Department of Electrical Engineering
Faculty of Engineering & Technology
University of Jordan

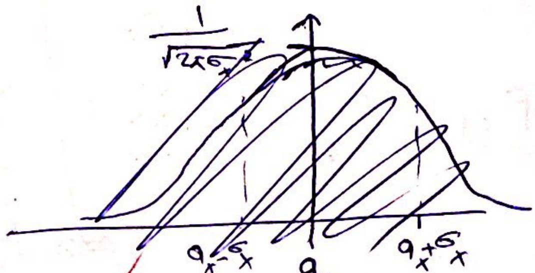
Q1) (7 marks) $1k\Omega \pm 0.05$

A factory produces resistors with 1 kΩ nominal values. It was clear that the probability of actual measured resistors follow Gaussian distribution. If about 60.7% of produced resistors have values within ~~±1.05 kΩ~~. Assume that 1,000,000 resistors are used in the analysis and only measured resistors within ±3σ are shipped to customers.

a) Calculate the tolerance of shipped resistors?
b) The total number of shipped resistors?
c) The probability of producing resistors with no more than 1.1 kΩ?

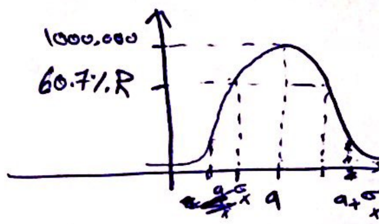
gaussian distribution :-

$$F_x(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{\frac{-(x-a_x)}{2\sigma_x^2}}$$



$R \simeq 1k\Omega$ $\quad a_x = \cancel{1}.05$ $\quad \sigma_x$

tolerance $= \sigma_x$ $\quad a_x = 1k\Omega$ $\quad \sigma_x = 0.05\ k\Omega$



a) tolerance for shiped $= \pm 3\sigma_x = \pm 3 * 0.05 = \pm 0.15$ 0.15 %

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b) ~~$F(x) = F\left(\frac{x-a_x}{\sigma_x}\right)$~~

shiped resistors $\rightarrow$ $1k \pm 0.15$

$a-3\sigma_x = 0.85\,k\Omega$

$a+3\sigma_x = 1.15\,k\Omega$

$a-\sigma_x = 1-0.05$
$= 0.95\,k\Omega$

$a+\sigma_x = 1.05\,k\Omega$

$$P[0.85 \leq x \leq 1.15] = F_x(1.15) - F_x(0.85)$$
$$= F\left(\frac{1.15-1}{0.15}\right) - F\left(\frac{0.85-1}{0.15}\right)$$
$$= 0.8413 -$$

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0.5

c) $P[x \leq 1.1k\Omega] = F_x(x \leq 1.1) = F\left(\frac{x-a_x}{\sigma_x}\right) = F\left(\frac{1.1-1}{0.15}\right) = \cancel{0.7454}$

$= F(0.6667) = 0.7454$

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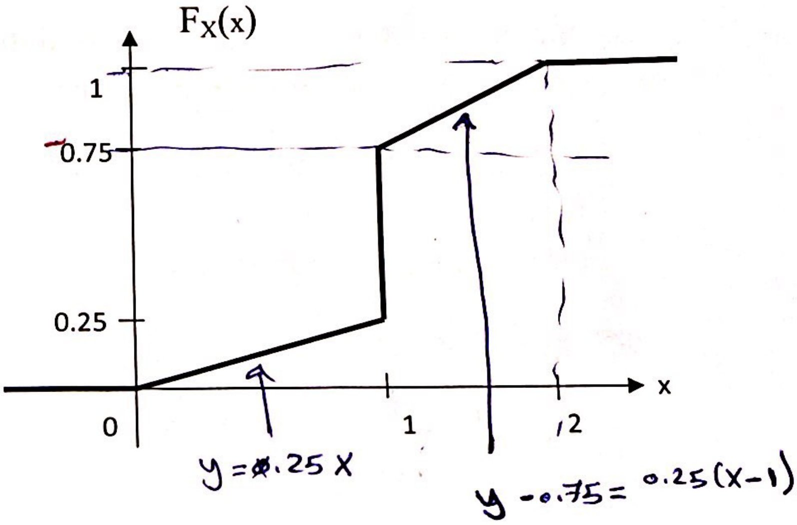
$f_X(x) = \frac{d}{dx} F_X(x)$

Q2) (5 marks)

a) Sketch the probability density function (pdf) $f_X(x)$ for the cumulative distribution function (cdf) shown in the figure below?

b) Validate the pdf?



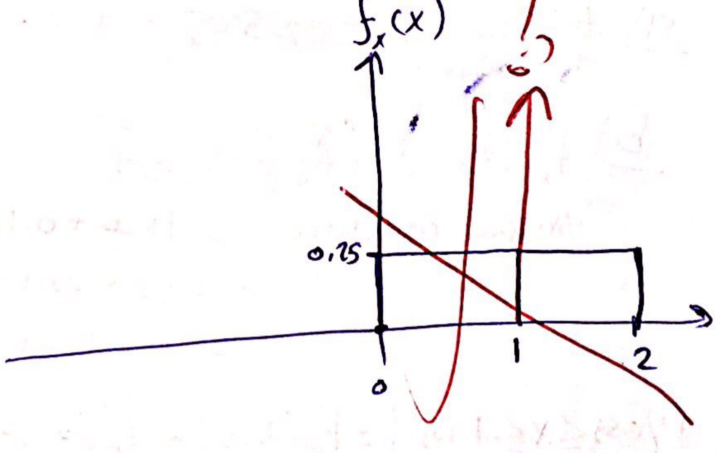
a)

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ 0.25x, & 0 < x \le 1 \\ 0.25x + 0.5, & 1 < x \le 2 \\ 1, & 2 < x \end{cases}$$

$$f_X(x) = \frac{dF_X(x)}{dx}$$

$$f_X(x) = \begin{cases} 0, & x \le 0 \\ 0.25, & 0 < x \le 1 \\ 0.25, & 1 < x \le 2 \\ 0, & x > 2 \end{cases}$$

not needed



b) $F_X(-\infty) = 0$

$F_X(\infty) = 1$

$F_X(x_1) < F_X(x_2)$

$f_X(x) = F_X(x_2) - F_X(x_1)$ where $x_1 < x < x_2$

~~at point 1 which we cannot take derivative so it is [illegible]~~

$F_X(x^+) = F_X(x)$

need PDF validation

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Q3) (8 marks)

a) Verify the validity of the following probability distribution function?
b) If it is not valid, go to part (c)? If it is valid go to part (d).
c) Modify the distribution function to make it valid?
d) Sketch its probability **density** function?
e) Find the P [$-\frac{1}{2} \leq X < \frac{1}{2}$]

$$F_x[x_i] = \begin{cases} 0 & x < -1 \\ \frac{1}{3} & -1 \leq x < -\frac{1}{2} \\ \frac{3}{4} & -\frac{1}{2} \leq x < 0 \\ \frac{7}{8} & 0 \leq x < \frac{1}{2} \\ \frac{15}{16} & \frac{1}{2} \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

a) $F_x(\infty) = 1$

$F_x(-\infty) = 0$

$F_x(x^+) = F_x(x)$

and there are a derivative for it to Find $f_x(x)$

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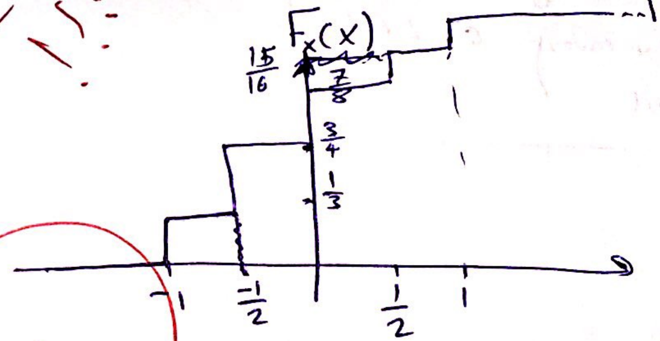
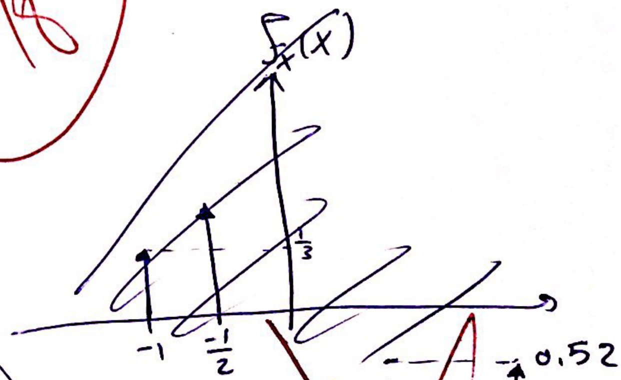
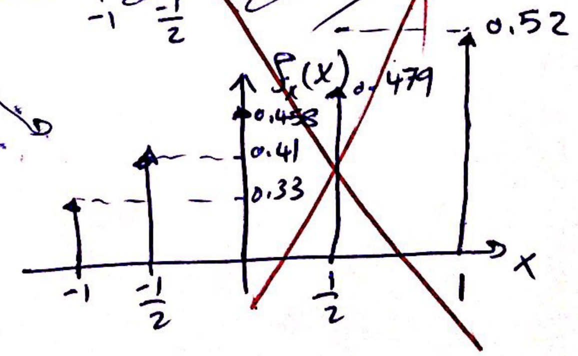
So it is valid



d)





e) $P[\frac{-1}{2} \leq x < \frac{1}{2}]$

since it is valid

$= F(\frac{-1}{2} \leq x \leq \frac{1}{2}) = F_x(\frac{1}{2}) - F_x(\frac{-1}{2})$

$= \frac{15}{16} - \frac{3}{4} = \frac{3}{16}$

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Q4) Bonus (3 marks)

At a certain military installation, six similar radars are placed in operation. It is known that a radar's probability of failing to operate before 500 hours of "ON" have accumulated is 0.06. What is the probability that; before 500 hours have elapsed, only one radar will fail?

P{Falling to operate} = 0.06

by bernoulli's

bernoulli's $P = \binom{N}{k} p^k (1-p)^{N-k}$

$P\{\text{only one radar will Fail}\} = \binom{6}{1} (0.06)^1 (0.94)^{6-1}$

$= \frac{6!}{(6-1)! * \cancel{1!}} (0.06)^1 (0.94)^5$

$= \frac{6 * \cancel{5!}}{\cancel{5!}} * (0.06) (0.94)^5$

$\boxed{P\{\text{onlyone radar Fail}\} = 0.264}$

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