DSP
FIRST EXAM

FALL-2012

POWER UNIT
ELECTRICAL ENGINEERING

Solution

University of Jordan
College of Engineering & Technology
Department of Electrical Engineering
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Digital Signal Processing – EE 424 First Exam

Name: Solution Student Number:

Q1.

2 a) The odd part of a real-valued sequence $x[n]$ is given by $x_{od}[n]=(\frac{1}{3})^3 u[n]$. If the average power of x[n] is $P_x = 10$, determine the average power of its even part $x_{ev}[n]$.

2 b) Compute the energy of length-N sequence $x[n]=\sin(2\pi kn/N)$, $0 \le n \le N-1$.

1 c) Determine if the system described by the following input-output equation $y[n]=y^2[n-1]+x[n]$ is stable or not.

a) $P_{od} = \lim_{k\to\infty} \frac{1}{2K+1} \sum_{0}^{K} (\frac{1}{3})^6$

$= (\frac{1}{3})^6 \lim_{K\to\infty} \frac{K+1}{2K+1} = (\frac{1}{3})^6 \cdot \frac{1}{2} = \frac{1}{2(3)^6} = \frac{1}{1458}$

$= 6.86 \times 10^{-4}$

$P_{even} = P_x - P_{od}$
$= 10 - 6.86 \times 10^{-4}$
$= 9.9993$ W.

$P_{even} = 10 - \frac{1}{2}(\frac{1}{3})^6$

b) $x[n] = \sin(\frac{2\pi kn}{N})$ $0 \le n \le N-1$

$E_x = \sum_{n=0}^{N-1} |x[n]|^2 = \sum_{n=0}^{N-1} \sin^2(2\pi k\frac{n}{N}) = \sum_{n=0}^{N-1} (\frac{1}{2} - \frac{1}{2}\cos\frac{4\pi kn}{N})$

$= \frac{N}{2} - \frac{1}{2}\underbrace{\cancel{\sum_{n=0}^{N-1} \cos(4\pi kn/N)}}_{C}$

$= \frac{N}{2}$

$C = \sum_{n=0}^{N-1} \cos(\frac{4\pi nk}{N})$

$S = \sum_{n=0}^{N-1} \sin(\frac{4\pi nk}{N})$

$C + jS = \sum^{N-1} e^{-j\frac{4\pi nk}{N}} \cdot \frac{1-e^{-j4\pi nk}}{1-e^{-j4\pi nk}}$

c) - Unstable

let $x[n] = u[n]$ $\qquad$ $x[n]$ i

$y[-1] = 0$

$y[0] = y^2[-1] + u[0]$

$y[0] = 1$

$y[1] = 1^2 + 1 = 2$

$y[2] = 2^2 + 1 = 5$

$y[3] = 5^2 + 1 = 26$

$y[n]$ is Unbounded.

Q2. Determine the DTFT of the following signals:

a) $x[n] = \cos(\omega_o n)u[n]$.

b) $x[n] = u[n] - u[n-6]$.

a)
$$x[n] = \cos(w_o n)\,u[n]$$
$$= \frac{e^{jw_o n} + e^{-jw_o n}}{2}\,u[n]$$
$$= \frac{1}{2} e^{jw_o n} u[n] + \frac{1}{2} e^{-jw_o n} u[n]$$
$$= \frac{1}{2}\left[\frac{1}{1 - e^{-j(w - w_o)}} + \frac{1}{1 - e^{-j(w + w_o)}}\right]$$
$$= \frac{1}{2}\,\frac{1 - e^{-j(w+w_o)} + 1 - e^{-j(w-w_o)}}{\left[1 - e^{-j(w-w_o)} - e^{-j(w+w_o)} + e^{-j2w}\right]}$$
$$= \frac{1 - e^{-jw}\left[\frac{e^{jw_o} + e^{-jw_o}}{2}\right]}{1 - 2e^{-jw}\left[\frac{e^{jw_o} + e^{-jw_o}}{2}\right] + e^{-j2w}}$$
$$= \frac{1 - e^{-jw}\cos w_o}{1 - 2e^{-jw}\cos w_o + e^{-j2w}} \qquad w \neq \pm w_o + 2\pi k, \quad k = 0, 1, \ldots$$

b)
$$x[n] = u[n] - u[n-6]$$
$$= \frac{1}{1 - e^{-jw}} - \frac{e^{-j6w}}{1 - e^{-jw}} = \frac{1 - e^{-j6w}}{1 - e^{-jw}}.$$

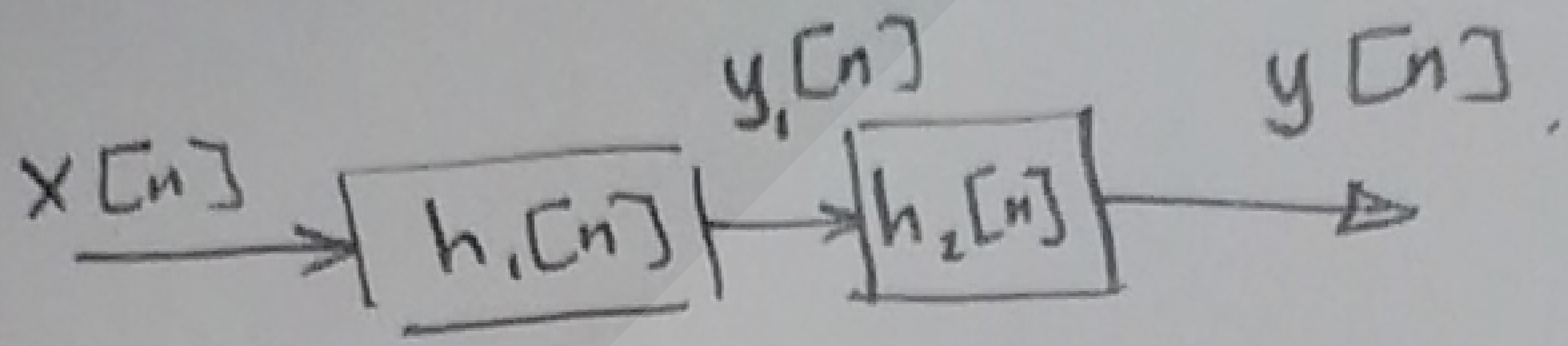
Q4. Two LTI systems are connected in cascade. Their impulse responses are:

$$h_1[n] = (\frac{1}{2})^n u[n] \qquad h_2[n] = (\frac{1}{4})^n u[n]$$

a) Prove that the impulse response of the cascaded system $h[n] = \sum_{k=-\infty}^{\infty} h_1[k]h_2[n-k]$.

b) Find the impulse response of the cascaded system h[n]. page 82

a)



2)

$y[n] = y_1[n] * h_2[n]$

$y_1[n] = x[n] * h_1[n]$

$\therefore y[n] = x[n] * h_1[n] * h_2[n]$

$= x[n] * h[n]$

where $h[n] = h_1[n] * h_2[n]$.

$$h[n] = \sum_{k=-\infty}^{\infty} h_1[k] h_2[n-k]$$

b)

$\boxed{h[n] = 0, \; n < 0}$.

3)

$$h[n] = \sum_{k=0}^{n} (\tfrac{1}{2})^k (\tfrac{1}{4})^{n-k}$$

$$= (\tfrac{1}{4})^n \sum_{k=0}^{n} 2^k$$

$$= (\tfrac{1}{4})^n \left[\frac{1-2^{n+1}}{1-2}\right] = (\tfrac{1}{4})^n (2^{n+1} - 1).$$

$$\boxed{h[n] = (\tfrac{1}{2})^n \left[2 - (\tfrac{1}{2})^n\right], \; n \geq 0}$$