

Student Name: \_\_\_\_\_

12<sup>th</sup> 2011

time allowed: 1 1/4 Hour

Student Number: \_\_\_\_\_

Section: \_\_\_\_\_

Date: \_\_\_\_\_

Problem 1 A three-phase 400 V, 50 Hz, 4-pole, Y-connected, induction motor. It has the following constants:  $R_1 = 0.6 \Omega$ ,  $X_1 = 0.6 \Omega$ ,  $R_2' = 0.3 \Omega$ ,  $X_2' = j 1 \Omega$ ,  $X_m = j 30 \Omega$ . The speed-torque curve shown in Figure (1). A resistance  $R_{add}$  is added to rotor resistance  $R_2'$  to increased starting torque as shown in dotted and dash-dotted. Find the two values  $R_{add,1}$  and  $R_{add,2}$ .

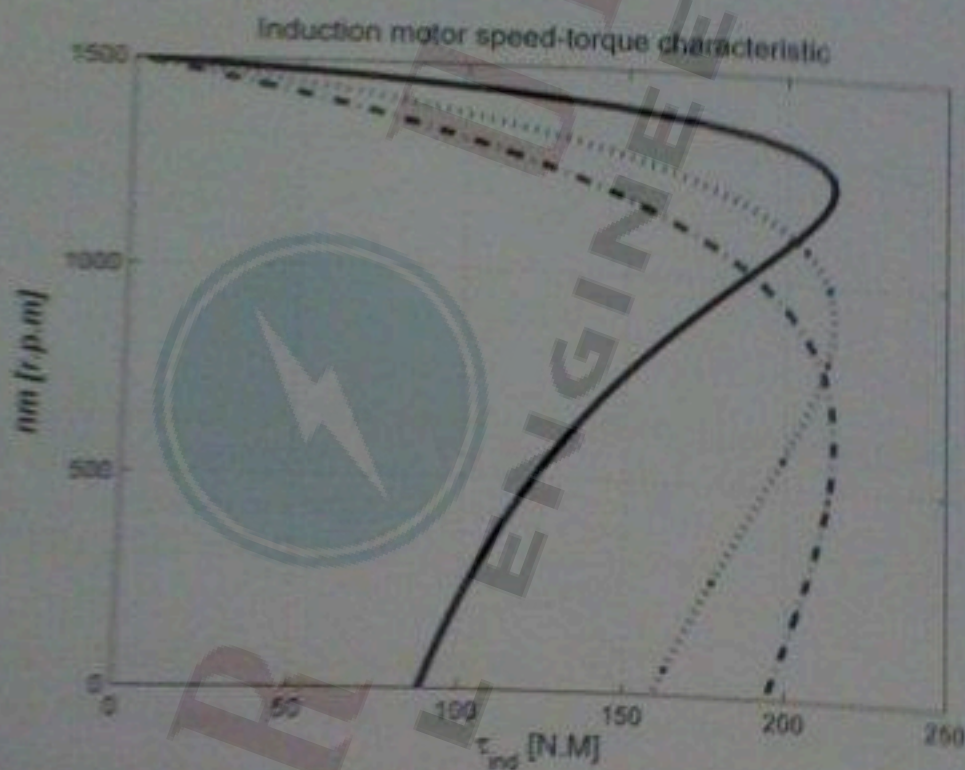


Figure 1

$$n_{r,max} = 1230 \Rightarrow s'_{max} = \frac{1500 - 1230}{1500} = 0.18$$

$$n_{r,max} = 900 \Rightarrow s''_{max} = \frac{1500 - 900}{1500} = 0.4$$

$$n_{r,max} = 630 \Rightarrow s'''_{max} = \frac{1500 - 630}{1500} = 0.58$$

$$\frac{s''}{s'} = \frac{R_2' + R_{a1}}{R_2'}$$

$$R_{a1} = \left(\frac{s''}{s'} - 1\right) R_2'$$

$$R_{a1} = (2.22 - 1) R_2' = 0.366 \Omega$$

$$R_{a2} = (3.22 - 1) R_2' = 0.667 \Omega$$



**Problem 2** A 400-V 55-hp four-pole  $\Delta$ -connected 50-Hz three-phase induction motor has a full-load slip of 6 percent, an efficiency of 90 percent, and a power factor of 0.85 lagging. This motor is to be started with an autotransformer to reduced voltage starter as shown in Fig. 3. The rotor losses is 2270 W. Find:

- (a) The rotating power losses at rated full load slip and value of  $R_2$ ?
  - (b) If  $V_r$  is the reduced voltage and  $x = \frac{V_r}{V_T}$  find  $x$  to obtain the lower curve of  $\tau$  versus  $n_m$
  - (c) Find the value of  $\tau$  after reduction the voltage
- Note: approximate the torque value up to  $\pm 10$  N.M and the value of current up to  $\pm 5$  A

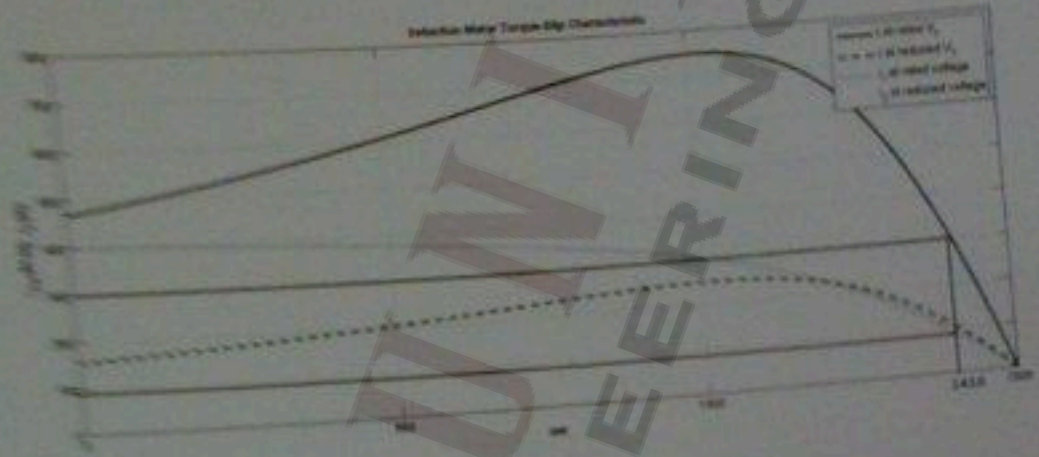


Figure 2

From the curve the  $\tau_{full}$  value opposite of 1410 r.p.m is 300 N.M, from which may find

$$P_{mech} = \tau_{full} \times \omega_{synchronous} = 300 \times 188.5 = 47124 \text{ W}$$

The converted power from electrical to mechanical is

$$P_{me} = P_{mech} + P_{rotor} = 47124 + 2270 = 49394 \text{ W}$$

The rotational losses

$$P_{rotor} = P_{me} - P_{mech} = 49394 - 47124 = 2270 \text{ W}$$

From the current curve may find  $I_{2,rated} = 75$  A. This leads to find the  $R_2 = \frac{P_{rotor}}{3I_{2,rated}^2} = \frac{2270}{3 \times 75^2} = 0.13452 \Omega$

From the original curve at rated voltage may find  $\tau_{started} = 468$  N.M. and  $\tau'_{started} = 156$  N.M. then  $\frac{156}{468} = 0.333$

the voltage ratio is  $x = \sqrt{\frac{\tau'_{started}}{\tau_{started}}} = \sqrt{\frac{156}{468}} = 0.577$

$$V_r = x \times V_T = 0.577 \times 400 = 231 \text{ V}$$

$$I_{sc} = \frac{1}{x} \times I_{sc,rated} = \frac{1}{0.577} \times 400 = 230 \text{ A}$$

$$I_{rat} = \frac{P_{me}}{\sqrt{3} \times V_T \times \text{PF}} = \frac{49394}{\sqrt{3} \times 400 \times 0.85} = 77.4 \text{ A}$$

$\frac{I_{sc}}{I_{rat}} = \frac{230}{77.4} = 2.98$  reduced to  $\frac{I_{sc}}{I_{rat}} = \frac{231}{77.4} = 2.98$



SOLUTION This problem is best solved with MATLAB, since it involves calculating the torque-speed values at many points. A MATLAB program to calculate and display both torque-speed characteristics is shown below. Note that this program shows the torque-speed curve for both positive and negative directions of rotation. Also, note that we had to avoid calculating the slip at exactly 0 or 2, since those numbers would produce divide-by-zero errors in  $Z_f$  and  $Z_b$  respectively.

```
% M-file: torque_speed_curve3.m
% M-file create a plot of the torque-speed curve of the
% single-phase induction motor of Problem 10-4.

% First, initialize the values needed in this program.
r1 = 1.80; % Stator resistance
x1 = 2.40; % Stator reactance
r2 = 2.50; % Rotor resistance
x2 = 2.40; % Rotor reactance
xm = 60; % Magnetization branch reactance
v = 120; % Single-Phase voltage
n_sync = 1800; % Synchronous speed (r/min)
w_sync = 188.5; % Synchronous speed (rad/s)

% Specify slip ranges to plot
s = 0:0.01:2.0;

% Offset slips at 0 and 2 slightly to avoid divide by zero errors
s(1) = 0.0001;
s(201) = 1.9999;

% Get the corresponding speeds in rpm
nm = (1 - s) * n_sync;

% Calculate Zf and Zb as a function of slip
zf = (r2 ./ s + j*x2) * (j*xm) ./ (r2 ./ s + j*x2 + j*xm);
zb = (r2 ./ (2-s) + j*x2) * (j*xm) ./ (r2 ./ (2-s) + j*x2 + j*xm);

% Calculate the current flowing at each slip
i1 = v ./ (r1 + j*x1 + zf + zb);

% Calculate the air-gap power
p_ag_f = abs(i1).^2 * 0.5 * real(zf);
p_ag_b = abs(i1).^2 * 0.5 * real(zb);
p_ag = p_ag_f - p_ag_b;

% Calculate torque in N-m.
t_ind = p_ag ./ w_sync;

% Plot the torque-speed curve
figure(1)
plot(nm, t_ind, 'Color', 'b', 'LineWidth', 2.0);
xlabel('\itn_{m} \rm(r/min)');
ylabel('\tau_{ind} \rm(N-m)');
title('Single Phase Induction motor torque-speed characteristic', 'FontSize', 12);
grid on;
hold off;
```



**Problem 3** A three-phase 400 V, 50 Hz, 4-pole, Y-connected, induction motor. The torque speed curve is given in Figure (3). Neglect rotational, core, friction and windage losses except the load losses (stator and rotor). It has the following constants:  $R_1 = 0.25 \Omega$ ,  $R_2 = 0.4 \Omega$ ,  $X_2 = j 1 \Omega$ ,  $X_m = j 50 \Omega$ . If the motor operates at  $\tau_d = 221.2 \text{ N.M}$  find  $X_1$

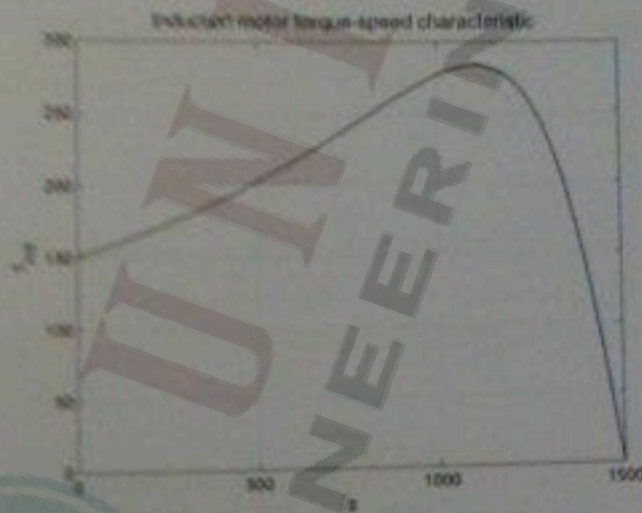


Figure 3

$$R_2 = 0.4; X_2 = j1; X_m = j50; R_1 = 0.25; s = 0.12; V_b = 400/\sqrt{3}$$

$$Z_F = \frac{jX_m \times (R_2/s + X_2)}{jX_m + R_2/s + X_2} = 3.19 + j1.19$$

From curve  $\tau_{ind} = 221.2 \text{ N.m}$  and The  $P_{ag} = \tau \times \omega_{sys} = 34728 \text{ W}$  then

$$I_A = \sqrt{\frac{P_{ag}}{3 \times R_F}} = 60.24 \text{ A}$$

$$Z_{eq} = \frac{V_b}{I_A} = \frac{400/\sqrt{3}}{60.24} = 3.834$$

$$Z_{eq} = Z_1 + Z_F = \sqrt{(R_1 + R_F)^2 + (X_1 + X_F)^2}$$

$$(X_1 + X_F) = \sqrt{Z_{eq}^2 - (R_1 + R_F)^2} = \sqrt{3.834^2 - (0.25 + 1.19)^2} = 3.553$$

$$X_1 = 3.553 - 1.19 = 2.36$$

$$I_2 = I_A \times \frac{jX_m}{j(X_m + X_2) + R_2} = 40.94 \text{ A}$$

$$Z_{F,sc} = \frac{jX_m \times (R_2/1 + X_2)}{j(X_m + R_2/1 + X_2)} = 0.2316 + j1.3491$$

$$P_{sc} = 3I_2^2 R_1 = 692 \text{ W}$$

$$P_m = P_{ag} + P_{sc} = 25870 \text{ W}$$

$$P_{rd} = 3I_2^2 R_2 = 1259 \text{ W}$$

$$P_{out} = P_{ag} - P_{rd} = 23919 \text{ W}$$

$$\eta = \frac{P_{out}}{P_m} = 0.925$$

$$\tau = 160 \text{ N.m}$$

$$I_{sc} = \sqrt{\frac{P_{sc}}{3 \times R_1} + \sqrt{\frac{P_{sc}}{180 \times 0.03}}} = 2.6$$

$$\tau_{sc} = 55 \text{ N.m}; P_{st} = \tau_{sc} \times \omega_{sys} = 8340 \text{ W}$$

$$I_{A,sc} = \sqrt{\frac{P_{st}}{3 \times R_F}} = 110 \text{ A}$$



The resulting torque-speed characteristic is shown below:



0-5. A 220-V, 1.5-hp 50-Hz, two-pole, capacitor-start induction motor has the following main-winding impedances:

$$R_1 = 1.40 \, \Omega$$

$$X_1 = 2.01 \, \Omega$$

$$X_M = 105 \, \Omega$$

$$R_2 = 1.50 \, \Omega$$

$$X_2 = 2.01 \, \Omega$$

At a slip of 0.05, the motor's rotational losses are 291 W. The rotational losses may be assumed constant over the normal operating range of the motor. Find the following quantities for this motor at 5 percent slip:

(a) Stator current

(b) Stator power factor

(c) Input power

(d)  $P_{AG}$

(e)  $P_{conv}$

(f)  $P_{out}$

(g)  $\tau_{ind}$

(h)  $\tau_{load}$

(i) Efficiency

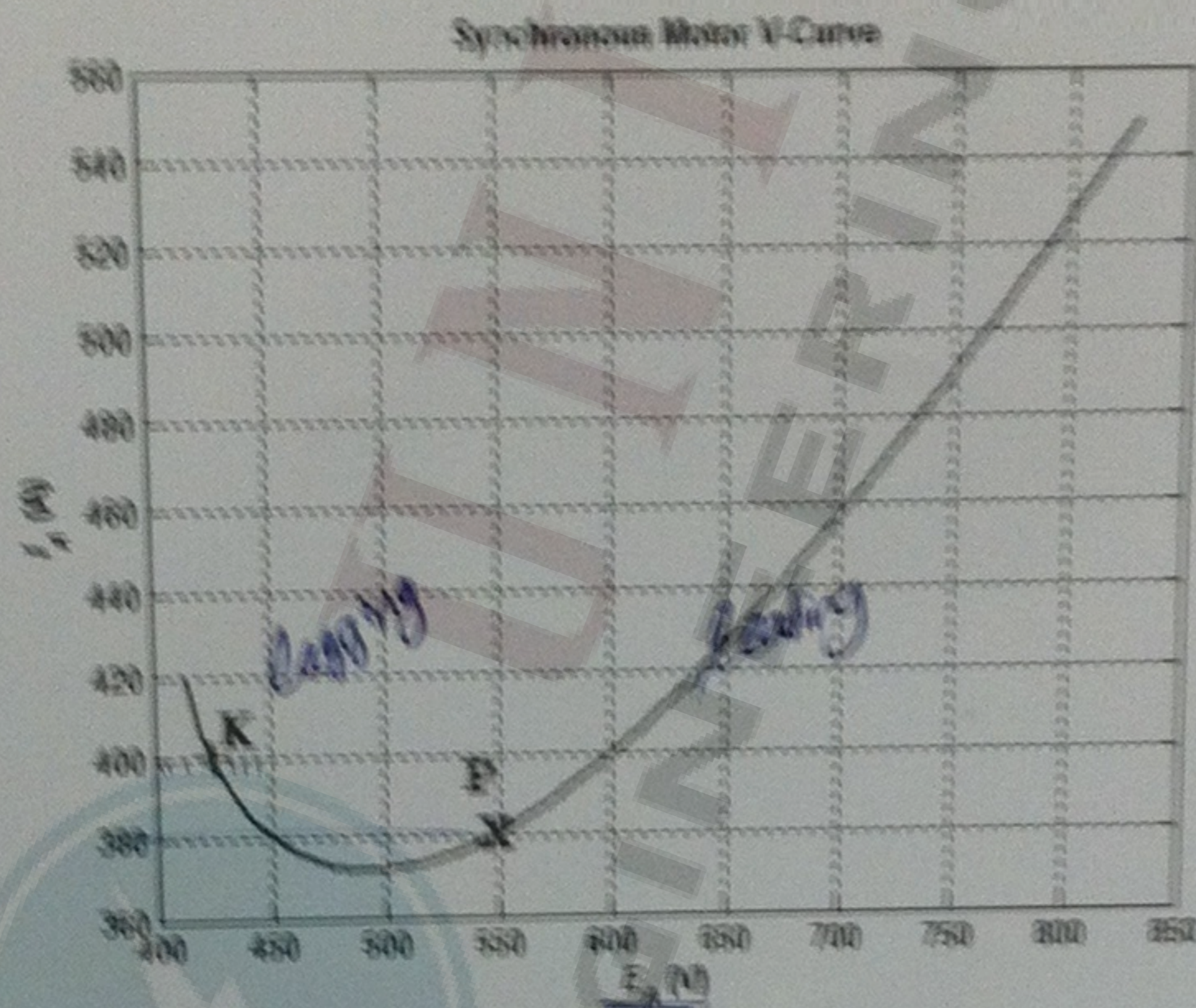
SOLUTION The equivalent circuit of the motor is shown below



The Fig. shows the V-curve of a 3 $\phi$ , 400-HP, 480 V, 50 Hz, 2-pole Y-connected synchronous motor has a reactance of 1.1  $\Omega$ . Neglect friction, windage, and core losses for this problem.

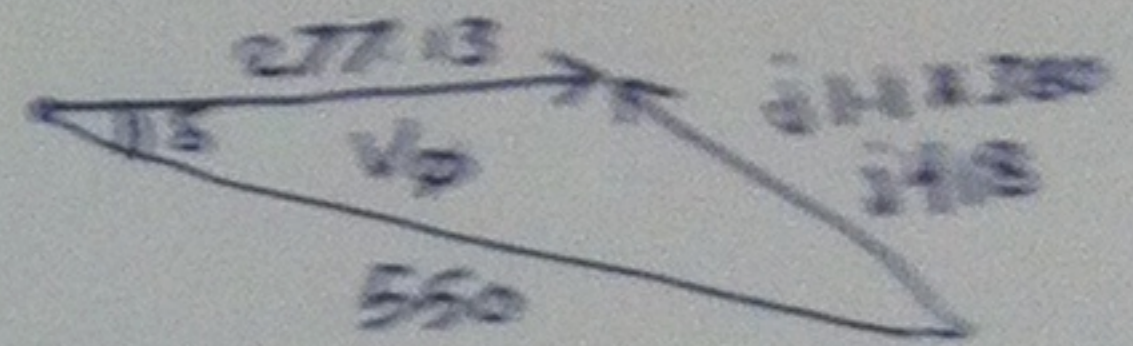
- (a) Find the torque angle  $\delta$  and the power factor if the motor operates at points P and K.  
 (b) Explain your answer by drawing exact phasor diagram.

$P_{out} = 400 \times 746 = 298400$   
 $V_{\phi} = \frac{480}{\sqrt{3}} = 277.13$   
 $X_s = 1.1$



9

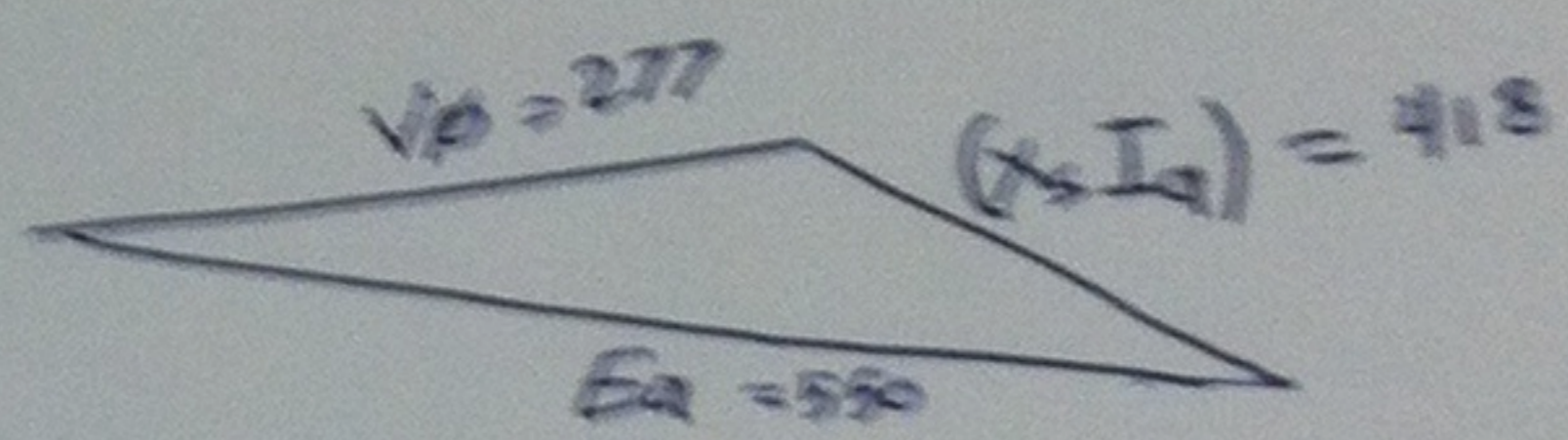
@ point P  $E_f = 550, I_a = 380, V_{\phi} = 277.13$



$c^2 = a^2 + b^2 - 2ab \cos \delta$  — cosine law

when  $\delta = 0$

$c = a + b$



$(X_s I_a)^2 = (V_{\phi})^2 + (E_f)^2 - 2 E_f V_{\phi} \cos \delta$

$(418)^2 = (277)^2 + (550)^2 - 2 \times 550 \times (277.13) \cos \delta$

$\Rightarrow \cos \delta = \frac{-204505}{-2 \times 550 \times 277.13} = 0.671$

$\Rightarrow \delta_P = 47.848^\circ$   
Leading

@ point K  $E_f = 425, I_a = 400, V_{\phi} = 277.13 \Rightarrow X_s I_a = 440$

$(X_s I_a)^2 = (V_{\phi})^2 + E_f^2 - 2 E_f V_{\phi} \cos \delta$

$\Rightarrow \cos \delta = \frac{440^2 - 425^2 - 277.13}{-2 \times 425 \times 277.13} = 0.27$

$\delta = \cos^{-1} 0.27 = 74.33^\circ$  Lagging

Changes

if you want to assume $V_{\phi} = 480$	$\Rightarrow \delta_K = 57.79^\circ$	pf = 0.9329
$E_{f_{min}} = \frac{480}{\sqrt{3}}$	$\Rightarrow \delta_K = 65.68^\circ$	pf = 0.41
$= \frac{550}{\sqrt{3}}$	$\Rightarrow \delta_P = 59^\circ$	pf = 0.66
$V_{\phi} = 480$		