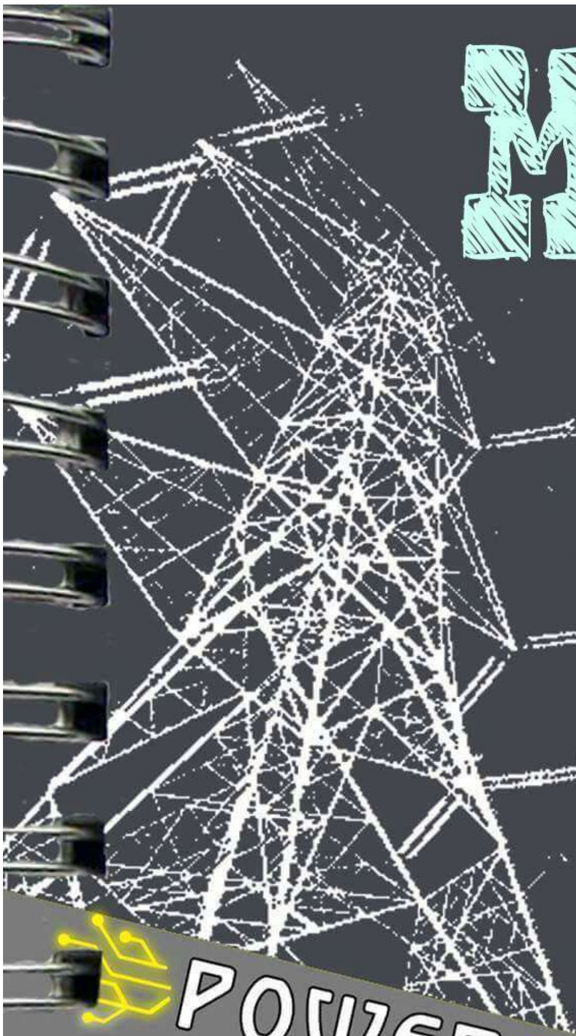
Machines1

By: Sara Audeh

Dr.Aldalabeeh



POWER UNIT

Fall2016

Machines :

No.

Magnetic Field and electric Circuits :-

What ? magnetic Field

why? Because magnetic Field is Fundamental for the operation of Transformers, Generators, and motors as follows :-

[1] any Current (i) in a Conductor generates Magnatic Field (ϕ) around it . [The direction of ϕ can be found by Using Right hand Rule → (RHR) التيار في أي موصل ينتج مجال مغناطيسي
(ϕ is $\propto$ i) يتم تحديد اتجاهه عن طريق قاعدة اليد اليمنى

[2] time varying magnetic field Passing through a coil or a Conductor will induce a voltage in this Coil or Conductor This is the Concept of transformer operation.

[3] If a Conductor cut a magnetic field then a voltage will be reduced in it. This is the Concept of generator operation عند قطع الموصل للمجال المغناطيسي سوف يقلل الجهد عليه

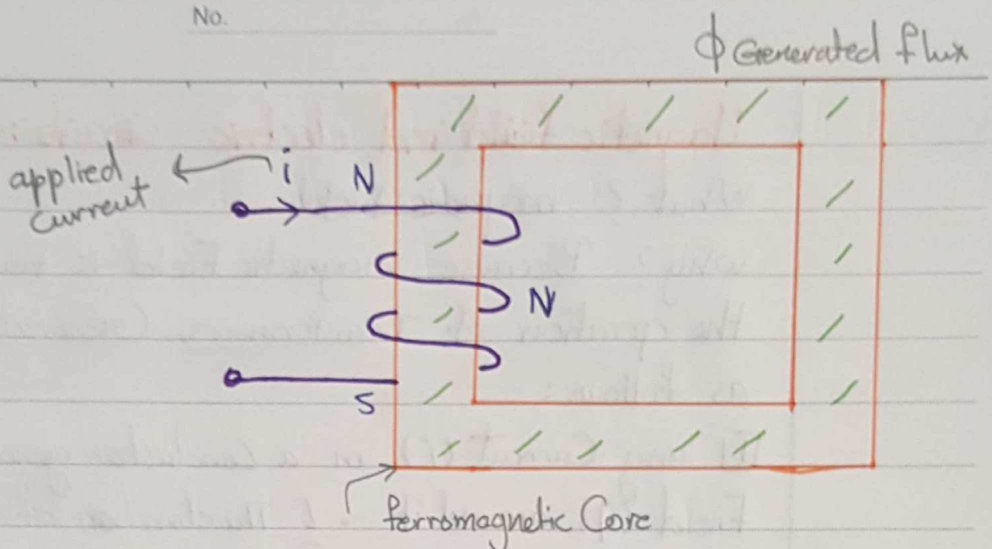
[4] A current carrying conductor in a magnetic field will experiance a force . This is the concept of motor operation

How ?? The objective is to find ϕ for a given i
OR to find i for a given ϕ

* This can be solved approximatly by the Concept of magnetic ckt. and introduced as follows :-

Basic mathmatical Laws and Relationships

Consider the following system



A core with N turns

Ampere's Law:-

$$\oint H \cdot dl = I$$ إيجاد التيار من مجال مغناطيسي

التكامل المغلق

$H \equiv$ Magnetic Field strength or intensity. المجال المغناطيسي

$l \equiv$ a closed path or loop. مجال مغلق

$I \equiv$ Total Current enclosed by the loop. التيار الكلي

∴ For Uniform Magnetic Field. and for the mean length of Flux path →

القوة الدافعة ← $Hl = Ni$ …①

$l \equiv$ Mean length of flux path (m)

$H \equiv A \cdot t / m$

Relation ship between flux density B and H is

$B = \mu H$

* Unit of B is wb/m² or Tesla.

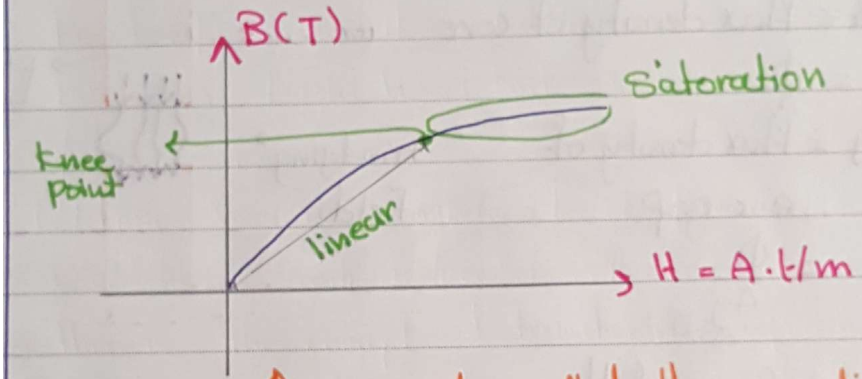
$\mu \equiv$ Called permeability (H/m) of ferromagnetic material or Core.

$\mu = \mu_0 \mu_r$

↳ for free space oR air $= 4\pi \times 10^{-7}$

$\mu_0 \equiv$ Permeability of free space.
$\mu_r \equiv$ Relative pemability of the material conce (2000-8000)

* However in practic the relationship between B and H is nonlinear as follows:-



↑ This is also called the magnetization curve

$\therefore B = \frac{\Phi}{A}$ A ≡ cross sectional Area of the core

$$Ni = Hl = \frac{B}{\mu} l = \frac{\Phi}{A\mu} l$$

$$\therefore \boxed{Ni} = \boxed{\frac{l}{A\mu}} \Phi$$

$\frac{l}{A\mu}$ → Reluctance ($\mathcal{R}$)

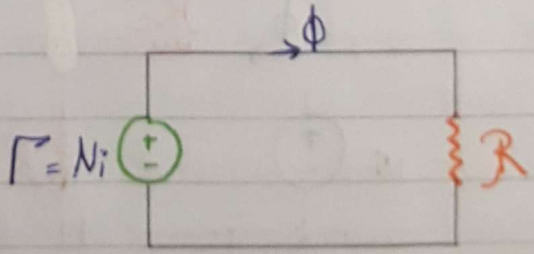
Ni → magnetiomotive force (mmf) $\mathcal{F}$

$Ni = \mathcal{R}\Phi$

I want Reluctance as small as possible

$\therefore \mathcal{F} = \mathcal{R}\Phi$

By using Anology with electrical Circuits, the equivalent magnetic circuit will be :-



ohms' law يطابق

All circuit laws (ohm's, KVL, KCL) are applicable to magnetic circuit with mmF ≡ emf

$\phi \equiv i$

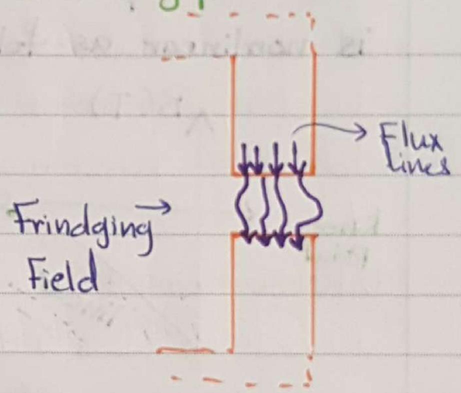
Magnetic structure with air gap:-

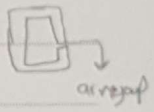
$B_c \equiv$ flux density of core

$= \frac{\phi}{A_c}$

$B_g \equiv$ flux density of air gap

$= \frac{\phi}{A_g}$





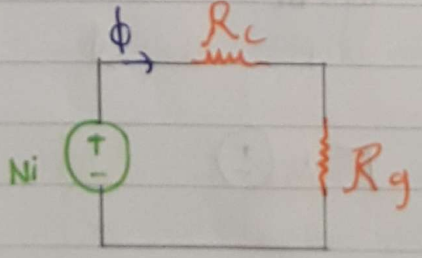
$A_g \equiv$ Area of the air gap

if the length of the air gap very small compare with the core + length of the gap

Note :- A_g can be found approximatly
for example: if $g << l_c$
Then all g to the sides of the core
OR multiply A_c by . . correction factor.

* If the frindging field is neglacted then $A_g = A_c$
$B_g = B_c$

Equivelent magnetic circuit

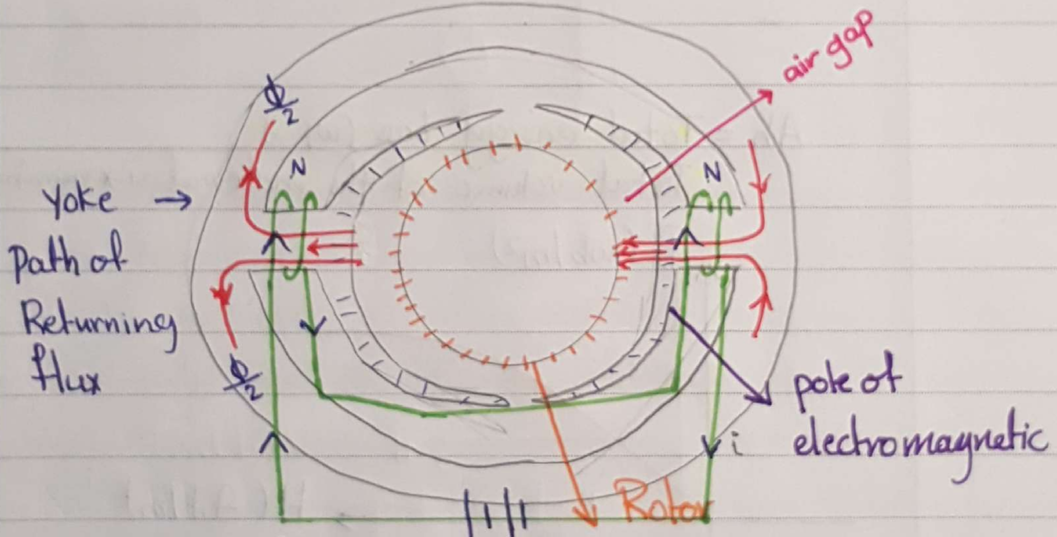


$$Ni = \phi(R_c + R_g)$$
$$= \phi\left(\frac{l_c}{A_c \mu_0 \mu_r} + \frac{g}{A_g \mu_0}\right)$$

Since air gap have high reluctance, then in electrical Machines, the air gap between the stator and the Rotor should be as small as possible (i.e Just a clearance)

Illustration:-

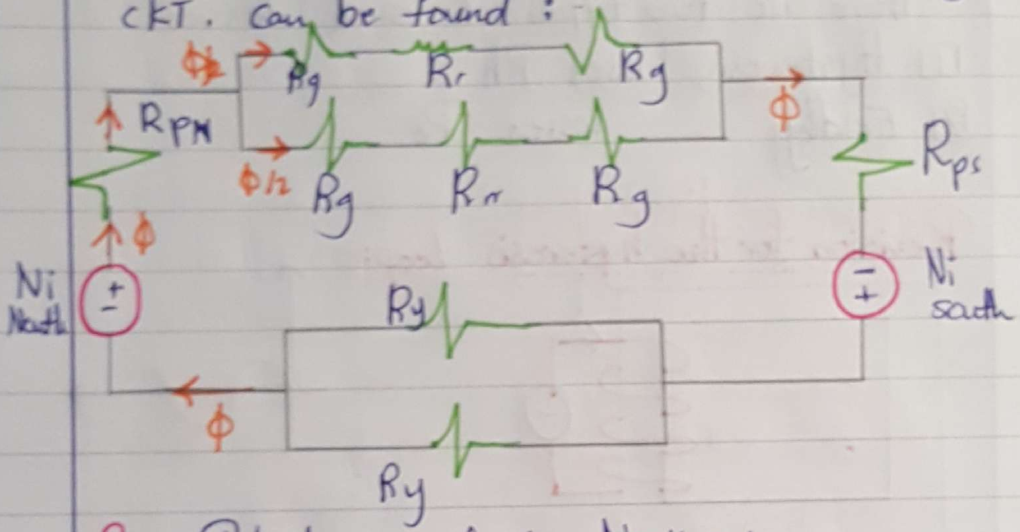
Find the equivalent Magnetic circuit of a 2-pole machine



Yoke: path of Returning Flux

Nx: number of Turns

Hence from the machine structure the following magnetic ckT. can be found :



R_{pn}: Reluctance of the North pole
R_g: Reluctance of the air gap
R_r: Reluctance of the rotor
R_{ps}: Reluctance of the south pole
R_y : Reluctance of the yoke

By applying the Magnetic circuit laws:
By KVL:

$$-Ni + \phi\,[(R_g+R_r+R_g)\,//\,(R_g+R_r+R_g)] + \phi\,[R_{ps}] - Ni + \phi\,[R_y // R_y] = 0 \quad \dots ①$$

- hence by using equation (1) we can calculate ϕ for a given Ni or Ni for a given ϕ.

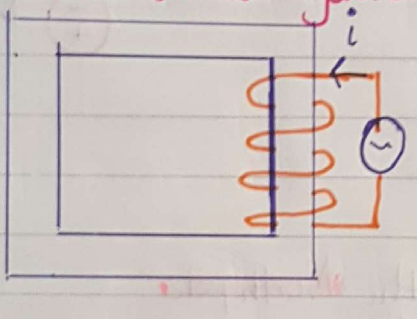
* there are some examples in the text Book

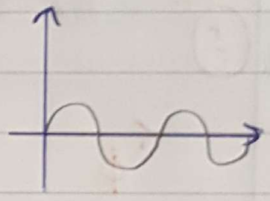
Losses in Ferromagnetic Materials:

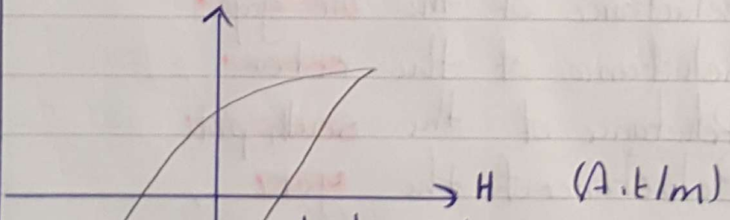
- There are two types of such losses.

1. Hysteresis losses Ph
2. Eddy losses Pe

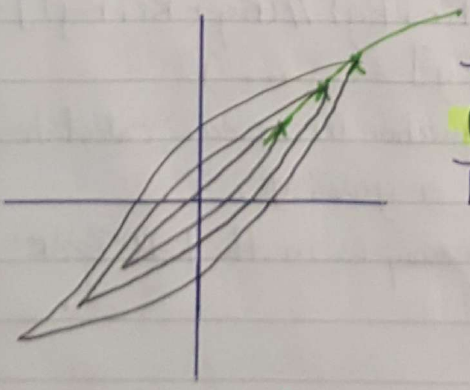
Revision for the hysteresis loop:







hysteresis loop for one cycle



* The area of hysteresis loop

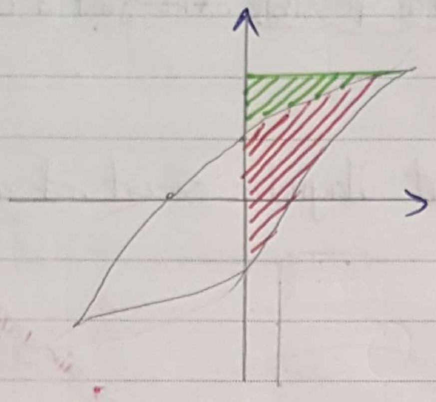
(B*H) = J/m³

This is called energy density

* The area of H. loop is Energy Density / cycle (Ah)

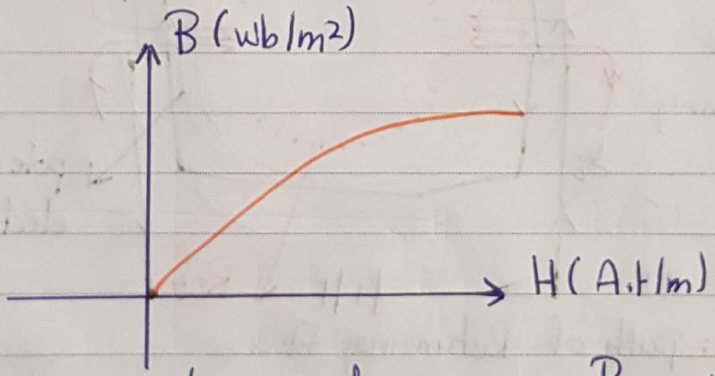
وحدة مش مهتمين

* the area of the hysteresis loop $(B * H) = J/m^3$ this is called energy density.



Area in red : Suplied energy
Area in Green : Area returned
Area of H-loop : Ah: energy density per cycle

$$A_h = \frac{\text{Total energy Loss } (w) \ J}{\text{Total volume of the material } v * \text{number of cycles}}$$



$$A_h = w/v * \# \text{ of cycles} = \frac{P_h * \text{time}}{V * \# \text{of cycles}}$$

P_h = Power losses (watt)

$$A_h = \frac{P_h}{V * \boxed{\frac{\text{number of cycles}}{\text{Time}}}} \rightarrow \text{frequency.}$$

* An Empirical formula was found for Ah

$A_h = K_h B_m^n$ n = constant $1.5 \leq n \leq 2.5$

K_h = An constant its value depends on the type of the material

B_m = Maximum Value of flux density.

So $P_h = K_h B_m^n * V * f$

Pc: Eddy Current losses

** To reduce the Eddy Current Losses we make the coil from eleminations ((شرائح) they should be isolated and parallel to the flux.
عازلة موازية

** i applied $\xrightarrow{\text{generate}}$ ϕ $\longrightarrow$ $\phi \propto i$ $\longrightarrow$ Since it's AC ϕ is AC $\rightarrow$ ϕ $\xrightarrow{\text{generate}}$ voltage (e) in the core, since the core is conducting. $\xrightarrow{\text{generate}}$ i_e

i_e will flow in such direction (accourding to lenz's) to generate ϕ_e opposing ϕ $\rightarrow$ i_e flow in sircular direction so it's called eddy Current $\rightarrow$ This i_e causes P_e.

$P_e \rightarrow$ reduced by using limitation for the core. عن طريق الصفائح

Since lamination are used then the total volume of the core is going to incrase. يزداد الحجم الكلي للملف

Hence, Stacking factor $\triangleq$

$$\triangleq \frac{\text{Volume of the ferromagnetic material}}{\text{Total volume of the core}}$$

$$= \frac{\text{Area of the ferro material}}{\text{Total Area of the core}}$$

Expression of P_e

e → for voltage

$P_e \propto i_e^2$, since $i_e \propto e_e$

frequency

$\therefore P_e \propto e_e^2$ since $e_e \propto B_m f$

maximum flux density

$\therefore P_e \propto B_m^2 \cdot f^2$

Hence P_e can be expressed as:

$$P_e = K_e B_m^2 f^2 \tau V$$

$K_e \equiv$ Constant depends on the type of ferromagnetic material

$\tau \equiv$ Lamination Thickness

$V \equiv$ Volume of the ferromaterial.

Core losses, $P_c \Rightarrow$

$$P_c \triangleq P_e + P_h$$

*Example: The total core losses for a given magnetic sheet steel is found to be 1800 W at 60 HZ, when the frequency is increased by 50% the core losses is found to be 3000 W.

$\xrightarrow{}$ 90 = 60 + 30

$$P_c = P_e + P_h$$

$$= \overbrace{K_2 f^2 B_m^2}^{\text{constant } K_b} + \overbrace{K_1 B_m^n}^{\text{constant } K_a} f$$

for the same flux density B_m

$$P_c = K_b f^2 + K_a f$$

$$\therefore \ 1800 = K_b \, 60^2 + K_a \, 60 \quad \dots①$$

$$3000 = K_b \, 90^2 + K_a \, 90 \quad \dots②$$

solving ① and ② it can be found:

$$K_b = \frac{1}{9}$$

f	Pe	Ph
60	$\frac{1}{9} \times 3600 = 400$	$1800 - 400 = 1400$
90	$\frac{1}{9} \times 90 \times 90 = 900$	$3000 - 900 = 2100$

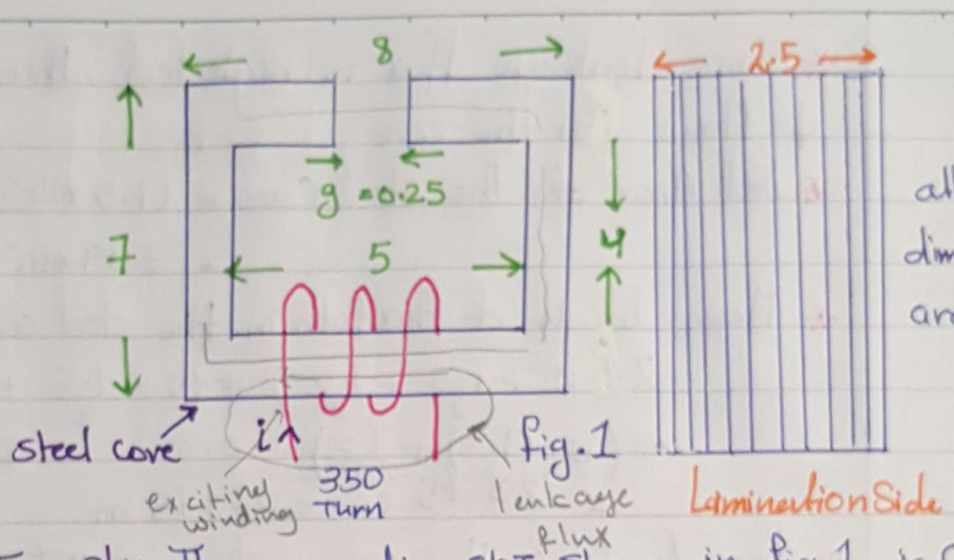


fig.1

all dimentions are in Cm

Example: The magnetic ckT shown in fig.1 is Composed of a laminated core of steel with air gap g and has an exciting winding of 350 turns. The magnetization curve of steel is shown in fig. 2. The Presence of laminations is taken into account by assuming stacking factor 0.93.

Neglect leakage flux, but take in to acount fringing find and calculate the required exciting Current to Produce a flux of $5*10^{-4}$ wb in the core.

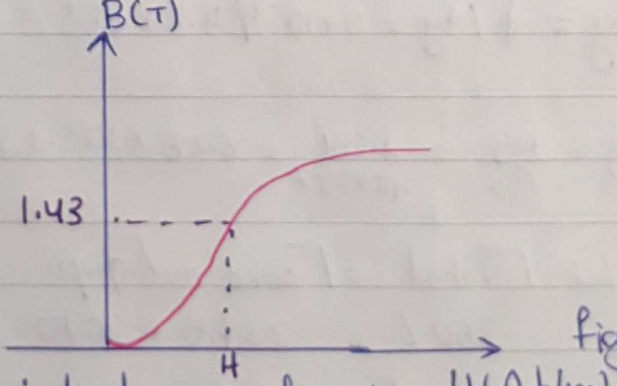
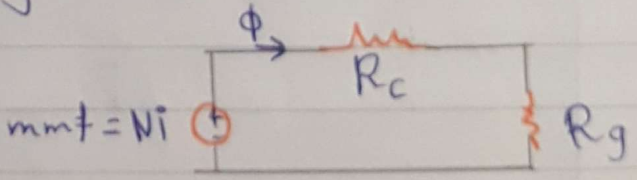


fig.2

* Equivalent magnetic ckT



* Since leukage flux is neglected, then all the flux ϕ flows in the core.

* Net Area of the steel Core $= (1.5 * 10^{-2} * 2.5 * 10^{-2} * 0.93)$

$= 3.49 * 10^{-4}\ m^2$

* Mean length of flux Path in the steel core $=$

$$2\left(4 + \frac{1.5}{2} + \frac{1.5}{2}\right) + \left(5 + \frac{1.5}{2} + \frac{1.5}{2}\right)$$

$$+ \left(5 + \frac{1.5}{2} + \frac{1.5}{2} - 0.25\right).$$

$$= 23.75\ cm = 02375\ m$$

* flux density in the core $= \frac{\phi}{A_c} = \frac{5*10^{-4}}{3.49*10^{-4}} = 1.43\ T$

By using Fig. 2: it can be found that for $B = 1.43$

$H_{core} = 10^3$

* Correction for air gap, add g to the side of the gap

$$A_g = (1.5 + 2.5)(2.5 + 0.25) * 10^{-4}$$

$$= 4.81 * 10^{-4}\ m^2$$

$$\therefore B_g = \phi / A_g = 5*10^{-4} / 4.8 * 10^{-4} = 1.04\ T$$

$$H_g = \frac{B_g}{\mu_o} = \frac{1.04}{4\pi * 10^{-7}} = 8.28 * 10^5\ A.t/m$$

By KVL: $\mathcal{F}_{total} = \mathcal{F}_{core} + \mathcal{F}_{gap}$

$$350\ i = 237.5 + 2070$$

$$i = 6.6\ A$$

Inductance L :

Flux linkage , $\lambda \triangleq N\phi$

$$\lambda \triangleq Li$$

$$L = \frac{N\phi}{i}$$

Since $Ni = \mathcal{R}\phi$

$$\therefore \quad L = \frac{N\phi}{\frac{\mathcal{R}\phi}{N}} = \frac{N^2}{\mathcal{R}} = \frac{N^2 \mu A}{l}$$

the end of the Concept of magnetic Ckts.

Chapter ② :-

Transformer :

what? Definition it's a device or Equipment which can be used to change the level or magnitude of AC voltage

why? It is used in the process of electrical energy transfer by ① stepping up the voltage and ② reducing the Current

↳ Consequently

*** Hence power losses will be decreased**

How? Analysis

***Construction** → هو القلب الأساس

البنية

Transformer consist of a Core with two or more windings

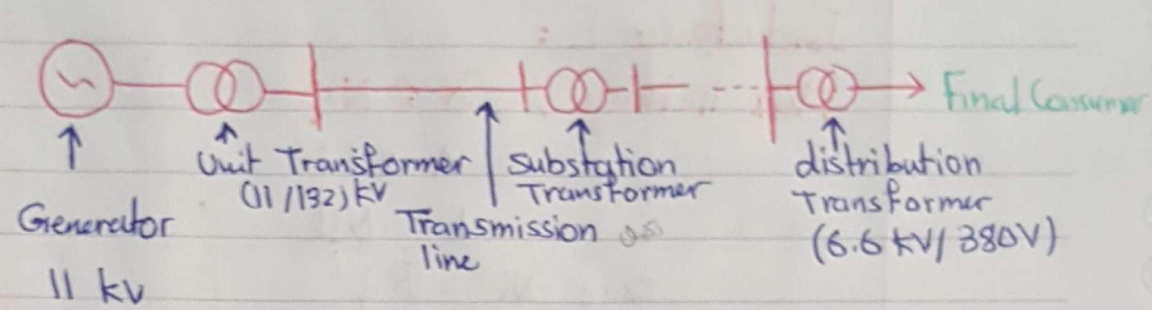
2 ملف

*** Clasification** التصنيف

- According to their application they can be classified into :-

I Power Transformer : This is the one used in the process of ¹· Generator Transmission and ²· distribution of electrical energy.

No.



[2] Instroment Transformer: These are the ones which are used in the process of electrical measurment of voltage and current:

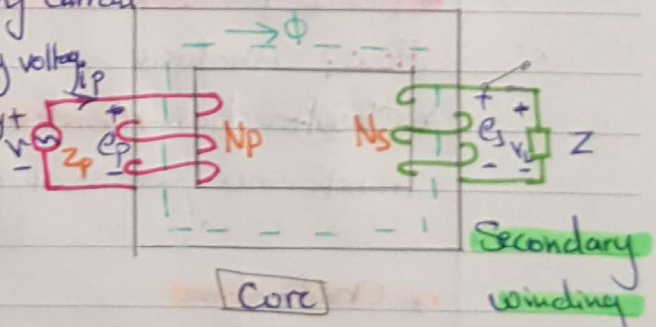
A. Potantial Transformer PT
B. Current Transformer CT

This Course Consernd with Power Transformer:-
Single Phase Transformer

Objective: To find Voltage, Current and power relationship

Ns, Np ≡ Primary and Secondary turns.

ip: applied primary Current
v: Applied Primary voltage
ep, es: Primary, Secondary Induced voltage.
Primary Coil or winding



VL: Load voltage

** Assume Ideal Transformer

Assumtion
① No Losses (electrical and core)
② $\mu_r \to \infty$
③ No Leakage flux (i.e. all generated flux flow in the core)
تسريب

[illegible]

No.

$e_p = N_p \frac{d\phi}{dt}$ $\qquad e = N \frac{d\phi}{dt}$

$e_s = N_s \frac{d\phi}{dt}$

$\frac{e_p}{e_s} = \frac{N_P}{N_s} = a$ = turns ratio

$i_p N_p = i_s N_s$

$\therefore \frac{i_p}{i_s} = \frac{N_s}{N_P} = \frac{1}{a}$

V_s: Secondary terminal voltage

Since a is a real number then e_p and e_s are in phase

Also i_p and i_s are in phase

* for Ideal Transformer

$e_p = V_p$ and $V_s = e_s$

* The voltage and the Current relationships Can be written in phaser form:

$\frac{V_p}{V_s} = \frac{I_s}{I_P} = a$

$\therefore V_p I_p = V_s I_s$

$\therefore$ Apparent power of primary and secondary is the same (KVL)

Power:

P supplied by Priamary $= P_p = V_p I_p \cos \Theta_p$

P taken by the secondary $= P_s = V_s I_s \cos \Theta_s$ → phase shift

* for ideal Transformer → $\Theta_p = \Theta_s = \Theta$

$\therefore P_p = (a V_s) * (\frac{I_s}{a}) * \cos\Theta = V_s I_s \cos\Theta = P_s$

$\therefore \eta$ of Trainsformer = 100%

efficiency $\eta \triangleq \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{losses}}$ $\qquad \frac{\text{output}}{\text{input}}$

No.

Impedance

$Z = \frac{V}{I}$ for Transformer

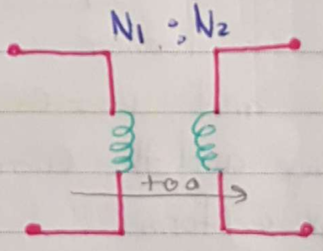
$$\rightarrow Z_p = \frac{V_P}{I_P} = \frac{aV_s}{\frac{I_s}{a}} = a^2\frac{V_s}{I_s} = a^2 Z_s$$

OR $Z_s = \frac{Z_p}{a^2}$

turns ratio ⟵ a^2

∴ To Reflect Z_s to primary * by a^2

∴ To Reflect Z_p to Secondary ÷ by a^2



$1 : \frac{N_2}{N_1} = a_1$ OR $\frac{N_1}{N_2} : 1 = a_2$

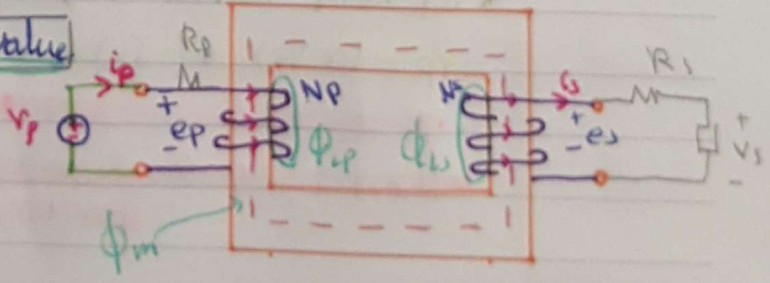
In general To reflect an impedance Z to the a side, multiply Z by a^2

To reflect an impedance Z to 1 side ÷ Z by a^2

* Real Transformer: (Practical)

Here:

1) losses will be taken into account
2) leakag flux
3) μ_r has a high value



No.

ϕ_m = Mutual flux
ϕ_{Lp} = leakage flux of primary
ϕ_{Ls} = Leakage flux of Secondary

$\phi_p = \phi_m + \phi_{Lp}$ $\qquad$ $e_p = e_{mp} + e_{Lp}$
$\phi_s = \phi_m + \phi_{Ls}$ $\qquad$ $e_s = e_{ms} + e_{Ls}$

$$\therefore e_p = N_p \frac{d\phi_p}{dt} = N_p \frac{d\phi_m}{dt} + N_p \frac{d\phi_{Lp}}{dt}$$

$$e_s = N_s \frac{d\phi_s}{dt} = N_s \frac{d\phi_m}{dt} + N_s \frac{d\phi_{Ls}}{dt}$$

$$\therefore \frac{e_{mp}}{e_{ms}} = \frac{N_p}{N_s}$$

fur Practical iron Core transformer $\phi_m \gg \phi_L$

$\therefore \phi_m \simeq \phi$

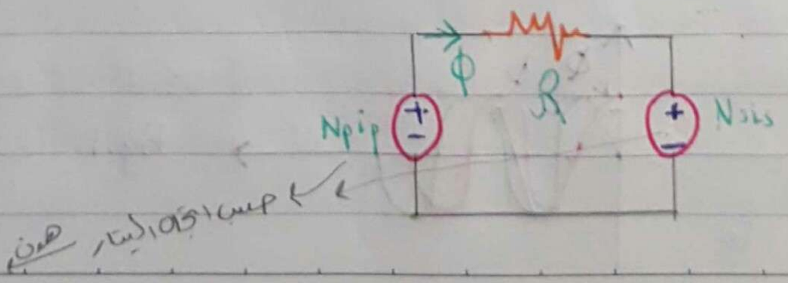
$\therefore \frac{e_p}{e_s} = \frac{N_p}{N_s}$

[redacted]

* If the resistance of the winding is neglected then:
أهمل

$V_p \simeq e_p$
$V_s \simeq e_s$

* Current Relationship $\qquad$ Equivelant CKT



No.

*مقدار الـ flux
في المحول الكهربائي
دائماً ثابت (لو زاد الـ current وشد الـ voltage لما يرجع ϕ للقيمة الأصلية)

$\therefore$ By KVL $\rightarrow N_p i_p - N_s i_s = \mathcal{R}\phi$

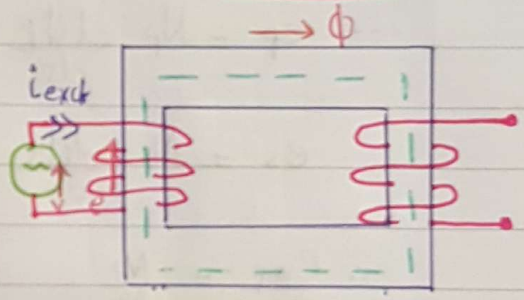
Since $\mathcal{R}$ is very small $\therefore \mathcal{R} \simeq 0$

$N_p i_p = N_s i_s$

$\therefore \frac{i_p}{i_s} = \frac{N_s}{N_p}$

Excitation Current :-

This is the primary current at $\boxed{\text{no-Load}}$



iext has 2 functions or 2 components :-

i) to supply the magnetizing current ⓘm for flux generation

ii) to account for the core losses ⓘc

*objective To find by qualitative means the shape of iext

Since $V = N \frac{d\phi}{dt}$

$\therefore \phi = \frac{1}{N} \int v\,dt$

if $v = V_m \cos \omega t$

$\therefore \phi = \phi_m \sin \omega t$

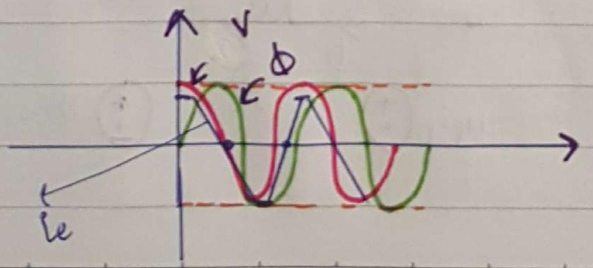


Fig.1

No.

i_m

If the magnetization curve of the Curve is given.

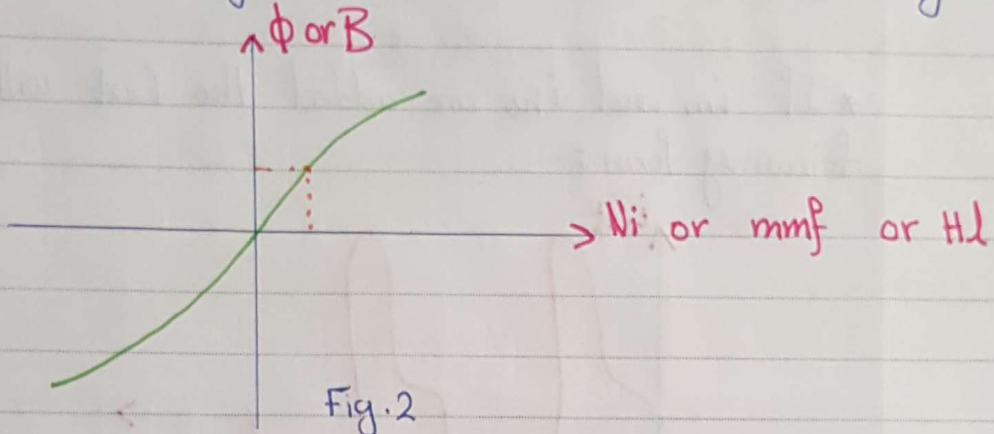
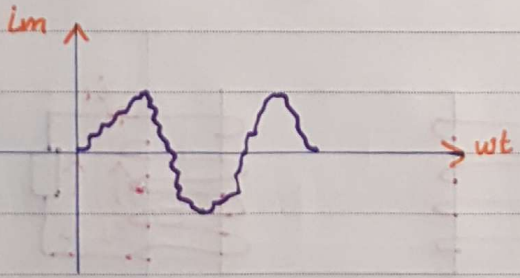


Fig. 2

* For each value of ϕ in Fig. 1, then by Projecting it in Fig. 2, the Corresponding value for $\boxed{i_m}$ Can be found.

* The shape of i_m will be as follows:-



∴ $\boxed{i_m}$ is a distorted sinusoidal, and by using Fourier Series it Can be analysed into fundamental + Harmonics

i_c

$i_c = i_e + i_h \equiv i_{he}$

$i_e \propto e_e \propto \frac{d\phi}{dt}$

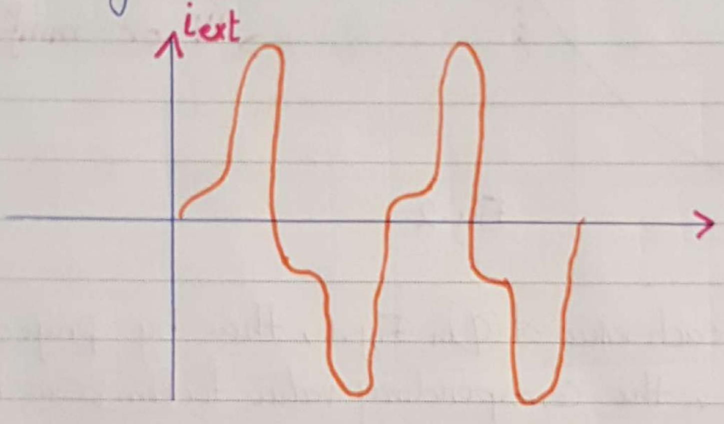
∴ i_e is nonsinusoidal, but it is periodic by Fourier Series

No.

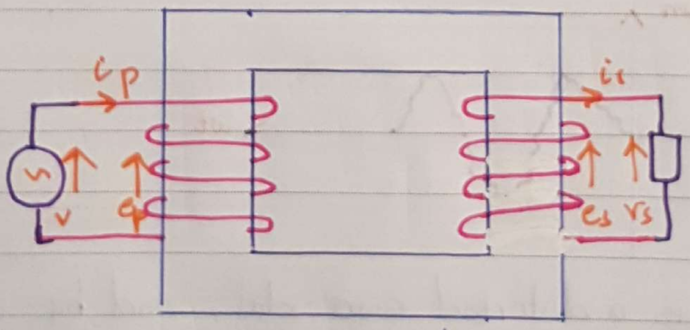
it can be decomposed into Fundamental + harmonic
[i_n is non linear Function of voltage]

core Losses

*If i_m and i_{he} are added the i_{ext} will have the following form:-



*Equivalent Circuit of Real Transformer :- (model)



The Equivalent circuit model should take into account the following:

[I] Copper Losses in the Primary and secondary windings (i.e heating Losses)
The Losses are represented by resistance in the primary and Secondary circuits (i.e Rp and Rs)

No.

$\boxed{2}$ leakage flux in the Primary and secondary windings (ϕ_{lp} and ϕ_{ls})

$$e_{lp} = N_p \frac{d\phi_{lp}}{dt} \qquad e_{ls} = N_s \frac{d\phi_{ls}}{dt}$$

$$= N_p \frac{d}{dt}\left(\frac{N_p i_p}{\mathcal{R}}\right) \qquad = N_s \frac{d}{dt}\left(\frac{N_s i_s}{R}\right)$$

$$\overset{L_p \leftarrow \text{self inductance}}{=} \frac{N_p^2}{\mathcal{R}} \frac{di_p}{dt} \qquad = \overset{L_s}{\frac{N_s^2}{R}} \frac{di_s}{dt}$$

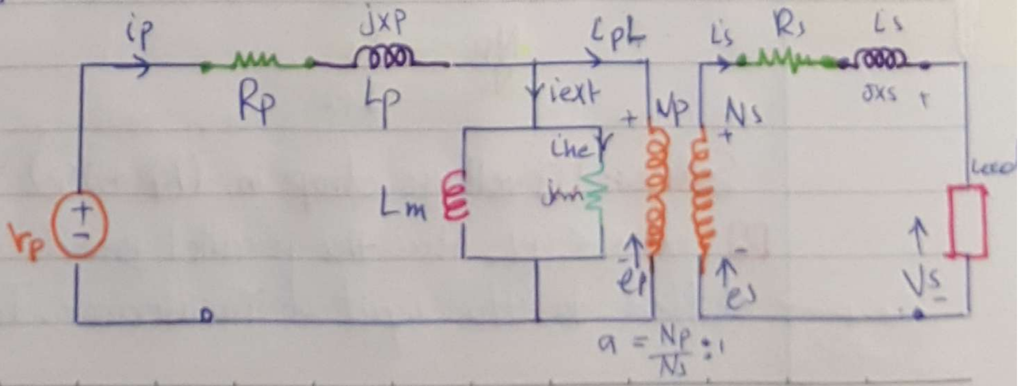
** $\boxed{\text{leakage flux}}$ in the Primary and secondary windings (ϕ_{lp} and ϕ_{ls}) are represented by self inductance in the Primary and secondary circuits (L_p and L_s)

$\boxed{3}$ iext : 1. im   2. ih+e

Represented by a reactance X_m / L_m
generate ϕ
Because im lags applied voltage by 90°

↳ this Represented by a resistance R_c

* Consequently the equivalent circuit can be deduced as follows:



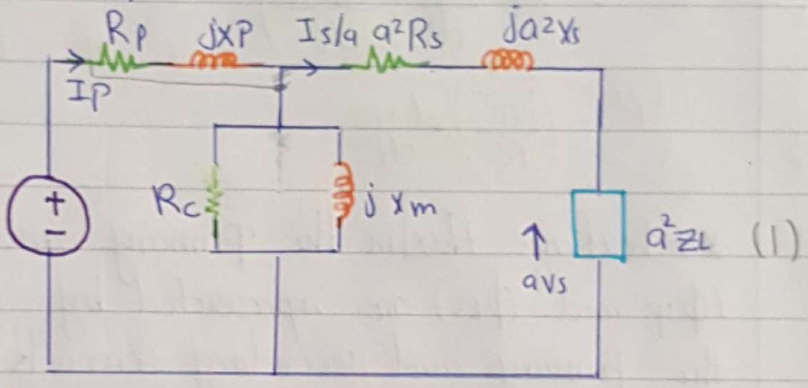
No.

i_p ≡ Total Primary Current

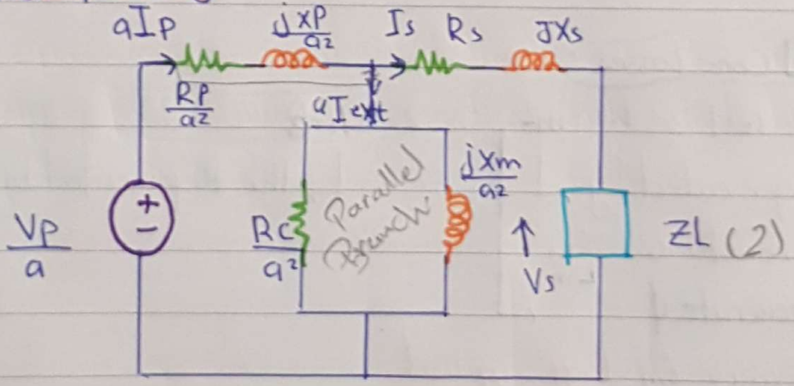
i_{ext} ≡ No-load Component of ip

i_{pL} ≡ Load component of ip

Reflect s → P



Reflect P → s



←

exact equation circuit of Transformer

its equivalent

$$\therefore a I_{pL} = I_s$$

$$a = \frac{I_s}{I_{pL}}$$

$$\frac{N_p}{N_s} = \frac{I_s}{I_{pL}}$$

Since:- $\boxed{1}$ voltage drop in $(R_p + jX_p)$ is very small and $\boxed{2}$ $i_{ext} << i_{pL}$ then the parallel Branch of $(R_C \parallel X_m)$ can shifted to the input of Tranformer, To Produce an approximate

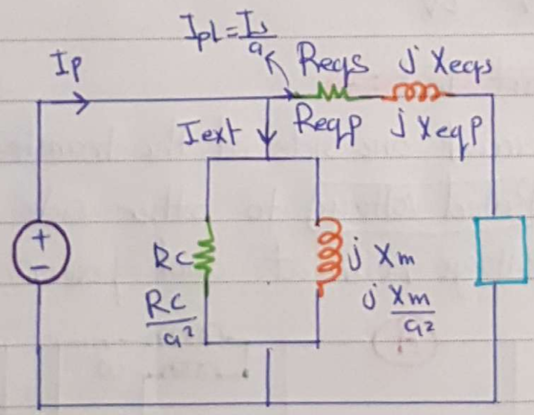
Scanned by CamScanner

equation circuit.

- $R_{eqp} = R_p + a^2 R_s$
  $X_{eqp} = X_p + a^2 X_s$

- $R_{eqs} = \frac{R_p}{a^2} + R_s$
  $X_{eqs} = \frac{X_p}{a^2} + X_s$

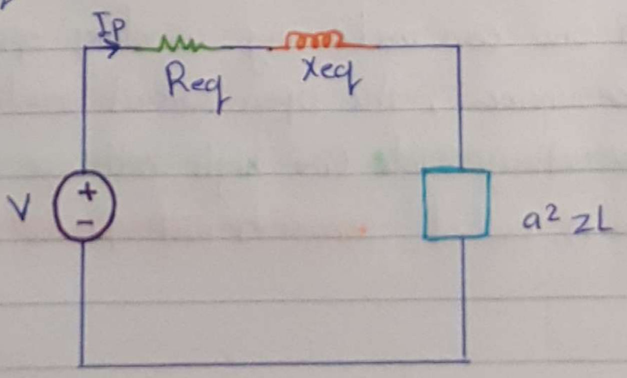
X_{eq}, R_{eq} = Called the equivalent Series Resistance and Reactance of the transformer.



Since: $I_{ext} << I_p$

Then Parallel branch from current voltage relationship can be neglected.

∴ equivalent circuit will be :-

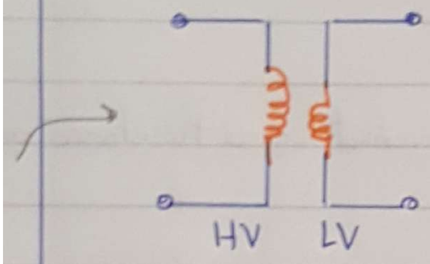


No.

Measurement of Transformer Paramaters (Rc, Xm, Req and Xeq)

why ?! These parameter are used in the Performance analysis of Transformer as will be shown later

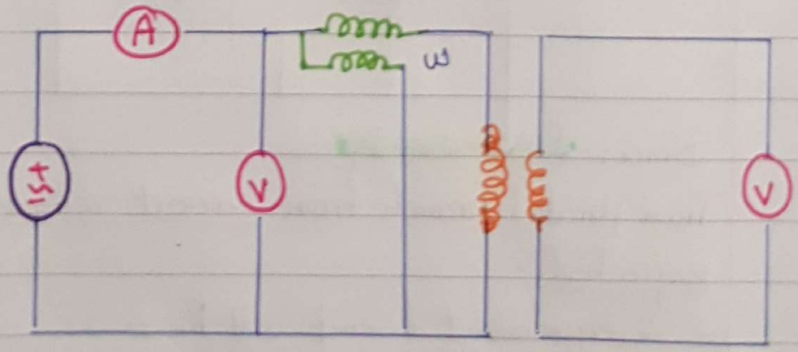
How ?! Procedure By performing open circuit and short circuit as follows.



high and low voltage

Open Circuit Test :-

1. open circuit one side of the transformer (i.e LV or HV)
2. Apply Rated voltage to other side and take the readings of input voltage, current and power.



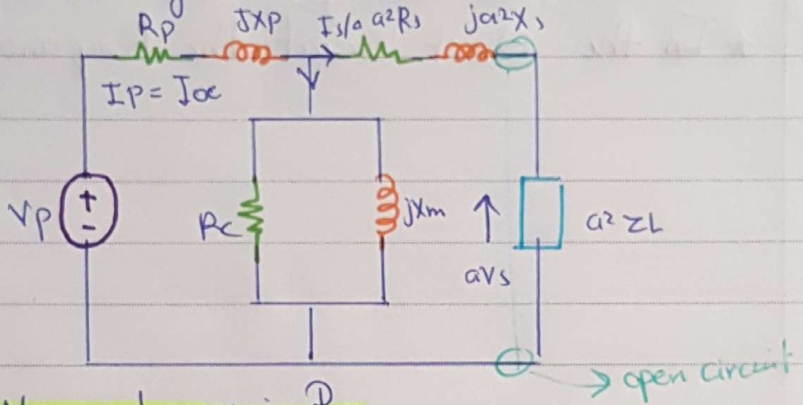
Although one can make any winding open circuit, but for easy measurement, the open circuit readings are taken on the low voltage side (i.e high voltage is open circuit)

↗ This is called conventional Procedure.

By using the equivalent circuit :-

Since $\boxed{I_{oc}}$ is very small, then drop in R_p and X_p is very small

∴ V_{oc} = Voltage accross $(R_m \,//\, X_m)$



∴ W_{oc} = Losses in R_c

∴ In the open circuit test one can determine $\underline{\underline{X_m}}$ and $\underline{\underline{R_c}}$ As follows:-

$$W_{oc} = I_{oc}^2 R_c$$

$$\therefore R_c = \frac{W_{oc}}{I_{oc}^2} = \frac{W.\ Reading}{(A.\ Reading)^2}$$

$$Y = \frac{1}{R_c} + \frac{1}{jX_m}$$

But $|Y| = \dfrac{I_{oc}}{V_{oc}} = \sqrt{\left(\dfrac{1}{R_c}\right)^2 + \left(\dfrac{1}{X_m}\right)^2}$

$$\therefore \frac{1}{X_m} = \sqrt{\left(\frac{1}{R_c}\right)^2 + \frac{1}{Y}}^{\,2} = \sqrt{\left|\frac{1}{Y}\right|^2 - \left(\frac{1}{R_c}\right)^2}$$

Short Circuit Test :

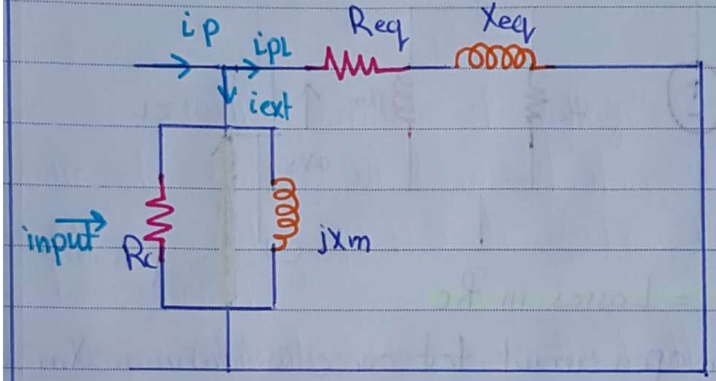
1. short circuit one side of the transformer
2. Apply $\boxed{\text{Rated Current}}$ to the other side.

Since:- High voltage has a lower rated current

Then as the convention, the readings are taken on the high voltage side.

Then take the readings of V, A and W.

* In the short circuit the Req and Xeq are measured.



Since $ipL \gg iext$

$ip = ipL$

$\therefore P_{sc} = I_{sc}^2 \, Req$

$\therefore Req = \frac{P_{sc}}{I_{sc}^2}$

$\therefore Xeq = \sqrt{|Zeq|^2 - Req^2}$

$Zeq = Req + jXeq$

$|Zeq| = \sqrt{Req^2 + Xeq^2}$

$|Zeq| = \frac{Vsc}{Isc}$

* Example:- 1380 VA, (230/115) V transformer has been tested to determine it's equivalent circuit. The results were as follows:-

O/C	S/C
V = 230 V	V = 13.2 V
I = 0.45 A	I = 6 A
P = 30 W	P = 20.1 W

No.

* Find the equivalent circuit refersed to Low volt side:-

Since open circuit Rated voltage applied.

Then in this example measurement were taken on the high voltage side

$P_{oc} = \frac{V_{oc}^2}{R_c}$ ∴ $R_c = \frac{V_{oc}^2}{P_{oc}} = \frac{(230)^2}{30} = 1763.3\ \Omega$

$|Y| = \frac{I_{oc}}{V_{oc}} = \frac{0.45}{230} = 0.001957$

$|Y| = \sqrt{\left(\frac{1}{R_c}\right)^2 + \left(\frac{1}{X}\right)^2}$ By substitution $X_m = 533.9\Omega$

Short Circuit Test Rated current is applied

$I_{LV} = \frac{1380}{115} = 12A$

$I_{HV} = \frac{1380}{230} = 6A$

∴ Readings are taken on the high voltage side

$P_{sc} = I_{sc}^2 R_{eq}$

$\therefore R_{eq} = \frac{20.1}{6^2}$

$= 0.558$

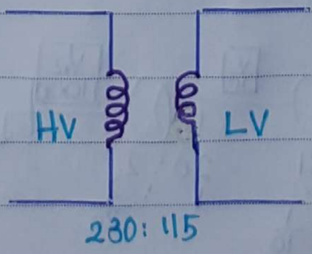
$|Z_{eq}| = V_{sc}/I_{sc}$

$= 13.2/6$

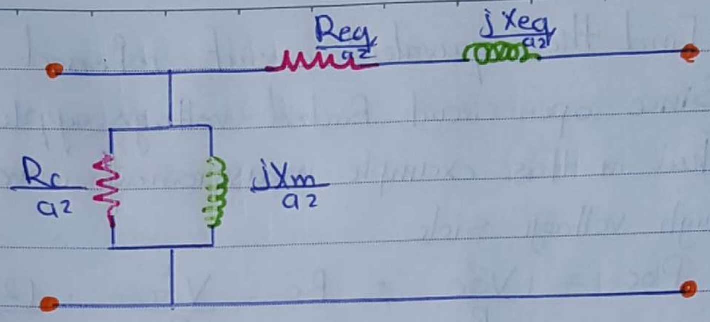
$= 2.2$

$\therefore Z_{eq} = \sqrt{|Z_{eq}|^2 - |R_{eq}|^2} = 2.128\ \Omega$

Refer all parameters to LV side



$a = \frac{230}{115} : 1 = 2$



Per-Unit System (PU)

What?! The variables of voltage, Current, Power and Impedance are not expressed in terms of it's SI Unit. However they are expressed with respect to certain Refrence values as will be shown

Why?! Advantages → see the textbook or any other reference.

$$\text{Quantity in PU} \triangleq \frac{\text{Actual value}}{\text{Base or Referance value}}$$

→ could be real or complex

↳ This is always a real value

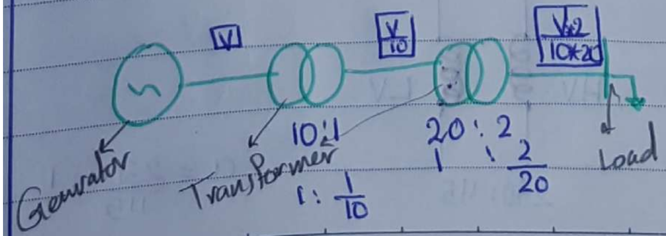
∴ PU value and actual value have the same Phase angle

Since there are 4 variables V, I, S, Z, any two are selected to be the Base values (usually the Base values of V and S are selected V_b and S_b)

And the base values of the other two are calculated from the selected ones as follows:-

$$S_b = V_b I_b \quad \therefore \quad I_b = \frac{S_b}{I_b}$$

$$Z_b = \frac{V_b}{I_b} = \frac{V_b}{S_b/V_b} = \frac{V_b^2}{S_b}$$



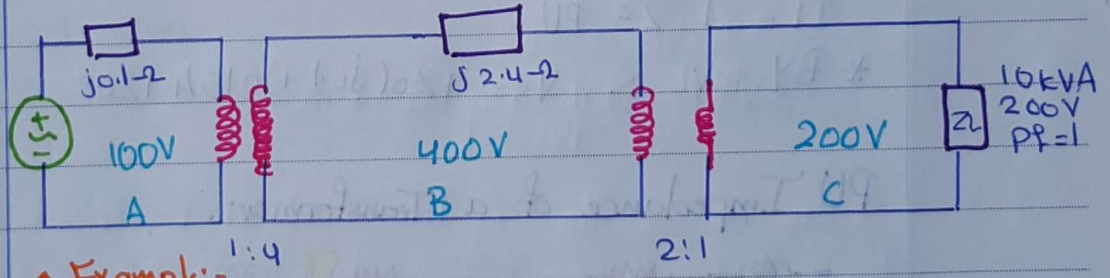
No.

*Since S for the high voltage and Low voltage does not change, the S_b remain the same for the system. However the Base value of voltage change accourding to transformer Ratio.

Procedure

1. Select Sb, Vb for the system
2. Identify Vb by using transformer ratio for each section
3. Find Zb for each section
4. evaluate PU system

* As a convention usually the Rating of the largest component are selected to be base value



→ Example:-

Find the PU equivalent system

let $S_b = 10$ kVA

let $V_b = 200$ V

$Z_b \triangleq V_b^2/S_b$

$$Z_{bA} = \frac{(100)^2}{10 * 10^3} = 1\ \Omega$$

$$Z_{bB} = \frac{(400)^2}{10 * 10^3} = 16\ \Omega$$

$$Z_{bC} = \frac{(200)^2}{10 * 10^3} = 4\ \Omega$$

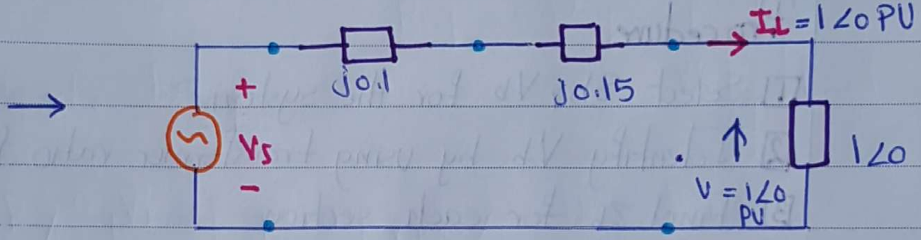
No.

$Z_{PU A} = \frac{j0.1}{1} = j0.1$

$Z_{PU B} = \frac{j2.4}{16} = j0.15$

$Z_{PU C} = \frac{Z_L}{4} = \frac{4}{4} = 1$ $\qquad |Z_L| = \frac{V^2}{S} = \frac{(200)^2}{10*10^3} = 4$

$\therefore Z_L = 1 \angle 0$ PU



$Z_L \rightarrow |I_L| = \frac{10*10^3}{200} = \frac{10^4}{200}$ A

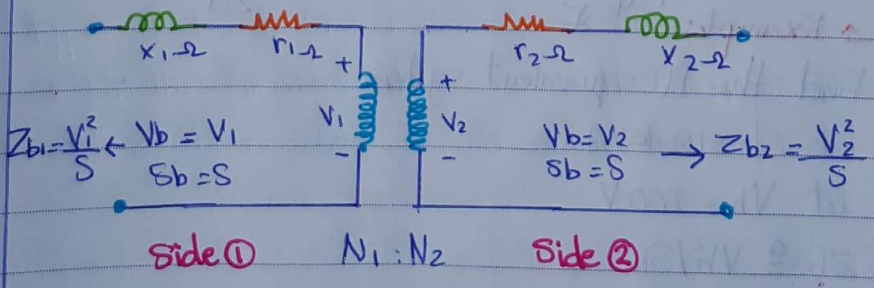
$I_b = \frac{S_b}{V_b} = \frac{10^4}{200}$ A

$|I_{L,PU}| = \frac{I_L}{I_b} = 1$

$I_L = 1 \angle 0°$ PU

* BY KVL → $V_s = 1\angle 0 (j0.1 + j0.15) + 1\angle 0°$

PU Impedance of a Transformer.



$a = \frac{N_1}{N_2} : 1$

Reflect Side 1 → Side 2

$\therefore R_{eq\,2} = r_2 + \frac{r_1}{a2}$

$X_{eq\,2} = X_2 + \frac{X_1}{a^2}$

$$R_{eq2}(PU) = \frac{R_{eq2}}{Z_{b2}}$$

$$R_{eq2}(PU) = \frac{S}{V_2^2}\left(r_2 + \frac{r_1}{a^2}\right)$$

But $\frac{V_1}{V_2} = \frac{N_1}{N_2} = a$

$$\therefore V_2 = \frac{V_1}{a}$$

$$\therefore R_{eq}(P) = \frac{Sa^2}{V_1^2}\left(r_2 + \frac{r_1}{a^2}\right)$$

$$R_{eq2}(PU) = \frac{S^2}{V_1^2}(r_2 a^2 + r_1) \cdots 1$$

→ Similarly

$$X_{eq2}(PU) = \frac{S}{V_1^2}(a^2 X_2 + X_1) \cdots 2$$

Reflect Side 2 → Side 1

$$\therefore R_{eq1} = r_1 + a^2 r_2$$

$$X_{eq1} = X_1 + a^2 X_2$$

$$\therefore R_{eq1}(PU) = \frac{R_{eq1}}{Z_{b1}} = \frac{S}{V_1^2}(r_1 + a^2 r_2) \cdots 3$$

→ Similarly

$$X_{eq1}(PU) = \frac{S}{V_1^2}(X_1 + a^2 X_2) \cdots 4$$

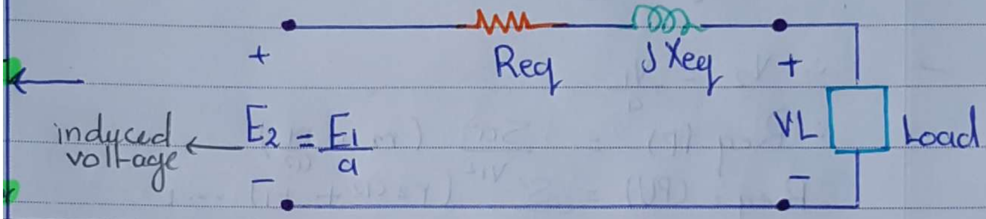
Hence it can be observed that 1 = 3 2 = 4

∴ PU Req and Xeq are the same for high value reflection or low value reflection.

No.

Voltage Regulation VR

* This measures the difference between input and output voltage at a given Load. and defined as follows by using the equivalent circuit



$VR\% \triangleq \frac{|V_{L,NL}| - |V_{L,FL}|}{|V_{L,FL}|} * 100\%$ at FL

NL = No Load

FL = Full Load

$\frac{E_1}{E_2} = \frac{N_1}{N_2} = a$

$E_2 = I_L\underbrace{(R_{eq} + jX_{eq})}_{Z_{eq}} + V_L$

$\therefore E_2 = \frac{E_1}{a}$

$\therefore V_{L,NL} = E_2 = \frac{E_1}{a}$

$\therefore VR\% = \frac{|E_1| - |V_{L,FL}|}{|V_{L,FL}|} * 100\%$

* One can see the significance of VR by considering varies types of Loads (i.e Inductive, Resistive and capacitive)

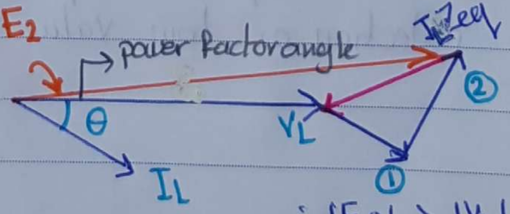
* In the Analysis, it's assumed that |VL| and |IL| are fixed.

* Here phasor diagram will be Used.

Inductive

1: $I_L R_{eq}$

2: $I_L X_{eq}$

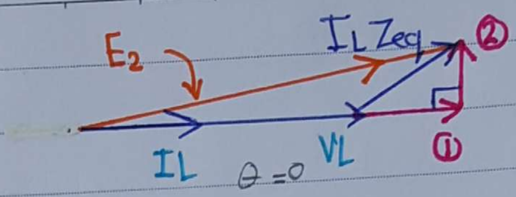


$\therefore |E_2| > |V_L|$

$\therefore$ VR is +ve

as a magnitude ✓

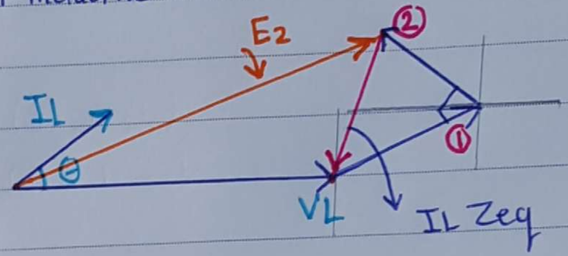
Resistive



VR is +ve

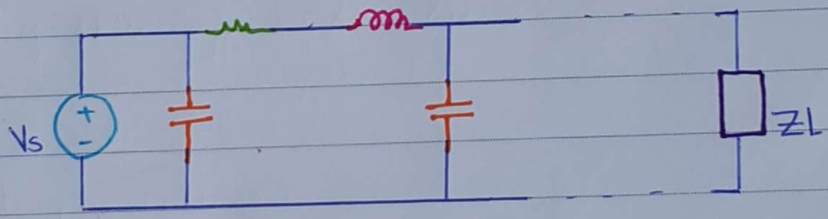
$|E_2|$ Resistive $<$ $|E_2|$ inductive

Capacitive



$|E_2| < |V_L|$

$\therefore$ VR is $-$ve



π- equivalent Circuit

Efficiency η

$$\eta \triangleq \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{Losses}}$$

output $= |V_L|\,|I_L| \cos\theta$ watt

$\theta \equiv$ PF angle of the load

Losses $\equiv$ electrical or Copper losses + Core losses

$\Downarrow$ $|I_L|^2 R_{eq}$

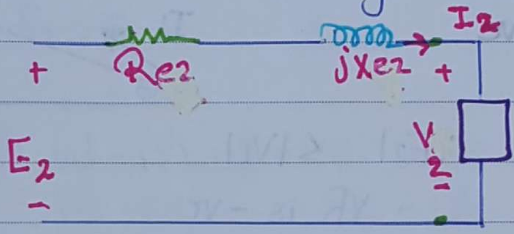
Core losses → P_e, P_h → are evaluated from o/c test

No.

Example: A 10 KVA (4800/240) V transformer was tested by o/c and s/c Test, and the results were as follows:-

o/c :	240 V	1.5 A	60 W	← LV (Readings)
S/C :	180 V	2.083 A	180 W	← HV (Readings)

$\boxed{1}$ Evaluate VR of the stepdown transformer at unity PF with full-Load. ∴ secondary is LV.



$I_2 = \frac{10*10^3}{240} \angle 0° A$

$V_2 = 240 \angle 0° V$

نحط الزاوية اللي بنحكي عليها

لكن الConvention = صفر

From the given data it can be found

$R_{e2} = 0.104 \Omega$

$X_{e2} = 0.19 \Omega$

$VR = \frac{|E_2| - |V_2|}{|V_2|}$

$E_2 = I_2(R_{e2} + jX_{e2}) + V_2$

$= (244.167 + j7.92)$ V

$\therefore |E_2| = 244.3$V

$\therefore = \frac{244.3 - 240}{240} * 100\%$

$= 1.79\%$

$\boxed{2}$ Evaluate η of the step down transformer at full-Load at 0.9PF Lagging.

$\eta = \frac{V_2 I_2 \cos\theta}{V_2 I_2 \cos\theta + \text{electrical Losses} + \text{core Losses}}$

$V_2 = 240$V

$I_2 = 41.67$ A

$\cos\theta = 0.9$

electrical losses : $I_2^2 R_{e2}$

↳ OR from short circuit data = 180 w

Core Losses → from open Circuit Test.

$$y = \frac{[240 * 41.67 * 09]}{(\quad) + 180 + 60} * 100\% = 97.4\%$$

١٢٠/١١٠   ١٢٠/١٠

No.

* Autotransformer :-

* this type is Used: when the required change in voltage is very small. example 110 → 120
13.2 → 13.8 KV

* This is introduced by considering the previous Conventional (isolation) transformer as follows:-

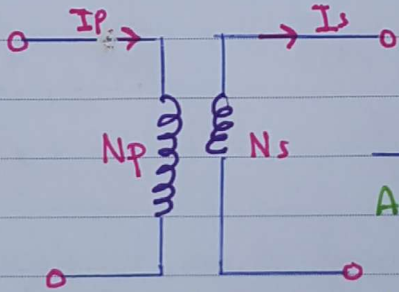


Fig.1

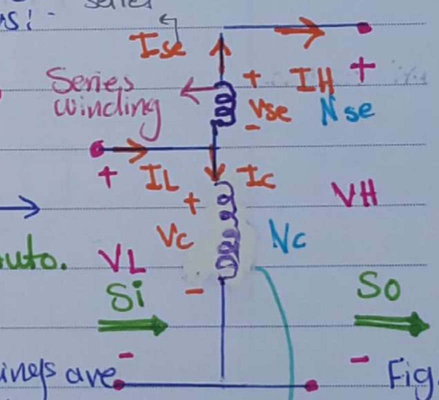


Fig.2

I_{se} ≡ Current in series winding.

I_c ≡ Current in common winding.

V_c ≡ Voltage in common winding.

V_{se} ≡ Voltage in series winding.

Two windings are connected electrically

common winding: because it is common between HV and LV sides

* Voltage and Current in the windings are related by turns Ratio as follows:

$$\frac{V_c}{V_{se}} = \frac{N_c}{N_{se}} \quad \text{....1}$$

$$I_c N_c = I_{se} N_{se}$$

$$\frac{I_c}{I_{se}} = \frac{N_{se}}{N_c} \quad \text{....2}$$

* Voltage and Current Relationship :-

objective: To find $\frac{V_H}{V_L}$ and $\frac{I_H}{I_L}$

Procedure :-

From fig.2

$V_H = V_c + V_{se}$ ... 3

$= V_c + V_c \frac{N_{se}}{N_c}$ by sub. ① in ③

$= V_c\left(\frac{N_c + N_{se}}{N_c}\right) = V_L\left(\frac{N_c + N_{se}}{N_c}\right)$

$\therefore \frac{V_H}{V_L} = \frac{N_c + N_{se}}{N_c}$ ... ⑤

From Fig.2

$I_L = I_c + I_{se}$ ... 4

Subtitute ② in ④

$I_L = I_{se}\frac{N_{se}}{N_c} + I_{se}$

$= \overset{I_H}{I_{se}}\left(\frac{N_c + N_{se}}{N_c}\right)$

$\therefore \frac{I_L}{I_H} = \frac{N_{se} + N_c}{N_c}$ ... ⑥

* Apparent power Relationship :-

$S_i \equiv$ input apparant power

$S_o \equiv$ output appearant power

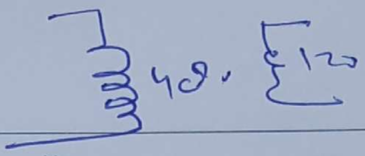
$S_i = V_L I_L$ ... ⑦

$S_o = V_H I_H$ ... ⑧

Find relation between S_i and S_o

Subtitute ⑤ and ⑥ in ⑦

$S_i = V_H\left(\frac{\cancel{N_c}}{\cancel{N_c + N_{se}}}\right) * \left(\frac{\cancel{N_c + N_{se}}}{\cancel{N_c}}\right) * I_H$



No.

$S_i = V_H I_H$ $\qquad\qquad$ $I_L = I_c + I_{se}$

$\therefore S_i = S_o = S_{io}$ $\qquad\qquad$ $\therefore I_c = I_L - I_{se}$

* Apparent Power advantage of Autotransformer :-

Apparent power of the winding, $S_w = V_c I_c$ OR $S_w = V_{se} I_{se}$

$$\therefore S_w = V_L(\overbrace{I_L - I_{se}}^{I_c})$$

$$= V_L(I_L - I_H)$$

$$= V_L\left(I_L - I_L \frac{N_c}{N_{se}+N_c}\right) = \underbrace{V_L I_L}_{S_{io}}\left(\frac{N_{se}}{N_c+N_{se}}\right)$$

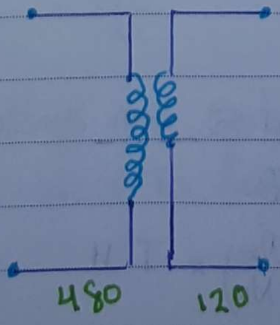
$$\frac{S_{io}}{S_w} = \frac{N_c+N_{se}}{N_{se}}$$

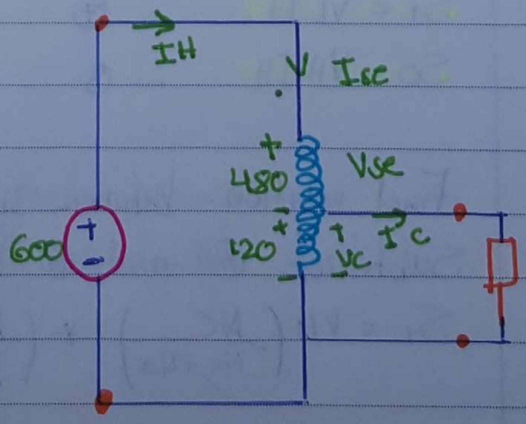
$$\therefore S_{io} > S_w$$

***Example:** A 5000VA, (480/120)V conventional transformer is to be Used to supply Power from 600V source to 120V Load. Consider that the transformer to be ideal and all insulation can handel 600V

a) Sketch the transformer connection that will do the job.

b) Find the kVA Rating of the Produced Autotransformer.





$$\frac{S_{io}}{S_w} = \frac{N_c + N_{se}}{N_{se}}$$

$N_c \propto V_c = 120$

$N_{se} \propto V_{se} = 480$

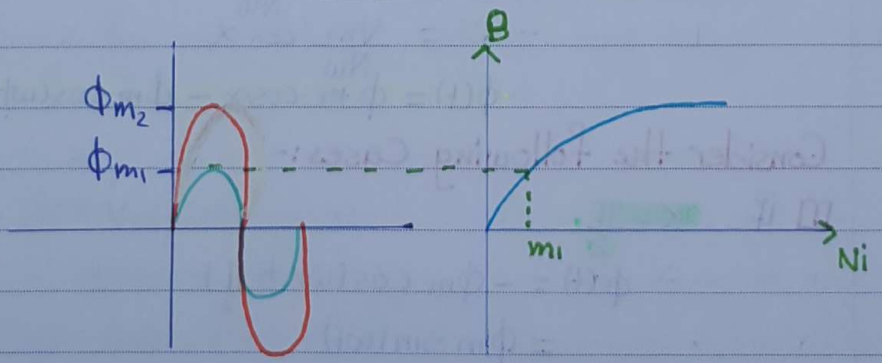
$$\therefore S_{io} = S_w\left(\frac{N_c + N_{se}}{N_{se}}\right)$$

$$= 5000\left(\frac{120 + 480}{480}\right) = 6250 \text{ VA}$$

c Find the max. primary and secondary current which may flow.

$$\therefore I_{p,max} = \frac{6250}{600} = 10.4 \text{ A}$$

$$I_{s,max} = \frac{6250}{120} = 52.1 \text{ A}$$

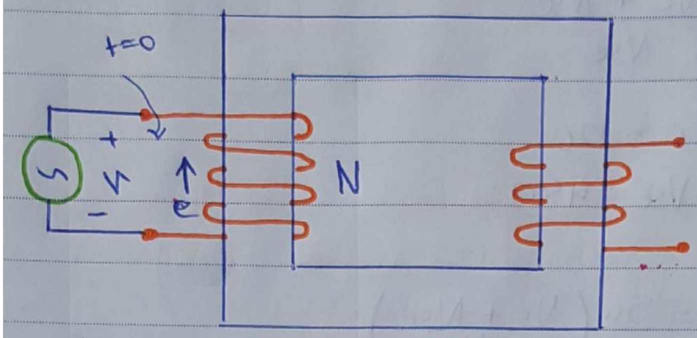


$I_{m2} \gg I_{m1}$

I_{m2} can exceed Rated current of Transformer.

*** Current Inrush**

This is a very high magnetizing current which may flow in the primary of a transformer when it is connected to the supply.



let $v = V_m \sin(\omega t + \alpha)$ $\alpha \equiv$ phase angle of the voltage.

$e \simeq V = V_m \sin(\omega t + \alpha)$

Since $e = N \frac{d\phi}{dt}$

$$\therefore \phi = \frac{1}{N}\int e\,dt = \frac{1}{N}\int V_m \sin(\omega t + \alpha)\,dt$$

$$\phi(t) = \boxed{-\frac{V_m}{N\omega}} \cos(\omega t + \alpha) + C$$

$\rightarrow$ constant of integration

ϕ_m: the Peak value of ϕ

$$\therefore \phi(0) = 0 = -\frac{V_m}{N\omega}\cos\alpha + C$$

$$\therefore C = \frac{V_m}{N\omega}\cos\alpha$$

$$\therefore \phi(t) = \phi_m \cos\alpha - \phi_m \cos(\omega t + \alpha)$$

Consider the following Cases:-

II if $\alpha = \frac{\pi}{2}$

$$\therefore \phi(t) = -\phi_m \cos(\omega t + \frac{\pi}{2})$$
$$= \phi_m \sin(\omega t)$$

$\therefore \phi(t)$ has it's normal Peak of ϕ_m

Since $V = V_m \sin(\omega t + \frac{\pi}{2})$

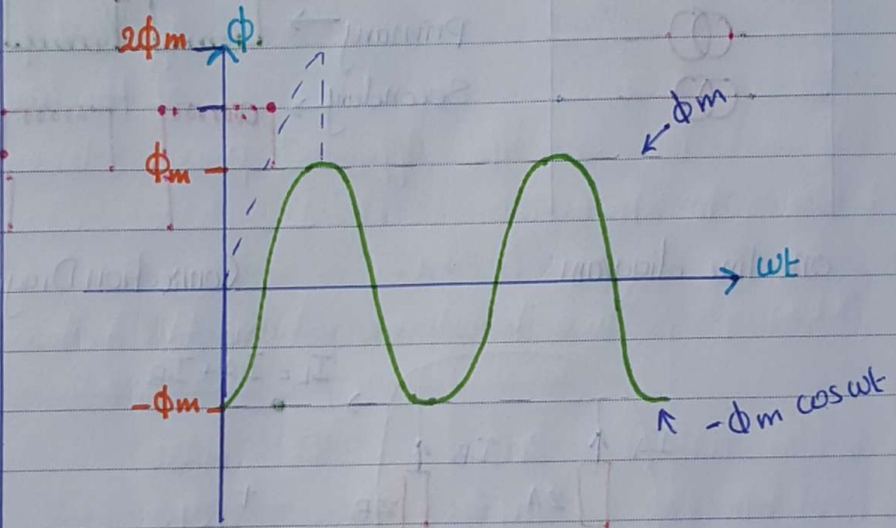
$\therefore$ at $t = 0$

$V = V_m \sin(\frac{\pi}{2}) = V_m$

Hence the voltage is applied at a value = +ve maximum then there is no problem in magnetizing Current.

$\boxed{2}$ $\alpha = 0$

$\therefore \phi(t) = \phi_m - \phi_m \cos(\omega t)$



* In this case flux is Twice the normal flux.

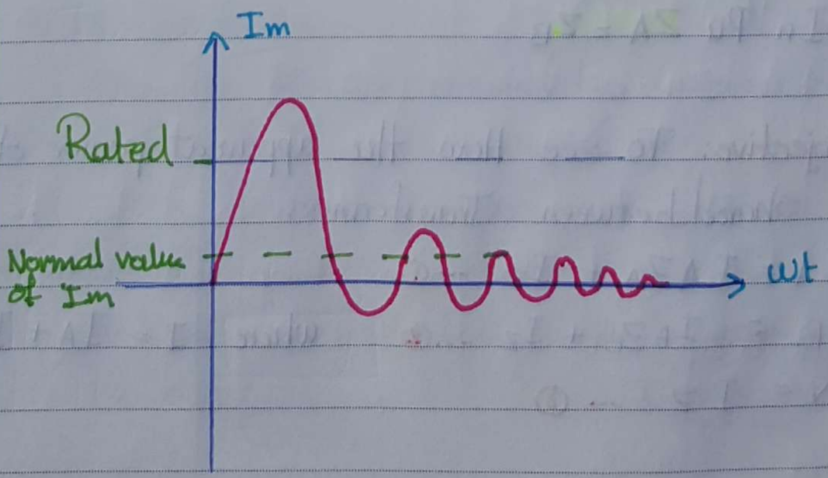
Hence the required I_m will be very large

and $I_m >> I_{rated}$

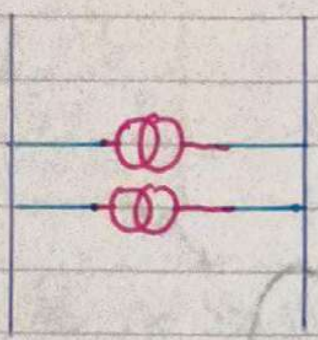
$v = V_m \sin(\omega t + 0°)$

$= V_m \sin \omega t$

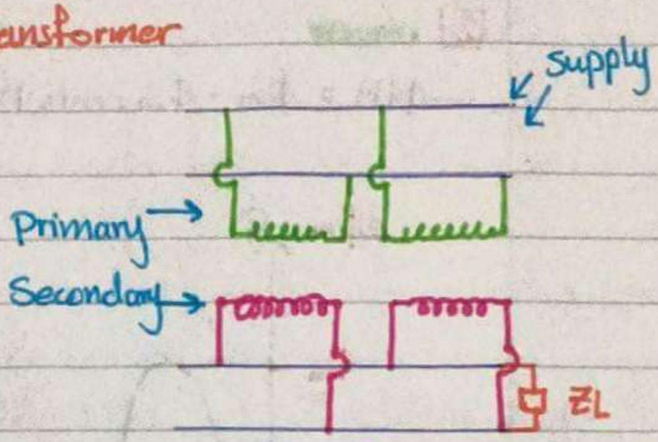
at $t = 0$, $v = 0$



Parallel operation of Transformer



one-line diagram



Connection Diagram

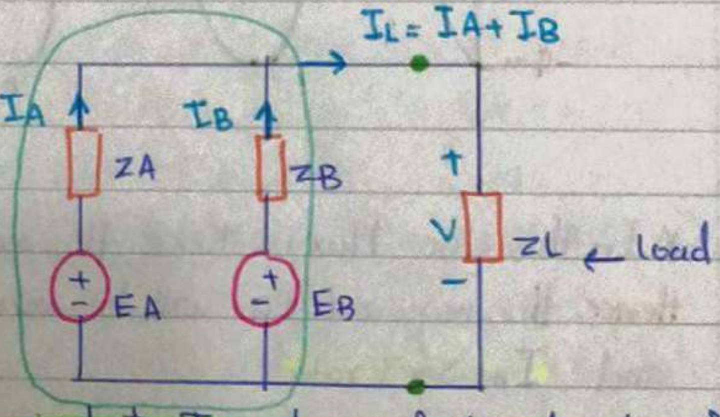


Fig.1

ZA, ZB = equivalent Impedance of the two transformer

* Among the major consider for Parallel operation:-

1. $E_A = E_B$
2. In Pu $Z_A = Z_B$

*Objective: To see How the apparent power of the load is shared between Transformer.

$E_A = I_A Z_A + I Z$ ...①

$E_B = I_B Z_B + I Z$ ...② where $I = I_A + I_B$

$V = I Z$ ...③

By using these equations it can be found that :-

$$I_A = \frac{E_A}{Z_A + Z + \frac{Z Z_A}{Z_B}} + \frac{E_A + E_B}{Z_A + Z_B + \frac{Z_A Z_B}{Z}} \quad \dots ④$$

representing the circulating current in the source

Similar expression for I_B can be found by replacing A By B in ④

$I_B =$ ...⑤

Since $V = IZ = (I_A + I_B) Z$ ...⑥ Then by substituting ④ and ⑤ into ⑥ it can be found that :

$$V = \frac{E_A / Z_A + E_B / Z_B}{1/Z_A + 1/Z_B + 1/Z} \quad \dots ⑦$$

The advantage of ⑦ that it can be modified when more transformers are added in parallel

For example :- if transformers C and D are added. then : ① Add the terms E_C / Z_C and E_D / Z_D to numerator in ⑦

② Add the terms $1/Z_C$ and $1/Z_D$ to Demominator in ⑦

* If $E_A = E_B$

Then By KVL : around the Green Loop in Fig. 1 ←

$$-\cancel{E}_A + I_A Z_A - I_B Z_B + \cancel{E}_B = 0$$

$$\therefore I_A Z_A = I_B Z_B \quad ⑧$$

Since $I = I_A + I_B$ ⑨

∴ By Using ⑧ and ⑨ it can be found :-

$$I_A = I\left(\frac{Z_B}{Z_A + Z_B}\right) \qquad I_B = I\left(\frac{Z_A}{Z_A + Z_B}\right)$$

∴ Apparent Power of (A) $S_A = VI_A = VI\left(\frac{Z_B}{Z_A+Z_B}\right)$ ...⑩

Apparent Power of (B) $S_B = VI_B = VI\left(\frac{Z_A}{Z_A+Z_B}\right)$ ...⑪

Apperant Power of the Load

[10] and [11] Give the Apparent power sharing (distribution) between Transformers.

*Example:- Two Transformers have the Following Data:-

	Transformer A	Transformer B
Rated Current (A)	200	600
E ← No load Voltage (V)	245 = EA	240 = EB
Req (PU)	0.02	0.025
Xeq (PU)	0.05	0.06

*Find the terminal voltage when they are connected in Parallel to supply a load $Z = (0.25 + j0.1)\ \Omega$

$$V = \frac{E_A/Z_A + E_B/Z_B}{1/Z_A + 1/Z_B + \frac{1}{Z}}$$

$\therefore E_A = 245\angle 0^\circ\ V$

$E_B = 240\angle 0^\circ V$

- Note: The Base values which used for Rpu, Xpu are the Rating of the Given Components.

$\therefore Z_{Ab} = \frac{245}{200}\ \Omega$

$Z_{Bb} = \frac{240}{600}\ \Omega$

No.

$$Z_A = (0.02 + j0.05) * \frac{245}{200} \ \Omega$$

$$Z_B = (0.025 + j0.06) * \frac{240}{600} \ \Omega$$

By substitution it can be found → $V = 230 \angle -3^\circ$ V

*Solve this example By using PU concept.

$$Z_{new}(PU) = Z_{old}(PU) * \left(\frac{V_{b,old}}{V_{b,new}}\right)^2 * \left(\frac{S_{b,new}}{S_{b,old}}\right)$$

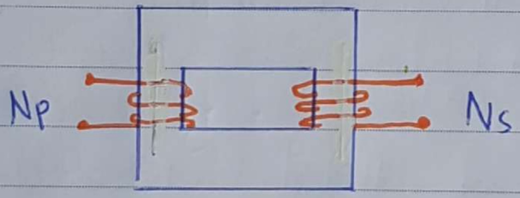
3-Phase Transformer:-

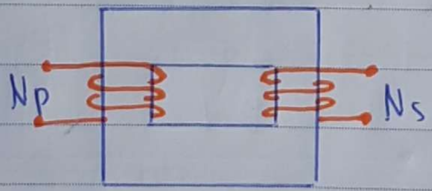
Why?? because electrical energy is **Generated**, **transmitted** and **distributed** as a 3-phase.

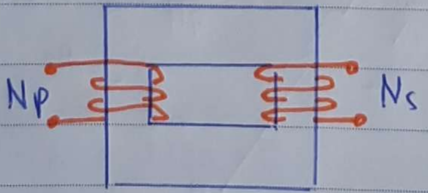
Construction:

It could be one of two types:-

1 Using 3 Single phase (1-ph) transformers, so called 3-phase transformer Bank





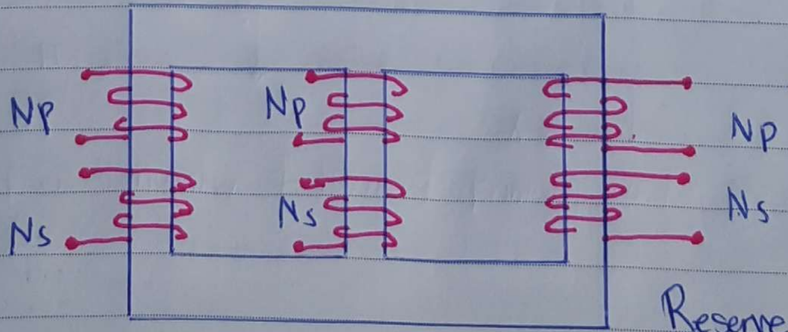


- more reliable:

Reserve = 1-ph terms

* IF one Transformer fail. still 3 phase can be supplied by the other 2 transformers So called V-connection But with a reduced Apparent power Capability

2 Single core with 3 pairs of windings:-



- lighter
- cheaper
- more efficient

Reserve = 3-ph - Terms

No.

Connections:-

*Since a 3phase winding can be connected as Y or Δ then there are 4-types of 3-phase transformer from connection point to view:-

1. Y - Y
2. Y - Δ
3. Δ - Y
4. Δ - Δ

*Objective

1. Ratio of voltage magnitude
2. phase shift in Transformer
3. Per phase equivalent circuit.

Convention

1. The phases of the HV side is represented by capital letters A B C
   The phase of the LV side is represented by small letters a b c

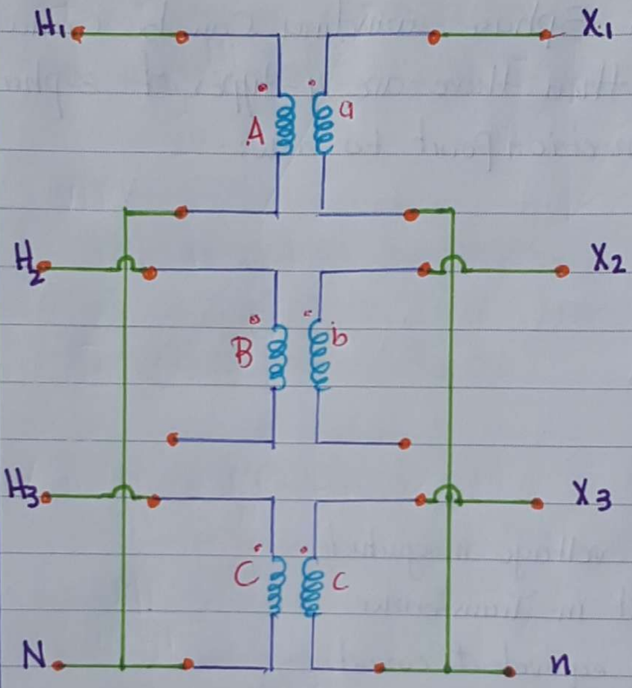
* See how various standards represent the HV and LV windings

2. Terminals of HV side are represented by letters H_1, H_2, H_3
   Terminals of LV side are represented by letters X_1, X_2, X_3

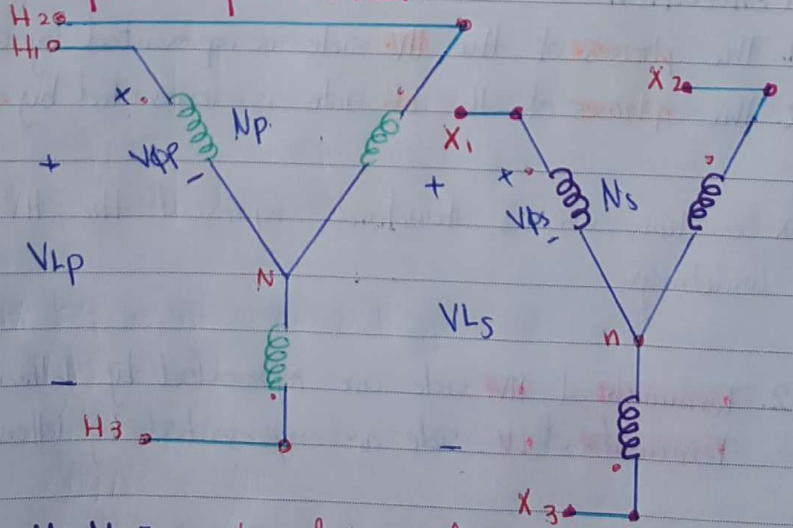
3. In Graphical representation the windings of each phase are drawn parallel to each other.

No.

Y-Y connection



Graphical representation:-



Np, Ns ≡ number of turns of Primary and secondary.

No.

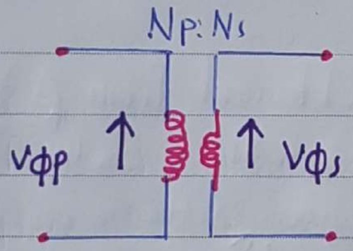
Objective:

Ratio of voltage magnitude.

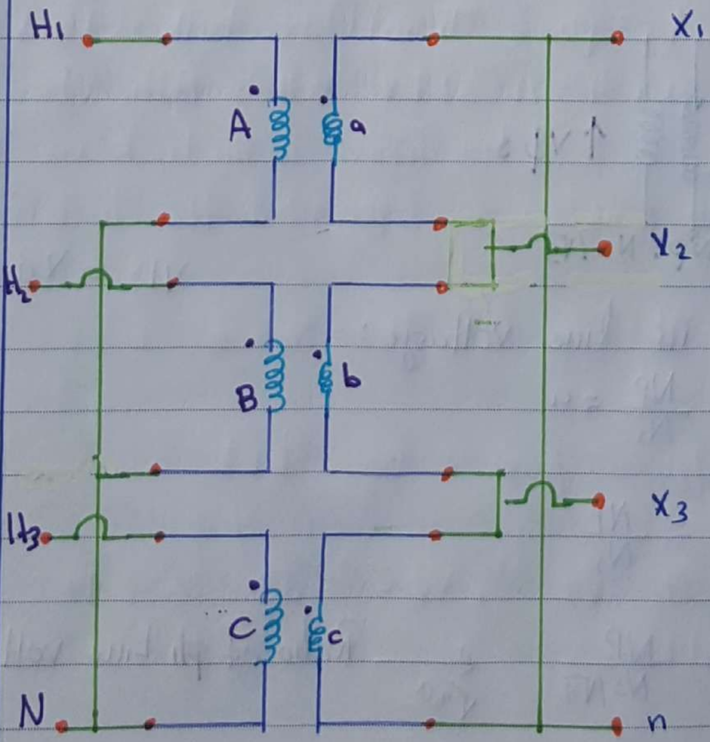
$$\frac{V\phi P}{V\phi s} = \frac{NP}{Ns} = a$$

$$\frac{VLP/\cancel{\sqrt{3}}}{VLs/\cancel{\sqrt{3}}} = \frac{NP}{Ns} = a$$

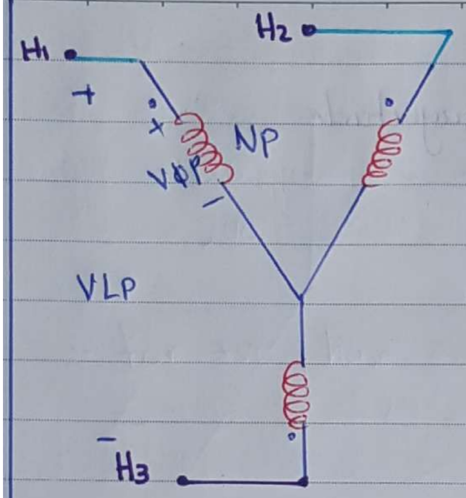
$$\therefore \frac{VLP}{VLS} = a$$

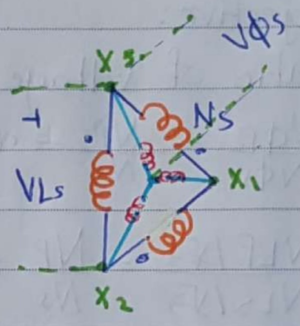


Y – Δ Connection



No.





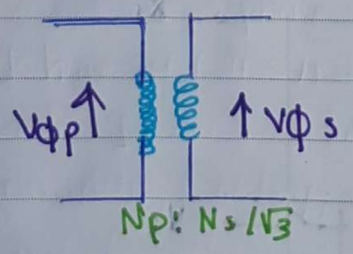
$$\frac{V\phi P}{V_{Ls}} = \frac{N_P}{N_s} = a$$

$$\frac{V\phi P}{\sqrt{3}\,V\phi_s} = \frac{N_P}{N_s}$$

$\sqrt{3}a$

$$\frac{V\phi P}{V\phi_s} = \frac{N_P}{N_s/\sqrt{3}}$$ ← Ratio of phase voltage.

→ effictive turns Ratio



VLS = Vφ.s ← Δ

* Express in line voltage :-

$$\frac{V\phi P}{V_{Ls}} = \frac{N_P}{N_s} = a$$

$$\frac{V_{Lp}/\sqrt{3}}{V_{Ls}} = \frac{N_P}{N_s}$$

$$\frac{V_{LP}}{V_{Ls}} = \frac{N_P}{N_s/\sqrt{3}}$$ ← Ratio of Line voltage

$\sqrt{3}a$

No.

By similar Procedure it can be shown for Δ-Y and Δ-Δ :-

The effective turns Ratio = line voltage Ratio.

*Phase shift in 3-phase Transformer :-

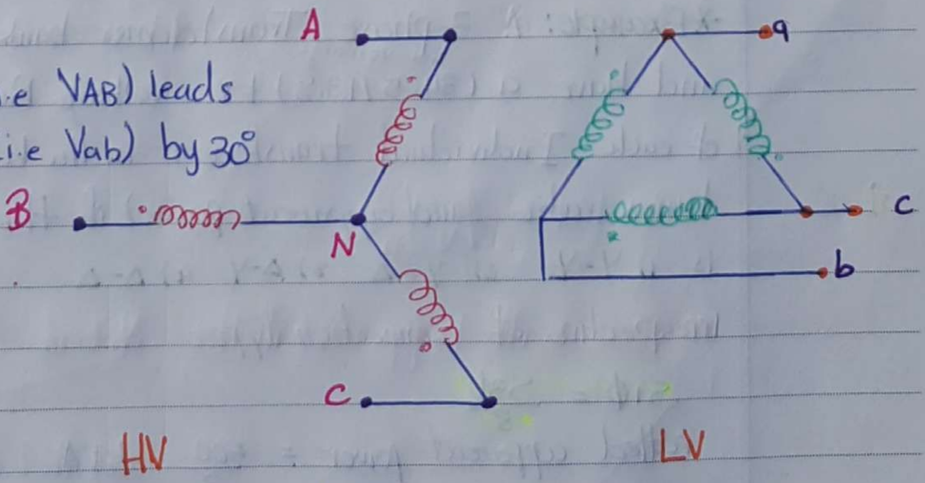
* Y-Y and Δ-Δ do not introduce phase shift between corresponding voltage. (e.g. V_{AB} and V_{ab} are in phase)

*However Δ-Y and Y-Δ create a phase shift of 30°

→ As a convention the terminal markings (H_1, H_2, H_3 and X_1, X_2, X_3) are made in such a way that HV Leads LV By 30° in the case of +ve phase sequence.

*To Illustrate consider the following Y-Δ connection.

∴ HV (i.e V_{AB}) leads LV (i.e V_{ab}) by 30°



*V_{AN} is in Phase with V_{ab}

if $\angle V_{AN} = 0°$, then $\angle V_{ab} = 0°$

In the positive Phase sequence

No.

In the +ve Phase Sequence

$\angle V_{an} = 0^\circ$ $\quad \angle V_{ab} = 0 + 30^\circ$

$\angle V_{bn} = 120^\circ$ $\quad \angle V_{bc} = 120^\circ + 30^\circ$

$\angle V_{cn} = 240^\circ$ $\quad \angle V_{ca} = 240 + 30^\circ$

∴ For +ve phase sequence

$\angle V_{AB} = \angle V_{AN} + 30^\circ$

$\angle V_{AB} = 0^\circ + 30^\circ = 30^\circ$

* The same Result can be obtained from Δ-Y

$\overset{\uparrow}{HV} \quad \overset{\uparrow}{LV}$

Note the → letters on the terminal should be in such away that V_{AB} leads V_{ab} by 30°

*Example: A 3-phase Transformer bank is to handle 500 kVA and have a (34.5/13.8) kV Voltage Ratio (line voltage). Find the rating of each Individual transformer in the bank (i.e its HV, LV, turns ratio and apparent power) if the bank is connected is 1) Y-Y 2) Y-Δ 3) Δ-Y 4) Δ-Δ

irrespective of connection type:-

$S_{1\phi} = \frac{S_{3\phi}}{3}$

ratted apparent power $= \frac{500}{3}$ kVA

1) Y-Y

$V_\phi HV = 34.5/\sqrt{3}$ $\quad , V_\phi LV = \frac{13.8}{\sqrt{3}} = 7.97$

$= 19.93$

$$a = \frac{V\phi\, HV}{V\phi\, LV} = 2.5$$

2) Y − Δ

$V\phi HV = 19.92$

$V\phi LV = 13.8 = V_L LV$

$a = 1.44$

3) Δ − Y

$V\phi HV = 34.5$

$V\phi LV = 13.8 / \sqrt{3}$

$a = 4.33$

4) Δ − Δ

$V\phi HV = 34.5$

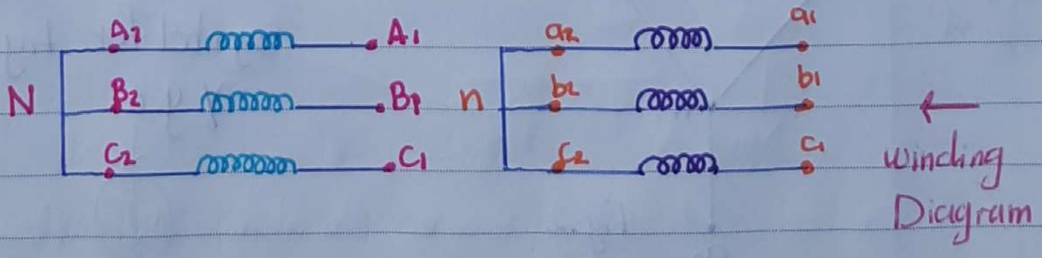
$V\phi LV = 13.8$

$a = 2.5$

Voltage Groups :-

Although the 3-phase winding, As seen before, can be connected as Δ or Y

Also the connection of these windings can be reversed as illustrated below:-



∴ Corresponding phasor Diagram:-

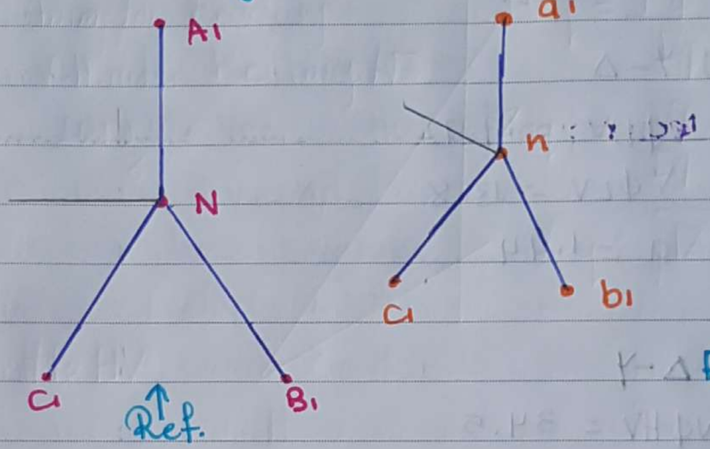
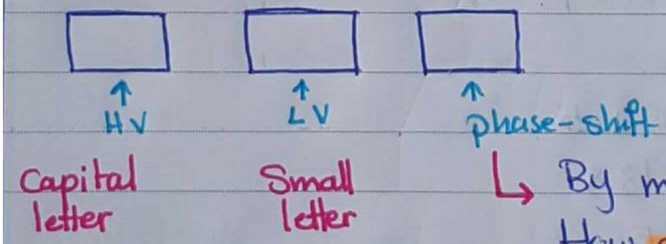


Fig.1

* Vector Diagram Symbol:-



↳ By means at the clock hours:-

How arm at 12 → HV

minute arm represent LV

Symbol = $Y_N y_n 0$ ← from Fig.1

* What will happen if the Neutral of LV is reversed?

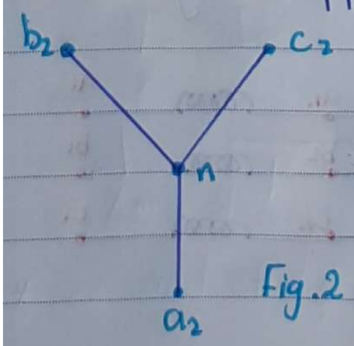
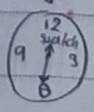


Fig.2

Symbol for Fig. 2 =
Y y 6
↓



Conclusion: If all possible connections of the LV windings are considered then the phase shift can be 0°, 180°, −30°, +30°. summerized in the following table

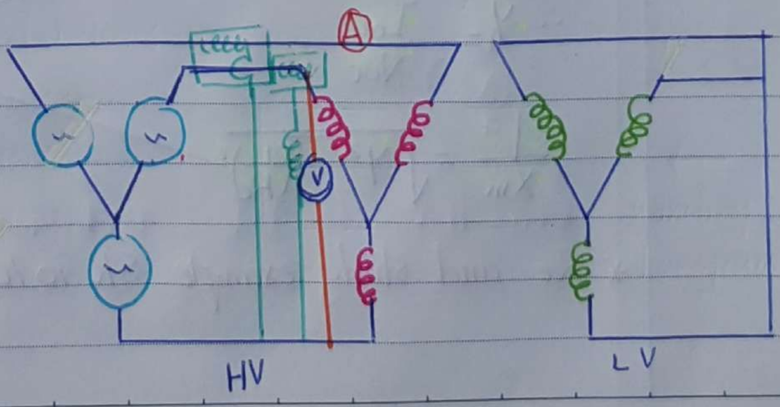
Vector Symbol	Phase Shift	Main Group
Yy0	0°	1
Dd0	0°	1
Yy6	180°	2
Dd6	180°	2
Dy1	−30°	3
Yd1	−30°	3
Dy11	30	4
Yd 11	30	4

Final Comment:- All calculation of 1-phase transformer (O/C, S/C, V.R., η and PU) are applicable to 3-phase transformers **but** using Per-Phase circuit.

Illustration:-

Consider O/C and S/C Tests.

S/C Test:



No.

From the Readings:-

I_ϕ = Ammeter Reading

V_ϕ = Voltmeter Reading $/\sqrt{3}$

$$P_\phi = \frac{W_1 + W_2}{3}$$

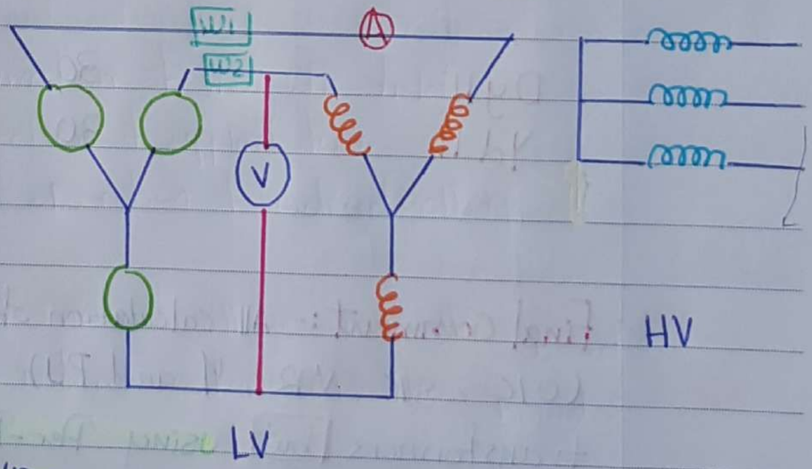
$$R_{eq} = \frac{P_\phi}{I_\phi^2}$$

$$|Z_{eq}| = \frac{V_\phi}{I_\phi}$$

$$\therefore X_{eq} = \sqrt{|Z_{eq}|^2 - R_{eq}^2}$$

Referred to HV

O/C Test:-



$\therefore P_{oc} = V_{oc}^2 / R_c$

$\therefore R_c = V_{oc}^2 / P_{sc}$

$I_{oc} = V_{oc} Y$

$$\therefore Y = \frac{I_{oc}}{V_{oc}}$$

$$\frac{1}{X_m} = \sqrt{Y^2 - \left(\frac{1}{R_c}\right)^2}$$

Refferred to LV

* See and study example S/C, O/C, Tests PU, VR and η

No.

AC Machines :- $\begin{cases}\text{Generators}\\ \text{Motors}\end{cases}$

* These machines can be classified in general into:

1. Synchronous machines: Magnetic Field winding is suppled by a dc current from an external source
2. Induction machines: Magnetic Field winding current is supplied by induction action (i.e transformer action)

* In this course, one is concern with 3-phase synchronous.

AC Machines Fundamental:-

Voltage induced in a coil

* Consider a stationary magnetic field, withen which a coil is rotating.

Objective: To find the magnitude and shape of the induced voltage in the coil.

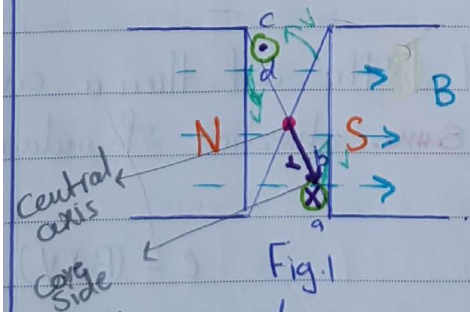
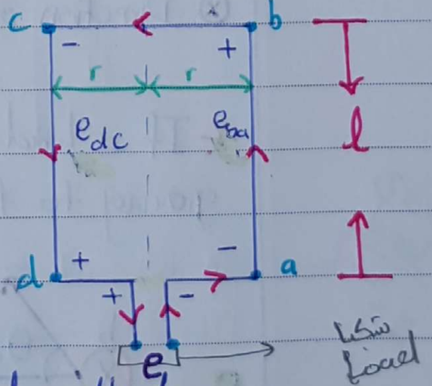


Fig.1



Voltage induce in a conductor of length l:

$$e = (B * V)\, l$$

B ≡ flux density

V ≡ linear velocity of the conductor

l ≡ length of the conductor

* The coil is rotated by means of an external mechanical force
Let the applied rotation is ccw

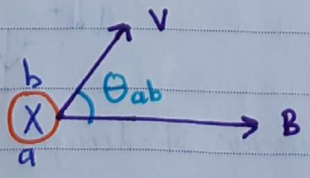
* Induced voltage across the coil terminals = Sum of voltages induced in ab, bc, cd, da.

* Usually bc (called Back end connection) and da (called Front end connection) are outside the magnetic Field. Hence no induced voltage in them.

Direction of induced voltage can be found by using RHR:-
Forefinger ≡ Direction of magnetic field
Thumb ≡ Direction of applied force
Middle finger ≡ Direction of induced voltage.

⊙ Direction of induced voltage out of the Board.
⊗ Direction of induced voltage into the Board.

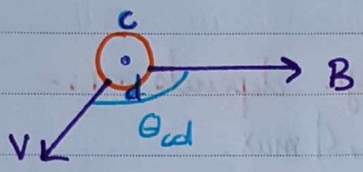
* If a load is connected to the coil then a current is going to flow in the same direction of induced voltage



$e = (B \times V) \cdot l$

$e_{ba} = (BV \sin\theta_{ab})\, l = BVl \sin\theta_{ab}$ ①

No.



∴ $e_{dc} = (BV \sin\theta_{cd})l$

$e_{dc} = BVl \sin\theta_{cd}$ ...②

∴ Induced voltage $e = e_{ba} + e_{dc}$

∴ $e = BVl \sin\theta_{ab} + BVl \sin\theta_{cd}$ ...③

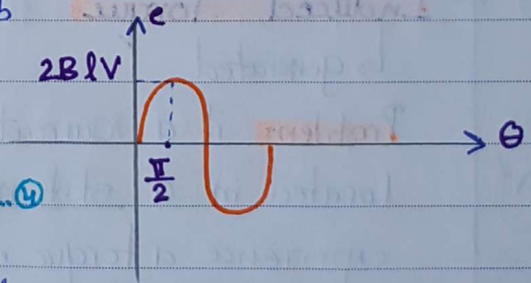
But $\theta_{cd} = 180 - \theta_{ab}$

∴ $e = BVl \sin\theta_{ab} + BVl \overset{\sin(\theta_{ab})}{\underline{\sin(180-\theta_{ab})}}$

$= 2BVl \sin\theta_{ab}$

* if let $\theta_{ab} = \theta$

∴ $e = 2BVl \sin\theta$ ...④



∴ The shape of induced voltage in a single turn coil is Sinusoidal with a magnitude = 2BlV

* ④ Can be Rewritten as follows :-

if Rotational Speed = ω (Rad/s)

∴ $\theta = \omega t$ ...⑤ $t \equiv$ time

$\omega r = V$ ...⑥ $r \equiv$ distance between coil side and central axis

Sub. ⑤, ⑥ into ④

$e = 2B(\omega r)l \sin\omega t = \boxed{(2rl)} B\omega \sin\theta$

↳ Area of the core plane = A

∴ $e = AB\omega \sin\omega t$ * when A is ⊥ to flux lines then $AB = \phi_{max}$

$e = \phi_{max}\ \omega\ \sin\omega t$ ...⑦

* In general induced voltage depends on :-

1. Flux in the machine, ϕ_{max}.
2. Speed of machine, ω.
3. Structure of the machine.

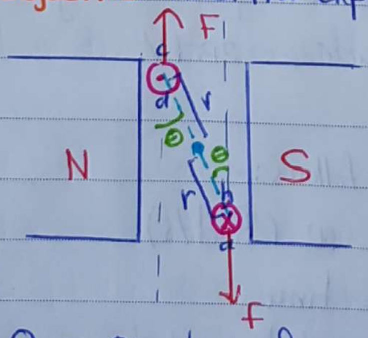
*Note:- This structure (Fig.1) is used in dc machines.

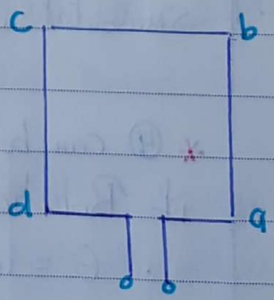
Induced Torque

↳ generated

Problem if a current is applied to a single turn coil located in a stationary magnetic field, then it will experience a torque.

Objective Find the expression for this torque.





⊗ ⊙ ≡ Direction of applied Current

Direction of developed force, F can be found by means of LHR ⟶ as follows:-

For Finger ≡ Direction of magnetic field.

Middle ≡ Direction of applied Current.

Thumb ≡ Direction of Developed force

Force الـ تعنى
تعنى B
تعنى ℓ
تعنى i

نفس اتجاه ℓ

By definition, $f = i\,(B * l)$

* Back and front end connections will not experience any torque. $\tau = f \times r$

$\therefore f_{ab} = f_{cd} = Bil$

$\tau_{ab} = f_{ab}\, r \sin\theta$

$= Bil\, r \sin\theta$

$\tau_{cd} = f_{cd}\, r \sin\theta$

$= Bil\, r \sin\theta$

$\therefore$ Total Torque, $\tau = \tau_{ab} + 0 + \tau_{cd} + 0$

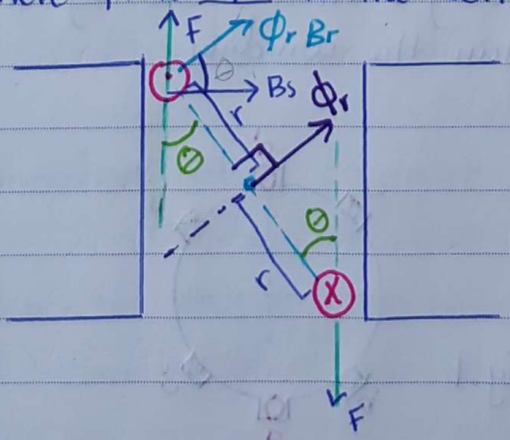
$\tau = 2Bil\, r \sin\theta$

$= (2rl)\, B\, i \sin\theta$

① ... $\tau = A\overset{\text{stationary}}{B}\, i \sin\theta$ A ≡ Coil's plane Area.

① can be Rewritten as follows:- where B is flux density of stationary (i.e stator) magnetic field Rewrite as B_s

* Current i in the rotating coil (i.e Rotor) generate flux ϕ_r, whose direction can be found by means of RHR where ϕ_r is ⊥ to the coil's plane.



* $Ni = \mathcal{R} \phi_r$

$Ni = \frac{l'}{A' \mu} \phi_r$

l' → mean length

A' → cross sectional Area.

Since $B_r \propto \phi_r$

$\therefore i = \frac{k}{\mu} B_r$ ... ②

$k \equiv$ Constant.

Substitute ② into ① →

stationary

induced

$T = A B_s \frac{k}{\mu} B_r \sin\theta = G B_s B_r \sin\theta$ ...③

$G(B_s \times B_r) \leftarrow$ sin لأن

Also it can be found that Angle between B_r and $B_s = \theta$

$\therefore T = G(B_s \times B_r)$

Comment:-

In a generator action there is a motor action and in the motor action there is a generator action

Rotating Magnetic Field

Definition: If a balance three phase voltage is applied to a 3-phase winding, then a rotating magnetic field is produced within the winding

Proof:-

3 coils

6 slots

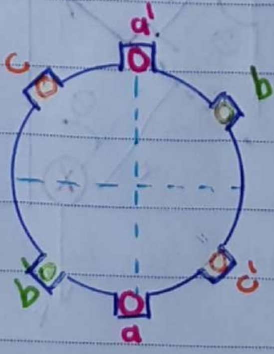
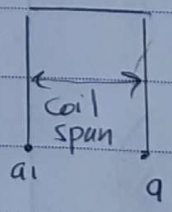


Fig. 1



No.

a, a' ≡ Terminals of coil a
b, b' ≡ Terminals of coil b
c, c' ≡ Terminals of coil c
In Fig. 1, coil span = 180 mechanical degrees

* let the Currents be:-

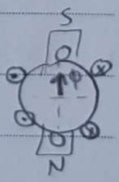
$i_a = I_m \sin \omega t$

$i_b = I_m \sin(\omega t - 120)$

$i_c = I_m \sin(\omega t - 240)$

where:
i_a enter terminal a and leave through a'
i_b enter terminal b and leave through b'
i_c enter terminal c and leave through c'

ωt	i_a	i_b	i_c	Direction of generated ϕ
0	0	رمز ⊗ −ve	⊙ +ve	↑
90	+ve	−ve	−ve	←
180	0	+ve	−ve	↓
270				
360				

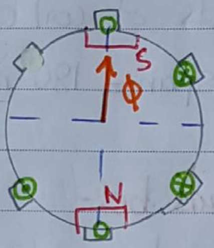


Convention +ve current ⊙
−ve Current ⊗

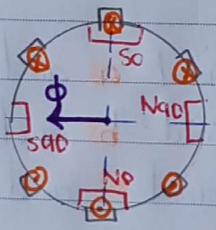
* Hence in this type of winding (2 pole - winding) when the Current make one cycle The magnetic field make one mechanical Rotation ∴ $f_e = f_m$

f_e ← electrical frequency
f_m ← mechanical frequency.

* when $w=0$ This type of stator winding is called 2pule-winding



* when $w=90$



* when $w=180$

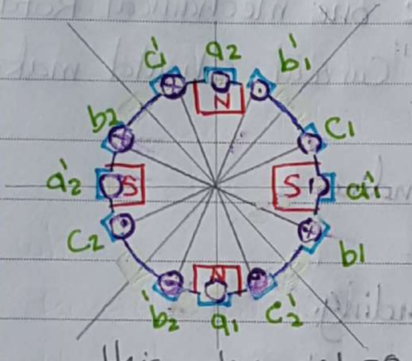
No.

* If the coil span and phase span are changed then one may change the n mber of winding-poles as follows:-

* let coil span $= \frac{180}{2} = 90^\circ$

phase span $= \frac{120}{2} = 60^\circ$

* let each phase represented by 2 coils connected in Series:-



this when ωt = 0

let: $i_a \equiv I_m \sin \omega t$

$i_b \equiv I_m \sin(\omega t - 120)$

$i_c \equiv I_m \sin(\omega t - 240)$

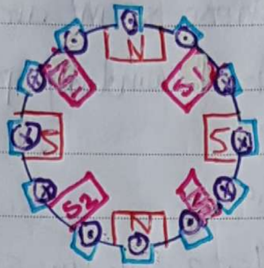
+ve Current ⊙

-ve Current ⊗

ωt	ia	ib	ic
0	0	-ve	+ve
90	+ve	-ve	-ve
180			
270			

← Represent 4-Pole winding

No.



← this when $\omega t = 90$

Check! :- To make one mechanical Rotation fo the Poles, then the applied Current should make 2 cycles.

∴ In the 4 Pole winding

$f_e = 2 f_m$

In the 2 Pole winding

$f_e = f_m$

∴ In General :-

$f_e = P f_m$

f_m → mechanical frequency (r.P.S) Rotation Per Second

P → No. of pole Pairs

f_e → electrical frequency (HZ)

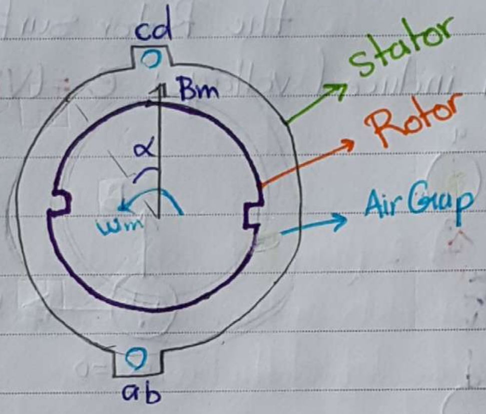
Induced or Generated Voltage in AC Machines:-

As a Balanced 3-phase voltage applied to stator windings Produce Rotating field. Then an applied Rotating magnetic field within stator windings will induced a balanced 3-phase voltage in the stator.

Objective:- To find an expression for the generated voltage in the stator winding due to the Rotating magnetic field.

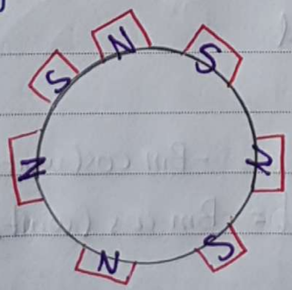
* In Ac machines, the Rotor windings are designed in such away to Produce a flux whose magnitude changes in Sinusoidal manner.

Procedure :- Consider a single turn coil on the stator.



Rotor are of two types:-

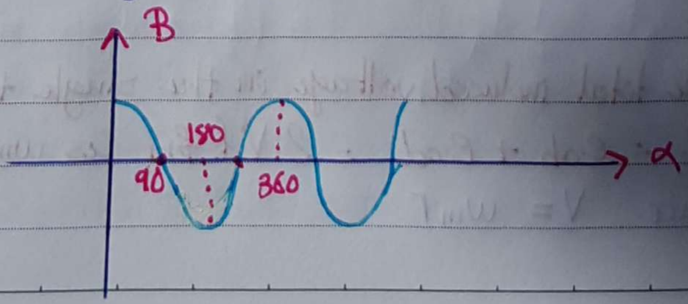
1. Cylindrical or non-salient.
2. noncylindrical or Salient.



$B = B_m \cos\alpha$

$\alpha \equiv$ the angle from the direction B_m

flux is sinusoidal let the direction of maximum flux density (B_m) along +ve y-direction



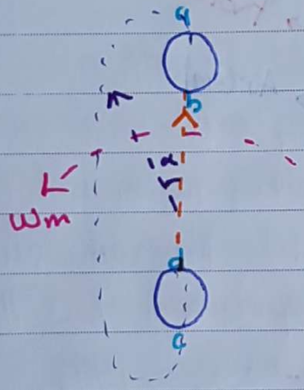
No.

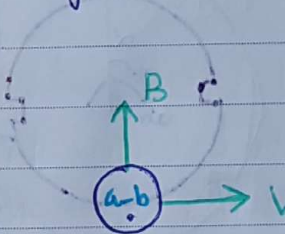
Since the Rotor is Rotating with angular speed ω_m (rad/s) Then the flux at a point on the Stator with angle α is

$$B = B_m \cos(\omega_m t - \alpha)$$

↳ This for flux Radially outward from the Rotor surface.

* To find induced voltage: $e = (V \times B)\, l$



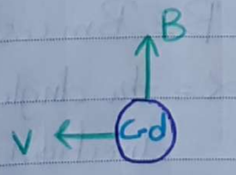


$\alpha = 0$

$B = B_m \cos(\omega_m t - \alpha)$ ← always assums that the flux is outward from Rotor surface.

$e = (B \;\; v) \cdot l$

$e_{ab} = (B_m \cos . \omega_m t)\, V l$



$e_{cd} = -B_m \cos(\omega_m t - \alpha)\, V l$

$e_{cd} = B_m \cos(\omega_m t)\, V l$

$\alpha = 180$

* There is no induced voltage in bc and cd because $(B \times V)$ is ⊥ on their length

∴ The total induced voltage in the single turn coil is

$$e = e_{ab} + e_{cd} = 2 V l B_m \cos \omega_m t \quad \text{....} ①$$

Since $V = \omega_m r$

No.

$\therefore e = 2 \omega_m r l B_m \cos \omega_m t$ 2.r.l ≡ Area of the plane

$= (2rl) B_m \omega_m \cos \omega_m t$

$= A B_m \omega_m \cos(\omega_m t)$

$\therefore e = \phi_m \omega_m \cos(\omega_m t)$

$\therefore$ For N turns coil ⟶

$e = N \phi_m \omega \cos(\omega_m t)$

For a 2-pole stator winding $\omega_s = \omega_m = \omega$

$e = \underbrace{N \phi_m \omega}_{E_m} \cos \omega t$

$\therefore$

***Comments:-**

1. ϕ_m is called flux per pole.
2. Peak or maximum value of Induced voltage,

   $E_m = N \phi_m \omega$

   $\therefore$ Em depends on 1. speed of rotor
   2. magnetic field of Rotor (i.e ϕ_m)
   3. Construction of Machine (i.e N)

3. $E_{RMS} = E_m / \sqrt{2}$

   $\therefore E_{RMS} = \frac{1}{\sqrt{2}} N \phi_m \omega$

   $= \frac{1}{\sqrt{2}} N \phi_m 2\pi f$

   ↳ frequency of Induced voltage.

   $E_{RMS} = 4.44 N \phi_m f$

   ↳ ولدتها الحركة

4. Hence for 3-phase Ac generator a Balanced 3 phase voltages will be generated

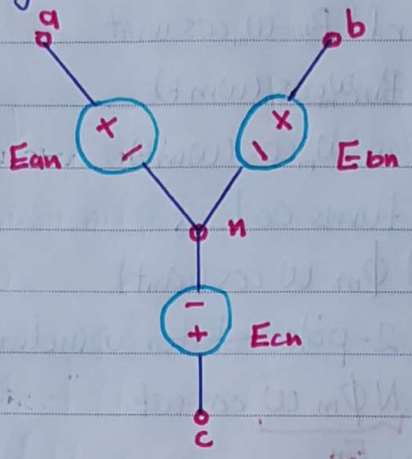
   $E_{an} = E_m \sin \omega t$

   $E_{bn} = E_m \sin(\omega t - 120)$

   $E_{cn} = E_m \sin(\omega t - 240)$

No.

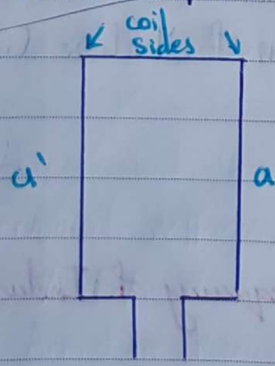
*usually the generator is Y - connected.

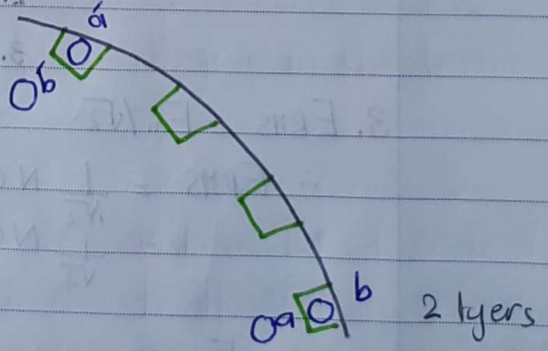


∴ line voltage $= \sqrt{3}\,(N\phi_m \omega)$

*Example:- A 3-phase, Y-connected, 4 pole winding is istalled in 24 slots in the stator. There are 40 turns of wire in each slot of the windings.
All coils in each phase are connected in series. The flux per pole is 0.06 wb and the speed of Rotation of magnetic field 1800 rpm.

تجمع الفولتية





1. what is the frequency of generated or induced voltage?

$$f_e = P\, f_m = \left(\frac{4}{2}\right) * \frac{1800}{60} = 60\ HZ$$

HZ, pole pairs, ps

2. what are the resulting phase and terminal voltages?

$E\phi = 4.44 N \phi_m f$

$= 4.44 * \left(\frac{24}{3}\right) * 40 * 0.06 * 60$

$= 3617.28V$ ← phase voltage

terminal , line Voltage $= \sqrt{3} E\phi = 6265.3 V$

* See the rating of generated voltage of 3-phase generator in power stations → (11-25) طلع تقريبا

$$E\phi = 4.44 N \phi_m f * k_p * k_d$$

k_d → Distribution factor.

↳ Pitch factor

* See Appendex B. in your textbook.

Machines: Chapter

3-phase Synchronous Generator:-

Construction

Rotor
(Carry the main magnetic field circuit)

Stator
(carry the Armature windings)
↳ this is the 3-phase winding (As explaine before)

*** Main magnetic field circuit:-**

-Usually the field Current is supplied from external DC Source to the field windings through Brushes and sliprings.

- other types of supplying field Current are:-
  1. Brushless exciters.
  2. Pilot exciters (use permenant Magnet)

two electromagnet
1. main
2. exciters magnet

* see the introduction and Block diagram in the textbook.

*** Equivalent circuit of the 3-phase generator →**

Terminal voltage, $V\phi$ = generated voltage $E\phi$ at No-Load.

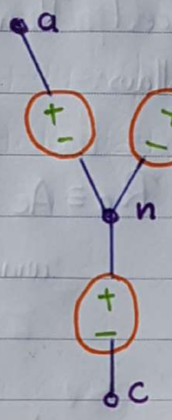
$Vo \neq E\phi$ at Load condition due to the following Reasons:-

A) Armature Reaction: Interaction between main flux of the rotor and the flux Produced by Armature Current.

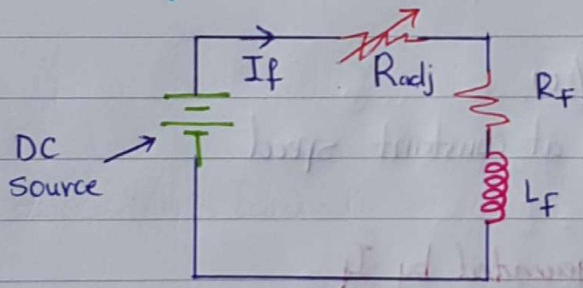
- This armature Reaction is represented by a reactance. jX

B) Self induced of Armature windings. jXa

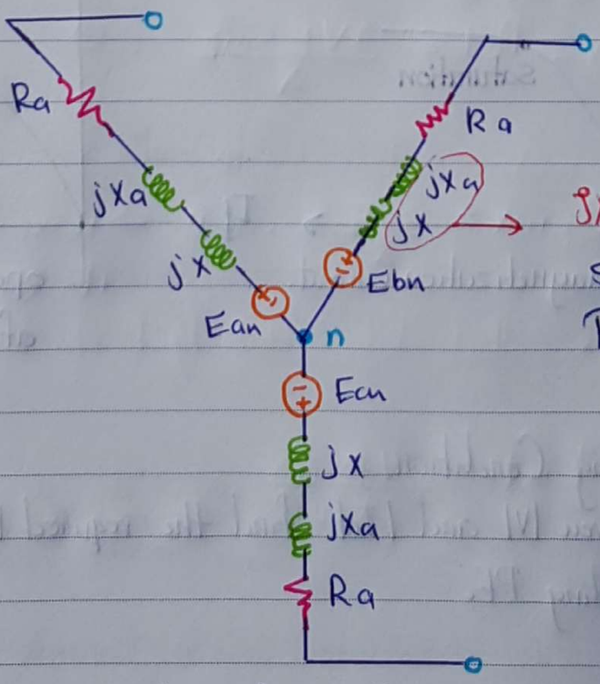
c) Resistance of Armature windings, Ra



∴ The equivalent circuit can be deduced as follows:



Field Circuit

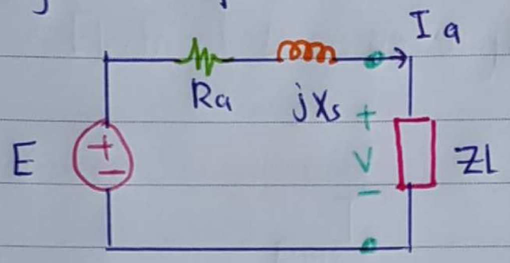


$jX + jX_a = jX_s$

Synchronous ↲
Reactance.

Armature Circuit

* Since the system is Balanced Then it Can be represented by Per-phase circuit as follows:-



$R_a \equiv$ Ac Resistor of Armature windings.

* Performance of the generator Under Various loading Condition:-

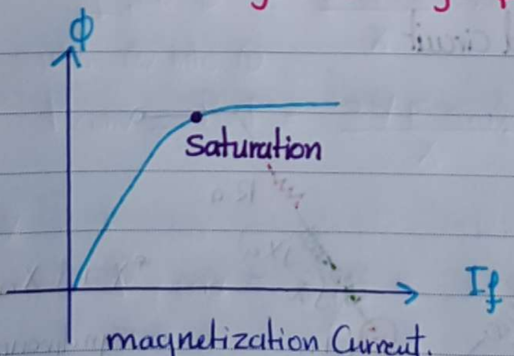
1) No-Load

$$V = E = N\phi\omega$$

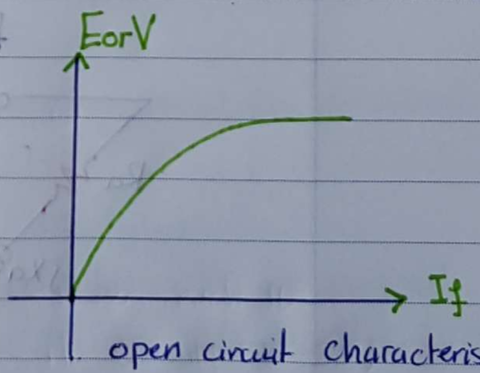
Running Rotor at constant speed

$$E = k\phi$$

constant ↵ ↳ generated by I_f



magnetization Curent.



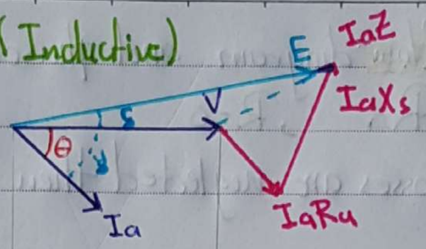
open circuit characteristic of generator (OCC)

2) Loading Condition

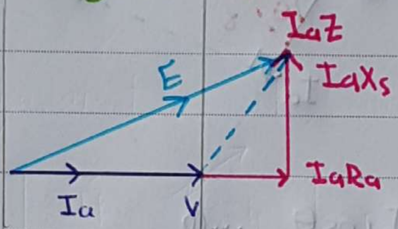
For a given |V| and |Ia| find the required |E| for lagging, Unity and Leading PF.

No.

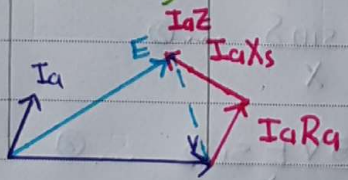
Lagging (Inductive)



Resistive (Unity PF)

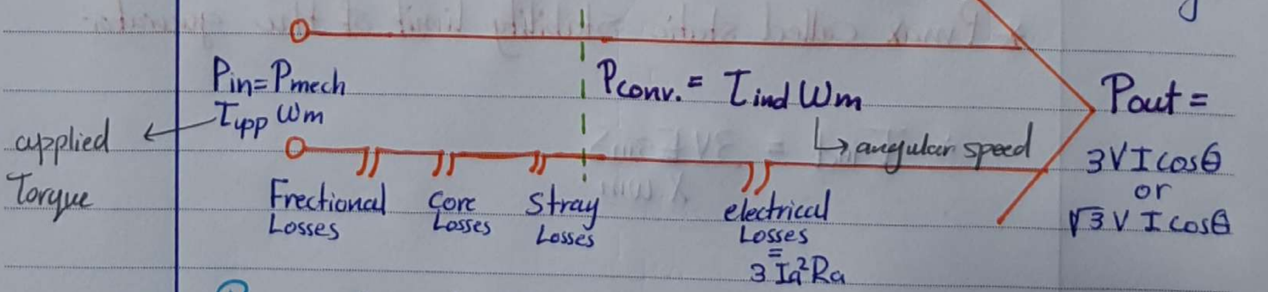


Capacitive (Leading PF)



*Power and Torque Relationship of 3-phase Synchronous generator:-

This can be explained and derived in terms of Power flow Diagram

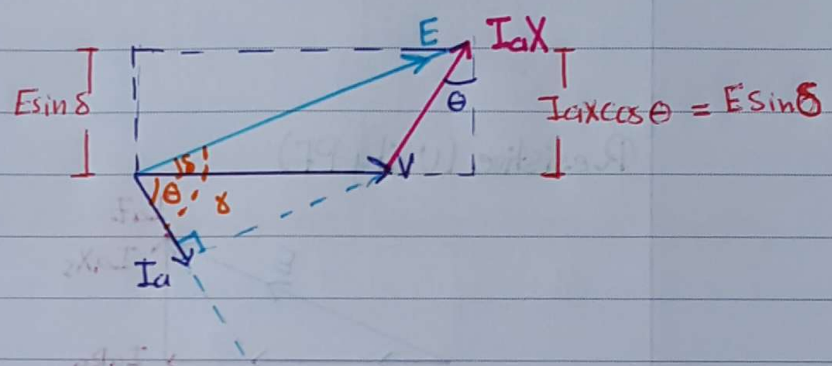


$P_{conv.} \equiv$ Amount of input Power which is converted to electrical.

No.

* Using power flow diagram and phasor diagram, Assuming that $X_s \gg R_a$, $R=0$

∴ Electrical Losses are neglected then $P_{con} = P_{out}$



$P_{conv} = P_{out} = 3VI\cos\theta$ ...①

From phasor diagram

$I_aX\cos\theta = E\sin\delta$

$I_a\cos\theta = \frac{E\sin\delta}{X}$ ...②

sub. ② into ①

$P_{conv} = P_{out} = \frac{3VE\sin\delta}{X}$ ...③

* $P_{out,max}$ is when $\delta = 90°$

$P_{max\,out} = \frac{3VE}{X}$ ...④

δ never reach 90°
it's usually (20°-30°)

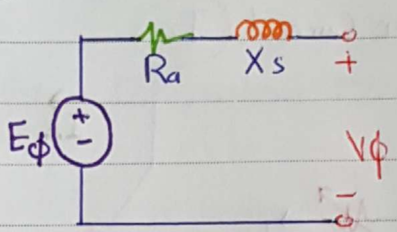
* Pmax called static stability limit of the generator

∴ $\tau_{ind} = \frac{3VE\sin\delta}{X\omega_m}$

*Measurement of Generator Parameters :-

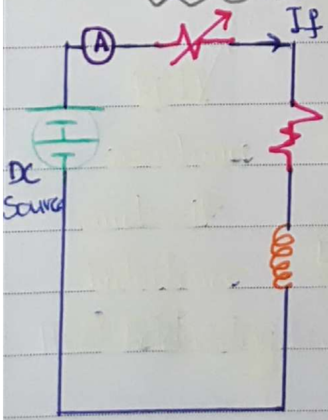
Objective: To measure R_a and X_s.

Procedure: This can be performed by means of $\boxed{O/C}$, $\boxed{S/C}$ and (dc) Tests as follows:-

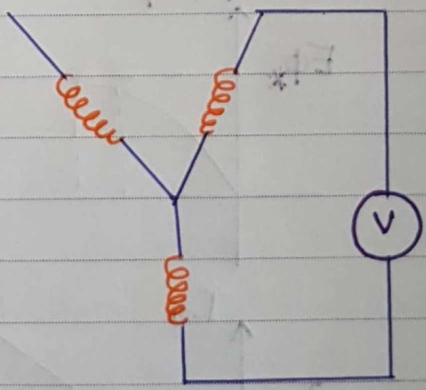


*Open circuit Test :-

1. open circuit The terminals of the armature
2. Run the $\boxed{\text{rotor}}$ at a constant $\overset{\omega}{\text{speed}}$
3. Vary the field Current, (I_f), $\boxed{\text{in steps}}$ and take the corresponding Readings of $V\phi$.
4. Plot $V\phi$ vs I_f

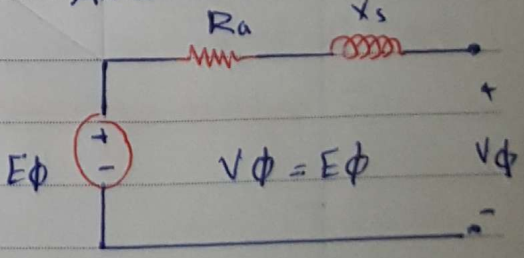


Field



Armature

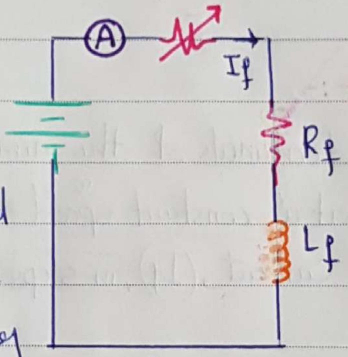
I_f [A. reading]	$V\phi = V/\sqrt{3}$
I_{f_1}	$V\phi_1$
I_{f_2}	$V\phi_2$
⋮	⋮



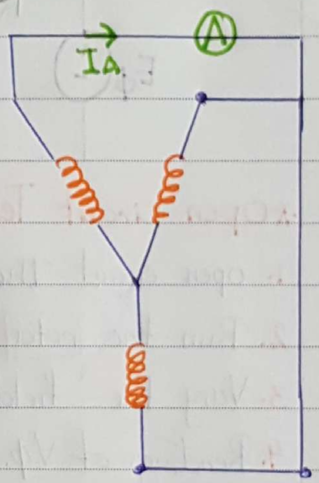
* Short circuit test:-

1. Short circuit The terminals of the Armature.
2. Run the rotor at a constant speed.
3. Vary the field Current (I_f) in steps and take the corresponding Readings of Armature Current (I_a)
4. plot I_a Vs I_f.

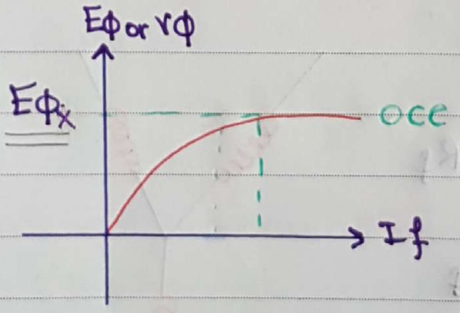
R_f, L_f = Resistance and self inductance of field winding

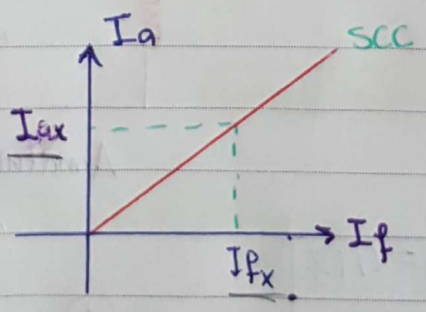


Field

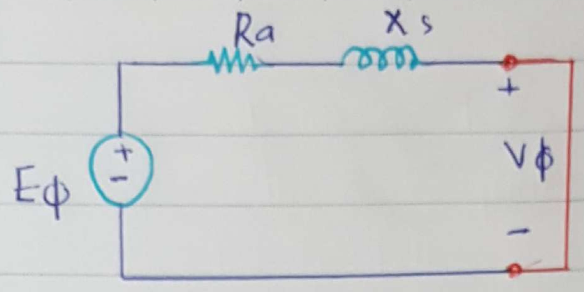


Armature





No.

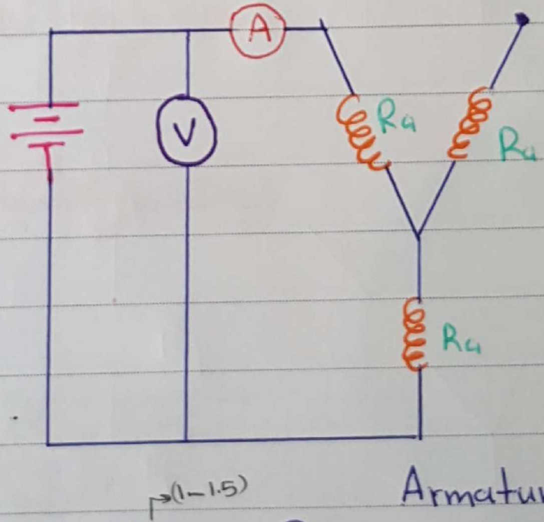


V_ϕ = zero in s/c ∴ for a given field current I_{fx}, then from the occ, one can fined the generated voltage $E_{\phi x}$ and from scc, one can find corresponding I_{ax}.

$$\therefore \frac{E_{\phi x}}{I_{ax}} = |Z| \text{, where } |Z| = \sqrt{R_a^2 + X_s^2} \quad \ldots (1) \quad Z = R_a + jX_s.$$

* Now if Ra is neglected then $X_s = E_{\phi x} / I_{ax}$ Unsl.

* If Ra is not neglected then it can be measured by means of $\boxed{\text{DC Test}}$ as follows:-



$$\therefore R_A = \frac{1}{2} \, \frac{\text{Voltmeter Reading}}{\text{Ammeter Reading}}$$

↳ DC resistance

$$\therefore R_{A,ac} = a \overset{(1-1.5)}{\text{factor}} * R_{A,dc} \quad \ldots ②$$

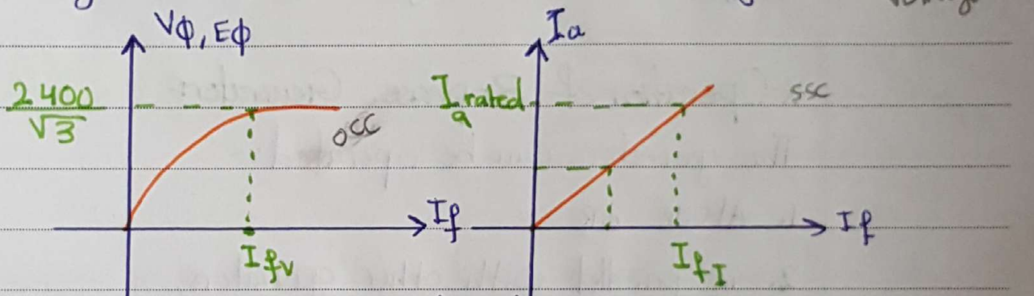
* Substitute (2) in (1) to find Xs

* If Xs is Calculated a Long Linear portion of the occ, it called Unsaturated Synchronous Reactance.

*Short Circuit Ratio (SCR) ≜ Field Current which produces Rated voltage at (O/C) divided by Field Current which produces Rated Armature Current at (S/C)

$$(SCR) = \frac{I_{fv}}{I_{fI}}$$

*Illustration Consider a 9375 kVA, 2pole, 60 Hz, $\overset{\text{Rated voltage}}{\underset{\text{line L-L voltage}}{2400\,V}}$ 3 phase generator. The occ and scc are given.



from the occ, it can be found that $I_{fv} = 116$ A

$I_{rated} = \frac{9375\,k}{\sqrt{3} * 2400} = 2255$ A $I_{fI} = 131$ A

$\therefore SCR = \frac{116}{131} = 0.89.$

*Significance of SCR:-

- $SCR = \frac{1}{X_s}$ ....①
- at short circuit if Ra is neglected then $E_\phi = I_a X$

  $\therefore X = \frac{E_\phi}{I_a}$ ....②

- It was found that $E_\phi = 4.44\, k_p k_d N \phi f$

  Since $\underset{\text{mmf}}{\mathcal{F}} = \underset{\text{Reluctance}}{\mathcal{R}}\ \underset{\text{flux}}{\phi}$

  $$E_\phi = \underbrace{4.44\, k_p k_d N \frac{\mathcal{F}}{\mathcal{R}} f}_{\text{constant}}$$

No.

$E\phi = k\Gamma$ ... ③ when $k \propto \frac{1}{R}$

substitute ③ into ②

$X = \frac{k\Gamma}{I_a}$ ... ④

* Large SCR → Smaller X_s → smaller k → Larger R → Longer Air gap between stator and Rotor → Hence a larger mmf is required from field windings → larger size of generator → higher cost per $\boxed{kVA}$ Rating.

*Operation of 3-phase Generator:-

The generator can be operated:-

1. Alone OR
2. in parallel with other generators.

***Alone:-** Assume that [1] $\boxed{\text{Rotor}}$ running at constant speed (ω constant)

[2] I_f is constant, (ϕ constant)

[3] R_a is neglected

∴ Since $E = \overset{N}{k}\,\overset{\text{from } I_f}{\phi}\,\omega$ ∴ E remain $\boxed{\text{Constant}}$

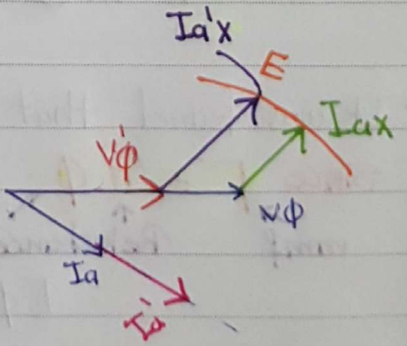
Objective:- what will happen to $V\phi$? when the load is changed keeping the power (PF) $\boxed{\text{constant}}$ for the three types Inductive Resistive and capacitive.

* Inductive :-

*let $|I_a|$ increases to $|I_a'|$

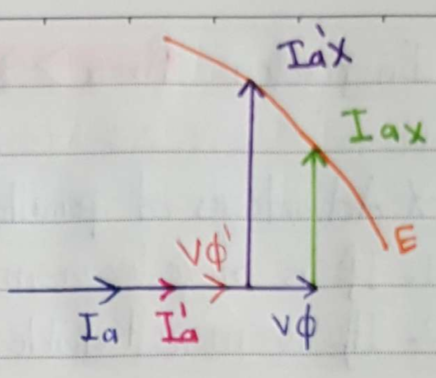
then $I_a'X > I_aX$

∴ $V'\phi < V\phi$



* Resistive :-

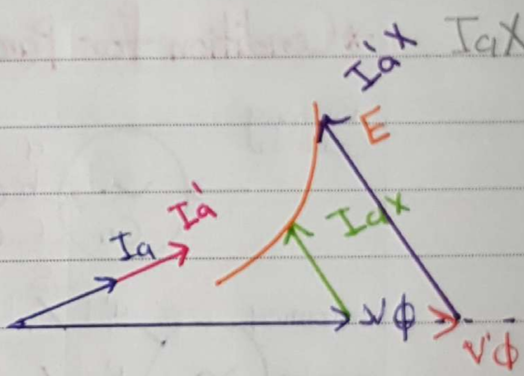
$V'_\phi < V_\phi$



$I_a X' > I_a X$

* Capacitive :-

$V'_\phi > V_\phi$



$I_a X' > I_a X$

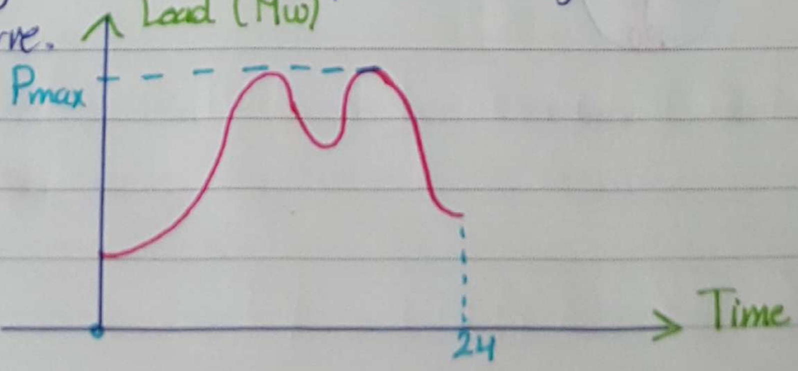
* However in practice V_ϕ is kept constant, Hence E is changed to achieve that. Since $E = k\phi\omega$

Since ω can not be changed in order to maintain f constant, that E is controlled by changing ϕ, through the adjustment of I_f

ضبط

* Parallel operation of generators:-

- Generator are used to supply a load, and this Load is changing with time represented by the so called Daily Load curve.



عدد ال generators

اذا خرب واحد ما بنفقد كل الطاقة

In Jordan $P_{max} > 3000$ Mw

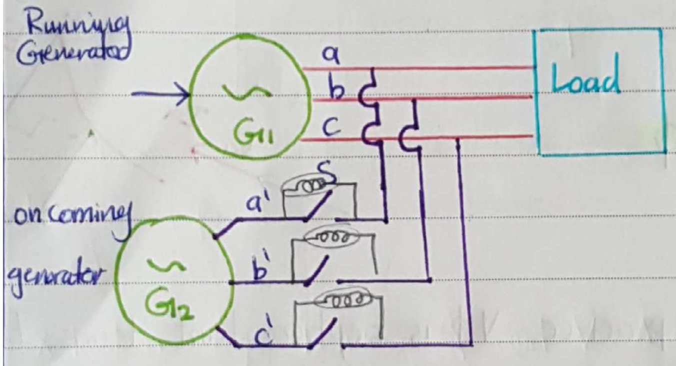
1 * 3000 MW

3 * 1000 MW
better

* Advantages of parallel operations:-

1. It is more economical.
2. It is more Reliable.

* Condition for Parallel Operations:-



This procces is called synchronization.

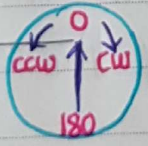
* what are the conditions to be satisfied before closing the switch S.

[1] G_1 and G_2 should have the same voltage (i.e magnitude and phase). i.e $V_{an} = V_{a'n}$, $V_{bn} = V_{b'n}$, $V_{cn} = V_{c'n}$

[2] G_1 and G_2 should have the same phase sequence

[3] frequency of G_2 should be slightly higher than that of G_1

$$f_{G_2} > f_{G_1}$$



*Procedure of synchronization:-

[1] Adjust I_{f_2} (i.e field current of G_2) until its terminal voltage equal that of G_1

[2] check the phase sequence by Using:-

A. a small 3-phase Induction motor (مثل التوضيح على الرسم ؟)

OR

B. 3 bulbs method.

عشان اقدر الاحظ التغيير

[3] Let f_2 to be slightly higher than f_1, hence her the phase angles of G_2 is changing slowly with respect to G_1, Then when the phase shift equal to Zero, close S.

*phase shift can be checked by:-

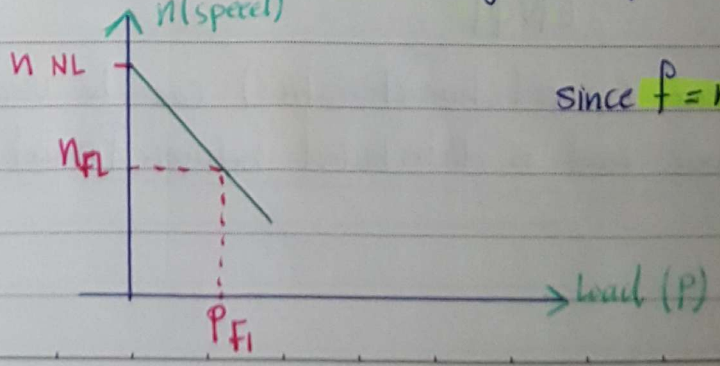
1. Synchroscope.
2. 3 bulbs method.

*Relationships between Active $\overset{P}{\text{Power}}$-$\overset{F}{\text{Frequency}}$ and Reactive Power-Voltage:-

*Relation between P and F:-

Since the Rotor is driven by a prime mover (Pm), then as the load in the generator ↑, then the speed of PM↓.

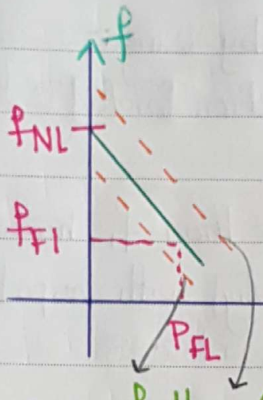
This decreases is usually non-Linear. However by Using a Governor mechanizm the Change in speed is made Linear.



since $f = nP$ ∴ $f \propto n$

Speed droop, SD $= \frac{n_{NL} - n_{FL}}{n_{Fl}}$

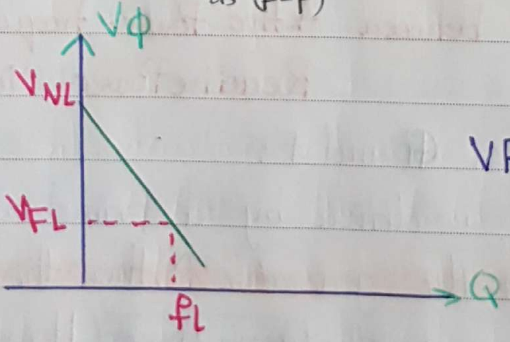
Typical values, SD $= (2\% - 4\%)$



* By means of the Governor one may change the setting of NO-Load point.

* Relationship between Q and V As explained before, when Q ↑ then V ↓ By using inductive load

Hence the same Relationship exists between V and Q.
as (f-P)



$$VR = \frac{|V_{NL}| - |V_{FL}|}{|V_{FL}|}$$

$$VR = \frac{|E\phi| - |V\phi|}{|V\phi|} * 100\%$$

* These Curves (characteristics) can be used to find how a given load is distributed between Parallel generators

Scanned by CamScanner

No.

* **Amount of Load sharing** This could be between

1. A Generator and the whole system

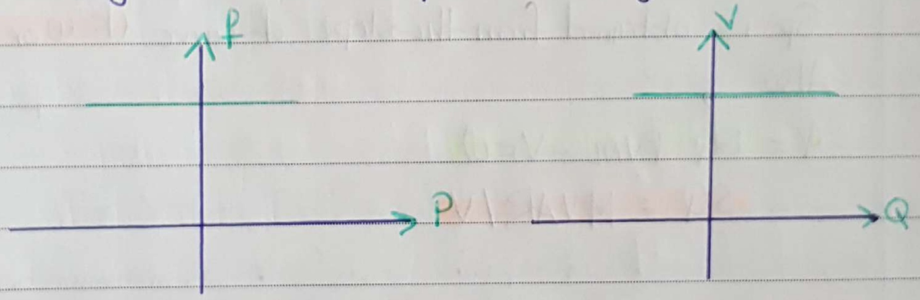
OR

2. 2 parallel Generators

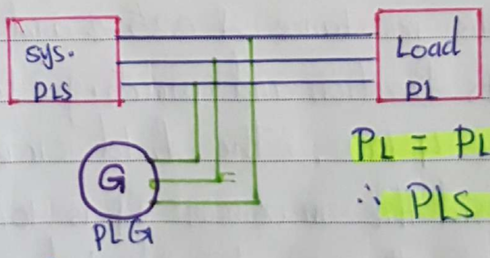
* **Procedure** It can be solved by means of the Hous diagram as follows

Case [I] :-

The system can be represented by the following characteristics:-

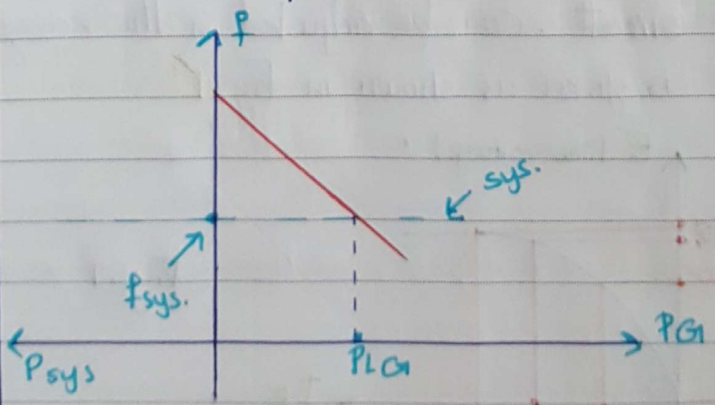


* This is called ∞ system

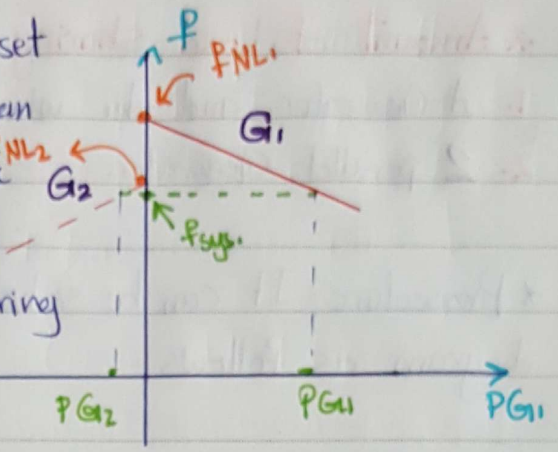


$P_L = P_{LG} + P_{LS}$

$\therefore P_{LS} = P_L - P_{LG}$



*by adjusting the set point of fNL one can shift the characteristic upword or downward. Consequently Power sharing can be controled



* In general:-

$P = S_p \, (f_{NL} - f_{FL} \text{ or } f_{sys})$

S_p is obtained from the slope of curve (KW or MW) / HZ

Also →

$Q = S_Q \, (V_{NL} - V_{FL})$

$\therefore S_Q = MVAR/V$

* Example:- A 480 V, 200 KVA, 0.8 PF lagging, 2-pole, Y-connected synch. generator, has synchronouse reactance and armature resistance of 0.25 Ω and 0.04 Ω respectively. At 60HZ, its fraction and windings losses = 6 KW and core losses = 4 KW. The field circuit has dc voltage of 200 V, and the maximum If is 10A. The resistance of the field circuit. can be adjusted in the range of (20-200) Ω. The OCC is given as shown in Fig. 1

→ minimum acceptable PF

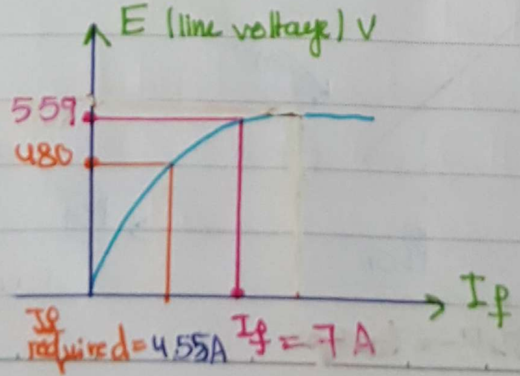


Fig. 1

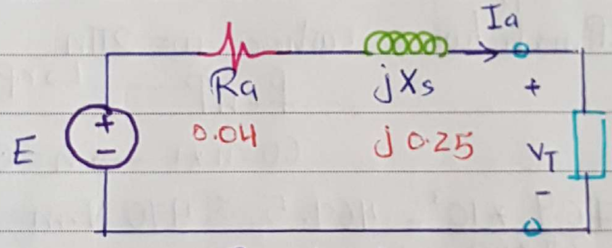
No.

$\boxed{1}$ How much I_f is required to generated terminal voltage V_T, equal to 480 V when the generator is running at NO-load.

$\therefore$ $VT = E = 480V$

$\therefore$ by using given OCC, find I_f which correspond to $E = 480V$

$\boxed{2}$ what is the internal generated voltage of this machine at Rated conditions.



Load: 480V → line voltage
200 KVA
0.8 PF lagging

$E = V_T + I_a(R_a + jX_s)$

$VT = \frac{480}{\sqrt{3}} \angle 0°$ ↳ انا [illegible]

$I_a = \frac{200*10^3}{\sqrt{3}*480} \angle \underset{\text{lagging}}{-} \cos^{-1}\underset{\text{PF}}{(0.8)} \longrightarrow \angle -36.87$

$\therefore$ By substitution

$E\phi = 322.7 \angle 7.4°$

$EL = \sqrt{3} * E\phi = 559V$

$\boxed{3}$ How much I_f is required to make $V_T = 480$ when the generator is running at rated conditions.

Rated Conditions

VT	E
(v)	559 (V)

$\therefore$ from open Circuit characteristic can be found, $I_f =$

No.

4 How much power and Torque must the generator's Prime mover must be capable of supplying at Rated conditions

$P_{\substack{mech\\input}} = P_{out} + \text{electrical Losses} + \text{Core Losses} + \text{friction Losses}$

$\downarrow$ $200 * 10^3 * 0.8$

$\downarrow$ $3 I_a^2 R_a = 3(240.6)^2 0.04$

$\downarrow$ $4 * 10^3$

$\downarrow$ $6 * 10^3$

$T_{mech} = 176.9 \text{ kw}$

$P_{meh} = \underset{\substack{or\\ T_{app}}}{T_m} * \omega$

$T_m = P_{mech} / \omega$ where $\omega = 2\pi \underset{\hookrightarrow r.p.s}{n}$

$f = nP$

$60 = n * 1 \longrightarrow n = 60 \text{ r.p.s}$

$$T_m = \frac{176.9 * 10^3}{2\pi * 60} = 469.5 \simeq 470 \text{ N.m}$$

5 find η

$$\eta = \frac{\text{output}}{\text{input}} = \frac{200 * 10^3 * 0.8}{176.9 * 10^3} = 90.4\ \%$$

*** Example:-** A 480 V, 200 kw, 2 pole, 3-phase, 50 Hz synchronous generator prime mover has $n_{NL} = 3040$ rpm and $n_{FL} = 2975$ rpm

a Find the SD of this generator

$$SD = \frac{n_{NL} - n_{FL}}{n_{FL}} * 100\ \% = 2.18\ \%$$

b This generator is connected to other generator with the following Data:-

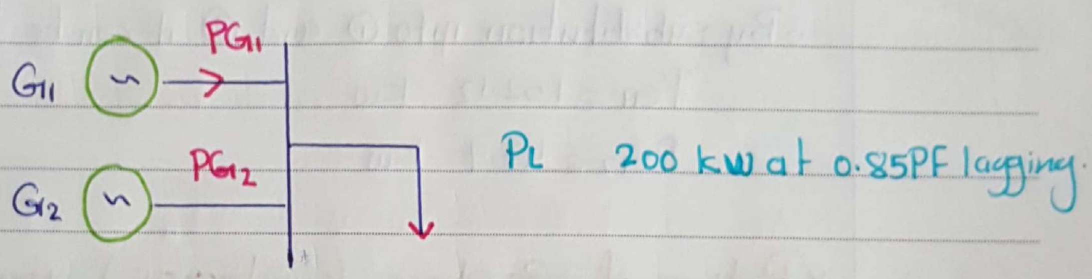
480 V, 150 kw, 4 pole, 50 Hz with prime mover of $n_{NL} = 1300$, $n_{FL} = 1485$ rpm

No.

The Load to be shared between the generator is 200 kW at 0.85 PF lagging.

b.1 Find SD of this generator ← By substitution SD= 12.45

b.2 Find the System's frequency and the amount of load sharing



Single-Line Diagram.

$$\therefore P_{G1} + P_{G2} = P_L$$

$$P_{G1} = S_{P1}(f_{NL1} - f_{sys.})$$

$$P_{G2} = S_{P2}(f_{NL2} - f_{sys})$$

→ SP_1 since $f = nP$

$$f_{N1} = \frac{3040}{60} * 1 = 50.67 \text{ Hz}$$

$$f_{FL} = \frac{2475}{60} = 49.58$$

$$S_{P1} = \frac{200}{50.67 - 49.58} = 183.5 \text{ kW/HZ}$$

→ SP_2

$$f_{NL} = \frac{1500}{60} * 2 = 50 \text{ Hz}$$

$$f_{FL} = \frac{148}{60} \times 2 = 49.5 \text{ HZ}$$

No.

$$\therefore SP_2 = \frac{150}{50-49.5} = 300 \text{ kw/HZ}$$

$$\therefore \underbrace{\overset{\textcircled{1}}{18.5(50.67-f_{sys})}}_{P_{G1}} + \underbrace{\overset{\textcircled{2}}{300(50-f_{sys})}}_{P_{G2}} = 200$$

$$\therefore f_{sys.} = 49.841 \text{ HZ}$$

∴ By substitution into ① and ② it can be found:-

$P_{G1} = 152.12$ kw

$P_{G2} = 47.7$ kw

* Ratings of 3-phase (synchronous) generator:-

- Since the generator is going to be used in a given country when it's frequency is specified (i.e 50 or 60 HZ)
- what type of prime mover to be used (i.e steam, hydro, gas, ...etc) Then the speed n is $\boxed{\text{fixed}}$.
- Since $E = k\phi\omega$, ϕ can be increased to a certain max. limit and since $\phi \propto I_f$ then here is a limit for I_f (i.e I_f max). In order to limit Rotor losses to a certain max. level.

∴ There is a limit on E (i.e Emax)

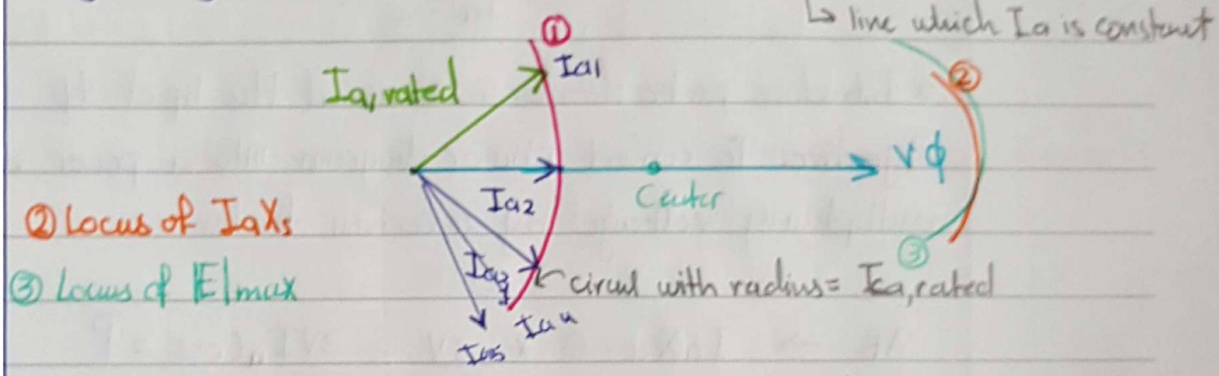
→ consequently Rated voltage, V_ϕ, is specified

- To limit the heating in the Armature circuit (i.e $3 I_a^2 R_a$) Then I_a is set at max. or rated value (i.e I rated)
- Having specified V_ϕ and I_ϕ then apperant power is specified.

$$S = 3 V_\phi I_\phi$$

$$= \sqrt{3} V_L I_L$$

- By using the previous specified values (i.e $|E|_{max}$, $V\phi$, $|I_a|_{max}$) then the Rated PF of the generator can be deduced as follows by using the phaser diagram and Neglecting R_a:-



- Since X_s is constant then the voltage drop (i.e $I_a X_s$) is constant.

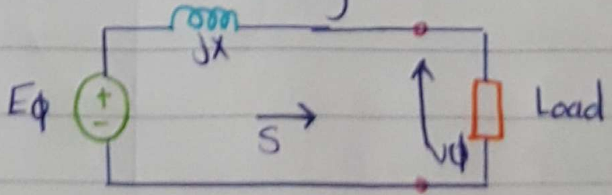
* For I_5 the required $E_5 > |E|_{max}$

* Hence a limit is specified on the PF. it's called the minimum acceptable PF. any PF below this limit will cause
E to be $> |E|_{max}$
Hence $I_f > |I_f|_{max}$.

*Power Capability Curve:- This is a graphical representation of the rated complex Power (i.e $S = P + jQ$) of the generator

why? Any Point within the diagram give safe operation of the generator.

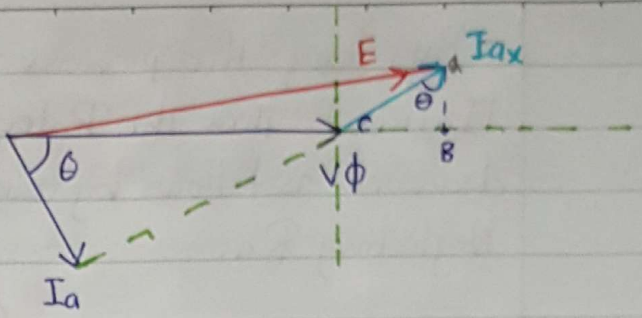
Procedure, How? This digram can be deduced by using phasor diagram and assuming $V\phi$ is constant, Hence neglect R_a.



$AB = I_aX\cos\theta$

$BC = I_aX\sin\theta$

$AC = I_aX$



*Intruduce perbendicular axis as at the tip of Vϕ

Objective: To convert phasor diagram into a power diagram, by multiplying voltage by a certain factor.

AB → $I_aX\cos\theta * \frac{3V}{X} = 3VI_a\cos\theta = P$

BC → $I_aX\sin\theta * \frac{3V}{X} = 3VI_a\sin\theta = Q$.

AC → $I_aX * \frac{3V}{X} = 3VI_a = |S|$

∴ Triangle ABC is a power Triangle

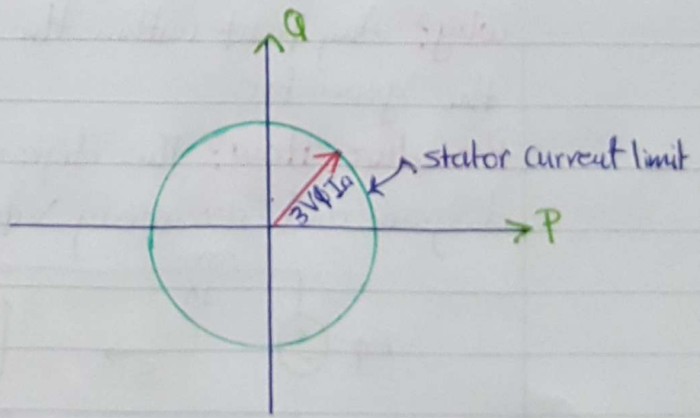
→ The new axis become power axis.

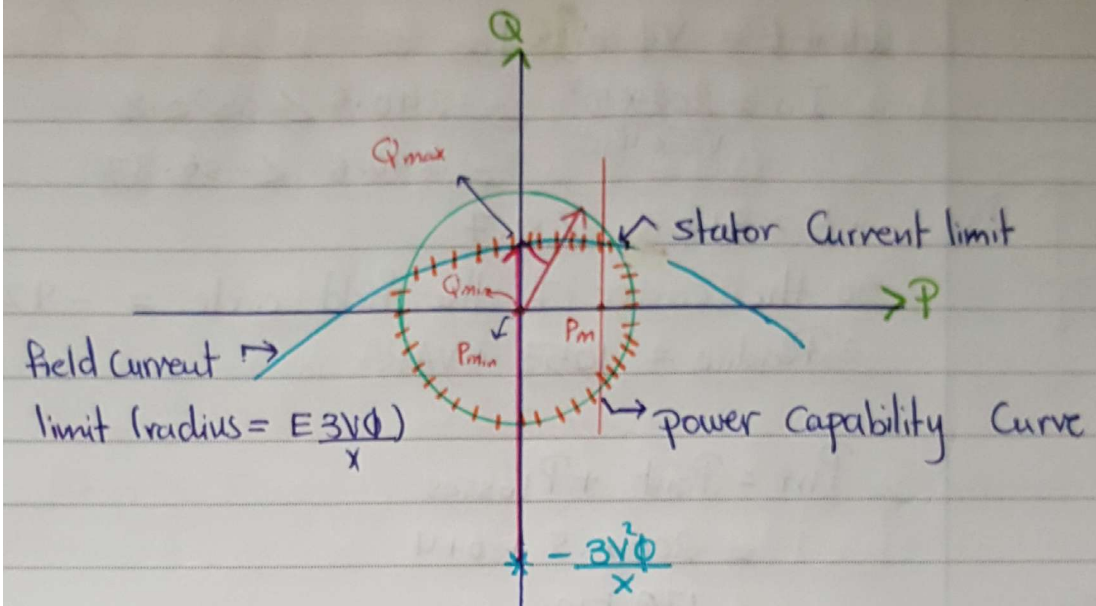
→ Also the origin of the phaser diagram, with respect to origin of power diagram $= -V\phi$

∴ This origin becomes $-V\phi * \frac{3V\phi}{X} = -\frac{3V^2\phi}{X}$

Also E → $E = \frac{3V\phi}{X}$

So → By using the previous information draw on the Power axis, a diagram which take into account the limits on Ia and If





* any point within the 2 circles is a safe operating Point

* one may add another constraints, for example: the max. Power supplied by the turbaine, say Pm

$$P_{gmin} \leq P_G \leq P_{gmax}$$

$$Q_{gmin} \leq Q_G \leq Q_{gmax}$$

* **Example:-** A 480 V, 200 KVA, 0.8 Pf lagging, 60 HZ 2pole Y-connected generator, has $X = 0.25\ \Omega$, $R_a = 0.04\ \Omega$, at 60Hz it's friction and windings losses = 6 kW it's core losses = 4 kW $V_f = 200V$, I_f. $X = 10A$. Construct the Power Capability Curve.

Stator Circle

Origin = (0,0) center

Radius = 200 KVA

field Circle

Center = $Q = -\frac{3V_\phi^2}{X}$

$V\phi = 480$

$X = 0.25$

Radius = $E * \frac{3V\phi}{X}$

No.

where $E = V\phi + jX I_a$

$I_a = \frac{200*10^3}{\sqrt{3}*480} = 240.6 \angle \cos^{-1} 0.8$

$= 240.6 \angle -36.87$

$\therefore E = 316.8 \angle 8.7$

so the center of the field circle = -920.7 KVAR

$\therefore$ Radius = 1053 KVAR

$P_m = P_{out} + P_{losses}$

$= 200*.8 + 6+4$

$= 170$ kw

*Pitch, distribution and winding Factor:-

It was found that

$$E\phi = 4.44\underbrace{(K_p\, K_d)}_{}^{K_w} N \phi f$$

↳ distribution factor

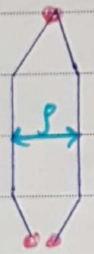
↳ Pitch factor

- winding factor:- $K_w = K_p K_d$

These are defined as follows:-

$$K_p \triangleq \sin\left(\frac{\rho}{2}\right)$$

$\rho \equiv$ Coil's Pitch in electrical degree

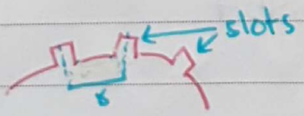


Fractional Pitch coil ≡ it's Pitch is < Pole's Pitch

$$K_d \hat{=} \frac{\sin\left(\frac{n\gamma}{2}\right)}{n \sin\left(\frac{\gamma}{2}\right)}$$

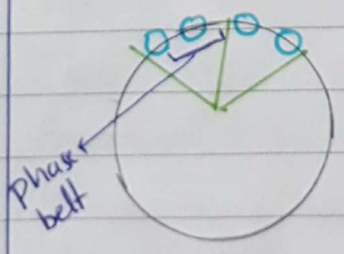
$\gamma \equiv$ space between adjacent slots

$n \equiv$ number of slots per phase belts



# of phases or groups = 3*P
belts

$P \equiv$ number of poles



No.

* Example:- 13.8 kV, 60 HZ, 12 Poles, 3-phase Y generator has:- 180 stator slots

Double layer winding

8 turns / coil

The coil pitch = 12 slots

→ Find kp, kd and kw.

$\rho = 12$ slots

Pole Pitch $= \frac{180}{12} = 15$ slots

$\therefore \rho = \frac{12}{15} * 180° = 144°$

$\therefore k_p = \sin\left(\frac{144°}{2}\right) = 0.95$

$\gamma = \frac{180°}{15} \frac{(\text{Pole Pitch})}{(\text{slots/Pole pitch})} = 12°$

$n = \frac{180 \text{ slots}}{3 * 12} = 5$

$\therefore k_d = \frac{\sin\left(\frac{5*12}{2}\right)}{5 * \sin\left(\frac{12}{2}\right)} = 0.957$

$k_w = k_p k_d = 0.95 * 0.957$

DC Machines

what? These are generators which generate DC voltages oR DC Motors into which DC current is applied.

why? They have various applications, for example

1. DC generator is used to supply I_f in 3-phase synchronous generator
2. DC motors are used as drives in electric trains

How?
- construction
- classification
- Operation
- Analysis

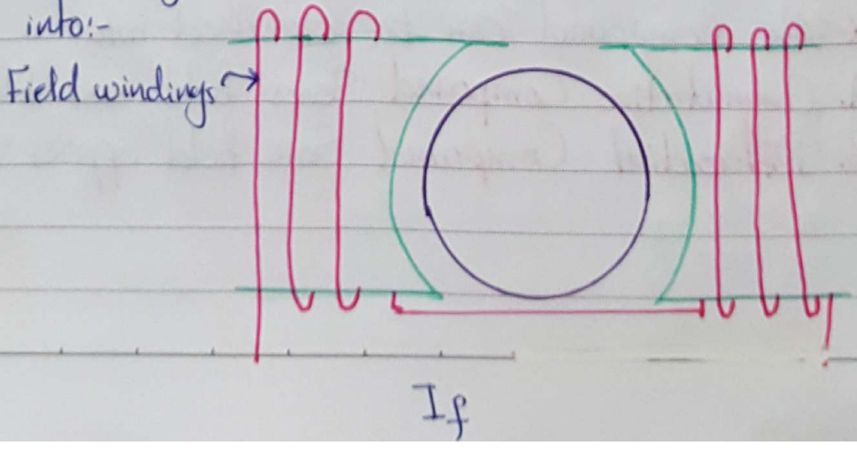
Construction:-

Rotor
Carry the Armature windings

Stator
Carry the magnetic field windings

Classification:-

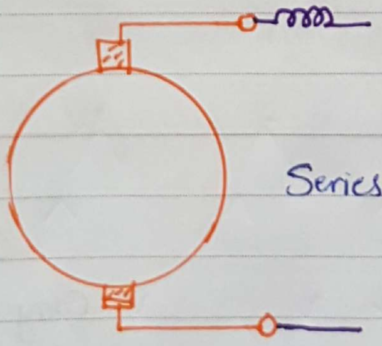
According to the source of I_f; DC machines can be clasified into:-

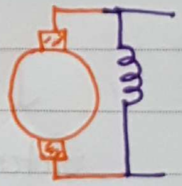


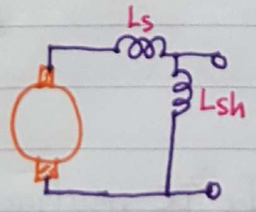
1. I_f is supplied from external sources, hence it is called Separately Excited DC machines.
2. Self excited DC machines In this case the field windings are conneted to Armature windings, In order to utilize the Rezidual magnetizum in the Field poles, Connection is made through Brushes and commutator Segments.

* According to the method of connection there are 3 types of self excited DC Machines:-

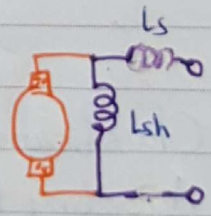
1. Series DC Machines.
2. Shunt DC Machines.
3. Compound DC Machines.
   - a. Long shunt
   - b. short shunt


Series


shunt



Long shunt



short shunt

* Also Compound can be classified into:-

جمع ← A. Commutative Compound Series field aid shunt field.

طرح ← B. Differential Compound Series field opposes shunt field.

* Operation:-

Operation of DC Machines was explained Before, during the Fundamental of AC Machines:-

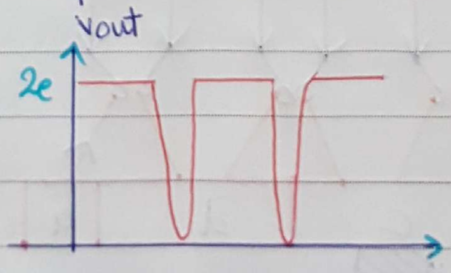
1. An applied mechanical input to the rotor generated an internal sinusoidal voltage, and by using Commutar and Brushes this AC voltage converted to DC voltage. (i.e DC generator action)
2. An applied DC current to Armature (through Brushes and commutor) a torque was produced (i.e DC Motor action)

* Commitation:-

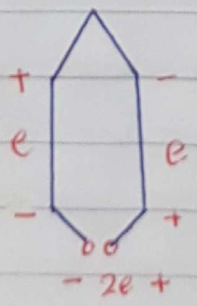
It is the process of converting the internal generated AC voltage to DC voltage by using Commutators and Brushes.

* Procedure:-

1. Previously one coil with a single turn was considered, The Brushes output:-



$e = B\ell V$ = voltage per conductors



* The dc output can be improved by increasing number of Armature coils.

2. let the Armature has 4 coils, with a double layer winding.

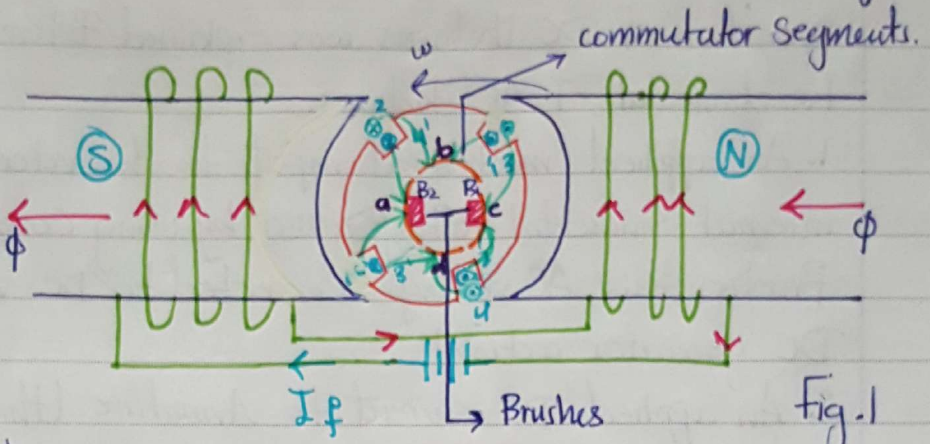


Fig. 1

1',2 → b

3,2' → c

4,3' → d

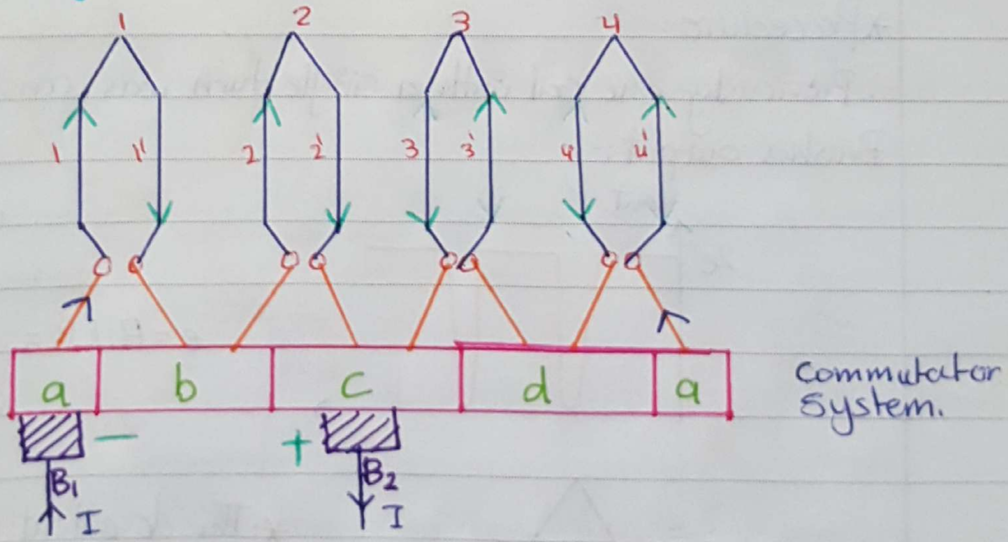
1,4' → a

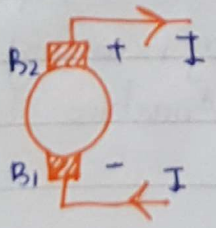
1,3',4',2 → ⊗

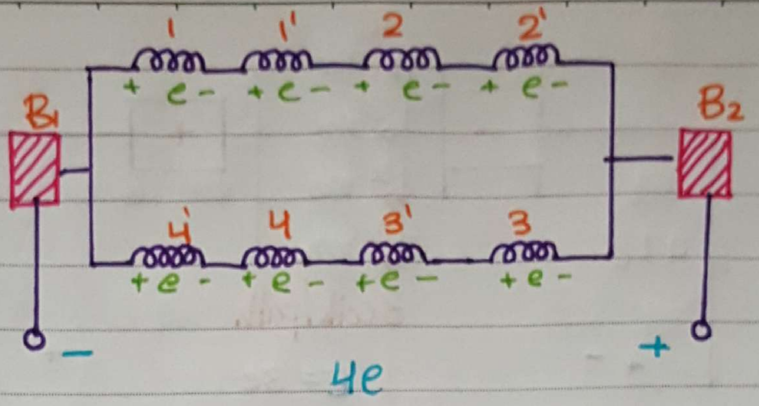
4,2',1',3 → ⊙

*there are 2 parallel paths

* Fig. 1 can be Redrawn as follows:-







$1-1'-2-2' \equiv$ path

$4'-4-3'-3 \equiv$ path

* There are two types of Armature windings:-

- Lab winding
- wave winding

* Number of parallel paths = a

For lab winding, $a = mp$

for wave winding, $a = 2m$

$P \equiv$ number of Poles

$m \equiv$ multiplicity of winding

$\equiv 1$ for simplex winding

$= 2$ for duplex winding

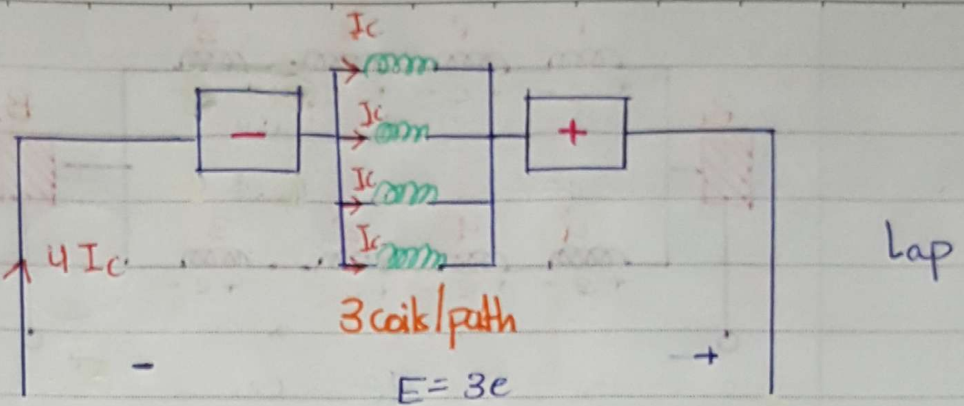
$= 3$ for Triplex winding

Illustration for 12 slots, double layer winding, number of coils =. 12

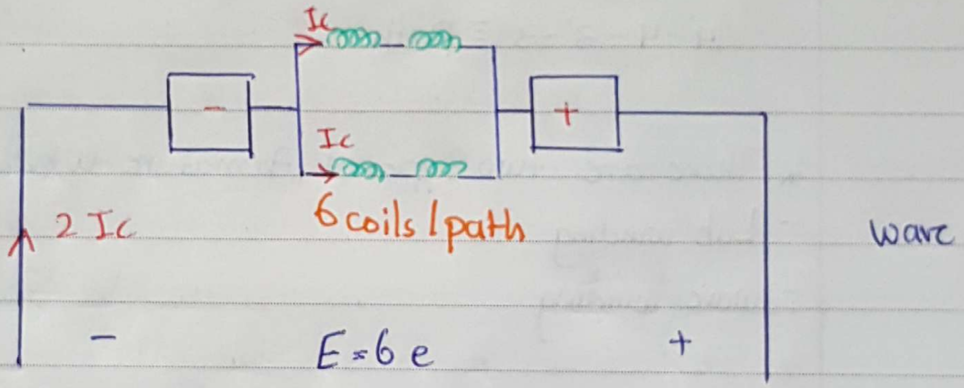
* if the machines has 4 poles.

∴ for simplex lap winding, $a = 1*4 = 4$

for simplex wave winding, $a = 2*1 = 2$



$P = 3e * 4 I_c = 12 e I_c$



$P = 6e * 2 I_c = 12 e I_c$

* if voltage / coil = e   * if Coils Current = I_c

* Expresion for the generated voltage. E. (i.e voltage across Brush)

if e_c = voltage generated per conductor. = BlV

* Single turn has 2 conductors.

if C = Number of Armature coils.

N ≡ Number of Turns per Coil.

Z = Total number of Armature conductors.

$$Z = 2 * N * C$$

$$E = \frac{Z}{a} * e_c = \frac{Z}{a} * B l V$$

$$E = \frac{Z B l V}{a} = \frac{Z B l w r}{a}$$

if flux per pole $= \phi$

Number of poles $= P$

$$\therefore B = \frac{P\phi}{Area} = \frac{P\phi}{2\pi r l}$$

$$E = \frac{ZP\phi}{2\pi r l} \frac{l w r}{a} = \frac{Z P w \phi}{2\pi a}$$

But $w = 2\pi n$ $\quad$ $n =$ Rotation speed (rps)

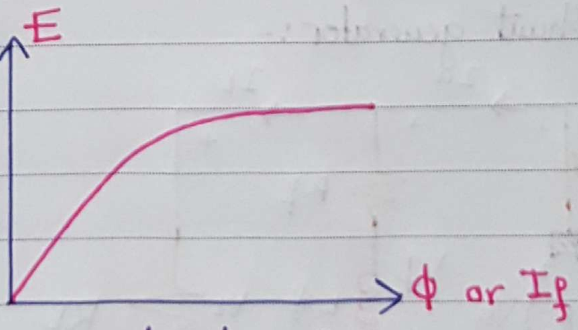
$$\therefore E = \frac{ZP\phi n}{a}$$

* For a given generator Z, P and a are constants.

$\therefore E = k \phi n$ $\quad$ $k = \frac{ZP}{a}$

* Comment :- For a constant speed (n)

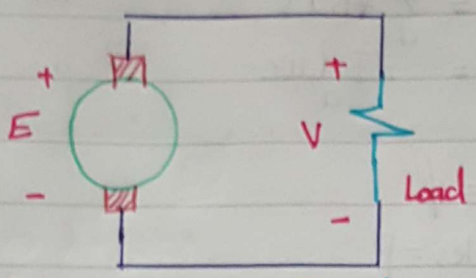
$\therefore E \propto \phi$



magnetization Curve

OR No-Load characteristic of DC generator

* Equivalent Circuit :-

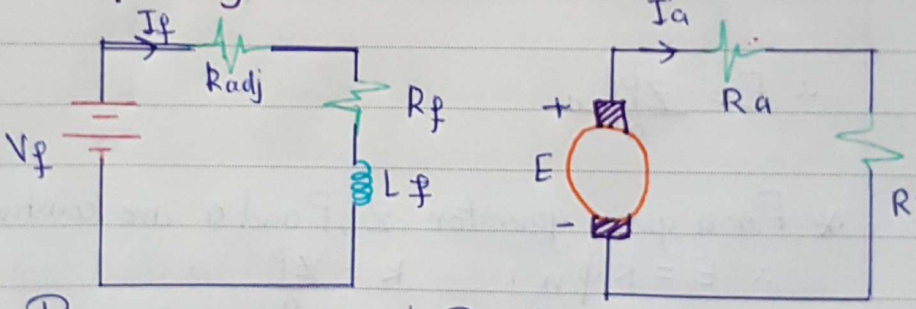


V= Terminal or Load voltage.

Relation between E and V depends on the type of generator.

* For Example :-

① Separately excited :-

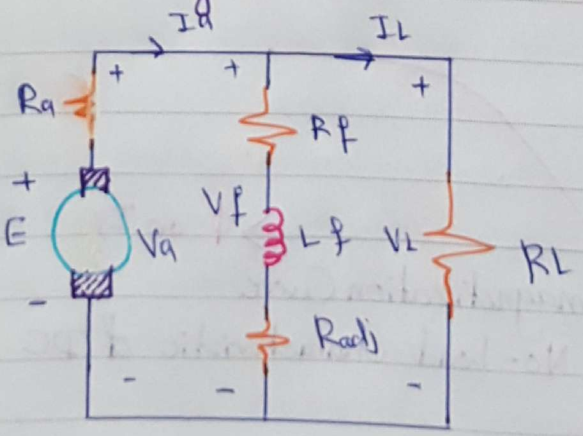


$R_a \equiv$ Armature Circuit Resistance.

$\therefore E = I_a R_a + V$

$I_a \equiv$ Armature Current.

② Shunt generator :-



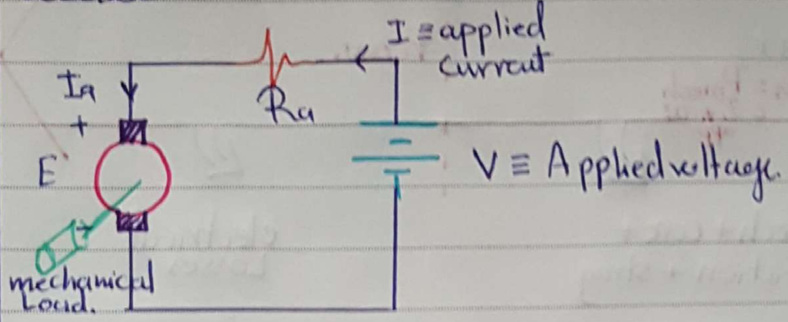
$\therefore V_L = V_a = V_f$

$I_a = I_f + I_L$

$\therefore E = I_a R_a + V_a$

No.

*** Note :-** E in the case of DC motor is called Back emf, because :-



E opposing applied voltage V

$\therefore V = E + I_a R_a$

*** Developed Torque By DC motor :-**

let $\tau_c \equiv$ Torque developed by a conductor $= B I_c l r$

$I_c \equiv$ conductor Current

$\therefore$ Developed Torque, $\tau = \tau_c Z$

$$\therefore \tau = B I_c l r Z = \frac{P\phi}{2\pi l r} * \frac{I_a}{a} * l r * Z$$

$$\therefore \tau = \frac{P \phi I_a Z}{2\pi a}$$

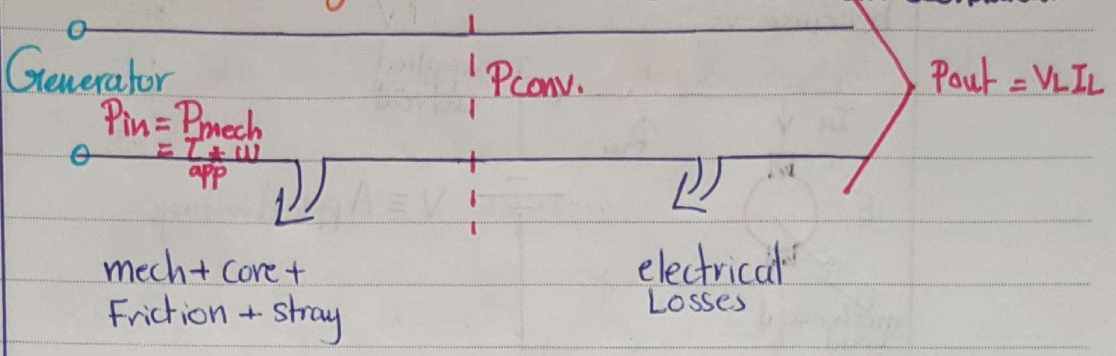
→ for a given motor P, Z and a are constant

$\therefore \tau = k \phi I_a$ $\quad k = \frac{PZ}{2\pi a}$

*** Note :-** in the case of generator τ is called τ ind or

عكسي < retarding Torque

***Power flow diagram:-** Pconv.:- Amount of electrical power converted from mechanical

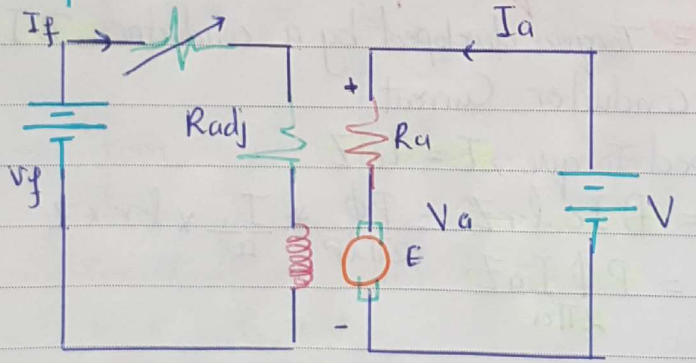


Various Types of DC Motors

objective:

To find their voltage and Current Relationships:-

Sep. excited



V_f = Applied field voltage

I_f = field Current

R_f = field Circuit Resistance

V = Applied voltage to Armature.

I_a = Armature Current

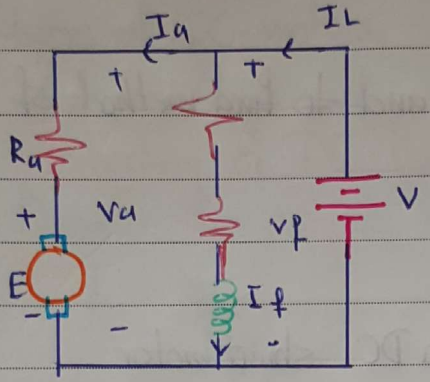
E = Back emf

V_a = Armature Terminal voltage.

$$I_f = V_f / R_f$$

$$V = E + I_a R_a \qquad V = V_a$$

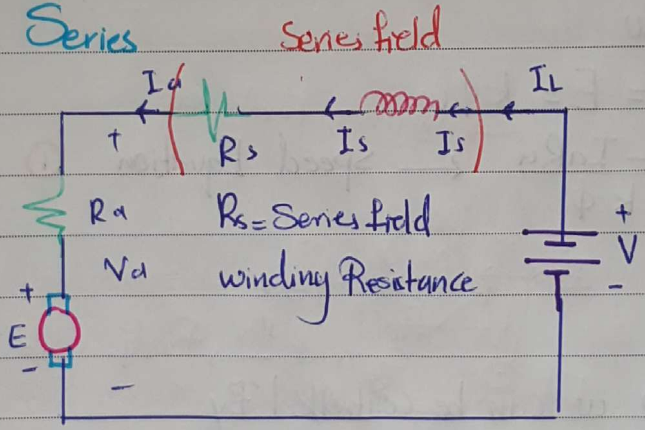
Shunt



$V = V_f = V_a$

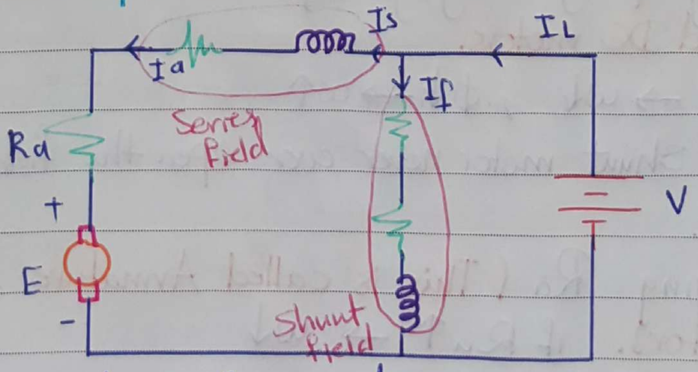
$I_L = I_f + I_a$

Series



R_s = Series field winding Resistance

Compound



Long Compound DC motor

$I_s = I_a$

$I_L = I_f + I_a$ $\quad V = I(R_s + R_a) + E$

*Speed Equation of DC Motors:-

Objective

To find such equation and to find method of controlling it's speed

Procedure

Consider for Example a DC shunt motor

$E \triangleq k\phi w$

$V - I_a R_a = E = k\phi w$

$\therefore w = \frac{V - I_a R_a}{k\phi}$ ← Speed Equation ① ٣. متغيرات

R_a → Amature Resistance

V → applied voltage

ϕ → flux → نغيرو عن طريق field current ال → Radj. عن طريق

∴ from ① w can be controlled By

1. Varying V (if V ↑ → ω↑)
2. Varing ϕ by changing I_f. This is called field Controlled DC motors.

* if ϕ↑ → ω↓ , ϕ↓ → ω↑

→ For Shunt motor never ever open the field Circuit Suddenly.

3. Changing Ra (This is called Armature Controlled DC motor). if Ra↑ → ω↓

→ most widely used are 1. and 2.

No.

*The same conclusions can be applied to the other types of DC motors.

*Starting of DC Motors:-

Consider for example DC shunt motor:-

$$I = \frac{V - E}{R_a}$$

* Initially at starting because of the Interia of the Rotor the motor does not rotate, Hence initially $E = k\phi\omega = 0$

$\therefore I_{starting} = \frac{V}{R_a}$ which much greater than Rated I_a.

$\therefore$ AT starting: I_a should be limited by using starting Resistance in series with R_a.

*Characteristics of DC motors:-

This gives Relationship between its 2 outputs:- ω and τ

Procedure:-

$E = k\phi\omega$

$E = V - I_aR_a = k\phi\omega$

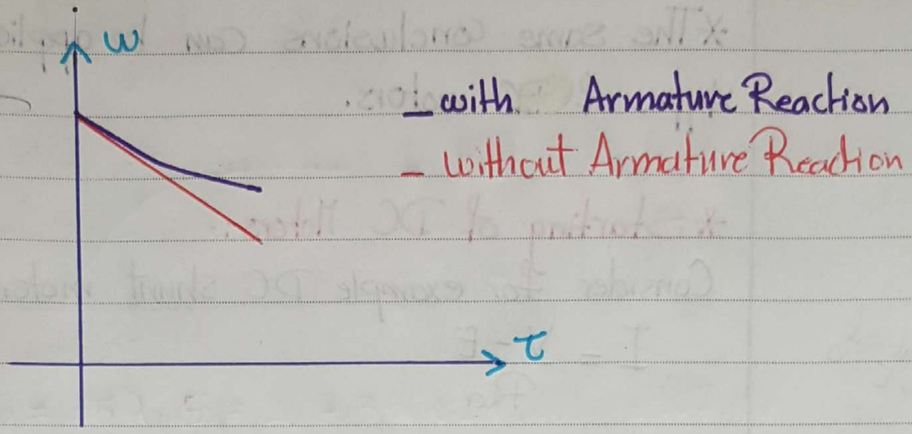
$\therefore \omega = \frac{V - I_aR_a}{k\phi}$ ...①

But $\tau = k'\phi I_a$

$\therefore I_a = \frac{\tau}{k\phi}$ ... ②

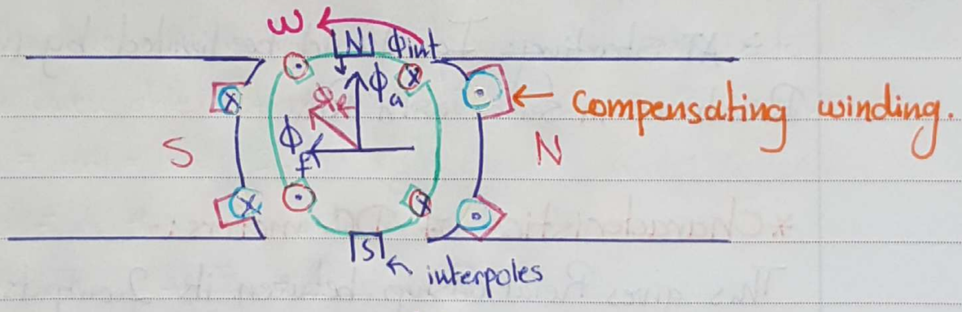
Sub. 2 into 1

$$\underset{Speed}{\omega} = \frac{V}{k\phi} - \frac{R_a \tau}{(k\phi)^2} \quad \xrightarrow{} Torque$$



*Methods of Reducing or Eliminating Armature Reaction

[1] Compensating Winding



DC Motor

* The compenstating windings are Connected in Series with Armature, in such a way that flux canceles the armature flux.

oR

[2] Using Interpoles

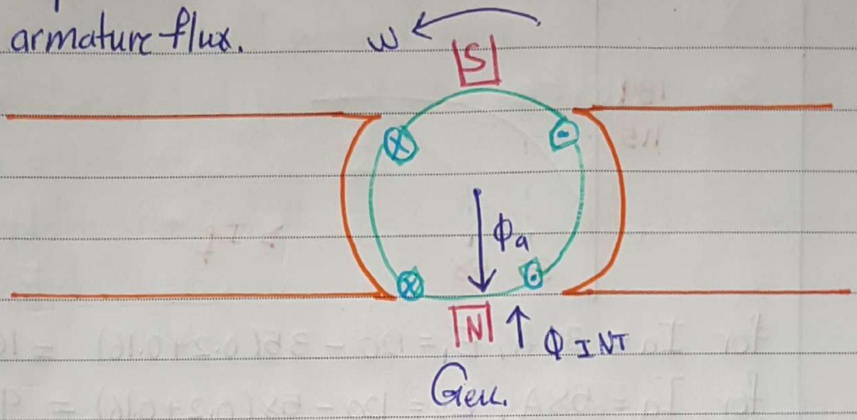
→ Polarity of Interpoles:-

In the case of motor it's equal to that to the previous main pole.

In the case of generator it's equal to the coming main pole.

No.

* Interpoles are introduced in order to produce a flux to cancel the armature flux.

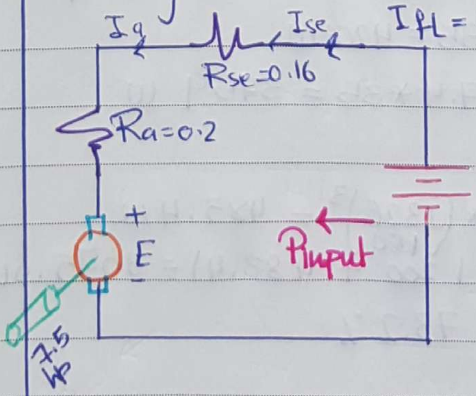


***Example:-** A 7.5 hp, 120V, Series dc motor has $R_a = 0.2\,\Omega$ and $R_{se} = 0.16\,\Omega$. At full load the Current input is 58A at the rated speed of 1050 rpm. The core losses are 200watt and the mech. losses are 240 watt at full load. Assume the mech. losses vary as the cube of motor's speed, where Core losses are constant. Given in the magnetization Curve at n= 1200 rpm, from which

I_f(A)	E(V)
35	115
58	134

→ Find the speed and efficiency of the motor if it is drawing 35A

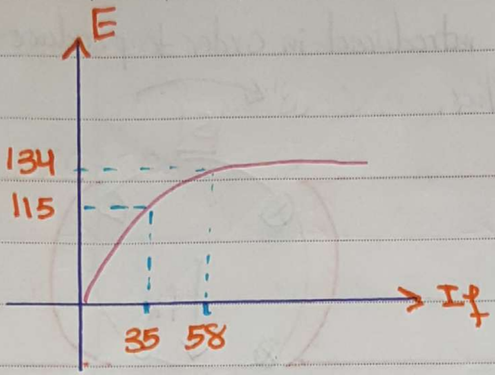
$I_{fL} = 58A$, $n_{fL} = 1050$ rpm



R_{se} = Series field Circuit Resistance

R_A = Armature circuit Resistance

No.



for $I_a = 35A$, $E_1 = 120 - 35(0.2+0.16) = 107.4V$

for $I_a = 58A$, $E_2 = 120 - 58(0.2+0.16) = 99.1V$

Since $E = k\phi n$

$\therefore \frac{E_1}{E_2} = \frac{\cancel{k}\phi_1 n_1}{\cancel{k}\phi_2 n_2}$ ① ⇒ n_1 = required speed at 35A

n_2 = 1050 rpm

Objective now is to find $\frac{\phi_1}{\phi_2}$ → This can be found from the given data of magnetization curve at $n = 1200$ rps

$E_1' = 115 = k\phi_1 1200$

$E_2' = 134 = k\phi_2 1200$

$\therefore \frac{E_1'}{E_2'} = \frac{115}{134} = \frac{\phi_1}{\phi_2}$ ... ②

Sub. ② in ① → $n = 1326$ rpm

$\eta = \frac{\text{output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{P_{conv} - (\text{mech} + \text{core})\text{Losses}}{\text{Input}}$

Input = VI = 120 * 35 = 4200 w

$P_{conv} = E I_a = 107.4 * 35 = 3759$ w

Core losses = 200

mech losses = $240 * \left(\frac{1326}{1050}\right)^3 = 483.4$ w

$\therefore P_{out} = 3759 - (200 + 483.4) = 3075.6$ w

$\therefore \eta = \frac{3075.6}{4200} = 73.2\%$

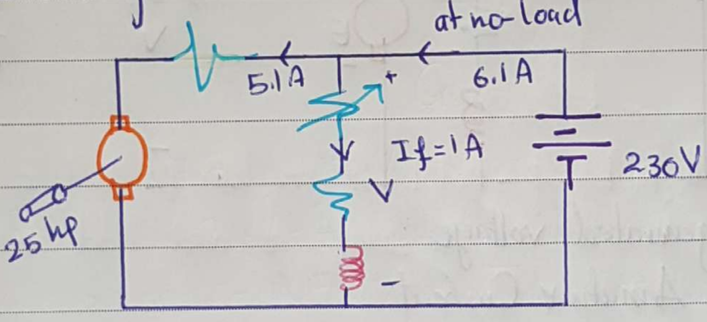
No.

*** Example:** when connected to a Source of Rated voltage, a 25hp, 230V shunt motor draw 6.1A at no-load. The field Current is 1A at no-load at a speed of 1300 rpm.

$R_a = 1\Omega$.

1. find the speed of the motor when $I_a = 80A$ (rated value).
2. Speed Regulation.

at no-load



$$E = V - I_a R_a$$

$$k\phi n = V - I_a R_a$$

$$n = \frac{V - I_a R_a}{k\phi}$$

$$\therefore \frac{n_1}{n_2} = \frac{V - I_{a1} R_a}{V - I_{a2} R_a}$$

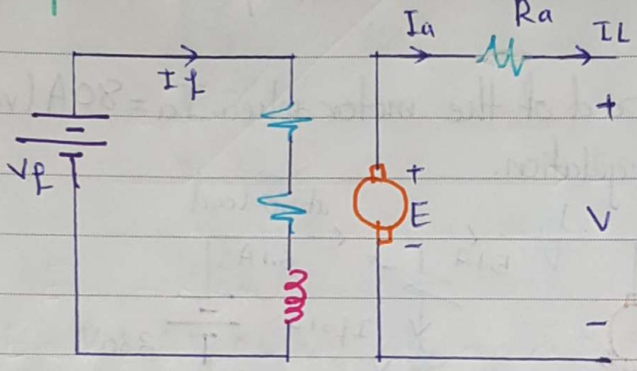
$$\frac{1300}{n_2} = \frac{230 - 5.1 * 1}{230 - 80 * 1} \rightarrow 867 \text{ rpm}$$

$$\text{Speed Regulation} = \frac{n_{nL} - n_{fl}}{n_{fl}} = \frac{1300 - 867}{867} * 100\% = 50\%$$

DC generators:-

Voltage - Current Relationship.

Sep. excited



$I_f = V_f / R_f$

$E = V + I_a R_a$

$I_a = I_L$

E ≡ generated voltage

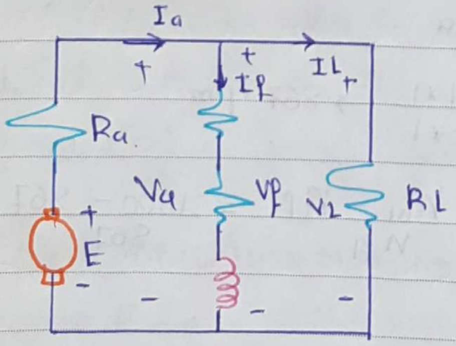
Ia ≡ Armature Current

Vf ≡ field voltage

If ≡ field Current

IL ≡ line or Load Current

Shunt

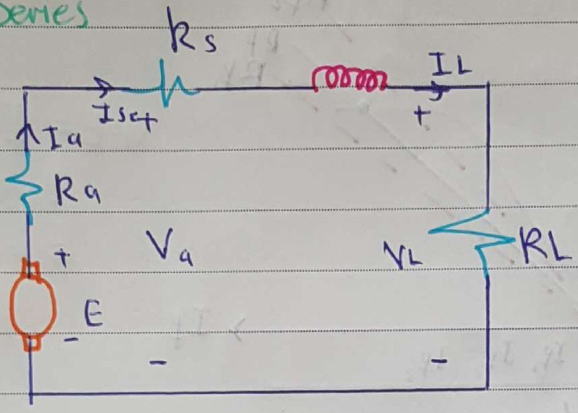


$I_a = I_f + I_L$

$V_a = V_f = V_L$

$E = V_a + I_a R_a$

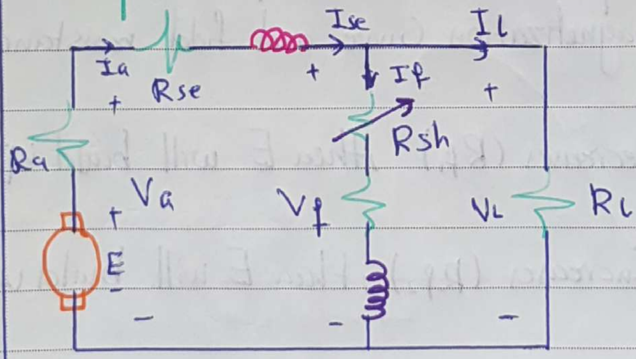
Series



$I_a = I_{se} = I_L$

$E = V_a + I_a R_a$

Compound



Long shunt

$I_a = I_{se} = I_f + I_L$

$V_L = V_f$

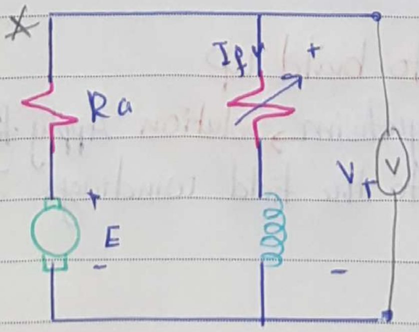
$E = I_a R_a + V_a$

$-V_a + I_{se} R_{se} + V_L = 0$

Build-up of voltage in self excited generator

The generation of voltage depends on the Existance of Residual Magnetism in the Pole.

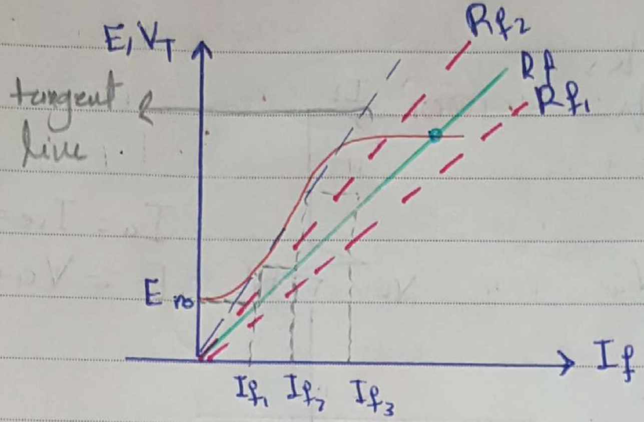
Consider for example a shunt generator.



$I_F = \frac{V_T}{R_F}$

$V_T = E - I_a R_a$

$I_f = \frac{V_T}{R_f}$



$E_{res} = k \phi_{res} n$

E will build up until it reaches point (i.e Intersective between magnetization Curve and field resistance line)

*If Rf decreases (R_{f1}), then E will build up to higher value.

*If Rf Increases (R_{f2}), then E will build up to lower value.

→ If Rf is increase to a higher value, then the generator may fail to build up. This is called the Critical field resistance (R_{fc})

*R_{fc} is tangent to magnetization Curve.

* Causes of failure to build up

1. lack to Residual Magnetisim → Solution Apply for a while and external dc current to the field windings. This is called "Flashing Out"

No.

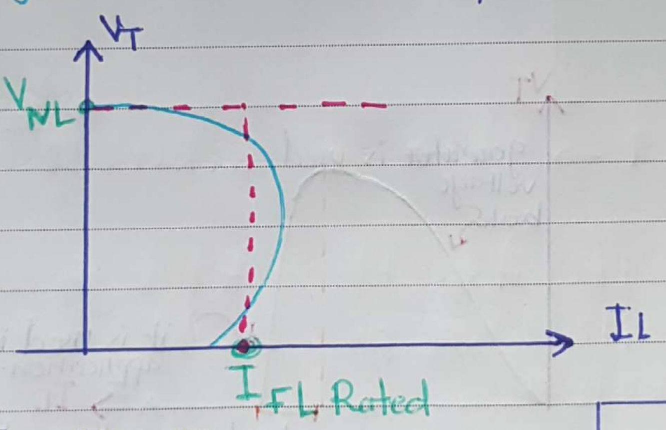
2. Field windings connection are in such a way that I_f Cancels $\phi_{res.}$ → Solution Reverse the Connections

3. R_f > critical field resistance (e.g o/c in the field windings)

4. o/c in the Armature windings

* Load characteristic of shunt generator :-

→ Objective: to find Relationship between V_T and I_L

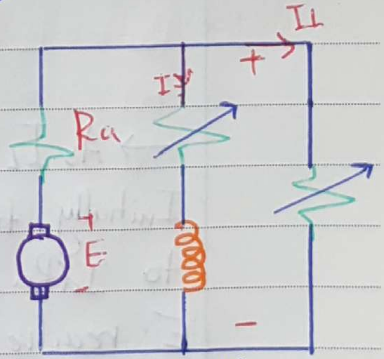


$I_a = I_f + I_L$

$V_T = E - I_aR_a$

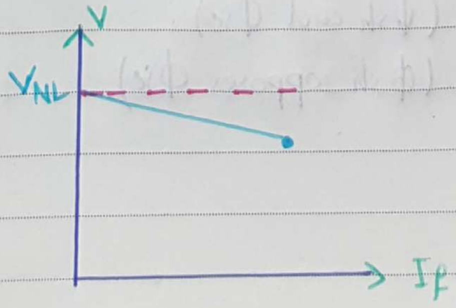
$I_f = V_T/R_f$

$E = k\phi s \quad \phi \propto I_f$

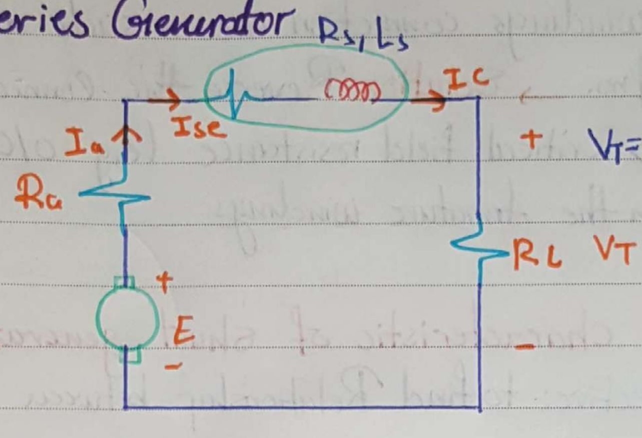


$I_L\uparrow \rightarrow I_aR_a\uparrow \rightarrow V_T\downarrow \longrightarrow I_f\downarrow \rightarrow \phi\downarrow \longrightarrow E\downarrow$

after a Certain Loading Point V_T is going to Collapse
↳ يسقط



* Series Generator

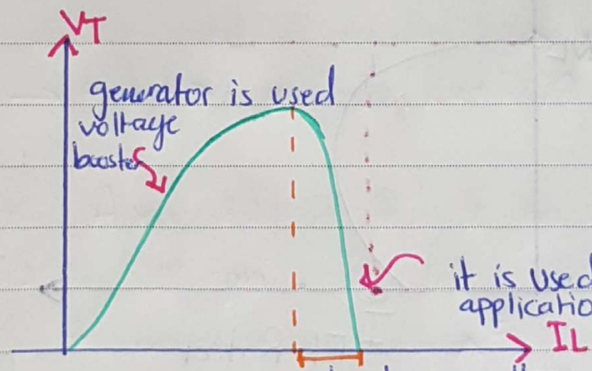


$V_T = E - I_a(R_a + R_s)$

$I_a = I_L = I_{se}$

$E = k\phi n$

$\phi \propto I_{se}$



it is used in the welding machine application.

↳ Large voltage drop and current is constant

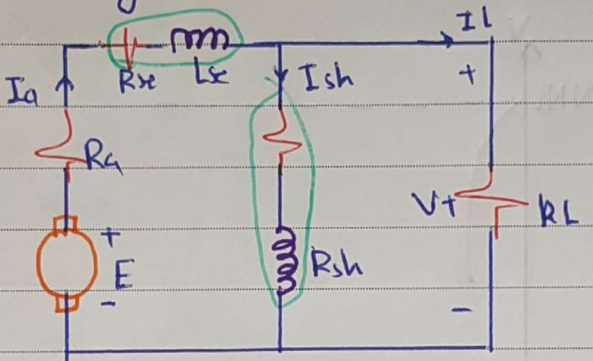
→ As I_L↑ ($I_L = I_a = I_{se}$) → E↑ and I_aR_a↑ → However Initially the Increase in E is greater than Reduction due to I_aR_a, Conequently V_T is going to increase. → when E reaches Saturation Value, then after that the increase in I_aR_a will be greater than increase in E

* Compound Generator :-

- Cummulative (ϕsh aid ϕse)
- Differential (ϕsh opposes ϕse)

1. Cummulative

Consider Long Shunt Comp. Generator



$V_T = E - I_a(R_s + R_a)$

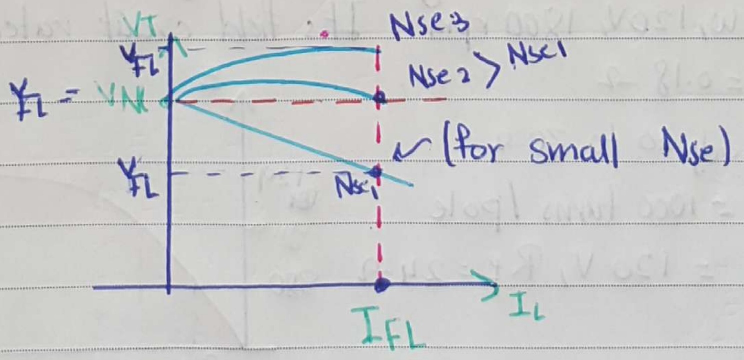
$I_a = I_L + I_{sh}$

$I_a = I_{se}$

Two mmf :- shunt $I_{sh} N_{sh}$

Series $I_{se} N_{se}$

$E = k(\phi_{sh} + \phi_{se}) * n$



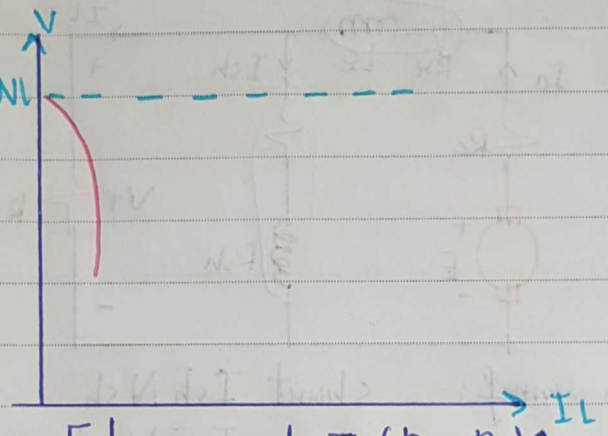
Nse 3 > Nse2 > Nse 1

→ Depending on the Number of turns of Series Field winding one may obtain :-

① Under Compound
② Flat Compound
③ Over Compound

2. Differential

$E = k(\phi_{sh} - \phi_{se})\, n$



$I_L \uparrow \rightarrow E \downarrow \rightarrow$ and $I_a(R_a + R_s) \uparrow$

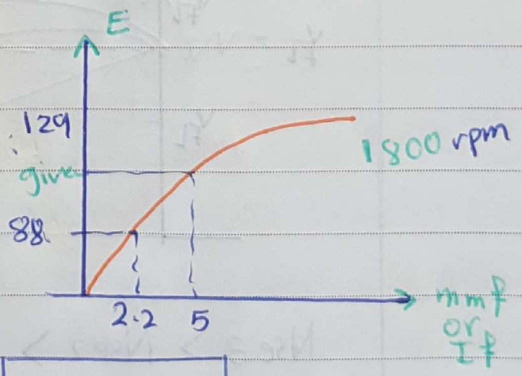
***Example:-** A separately excited dc generator is rated at:-
6 kW, 120V, 1800 rpm. It's field circuit ratedat 5A

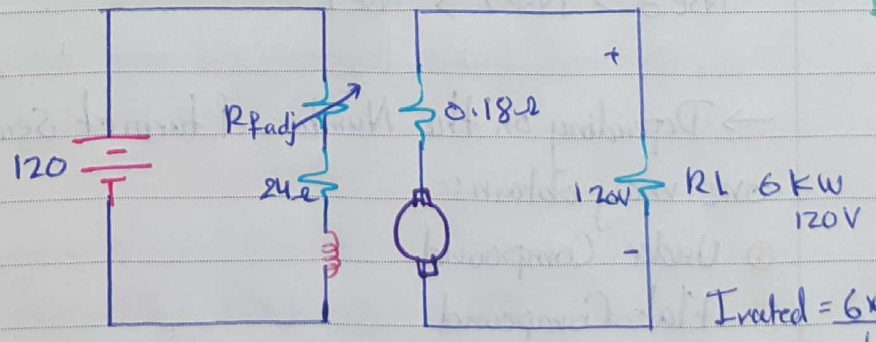
$R_a = 0.18\ \Omega$

$R_{f\,adj} = 0$ to $30\ \Omega$

$N_f = 1000$ turns / pole

$V_f = 120$ V, $R_f = 24\ \Omega$





$I_{rated} = \frac{6 \times 10^3}{120}$
$= 50A$

a. If generator operating at no-Load what is the range of voltage adjustments that can be made or achieved by changing Radj

$R_{adj} = 0$, $I_f = \frac{120}{24} = 5A \rightarrow 5 * 1000 = 5000$ A-t

$R_{adj} = 30\Omega$, $I_f = \frac{120}{24+30} = 2.2 \longrightarrow 2.2 * 1000 = 2200$ A.t

b. If Radj is allowed to vary from $(0 \rightarrow 30)\Omega$ and generator speed allowed to vary from (1500 - 2000) rpm what are the max. and min. no-load voltage?

Assuming ϕ to be constant , $E = kn$

$$\frac{E_1}{E_2} = \frac{n_1}{n_2} \rightarrow E_2 = E_1 \frac{n_2}{n_1}$$

for 1500 rpm, $E_{min} = 88 \frac{1500}{1800} = 73.3V$

$E_{max} = 129 \frac{1500}{1800} = 107.5V$

for 2000 rpm, $E_{min} = 88 \frac{2000}{1800} = 97.8V$

$E_{max} = 129 \frac{2000}{1800} = 143.3V$

Scanned by CamScanner