

MACHINES I SUMMARY

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Revision of magnetic circuits

* Principles of operation of Electrical Machines:-

1. Any current carrying conductor or coil generate magnetic field (ϕ) or (H) around it.
2. A time varying field (ϕ) will induce a voltage in any coil cutting the field. (Principle of operation of Transformer)
3. If a conductor cut ϕ , then a voltage will be induced in that conductor. (This is principle of operation of Generator.)
4. If a current carrying conductor is located in ϕ , then it will develop a torque. (Principle of operation of Motor)

* Magnetic Circuits:-

N : # of turns.

i : Applied current.

ϕ : generated flux (ϕ) wb.

From Ampere's law

$$H L_c = N i$$

Field strength $H = \frac{N i}{L_c}$ (A.t/m) --- [1]
 $L_c \rightarrow$ Mean length of flux path

* $B = \frac{\phi}{A}$ (Tesla = wb/m²) --- [2]

and $B = \mu H$ --- [3]
 $\mu \rightarrow$ permeability

* From [1], [2] and [3]

MMF \rightarrow $N i = \frac{L_c}{\mu} \phi$

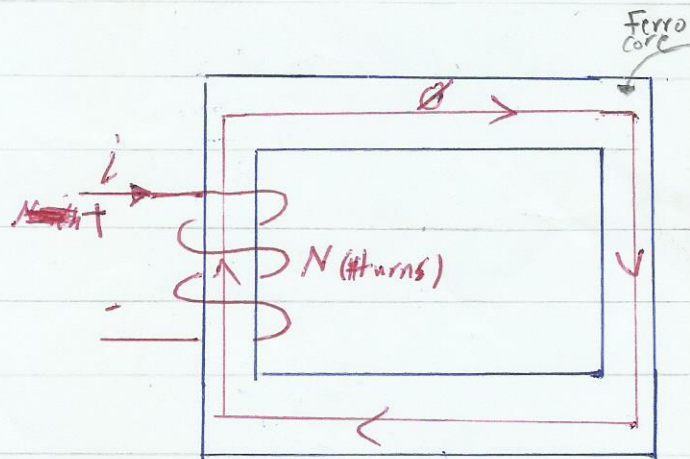


Fig 1

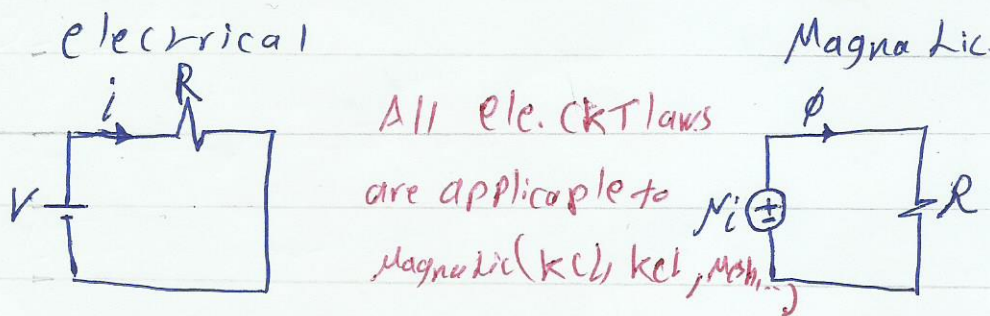
Note:-

$M = \mu_0 \mu_r$
 $\mu_r \rightarrow$ relative permeability
 $\mu_0 \rightarrow$ Permeability of free space = $4\pi \times 10^{-7}$

* Analogy Between Electrical and Magnetic Circuits:

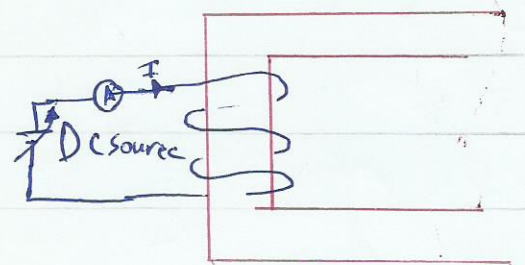
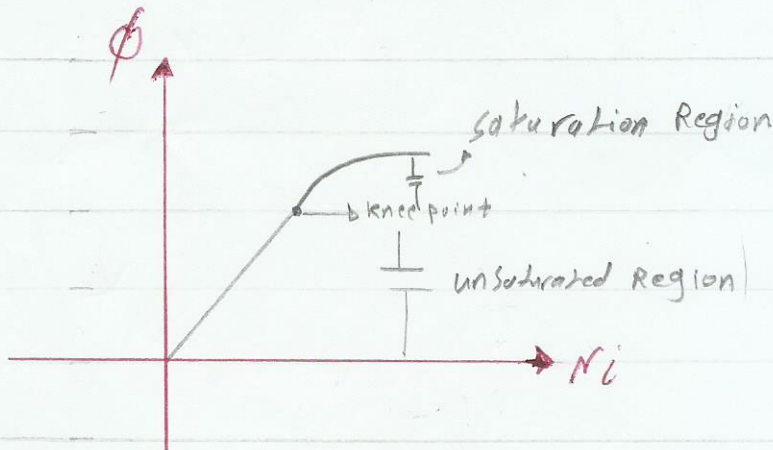
Electrical	Magnetic
1. emf, V	MMF (Ni), A.t
2. i & A	ϕ , wb
3. Resistance, Ω	Reluctance, A.t/wb

* Equivalent circuits:



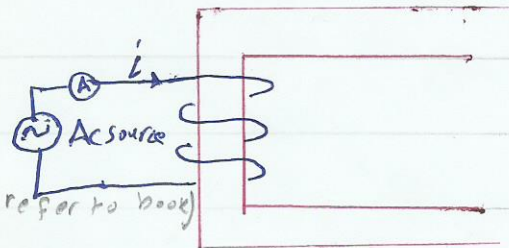
* Magnetization curves: (The Relation between ϕ or B vs Ni or H)

1- For DC Case:



2- For AC Source:

* The Magnetization Curve cuts the tips of **Hysteresis loop** (More about Hysteresis refer to book)



* There are 2-types of losses through the core:-

- 1- hysteresis loss
- 2- eddy current losses

* Examples (Magnetic Circuits) :-

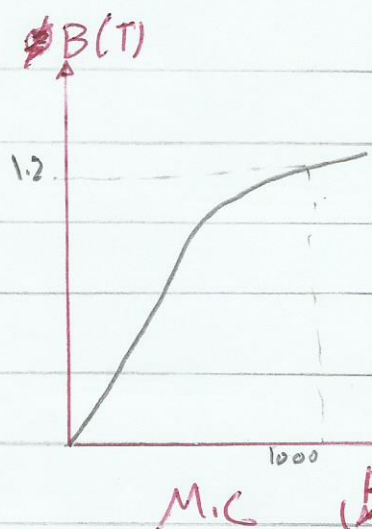
Ex 1:- The toroidal (Circular cross section) core shown below is made from cast steel. (Magnetization curve given below)

a) Calculate the coil current required to produce a core density of 1.2 T at the mean radius of the toroid?

Sol:- Mean length of core, $l_c = 2\pi \frac{(10+6)}{2} \times 10^{-2} \text{ m}$

$l_c = 0.503 \text{ m}$, From M.C $H = 1000 \text{ A.t/m}$ at $B = 1.2$

$$I = \frac{H l_c}{N} = \frac{1000 \times 0.503}{200} = 2.51 \text{ A}$$



b) What is the core flux?

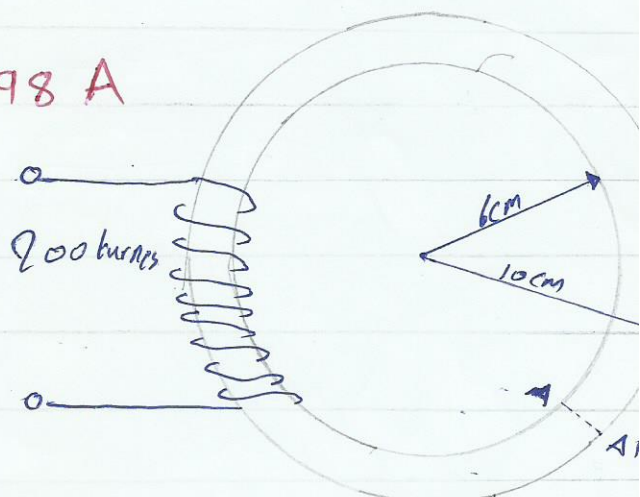
Sol:- $A = (2 \times 10^{-2})^2 \pi = 1.26 \times 10^{-3} \text{ m}^2$

$$\phi = BA = 1.2 \times 1.26 \times 10^{-3} = 1.52 \times 10^{-3} \text{ Wb}$$

c) If a 2-mm wide air gap is made in the toroid (across A-A'), determine the new coil current required to maintain a core flux density of 1.2 T.

Sol:- $Ni = H_c l_c + H_a l_a = H_c l_c + \frac{B l_g}{\mu_0}$

$$I = \frac{1000 \times 0.503}{200} + \frac{1.2 \times 2 \times 10^{-3}}{4\pi \times 10^{-7} \times 200} = 98 \text{ A}$$



Ex 2:- In the magnetic circuit shown below, the relative permeability of the ferromagnetic material is 1200. Neglect magnetic leakage and fringing. All dimensions are in cm, and the core has a square cross-sectional area. Determine the air gap flux, the air gap density, and the magnetic field intensity.

Sol: $F_1 = N_1 i_1 = 5000 \text{ A}\cdot\text{t}$

$F_2 = N_2 i_2 = 5000 \text{ A}\cdot\text{t}$

$M_C = 1200 \text{ m} = 1.51 \times 10^{-3} \text{ (T}\cdot\text{m/A}\cdot\text{t)}$

$R_1 = \frac{L_C}{\mu_0 \mu_r} = \frac{3 \times 52 \times 10^{-2}}{1.51 \times 10^{-3} \times 4 \times 10^{-4}}$

$R_1 = 2.58 \times 10^6 \text{ A}\cdot\text{t/Wb}$

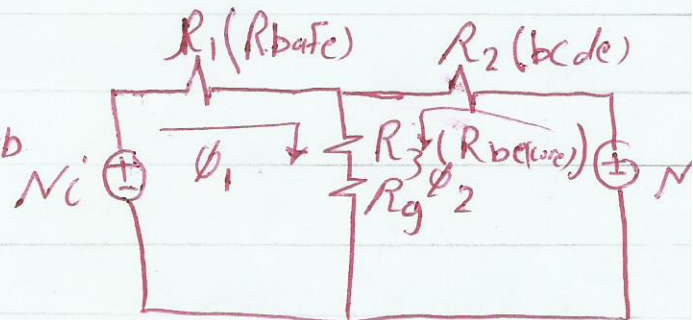
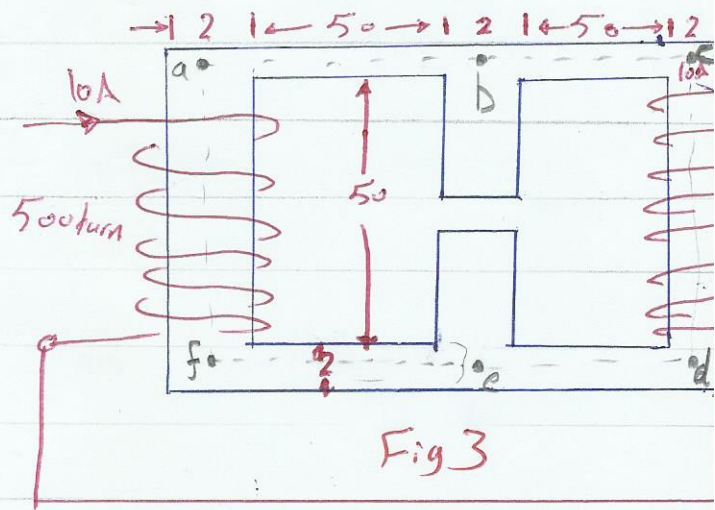
From symmetry

$R_1 = R_2 = 2.58 \text{ MA}\cdot\text{t/Wb}$

$R_g = \frac{L_g}{\mu_0 \mu_r} = \frac{5 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 10^{-4}}$

$R_g = 9.94 \text{ MA}\cdot\text{t/Wb}$

$R_3 = \frac{51.5 \times 10^{-2}}{1.51 \times 10^{-3} \times 4 \times 10^{-4}} = 0.82 \text{ MA}\cdot\text{t/Wb}$



Mesh:-

$\phi_1 (R_1 + R_3 + R_g) + \phi_2 (R_g + R_3) = F_1$ --- (1) Equation 1

$\phi_2 (R_2 + R_3 + R_g) + \phi_1 (R_g + R_3) = F_2$ --- (2) Equation 2

Solving (1), (2) ...

$\phi_1 = \phi_2 = 2.067 \times 10^{-4} \text{ Wb}$

$\phi_g = \phi_1 + \phi_2 = 4.134 \times 10^{-4} \text{ Wb}$

$B_g = \frac{\phi_g}{A_g} = 1.034 \text{ T}$

$H_g = \frac{B_g}{\mu_0} = 0.822 \times 10^6 \text{ A}\cdot\text{t/m}$

Transformers.

* Classification:

- 1) Power transformer
- 2) Instrument transformer
- 3) Impedance matching transformers.

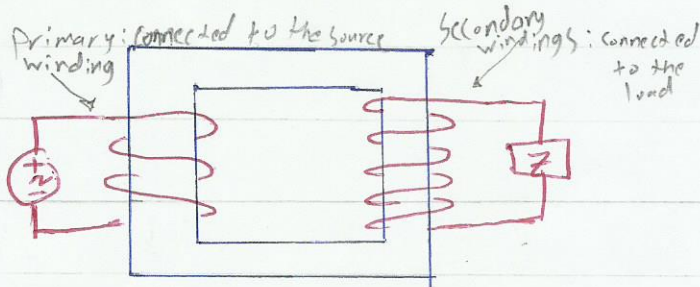
* Construction:

It consists of the following two parts:

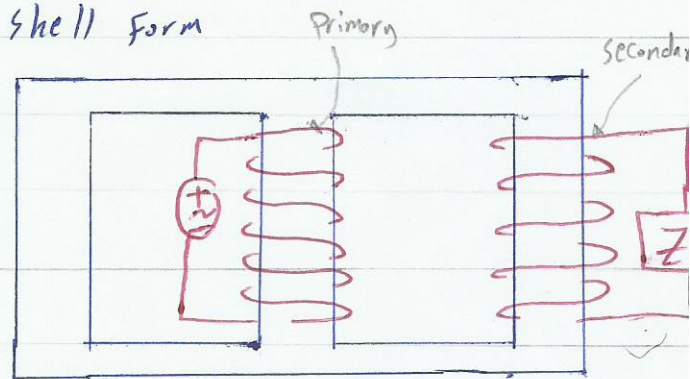
i) Core: to carry the windings

two types of cores:

1) Core form



2) Shell form



ii) windings of coils: 1) Primary windings 2) Secondary windings

* Note: The principle of operation is based on Faraday's law. $(e = -N \frac{d\phi}{dt})$

* Ideal Transformers:

assume the following:

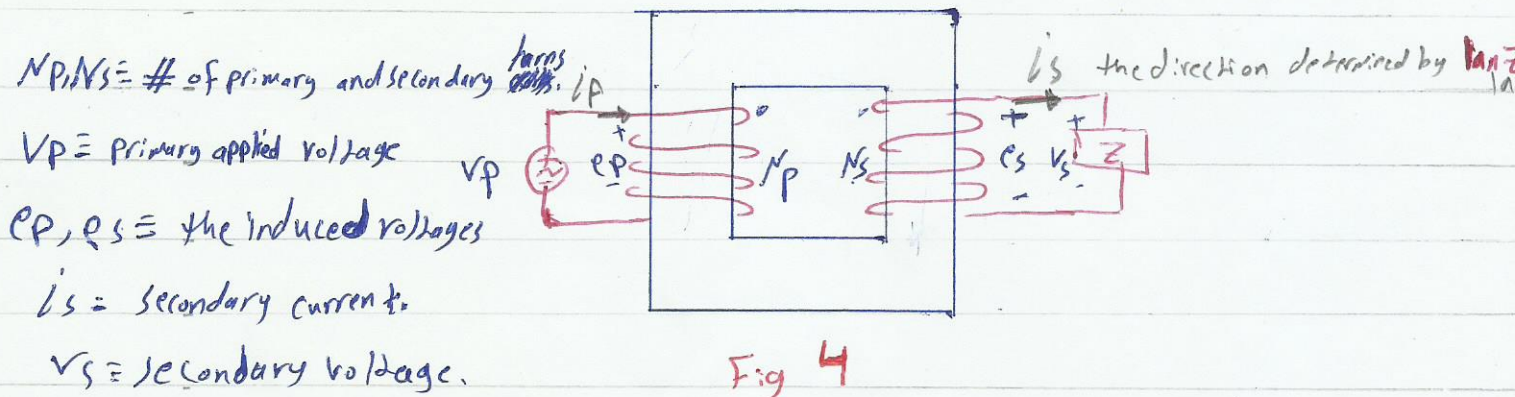
1. No losses \rightarrow Core losses (eddy current and hysteresis)
 \rightarrow electrical losses (resistance and self inductance)

2. No leakage flux.

3. Core has very high μ_r . ($\mu_r = \infty$)

Cont. Ideal transformers

* We will find the V, I, Z relations of primary and secondary windings.



$$V_P = e_P = N_P \frac{d\phi}{dt} \quad V_S = e_S = N_S \frac{d\phi}{dt}$$

$$\boxed{\frac{V_P}{V_S} = \frac{N_P}{N_S} = a} \quad \text{--- [1]}$$

Note: $\frac{N_P}{N_S} = a$

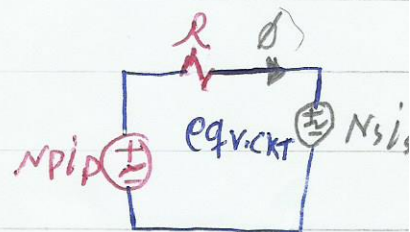
From equivalent CKT:

$$-N_P i_P + \phi R + N_S i_S = 0$$

$R=0$ (Ideal)

$$\therefore N_P i_P = N_S i_S$$

$$\boxed{\frac{i_P}{i_S} = \frac{N_S}{N_P} = \frac{1}{a}} \quad \text{--- [2]}$$



* Note :-

- 1) If $a > 1 \Rightarrow$ Step up transformer
- 2) If $a < 1 \Rightarrow$ Step down transformer

Ⓐ $Z_P = \frac{V_P}{I_P}$ / since $\frac{V_P}{V_S} = \frac{I_S}{I_P} = \frac{N_P}{N_S} = a$ --- Ⓑ From Ⓐ and Ⓑ

$$Z_P = a^2 \frac{V_S}{I_S} \Rightarrow \boxed{Z_P = a^2 Z_L} \quad \text{--- [3]} \quad \text{(6)}$$

Practical OR = Real Single-Phase transformer

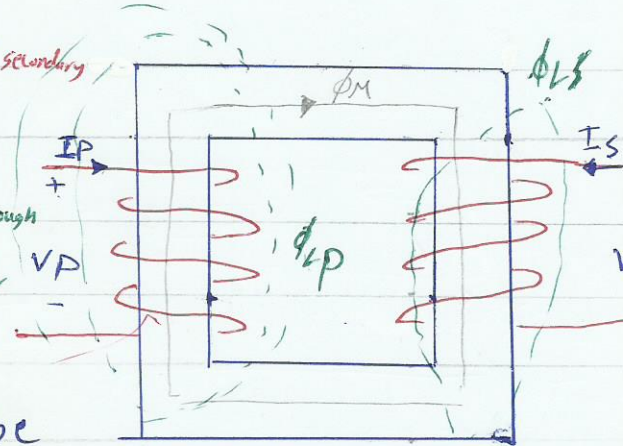
* operation:

ϕ_M : Flux Component linking both primary and secondary coils (mutual flux)

ϕ_{Ls} : secondary leakage flux.

ϕ_{Lp} : primary leakage flux.

passes through
→ only winding
and the air



* From Fig 5 the flux in both coils can be divided into 2 components; mutual flux and leakage flux. Fig 5

$\therefore \phi_p = \phi_{Lp} + \phi_M$ --- [1] and $\phi_s = \phi_{Ls} + \phi_M$ --- [2]

* From Faraday's law:

$e_c = \frac{d\lambda}{dt}$ --- [3]

where λ is the flux linkage in the coil

Flux passing through the coil
 $\lambda = \sum_{n=1}^N \phi_n$

* the average flux/turn can be determined:

$\bar{\phi} = \frac{\lambda}{N}$; the eq [3] become $e_c = N \frac{d\bar{\phi}}{dt}$ --- [4]

Note: in Real transformer the total flux is not $N\bar{\phi}$ because not every turn has the same flux.

Now plug [4] in [3]

$e_p = N_p \frac{d\bar{\phi}_p}{dt} = N_p \frac{d\phi_M}{dt} + \cancel{N_p \frac{d\phi_{Lp}}{dt}} = e_{pm} + e_{pl}$ --- [5]

In ideal transformer No epl

also plug [2] in [4]

$e_s = N_s \frac{d\phi_M}{dt} + N_s \frac{d\phi_{Ls}}{dt} = e_{sm} + e_{sz}$ --- [6]

Now we need to model both primary and secondary CKTs taking leakage flux in account

Cont. Practical 1. of transformer former

* The losses that occur in real transformer that have to be accounted

A. Copper losses. Copper losses are resistive heating losses in both coils. These losses proportional to I^2 ($I^2 R$)

B. Eddy Current losses. also resistive heating that proportional to the square of voltage square that applied to the transformer

C. Hysteresis losses (explained previous).

Note: - Copper losses represented by R series with the leakage flux inductance.

D. Leakage flux: represented as Inductance ??!! why??

was shown previous :- (eq 5 and 6 at 7th page)

$$e_{p1} = N_p \frac{d\phi_{p1}}{dt} \quad \text{and} \quad e_{s1} = N_s \frac{d\phi_{s1}}{dt}$$

* Note: much of leakage flux path is through air and air has constant reluctance higher than the core, $\phi_{p1} = \frac{L_p}{\mu_0 \mu_r} i_p$ and $\phi_{s1} = \frac{L_s}{\mu_0 \mu_r} i_s$

$$\therefore \phi_{p1} = \left(\frac{\mu_0 \mu_r N_p^2}{R} \right) i_p \quad \text{and} \quad \phi_{s1} = \left(\frac{\mu_0 \mu_r N_s^2}{R} \right) i_s$$

From 1, 2, 3 and 4:

$$e_{p1} = \frac{N_p^2}{R} \frac{d i_p}{dt} \quad \text{and} \quad e_{s1} = \frac{N_s^2}{R} \frac{d i_s}{dt}$$

Note: $P = \frac{1}{R}$

* The Magnetization current in a Real transformer:

at No-load (secondary circuit is open): the primary current has 2 components

1) The Magnetization current i_m (the current required to produce the flux in the core)

Note: $i_m + i_{we} = i_{ex}$

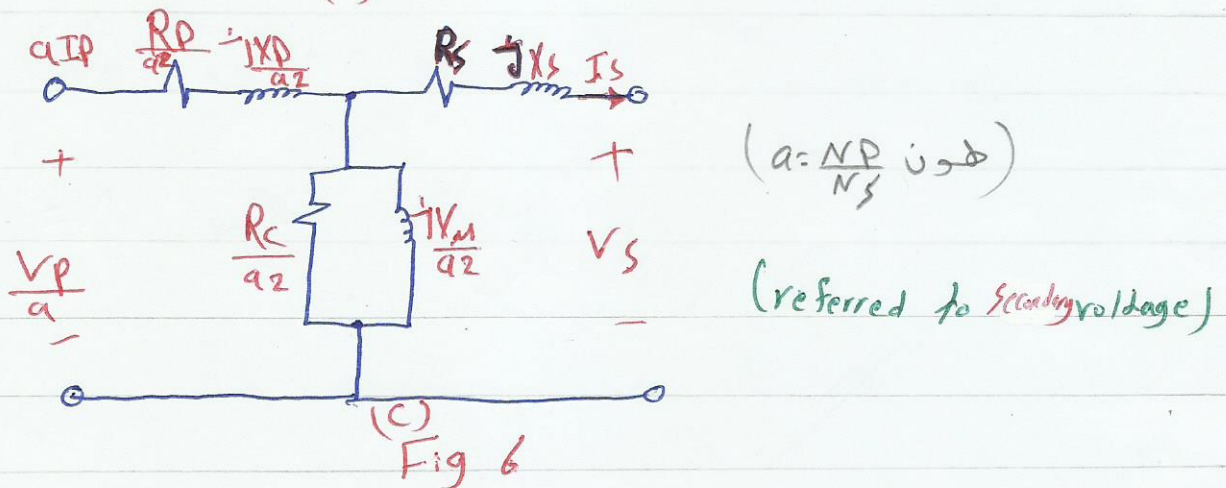
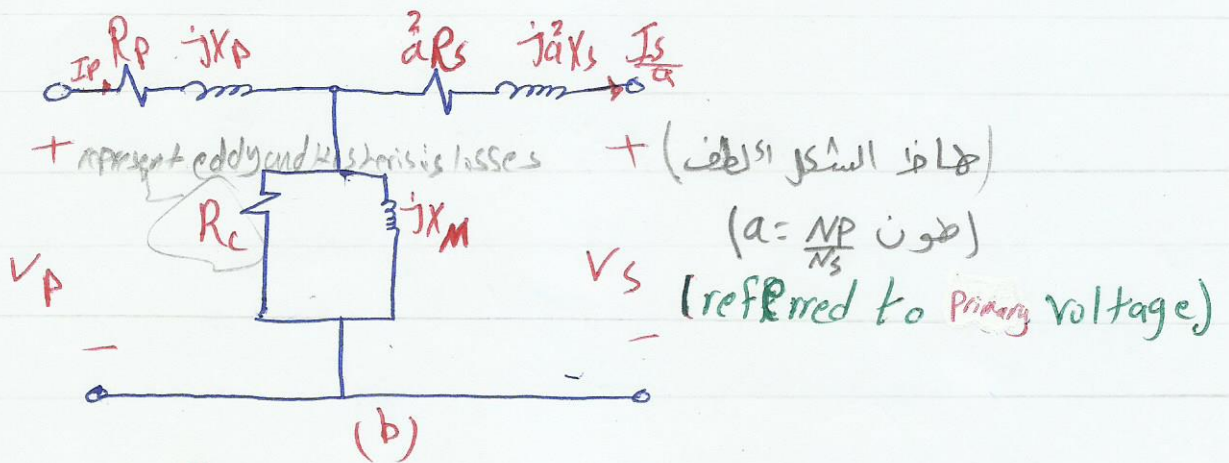
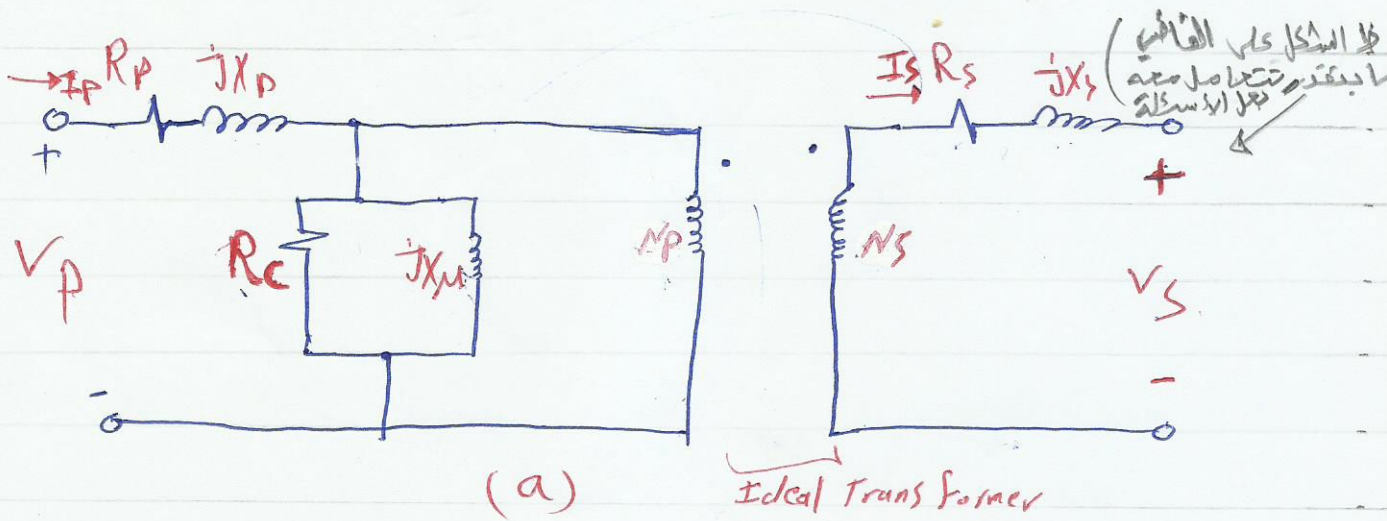
2) The core-losses current i_{we} . (the current which is required to make up for hysteresis and eddy current losses)

For more details refer to your book (electric machinery fundamentals by Chapman 5th edition pages (81, 82, 83).

Equivalent circuits of Transformer

Now we can construct our model:- (الآن يمكننا بناء النموذج)

1) Exact equivalent circuit:-



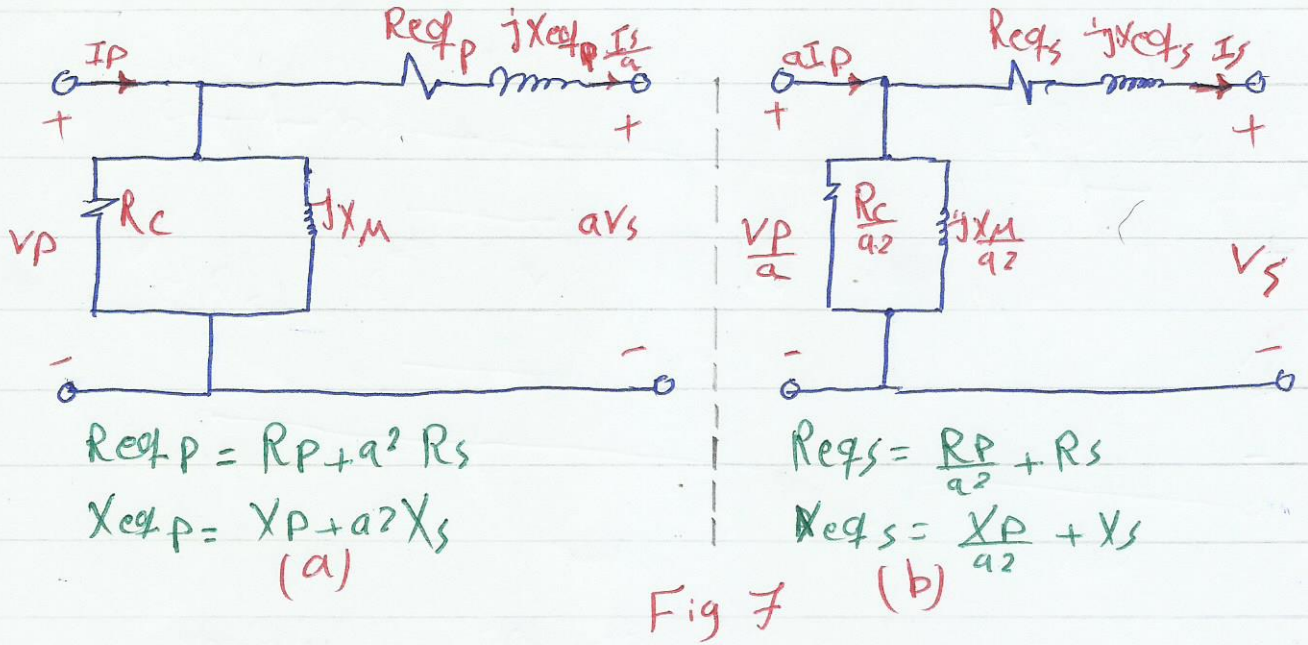
2) Approximate equivalent circuit:- (النموذج التقريبي)

* The exact form is complex than necessary.

* Since the excitation current (I_{exc}) is only about 2% - 3% of full current so the excitation branch is moved to front of transformer and primary and secondary impedances are left in series with each other.

Cont. Equivalent circuit of Transformer

So the Approximate eq uivalent C.K.T:-



* Notes:

1) $N_P : N_S \Rightarrow \frac{N_P}{N_S} = a \Rightarrow a \neq 1$ If we referred \rightarrow to a then we multiply Z by a^2 , V by a and divide I by

2) $a \neq 1$ If we referred \rightarrow to 1 then we divide Z by a^2 , V by a and multiply I by

2) $R_{eqP} \neq R_{eqS}$ and $X_{eqP} \neq X_{eqS}$ (So we will use pu system as will be explained later)

* Evaluation of transformers parameters: (نقطة التقييم)

- We want measure the parameters R_C , X_M and Z_{eq} experimentally.

How??!!

by performing 2 tests:-

1) open CKT test :- used to measure R_C and X_M (o/c)

2) short CKT test :- it is used to measure Z_{eq} (s/c)

Evaluation of Transformer Parameter

1) O/C Test:

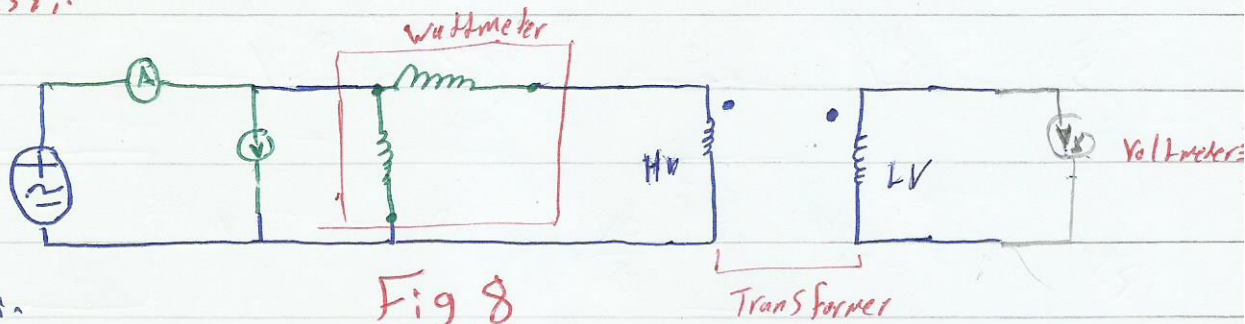


Fig 8

* Procedures:

A) Apply open circuit at secondary by keeping the terminal **O/C** or by connecting **voltmeter**.

B) Apply **rated** voltage at **the primary**, then measure V_{OC}/I_{OC} and

* Calculations:

$$Y_F = \frac{1}{R_c} - \frac{j}{X_M} = G_c - jB_X \quad \text{--- [1]}$$

Conductance Susceptance

X_M, R_c **مقرون بطرف**
Referred to LV

$$|Y_F| = \frac{I_{OC}}{V_{OC}} \quad \text{--- [2]}$$

Note: Here
- It's better to use
Low voltage as p

$$PF = \cos \theta = \frac{P_{OC}}{V_{OC} I_{OC}} \Rightarrow \theta = \cos^{-1} \frac{P_{OC}}{V_{OC} I_{OC}} \quad \text{--- [3]}$$

$$Y_F = \frac{I_{OC}}{V_{OC}} \angle -\theta \quad \text{--- [4]}$$

plug [4] in [2] to find R_c, X_M

2) S/C test =

(Connection as Fig 8 but we connect a meter inside the voltmeter)

* procedure:-

A) Apply **S/C** at secondary or connect **ammeter**

B) Apply **rated current** to (S/C)

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}} \quad \text{--- [1]}$$

$$Z_{SE} = R_{eq} + jX_{eq} \quad \text{--- [2]}$$

$$PF = \cos \theta = \frac{P_{SC}}{I_{SC} V_{SC}}$$

$$\theta = \cos^{-1} \frac{P_{SC}}{I_{SC} V_{SC}} \quad \text{--- [3]}$$

Here, it's better to use
High voltage as primary

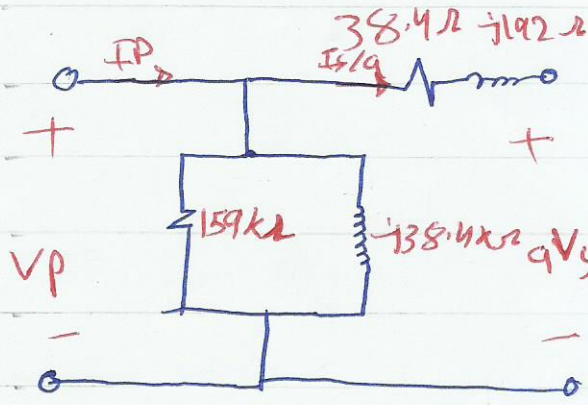
$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \theta \quad \text{--- [4]}$$

plug [4] in [2] to find R_{eq}, X_{eq}

Ex: 20 kVA, (8000/240), 60 Hz transformer has the following parameters referred to HV side:
 $R_C = 159 \text{ k}\Omega$ $X_M = 38.4 \text{ k}\Omega$ $R_{eq} = 38.4$ $X_{eq} = 192$
 If an o/c test was performed with LV as the primary, what would be the reading of its measuring instruments

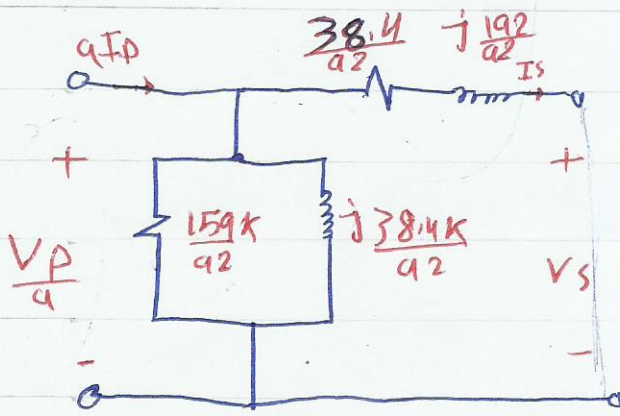
(السؤال في امتحان 2013 / 2014)

Sol: First of all we will refer the parameters to LV:-



33.34:1

referred to H.V side



33.34:1

referred to L.V side

Fig(9)

$V_{o/c} = 240 \text{ V}$ because we applied rated voltage at primary

$$I_{o/c} = I_C + I_M$$

$$I_C = \frac{V_{o/c}}{R_C} = 1.68 \text{ A}$$

$$I_M = \frac{V_{o/c}}{jX_M} = 6.95 \angle -90^\circ$$

$$I_{o/c} = 7.15 \angle -6.41^\circ$$

$$P_{o/c} = |I_{o/c}| |V_{o/c}| \cos(\theta_{V_{o/c}} - \theta_{I_{o/c}})$$

$$P_{o/c} = 403.2 \text{ W}$$

Note: Example 2-2 page 92
 بيان الامتحان

Per-unit (pu) System

Definition: instead of expressing the basic quantities of voltage, current, Impedance and power by their units of (V, A, Ω , VA), they are expressed as a fraction of Base value.

$$\text{Pu value} = \frac{\text{Actual Value}}{\text{Base Value}}$$

why we use it!!

* we use pu system because the following advantages:

- 1) it simplifies the analysis of power system
- 2) the Resistance or Reactance of transformers or generators in pu fall in a certain narrow range, hence this may help in checking answers

* Base value selected for S and V (S_B, V_B) then I_B, Z_B calculated as follow:-

$$I_B = \frac{S_B}{V_B}$$

$$Z_B = \frac{(V_B)^2}{S_B}$$

Transformer Voltage Regulation:

* voltage Regulation:- is a quantity that compares the output voltage of the transformer at no-load with the output voltage at full loads.

$$VR = \frac{V_P - V_{S, FL}}{V_{S, FL}} \times 100\%$$

$$VR = \frac{V_{P, pu} - V_{S, pu}}{V_{S, pu}} \times 100\%$$

Transformer efficiency

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$= \frac{P_{out}}{P_{out} + \text{losses}} \times 100\%$$

$$\eta = \frac{V_S I_S \cos \phi}{P_{cu} + P_{core} + V_S I_S \cos \phi}$$

Example 2-5 page 102

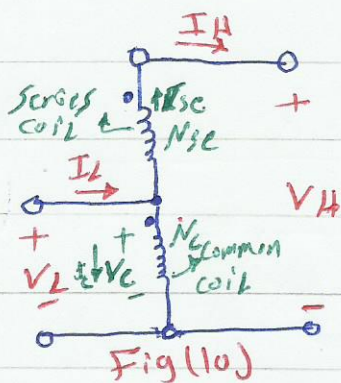
(ada jala)

The auto transformer

- * We use this transformer to change voltage levels by small amount.
- * It's too expensive to build conventional transformer with windings almost having the same voltage rating \Rightarrow use auto transformer

* Construction:

The coils are connected in series. (additive manner) as follows:-



Note:

$$I_L = I_c + I_{sc}$$

$$I_H = I_{sc}$$

$$V_c = V_c$$

$$V_H = V_c + V_{sc}$$

* analysis:-

- voltage-current relationships:-

A) Voltage relation:-

$$\frac{V_c}{V_{sc}} = \frac{N_c}{N_{sc}} \quad \text{--- [1] (From previous note)}$$

$$V_H = V_c + \frac{N_{sc}}{N_c} V_c \quad \text{--- [2]}$$

\downarrow $V_c = V_c$

$$\frac{V_H}{V_c} = \left(1 + \frac{N_{sc}}{N_c}\right) \quad \text{--- [3]}$$

* B) Current relation:-

$$\frac{I_c}{I_{sc}} = \frac{N_{sc}}{N_c} \quad \text{--- [4]}$$

$$I_L = \frac{N_{sc}}{N_c} I_{sc} + I_{sc} \quad \text{--- [5]}$$

\downarrow $I_{sc} = I_H$

$$\frac{I_L}{I_H} = \left(1 + \frac{N_{sc}}{N_c}\right) \quad \text{--- [6]}$$

Cont. autotransformer

* Apparent power advantage of Autotransformer :-

* If a conventional transformer is connected as autotransformer, then the apparent power of auto will be greater than the apparent power of the conventional

Input apparent power $S_I = V_L I_L$ --- [5]

output apparent power $S_O = V_H I_H$ --- [6] → assuming step-up

Now sub into [5] or [6]

$$S_O = V_L \left[\frac{N_{sc} + N_c}{N_c} \right] \times I_L \left[\frac{N_c}{N_L + N_{sc}} \right] = V_L I_L = S_I$$

$$S_I = S_O = S_{IO}$$

* S_w (apparent power of the windings) = the apparent power of conventional

$$S_w = V_c I_c = V_{sc} I_{sc}$$

$$S_w = V_{sc} I_{sc}$$

$$(V_{sc} = V_H - V_L \text{ and } I_{sc} = I_H)$$

$$S_w = (V_H - V_L) I_H$$

$$S_w = V_H I_H - V_L I_H$$

$$(V_L = \frac{N_c}{N_{sc} + N_c} V_H)$$

$$S_w = V_H I_H \left(1 - \frac{N_c}{N_{sc} + N_c} \right)$$

$$S_w = S_{IO} \left(\frac{N_{sc}}{N_{sc} + N_c} \right)$$

$$\frac{S_{IO}}{S_w} = \frac{N_c + N_{sc}}{N_{sc}} \text{ --- [7]}$$

Notes: the increase of S is due to:-

- transformation action
- conduction action

Cont. The internal impedance of auto transformer

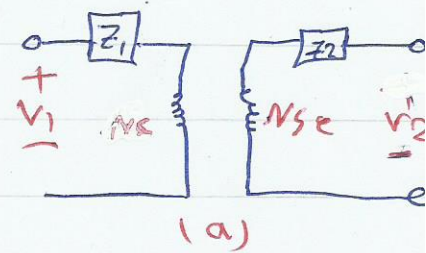
* Compared to a given transformer connected in the conventional manner, the effective per-unit impedance of an auto transformer is smaller by a factor equal to the reciprocal of the power advantage of the auto transformer (why???)

(Problem 2-16)

Ex: prove that $Z_{eq}' = \frac{N_{se}}{N_{se} + N_c} Z_{eq}$, If a transformer having a series impedance Z_{eq} is connected as auto transformer, Z_{eq}' is its per-unit impedance of auto transformer.

Sol: For the transformer connected as conventional, the impedance referred to the primary (N_c) is:

$$Z_{eq} = Z_1 + \left(\frac{N_c}{N_{se}}\right)^2 Z_2$$



When this transformer is connected as an auto transformer, the circuit is shown at Fig (11, c).

If the output windings of the are shorted, the voltage $V_H = 0$, and the voltage V_L will be

$$V_L = I_C Z_{eq} \quad \text{--- [2]}$$

$$I_L = I_C + I_{se} = I_C + \frac{N_c}{N_{se}} I_C$$

$$I_L = \frac{N_{se} + N_c}{N_{se}} I_C \quad \text{--- [3]}$$

$$Z_{eq}' = \frac{V_L}{I_L} = \frac{N_{se}}{N_{se} + N_c} Z_{eq} \quad \#$$

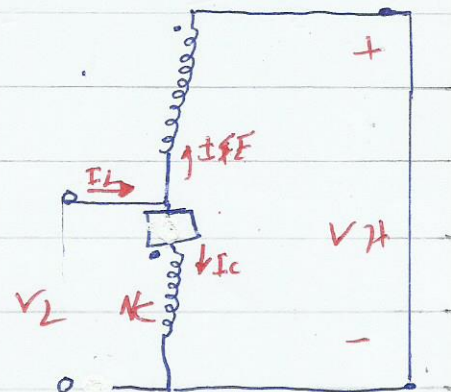


Fig (11)

Example 2-8 page 116

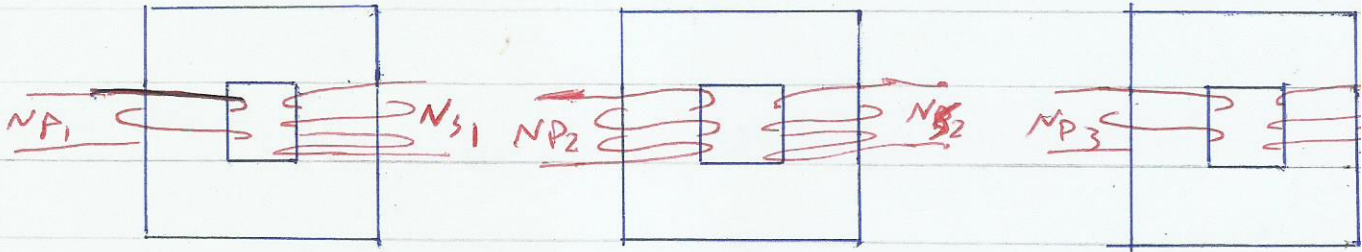
(سیمی کنڈکٹور)

Three-Phase Transformer

نظام التحويل الكهربائي
 يتم تحويله على السلك

* There are 2 basic constructions:-

1) Using 3-Single Phase transformers as Fig (12):



Bank of 3- ϕ transformer

More reliable (each unit could be replaced individually)

Fig (12)

2) Three sets of windings wrapped on common core as shown in Fig (13)

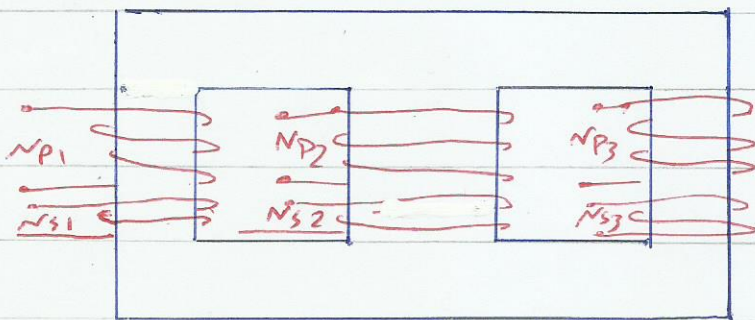


Fig (13)

(lighter, cheaper, more economical and smaller)

* Three-Phase transformer connections:-

* The primaries or secondaries of any 3- ϕ transformer can be independently connected in either a wye (γ) or a delta (Δ). This gives 4 possibilities:

1) γ - γ

2) γ - Δ

3) Δ - γ

4) Δ - Δ

1) Y-Y Connection:

Y-Y Connection is shown in Fig (14)

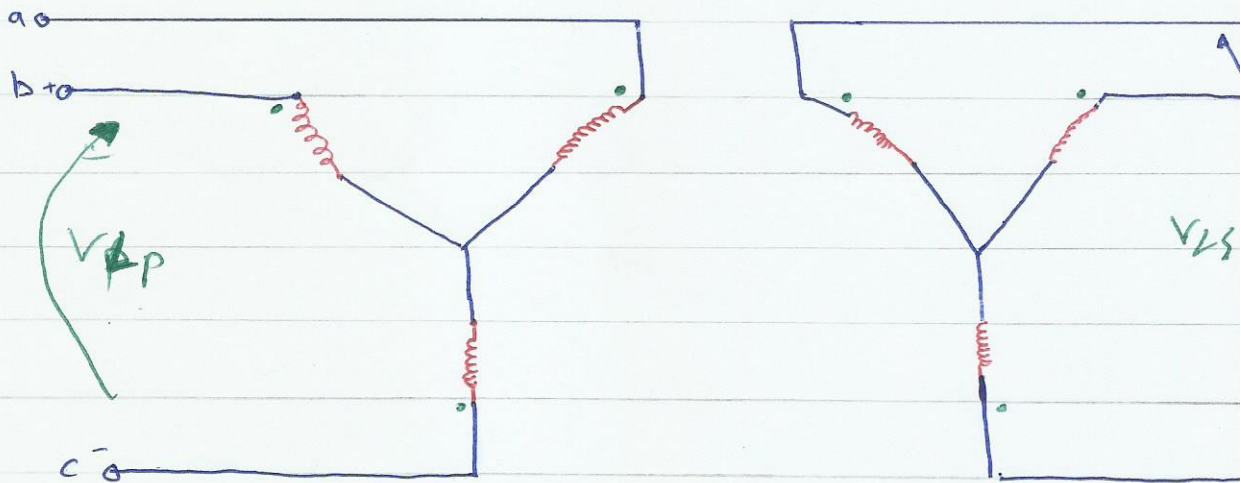


Fig 14

$$V_{LP} = \sqrt{3} V_{\phi P}$$

$$V_{LS} = \sqrt{3} V_{\phi S}$$

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3} V_{\phi P}}{\sqrt{3} V_{\phi S}} = a$$

Y-Y Connection has 2 problems:

1) If load on the transformer circuit unbalanced, then the voltages on the phases can become unbalanced.

2) Third-harmonic voltage can be large

* Harmonics in Y-Connections:

If 3- ϕ set of voltages is applied to Y-Y transformer, the voltages in any phase will be 120° apart from the voltage in any other phase, however the third-harmonic components of each of three phases will be in phase with each other. why!!!

in phase with each other.

$$V_A = V_{m1} \sin \omega t + V_{m3} \sin 3\omega t$$

$$V_B = V_{m1} \sin(\omega t - 120^\circ) + V_{m3} \sin(3\omega t - 120^\circ)$$

$$V_C = V_{m1} \sin(\omega t - 240^\circ) + V_{m3} \sin(3\omega t - 240^\circ)$$

Cont. 3- ϕ transformer

So there are **three cycle** in **third harmonic** For each cycle of the fundamental frequency. There are **always** some **third-harmonic** component in a transformer because of the **nonlinearity** of the core.

These components **add up**, and the result is very large **third-harmonic** component of voltage on top of the **50/60 Hz** fundamental voltage.

Both of the unbalanced problem and **third-harmonic** can be solved by using one of the following two techniques:-

1) **Solidly ground** the neutrals of the transformer; especially the **primary** winding's neutral. This connection **prevents** the **additive** **third-harmonic** component to cause a current flow in the neutral instead of building up large voltage. It's also provide **a return path** for any imbalance in the load.

2) **Add third windings** connected in Δ to the transformer called **tertiary** windings; this winding must handle the **circulating current** so it made about **one-third** power rating of two main winding.

* these techniques are shown below:

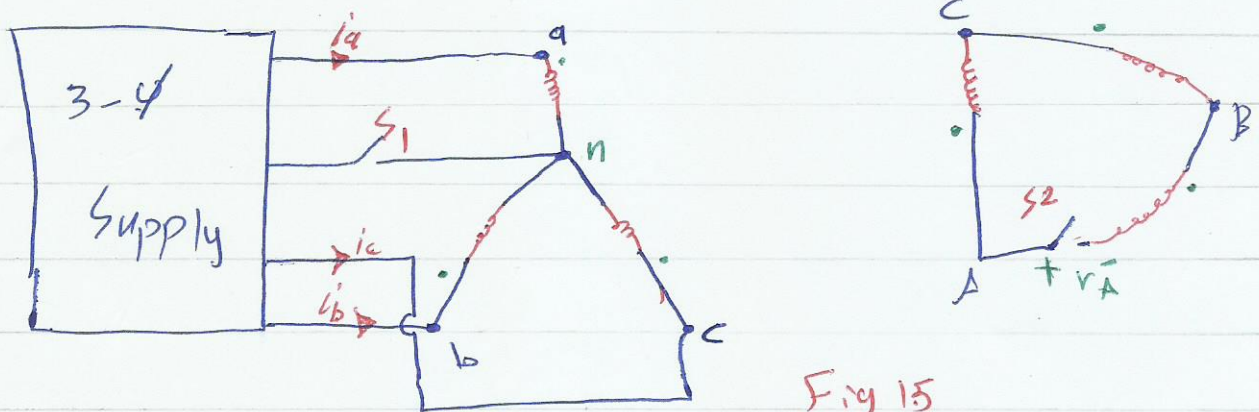


Fig 15

Case 1: S_1 closed, S_2 open:-

$$i_a = I_{m1} \sin \omega t + I_{m3} \sin 3\omega t$$

$$i_b = I_{m1} \sin(\omega t - 120^\circ) + I_{m3} \sin 3(\omega t - 120^\circ)$$

$$i_c = I_{m1} \sin(\omega t - 240^\circ) + I_{m3} \sin 3(\omega t - 240^\circ)$$

$$i_n = i_a + i_b + i_c = 3I_{m3} \sin 3\omega t$$

Cont. 3- ϕ transformer

Case 2: S_1 open, S_2 open

In this case the harmonic current can't flow in the primary ckt. So E_a, I_b, I_c will be pure sinusoidal which leads to non-sinusoidal flux so the voltage in Δ windings will be non-sinusoidal:

Phase voltages

$$\left. \begin{aligned} V_A &= V_{m1} \sin \omega t + V_{m3} \sin 3\omega t \\ V_B &= V_{m1} \sin(\omega t - 120^\circ) + V_{m3} \sin(3(\omega t - 120^\circ)) \\ V_C &= V_{m1} \sin(\omega t - 240^\circ) + V_{m3} \sin(3(\omega t - 240^\circ)) \end{aligned} \right\}$$

$$V_{AB} = V_{m1} \sin \omega t - V_{m1} \sin(\omega t - 120^\circ)$$

line voltage sinusoidal !!

Case 3: S_1 open, S_2 closed

* V_A will generate 3rd harmonic current, these currents will substitute for the missing 3rd harmonic currents in the primary

* $Y-\Delta$ Connections:

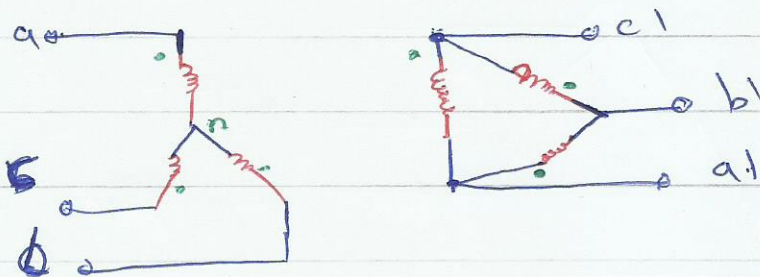


Fig 16

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3} V_{\phi P}}{V_{\phi S}} = \sqrt{3} a$$

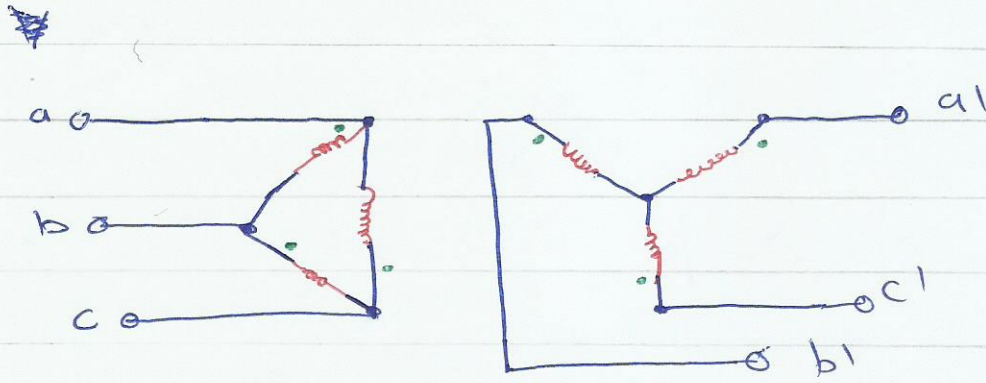
Assuming +ve phase sequence, if $\angle_{\text{van}} = 0^\circ$

$$\therefore \angle_{V_{ab}} = \angle_{V_{LP}} = 30^\circ \quad \text{since } V_{\phi P} \text{ and } V_{\phi S} \text{ are in-phase } \therefore \angle_{V_{ab}}$$

\therefore $Y-\Delta$ connection introduce a phase shift = 30°
(20)

Cont 3- ϕ transformer

3) Δ - Y connection



$$\frac{V_{LP}}{V_{2\phi}} = \frac{V_{\Delta P}}{V_{3\phi S}} = \frac{a}{\sqrt{3}}$$

Fig 17

* This connection has same phase shift as Y - Δ connection (3)

4) Δ - Δ connection

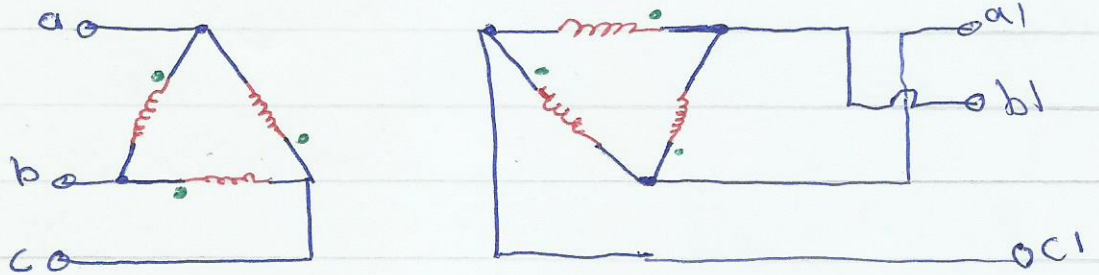


Fig 18

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\Delta P}}{V_{\Delta S}} = a \quad (\text{No-phase shift})$$

* PU system for 3- ϕ transformer

* applicable to 3- ϕ transformer, by using per-phase CKT Concept.

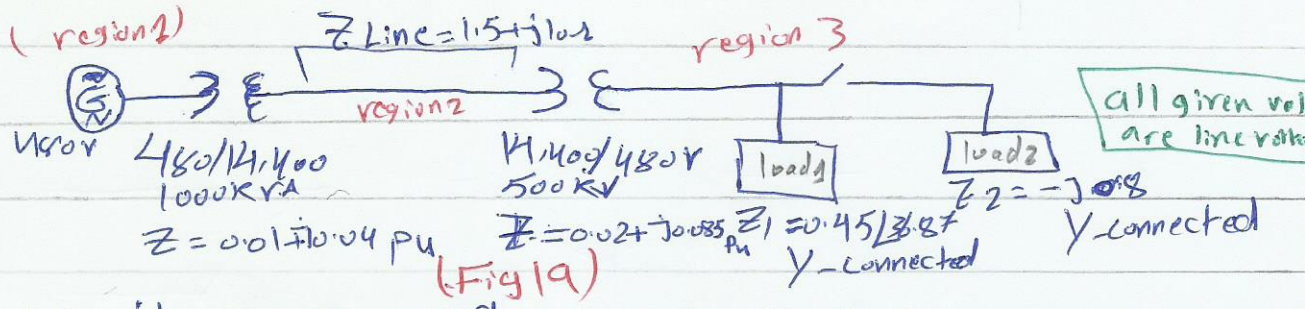
$$S_{1\phi \text{ base}} = \frac{S_{3\phi}}{3}$$

$$Z_{1\phi, b} = \frac{(V_b)^2}{S_{1\phi b}}$$

(Problem 2-24)

EX: Power system consisting of 3- ϕ , 480-V, 60Hz generator supplying 2-loads through transmission line with a pair of transformer at either end. as shown in Fig 19

Cont 3-φ transformer

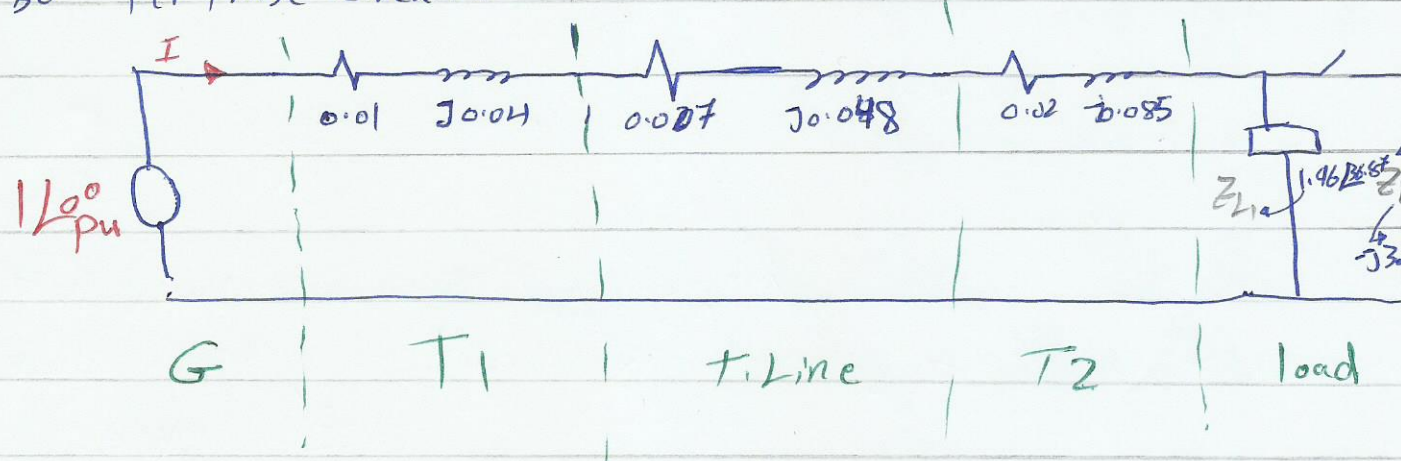


a) sketch the per-phase equivalent circuit of this power system.

$S_{3\phi,b} = 1000 \text{ KVA}$

Region	Z base
1	$Z_b = \frac{(1480/\sqrt{3})^2}{(1000 \times 10^3)/3} = 0.23 \Omega$
2	$Z_b = \frac{(14400/\sqrt{3})^2}{(1000 \times 10^3)/3} = 207.36 \Omega$
3	as region 1 $Z_b = 0.23 \Omega$

So Per phase circuit:-



b) with switch open find P, Q, S and PF for the generator?

KVL to find I

$$1 \angle 0^\circ = (0.01 + j0.04 + 0.007 + j0.048 + 0.02 + j0.085 + 1.96 \angle 36.87^\circ) I$$

$$I = 0.477 \angle -40.05^\circ$$

$$S_{pu} = \frac{V I^*}{P_u I_{pu}^*}$$

$$S_{pu} = 0.477 \angle +40.05 \text{ pu}$$

$$PF = \cos 40.05$$

$$PF = 0.765 \text{ lag}$$

~~$S = 477.26 + j365$~~

$$S = 0.477 \angle +40.05 \times 1000 \text{ KVA}$$

$$S_G = 477 \text{ KVA} \angle 40.05 \quad P_G = \text{real}\{S_G\} = 365 \text{ KW} \quad Q_G = 306 \text{ KVAR}$$

Cont 3-4 transformer

c) with switch closed, Find P, Q, S and PF of Generator?

KVL to find I:

$$1 \angle 0^\circ = (0.01 + j0.04 + 0.007 + j0.048 + 0.02 + j0.08) + (1.568 + j1.761) I$$

$$I = 0.4 \angle -6.56^\circ$$

$$S_{pu} = V_{pu} I_{pu}^*$$

$$S_{pu} = 0.4 \angle 6.56$$

$$S_G = 400 \text{ kVA} \quad P_G = 397.4 \text{ kW} \quad Q = 45.7 \text{ kVAR}$$

$$PF = \cos 6.56 = 0.9935 \text{ lagging}$$

d) what are the transmission losses with switch open? with switch closed? what is the effect of adding load 2 to the system?

1) switch open:

$$I = 0.477 \angle -40.05 \text{ pu}$$

$$P_{pu} = I^2 Z_{line+transformers}$$

$$P_{losses pu} = 7.39 \times 10^{-3} \text{ pu}$$

$$P_{losses} = 7.4 \text{ kW}$$

2) switch closed:

$$I = 0.4 \angle -6.56^\circ$$

$$P_{pu} = I^2 Z$$

$$P_{losses} = 5.92 \times 10^{-3} \text{ pu}$$

$$P_{losses} = 5.92 \text{ kW}$$

adding load 2 leads to:-

1) decreasing power losses in transmission process

2) increasing power factor