_8 & +x_9 & +2x_{10} & = & 1 \\
3x_1 & +x_2 & -x_3 & +3x_4 & -x_5 & +3x_6 & & & & -x_{10} & = & 0 \\
-x_1 & +2x_2 & +x_3 & +x_4 & +3x_5 & -x_6 & & +x_8 & -x_9 & -x_{10} & = & -1 \\
-x_1 & +2x_2 & & +3x_4 & -x_5 & +3x_6 & +x_7 & -x_8 & -x_9 & & = & 2
\end{array}
$$

- Humans are note good at this.
  MATLAB (a computer software) is!

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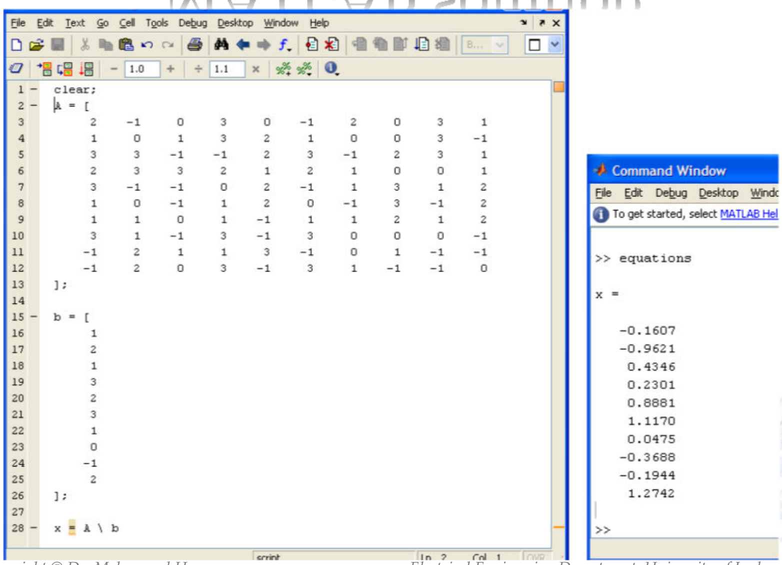
MATLAB solution



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MATLAB is powerful!

- We often need to solve systems with 10,000 or 100,000 simultaneous equations (could be non-linear or differential equations too)
- Can be done very quickly using a computer
- This is common in engineering
  – Electrical circuits
  – Image recognition
  – Communication systems (MIMO, OFDM, etc)
  – Operations research
  – Mechanics and dynamics, etc

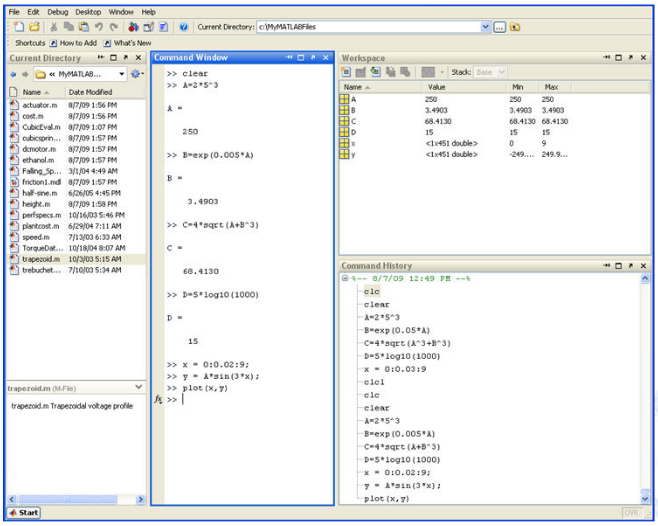
MATLAB vs. Programming languages

- MATLAB is a vector-based numerical analysis language:
  – Can be used as an advanced calculator and graphing tool
  – Also can be used as a programming language
- This is different than the programming languages you are familiar with (C, C++, …)
  – Can be a little frustrating since it takes time and effort to write code in MATLAB
  – But the code is very effective and can be refined gradually

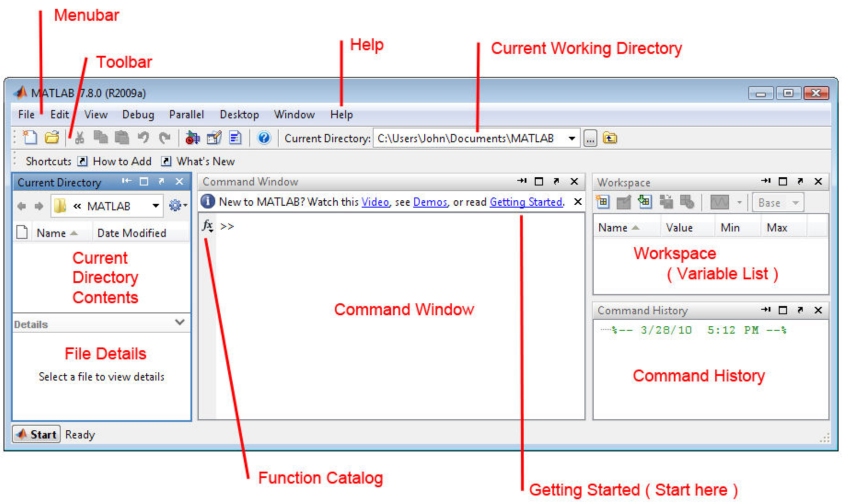
Know about MATLAB

- MATLAB is easy to begin with but needs hard work to master.
- MATLAB is optimized for performing matrix operations.
- MATLAB is interpreted
  – for the most part slower than a compiled language such as C++
  – but interactive and simplifies fixing errors
- Although primarily procedural, MATLAB does have some object-oriented elements.
- MATLAB is NOT a general purpose programming language
- MATLAB is designed for scientific computation and is not suitable for some things (such as parsing text)
- MATLAB is very useful for data analysis and rapid prototyping, but is not designed for large-scale system development.

Let us run MATLAB …



MATLAB Environment



MATLAB as a Calculator

- You can enter expressions at the command line and evaluate them right away.
- The >> symbols indicate where commands are typed.

previous command

```
>> 3 + 5 * 8

ans =

      43

>>
```

next command

Mathematical Operators

Operator	MATLAB	Algebra
+	+	5 + 4 = 9
–	-	5 - 4 = 1
×	*	5 * 4 = 20
÷	/	5 / 4 = 1.25
a^b	a^b	5^4 = 625

Order of Precedence (BEDMAS)

- B = Brackets
- E = Exponentials
- D = Division
- M = Multiplication
- A = Addition
- S = Subtraction

- Careful using brackets: check that opening and closing brackets are matched up correctly.

```
>> 3*4 + 2

ans =

    14

>> 3*(4+2)

ans =

    18
```

Order of Precedence

Precedence	Operation
First	Parentheses ( ), evaluated starting with the innermost pair.
Second	Exponentiation (power) ^ , evaluated from left to right.
Third	Multiplication * and division / with equal precedence, evaluated from left to right.
Fourth	Addition + and subtraction - with equal precedence, evaluated from left to right.

Exercise: Try it yourself

```
>> 8 + 3*5
ans =
    23

>> 8 + (3*5)
ans =
    23

>> (8 + 3)*5
ans =
    55

>> 4^2-12- 8/4*2
ans =
     0

>> 4^2-12- 8/(4*2)
ans =
     3
```


Entering Commands

- MATLAB retains your previous keystrokes.
- Use the ↑ key to scroll back through previous commands.
- Press the ↑ key once to see the previous entry, and so on.
- Use the ↓ key to scroll forward.
- Edit a line using the ← and → arrow keys, the Backspace key, and the Delete key.
- Press the Enter key to execute the command.
- You can copy (highlight & ctrl+c) from Command History window to the Command Window.

Built-in Math Constants

`pi`	π: ratio of circle's circumference to its diameter
`i`	$\sqrt{-1}$: Imaginary unit
`j`	$\sqrt{-1}$: Imaginary unit
`Inf`	∞: Infinity
`NaN`	Not-a-Number
`intmax`	Largest value of integer type
`intmin`	Smallest value of integer type
`ans`	Temporary variable containing the most recent answer
`eps`	The accuracy of floating point precision
	…

```
>> 2*pi
ans =
    6.2832

>> Inf+100000
ans =
   Inf

>> format long g

>> 2*pi
ans =

6.28318530717959

>> 1+ans
ans =

7.28318530717959
```

Exercise

```
>> 1/0
ans =
   ???

>> 0/0
ans =
   ???

>> 7/2*i
ans =
    ???

>> 7/2i
ans =
    ???
```


Exercise: Answers

```
>> 1/0
ans =
   Inf

>> 0/0
ans =
   NaN

>> 7/2*i
ans =
   0 + 3.5000i

>> 7/2i
ans =
   0 - 3.5000i
```

Possible Formats

Command	Description and example
`format short`	Four decimal digits (the default); 13.6745.
`format long`	16 digits; 17.27484029463547.
`format short e`	Five digits (four decimals) plus exponent; 6.3792e+03.
`format long e`	16 digits (15 decimals) plus exponent; 6.379243784781294e−04.
`format bank`	Two decimal digits; 126.73.
`format +`	Positive, negative, or zero; +.
`format rat`	Rational approximation; 43/7.
`format compact`	Suppresses some blank lines.
`format loose`	Resets to less compact display mode.

Built-in Functions

- Like a calculator, MATLAB has many built-in mathematical functions.

```
>> log2(131072)
ans =
    17

>> sqrt(4)
ans =
     2

>> abs(-3)
ans =
     3

>> exp(-1)
ans =

0.367879441171442
```

Common Built-in Functions

Function	MATLAB syntax*
e^x	`exp(x)`
$\sqrt{x}$	`sqrt(x)`
$\ln x$	`log(x)`
$\log_{10} x$	`log10(x)`
$\cos x$	`cos(x)`
$\sin x$	`sin(x)`
$\tan x$	`tan(x)`
$\cos^{-1} x$	`acos(x)`
$\sin^{-1} x$	`asin(x)`
$\tan^{-1} x$	`atan(x)`

*The MATLAB trigonometric functions listed here use radian measure. Trigonometric functions ending in d, such as `sind(x)` and `cosd(x)`, take the argument x in degrees. Inverse functions such as `atand(x)` return values in degrees.

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Exercise: Discussed Later…

```
x = 0:pi/100:2*pi;
y = sin(x);
plot(x,y)
```

- By the way, what is the purpose of the semicolon at the end of the command?

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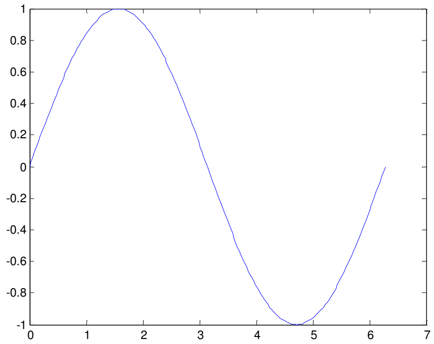
Exercise: Discussed Later…

```
x = 0:pi/100:2*pi;
y = sin(x);
plot(x,y)
```

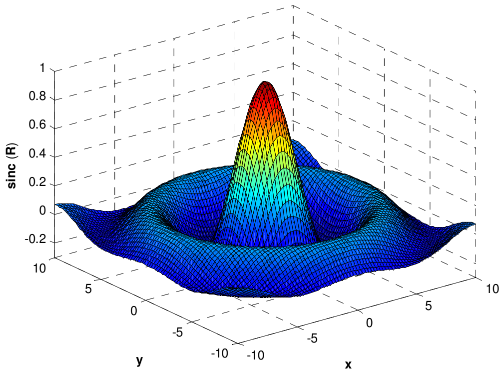


Exercise 2: Discussed Later…

```
[X,Y] = meshgrid(-10:0.25:10,-10:0.25:10);
f = sinc(sqrt((X/pi).^2+(Y/pi).^2));
surf(X,Y,f);
axis([-10 10 -10 10 -0.3 1])
```


Exercise 2: Discussed Later…

```
[X,Y] = meshgrid(-10:0.25:10,-10:0.25:10);
f = sinc(sqrt((X/pi).^2+(Y/pi).^2));
surf(X,Y,f);
axis([-10 10 -10 10 -0.3 1])
```



To Know More: `help`

```
>> help
HELP topics:

matlab\general        - General purpose commands.
matlab\ops            - Operators and special characters.
matlab\lang           - Programming language constructs.
matlab\elmat          - Elementary matrices and matrix manipulation.
matlab\randfun        - Random matrices and random streams.
matlab\elfun          - Elementary math functions.
matlab\specfun        - Specialized math functions.
matlab\matfun         - Matrix functions - numerical linear algebra.
matlab\datafun        - Data analysis and Fourier transforms.
matlab\polyfun        - Interpolation and polynomials.
matlab\funfun         - Function functions and ODE solvers.
matlab\sparfun        - Sparse matrices.
matlab\scribe         - Annotation and Plot Editing.
matlab\graph2d        - Two dimensional graphs.
matlab\graph3d        - Three dimensional graphs.
matlab\specgraph      - Specialized graphs.
matlab\graphics       - Handle Graphics.
matlab\uitools        - Graphical User Interface Tools.
matlab\strfun         - Character strings.
matlab\imagesci       - Image and scientific data
matlab\plottools      - Graphical User Interface Tools.
fuzzy\fuzzy           - Fuzzy Logic Toolbox
images\images         - Image Processing Toolbox
signal\signal         - Signal Processing Toolbox
wavelet\wavelet       - Wavelet Toolbox
...
```

Go inside: `help`

```
>> help elfun
  Elementary math functions.

  Trigonometric.
    sin         - Sine.
    sind        - Sine of argument in degrees.
    sinh        - Hyperbolic sine.
    asin        - Inverse sine.
    asind       - Inverse sine, result in degrees.
    asinh       - Inverse hyperbolic sine.
    cos         - Cosine.
    ...

  Exponential.
    exp         - Exponential.
    expm1       - Compute exp(x)-1 accurately.
    log         - Natural logarithm.
    log1p       - Compute log(1+x) accurately.
    log10       - Common (base 10) logarithm.
    log2        - Base 2 logarithm and dissect floating point num.
    pow2        - Base 2 power and scale floating point number.
    realpow     - Power that will error out on complex result.
    reallog     - Natural logarithm of real number.
    ...

  Rounding and remainder.
    fix         - Round towards zero.
    floor       - Round towards minus infinity.
    ceil        - Round towards plus infinity.
    round       - Round towards nearest integer.
    mod         - Modulus (signed remainder after division).
    rem         - Remainder after division.
    sign        - Signum.
```


For a specific function: `help exp`

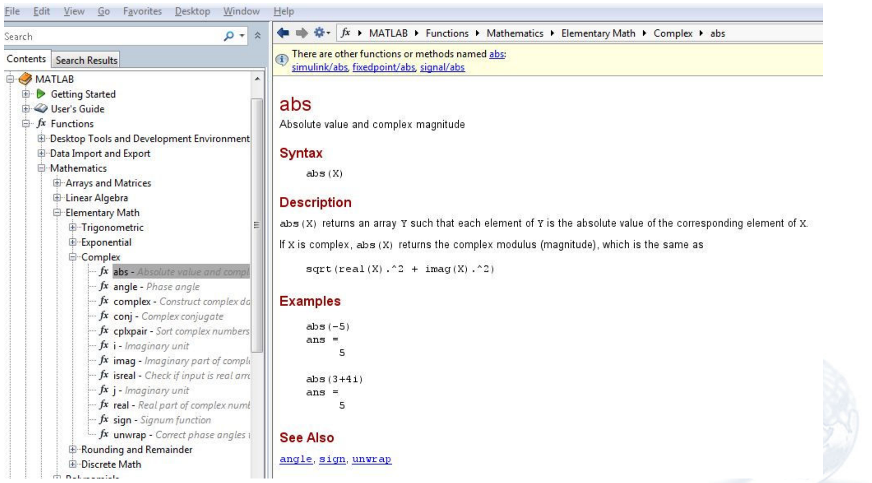
```
>> help exp
 EXP    Exponential.
    EXP(X) is the exponential of the elements of X, e to the X.
    For complex Z=X+i*Y, EXP(Z) = EXP(X)*(COS(Y)+i*SIN(Y)).

    See also expm1, log, log10, expm, expint.

    Overloaded methods:
       codistributed/exp
       fints/exp

    Reference page in Help browser
       doc exp
```

To Know More: `doc abs`



Where do you get more help?

- Read your textbook.
- Practice the end-of-chapter examples.
- References in the syllabus.
- MATLAB Central: http://www.mathworks.com/matlabcentral/
- Google
- YouTube

Lecture 2: Variables, Vectors and Matrices in MATLAB

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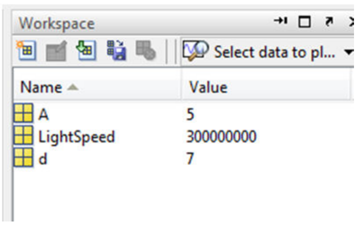
EE201: Computer Applications. See Textbook Chapter 1 and Chapter 2.

Variables in MATLAB

- Just like other programming languages, you can define variables in which to store values.
- All variables can by default hold matrices with scalar or complex numbers in them.
- You can define as many variables as your PC memory can hold.
- Values in variables can be inspected, used and changed
- Variable names are case-sensitive, and show up in the Workspace.

```
>> A = 5
A =
     5

>> d = 7
d =
     7

>> LightSpeed = 3e8
LightSpeed =
   300000000
```



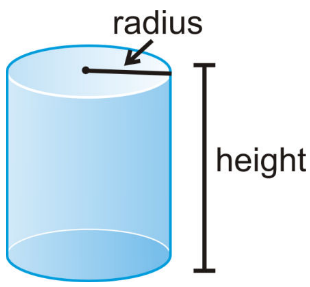
Variables

- You can change the value in the variable by over-writing it with a new value
- Remember that variables are case-sensitive (easy to make a mistake)
- Always left-to right
  >> variable = expression

```
>> a = 7
a =
     7

>> b = 12
b =
    12

>> b = 14
b =
    14

>> B = 88
B =
    88

>> c = a + b
c =
    21

>> c = a / b
c =
    0.5000
```


Exercise

- Develop MATLAB code to find Cylinder volume and surface area.
- Assume radius of 5 m and height of 13 m.



$$V = \pi r^2 h$$

$$A = 2\pi r^2 + 2\pi r h = 2\pi r(r + h)$$

Solution

```
>> r = 5
r =
     5

>> h = 13
h =
    13

>> Volume = pi * r^2 * h
Volume =
  1.0210e+003

>> Area = 2 * pi * r * (r + h)
Area =
  565.4867
```


Useful MATLAB commands

Command	Description
`clc`	Clears the Command window.
`clear`	Removes all variables from memory.
`clear var1 var2`	Removes the variables `var1` and `var2` from memory.
`exist('name')`	Determines if a file or variable exists having the name 'name'.
`quit`	Stops MATLAB.
`who`	Lists the variables currently in memory.
`whos`	Lists the current variables and sizes, and indicates if they have imaginary parts.
`:`	Colon; generates an array having regularly spaced elements.
`,`	Comma; separates elements of an array.
`;`	Semicolon; suppresses screen printing; also denotes a new row in an array.
`...`	Ellipsis; continues a line.

Vectors and Matrices (Arrays)

- So far we used MATLAB variables to store a single value.
- We can also create MATLAB arrays that hold multiple values
  - List of values (1D array) called **Vector**
  - Table of values (2D array) called **Matrix**
- Vectors and matrices are used extensively when solving engineering and science problems.

Row Vector

- Row vectors are special cases of matrices.
- This is a 7-element row vector (1×7 matrix).
- Defined by enclosing numbers within square brackets `[ ]` and separating them by `,` or a space.

```
>> C = [10, 11, 13, 12, 19, 16, 17]

C =
    10    11    13    12    19    16    17


>> C = [10 11 13 12 19 16 17]

C =
    10    11    13    12    19    16    17
```

Column Vector

- Column vectors are special cases of matrices.
- This is a 7-element column vector (7×1 matrix).
- Defined by enclosing numbers within `[ ]` and separating them by semicolon `;`

```
>> R = [10; 11; 13; 12; 19; 16; 17]

R =
    10
    11
    13
    12
    19
    16
    17
```


Matrix

- This is a 3×4-element matrix.
- It has 3 rows and 4 columns (dimension 3×4).
- Spaces or commas separate elements in different columns, whereas semicolons separate elements in different rows.
- A dimension $n \times n$ matrix is called *square* matrix.

```
>> M = [1, 3, 2, 9; 6, 7, 8, 1; 7, 4, 6, 0]
M =
     1     3     2     9
     6     7     8     1
     7     4     6     0


>> M = [1 3 2 9; 6 7 8 1; 7 4 6 0]
M =
     1     3     2     9
     6     7     8     1
     7     4     6     0
```

Transpose of a Matrix

- The transpose operation interchanges the rows and columns of a matrix.
- For an $m \times n$ matrix $\mathbf{A}$ the new matrix $\mathbf{A}^T$ (read "A transpose") is an $n \times m$ matrix.
- In MATLAB, the `A'` command is used for transpose.

$$\mathbf{A} = \begin{bmatrix} -2 & 6 \\ -3 & 5 \end{bmatrix} \qquad \mathbf{A}^T = \begin{bmatrix} -2 & -3 \\ 6 & 5 \end{bmatrix}$$

Exercise

```
>> A = [1 2 3; 5 6 7]
A =
     1     2     3
     5     6     7

>> A'
ans =
     1     5
     2     6
     3     7
```

```
>> B = [5 6 7 8]
B =
     5     6     7     8

>> B'
ans =
     5
     6
     7
     8
```

- What happens to a row vector when transposed?
- What happens to a column vector when transposed?

Useful Functions

`length(A)`	Returns either the number of elements of A if A is a vector or the largest value of m or n if A is an $m \times n$ matrix
`size(A)`	Returns a row vector `[m n]` containing the sizes of the $m \times n$ matrix A.
`max(A)`	For vectors, returns the largest element in A. For matrices, returns a row vector containing the maximum element from each column. If any of the elements are complex, `max(A)` returns the elements that have the largest magnitudes.
`[v,k] = max(A)`	Similar to `max(A)` but stores the maximum values in the row vector `v` and their indices in the row vector `k`.

`min(A)`
and
`[v,k] = min(A)`
	Like `max` but returns minimum values.

More Useful Functions

`sort(A)`	Sorts each column of the array `A` in ascending order and returns an array the same size as `A`.
`sort(A,DIM,MODE)`	Sort with two optional parameters: DIM selects a dimension along which to sort. MODE is sort direction ('ascend' or 'descend').
`sum(A)`	Sums the elements in each column of the array `A` and returns a row vector containing the sums.
`sum(A,DIM)`	Sums along the dimension DIM.

Exercises

```
>> X = [4 9 2 5]
X =
     4     9     2     5

>> length(X)
ans =
     4

>> size(X)
ans =
     1     4

>> min(X)
ans =
     2
```

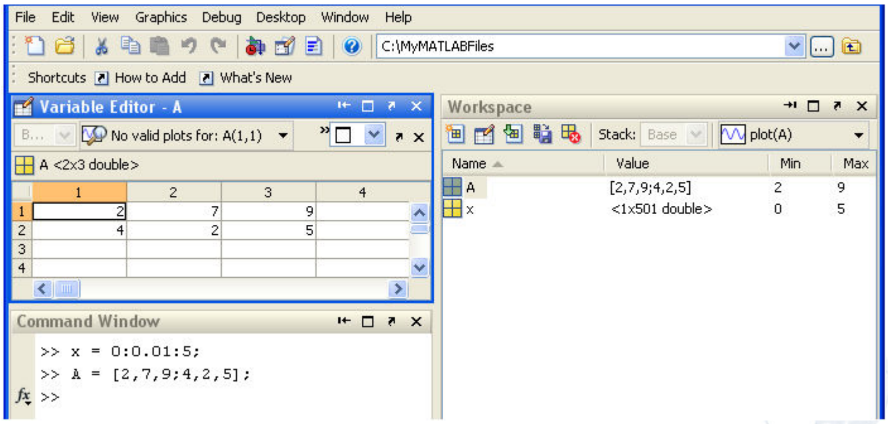
```
>> M = [1 6 4; 3 7 2]

>> size(M)

>> length(M)

>> max(M)

>> [a,b] = max(M)

>> sort(M)

>> sort(M, 1, 'descend')

>> sum(M)

>> sum(M, 2)
```


Solution

```
>> M = [1 6 4; 3 7 2]
M =
     1     6     4
     3     7     2

>> size(M)
ans =
     2     3

>> length(M)
ans =
     3

>> max(M)
ans =
     3     7     4

>> [a,b] = max(M)
a =
     3     7     4
b =
     2     2     1
```

```
>> sort(M)
ans =
     1     6     2
     3     7     4

>> sort(M, 1, 'descend')
ans =
     3     7     4
     1     6     2

>> sum(M)
ans =
     4    13     6

>> sum(M, 2)
ans =
    11
    12
```

The Variable Editor [from Workspace or `openvar('A')`]



Creating Big Matrices

- What if you want to create a Matrix that contains 1000 element (or more)?
- Writing each element by hand is difficult, time-consuming and error-prone.
- MATLAB allows simple ways to quickly create matrices, such as:
- Using the colon : operator (very popular).
- Using `linspace()` and `logspace()` functions (less popular, but useful).

Using the colon operator

- MATLAB command `X = J:D:K` creates vector X = [J, J+D, ..., J+m*D] where m = fix((K-J)/D).
- In other words, it creates a vector X of values **starting** at J, **ending** with K, and with **spacing** D.
- Notice that the last element is K if K - J is an integer multiple of D. If not, the last value is *less than* J.
- MATLAB command `J:K` is the same as `J:1:K`.
- Note:
  - `J:K` is empty if J > K.
  - `J:D:K` is empty if D == 0, if D > 0 and J > K, or if D < 0 and J < K.

Example 1

```
>> x = 0:2:8
x =
     0     2     4     6     8

>> x = 0:2:7
x =
     0     2     4     6

>> x = 4:7
x =
     4     5     6     7

>> x = 7:2
x =
   Empty matrix: 1-by-0
```

Example 2

```
>> x = 7:-1:2
x =
     7     6     5     4     3     2

>> x = 5:0.1:5.9
x =
  Columns 1 through 5
    5.0000    5.1000    5.2000    5.3000    5.4000

  Columns 6 through 10
    5.5000    5.6000    5.7000    5.8000    5.9000

>> y = 5:0.1:5.9; % what happened here?!
>>
>> % now create a 'column' vector from 1 to 10 using :
```


Alternatives to colon

- `linspace` command creates a linearly spaced row vector, but instead you specify the number of values rather than the increment.
- The syntax is `linspace(x1,x2,n)`, where `x1` and `x2` are the lower and upper limits and `n` is the number of points.
- If `n` is omitted, the number of points defaults to 100.
- `logspace` command creates an array of logarithmically spaced elements.
- Its syntax is `logspace(a,b,n)`, where `n` is the number of points between 10^a and 10^b.
- If `n` is omitted, the number of points defaults to 50.

Exercise

```
>> x = linspace(5,8,3)

x =

    5.0000    6.5000    8.0000

>> x = logspace(-1,1,4)

x =

    0.1000    0.4642    2.1544   10.0000
```


Special: ones, zeros, rand

```
>> a = ones(2,4)
a =
     1     1     1     1
     1     1     1     1

>> b = zeros(4, 3) % null matrix
b =
     0     0     0
     0     0     0
     0     0     0
     0     0     0

>> c = rand(2, 4)
c =
    0.8147    0.1270    0.6324    0.2785
    0.9058    0.9134    0.0975    0.5469

% random values drawn from the standard
% uniform distribution on the open
% interval(0,1)
```

```
>> eye(4) % identity matrix
ans =
      1      0      0      0
      0      1      0      0
      0      0      1      0
      0      0      0      1

>> A = [1 2 3; 4 5 6; 7 8 9]
A =
      1      2      3
      4      5      6
      7      8      9

>> I = eye(3)
I =
      1      0      0
      0      1      0
      0      0      1

>> A*I
ans =
      1      2      3
      4      5      6
      7      8      9
```

Null and Identity Matrix

$$\mathbf{0A} = \mathbf{A0} = \mathbf{0}$$

$$\mathbf{IA} = \mathbf{AI} = \mathbf{A}$$

Matrix Determinant & Inverse

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$= aei + bfg + cdh - ceg - bdi - afh.$$

$$\mathrm{A}^{-1} = \frac{1}{|\mathrm{A}|}\begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}.$$

```
>> A = [1 2 3; 2 3 1; 3 2 1]
A =
     1     2     3
     2     3     1
     3     2     1

>> det(A) % determinant
ans =
   -12

>> inv(A) % inverse
ans =
   -0.0833   -0.3333    0.5833
   -0.0833    0.6667   -0.4167
    0.4167   -0.3333    0.0833

>> A^-1
ans =
   -0.0833   -0.3333    0.5833
   -0.0833    0.6667   -0.4167
    0.4167   -0.3333    0.0833
```


Accessing Matrix Elements

```
>> C = [10, 11, 13, 12, 19, 16, 17]

C =
    10    11    13    12    19    16    17

>> C(4)
ans =
    12

>> C(1,4)
ans =
    12

>> C(20)
??? Index exceeds matrix dimensions.
```


Notes

- Use `()` not `[]` to access matrix elements.
- The row and column indices are NOT zero-based, like in C/C++.
- The first is row number, followed by the column number.
- For matrices and vectors, you can use one of three indexing methods: matrix row and column indexing; linear indexing; and logical indexing.
- You can also use ranges (shown later).

Accessing Matrix Elements

```
>> M = [1, 3, 2, 9; 6, 7, 8, 1; 7, 4, 6, 0]
M =
     1     3     2     9
     6     7     8     1
     7     4     6     0

>> M(2, 3)
ans =
     8

>> M(3, 1)
ans =
     7

>> M(0, 1)
??? Subscript indices must either be real
positive integers or logicals.

>> M(9)
ans =
     6
```


Matrix Linear Indexing

A =

Columns (n)

Rows (m)	1	2	3	4	5
1	4^{1}	10^{6}	1^{11}	6^{16}	2^{21}
2	8^{2}	1.2^{7}	9^{12}	$\boxed{4^{17}}$	25^{22}
3	7.2^{3}	5^{8}	7^{13}	1^{18}	11^{23}
4	0^{4}	0.5^{9}	4^{14}	5^{19}	56^{24}
5	23^{5}	83^{10}	13^{15}	0^{20}	10^{25}

A (2,4)

A (17)

A = 5 x 5 matrix.

Rectangular Matrix:
Scalar: 1-by-1 array
Vector: m-by-1 array
1-by-n array
Matrix: m-by-n array

Indexing: Sub-matrix

- `v(2:5)` represents the second through fifth elements
  - i.e., v(2), v(3), v(4), v(5).
- `v(2:end)` represents the second till last element of v.
- `v(:)` represents all the row or column elements of vector v.

- `A(:,3)` denotes all elements in the third column of matrix A.
- `A(:,2:5)` denotes all elements in the second through fifth columns of A.
- `A(2:3,1:3)` denotes all elements in the second and third rows that are also in the first through third columns.
- `A(end,:)` all elements of the last row in A.
- `A(:,end)` all elements of the last column in A.
- `v = A(:)` creates a vector v consisting of all the columns of A stacked from first to last.

Exercise

```
>> v = 10:10:70
v =
    10    20    30    40    50    60    70

>> v(2:5)
ans =
    20    30    40    50

>> v(2:end)
ans =
    20    30    40    50    60    70

>> v(:)
ans =
    10
    20
    30
    40
    50
    60
    70
```

Exercise

```
>> A = [4 10 1 6 2; 8 1.2 9 4 25; 7.2 5 7 1
11; 0 0.5 4 5 56; 23 83 13 0 10]

A =
    4.0000  10.0000  1.0000  6.0000   2.0000
    8.0000   1.2000  9.0000  4.0000  25.0000
    7.2000   5.0000  7.0000  1.0000  11.0000
         0   0.5000  4.0000  5.0000  56.0000
   23.0000  83.0000 13.0000       0   0.0000

>> A(:,3)
ans =
     1
     9
     7
     4
    13

>> A(:,2:5)
ans =
   10.0000    1.0000    6.0000    2.0000
    1.2000    9.0000    4.0000   25.0000
    5.0000    7.0000    1.0000   11.0000
    0.5000    4.0000    5.0000   56.0000
   83.0000   13.0000         0   10.0000

>> A(2:3,1:3)
ans =

    8.0000    1.2000    9.0000
    7.2000    5.0000    7.0000
```

```
>> A(end,:)
ans =
    23    83    13     0    10

>> A(:,end)
ans =
     2
    25
    11
    56
    10

>> v = A(:)
v =
    4.0000
    8.0000
    7.2000
         0
   23.0000
   10.0000
    1.2000
    5.0000
    0.5000
   83.0000
    1.0000
    9.0000
    7.0000
    4.0000
   13.0000
    6.0000
    4.0000
    1.0000
    5.0000
         0
    2.0000
   25.0000
   11.0000
   56.0000
   10.0000
```


Linear indexing: *Advanced*

```
>> A = 5:5:50
A =
   5  10  15  20  25  30  35  40  45  50

>> A([1 3 6 10])
ans =
      5     15     30     50

>> A([1 3 6 10]')
ans =
      5     15     30     50

>> A([1 3 6; 7 9 10])
ans =
      5     15     30
     35     45     50
% indexing into a vector with a nonvector,
the shape of the indices is honored
```

Linear indexing is useful: `find`

```
>> A = [1 2 3; 4 5 6; 7 8 9]
A =
      1      2      3
      4      5      6
      7      8      9

>> B = find(A > 5) % returns linear index
B =
      3
      6
      8
      9

>> A(B) % same as A( find(A > 5) )
ans =
      7
      8
      6
      9
```

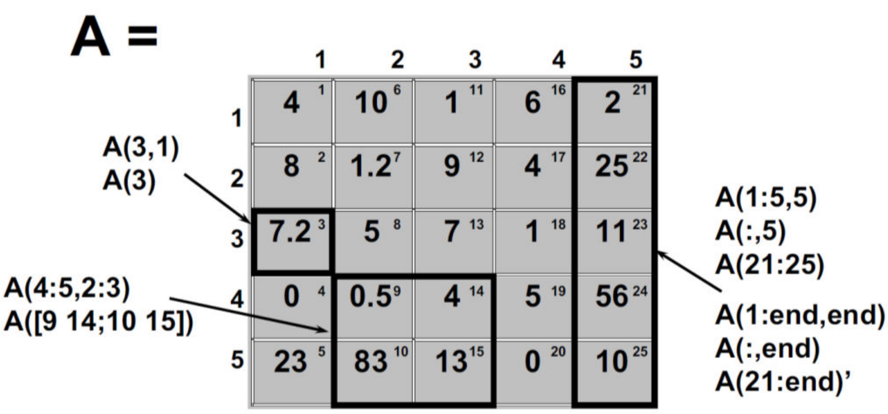

Advanced: Logical indexing

```
>> A = [1 2 3; 4 5 6; 7 8 9]
A =
      1      2      3
      4      5      6
      7      8      9

>> B = logical([0 1 0; 1 0 1; 0 0 1])
B =
      0      1      0
      1      0      1
      0      0      1

>> A(B)
ans =
      4
      2
      6
      9
```

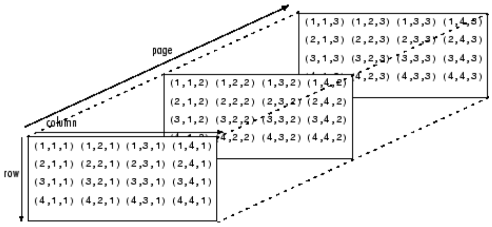

Logical indexing is also useful!

```
>> A = [1 2 3; 4 5 6; 7 8 9]
A =
     1     2     3
     4     5     6
     7     8     9

>> B = (A > 5) % true or false
B =
     0     0     0
     0     0     1
     1     1     1

>> A(B) % same as A( A > 5 )
ans =
     7
     8
     6
     9
```


Subscripting Examples



More dimensions possible



- The first index references array dimension 1, the row.
- The second index references dimension 2, the column.
- The third index references dimension 3, the page.

```
>> rand(4,4,3)

ans(:,:,1) =

    0.7431    0.7060    0.0971    0.9502
    0.3922    0.0318    0.8235    0.0344
    0.6555    0.2769    0.6948    0.4387
    0.1712    0.0462    0.3171    0.3816


ans(:,:,2) =

    0.7655    0.4456    0.2760    0.1190
    0.7952    0.6463    0.6797    0.4984
    0.1869    0.7094    0.6551    0.9597
    0.4898    0.7547    0.1626    0.3404


ans(:,:,3) =

    0.5853    0.5060    0.5472    0.8407
    0.2238    0.6991    0.1386    0.2543
    0.7513    0.8909    0.1493    0.8143
    0.2551    0.9593    0.2575    0.2435
```


Extending Matrices

- You can add extra elements to a matrix by creating them directly using `()`
- Or by concatenating (appending) them using `[ , ]` or `[ ; ]`
- If you don't assign array elements, MATLAB gives them a default value of 0

```
>> h = [12 11 14 19 18 17]
h =
   12   11   14   19   18   17

>> h = [h 13]
h =
   12   11   14   19   18   17   13

>> h(10) = 1
h =
   12   11   14   19   18   17   13    0    0    1
```


Example

```
>> a = [2 4 20]
a =
     2     4    20

>> b = [9, -3, 6]
b =
     9    -3     6

>> [a b]
ans =
     2     4    20     9    -3     6

>> [a, b]
ans =
     2     4    20     9    -3     6

>> [a; b]
ans =
     2     4    20
     9    -3     6
```

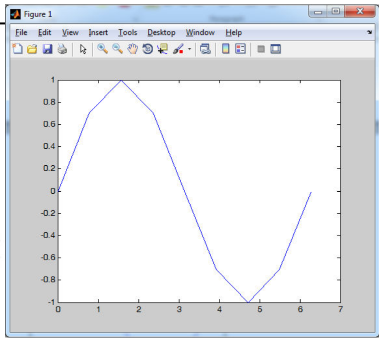

Functions on Arrays

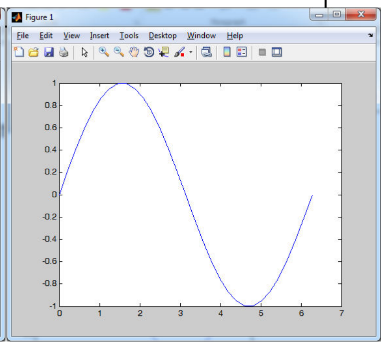
- Standard MATLAB functions (sin, cos, exp, log, etc) can apply to vectors and matrices as well as scalars.
- They operate on array arguments to produce an array result the same size as the array argument x.
- These functions are said to be vectorized functions.
- In this example y is [sin(1), sin(2), sin(3)]
- So, when writing functions (later lectures) remember input might be a vector or matrix.

```
>> x = [1, 2, 3]
x =
     1     2     3

>> y = sin(x)
y =
    0.8415    0.9093    0.1411
```

Exercise

```
>> x = linspace(0, 2*pi, 9) % OR x = linspace(0, 2*pi, 31)
x =
         0    0.7854    1.5708    2.3562    3.1416    3.9270    4.7124    5.4978    6.2832

>> y = sin(x)
y =
         0    0.7071    1.0000    0.7071    0.0000   -0.7071   -1.0000   -0.7071   -0.0000

>> plot(x,y)
```





Matrix vs. Array Arithmetic

- Multiplying and dividing vectors and matrices is different than multiplying and dividing scalars (or arrays of scalars).
- This is why MATLAB has two types of arithmetic operators:
  - **Array** operators: where the arrays operated on have the same size. The operation is done element-by-element (for all elements).
  - **Matrix** operators: dedicated for matrices and vectors. Operations are done using the matrix as a whole.

Matrix vs. Array Operators

Symbol	Operation	Symbol	Operation
+	Matrix addition	+	Array addition
−	Matrix subtraction	−	Array subtraction
*	Matrix multiplication	.*	Array multiplication
/	Matrix division	./	Array division
\	Left matrix division	.\	Left array division
^	Matrix power	.^	Array power

* idivide() allows integer division with rounding options

Matrix/Array Addition/Subtraction

- Matrices and arrays are treated the same when adding and subtracting.
- The two matrices should have identical size.
- Their sum or difference has the same size, and is obtained by adding or subtracting the corresponding elements.
- Addition and subtraction are associative and commutative.

$$\begin{bmatrix} 6 & -2 \\ 10 & 3 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ -12 & 14 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -2 & 17 \end{bmatrix}$$

```
>>A = [6,-2;10,3];
>>B = [9,8;-12,14]
>>A+B
ans =
      15   6
      -2   17
```

$$(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$$

$$\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{B} + \mathbf{C} + \mathbf{A} = \mathbf{A} + \mathbf{C} + \mathbf{B}$$

More …

- A scalar value at either side of the operator is expanded to an array of the same size as the other side of the operator.

```
[6,3]+2=[8,5]
[8,3]−5=[3,−2]
[6,5]+[4,8]=[10,13]
[6,5]−[4,8]=[2,−3]
```


Array Multiplication

- Element-by-element multiplication.
- Only for arrays that are the same size.
- Use the `.*` operator not the `*` operator.
- Not the same as matrix multiplication.
- Useful in programming, but students make the mistake of using *

$$\mathbf{A} = \begin{bmatrix} 11 & 5 \\ -9 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -7 & 8 \\ 6 & 2 \end{bmatrix}$$

```
C = A.*B
```

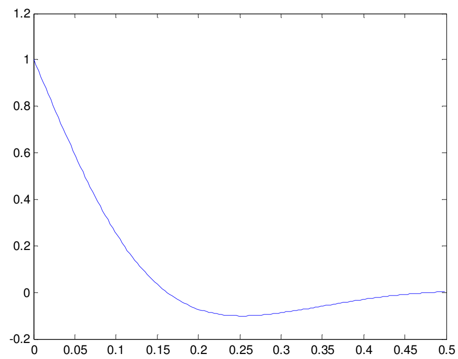
$$\mathbf{C} = \begin{bmatrix} 11(-7) & 5(8) \\ -9(6) & 4(2) \end{bmatrix} = \begin{bmatrix} -77 & 40 \\ -54 & 8 \end{bmatrix}$$

Using Array Multiplication (Plot)

- Plot the following function:
- Notice the use of `.*` operator

```
>> t = 0:0.003:0.5;
>> y = exp(-8*t).*sin(9.7*t+pi/2);
>> plot(t,y)
```

$$y(t) = e^{-8t} \sin\left(9.7t + \frac{\pi}{2}\right)$$



Matrix Multiplication

- If A is an $n \times m$ matrix and B is a $m \times p$ matrix, their matrix product AB is an $n \times p$ matrix, in which the m entries across the rows of A are multiplied with the m entries down the columns of B.
- In general, AB ≠ BA for matrices. Be extra careful.

$$\begin{bmatrix} 2 & 7 \\ 6 & -5 \end{bmatrix} \begin{bmatrix} 3 \\ 9 \end{bmatrix} = \begin{bmatrix} 2(3) + 7(9) \\ 6(3) - 5(9) \end{bmatrix} = \begin{bmatrix} 69 \\ -27 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = u_1 w_1 + u_2 w_2 + u_3 w_3$$

Matrix Multiplication

$$\begin{bmatrix} 6 & -2 \\ 10 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 9 & 8 \\ -5 & 12 \end{bmatrix} = \begin{bmatrix} (6)(9)+(-2)(-5) & (6)(8)+(-2)(12) \\ (10)(9)+(3)(-5) & (10)(8)+(3)(12) \\ (4)(9)+(7)(-5) & (4)(8)+(7)(12) \end{bmatrix}$$

$$= \begin{bmatrix} 64 & 24 \\ 75 & 116 \\ 1 & 116 \end{bmatrix} \qquad (2.4\text{–}4)$$

```
>> A = [6,-2;10,3;4,7];
>> B = [9,8;-5,12];
>> A*B
ans =
    64    24
    75   116
     1   116
```

$$3\begin{bmatrix} 2 & 9 \\ 5 & -7 \end{bmatrix} = \begin{bmatrix} 6 & 27 \\ 15 & -21 \end{bmatrix}$$

```
>>A = [2,9;5,-7];
>>3*A
```


Array Division

- Element-by-element division.
- Only for arrays that are the same size.
- Use the ./ operator not the / operator.
- Not the same as matrix division.
- Useful in programming, but students make the mistake of using /

$$\mathbf{A} = \begin{bmatrix} 24 & 20 \\ -9 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -4 & 5 \\ 3 & 2 \end{bmatrix}$$

```
C = A./B
```

$$\mathbf{C} = \begin{bmatrix} 24/(-4) & 20/5 \\ -9/3 & 4/2 \end{bmatrix} = \begin{bmatrix} -6 & 4 \\ -3 & 2 \end{bmatrix}$$

Matrix Division

- An $n \times n$ square matrix **B** is called invertible (also nonsingular) if there exists an $n \times n$ matrix $\mathbf{B}^{-1}$ such that their multiplication is the identity matrix.

$$\frac{\mathbf{A}}{\mathbf{B}} = \mathbf{A}\,\mathbf{B}^{-1}$$

$$\mathbf{B}\,\mathbf{B}^{-1} = \mathbf{I}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 5 & 6 \\ 6 & 5 & 4 \\ 4 & 6 & 5 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} \frac{1}{30} & \frac{11}{30} & \frac{-1}{3} \\ \frac{-7}{15} & \frac{-2}{15} & \frac{2}{3} \\ \frac{8}{15} & \frac{-2}{15} & \frac{-1}{3} \end{bmatrix}$$

wikiHow

Matrix Division

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 5 & 6 \\ 6 & 5 & 4 \\ 4 & 6 & 5 \end{bmatrix}$$

$$A \cdot B^{-1}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{30} & \frac{11}{30} & \frac{-1}{3} \\ \frac{-7}{15} & \frac{-2}{15} & \frac{2}{3} \\ \frac{8}{15} & \frac{-2}{15} & \frac{-1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{10} & \frac{-3}{10} & 0 \\ \frac{-3}{10} & \frac{7}{10} & 0 \\ \frac{6}{5} & \frac{1}{5} & -1 \end{bmatrix}$$

```
>> A = [1 2 3; 3 2 1; 2 1 3];
>> B = [4 5 6; 6 5 4; 4 6 5];
>> A/B
ans =
    0.7000   -0.3000         0
   -0.3000    0.7000    0.0000
    1.2000    0.2000   -1.0000

>> format rat
>> A/B
ans =
     7/10         -3/10           0
    -3/10          7/10           *
     6/5           1/5           -1
```

Matrix Left Division

- Use the left division operator (\) (back slash) to solve sets of linear algebraic equations.
- If A is $n \times n$ matrix and B is a column vector with n elements, then x = A\B is the solution to the equation Ax = B.
- A warning message is displayed if A is badly scaled or nearly singular.

$$6x + 12y + 4z = 70$$
$$7x - 2y + 3z = 5$$
$$2x + 8y - 9z = 64$$

```
>>A = [6,12,4;7,-2,3;2,8,-9];
>>B = [70;5;64];
>>Solution = A\B
Solution =
     3
     5
    -2
```

The solution is $x = 3$, $y = 5$, and $z = -2$.

Homework: Mesh Analysis

KVL @ mesh 2:

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

KVL @ supermesh 1/3:

$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$$

@ current source:

$$7 = i_1 - i_3$$

Three equations:

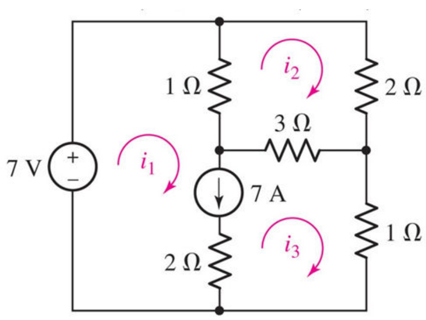
$-i_1 + 6i_2 - 3i_3 = 0$

$i_1 - 4i_2 + 4i_3 = 7$

$i_1 - i_3 = 7$

Solution:

$i_1 = 9A$, $i_2 = 2.5A$, $i_3 = 2A$



Just between us…

- Matrix division and matrix left division are related in MATLAB by the equation:

```
B/A = (A'\B')' % reversing
```

- To see the details, type: `doc mldivide` or type: `doc mrdivide`

Array Left Division

- The array left division `A.\B` (back slash) divides each entry of B by the corresponding entry of A.
- Just like `B./A`
- A and B must be arrays of the same size.
- A scalar value for either A or B is expanded to an array of the same size as the other.

```
>> A = [-4 5; 3 2];
>> B = [24 20; -9 4];

>> A.\B % notice the back slash
ans =
      -6              4
      -3              2

>> B./A
ans =
      -6              4
      -3              2
```

Array Power

`B = A.^3`

$$\mathbf{B} = \begin{bmatrix} 4^3 & (-5)^3 \\ 2^3 & 3^3 \end{bmatrix} = \begin{bmatrix} 64 & -125 \\ 8 & 27 \end{bmatrix}$$

```
[3,5].^2=[3^2,5^2]
2.^[3,5]=[2^3,2^5]
[3,5].^[2,4]=[3^2,5^4]
```

```
p = [2, 4, 5]

3.^p
3.0.^p
3..^p
(3).^p
3.^[2,4,5]
```


Matrix Power

- `A^k` computes matrix power (exponent).
- In other words, it multiplies matrix **A** by itself *k* times.
- The exponent *k* requires a positive, real-valued integer value.
- Remember: this is repeated *matrix* multiplication

```
>> A = [1 2; 3 4];
>> A^3
ans =
    37    54
    81   118

>> A*A*A
ans =
    37    54
    81   118
```

Matrix Manipulation Functions

- `diag`: Diagonal matrices and diagonal of a matrix.
- `det`: Matrix determinant
- `inv`: Matrix inverse
- `cond`: Matrix condition number (for inverse)
- `fliplr`: Flip matrices left-right
- `flipud`: Flip matrices up and down
- `repmat`: Replicate and tile a matrix

Matrix Manipulation Functions

- `rot90`: rotate matrix 90^o
- `tril`: Lower triangular part of a matrix
- `triu`: Upper triangular part of a matrix
- `cross`: Vector cross product
- `dot`: Vector dot product
- `eig`: Evaluate eigenvalues and eigenvectors
- `rank`: Rank of matrix

Exercise

```
>> A = [1 2 3; 4 5 6; 7 8 9]
A =
     1     2     3
     4     5     6
     7     8     9

>> diag(A)
ans =
     1
     5
     9

>> det(A)
ans =
   6.6613e-016
```

```
>> fliplr(A)
ans =
     3     2     1
     6     5     4
     9     8     7

>> flipud(A)
ans =
     7     8     9
     4     5     6
     1     2     3

>> rot90(A)
ans =
     3     6     9
     2     5     8
     1     4     7
```


Exercise

```
>> A = [1 2 3; 4 5 6; 7 8 9]
A =
     1     2     3
     4     5     6
     7     8     9

>> tril(A)
ans =
     1     0     0
     4     5     0
     7     8     9

>> triu(A)
ans =
     1     2     3
     0     5     6
     0     0     9
```

```
>> [V, D] = eig(A)

V =
  -0.2320   -0.7858    0.4082
  -0.5253   -0.0868   -0.8165
  -0.8187    0.6123    0.4082


D =
  16.1168         0         0
        0   -1.1168         0
        0         0   -0.0000
```

Exercise

- Define matrix A of dimension 2 by 4 whose (i,j) entries are A(i,j) = i+j
- Extract two 2 by 2 matrices A1 and A2 out of matrix A.
  - A1 contains the first two columns of A
  - A2 contains the last two columns of A
- Compute matrix B to be the sum of A1 and A2
- Compute the eigenvalues and eigenvectors of B
- Solve the linear system B x = b, where b has all entries = 2
- Compute the determinant of B, inverse of B, and the condition number of B
- NOTE: Use only MATLAB native functions for all above.

Solution

```
>> A =[0 1 2 3; 1 2 3 4]
A =
     0     1     2     3
     1     2     3     4

>> A1 = A(:,1:2)
A1 =
     0     1
     1     2

>> A2 = A(:,3:4)
A2 =
     2     3
     3     4

>> B = A1 + A2
B =
     2     4
     4     6
```

```
>> b = [2; 2]
b =
     2
     2

>> B\b
ans =
   -1.0000
    1.0000

>> det(B)
ans =
    -4

>> inv(B)
ans =
   -1.5000    1.0000
    1.0000   -0.5000

>> cond(B)
ans =
   17.9443
```

Homework

- Solve as many problems from Chapter 1 as you can
- Suggested problems:
- 1.3, 1.8, 1.15, 1.26, 1.30
- Solve as many problems from Chapter 2 as you can
- Suggested problems:
- 2.3, 2.10, 2.13, 2.25, 2.26

Lecture 3: Array Applications, Cells, Structures & Script Files

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EE201: Computer Applications. See Textbook Chapter 2 and Chapter 3.

Euclidean Vectors

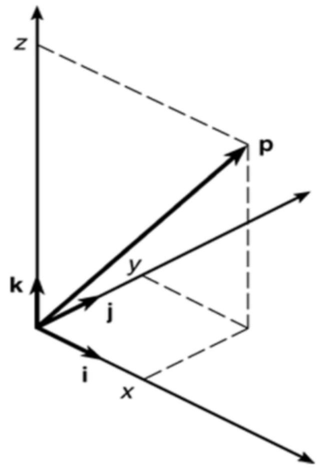
- An Euclidean vector (or geometric vector, or simply a vector) is a geometric entity that has both **magnitude** and **direction**.
- In physics, vectors are used to represent physical quantities that have both magnitude and direction, such as force, acceleration, electric field, etc.
- Vector algebra: adding and subtracting vectors, multiplying vectors, scaling vectors, etc.

Euclidean Vectors in MATLAB

- We specify a vector using its Cartesian coordinates.
- Hence, the vector **p** can be specified by three components: x, y and z, and can be written in MATLAB as:

```
p = [x, y, z];
```

- MATLAB supports 2-D and 3-D vectors, and even higher dimensional ones.



Magnitude, Length, Absolute Value

- In MATLAB, `length()` of a vector is **not** its magnitude. It is the number of elements in the vector.
- The **absolute value** of a vector **a** is a vector whose elements are the absolute values of the elements of **a**.
- The **magnitude** of a vector is its Euclidean norm or geometric length as shown:

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

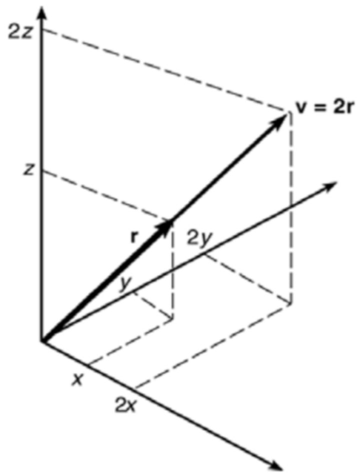
$$\|\mathbf{a}\| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

```
>> a = [2, -4, 5]
a =
     2    -4     5

>> length(a)
ans =
     3

>> abs(a)
ans =
     2     4     5

>> sqrt(a*a') % magnitude
ans =
    6.7082

>> sqrt(sum(a.*a)) %magnitude
ans =
    6.7082
```

$$\|\mathbf{a}\| = \sqrt{2^2 + (-4)^2 + 5^2} = \sqrt{\begin{bmatrix} 2 & -4 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix}} = 6.7082$$

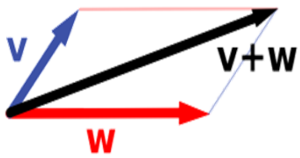
Vector Scaling

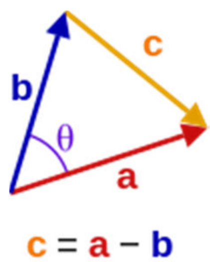
- For vector:
  $\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$
- Scaling this vector by a factor of 2 gives:
- $\mathbf{v} = 2\mathbf{a}$
  $= 2a_x\mathbf{i} + 2a_y\mathbf{j} + 2a_z\mathbf{k}$
- This is just like MATLAB scalar multiplication of a vector:
- `v = 2*[x, y, z];`



Adding and Subtracting Vectors

- Vector addition by geometry: The parallelogram law.
- Or, mathematically:
  $\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$
  $\mathbf{b} = b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}$
  $\mathbf{a} + \mathbf{b} = (a_x + b_x)\mathbf{i}$
  $+ (a_y + b_y)\mathbf{j}$
  $+ (a_z + b_z)\mathbf{k}$
- Same as vector addition and subtraction in MATLAB.



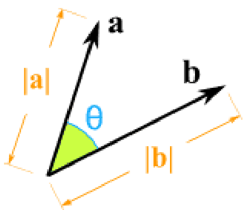


Exercise

```
>> a = [2 -4 6]
a =
     2    -4     6

>> b = [3 -1 -1]
b =
     3    -1    -1

>> c = a + b
c =
     5    -5     5

>> d = a - b
d =
    -1    -3     7

>> e = 2*a
e =
     4    -8    12
```

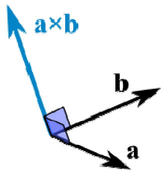

Dot Product

- The dot product of vectors results in a scalar value.
- $\mathbf{a} \cdot \mathbf{b}$
  $= (a_x b_x + a_y b_y + a_z b_z)$
  $= \|\mathbf{a}\|\|\mathbf{b}\| \cos(\theta)$




```
>> a = [2 -4 6];
>> b = [3 -1 -1];
>> c = a * b'
c =
     4

>> c = sum(a .* b)
c =
     4

>> c = dot(a, b)
c =
     4
```

Cross Product

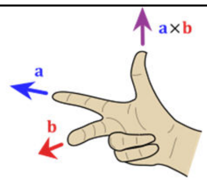


$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \sin(\theta)\, \mathbf{n}$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \mathbf{k}$$

```
>> a = [2 -4 6];
>> b = [3 -1 -1];
>> cross(a, b)
ans =
    10    20    10

>> syms x y z
>> det([x y z; 2 -4 6; 3 -1 -1])
ans =
 10*x + 20*y + 10*z

>> cross([1 0 0], [0 1 0])
ans =
     0     0     1
```



Complex Numbers

```
>> a = 7 + 4j
a =
   7.0000 + 4.0000i

>> [theta, rho] = cart2pol(real(a), imag(a))
theta =
    0.5191
rho =
    8.0623

>> rho = abs(a) % magnitude of complex number
rho =
    8.0623

>> theta = atan2(imag(a), real(a))
theta =
    0.5191
% atan2 is four quadrant inverse tangent

>> b = 3 + 4j
b =
   3.0000 + 4.0000i

>> a+b
ans =
  10.0000 + 8.0000i

>> a*b
ans =
   5.0000 + 40.0000i
```

Polynomials

- A polynomial can be written in the form:
  $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$
- Or more concisely:

$$\sum_{i=0}^{n} a_i x^i$$

- We can use MATLAB to find all the roots of the polynomial, i.e., the values of x that makes the polynomial equation equal 0.

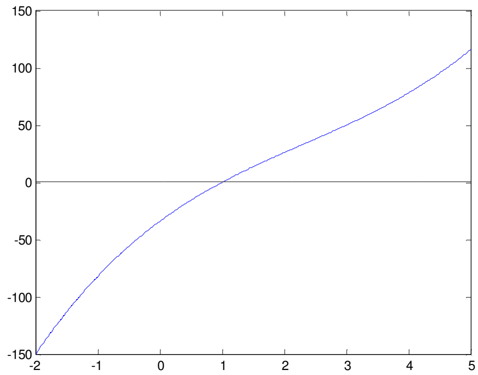
Exercise

- Polynomial Roots:
  $x^3 - 7x^2 + 40x - 34 = 0$
- Roots are $x = 1$, $x = 3 \pm 5i$.
- We can also build polynomial coefficients from its roots.
- We can also multiply (convolution) and divide (deconvolution) two polynomials.

```
>> a = [1 -7 40 -34];

>> roots(a)
ans =
   3.0000 + 5.0000i
   3.0000 - 5.0000i
   1.0000

>> poly([1 3+5i 3-5i])
ans =
     1    -7    40   -34
```

Just for fun… Plot…

```
>> x = -2:0.01:5;
>> f = x.^3 - 7*(x.^2) + 40*x - 34;
>> plot(x, f)
```



Cell Array

- The cell array is an array in which each element is a cell. Each cell can contain an array.
- So, it is an array of different arrays.
- You can store different classes of arrays in each cell, allowing you to group data sets that are related but have different dimensions.
- You access cell arrays using the same indexing operations used with ordinary arrays, but using `{}` not `()`.

Useful functions

`C = cell(n)`	Creates n × n cell array C of empty matrices.
`C = cell(n,m)`	Creates n × m cell array C of empty matrices.
`celldisp(C)`	Displays the contents of cell array C.
`cellplot(C)`	Displays a graphical representation of the cell array C.
`C = num2cell(A)`	Converts a numeric array A into a cell array C.
`iscell(C)`	Returns a 1 if C is a cell array; otherwise, returns a 0.

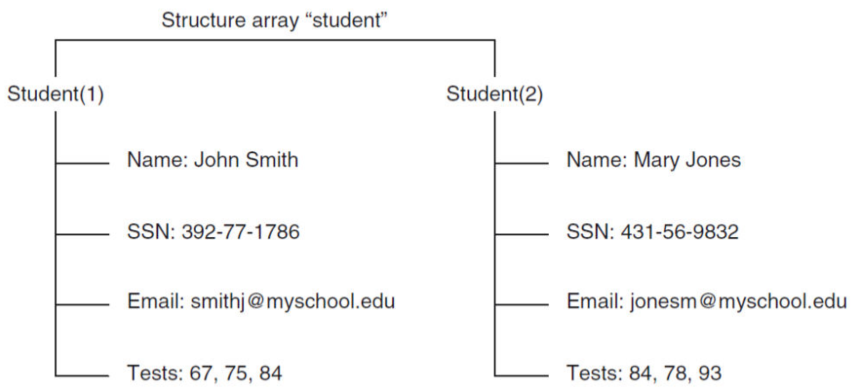
Exercise

```
>> C = cell(3)
C =
    []    []    []
    []    []    []
    []    []    []

>> D = cell(1, 3)
D =
    []    []    []

>> A(1,1) = {'Walden Pond'};
>> A(1,2) = {[1+2i 5+9i]};
>> A(2,1) = {[60,72,65]};
>> A(2,2) = {[55,57,56;54,56,55;52,55,53]};

>> A
A =
    'Walden Pond'    [1x2 double]
     [1x3 double]    [3x3 double]
```

Exercise (Continue)

```
>> celldisp(A)
A{1,1} =
Walden Pond

A{2,1} =
    60    72    65

A{1,2} =
   1.0000 + 2.0000i   5.0000 + 9.0000i

A{2,2} =
    55    57    56
    54    56    55
    52    55    53

>> B = {[2,4], [6,-9;3,5]; [7;2], 10}
B =
    [1x2 double]    [2x2 double]
    [2x1 double]    [         10]

>> B{1,2}
ans =
     6    -9
     3     5
```


Structures (*strcut.memebr*)



Create and Add to Structure

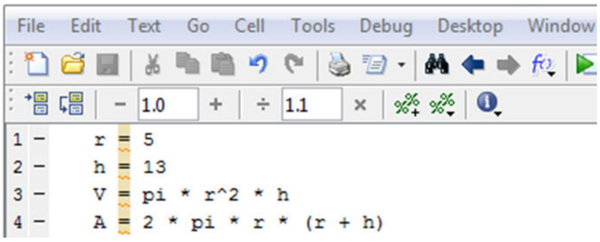
```
>> student.name = 'John Smith';
>> student.SSN = '392-77-1786';
>> student.email = 'smithj@myschool.edu';
>> student.exam_scores = [67,75,84];

>> student
student =
           name: 'John Smith'
            SSN: '392-77-1786'
          email: 'smithj@myschool.edu'
    exam_scores: [67 75 84]

>> student(2).name = 'Mary Jones';
>> student(2).SSN = '431-56-9832';
>> student(2).email = 'jonesm@myschool.edu';
>> student(2).exam_scores = [84,78,93];

>> student
student =
1x2 struct array with fields:
    name
    SSN
    email
    exam_scores
```


Investigate Structure

```
>> student(2)
ans =
           name: 'Mary Jones'
            SSN: '431-56-9832'
          email: 'jonesm@myschool.edu'
    exam_scores: [84 78 93]

>> fieldnames(student)
ans =
    'name'
    'SSN'
    'email'
    'exam_scores'

>> max(student(2).exam_scores)
ans =
    93

>> isstruct(student)
ans =
     1
```

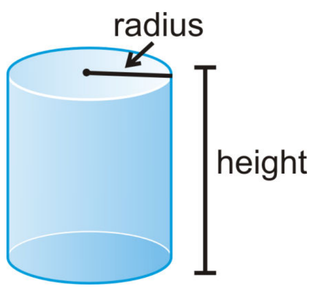
Script files

- You can save a particular sequence of MATLAB commands for reuse later in a script file (.m file)
- Each line is the same as typing a command in the command window.
- From the main menu, select File | New | Script, then save the file as `mycylinder.m`



Remember Example?

- Develop MATLAB code to find Cylinder volume and surface area.
- Assume radius of 5 m and height of 13 m.



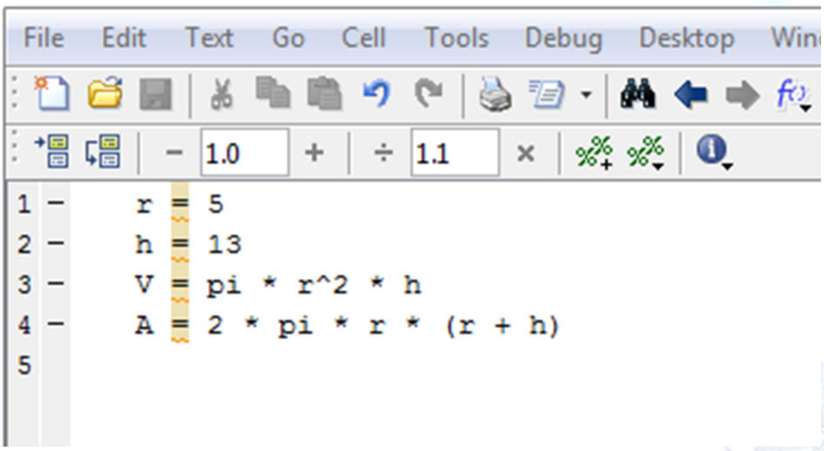
$$V = \pi r^2 h$$

$$A = 2\pi r^2 + 2\pi r h = 2\pi r(r + h)$$

Solution

```
>> r = 5
r =
     5

>> h = 13
h =
    13

>> V = pi * r^2 * h
V =
  1.0210e+003

>> A = 2 * pi * r * (r + h)
A =
  565.4867
```


Exercise



Be ware…

- Script File names MUST begin with a letter, and may include digits and the underscore character.
- Script File names should NOT:
  - include spaces
  - start with a number
  - use the same name as a variable or an existing command
- If you do any of the above you will get unusual errors when you try to run your script.
- You can check to see if a command, function or file name already exists by using the `exist` command.

Running .m files

- Run sequence of commands by typing

  `mycylinder`

  in the command window
- Make sure the current folder is set properly

```
>> mycylinder
r =
     5

h =
    13

V =
  1.0210e+003

A =
  565.4867
```

When you type `mycylinder`

When multiple commands have the same name in the current scope (scope includes current file, optional private subfolder, current folder, and the MATLAB path), MATLAB uses this precedence order:

1. **Variables** in current workspace: Hence, if you create a variable with the same name as a function, MATLAB cannot run that function until you clear the variable from memory.
2. **Nested functions** within current function
3. **Local functions** within current file
4. **Functions** in current folder
5. **Functions** elsewhere on the path, in order of appearance

Precedence of functions within the same folder depends on file type:

1. MATLAB **built-in** functions have precedence
2. Then **Simulink** models
3. Then program files with **.m extension**

Comments in MATLAB

- Comment lines start with a `%` not `//`
- Comments are not executed by MATLAB; it is there for people reading the code.
- Helps people understand what the code is doing and why!
- Comments are VERY IMPORTANT.
- Comment anything that is not easy to understand.
- Good commenting is a huge help when maintaining/fixing/extending code.
- Header comments show up when typing the `help` command.

Bad vs. Good Comments/Code

```
% set x to zero
x = 0
% calculate y
y = x * 9/5 + 32
```

```
% Convert freezing point of
% water from celsius to
% farenheit
c = 0
f = c * 9/5 + 32
```


Exercise

Editor - D:\EE 201 Computer Applications\Book Chapters\Lecture3 Arrays and Script Files\tem

File Edit Text Go Cell Tools Debug Desktop Window Help

```
% temperature.m Convert the boiling point for
% water from degrees Celsius (C) to Farenheit (F)
% Author: Dr. Mohammed Hawa

% Convert freezing point of water
C = 100
F = C * 9/5 + 32
```

Header comments

```
>> help temperature
  temperature.m Convert the boiling point for
  water from degrees Celsius (C) to Farenheit (F)
  Author: Dr. Mohammed Hawa

>> temperature

C =
   100

F =
   212
```


Simple User Interaction: I/O

- Use `input` command to get input from the user and store it in a variable:

```
h = input('Enter the height:')
```

- MATLAB will display the message enclosed in quotes, wait for input and then store the entered value in the variable

Simple User Interaction: I/O

- Use `disp` command to show something to a user

```
disp('The area of the cylinder is: ')
disp(A)
```

- MATLAB will display any message enclosed in quotes and then the value of the variable.

Exercise

```
r = input('Enter the radius:');
h = input('Enter the height:');

V = pi * r^2 * h;
A = 2 * pi * r * (r + h);

disp('The volume of the cylinder is: ');
disp(V);
disp('The area of the cylinder is: ');
disp(A);
```

```
>> mycylinder
Enter the radius:5
Enter the height:13
The volume of the cylinder is:
  1.0210e+003

The area of the cylinder is:
  565.4867
```

Summary

`disp(A)`	**Displays the contents, but not the name, of the array A.**
`disp('text')`	**Displays the text string enclosed within quotes.**
`x = input('text')`	**Displays the text in quotes, waits for user input from the keyboard, and stores the *value* in x.**
`x = input('text','s')`	**Displays the text in quotes, waits for user input from the keyboard, and stores the input as a *string* in x.**

Homework

- The speed v of a falling object dropped with zero initial velocity is given as a function of time t by $v = gt$, where g is the gravitational acceleration.
- Plot v as a function of t for $0 \ll t \ll t_f$, where t_f is the final time entered by the user.
- Use a script file with proper comments.

Solution

```
% Plot speed of a falling object
% Author: Dr. Mohammed Hawa

g = 9.81; % Acceleration in SI units

tf = input('Enter final time in seconds:');

t = [0:tf/500:tf]; % array of 501 time instants
v = g*t; % speed

plot(t,v);
xlabel('t (sseconds)');
ylabel('v m/s)');
```


Homework

- Solve as many problems from Chapter 2 as you can
- Suggested problems:
- 2.33, 2.34, 2.35, 2.36, 2.39, 2.41, 2.45, 2.48

Lecture 4: Complex Numbers Functions, and Data Input

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EE201: Computer Applications. See Textbook Chapter 3.

What is a Function?

- A MATLAB Function (e.g. `y = func(x1, x2)`) is like a script file, but with inputs and outputs provided automatically in the commend window.
- In MATLAB, functions can take zero, one, two or more inputs, and can provide zero, one, two or more outputs.
- There are built-in functions (written by the MATLAB team) and functions that you can define (written by you and stored in .m file).
- Functions can be called from command line, from wihtin a script, or from another function.

Table 3.1–1 Some common mathematical functions

Exponential	
`exp(x)`	Exponential; e^x.
`sqrt(x)`	Square root; $\sqrt{x}$.
Logarithmic	
`log(x)`	Natural logarithm; $\ln x$.
`log10(x)`	Common (base-10) logarithm; $\log x = \log_{10} x$.
Complex	
`abs(x)`	Absolute value; x.
`angle(x)`	Angle of a complex number x.
`conj(x)`	Complex conjugate.
`imag(x)`	Imaginary part of a complex number x.
`real(x)`	Real part of a complex number x.
Numeric	
`ceil(x)`	Round to the nearest integer toward ∞.
`fix(x)`	Round to the nearest integer toward zero.
`floor(x)`	Round to the nearest integer toward $-\infty$.
`round(x)`	Round toward the nearest integer.
`sign(x)`	Signum function: $+1$ if $x > 0$; 0 if $x = 0$; -1 if $x < 0$.

Functions are Helpful

- Enable "divide and conquer" strategy
  – Programming task broken into smaller tasks
- Code reuse
  – Same function useful for many problems
- Easier to debug
  – Check right outputs returned for all possible inputs
- Hide implementation
  – Only interaction via inputs/outputs, how it is done (implementation) hidden inside the function.

Finding Useful Functions

- You can use the `lookfor` command to find MATLAB functions that are relevant to your application.
- Example: `>> lookfor imaginary`
- Gets a list of functions that deal with imaginary numbers.
- `i       - Imaginary unit.`
- `j       - Imaginary unit.`
- `complex - Construct complex result from real and imaginary parts.`
- `imag    - Complex imaginary part.`

Calling Functions

- Function names are case sensitive (meshgrid, meshGrid and MESHGRID are interpreted as different functions).
- Inputs (called function *arguments* or function *parameters*) can be either numbers or variables.
- Inputs are passed into the function inside of parentheses `()` separated by commas.
- We usually assign the output to variable(s) so we can use it later. Otherwise it is assigned to the built-in variable `ans`.

Rules

- To evaluate sin 2 in MATLAB, we type `sin(2)`, not `sin[2]`
- For example `sin[x(2)]` gives an error even if x is defined as an array.
- Inputs to functions in MATLAB can be sometimes arrays.

```
>> x = -3 + 4i;
>> mag_x = abs(x)
mag_x =
     5

>> mag_y = abs(6 - 8i)
mag_y =
    10

>> angle_x = angle(x)
angle_x =
    2.2143

>> angle(x)
ans =
    2.2143

>> x = [5,7,15]
x =
     5     7    15

>> y = sqrt(x)
y =
    2.2361    2.6458    3.8730
```


Function Composition

- Composition: Using a function as an argument of another function
- Allowed in MATLAB.
- Just check the number and placement of parentheses when typing such expressions.
- `sin(sqrt(x)+1)`
- `log(x.^2 + sin(5))`

Which expression is correct?

- You want to find $\sin^2(x)$. What do you write?
- `(sin(x))^2`
- `sin^2(x)`
- `sin^2x`
- `sin(x^2)`
- `sin(x)^2`
- *Solution*: Only first and last expressions are correct.

Trigonometric Functions

Trigonometric*	
`cos(x)`	Cosine; $\cos x$.
`cot(x)`	Cotangent; $\cot x$.
`csc(x)`	Cosecant; $\csc x$.
`sec(x)`	Secant; $\sec x$.
`sin(x)`	Sine; $\sin x$.
`tan(x)`	Tangent; $\tan x$.
Inverse trigonometric†	
`acos(x)`	Inverse cosine; $\arccos x = \cos^{-1} x$.
`acot(x)`	Inverse cotangent; $\operatorname{arccot} x = \cot^{-1} x$.
`acsc(x)`	Inverse cosecant; $\operatorname{arccsc} x = \csc^{-1} x$.
`asec(x)`	Inverse secant; $\operatorname{arcsec} x = \sec^{-1} x$.
`asin(x)`	Inverse sine; $\arcsin x = \sin^{-1} x$.
`atan(x)`	Inverse tangent; $\arctan x = \tan^{-1} x$.
`atan2(y,x)`	Four-quadrant inverse tangent.

*These functions accept x in radians.
†These functions return a value in radians.

Hyperbolic functions

Hyperbolic	
`cosh(x)`	Hyperbolic cosine; $\cosh x = (e^x + e^{-x})/2$.
`coth(x)`	Hyperbolic cotangent; $\cosh x/\sinh x$.
`csch(x)`	Hyperbolic cosecant; $1/\sinh x$.
`sech(x)`	Hyperbolic secant; $1/\cosh x$.
`sinh(x)`	Hyperbolic sine; $\sinh x = (e^x - e^{-x})/2$.
`tanh(x)`	Hyperbolic tangent; $\sinh x/\cosh x$.
Inverse hyperbolic	
`acosh(x)`	Inverse hyperbolic cosine
`acoth(x)`	Inverse hyperbolic cotangent
`acsch(x)`	Inverse hyperbolic cosecant
`asech(x)`	Inverse hyperbolic secant
`asinh(x)`	Inverse hyperbolic sine
`atanh(x)`	Inverse hyperbolic tangent

User-Defined Functions

- Functions must be saved to a file with .m extension.
- Filename (without the .m) must match EXACTLY the function name.
- First line in the file must begin with a function definition line that illustrates inputs and outputs.

```
function [output variables] = name(input variables)
```

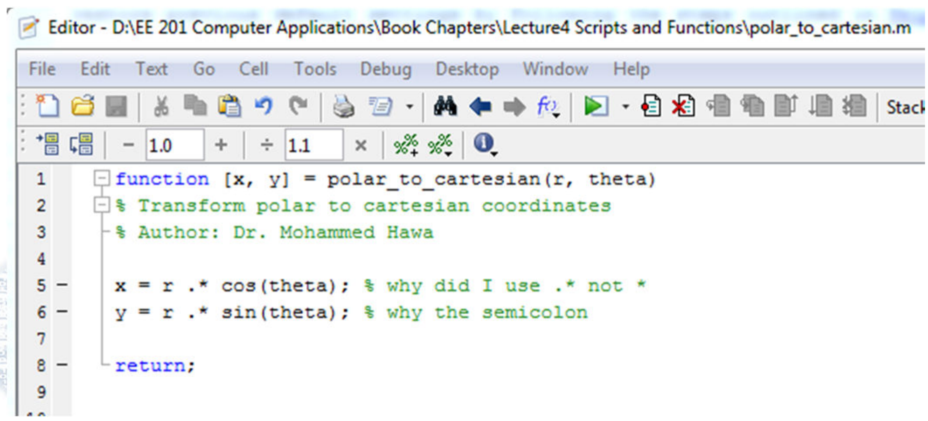
- This line distinguishes a function M-file from a script M-file.
- Output variables are enclosed in square brackets.
- Input variables must be enclosed with parentheses.

Functions Names

- Function names may only use alphanumeric characters and the underscore.
- Functions names should NOT:
  - include spaces
  - start with a number
  - use the same name as an existing command
- Consider adding a header comment, just under the function definition (for `help`).

Exercise: Your Own `pol2cart`

- Make sure you set you Current Folder to Desktop (or where you saved the .m file).

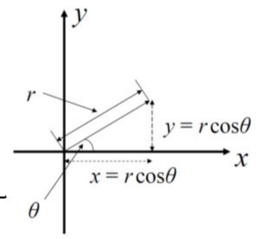


Test your newly defined function

```
>> [a, b] = polar_to_cartesian(3, pi)
a =
    -3
b =
  3.6739e-016

>> polar_to_cartesian(3, pi)
ans =
    -3

>> [a, b] = polar_to_cartesian(3, pi/4)
a =
    2.1213
b =
    2.1213

>> [a, b] = polar_to_cartesian([3 3 3], [pi pi/4 pi/2])
a =
   -3.0000    2.1213    0.0000
b =
    0.0000    2.1213    3.0000
```


MATLAB has pol2cart

```
>> help pol2cart

 POL2CART Transform polar to Cartesian coordinates.
    [X,Y] = POL2CART(TH,R) transforms corresponding elements of data
    stored in polar coordinates (angle TH, radius R) to Cartesian
    coordinates X,Y.  The arrays TH and R must the same size (or
    either can be scalar).  TH must be in radians.

    [X,Y,Z] = POL2CART(TH,R,Z) transforms corresponding elements of
    data stored in cylindrical coordinates (angle TH, radius R, height
    Z) to Cartesian coordinates X,Y,Z. The arrays TH, R, and Z must be
    the same size (or any of them can be scalar).  TH must be in radians.

    Class support for inputs TH,R,Z:
       float: double, single

    See also cart2sph, cart2pol, sph2cart.

    Reference page in Help browser
       doc pol2cart
```

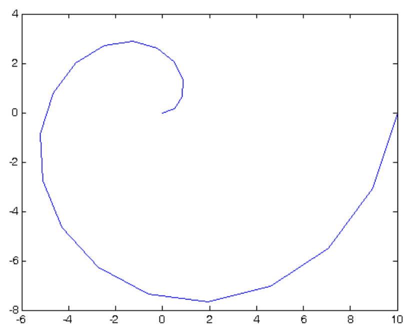


Just like your code!

```
>> type pol2cart

function [x,y,z] = pol2cart(th,r,z)
%POL2CART Transform polar to Cartesian coordinates.
%   [X,Y] = POL2CART(TH,R) transforms corresponding elements of data
%   stored in polar coordinates (angle TH, radius R) to Cartesian
%   coordinates X,Y.  The arrays TH and R must the same size (or
%   either can be scalar).  TH must be in radians.
%
%   [X,Y,Z] = POL2CART(TH,R,Z) transforms corresponding elements of
%   data stored in cylindrical coordinates (angle TH, radius R, height
%   Z) to Cartesian coordinates X,Y,Z. The arrays TH, R, and Z must be
%   the same size (or any of them can be scalar).  TH must be in radians.
%
%   Class support for inputs TH,R,Z:
%      float: double, single
%
%   See also CART2SPH, CART2POL, SPH2CART.

%   L. Shure, 4-20-92.
%   Copyright 1984-2004 The MathWorks, Inc.
%   $Revision: 5.9.4.2 $  $Date: 2004/07/05 17:02:08 $

x = r.*cos(th);
y = r.*sin(th);
```


Exercise: Spiral

```
>> r = linspace(0, 10, 20);
>> theta = linspace(0, 2*pi, 20);
>> [x, y] = polar_to_cartesian(r, theta);
>> plot(x,y);
```



Possible Cases

- One input:

```
function [o1, o2, o3] = myfunc(i1)
```

- Three inputs:

```
function [o1, o2, o3] = myfunc(i1, i2, i3)
```

- No inputs:

```
function [o1, o2, o3] = myfunc()
function [o1, o2, o3] = myfunc
```

- One output:

```
function [o1] = myfunc(i1, i2, i3)
function o1 = myfunc(i1, i2, i3)
```

- No output:

```
function [] = myfunc(i1, i2, i3)
function myfunc(i1, i2, i3)
```


Local Variables

```
function z = fun(x,y)

u = 3*x;
z = u + 6*y.^2;

% return missing is fine at end of file
```

- The variables x, y, u, z are **local** to the function `fun`, so their values will not be available in the workspace outside the function.
- See example below.

Example

```
>> x = 3;
>> b = 7;
>> q = fun(x, b);

>> x
x =
     3

>> y
??? Undefined function or variable 'y'.

>> u
??? Undefined function or variable 'u'.

>> z
??? Undefined function or variable 'z'.

>> q
q =
   303
```


Exercise

```
function show_date
clear
clc
date

% how many inputs and outputs do we have?
```

Homework

```
function [dist, vel] = drop(v0, t)
% Compute the distance travelled and the
% velocity of a dropped object, from
% the initial velocity v0, and time t
% Author: Dr. Mohammed Hawa

g = 9.80665; % gravitational acceleration (m/s^2)
vel = g*t + v0;
dist = 0.5*g*t.^2 + v0*t;
```

```
>> t = 0:0.1:5;
>> [distance_dropped, velocity] = drop(10, t);
>> plot(t, velocity)
```


Local vs. Global Variables

- The variables inside a function are local. Their scope is only inside the function that declares them.
- In other words, functions create their own workspaces.
- Function inputs are also created in this workspace when the function starts.
- Functions do not know about any variables in any other workspace.
- Function outputs are copied from the function workspace when the function ends.
- Function workspaces are destroyed after the function ends.
  - Any variables created inside the function "disappear" when the function ends.

Local vs. Global Variables

- You can, however, define global variables if you want using the `global` keyword.
- Syntax: `global a x q`
- Global variables are available to the basic workspace and to other functions that declare those variables global (allowing assignment to those variables from multiple functions).

Subfunctions

- An M-file may contain more than one user-defined function.
- The first defined function in the file is called the *primary* function, whose name is the same as the M-file name.
- All other functions in the file are called *subfunctions*. They can serve as subroutines to the primary function.
- Subfunctions are normally "visible" only to the primary function and other subfunctions in the same file; that is, they normally cannot be called by programs or functions outside the file.
- However, this limitation can be removed with the use of function handles.
- We normally use the same name for the primary function and its file, but if the function name differs from the file name, you must use the file name to invoke the function.

Exercise

- The following example shows how the MATLAB M-function `mean` can be superceded by our own definition of the mean, one which gives the root-mean square value.

```
function y = subfun_demo(a)
y = a - mean(a);

function w = mean(x)
w = sqrt(sum(x.^2))/length(x);
```


Example

- A sample session follows.

```
>>y = subfn_demo([4 -4])
y =
     1.1716     -6.8284
```

- If we had used the MATLAB M-function mean, we would have obtained a different answer; that is,

```
>>a = [4 -4];
>>b = a - mean(a)
b =
     4       -4
```

Function Handles

- You can create a function handle to any function by using the @ sign before the function name.
- You can then use the handle to reference the function.

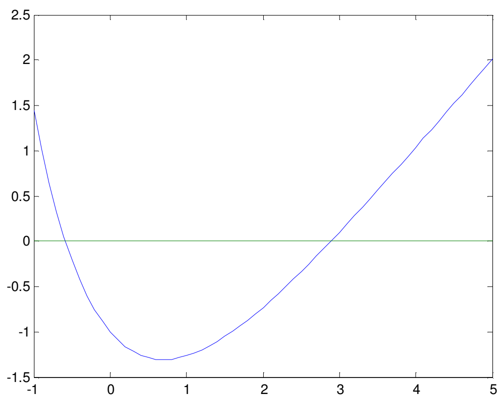
```
function y = f1(x)
y = x + 2*exp(-x) - 3;
```

- You can pass the function as an argument to another function using the handle. Example: `fzero` function finds the zero of a function of a single variable x.
- `>> x0 = 3; % initial guess`
- `>> fzero(@f1, x0)`

Handle vs. Return Value

```
t = -1:0.1:5;
plot(t, f1(t));
```

- There is a zero near $x = -0.5$ and one near $x = 3$.



Exercise

fzero(@function,x0)

- where @function is the function handle, and x0 is a user-supplied initial guess for the zero.

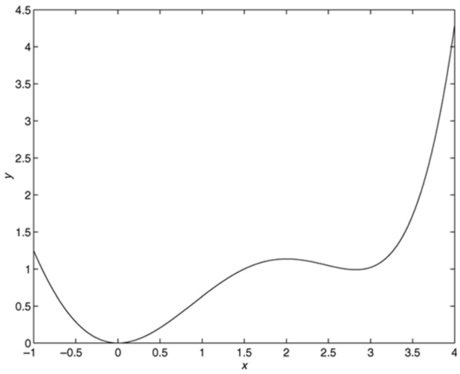
```
>> fzero(@f1, -0.5)
ans =
   -0.5831

>> fzero(@f1, 3)
ans =
    2.8887

>> fzero(@sin, 0.1)
ans =
  6.6014e-017

>> fzero(@cos, 2)
ans =
    1.5708

>> pi/2
ans =
    1.5708
```


Finding the Minimum

- The `fminbnd` function finds the minimum of a function of a single variable x in the interval x1 ≤ x ≤ x2.
- `fminbnd(@function, x1, x2)`
- `fminbnd(@cos, 0, 4)` returns 3.1416
- `function y = f2(x)`
- `y = 1-x.*exp(-x);`
- `x = fminbnd(@f2, 0, 5)` returns x = 1
- How would I find the min value of f2? (i.e. 0.6321)

Exercise

- For the function:
- $y = 0.025x^5 - 0.0625x^4 - 0.333x^3 + x^2$
- Find the minimum in the intervals:
- $x \in [-1, 4]$
- $x \in [1, 4]$
- $x \in [2, 4]$
- $x \in [-1, 1]$



Old vs. New

- New syntax for function handles:

```
fzero(@f1, -0.5)
```

- Older syntax for function handles :

```
fzero('f1', -0.5)
```

- The new syntax is preferred, though both will work just fine.
- Which one gives the correct answer: `fzero('sin', 3)` or `fzero(@sin, 3)`

The `fminsearch` function

- `fminsearch` finds minimum of a function of *more than one* variable.
- To find where the minimum of $f = xe^{-(x^2+y^2)}$, define it in an M-file, using the vector x whose elements are x(1) = x and x(2) = y.

```
function f = f4(x)
f = x(1).*exp(-x(1).^2-x(2).^2);
```

- Suppose we guess that the minimum is near $x = 0, y = 0$.

```
>>fminsearch(@f4,[0,0])
ans =
   -0.7071   0.000
```

- Thus the minimum occurs at $x = -0.7071, y = 0$.

Inline Function

- No need to save the function in an M-file.
- Useful for small size functions defined on the fly.
- You can use a string array to define the function.
- Anonymous functions are similar (see next).

```
>> f4 = inline('x.^2-4')
f4 =
     Inline function:
     f4(x) = x.^2-4

>> [x, value] = fzero(f4, 0)
x =
    -2
value =
     0

>> f5str = 'x.^2-4'; % string array
>> f5 = inline(f5str)
f5 =
     Inline function:
     f5(x) = x.^2-4

>> x = fzero(f5, 3)
x =
     2

>> x = fzero('x.^2-4', 3)
x =
     2

>> f6 = inline('x.*y')
f6 =
     Inline function:
     f6(x,y) = x.*y
```

Anonymous functions

- Here is a simple function called `sq` to calculate the square of a number.

```
>> sq = @(x) x.^2;

>> sq = @(x) (x.^2)
sq =
    @(x)(x.^2)

>> sq([5 7])
ans =
    25    49

>> fminbnd(sq, -10, 10)
ans =
     0
```


Exercise

```
>> sqrtsum = @(x,y) sqrt(x.^2 + y.^2);

>> sqrtsum(3, 4)
ans =
     5

>> A = 6; B = 4;

>> plane = @(x,y) A*x + B*y;

>> z = plane(2,8)
z =
    44

>> f = @(x) x.^3; % try by hand!
>> g = @(x) 5*sin(x);
>> h = @(x) g(f(x));
>> h(2)
ans =
    4.9468
```

Variables in Anonymous Functions

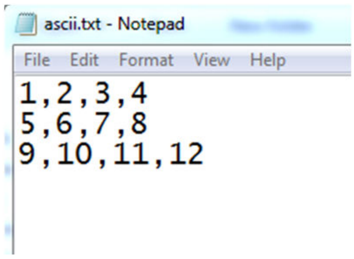
- When the function is created MATLAB, it captures the values of these variables and retains those values for the lifetime of the function handle. If the values of A or B are changed after the handle is created, their values associated with the handle do not change.
- This feature has both advantages and disadvantages, so you must keep it in mind.

For Speed Use Handles

- The function handle provides speed improvements.
- Another advantage of using a function handle is that it provides access to subfunctions, which are normally not visible outside of their defining M-file.

Importing Data: ASCII

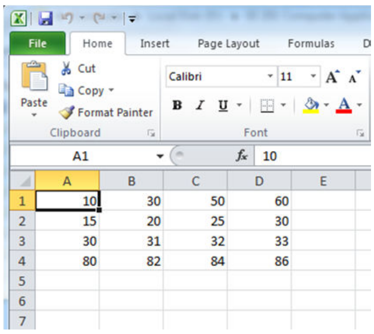
- Make the ‘data’ folder your Current Folder.
- Delimited ASCII files are common to save data from experiments.
- `dlmread/dlmwrite`




```
>> a = dlmread('ascii.txt')
a =
     1     2     3     4
     5     6     7     8
     9    10    11    12
```


Importing Data: Excel

- Make the ‘data’ folder your Current Folder.
- MATLAB can also read and write to Excel Files.
- `xlsread/xlswrite`




```
>> a = xlsread('data.xls')
a =
    10    30    50    60
    15    20    25    30
    30    31    32    33
    80    82    84    86
```

Importing Data: Images

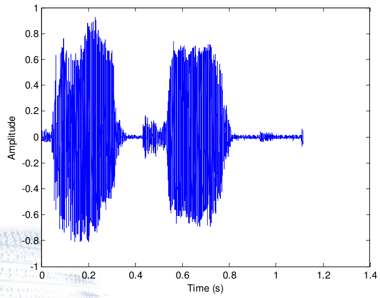
- Make the 'data' folder your Current Folder.
- Read and write images:
- `imread/imwrite`

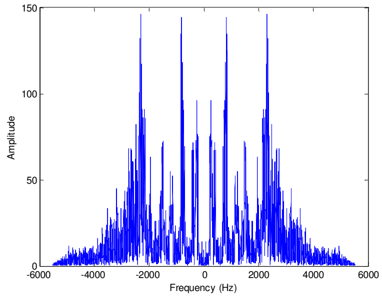
```
>> c = imread('cat.jpg');
>> imshow(c);
>>
>> imshow(255-c); % inverse
```


Importing Data: Sound Files

```
% use a script file (fourier.m)
[y,Fs,bits] = wavread('bequiet');
N = length(y);
t = (1/Fs)*(1:N);

plot(t, y);
xlabel('Time (s)');
ylabel('Amplitude');

f = Fs*(-N/2:N/2-1)/N;
y_fft = fftshift(abs(fft(y)));

figure;
plot(f, y_fft);
xlabel('Frequency (Hz)');
ylabel('Amplitude');
```


bequiet.wav (BW of human voice!)





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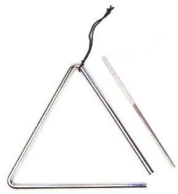
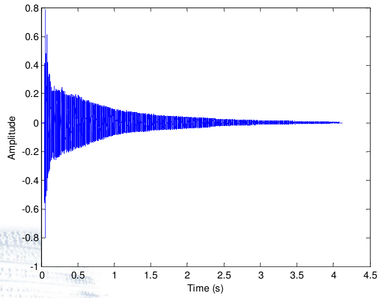
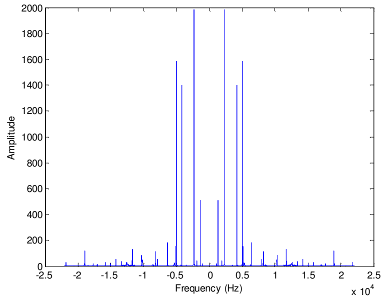
triangle.wav







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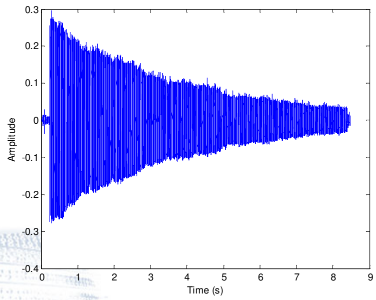
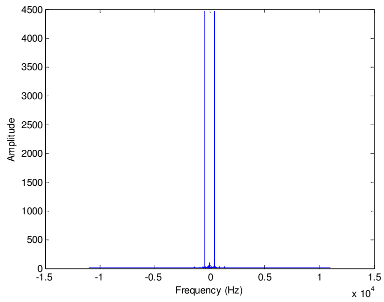
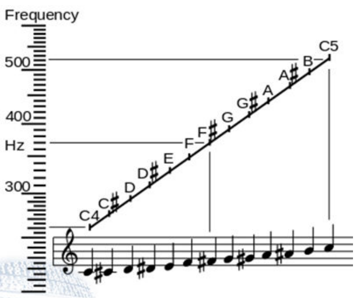
tuningA4.wav (frequency?)



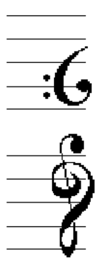




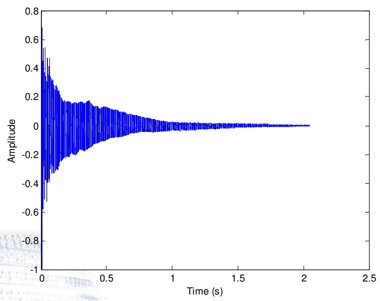
MIDI number	Note name	Frequency Hz	Period ms
21	A0	27.500	36.36
22		29.135	34.32
23	B0	30.868	32.40
24	C1	32.703	30.58
25		34.648	28.86
26	D1	36.708	27.24
27		38.891	25.71
28	E1	41.203	24.27
29	F1	43.654	22.91
30		46.249	21.62
31	G1	48.999	20.41
32		51.913	19.26
33	A1	55.000	18.18
34		58.270	17.16
35	B1	61.735	16.20
36	C2	65.406	15.29
37		69.296	14.29
38	D2	73.416	13.62
39		77.782	12.86
40	E2	82.407	12.13
41	F2	87.307	11.45
42		92.499	10.81
43	G2	97.999	10.20
44		103.83	9.631
45	A2	110.00	9.091
46		116.54	8.581
47	B2	123.47	8.099
48	C3	130.81	7.645
49		138.59	7.216
50	D3	146.83	6.811
51		155.56	6.428
52	E3	164.81	6.068
53	F3	174.61	5.727
54		185.00	5.405
55	G3	196.00	5.102
56		207.65	4.816
57	A3	220.00	4.545
58		233.08	4.290
59	B3	246.94	4.050
60	**C4**	**261.63**	**3.822**
61		277.18	3.608
62	D4	293.67	3.405
63		311.13	3.214
64	E4	329.63	3.034
65	F4	349.23	2.863
66		369.99	2.703
67	G4	392.00	2.551
68		415.30	2.408
69	**A4**	**440.00**	**2.273**
70		466.16	2.145
71	B4	493.88	2.025
72	C5	523.25	1.910
73		554.37	1.804
74	D5	587.33	1.703
75		622.25	1.607
76	E5	659.26	1.517
77	F5	698.46	1.432
78		739.99	1.351
79	G5	783.99	1.276
80		830.61	1.204
81	A5	880.00	1.136
82		932.33	1.073
83	B5	987.77	1.012
84	C6	1046.5	0.9556
85		1108.7	0.9020
86	D6	1174.7	0.8513
87		1244.5	0.8034
88	E6	1318.5	0.7584
89	F6	1396.9	0.7159
90		1480.0	0.6757
91	G6	1568.0	0.6378
92		1661.2	0.6020
93	A6	1760.0	0.5682
94		1864.7	0.5363
95	B6	1975.5	0.5062
96	C7	2093.0	0.4778
97		2217.5	0.4510
98	D7	2349.3	0.4257
99		2489.0	0.4018
100	E7	2637.0	0.3792
101	F7	2793.0	0.3580
102		2960.0	0.3378
103	G7	3136.0	0.3189
104		3322.4	0.3010
105	A7	3520.0	0.2841
106		3729.3	0.2681
107	B7	3951.1	0.2531
108	C8	4186.0	0.2389

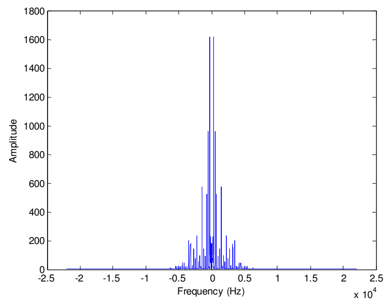
Keyboard

J. Wolfe, UNSW



guitar.wav





Homework

- Solve as many problems from Chapter 3 as you can
- Suggested problems:
- 3.1, 3.3, 3.6, 3.9, 3.14, 3.18, 3.24

Lecture 5: Programming using MATLAB

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EE201: Computer Applications. See Textbook Chapter 4.

Algorithms and Control Structures

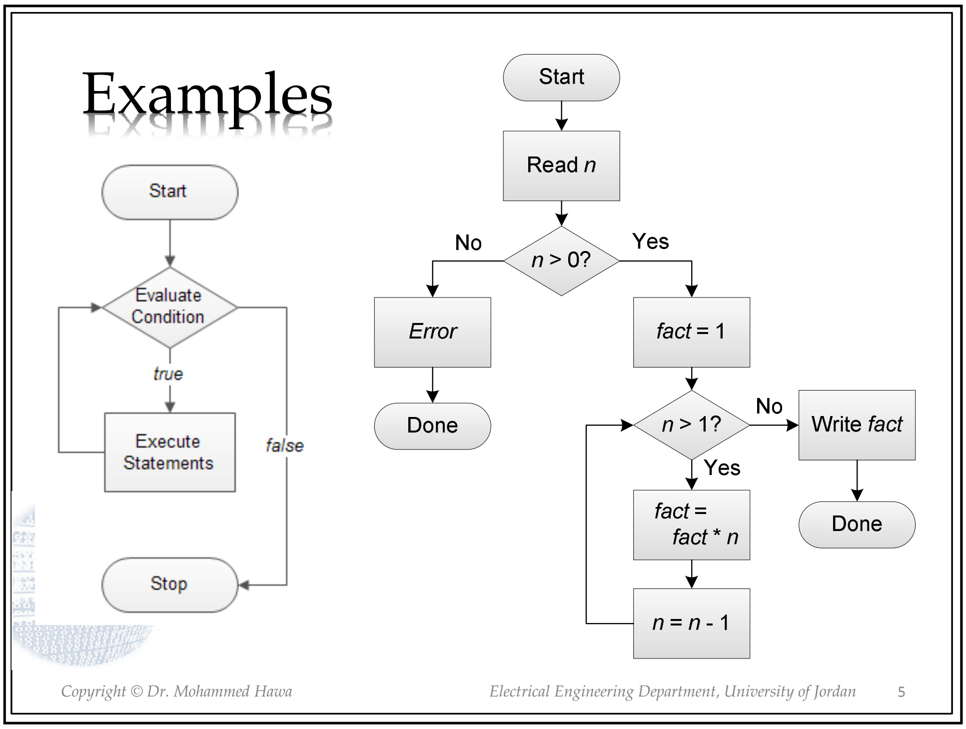
- **Algorithm**: a sequence of instructions that performs some task in a finite amount of time.
- The algorithm uses a *control structure* to execute instructions in a certain order.
- Control structure categories:
  - **Sequential operations**: Instructions executed in order.
  - **Conditional operations**: First ask a question to be answered with a true/false answer and then select the next instruction based on the answer.
  - **Iterative operations (loops)**: Repeat the execution of a block of instructions.

Before Programming

- Before writing a program, we need a plan.
- A plan helps us focus on the problem, not the code.
- Common methods to show a plan are:
  - **Flowchart**: A graphical description of the program flow.
  - **Pseudocode**: A verbal description of the program details.

Flowcharts

- Flowcharts are geometric symbols to describe the program steps.
- They capture the "flow" of the program.
- Flowcharts are useful for developing and documenting programs that contain conditional statements, because they can display the various paths (called "branches") that a program can take, depending on how the conditional statements are executed.


Examples
Start
Evaluate Condition
true
Execute Statements
false
Stop
Start
Read n
n > 0?
No
Yes
Error
Done
fact = 1
n > 1?
No
Write fact
Done
Yes
fact = fact * n
n = n - 1
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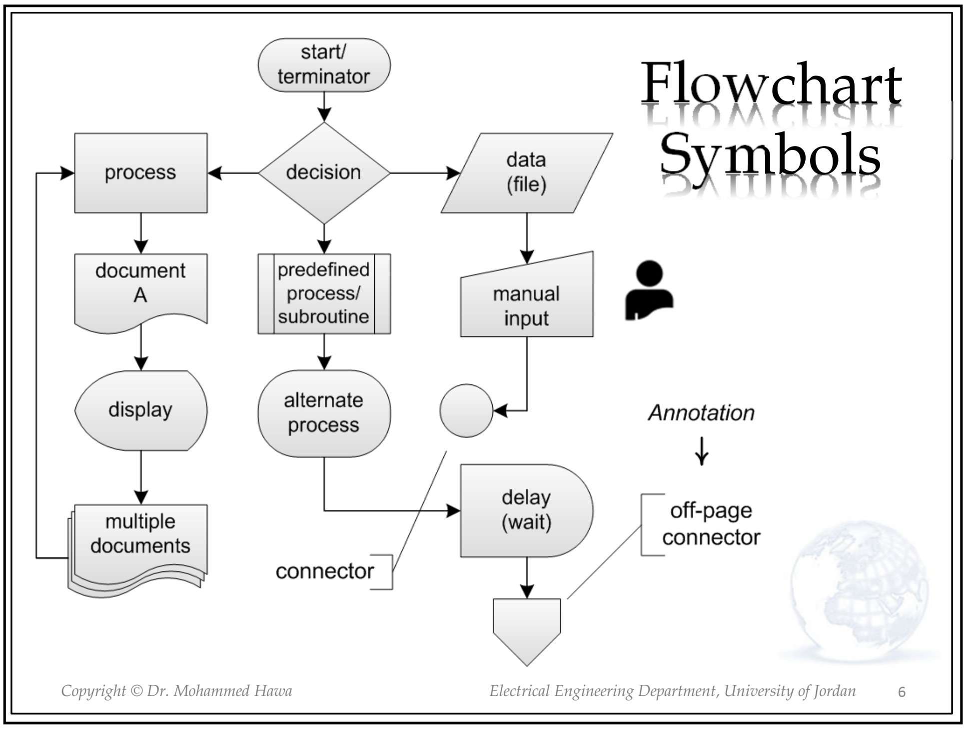
Flowchart Symbols
start/ terminator
process
decision
data (file)
document A
predefined process/ subroutine
manual input
display
alternate process
Annotation
delay (wait)
off-page connector
multiple documents
connector
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Pseudocode

- In pseudocode, natural language and mathematical expressions are used to construct statements that look like computer statements but without detailed syntax.
- Each pseudocode instruction may be numbered, but should be unambiguous and computable.
- Similar to a recipe.

Pseudocode Example

Input: A nonempty string of characters $S_1S_2\ldots S_n$, and a positive integer n giving the number of characters in the string.
Output: See the related problem below.
Procedure:

```
Get n
Get S1S2...Sn
Set count = 1
Set ch = S1
Set i = 2
While i ≤ n
   If Si equals ch
      Set count = count + 1
   Set i = i + 1
Print ch, ' appeared ', count, ' times.'
Stop
```

Problem 1.1 *What is printed if the input string is* pepper*?*

Problem 1.2 *What is printed if the input string is* CACCTGGTCCAAC*?*

Algorithm Distribute

Input: $(G^*, f, edge)$, where $G^* = (N, M, s, t, E^*, w)$, f is a set of flows f_e^v
and $edge$ is the edge that is being distributed.

Initialize $scan(v) = 0, label(v) = 0, scan(e) = 0, label(e) = 0$ for all $v \in N, e \in M$

$vert = 0,\ capvert = 0$
$label(edge) = 1,\ pathcap(edge) = w(edge)$
while $(w(edge) > \sum_v f_{edge}^v)$ **or** not all labeled nodes have been scanned
  for all labeled $e \in M$, with $scan(e) = 0$
    label unlabeled neighbors of e (i.e $v \in N$)
    $scan(e) = 1,\ pred(v) = e,\ pathcap(v) = pathcap(e)$
  endfor
  for all labeled $v \in N$ with $scan(v) = 0$
    if $\min(w(v) - \sum_e f_e^v,\ pathcap(v)) > capvert$ **then**
      $vert = v,\ capvert = min(w - \sum_e f_e^v,\ pathcap(v))$
      else
        label all unlabeled $e' \in M$ s.t $f_{e'}^v > 0$
    endif
      $scan(v) = 1$
  endfor
  if $vert > 0$ **then**
    An augmenting path from s to t has been found: backtrack from
  $vert$ using $pred()$ and change the values of f_e^v as requirted.
    for all $e \in M,\ v \in N$
      $label(e) = 0, scan(e) = 0, label(v) = 0, scan(v) = 0$
    endfor
    $vert = 0, capvert = 0, label(edge) = 1$
    $pathcap(edge) = w(edge) - \sum_v f_{edge}^v$
  endif
endwhile

During and After Programming

- Make sure to provide effective documentation along with the program. This can be accomplished using:
  - Proper selection of variable names to reflect the quantities they represent.
  - Using comments within the program.
- Debugging a program is the process of finding and removing the "bugs" or errors in a program.

Bugs

Bugs usually fall into one of two categories:

1. **Syntax errors:** such as omitting a parenthesis or comma, or spelling a command name incorrectly. MATLAB usually detects the more obvious errors and displays a message describing the error and its location.
2. Errors due to an incorrect mathematical procedure. These are called **runtime errors**. They do not necessarily occur every time the program is executed; their occurrence often depends on the particular input data. A common example is division by zero.

Finding Bugs: Debugging

To locate runtime errors, try the following:

1. Always test your program with a simple version of the problem, whose answers can be checked by hand calculations.
2. Display any intermediate calculations by removing semicolons at the end of statements.

Relational Operators

Operator	Meaning
<	Less than.
<=	Less than or equal to.
>	Greater than.
>=	Greater than or equal to.
==	Equal to.
~=	Not equal to.

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Examples

```
>> a = 3;
>> b = 4;
>> a == b
ans =
     0

>> a ~= b
ans =
     1

>> a < b
ans =
     1

>> b >= -4
ans =
     1
```

```
>> x = [6 3 9];
>> y = [14 2 9];
>> z = (x < y)
z =
     1     0     0

>> z = x ~= y
z =
     1     1     0

>> z = x > 8
z =
     0     0     1
```

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Relational operators can be used for array addressing.

For example

```
>> x = [6,3,9];
>> y = [14,2,9];
>> x<y
ans =
     1     0     0
>> z = x(x<y)
z =
     6
```

finds all the elements in x that are less than the corresponding elements in y. The result is `z = 6`.

The arithmetic operators +, -, *, /, and \ have precedence over the relational operators. Thus the statement

```
z = 5 > 2 + 7
```

is equivalent to

```
z = 5 > (2+7)
```

and returns the result `z = 0`.

We can use parentheses to change the order of precedence; for example, `z = (5 > 2) + 7` evaluates to `z = 8`.

The logical Class

When the relational operators are used, such as

```
x = (5 > 2)
```

they create a *logical* variable, in this case, `x`.

Logical variables may have only the values 1 (true) and 0 (false).

Just because an array contains only 0s and 1s, however, it is not necessarily a logical array. For example, in the following session `k` and `w` appear the same, but `k` is a logical array and `w` is a numeric array, and thus an error message is issued.

```
>>x = -2:2;
>>k = (abs(x)>1)
k =
   1   0   0    0    1
>>z = x(k)
z =
   -2   2
>>w = [1,0,0,0,1]; v = x(w)
??? Subscript indices must either be real
  positive... integers or logicals.
```

Accessing Arrays Using Logical Arrays

When a logical array is used to address another array, it extracts from that array the elements in the locations where the logical array has 1s.

So typing `A(B)`, where `B` is a logical array of the same size as `A`, returns the values of `A` at the indices where `B` is 1.

Given `A =[5,6,7;8,9,10;11,12,13]` and `B = logical(eye(3))`, we can extract the diagonal elements of `A` by typing `C = A(B)` to obtain `C = [5;9;13].`

See our earlier discussion of logical indexing.

Logical Operators

Operator	Name	Definition
`~`	NOT	`~A` returns an array the same dimension as `A`; the new array has ones where `A` is zero and zeros where `A` is nonzero.
`&`	AND	`A & B` returns an array the same dimension as `A` and `B`; the new array has ones where both `A` and `B` have nonzero elements and zeros where either `A` or `B` is zero.
`|`	OR	`A | B` returns an array the same dimension as `A` and `B`; the new array has ones where at least one element in `A` or `B` is nonzero and zeros where `A` and `B` are both zero.
`&&`	Short-Circuit AND	Short-circuiting means the second operand (right hand side) is evaluated only when the result is not fully determined by the first operand (left hand side) `A & B` (A and B are evaluated) `A && B` (B is only evaluated if A is true)
`||`	Short-Circuit OR	`|` can operate on arrays but `||` only operates on scalars

Examples

```
>> a = 3;
>> b = 4;
>> c = 5;
>> x = ~(a == b)
x =
     1

>> (a < b) & (b < c)
ans =
     1

>> (a < b) && (b < c)
ans =
     1

>> 5 && 0
ans =
     0

>> [1 2] && [3 4]
??? Operands to the || and && operators must
be convertible to logical scalar values.
```


Order of precedence for operators

Precedence	Operator type
First	Parentheses; evaluated starting with the innermost pair.
Second	Arithmetic operators and logical NOT (~); evaluated from left to right.
Third	Relational operators; evaluated from left to right.
Fourth	Logical AND.
Fifth	Logical OR.

Logical functions

Logical function	Definition
`ischar(A)`	Returns a 1 if `A` is a character array and 0 otherwise.
`isempty(A)`	Returns a 1 if `A` is an empty matrix and 0 otherwise.
`isinf(A)`	Returns an array of the same dimension as `A`, with ones where `A` has ‘`inf`’ and zeros elsewhere.
`isnan(A)`	Returns an array of the same dimension as `A` with ones where `A` has ‘`NaN`’ and zeros elsewhere. (‘`NaN`’ stands for “not a number,” which means an undefined result.)

Logical Functions

`isnumeric(A)`	Returns a 1 if A is a numeric array and 0 otherwise.
`isreal(A)`	Returns a 1 if A has no elements with imaginary parts and 0 otherwise.
`logical(A)`	Converts the elements of the array A into logical values.
`xor(A,B)`	Returns an array the same dimension as A and B; the new array has ones where either A or B is nonzero, but not both, and zeros where A and B are either both nonzero or both zero.

Logical Operators and the `find` Function

Consider the session

```
>> x = [5, -3, 0, 0, 8];
>> y = [2, 4, 0, 5, 7];
>> x&y
ans =
     1     1     0     0     1
>> z = find(x&y)
z =
   1    2    5
```

Note that the find function returns the *indices,* and not the *values*.

Conditional Statements: The `if` Statement

The `if` statement's basic form is

`if` *logical expression*
  statements
`end`

Every `if` statement must have an accompanying `end` statement. The end statement marks the end of the *statements* that are to be executed if the *logical expression* is true.

The `else` Statement

The basic structure for the use of the `else` statement is

```
if logical expression
  statement group 1
else
  statement group 2
end
```

When the test, if *logical expression,* is performed, where the logical expression may be an *array,*
the test returns a value of true only if *all* the elements of the logical expression are true!

The `elseif` Statement

The general form of the `if` statement is

```
if logical expression 1
     statement group 1
elseif logical expression 2
     statement group 2
else
     statement group 3
end
```

The `else` and `elseif` statements may be omitted if not required. However, if both are used, the `else` statement must come after the `elseif` statement to take care of all conditions that might be unaccounted for.

Exercise

File: test.m

```
a = 5;
b = 4;

if a == b
    disp(a);
    disp(b);
elseif a < b
    disp(a);
else
    disp(b);
end
```

Matlab command prompt

```
>> test
     4
>>
```


Example

- Suppose that we want to compute y, which is given by the equation:

$$y = \begin{cases} 15\sqrt{4x} + 10 & \text{if } x \geq 9 \\ 10x + 10 & \text{if } 0 \leq x < 9 \\ 10 & \text{if } x < 0 \end{cases}$$

```
function y = test(x)
if x >= 9
  y = 15*sqrt(4*x) + 10
elseif x >= 0 % already less than 9
  y = 10*x + 10
else
  y = 10
end
```

Example: if we fail to recognize how the test works, the following statements do not perform the way we might expect.

```
x = [4 -9 25];
if x < 0
  disp('Cant find square root of negative.')
else
  y = sqrt(x)
end
```

When this program is run it gives the result

```
y =
 2   0 + 3.000i   5
```


Instead, consider what happens if we test for `x` positive.

```
x = [4, -9, 25];
if x >= 0
  y = sqrt(x)
else
  disp('Cant find square root of negative.')
end
```

When executed, it produces the following message:

```
Cant find square root of negative.
```

The test if `x < 0` is false, and the test if `x >= 0` also returns a false value because `x >= 0` returns the vector `[1,0,1]`.

Loops

- Often in your programs you will want to "loop"
  - repeat some commands multiple times
- If you know how many times you want to loop
  - use a `for` loop
- If you want to loop until something happens (a condition is satisfied)
  - use a `while` loop
- If you find yourself typing similar lines more than a couple of times, use a loop

`for` Loops

A simple example of a for loop is:

```
m = 0;
x(1) = 10;
for k = 2:3:11;
  m = m + 1;
  x(m+1) = x(m) + k^2;
end
```

k takes on the values 2, 5, 8, 11. The variable m indicates the index of the array x. When the loop is finished the array x will have the values x(1)=14, x(2)=39, x(3)=103, x(4)=224.

Note the following rules when using for loops with the loop variable expression `k = m:s:n`:

- The step value `s` may be negative.
  Example: `k = 10:-2:4` produces `k` = 10, 8, 6, 4.
- If `s` is omitted, the step value defaults to 1.
- If `s` is positive, the loop will not be executed if `m` is greater than `n`.
- If `s` is negative, the loop will not be executed if `m` is less than `n`.
- If `m` equals `n`, the loop will be executed only once.
- If the step value s is not an integer, round-off errors can cause the loop to execute a different number of passes than intended.

Exercise

File: loop.m

```
for i = 1:1:5
    disp(i)
end
```

Matlab command prompt

```
>> loop
     1
     2
     3
     4
     5
>>
```


Strings and Conditional Statements

A *string* is a variable that contains characters. Strings are useful for creating input prompts and messages and for storing and operating on data such as names and addresses.

To create a string variable, enclose the characters in single quotes. For example, the string variable name is created as follows:

```
>>name = 'Mohammed Ali'
name =
   Mohammed Ali
```


The following string, `number`, is *not* the same as the variable number created by typing number = 123.

```
>>number = '123'
number =
   123
```

The following prompt program is a script file that allows the user to answer *Yes* by typing either `Y` or `y` or by pressing the **Enter** key. Any other response is treated as a No answer.

```
response = input('Continue? Y/N [Y]: ','s');
if (isempty(response))|(response ==
'Y')|(response == 'y')
  response = 'Y'
else
  response = 'N'
end
```


Programming Exercise #1

- Write a MATLAB program that does the following:
- The program asks you to enter your name.
- It waits for you to enter your name and hit Enter.
- The program reads your name, counts its characters and any blank spaces in the name, then displays something like this:
- You name is "Mohammed Ali". It has 11 characters and 1 blank space.

Using loops is slower than arrays in MATLAB

We can use the mask technique to compute the square root of only those elements of `A` that are no less than 0 and add 50 to those elements that are negative. The program is

```
A = [0, -1, 4; 9, -14, 25; -34, 49, 64];
C = (A >= 0);
A(C) = sqrt(A(C))
A(~C) = A(~C) + 50
```


`while` Loops

The `while` loop is used when the looping process terminates because a specified condition is satisfied, and thus the number of passes is not known in advance.
A simple example of a while loop is

```
x = 5;
while x < 25
   disp(x)
   x = 2*x - 1;
end
```

The results displayed by the `disp` statement are 5, 9, 17.

The typical structure of a while loop follows.

```
while logical expression
    statements
end
```

For the `while` loop to function properly, the following two conditions must occur:

1. The loop variable must have a value before the while statement is executed.

2. The loop variable must be changed somehow by the *statements.*

Exercise

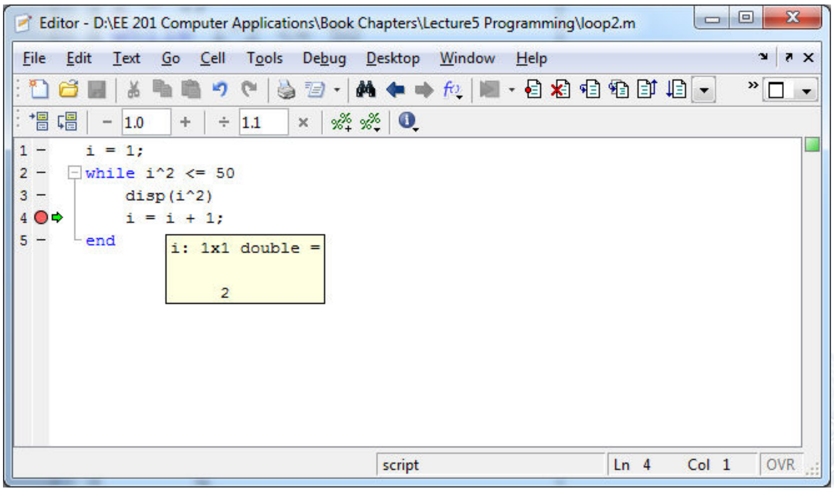
File: loop2.m

```
i = 1;
while i^2 <= 50

    disp(i^2)
    i = i + 1;

end
```

Matlab command prompt

```
>> loop2
     1
     4
     9
    16
    25
    36
    49
>>
```

Editor/Debugger containing program to be analyzed



The `break` statement

- `break` terminates the execution of a loop, so if you have a nested loop, break will only quit the innermost loop, and the program will continue running.

```
s=6;                % initialize s to 6
while s~=1          % as long as s is not equal to 1 stay in loop
     if s==17       % if s equals 17
          sprintf('Found 17 in the loop!!')
          break;
     end
     if mod(s,2) % the actual "brains" of the iteration
          s=s/2;
     else
          s=3*s+1;
     end
end
```

The `continue` statement

The following code uses a `continue` statement to avoid computing the logarithm of a negative number.

```
x = [10,1000,-10,100];
y = NaN*x;
for k = 1:length(x)
  if x(k) < 0
    continue
  end
  y(k) = log10(x(k));
end
y
```

The result is `y = [1   3   NaN   2].`

Homework

- Write a script file to determine how many terms are required for the sum of the series $5k^2 - 2k, k = 1, 2, 3, \ldots$ to exceed 10,000. What is the sum for this many terms?

```
total = 0; k = 0;
while total < 1e4
      k = k + 1;
      total = total + 5*k^2 - 2*k;
end
disp('The number of terms is:')
disp(k)
disp('The sum is:')
disp(total)
```

- The sum is 10,203 after 18 terms.

Exercise: Fourier Series

- $x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t - \theta_n)$
- Discover the following periodic function:
- $x(t) = 0.5 + \frac{2}{\pi}\left[\cos(t) + \frac{1}{3}\cos(3t) + \frac{1}{5}\cos(5t) + \frac{1}{7}\cos(7t) + \cdots\right]$
- Use a `for` or `while` loop. Use `n` as the loop parameter to add certain terms then plot the result versus time $-10 \le t \le 10$.
- On one figure, draw the result of 3 terms.
- On one figure, draw the result of 10 terms.
- On one figure, draw the result of 100 terms.

Infinite Loops

- "Infinite loop" = piece of code that will execute again and again … without ever ending.
- Possible reasons for infinite loops:
  - getting the conditional statement wrong
  - forgetting the update step
- If you are in an infinite loop then ctrl-c stops MATLAB executing your program.

The `switch` statement

The `switch` statement provides an alternative to using the `if`, `elseif`, and `else` commands.

Anything programmed using `switch` can also be programmed using `if` statements.

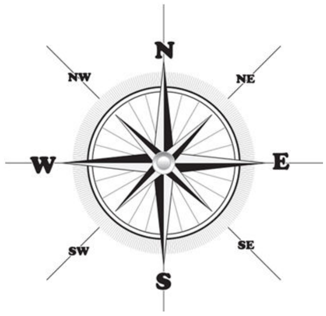
However, for some applications the `switch` statement is more readable than code using the `if` structure.

Syntax of `switch`

```
switch input expression (can be a scalar or string).
  case value1
       statement group 1
  case value2
       statement group 2
  .
  .
  .
  otherwise
       statement group n
end
```

The following switch block displays the point on the compass that corresponds to that angle.

```
switch angle
  case 45
     disp('Northeast')
  case 135
     disp('Southeast')
  case 225
     disp('Southwest')
  case 315
     disp('Northwest')
  otherwise
     disp('Direction Unknown')
end
```



Boolean Variables

- MATLAB allows boolean variables that take true/false values

```
if (atUniversity & stillAStudent)
    needMoreMoney = true;
end
```

Programming Exercise #2

- Write a MATLAB program to solve this:
- One investment opportunity pays 5.5% annual profit, while a second investment opportunity pays 4.5% annual profit.
- Determine how much longer it will take to accumulate at least $50,000 in the second investment opportunity compared to the first if you invest $1000 initially and $1000 at the end of each year.

Programming Exercise #3

- Write a MATLAB program that asks you for a hexadecimal integer number.
- The program should read that number and convert it to decimal.
- Example: 84CD hexadecimal is 33997 decimal.
- Can you improve on your program so it accepts binary or hexadecimal or decimal and converts it to all other formats? You need to accept numbers written in something like this: 94CA**h** or 110110001**b**.

Homework

- Solve as many problems from Chapter 4 as you can
- Suggested problems:
- 4.2, 4.4, 4.5, 4.11, 4.13, 4.15, 4.16, 4.17, 4.23, 4.24, 4.25, 4.26, 4.33, 4.37, 4.39, 4.47

Lecture 6: Plotting in MATLAB

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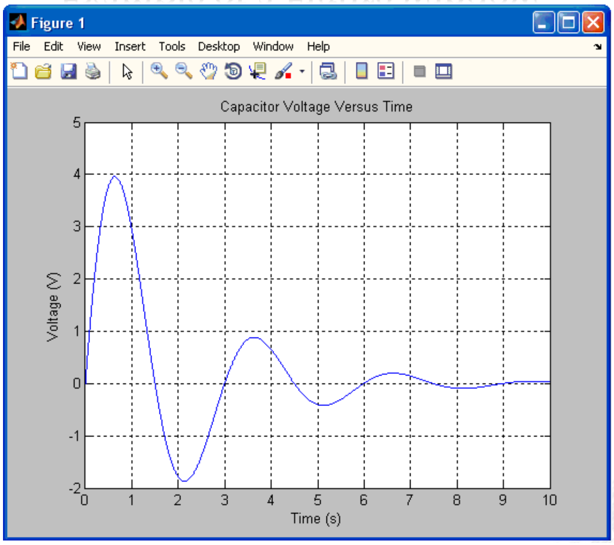
EE201: Computer Applications. See Textbook Chapter 5.

A picture is worth a thousand words

- MATLAB allows you to plot data sets for better visualization and interpretation.
- There are different types of plots available in MATLAB (*see next*) including 2D and 3D plots.
- You can control all aspects of the plot: lines, colors, grids, labels, etc.
- Plotting clear and easy-to-read figures is an important skill, which you gain from experience.
- For pointers, read in your textbook the *Requirements for a Correct Plot* (Table 5.1-1, page 221), and *Hints for Improving Plots* (Table 5.1-3, page 226).

Example of a Figure window



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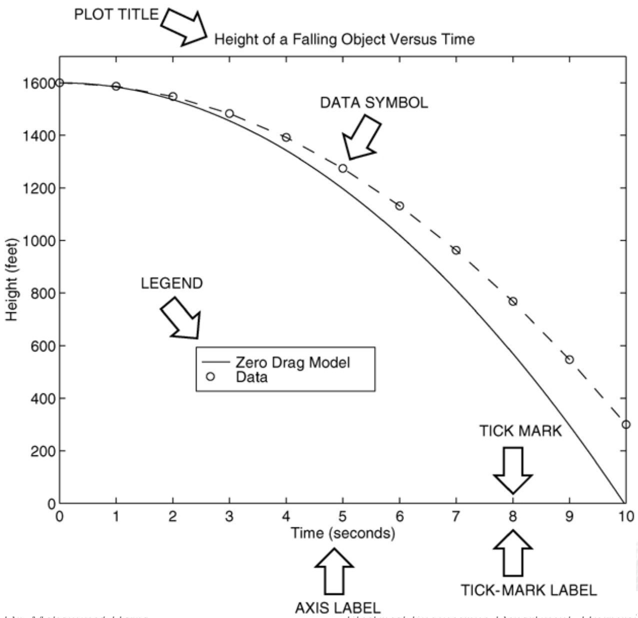
Nomenclature for a typical *xy* two-dimensional plot.

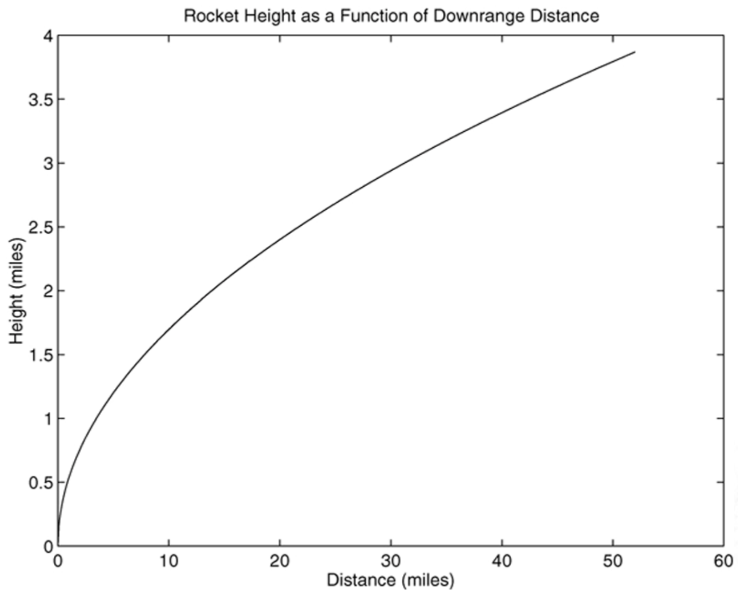


Example: Plot $y = 0.4 \times \sqrt{1.8x}$ for $0 \leq x \leq 52$, where y represents the height of a rocket after launch, in miles, and x is the horizontal (downrange) distance in miles.

```
>> x = 0:0.1:52;
>> y = 0.4*sqrt(1.8*x);
>> plot(x,y);
>> xlabel('Distance (miles)');
>> ylabel('Height (miles)');
>> title('Rocket Height vs. Distance');
```

Notice that for each x there is y; so MATLAB plots one array against another.
Also notice how we added the axes labels and plot title.
The resulting plot is shown on the next slide.

The autoscaling feature in MATLAB selects tick-mark spacing.



The plot will appear in the Figure window. You can use the plot in other applications in several ways:

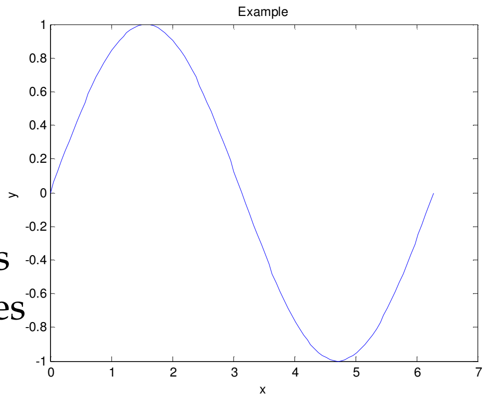
1. You can print a hard copy of the figure by selecting **File | Print** menu item in the Figure window.
2. You can save the plot to a file to be used later. You can save the plot by selecting **File | Save As** menu item. Possible file formats include: *.fig (MATLAB format), *.bmp, *.eps, *.jpg, *.png, *.tif, *.pdf, …. Another way to save is **File | Export Setup** that allows specifying options for the output file, then selecting **Export**.
3. You can copy a figure to the clipboard and then paste it into another application using the **Edit | Copy Figure** menu item. For options, use **Edit | Copying Options** menu item.

When you have finished with the plot, close the figure window by selecting **File | Close** menu item in the figure window.

If you do not close the window, it will not re-appear when a new `plot` command is executed. However, the figure will still be updated.

One Data Set: plot

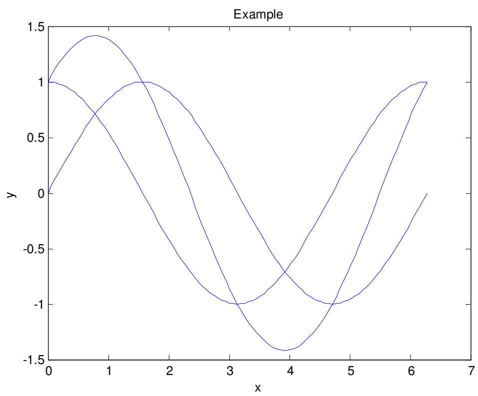
```
x = 0:2*pi/100:2*pi;
y1 = sin(x);
plot(x,y1);
xlabel('x');
ylabel('y');
title('Example');
```



`plot(y1)`: Plots values of y1 versus their indices if y1 is a vector.

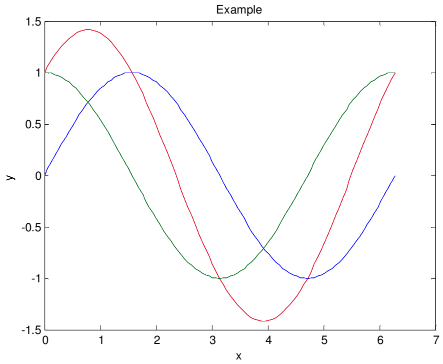
Multiple Data Sets: plot, hold

```
x = 0:2*pi/100:2*pi;
y1 = sin(x);
y2 = cos(x);
y3 = sin(x)+cos(x);
plot(x,y1);
hold on;
plot(x,y2);
plot(x,y3);
xlabel('x');
ylabel('y');
title('Example');
hold off;
```



Or better use one `plot` command

```
x = 0:2*pi/100:2*pi;
y1 = sin(x);
y2 = cos(x);
y3 = sin(x)+cos(x);
plot(x,y1,x,y2,x,y3);
xlabel('x');
ylabel('y');
title('Example');
% Notice the auto coloring
% by  MATLAB
```

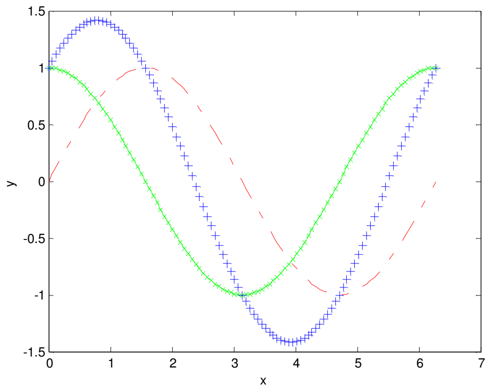


Colors, Data Markers & Line Types

- You can also specify your own line styles in the plot command.
- For full details enter `help plot` in MATLAB.

```
b     blue          .     point              -      solid
g     green         o     circle             :      dotted
r     red           x     x-mark             -.     dashdot
c     cyan          +     plus               --     dashed
m     magenta       *     star             (none)   no line
y     yellow        s     square
k     black         d     diamond
w     white         v     triangle (down)
                    ^     triangle (up)
                    <     triangle (left)
                    >     triangle (right)
                    p     pentagram
                    h     hexagram
```

```
x = 0:2*pi/100:2*pi;
y1 = sin(x);
y2 = cos(x);
y3 = sin(x)+cos(x);
plot(x,y1,'r-.',x,y2,'g-x',x,y3,'b+');
xlabel('x');
ylabel('y');
```



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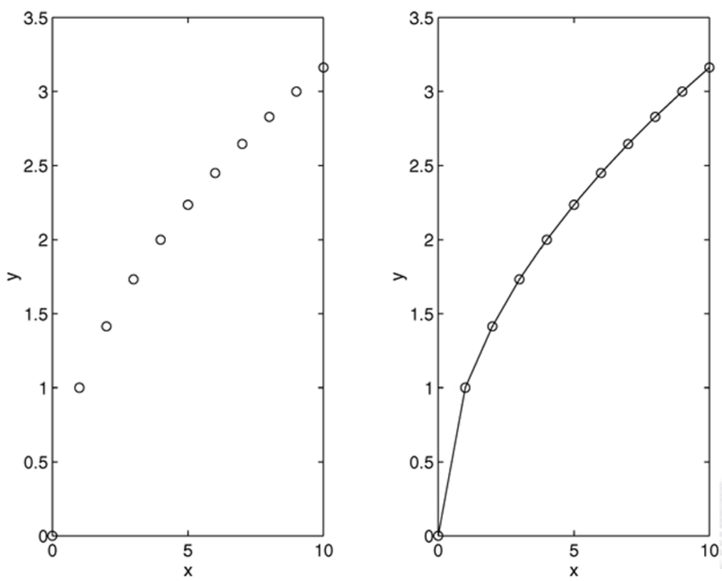
Exercise: How did we use different data markers below?



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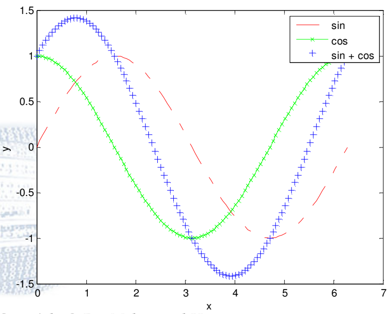
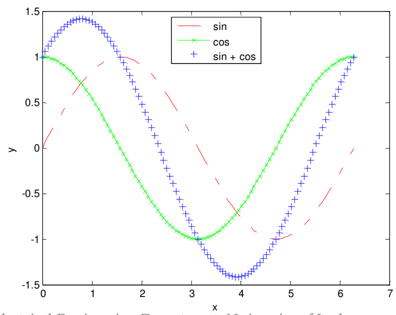
Legends

- With multiple lines on the same plot it is a good idea to add a legend.

```
legend('sin','cos','sin + cos');
legend('sin','cos','sin+cos','Location','North');
```

- You can also move the legend with the mouse.





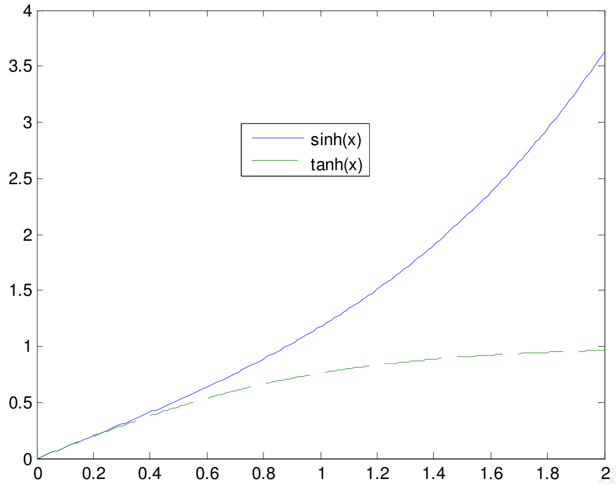
Labeling Curves and Data

The `legend` command automatically obtains from the plot the line type used for each data set and displays a sample of this line type in the legend box next to the string you selected. The following script file produced the plot in the next slide.

```
x = 0:0.01:2;
y = sinh(x);
z = tanh(x);
plot(x,y,x,z,'--');
legend('sinh(x)', 'tanh(x)');
```

`gtext('text')`: Places a string in the Figure window at a point specified by the mouse.
`text(x,y,'text')`: Places a string in the Figure window at a point specified by coordinates x, y.

Application of the `legend` command.
I moved the legend to an empty space using the mouse.



The `grid` and `axis` Commands

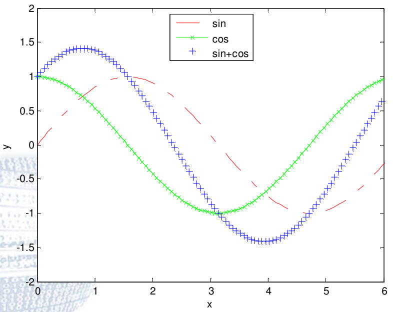
MATLAB will automatically determine the maximum and minimum values for the axes. You can use the `axis` command to override the MATLAB selections for the axis limits. The syntax is `axis([xmin xmax ymin ymax])`. This command sets the scaling for the *x*- and *y*-axes to the minimum and maximum values indicated.

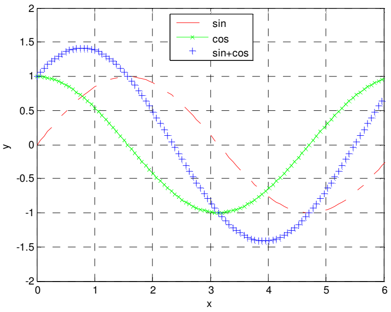
The `grid` command displays gridlines at the tick marks corresponding to the tick labels. Type `grid on` to add gridlines; type `grid off` to stop plotting gridlines. When used by itself, `grid` toggles this feature on or off, but you might want to use `grid on` and `grid off` to be sure.

`axis` and `grid` commands

```
axis([0 9 -2 2]);
axis([0 6 -2 2]);
```

```
grid on;
grid off;
```

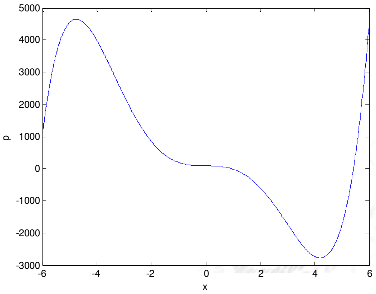




Homework #1
Plotting Polynomials with the `polyval` Function.

To plot the polynomial $3x^5 + 2x^4 - 100x^3 + 2x^2 - 7x + 90$ over the range $-6 \le x \le 6$ with a spacing of 0.01, you type

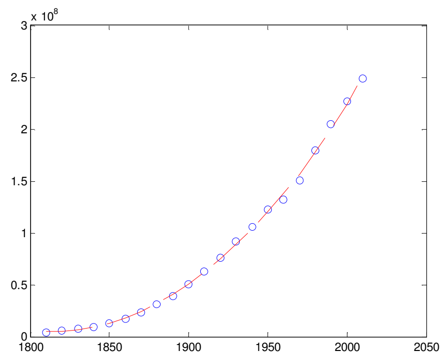
```
>> x = -6:0.01:6;
>> p = [3,2,-100,2,-7,90];
>> plot(x,polyval(p,x));
>> xlabel('x');
>> ylabel('p');
```



Homework #2

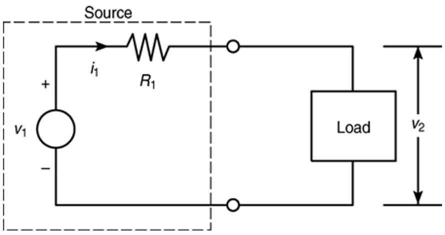
- The `polyfit` function is based on the least-squares method. It fits a polynomial of degree n to data described by the vectors x and y, where x is the independent variable.
- Syntax: `p = polyfit(x,y,n)`
- It returns a row vector p of length n+1 that contains the polynomial coefficients in order of descending powers.
- For the following census data, draw the actual points and the best 5^{th} order polynomial fit for such data.


```
year = 1810:10:2010;
population = 1e6*[3.9 5.3 7.2 9.6 12.9 17.1
23.1 31.4 38.6 50.2 62.9 76. 92. 105.7 122.8
131.7 150.7 179. 205. 226.5 248.7];
coeff = polyfit(year, population, 5)
f = polyval(coeff, year);
plot(year, population, 'bo', year, f, 'r--');
```



Homework #3
Graphical solution of an Electrical System

- Load is governed by:
- $i1 \ = \ 0.16 \ (e^{0.12 v_2} - \ 1)$
- What is the equation for the practical source? Assume:
- $R1 \ = \ 30\Omega, v_1 \ = 15V$
- Find the correct value for $v2$ between 0 and 20V, and also i_1 value



Solution

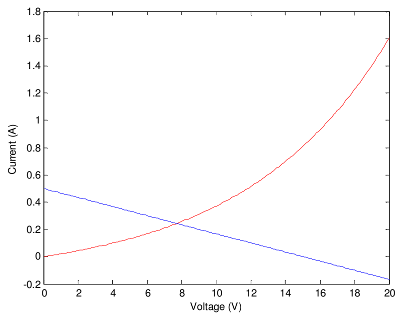
- The equation for the power supply is:

$$v_2 = v_1 - Ri_1$$

$$i_1 = \frac{15 - v_2}{30}$$

- If we draw both equations we can see the solution point (the one that satisfies both equations).

```
v2 = [0:0.1:20];
i_load = ...
0.16*(exp(0.12*v2) - 1);
i_source = (15-v2)/30;
plot(v2, i_load, 'r', ...
v2, i_source, 'b');
```



More Than One Figure Window

- What happens if you enter the following?

```
x = 0:2*pi/100:2*pi;
y1 = sin(x);
y2 = cos(x);
plot(x,y1);
title('Plot #1');
plot(x,y2);
title('Plot #2');
```

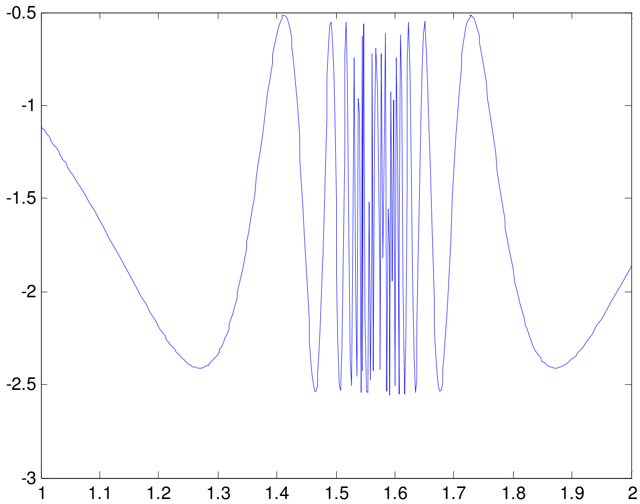

More Than One Figure Window

- … you end up with one figure window and it contains a plot of $y = \cos(x)$.
- To open a new figure window enter the command `figure` before making the second plot.

```
plot(x,y1);
title('Plot #1');
figure;
plot(x,y2);
title('Plot #2');
```

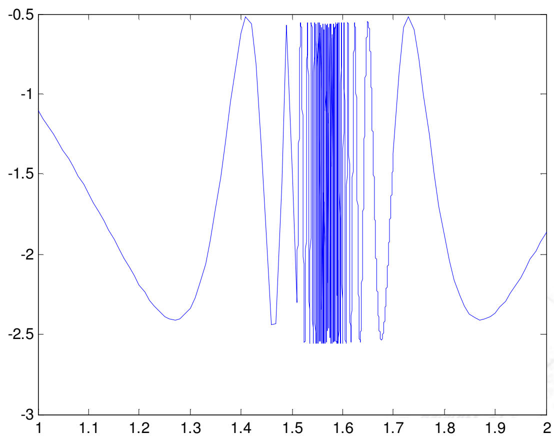

The `fplot` command is a “smart” plotting function. Example:

```
f = @(x) (cos(tan(x)) - tan(sin(x)));
fplot(f,[1 2]);
```



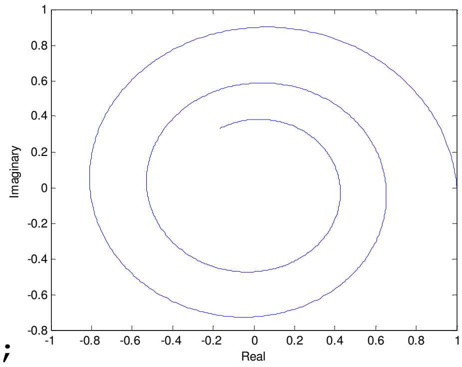
The `plot` command is more common than the `fplot` command because it gives more control. Also when you `type fplot` you see it actually uses `plot`.

```
f = @(x) (cos(tan(x)) - tan(sin(x)));
t=[1:0.01:1.5, 1.51:0.0001:1.7, 1.71:0.01:2];
plot(t, f(t));
```



Complex Plot: Real vs. Imaginary

```
n = [0:0.01:10];
y = (0.1+0.9j).^n;
plot(y);
xlabel('Real');
ylabel('Imaginary');
```



- Similar to:

```
plot(real(y),imag(y));
```

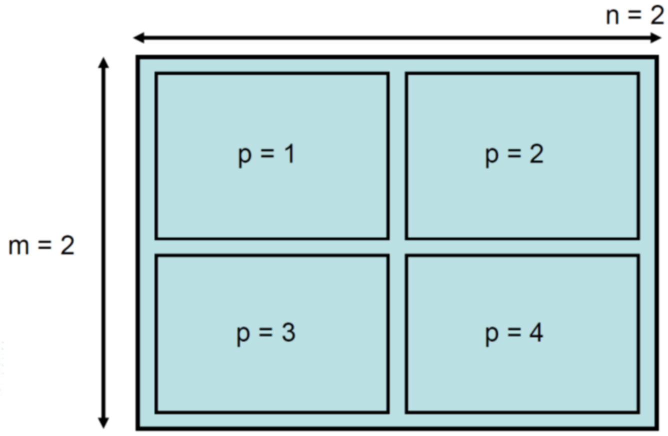

Subplots

You can use the `subplot` command to obtain several smaller "subplots" in the same figure. The syntax is `subplot(m,n,p)`. This command divides the Figure window into an array of rectangular panes with *m* rows and *n* columns. The variable `p` tells MATLAB to place the output of the `plot` command following the `subplot` command into the *p*th pane.

For example, `subplot(3,2,5)` creates an array of six panes, three panes deep and two panes across, and directs the next plot to appear in the fifth pane (in the bottom-left corner).

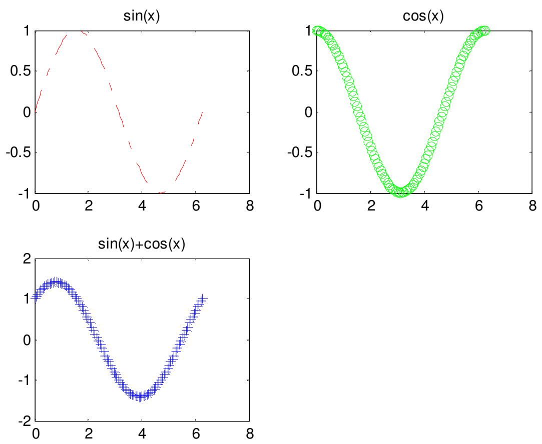
Subplots

- `subplot(m,n,p)`



Example

```
x = 0:2*pi/100:2*pi;
y1 = sin(x);
y2 = cos(x);
y3 = sin(x)+cos(x);
subplot(2,2,1);
plot(x,y1,'r-.');
title('sin(x)');
subplot(2,2,2);
plot(x,y2,'go');
title('cos(x)');
subplot(2,2,3);
plot(x,y3,'b+');
title('sin(x)+cos(x)');
```

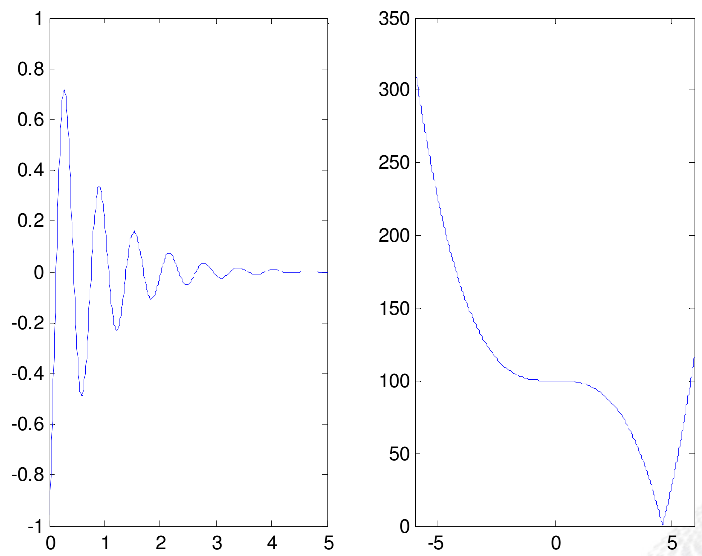


Homework:
The following script file shows two plots of the functions
$y = e^{-1.2x} \sin(10x + 5)$ for $0 \le x \le 5$
and $y = |x^3 - 100|$ for $-6 \le x \le 6$.

```
x = 0:0.01:5;
y = exp(-1.2*x).*sin(10*x+5);
subplot(1,2,1);
plot(x,y);
axis([0 5 -1 1]);
x = -6:0.01:6;
y = abs(x.^3-100);
subplot(1,2,2);
plot(x,y);
axis([-6 6 0 350])
```

The figure is shown on the next slide.

Application of the `subplot` command.

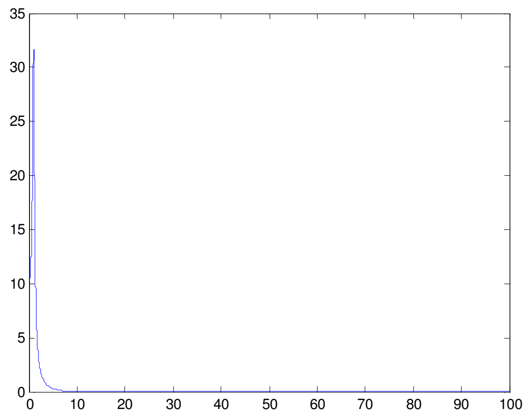


Log-scale Plots

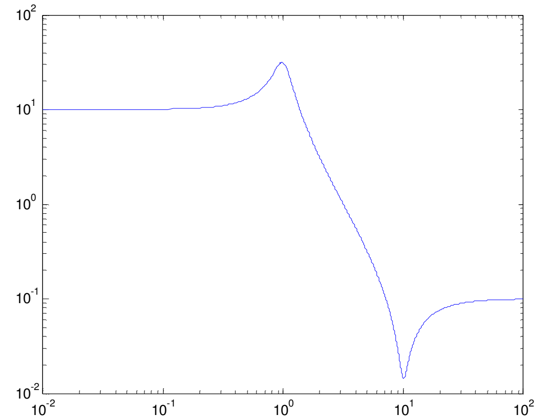
- Why use log scales? Linear scales cannot properly display wide variations in data values.
- MATLAB has three commands. The appropriate command depends on which axis you want to be a log scale.
- `loglog(x,y)`: both scales logarithmic.
- `semilogx(x,y)`: x-axis is logarithmic and y-axis is rectilinear.
- `semilogy(x,y)`: y-axis is logarithmic and x-axis is rectilinear.
- The syntax is similar to the `plot` command.

$$y = \sqrt{\frac{100(1 - 0.01x^2)^2 + 0.02x^2}{(1 - x^2)^2 + 0.1x^2}} \qquad 0.1 \leq x \leq 100$$


```
x = [0.1:0.01:100];
y = sqrt((100*(1-0.01*x.^2).^2 ...
+0.02*x.^2) ...
./ ((1-x.^2).^2+0.1*x.^2));
plot(x,y);
```



```
x = [0.1:0.01:100];
y = sqrt((100*(1-0.01*x.^2).^2 ...
+0.02*x.^2) ...
./ ((1-x.^2).^2+0.1*x.^2));
loglog(x,y);
```



Logarithmic Plots

It is important to remember the following points when using log scales:

1. You cannot plot negative numbers on a log scale, because the logarithm of a negative number is not defined as a real number.
2. You cannot plot the number 0 on a log scale, because $\log_{10} 0 = \ln 0 = -\infty$. You must choose an appropriately small number as the lower limit on the plot.

(continued…)

Logarithmic Plots (continued)

3. The tick-mark labels on a log scale are the actual values being plotted; they are not the logarithms of the numbers. For example, the range of *x* values in the plot in the above Figure is from $10^{-2} = 0.01$ to $10^{2} = 100$.

4. Gridlines and tick marks within a decade are unevenly spaced. If 8 gridlines or tick marks occur within the decade, they correspond to values equal to 2, 3, 4, . . . , 8, 9 times the value represented by the first gridline or tick mark of the decade.

(continued…)

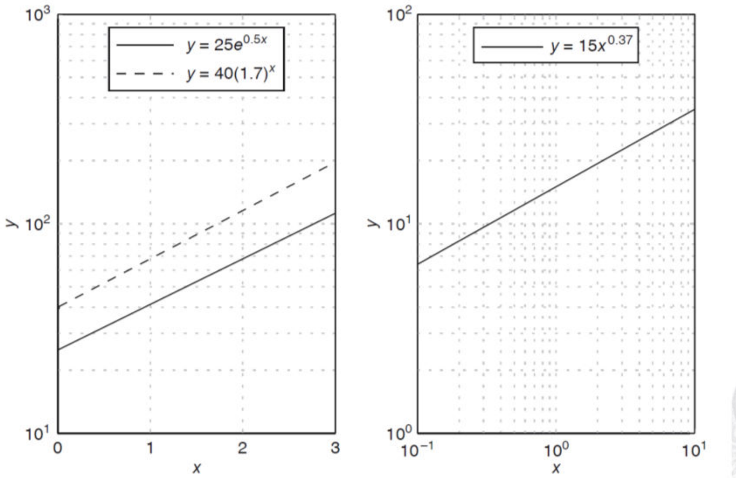
Logarithmic Plots (continued)

5. Equal distances on a log scale correspond to multiplication by the same constant (as opposed to addition of the same constant on a rectilinear scale).

For example, all numbers that differ by a factor of 10 are separated by the same distance on a log scale. That is, the distance between 0.3 and 3 is the same as the distance between 30 and 300. This separation is referred to as a *decade* or *cycle.*

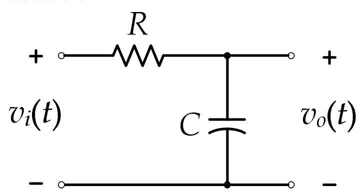
The plot shown in the above Figure covers four decades in *x* (from 0.01 to 100) and four decades in *y.*

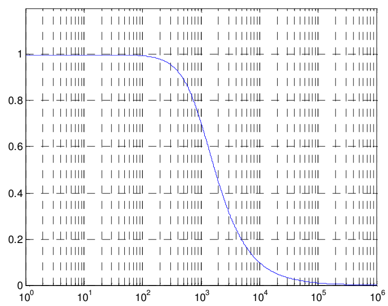
Homework: reproduce the following plots.
What commands did you use?



Homework

- For the first-order RC circuit, which acts as a LPF, the output to input ratio is:
- $|H(\omega)| = \frac{|V_o(\omega)|}{|V_i(\omega)|} = \left|\frac{1}{1+j\omega RC}\right|$
- Sketch this frequency response function using `semilogx`. Assume: $R = 1k\Omega, C = 1\mu F$





Solution

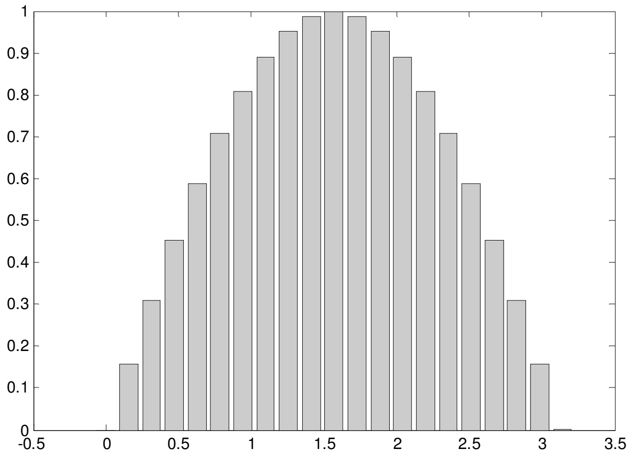
```
omega = 0:1:1e6;
h = abs(1./(1+i*omega*1e3*1e-6));
semilogx(omega, h);
axis([0 1e6 0 1.2]);
grid on;
```

Q. What is the bandwidth of this LPF?

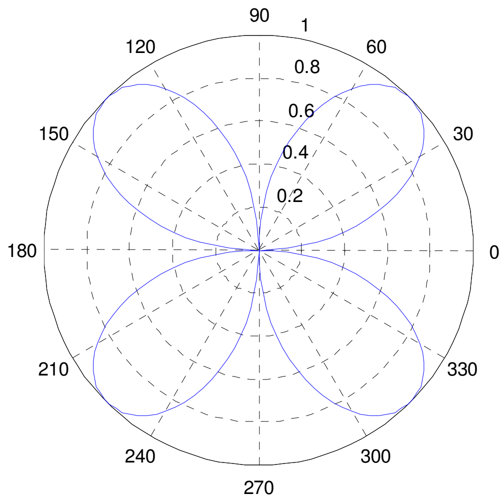
Specialized plot commands.

Command	Description
`bar(x,y)`	Creates a bar chart of `y` versus `x`.
`plotyy(x1,y1,x2,y2)`	Produces a plot with two *y*-axes, `y1` on the left and `y2` on the right.
`polar(theta,r,'type')`	Produces a polar plot from the polar coordinates `theta` and `r`, using the line type, data marker, and colors specified in the string `type`.
`stairs(x,y)`	Produces a stairs plot of `y` versus `x`.
`stem(x,y)`	Produces a stem plot of `y` versus `x`.

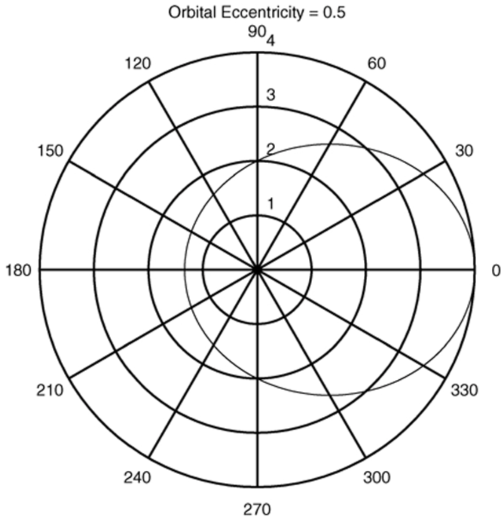

```
x = [0:pi/20:pi];
bar(x,sin(x));
```




```
theta = [0:pi/90:2*pi];
polar(theta , sin(2*theta));
grid;
```

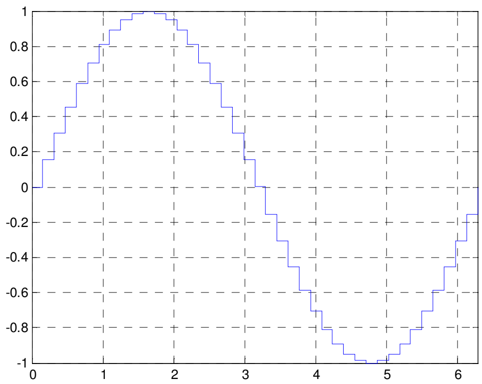


Homework: Reproduce the following plot for an orbit with an eccentricity of 0.5.

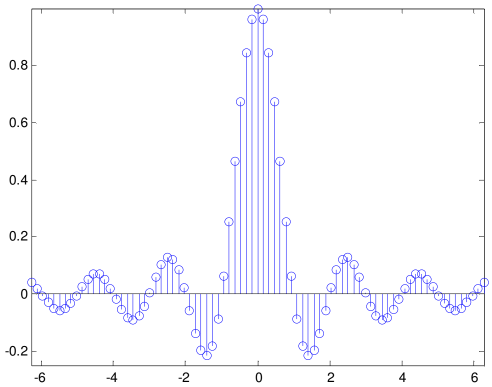


$$r = \frac{2}{1 - 0.5\cos(\theta)}$$

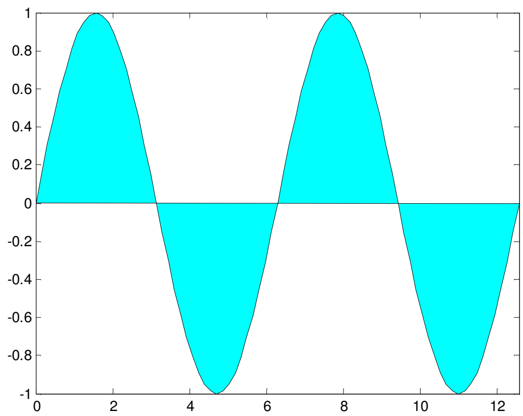

```
x = [0:pi/20:2*pi];
stairs(x,sin(x));
grid;
axis([0 2*pi -1 1]);
```



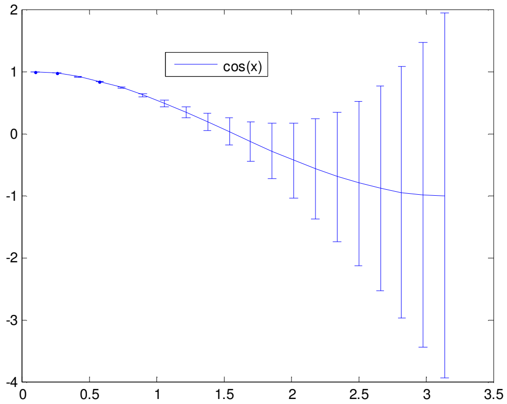

```
x = [-2*pi:pi/20:2*pi];
x = x + (~x)*eps;
y = sin(pi*x)./(pi*x);
stem(x,y);
axis([-2*pi 2*pi -.25 1]);
```




```
x = [-2*pi:pi/20:4*pi];
fill(x,sin(x),'c');
axis([0 4*pi -1 1]);
```




```
x = linspace(0.1, pi, 20);
approx = 1 - x.^2/2;
error = approx - cos(x);
errorbar(x, cos(x), error);
legend('cos(x)');
```

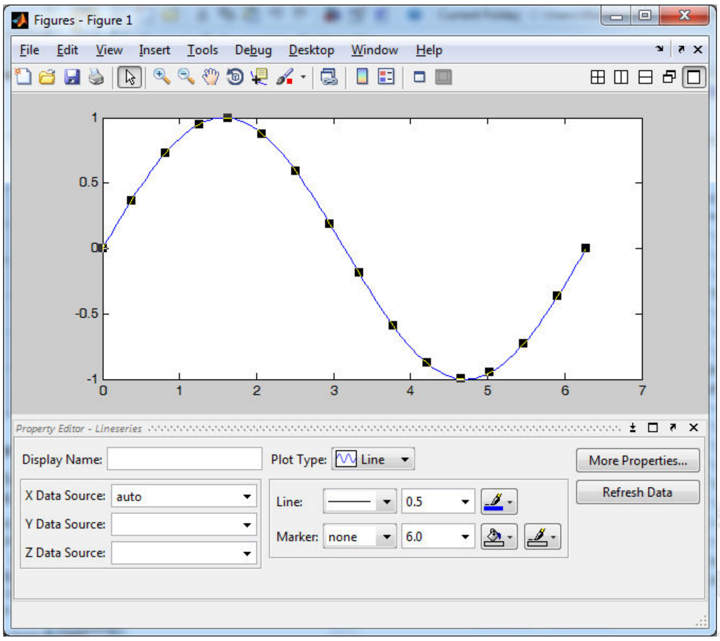


Interactive Editing of Plots in MATLAB

This interface can be advantageous in situations where:

- You want to add annotations such as lines, arrows, text, rectangles, and ellipses.
- You want to change plot characteristics such as tick spacing, fonts, bolding, colors, line weight, etc.

Select the Arrow (or **Tools| Edit Plot** from the menu) then double click on the portion you want to edit.

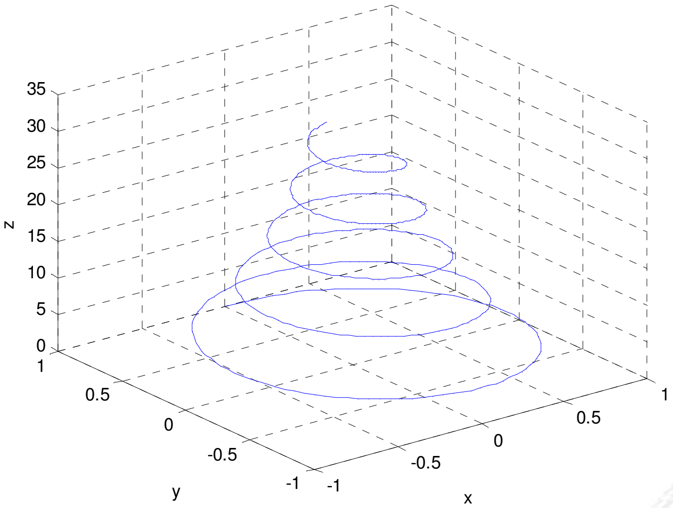


Three-Dimensional Line Plots

The following program uses the `plot3` function to generate the spiral curve shown in the next slide.

```
t = 0:pi/50:10*pi;
x = exp(-0.05*t).*sin(t);
y = exp(-0.05*t).*cos(t);
z = t;
plot3(x, y, z);
xlabel('x'),ylabel('y'),zlabel('z'),grid;
```

The curve $x = e^{-0.05t} \sin t$, $y = e^{-0.05t} \cos t$, $z = t$ plotted with the `plot3` function.



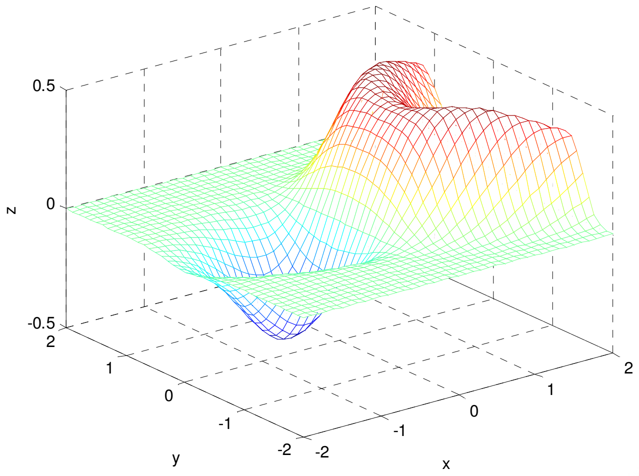
Surface Plots: `mesh` and `surf`

The following session shows how to generate the surface plot of the function $z = xe^{-[(x-y^2)^2+y^2]}$, for $-2 \le x \le 2$ and $-2 \le y \le 2$, with a spacing of 0.1. This plot appears in the next slide.

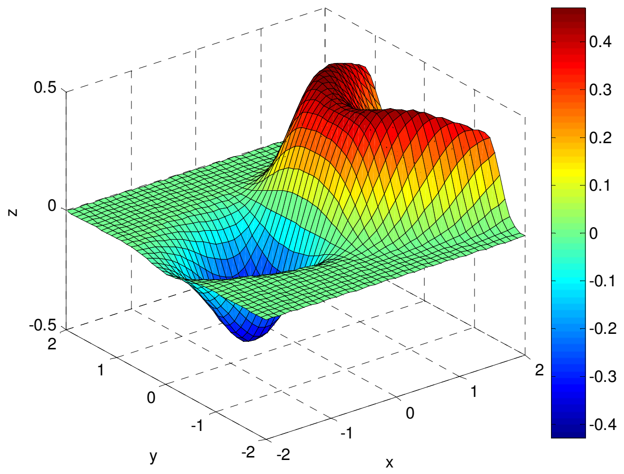
```
[X,Y] = meshgrid(-2:0.1:2);
Z = X.*exp(-((X-Y.^2).^2+Y.^2));
mesh(X,Y,Z);
xlabel('x'),ylabel('y'),zlabel('z');

[X,Y] = meshgrid(-2:0.1:2);
Z = X.*exp(-((X-Y.^2).^2+Y.^2));
surf(X,Y,Z);
xlabel('x'),ylabel('y'),zlabel('z'),colorbar
```

A plot of the surface $z = xe^{-[(x-y^2)^2+y^2]}$ created with the `mesh` function.



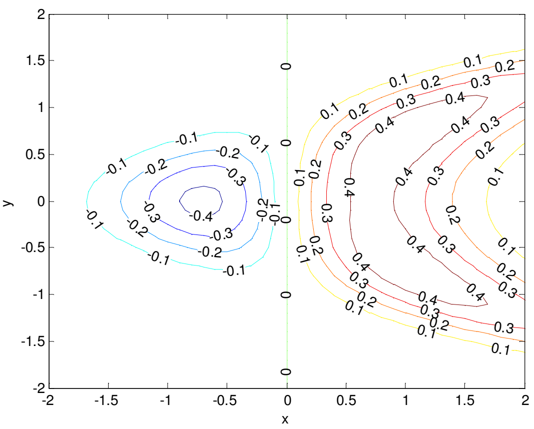
A plot of the surface $z = xe^{-[(x-y^2)^2+y^2]}$ created with the `surf` function.

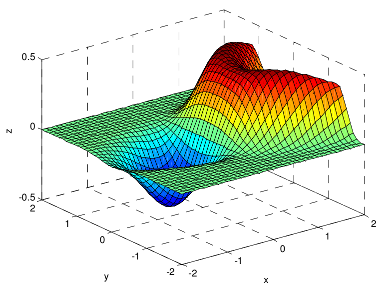


The following session generates the contour plot of the function whose surface plot is shown above; namely, $z = xe^{-[(x-y^2)^2+y^2]}$, for $-2 \le x \le 2$ and $-2 \le y \le 2$, with a spacing of 0.1. This plot appears in the next slide.

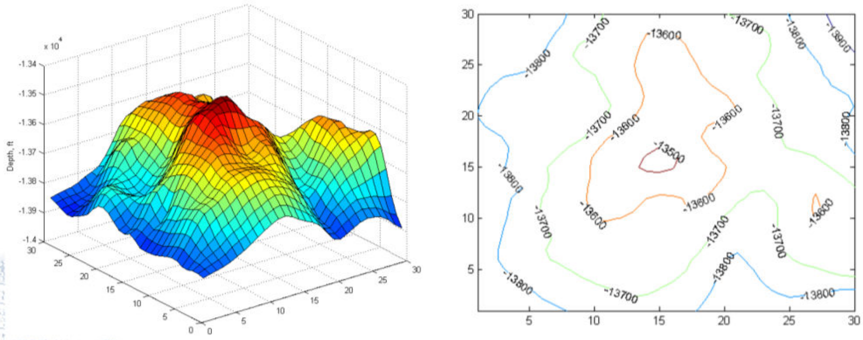
```
[X,Y] = meshgrid(-2:0.1:2);
Z = X.*exp(-((X-Y.^2).^2+Y.^2));
[cs, h] = contour(X,Y,Z);
xlabel('x'),ylabel('y'),zlabel('z');
clabel(cs, h, 'labelspacing', 72);
```


A contour plot of the surface $z = xe^{-[(x-y^2)^2+y^2]}$ created with the `contour` function.





Contours are useful for Terrain



Vector fields: `quiver`

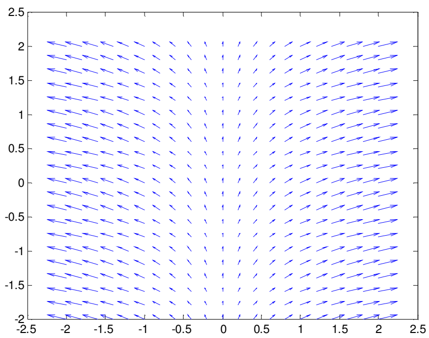
- `quiver` draws little arrows to indicate a gradient or other vector field.
- Although it produces a 2-D plot, it is often used in conjunction with contour. As an example, consider the scalar function of two variables: $V = x^2 + y$.
- The gradient of V is defined as the vector field: $\nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}\right) = (2x, 1)$

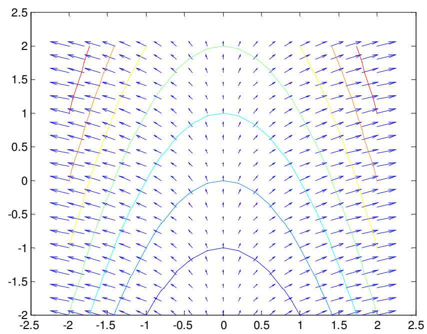
quiver

- The following statements draw arrows indicating the direction of the vector ∇V at points in the x-y plane (see next slide).

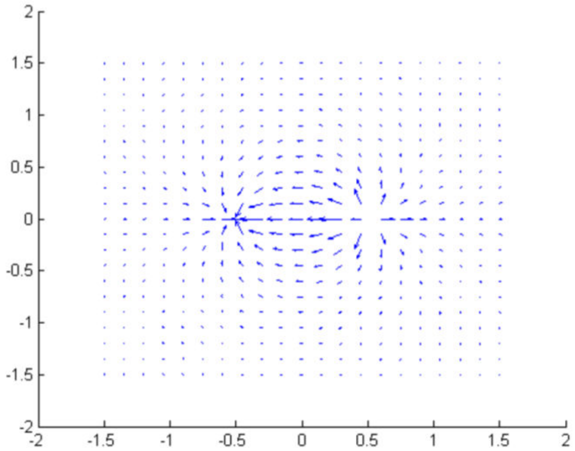
```
[x y] = meshgrid(-2:0.2:2, -2:0.2:2);
V = x.^2 + y;
dx = 2*x;
dy = ones(size(dx)); % dy same size as dx
quiver(x, y, dx, dy);
hold on;
contour(x, y, V);
hold off;
```


quiver alone; and with contour



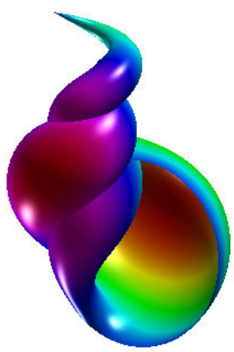


Useful for Electromagnetic Fields



Homework

```
% What is the output of this MATLAB code? Use help if you need.
figure;
t = linspace(0, 2*pi, 512);
[u,v] = meshgrid(t) ;
a = -0.2 ; b = .5 ; c = .1 ;
x = (a*(1-v/(2*pi)) .* (1+cos(u)) + c) .* cos(2*v);
y = (a*(1-v/(2*pi)) .* (1+cos(u)) + c) .* sin(2*v);
z = b*v/(2*pi) + a*(1-v/(2*pi)) .* sin(u);
surf(x,y,z,y);
shading interp;
axis off;
axis equal;
colormap(hsv(1024));
material shiny;
lighting gouraud;
lightangle(80, -40);
lightangle(-90, 60);
view([-150 10]);
```



Animation and Movies!

- A movies is just successive plots seen in quick succession.
- We can plot data repeatedly on a single figure.
- For example the function $y = \sin(x + t)$

```
x = 0:2*pi/100:2*pi;
for t = 0:0.05:5 % 5 seconds
  y = sin(x+t);
  plot(x,y,'k')
  pause(0.2); % 200 ms between frames
end
```


Homework: Creating Movies

- To create a movie a sequence of frames are "grabbed" from the figure, stored in an array and written out as .avi file.

```
nFrame = 1; % frame counter
x = 0:2*pi/100:2*pi;
for t=0:0.05:5
   y=sin(x+t);
   plot(x,y);
   pause(0.2);
   movie(nFrame) = getframe; % grab frame & store it
   nFrame = nFrame + 1;
end
movie2avi(movie,'animation.avi'); % save movie
```

Homework

- Solve as many problems from Chapter 5 as you can
- Suggested problems:
- Solve: 5.3, 5.5, 5.9, 5.11, 5.15, 5.20, 5.27, 5.29, 5.35, 5.36, 5.39.

Lecture 8: Calculus and Differential Equations

Dr. Mohammed Hawa
Electrical Engineering Department
University of Jordan

EE201: Computer Applications. See Textbook Chapter 9.

Numerical Methods

- MATLAB provides many functions that support numerical solutions to common math problems:
  - Integration and Differentiation (Calculus)
  - Finding zeros of a function
  - Solving ordinary differential equations
  - Many others
- Numerical analysis provides answers as numbers, not closed-form solutions as in analytical solutions (see *next* lecture for symbolic math in MATLAB).

The integral of *f(x)* is the area *A* under the curve of *f* (*x*) from *x* = *a* to *x* = *b*.

$$A = \int_a^b f(x)dx$$

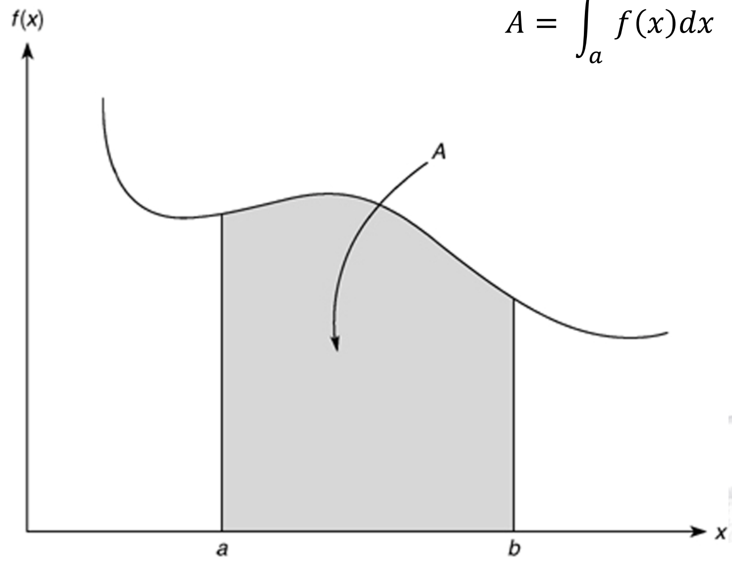


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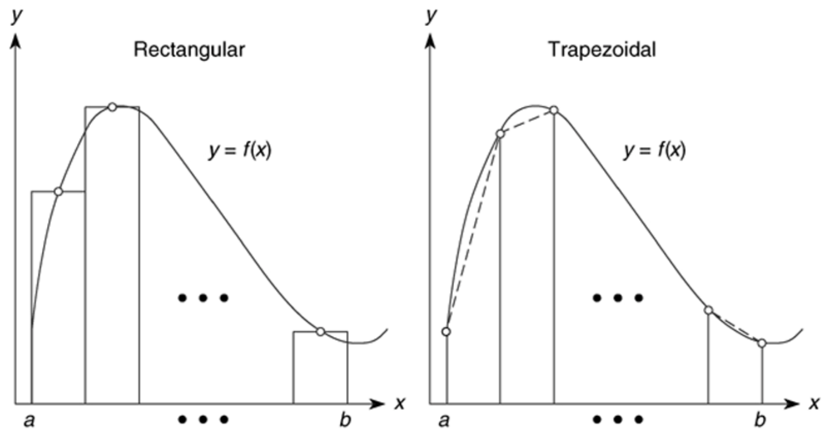
Illustration of Numerical Integration: (a) rectangular method and (b) more accurate trapezoidal method.



(a) (b)

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Example

$$A = \int_0^{\pi} \sin(x)\,dx = [-\cos(x)]_0^{\pi} = 1-(-1) = 2$$

`trapz(x,y)`

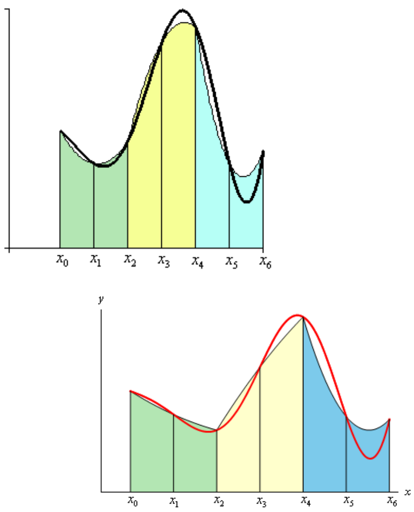
Uses trapezoidal integration to compute the integral of `y` with respect to `x`, where the array `y` contains the function values at the points contained in the array `x`.

```
>> x = linspace(0,pi,10);
>> y = sin(x);
>> A = trapz(x,y)
A =
     1.9797

>> x = linspace(0,pi,100);
>> y = sin(x);
>> A = trapz(x,y)
A =
     1.9998
```


Simpson's Rule

- Another approach to numerical integration is Simpson's Rule, which divides the integration range [a, b] into an even number of sections and uses a different quadratic function to represent the integrand for each panel.



Important numerical integration functions:

`quad(fun, a, b)` `quad(fun, a, b, tol)`	Uses an adaptive Simpson's rule to compute the integral of the function whose handle is `fun`, with `a` the lower limit and `b` the upper limit. The function `fun` must accept a vector argument. The parameter `tol` is optional, and indicates the specified error tolerance.
`quadl(fun,a,b)`	Uses Lobatto quadrature to compute the integral of the function `fun`. The rest of the syntax is identical to `quad`.
`dblquad(fun, a, b, c, d)`	computes the integral of f(x,y) from x = a to b, and y = c to d. The function `fun` must accept a vector argument x and scalar y, and it must return a vector result.
`triplequad(fun,a,b,c,d,e,f)`	computes the integral of f(x,y,z) from x = a to b, y = c to d, and z = e to f. The function must accept a vector x, and scalar y and z.

Although the `quad` and `quadl` functions are more accurate than `trapz`, they are restricted to computing the integrals of functions and cannot be used when the integrand is specified by a set of points. For such cases, use the `trapz` function.

MATLAB function `quad` implements an adaptive version of Simpson's rule, while the `quadl` function is based on an adaptive Lobatto integration algorithm.

To compute the integral of sin(*x*) from 0 to π, type

```
>> A = quad(@sin,0,pi)
```

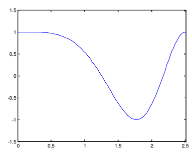
The answer given by MATLAB is 2.0000, which is correct. We use `quadl` the same way; namely,

```
>> A = quadl(@sin,0,pi).
```


To integrate cos(x^2) from 0 to $\sqrt{2\pi}$, create the function in an m-file:

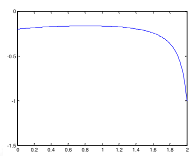
```
function yy = cossq(x)
yy = cos(x.^2);
```

Note that we must use array exponentiation. Then `quad` function is called as follows:



```
>> quad(@cossq, 0, sqrt(2*pi))
ans =
   0.6119
```

Or you can use an anonymous function:



```
>> f = @(x)(1./(x.^3 - 2*x - 5));
>> quad(f, 0, 2)
ans =
   -0.4605
```

Double and Triple Integrals

`A = dblquad(fun, a, b, c, d)` computes the integral of *f(x,y)* from *x* = *a* to *b*, and *y* = *c* to *d*. Example: $f(x,y) = xy^2$.

```
>> fun = @(x,y) x.*y^2;
>> A = dblquad(fun, 1, 3, 0, 1)
A =
    1.3333
```

$$\int_c^d \int_a^b f(x,y)dxdy$$

`A = triplequad(fun, a, b, c, d, e, f)` computes the triple integral of *f(x,y, z)* from *x* = *a* to *b*, *y* = *c* to *d*, and *z* = *e* to *f*. Example: $f(x,y,z) = (xy - y^2)/z$.

```
>> fun = @(x,y,z)(x*y - y^2)/z;
>> A = triplequad(fun, 1,3, 0,2, 1,2)
A =
    1.8484
```

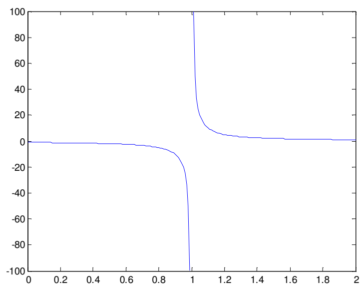
$$\int_e^f \int_c^d \int_a^b f(x,y,z)dxdydz$$

Note: The function must accept a vector *x*, but scalar *y* and *z*.

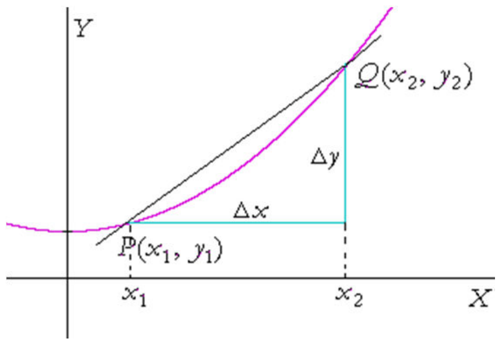
Be careful: function singularity

```
>> f = @(x) ( 1./(x-1));
>> quad(f, 0, 2)
Warning: Infinite or Not-
a-Number function value
encountered.
> In quad at 113
ans =
    NaN
```

$$\int_0^2 \frac{1}{1-x} dx$$



Numerical differentiation: Illustration of estimating the derivative *dy*/*dx*.



$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$\frac{dy}{dx} \approx \frac{y_2 - y_1}{x_2 - x_1}$$

MATLAB provides the `diff` function to use for computing derivative estimates.

`d = diff(y)`, where `y` is a vector of *n* elements, the result is a vector `d` containing *n* – 1 elements that are the differences between adjacent elements in `y`. That is:

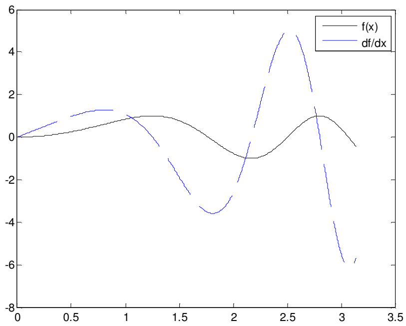
d=`[`y`(2)-`y`(1), `y`(3)-`y`(2),..., `y`(`n`)-`y`(`n`-1)]`

For example:

```
>> y = [5, 7, 12, -20];
>> diff(y)
ans =
     2     5   -32
```

Example

```
step = 0.001;
x = 0 : step : pi;
y = sin(x.^2);
d = diff(y)/step;
% an approximation
% to derivative
% 2.*x.*cos(x.^2)
plot(x,y,'k',x(2:end),d,'--');
legend('f(x)', 'df/dx');
```



Ordinary Differential Equations

- An ordinary differential equation (ODE) is an equation containing ordinary derivatives of the dependent variable.
- An equation containing partial derivatives with respect to two or more independent variables is a partial differential equation (PDE).
- We limit ourselves to ODE that must be solved for a given set of initial conditions.
- Solution methods for PDEs are an advanced topic, and we do not look at them.

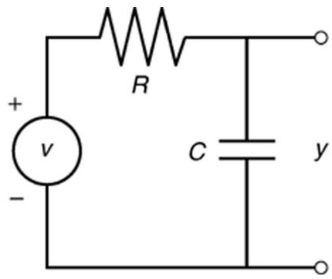
Several Methods

- Several numerical methods to solve ODEs.
- Examples include:
  - **Euler and Backward Euler methods**
  - Predictor-Corrector method
  - First-order exponential integrator method
  - **Runge-Kutta methods**
  - Adams-Moulton methods
  - Gauss-Radau methods
  - Adams-Bashforth methods
  - Hermite–Obreschkoff methods
  - Fehlberg methods
  - Parker–Sochacki methods
  - Nyström methods
  - Quantized State Systems methods

Multiple Solvers

- MATLAB offers multiple ODE solvers, each uses different methods.
- `Ode23`: Solves non-stiff differential equations, low order method.
- `ode45`: Solves non-stiff differential equations, medium order method: *uses a combination of fourth- and fifth-order Runge-Kutta methods*.
- `ode23s`: Solves stiff differential equations, low order method.
- `ode15i`: Solves fully implicit differential equations, variable order method.
- And so on.
- We will limit ourselves to the `ode45` solver.

Example: Find the response of the first-order RC circuit .



$$\tau\frac{dy}{dt} + y = 0$$

$$y(0) = V_c \quad (I.C.)$$

$$y(t) = y(0)e^{-t/\tau} \quad (natural\ response)$$

$$\tau\frac{dy}{dt} + y = V_s$$

$$y(0) = V_c \quad (I.C.)$$

$$y(t) = V_s + (y(0) - V_s)e^{-t/\tau} \quad (total\ response)$$

$$\dot{y}(t) = \frac{dy}{dt}$$

$$\ddot{y}(t) = \frac{d^2y}{dt^2}$$

Solving First-Order Differential Equations

First write the equation as *dy/dt = f(t,y)* then solve it using this syntax:

```
[t,y] = ode45(@f,tspan,y0)
```

where `@f` is the handle of the function file whose inputs must be *t* and *y*, and whose output must be a column vector representing *dy/dt*; that is, *f(t,y)*. The number of rows in the output column vector must equal the order of the equation.

The array `tspan` contains the starting and ending values of the independent variable *t*, and optionally any intermediate values.

The array `y0` contains the initial values of *y*. If the equation is first order, then `y0` is a scalar.

The circuit model for zero input voltage V_s and $\tau = 0.1$ is:

$$0.1 \times \frac{dy}{dt} + y = 0$$

And the i.c. is $y(0) = 2$ V.

First re-write the equation in the required format:

$$\frac{dy}{dt} = -10y$$

Next define the following function file. Note that the order of the input arguments must be *t* and *y*.

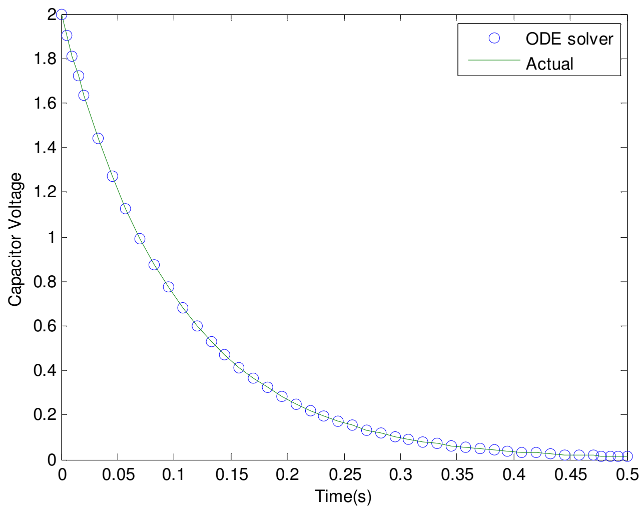
```
f = @(t,y) -10*y;
```


The solver is called as follows, and the solution plotted along with the analytical solution `y_true`. The initial condition is $y(0) = 2$.

```
f = @(t,y) -10*y;
[t, y] = ode45(f, [0 0.5], 2);
y_analytical = 2*exp(-10*t);
plot(t,y,'o', t, y_analytical);
legend('ODE solver', 'Actual');
xlabel('Time(s)');
ylabel('Capacitor Voltage');
```

Note that we need not generate the array `t` to evaluate `y_analytical`, because `t` is generated by the `ode45` function.
The plot is shown on the next slide.

Free (natural) response of an RC circuit (decaying exponential).



The circuit model for input voltage $V_s = 10V$ and $\tau = 0.1$:

$$0.1 \times \frac{dy}{dt} + y = 10$$

And the i.c. is $y(0) = 2$ V.

First re-write the equation in the required format:

$$\frac{dy}{dt} = -10y + 100$$

Next define the following function file. Note that the order of the input arguments must be *t* and *y*.

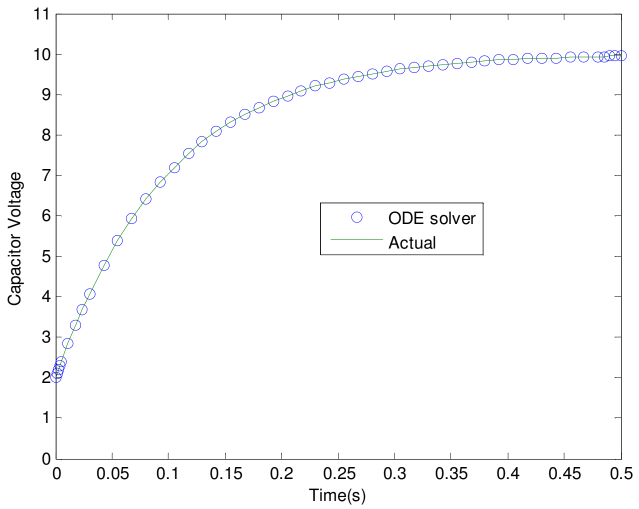
```
f = @(t,y) -10*y+100;
```


The solver is called as follows, and the solution plotted along with the analytical solution `y_true`. The initial condition is $y(0) = 2$.

```
f = @(t,y) -10*y+100;
[t, y] = ode45(f, [0 0.5], 2);
y_analytical = 10+(2-10)*exp(-10*t);
plot(t,y,'o', t, y_analytical);
legend('ODE solver', 'Actual');
xlabel('Time(s)');
ylabel('Capacitor Voltage');
```

Note that we need not generate the array `t` to evaluate `y_analytical`, because `t` is generated by the `ode45` function.
The plot is shown on the next slide.

Natural plus forced (total) response of an RC circuit (increasing exponential).



The circuit model for input voltage $V_s = 10e^{-t/0.3}\sin\left(\frac{2\pi t}{0.03}\right)$ and $\tau = 0.1$:

$$0.1 \times \frac{dy}{dt} + y = 10e^{-t/0.3}\sin\left(\frac{2\pi t}{0.03}\right)$$

And assume the i.c. is $y(0) = 0$ V.

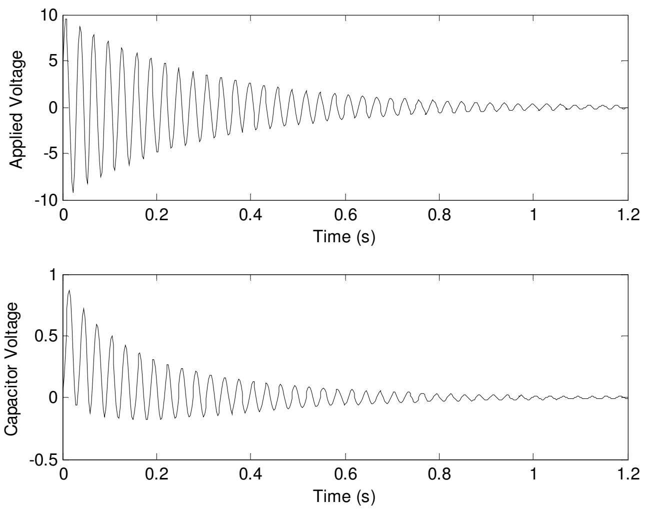
First re-write the equation in the required format:

$$\frac{dy}{dt} = -10y + 100e^{-t/0.3}\sin\left(\frac{2\pi t}{0.03}\right)$$

Next define the following function file. Note that the order of the input arguments must be *t* and *y*.

```
f = @(t,y) -10*y+100* ...
exp(-1*t/0.3).*sin(2*pi*t/0.03);
```


Result



Extension to Higher-Order Equations

To use the ODE solvers to solve an equation of 2nd order or higher, you must first write the equation as a set of first-order equations.

Example:

$$5\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 4y = f(t)$$

By re-arranging to get the highest derivative:

$$\frac{d^2y}{dt^2} = \frac{1}{5}f(t) - \frac{4}{5}y - \frac{7}{5}\frac{dy}{dt}$$

Example (*Continue*)

$$\frac{d^2y}{dt^2} = \frac{1}{5}f(t) - \frac{4}{5}y - \frac{7}{5}\frac{dy}{dt}$$

We then change variables: $x_2 = dy/dt$
Hence: $dx_2/dt = d^2y/dt^2$
Also: $x_1 = y$. Hence we have two equations:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{1}{5}f(t) - \frac{4}{5}x_1 - \frac{7}{5}x_2$$

Example (*Continue*)

$$\frac{dx_1}{dt} = x_2$$
$$\frac{dx_2}{dt} = \frac{1}{5}f(t) - \frac{4}{5}x_1 - \frac{7}{5}x_2$$

This form is sometimes called the Cauchy form or the state-variable form.

We now define a function that accepts two values of x and then computes the values of dx_1/dt and dx_2/dt and stores them in a column vector.

Example (Code)

$$\frac{dx_1}{dt} = x_2$$
$$\frac{dx_2}{dt} = \frac{1}{5}\sin(t) - \frac{4}{5}x_1 - \frac{7}{5}x_2$$

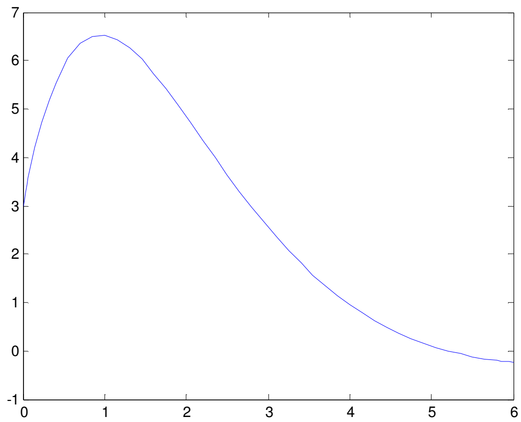
```
d = @(t,x) [x(2); sin(t)/5-4*x(1)/5-7*x(2)/5];
[t, x] = ode45(d, [0 6], [3 9]);
```

Here $x(0) = 3$ and $\dot{x}(0) = 9$, and we solve for $0 \leq t \leq 6$. Also $f(t) = \sin(t)$.

Note `x` is a matrix with two columns. The first column contains the values of x_1 at the various times generated by the solver; the second column contains the values of x_2.

If you type `plot(t, x)`, you will obtain a plot of both x_1 and x_2 versus `t`. Thus, type `plot(t, x(:,1))` to see the result for `y`.

Result



HW: Alternative Solution

Define the function in an m-file:

```
function xdot = d(t, x)
xdot(1) = x(2);
xdot(2) = (1/5)*(sin(t)-4*x(1)-7*x(2));
xdot = [xdot(1); xdot(2)];
```

Use the function to solve the ODE:

```
[t, x] = ode45(@d, [0 6], [3 9]);
% notice the need to use handles
plot(t, x(:,1));
```

Homework

- Solve as many problems from Chapter 9 as you can
- Suggested problems:
- Solve: 9.1, 9.4, 9.14, 9.16, 9.23, 9.27, 9.31, 9.34.

Lecture 9: Symbolic Processing in MATLAB

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EE201: Computer Applications. See Textbook Chapter 11.

The `sym` function can be used to create "symbolic objects" in MATLAB.

If the input argument to `sym` is a string, the result is a symbolic number or variable. If the input argument is a numeric scalar or matrix, the result is a symbolic representation of the given numeric values.

For example, typing `x = sym('x')` creates the symbolic variable with name `x`, and typing `y = sym('y')` creates a symbolic variable named `y`.

Typing `x = sym('x', 'real')` tells MATLAB to assume that `x` is real. Typing `x = sym('x', 'unreal')` tells MATLAB to assume that `x` is not real.

The `syms` function enables you to combine more than one such statement into a single statement.

For example, typing `syms x` is equivalent to typing `x = sym('x')`, and typing `syms x y u v` creates the four symbolic variables `x`, `y`, `u`, and `v`.

Symbolic vs. Numeric Objects

```
>> x = sym('x')
x =
 x
>> class(x)
ans =
 sym


>> syms y
>> class(y)
ans =
 sym
```

```
>> a = 5
a =
     5
>> class(5)
ans =
 double

>> b = 't'
b =
 t
>> class(b)
ans =
 char
```

You can use the `sym` function to create *symbolic constants* by using a numerical value for the argument. For example, typing

```
fraction = sym('1/3')

sqroot2 = sym('sqrt(2)')

pi = sym('pi')
```

will create symbolic constants that avoid the floating-point approximations inherent in the values of π, 1/3, and $\sqrt{2}$.

Symbolic Expressions

You can use symbolic variables in expressions and as arguments of functions. You use the operators `+ – * / ^` and the built-in functions just as you use them with numerical calculations. For example, typing

```
>> syms x y
>> s = x + y;
>> r = sqrt(x^2 + y^2);
```

creates the symbolic variables `s` and `r`. The terms `s = x + y` and `r = sqrt(x^2 + y^2)` are examples of symbolic *expressions*.

The vector and matrix notation used in MATLAB also applies to symbolic variables. For example, you can create a symbolic matrix `A` as follows:

```
>> n = 3;
>> syms x;
>> A = x.^((0:n)'*(0:n))
A =
   [ 1, 1, 1, 1]
   [ 1, x, x^2, x^3]
   [ 1, x^2, x^4, x^6]
   [ 1, x^3, x^6, x^9]
```


The `expand` and `simplify` functions.

```
>> syms x y
>> expand((x+y)^2) % applies algebra rules
ans =
     x^2 + 2*x*y + y^2

>> syms x y
>> expand(sin(x+y)) % applies trig identity
ans =
     cos(x)*sin(y) + cos(y)*sin(x)

>> syms x
>> simplify(6*((sin(x))^2+(cos(x))^2))
% applies another trig identity
ans =
     6
```



```
>> syms x
>> E1 = x^2+5;
>> E2 = x^3+2*x^2+5*x+10;
>> S = E1/E2;
>> simplify(S)
ans =
   1/(x + 2)
```

The `factor` function.

```
>> syms x
>> factor(x^2-1)
ans =
     (x - 1)*(x + 1)
```


The function `subs(E,old,new)` substitutes `new` for `old` in the expression `E`, where `old` can be a symbolic variable or expression and `new` can be a symbolic variable, expression, or matrix, or a numeric value or matrix. For example,

```
>> syms x y
>> E = x^2+6*x+7;
>> F = subs(E,x,y)
F =
   y^2 + 6*y + 7

>> G = subs(E,x,y+3)
G =
   6*y + (y + 3)^2 + 25
```

If you want to tell MATLAB that f is a function of the variable t, type `f = sym('f(t)')`. Thereafter, `f` behaves like a function of `t`, and you can manipulate it with the toolbox commands. For example, to create a new function $g(t) = f(t + 2) - f(t)$, the session is

```
>> syms t
>> f = sym('f(t)');
>> g = subs(f,t,t+2)-f
g =
   f(t+2)-f(t)
```

Once a specific function is defined for $f(t)$, the function $g(t)$ will be available.

Use the `subs` and `double` functions to evaluate an expression numerically. Use `subs(E,old,new)` to replace `old` with a numeric value `new` in the expression `E`. The result is of class double. For example,

```
>> syms x
>> E = x^2+6*x+7;
>> G = subs(E,x,2)
G =
   23
>> class(G)
ans =
     double
```

The MATLAB function `ezplot(E)` generates a plot of a symbolic expression `E`, which is a function of one variable. The default range of the independent variable is the interval $[-2\pi, 2\pi]$ unless this interval contains a singularity.

The optional form `ezplot(E,[xmin xmax])` generates a plot over the range from `xmin` to `xmax`.

Example:

```
>> syms x
>> E = x^2 - 6*x + 7;
>> ezplot(E, [-2 6]);
```

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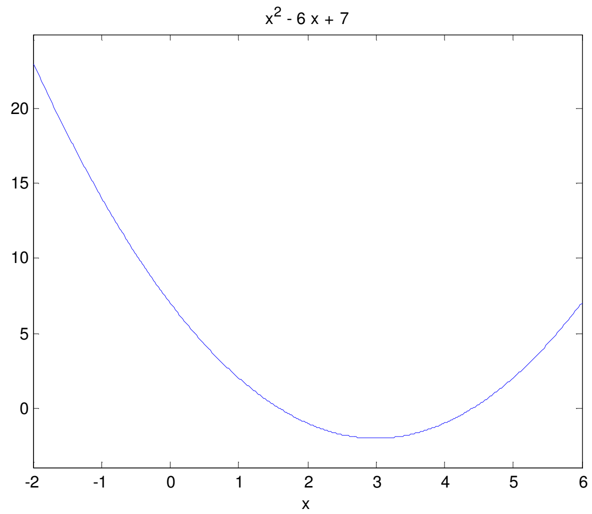
Result



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Order of Precedence.

MATLAB does not always arrange expressions in a form that we normally would use.

For example, MATLAB might provide an answer in the form `-c+b`, whereas we would normally write `b-c`.

The order of precedence used by MATLAB must be constantly kept in mind to avoid misinterpreting the MATLAB output (see earlier slides).

MATLAB frequently expresses results in the form `1/a*b`, whereas we would normally write `b/a`.

The `solve` function.

There are three ways to use the `solve` function. For example, to solve the equation $x + 5 = 0$, one way is

```
>> eq1 = 'x+5=0';
>> solve(eq1)
ans =
      -5
```

The second way is

```
>> solve('x+5=0')
ans =
      -5
```

The `solve` function (continued).

The third way is

```
>> syms x
>> solve(x+5)
ans =
     -5
```

You can store the result in a named variable as follows:

```
>>syms x
>>x = solve(x+5)
x =
   -5
```


To solve the equation $e^{2x} + 3e^{x} = 54$, the session is

```
>> solve('exp(2*x)+3*exp(x) = 54')
ans =
         log(6)
 log(9) + pi*I


>> syms x
>> solve(exp(2*x)+3*exp(x)-54)
ans =
        log(6)
 log(9) + pi*i
```

Other examples:

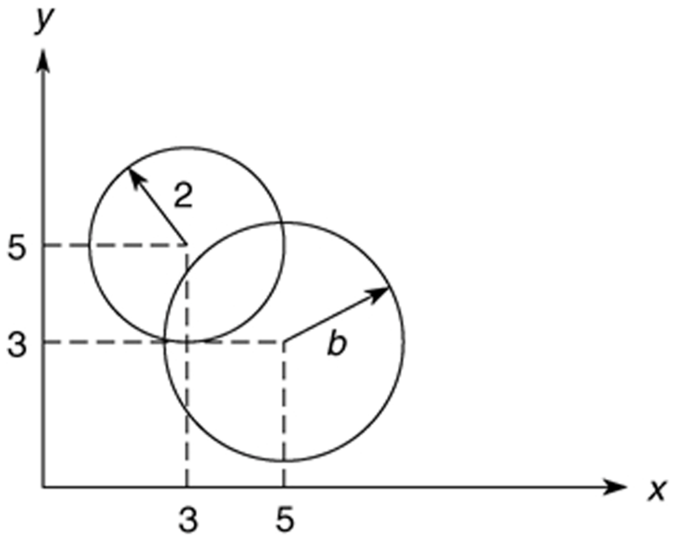
```
>> eq2 = 'y^2+3*y+2=0'; % quadratic eq
>> solve(eq2)
ans =
     [-2]
     [-1]

>> eq3 = 'x^2+9*y^4=0'; % x is squared
>> solve(eq3) % x is assumed the unknown
ans =
     [ 3*i*y^2]
     [-3*i*y^2]
```


When more than one variable occurs in the expression, MATLAB assumes that the variable closest to x in the alphabet is the variable to be found. You can specify the solution variable using the syntax
`solve(E, 'v')`, where `v` is the solution variable.

```
>> eq3 = 'x^2+9*y^4=0'; % y is to power 4
>> solve(eq3,'y')
ans =
   -((-1)^(1/4)*9^(3/4)*x^(1/2))/9
    ((-1)^(1/4)*9^(3/4)*x^(1/2))/9
 -((-1)^(1/4)*9^(3/4)*x^(1/2)*i)/9
  ((-1)^(1/4)*9^(3/4)*x^(1/2)*i)/9
```


Application of the `solve` function: Find the two Intersection points of the following two circles. Keep b unknown.



Solution

```
>> S = solve('(x-3)^2+(y-5)^2=4, (x-5)^2+(y-3)^2=b^2')
S =
    x: [2x1 sym]
    y: [2x1 sym]

>> S.x
ans =
 (- b^4/16 + (3*b^2)/2 - 1)^(1/2)/2 - b^2/8 + 9/2
 9/2 - b^2/8 - (- b^4/16 + (3*b^2)/2 - 1)^(1/2)/2

>> S.y
ans =
 (- b^4/16 + (3*b^2)/2 - 1)^(1/2)/2 + b^2/8 + 7/2
 b^2/8 - (- b^4/16 + (3*b^2)/2 - 1)^(1/2)/2 + 7/2
```

Differentiation with the `diff` function.

```
>> syms n x y
>> diff(x^n)
ans =
     x^n*n/x
>> simplify(ans)
ans =
     x^(n-1)*n
>> diff(log(x)) % means ln
ans =
     1/x
>> diff((sin(x))^2)
ans =
     2*sin(x)*cos(x)
```


If the expression contains more than one variable, the `diff` function operates on the variable *x*, or the variable closest to *x*, unless told to do otherwise. When there is more than one variable, the `diff` function computes the *partial* derivative.

```
>> syms x y
>> diff(sin(x*y))
ans =
     cos(x*y)*y
```

The function `diff(E,v)` returns the derivative of the expression `E` with respect to the variable `v`.

```
>> syms x y
>> diff(x*sin(x*y),y)
ans =
     x^2*cos(x*y)
```

The function `diff(E,n)` returns the *n*th derivative of the expression `E` with respect to the default independent variable.

```
>> syms x
>> diff(x^3,2)
ans =
     6*x
```

The function `diff(E,v,n)` returns the *n*th derivative of the expression `E` with respect to the variable `v`.

```
>> syms x y
>> diff(x*sin(x*y),y,2)
ans =
     -x^3*sin(x*y)
```


Integration with the `int` function.

```
>> syms x
>> int(2*x)
ans =
     x^2
```

The function `int(E)` returns the integral of the expression `E` with respect to the default independent variable.


```
>> syms n x y

>> int(x^n)
ans =
      x^(n+1)/(n+1)

>> int(1/x)
ans =
      log(x)

>> int(cos(x))
ans =
      sin(x)
```

$$\int x^n dx$$

$$\int \frac{1}{x} dx = \ln(x)$$

The form `int(E,v)` returns the integral of the expression `E` with respect to the variable `v`.

```
>>syms n x
>>int(x^n,n)
ans =
      1/log(x)*x^n
```

$$\int x^n dn$$

The form `int(E,a,b)` returns the integral of the expression `E` with respect to the default independent variable evaluated over the interval [*a*, *b*], where `a` and `b` are numeric expressions.

```
>>syms x
>>int(x^2,2,5)
ans =
     39
```

$$\int_{2}^{5} x^2 dx$$

The form `int(E,v,a,b)` returns the integral of the expression `E` with respect to the variable `v` evaluated over the interval [*a*, *b*], where `a` and `b` are numeric quantities.

```
>> syms x y
>> int(xy^2,y,0,5)
ans =
     125/3*x
```

The form `int(E,m,n)` returns the integral of the expression `E` with respect to the default independent variable evaluated over the interval [*m*, *n*], where `m` and `n` are symbolic expressions.

```
>> syms t x
>> int(x,1,t)
ans =
     t^2/2 - 1/2
```

$$\int_1^t x\,dx$$

```
>> syms t x
>> int(sin(x),t,exp(t))
ans =
     cos(t) - cos(exp(t))
```


The following session gives an example for which no integral can be found. The indefinite integral exists, but the definite integral does not exist if the limits of integration include the singularity at $x = 1$.

```
>> syms x
>> int(1/(x-1))
ans =
     log(x - 1)

>> syms x
>> int(1/(x-1),0,2)
ans =
    NaN
```

Taylor Series. $f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(t) + \frac{(x-a)^3}{3!}f^{(3)}(t) + \cdots$

The `taylor(f,n,a)` function gives the first `n-1` terms in the Taylor series for the function defined in the expression `f`, evaluated at the point $x = a$. If the parameter `a` is omitted the function returns the series evaluated at $x = 0$.

```
>> syms x
>> f = exp(x);
>> taylor(f,3,2)
ans =
     exp(2)+exp(2)*(x-2)+(exp(2)*(x-2)^2)/2

>> taylor(f,4)
ans =
     x^3/6 + x^2/2 + x + 1
```


Series summation.

The `symsum(E,a,b)` function returns the sum of the expression `E` as the default symbolic variable varies from `a` to `b`.

```
>> syms k n
>> symsum(k,0,10)
ans =
     55
>> symsum(k^2, 1, 4)
ans =
     30
>> symsum(k,0,n-1)
ans =
     (n*(n - 1))/2
```

$$\sum_{k=0}^{10} k$$

$$\sum_{k=1}^{4} k^2$$

Finding limits.

The basic form `limit(E)` finds the limit as $x \to 0$.

```
>> syms a x
>> limit(sin(a*x)/x)
ans =
     a
```


The form `limit(E,v,a)` finds the limit as $\upsilon \to a$.

```
>>syms h x

>>limit((x-3)/(x^2-9),3)
ans =
     1/6

>>limit((sin(x+h)-sin(x))/h,h,0)
ans =
     cos(x)
```

The forms `limit(E,v,a,'right')` and `limit(E,v,a,'left')` specify the direction of the limit.

```
>> syms x
>> limit(1/x,x,0,'left')
ans =
     -inf

>> syms x
>> limit(1/x,x,0,'right')
ans =
     inf
```


Solving differential equations with `dsolve`

The `dsolve` syntax for solving a single equation is `dsolve('eqn')`. The function returns a symbolic solution of the ODE specified by the symbolic expression `eqn`.

```
>> dsolve('Dy+2*y=12')
ans =
     6+C1*exp(-2*t)
```

There can be symbolic constants in the equation.

```
>> dsolve('Dy=sin(a*t)')
ans =
     (-cos(a*t)+C1*a)/a
```


Here is a second-order example:

```
>> dsolve('D2y=c^2*y')
ans =
     C1*exp(-c*t) + C2*exp(c*t)
```


Sets of equations can be solved with `dsolve`. The appropriate syntax is `dsolve('eqn1','eqn2',...)`.

```
>>[x, y]=dsolve('Dx=3*x+4*y','Dy=-4*x+3*y')
   x =
C1*exp(3*t)*cos(4*t)+C2*exp(3*t)*sin(4*t)
   y = -
C1*exp(3*t)*sin(4*t)+C2*exp(3*t)*cos(4*t)
```

Conditions on the solutions at specified values of the independent variable can be handled as follows.

The form

`dsolve('eqn', 'cond1', 'cond2',...)`

returns a symbolic solution of the ODE specified by the symbolic expression `eqn`, subject to the conditions specified in the expressions `cond1`, `cond2`, and so on.

If `y` is the dependent variable, these conditions are specified as follows: `y(a) = b`, `Dy(a) = c`, `D2y(a) = d`, and so on.

Example:

```
>> dsolve('D2y=c^2*y','y(0)=1','Dy(0)=0')
ans =
     1/2*exp(c*t)+1/2*exp(-c*t)
```

Example:

```
>> [x,y]=dsolve('Dx=3*x+4*y','Dy=-4*x+3*y',
'x(0)=0','y(0)=1')

x =
  sin(4*t)*exp(3*t)
y =
  cos(4*t)*exp(3*t)
```

It is not necessary to specify only initial conditions. The conditions can be specified at different values of *t*.

```
>> dsolve('D2y+9*y=0','y(0)=1','Dy(pi)=2')
ans =
     cos(3*t) - (2*sin(3*t))/3
```


Laplace and Fourier Transform

```
>> syms b t
>> laplace(t^3)
ans =
     6/s^4

>> laplace(exp(-b*t))
ans =
     1/(s+b)

>> laplace(sin(b*t))
ans =
     b/(s^2+b^2)

>> fourier(exp(-t^2))
ans =
     pi^(1/2)/exp(w^2/4)
```

Laplace Inverse Transform

```
>>syms b s

>>ilaplace(1/s^4)
ans =
     1/6*t^3

>>ilaplace(1/(s+b))
ans =
     exp(-b*t)

>>ilaplace(b/(s^2+b^2)
ans =
     sin(b*t)
```


You can use the `inv(A)` and `det(A)` functions to invert and find the determinant of a matrix symbolically.

```
>> syms k
>> A = [0 ,1;-k, -2];
>> inv(A)
ans =
     [ -2/k, -1/k ]
     [ 1, 0 ]
>> A*ans     % verify inverse is correct
ans =
     [ 1, 0 ]
     [ 0, 1 ]
>> det(A)
ans =
     k
```

You can use matrix methods in MATLAB to solve linear algebraic equations symbolically. You can use the matrix inverse method, if the inverse exists, or the left-division method.

```
>> syms c
>> A = sym([2, -3; 5, c]);
>> b = sym([3; 19]);
>> x = inv(A)*b   % matrix inverse method
x =
 (3*c)/(2*c + 15) + 57/(2*c + 15)
 23/(2*c + 15)

>> x = A\b   % left-division method
x =
 (3*c)/(2*c + 15) + 57/(2*c + 15)
 23/(2*c + 15)
```


Homework

- Solve as many problems from Chapter 11 as you can
- Suggested problems:
- Solve: 11.3, 11.4, 11.12, 11.18, 11.22, 11.23, 11.28, 11.31, 11.32, 11.35, 11.37, 11.41, 11.42, 11.50, 11.51.

Lecture 10: Simulink

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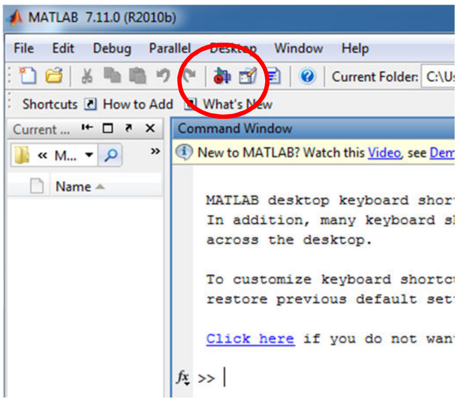
EE201: Computer Applications. See Textbook Chapter 10.

What is Simulink?

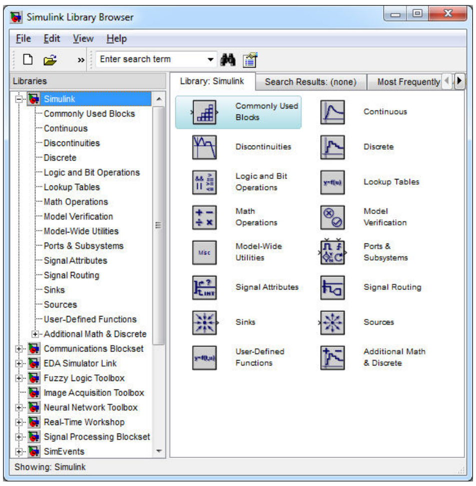
- Simulink is a tool for modeling, simulating and analyzing dynamic systems.
- Its primary interface is a graphical block diagramming tool and a customizable set of block libraries.
- It supports linear and nonlinear systems, modeled in continuous time, discrete time, or a hybrid of both.
- It easily integrates with the rest of the MATLAB environment.
- Simulink is widely used in control theory and digital signal processing for simulation and model-based design.

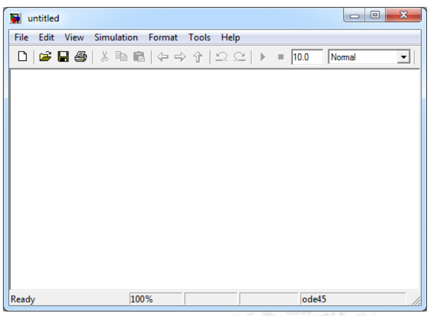
Starting Simulink

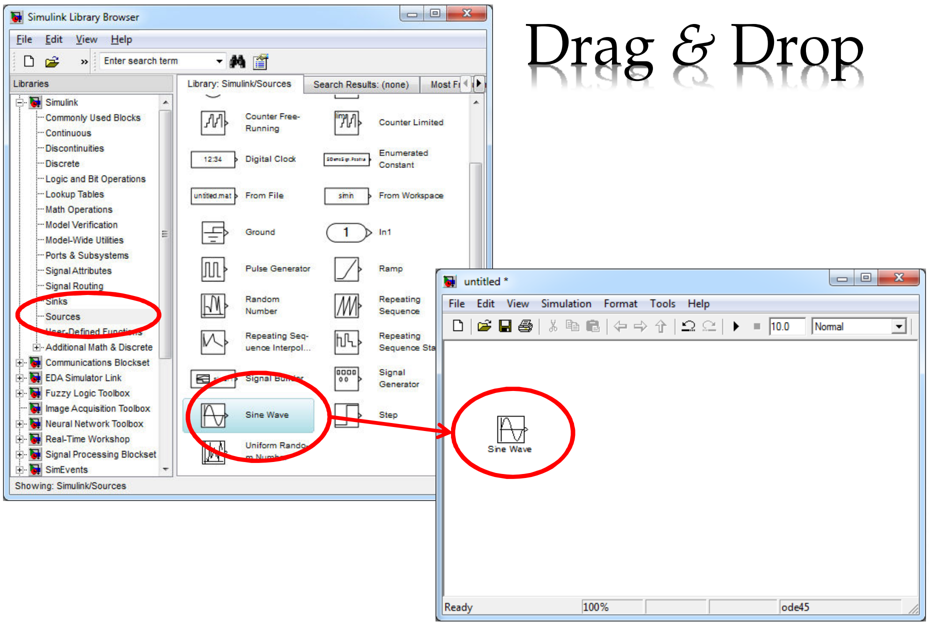
- To build a Simulink model, choose File | New | Model.
- To see the Simulink library of blocks click on the Simulink icon in MATLAB.



Library Browser & Model Window

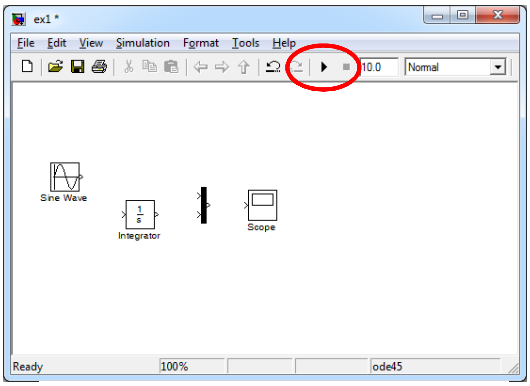


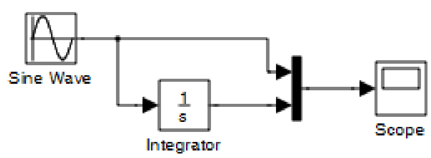




Drag & Drop

- Sources | *Sine Wave*
- Continuous | *Integrator*
- Signal Routing | *Mux*
- Sinks | *Scope*
- To connect blocks, move the cursor to the output port represented by ">" sign. Once placed at a port, the cursor will turn into a cross "+" enabling you to make the connection between blocks.
- Run the simulation of the simple system shown by clicking on the play icon.

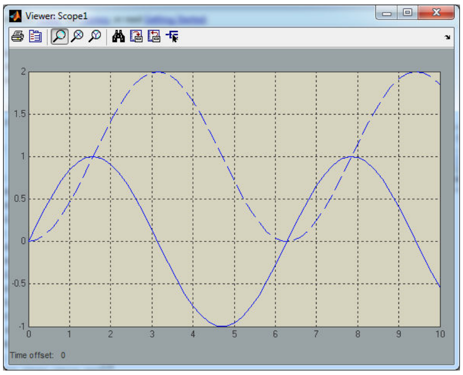




$$\mathcal{L}\left\{\int_0^t f(v)dv\right\} = \frac{1}{s}F(s)$$

Scope Results

- Double click on the scope block to see the results of the simulation.
- To view/edit the parameters of a block, double click on the block to see the *Block Parameters* window.
- Try changing the initial condition of the Integrator from 0 to -1.



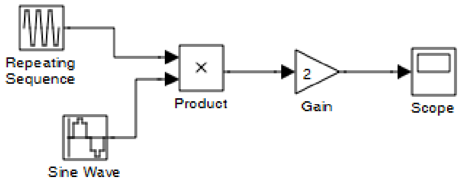
Blocks & Model File

- MATLAB uses the default values of the block parameters, except where you explicitly change them.
- You can always click on Help within the Block Parameters window to obtain more information.
- You can edit the label of a block by clicking on the text and making the changes.
- You can search for Blocks in the Simulink search window.
- You can save the Simulink model as **.mdl** file by selecting File | Save menu item in Simulink.

Exercise: Modulation

Blocks:

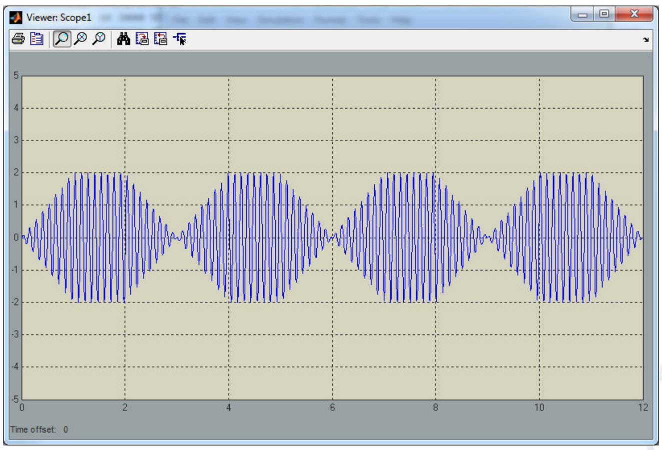
- Sources: Repeating Sequence
- Sources: Since Wave
- Math Operation: Produc
- Math Operation: Gain
- Sinks: Scope



Edit the following properties:

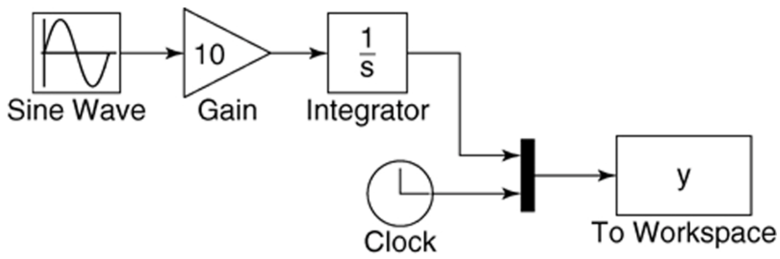
- Repeating Sequence:
  - Time Values: [0 1 2 3 4 5 6]
  - Output Values: [0 1 1 0 -1 -1 0]
- Sine Wave:
  - Frequency: 50 rad/s
  - Sample time: 0.01
- Gain: 2
- Simulation Stop Time:
  - 12 seconds

Results



Exercise: Sending data to Workspace.

Notice the “Clock” and “To Workspace” blocks.
Set simulation time to 13 seconds.



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Double-click on the To Workspace block. You can specify any variable name you want as the output; the default is `simout`. Change its name to `y`.

The output variable `y` will have as many rows as there are simulation time steps, and as many columns as there are inputs to the block.

The second column in our simulation will be time, because of the way we have connected the Clock to the second input port of the Mux.

Specify the **Save format** as **Array**. Use the default values for the other parameters (these should be `inf`, `1`, and `-1` for Maximum number of rows, Decimation, and Sample time, respectively). Click on **OK.**

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Simulink can be configured to put the time variable `tout` into the MATLAB workspace automatically when you are using the To Workspace block.

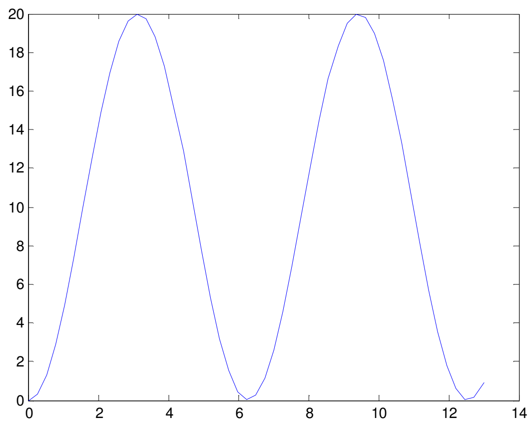
This is done with the Data I/O tab under **Configuration Parameters** on the Simulation menu.

The alternative is to use the Clock block to put `tout` into the workspace.

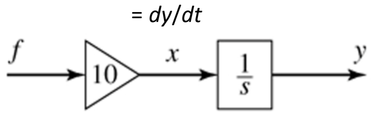
The Clock block has one parameter, **Decimation**. Set this parameter to **1**, which means the Clock block will output the time every time step; if set to 10 for example, the block will output every 10 time steps, and so on.

In MATLAB, try: **plot(y(:,2), y(:,1))**

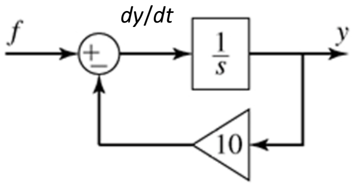
Result



Simulation diagrams for $x = dy/dt = 10\,f(t)$



Simulation diagram for $dy/dt = f(t) - 10y$



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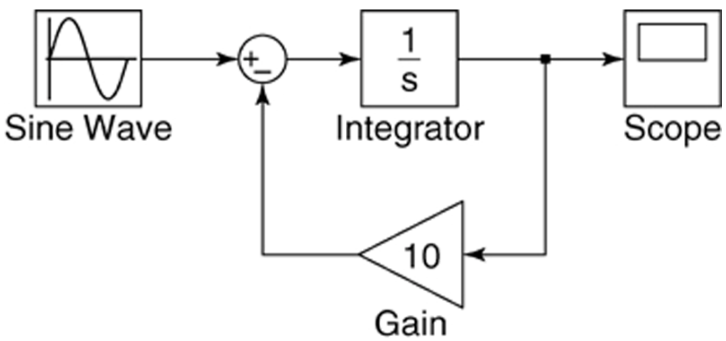
Exercise: Simulink model to solve the first-order ODE

$$dy/dt = -10y + 2\sin(4t) \quad 0 \le t \le 3$$



Homework: Use Simulink to solve the second-order ODE

$$d^2x/dt^2 = 5\cos(2t) - 3\,dx/dt - 4x \qquad 1 \le t \le 3$$

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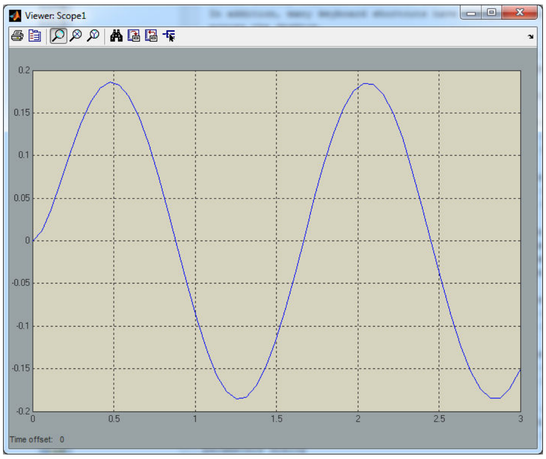
Result



Homework

- Solve as many problems from Chapter 10 as you can
- Suggested problems:
- Solve: 10.1, 10.3, 10.4.

Lecture 11: MATLAB Exercises

Dr. Mohammed Hawa
Electrical Engineering Department
University of Jordan

EE201: Computer Applications. See Textbook Chapter 4.

Exercise 1

- Write a MATLAB m-file function (called `fact.m`) which takes a single argument (an integer), computes the factorial and returns the answer.
- Hint: For better performance, do *not* use loops!

Exercise 2

- Write a MATLAB m-file function (called `grades.m`) which accepts student grades as argument (*hint*: number array) and then determines the lowest, highest and average of such scores.
- E.g., `grades([11 10 99 5 19 3 17])`
- `Total: 7 scores`
- `Min value: 3`
- `Max value: 99`
- `Average value: 23.43`

Exercise 3

- Write a MATLAB m-file function (`dice.m`) which simulates one or more dice with each die giving values from 1 to 6.
- The program takes a single argument which is the number of dice.
- The output should contain the values of the dice and also the probability for this combination of dice to occur. The probability is expressed as a decimal value between 0 and 1 with five decimal points.
- E.g., Rolling 3 dice: 4 1 6 (Probability: 0.00463)

Exercise 4

- Write a MATLAB script (called `rev.m`) which reads a number of strings from standard input and prints them in reverse order on the command window.
- The input sequence is terminated with the string END.
- Hint: Use a cell array!

```
>> rev
one
two
three
END
-> three
-> two
-> one
```


Exercise 5

- Write a MATLAB script (called `count.m`) which reads a string from standard input and then counts the number of words in that string.
- E.g., "Everyone loves MATLAB" contains 3 words.

Exercise 6

- The sum of the squares of the first ten integers is:
- $1^2 + 2^2 + \dots + 10^2 = 385$
- The square of the sum of the first ten integers is:
- $(1 + 2 + \dots + 10)^2 = 55^2 = 3025$
- Hence the difference between the sum of the squares of the first ten integer numbers and the square of the sum is 3025 – 385 = 2640.
- Find the difference between the sum of the squares of the first one hundred integer numbers and the square of the sum.

Exercise 7

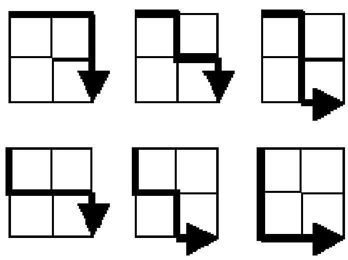
- A prime number (or a prime) is an integer number greater than 1 that has no positive divisors other than 1 and itself.
- The first six prime numbers are: 2, 3, 5, 7, 11, and 13.
- We can see that the 6th prime is 13.
- Write a MATLAB script to print the first 50 prime numbers.

Exercise 8

- A Pythagorean triplet is a set of three positive integer numbers, $a < b < c$, for which: $a^2 + b^2 = c^2$
- For example, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.
- There exists exactly one Pythagorean triplet for which $a + b + c = 1000$.
- Write a MATLAB script to find this triplet.

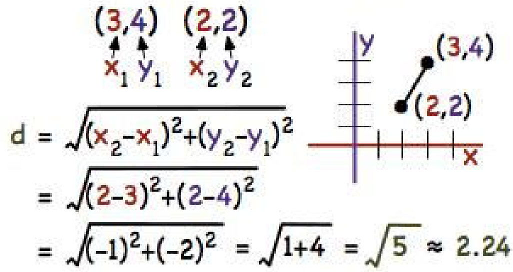
Exercise 9

- Starting in the top left corner of a 2×2 grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner (see the figure below).
- How many such routes are there through a 10×10 grid?



Exercise 10

- Write a MATLAB script file that asks the user to type the coordinate of two points: A and B (in a plane), and then displays the distance between A and B.



The University of Jordan
School of Engineering
Department of Electrical Engineering
1st Semester – A.Y. 2014/2015

Course: Computer Applications – 0903201 (1 Cr. – Core Course)

Instructor: Dr. Mohammed Hawa
Office: E306, *Telephone:* 5355000 ext 22857, *Email*: hawa@ju.edu.jo
Office Hours: will be posted soon

Course Website: http://fetweb.ju.edu.jo/staff/EE/mhawa/201/

Catalog Data: Computer packages for mathematical and symbolic manipulations (MATLAB, Mathematica). Windows environment. Graphics packages. INTERNET and its use in literature survey and information acquisition. Library search via computer. Engineering packages for computation. Data processing and statistical packages. Standard computer libraries.

Prerequisites by Course: **EE 1901102 – Computer Skills 2 (C++) (pre-requisite)**

Prerequisites By Topic: Students are assumed to have a background in the following topics:

- Basic computer and software skills.
- Basic programming language skills, such as C/C++.
- Basic mathematics, calculus and linear algebra.
- Basic scalar, array, vector and matrix operations.
- Solution of ordinary differential equations.
- Basic electric circuit analysis.

Textbook: **Introduction to MATLAB for Engineers by William J. Palm III, McGraw-Hill, 3rd Edition, 2011.**

References:

- *Essential MATLAB for Engineers and Scientists* by Brian Hahn and Daniel Valentine, Academic Press, 5th Edition, 2013.
- *MATLAB for Engineers* by Holly Moore, Prentice Hall, 3rd Edition, 2011.
- *Getting Started with MATLAB 7: A Quick Introduction for Scientists and Engineers* by Rudra Pratap, Oxford University Press, 1st Edition, 2005.
- *MATLAB Programming with Applications for Engineers* by Stephen J. Chapman, CL-Engineering, 1st Edition, 2012.
- *An Engineers Guide to MATLAB* by Edward B. Magrab, et. al., Prentice Hall, 3rd Edition, 2010.
- *Mastering MATLAB* by Duane C. Hanselman and Bruce L. Littlefield, Prentice Hall, 1st Edition, 2011.
- *Modeling and Simulation in SIMULINK for Engineers and Scientists* by Mohammad Nuruzzaman, AuthorHouse; 1st Edition, 2005.
- *Mastering Simulink* by James B. Dabney and Thomas L. Harman, Prentice Hall, 1st Edition, 2003.

Schedule & Duration: 16 Weeks, 45 lectures (50 minutes each) plus exams.

Minimum Student Material: Textbook, class handouts, scientific calculator, and an access to a personal computer.

Minimum College Facilities: Classroom with whiteboard and projection display facilities, library, computational facilities with the MATLAB program.

Course Objectives: The overall objective is to introduce the student to solving engineering problems using computers and scientific programming packages.

Course Learning Outcomes and Relation to ABET Student Outcomes:

Upon successful completion of this course, a student should:

1.	Use MATLAB to solve computational problems and generate publishable graphics	[e, k]
2.	Use complex arithmetic and complex functions to describe applied problems. Describe complex numbers and functions in rectangular and exponential forms. Graph the magnitude and phase of complex functions	[a]
3.	Use matrix forms to describe and solve linear systems of equations and systems of differential equations	[e]
4.	Determine the system of linear equations required to find the coefficients that define an interpolating function that matches a set of data samples.	[a, e]
5.	Solve first and second order linear differential equations with constant coefficients both analytically and numerically. Use the MATLAB routine ODE23 to solve differential equations numerically.	[a, k]
6.	Define the Fourier series for a periodic signal. Define the Fourier transform of an aperiodic signal.	[a, k]
7.	Compute the Fourier series and transform from their definition as integrals.	[a, k]
8.	Use the properties of linearity, time-shifting and time-scaling to compute the Fourier series/transform of complex functions from the Fourier series/transforms of simple functions.	[a, k]
9.	Use the Simulink simulation package to simulate some electric and electronic circuits	[k]

Course Topics:

	Topic Description	Hrs
1	Introduction to MATLAB and its use cases. Using the workspace to explore MATLAB features regarding ease of use and versatility. Entering commands. Using MATLAB help.	**2**
2	General number formatting. Variables, Vectors and Matrices. Built-in MATLAB engineering functions. Matrix-related functions. Operator precedence. Matrix indexing: row and column versus linear versus logical indexing. Matrix versus element-by-elemtn operations.	**3**
3	Solving a system of linear equations. The concept of vectorization and its use in speeding computations.	**2**
4	Euclidean Vectors and their operations. Complex numbers. Polynomials. Cells arrays. Structures.	**2**
5	Script Files. Header comments. User Input/Output commands. The concept of functions in MATLAB and how to build user defined functions. Local vs. global variables. Subfunctions. Inline functions and function handles. Importing data: text, Excel, images, audio, etc.	**3**
6	Writing general-purpose programs in MATLAB. Flowchart versus pseudocode. Relational operators and conditional statements. Flow control structures and loops. Practical exercises.	**4**
7	**Midterm Exam**	**1**
8	Plotting. The different plot types available. Figure annotations. Three dimensional plots.	**3**
9	Using MATLAB buil-in functions to obtain numerical solutions for various calculus problems: differentiation, integration, ordinary differential equations, etc.	**2**
10	MATLAB symbolic engine. Using symbolic notation to define and plot functions. Using symbolic capapilities for liner algebra, calcuals and other problems. Introduction to MuPAD.	**2**
11	Introduction to Simulink and its libraries. Simulating some engineering systems and finding solutions. Linking Simulink with the MATLAB workspace.	**2**

Ground Rules: **Attendance is required** and highly encouraged. To that end, attendance will be taken every lecture. All exams (including the final exam) should be considered **cumulative**. Exams are closed book. No scratch paper is allowed. You will be held responsible for all reading material assigned, even if it is not explicitly covered in lecture notes.

Assessments: Exams, Quizzes, Projects, and Assignments.

Grading policy:

Assignments, projects, quizzes	**20 %**
Midterm Exam	**30 %**
Final Exam	**50 %**
Total	**100%**

Last Updated: January 2015