diff First Exam
2nd summer semester 2017

Question 1: Fill in the blanks to get acorrect sentence (2marks each).

[1] The value of k thats makes the ODE

$$(y^3 + kxy^4 - 2x)\,dx + (3xy^2 + 20x^2y^3)\,dy = 0 \text{ exact is:-}$$

↳ solution

$M = y^3 + kxy^4 - 2x$ $\qquad$ $N = 3xy^2 + 20x^2y^3$

$M_y = 3y^2 + 4kxy^3$ $\qquad$ $N_x = 3y^2 + 40xy^3$

$N_x = M_y$

$4K = 40$ $\qquad$ $\boxed{k = 10}$

[2] an integrating factor for the ODE ~~...~~

$$\cos x\,dx + \left(1 + \frac{2}{y}\right)\sin x\,dy = 0 \quad \text{is.}$$

$$\boxed{y^2e^y}$$

[3] A general form for the particular solution using undetermined coefficients method for the OD (do not determine the coefficients)

$$y'' - 5y' - 6y = 3e^{6x} - e^x \quad \text{is}$$

$$\boxed{y_p = Be^x + Dxe^{6x}}$$

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4 the 2^{nd} order Cauchy-Euler ODE with general Solution $y = c_1 x + c_2 x \ln x$ is

$$\boxed{x^2 y'' - x y' + y = 0}$$

5 if $y = \frac{\cos x}{x}$ is a solution for the ODE

$x y'' + 2y' + xy = 0$ then the general Solution ODE is:-

$$\boxed{y = c_1 \frac{\cos x}{x} + c_2 \frac{\sin x}{x}}$$

Question 2:- (5 marks)

Solve $\boxed{y'' + 6y' + 9y = \frac{16 e^{-3x}}{1+x^2}}$

Solution $y'' + 6y' + 9y = 0$

$\lambda^2 + 6\lambda + 9 = 0$

$(\lambda + 3)(\lambda + 3) = 0$

$\lambda = -3, -3 \implies y = c_1 e^{-3x} + c_2 x e^{-3x}$

$y_p = u(x) e^{-3x} + V(x) x e^{-3x}$.

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$$W = \begin{vmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & -3xe^{-3x} + e^{-3x} \end{vmatrix}$$

$$W = e^{-6x}$$

$$u = \int \frac{-x \cancel{e^{-3x}}}{\cancel{e^{-6x}}} \cdot \frac{16 \cancel{e^{-3x}}}{1+x^2}$$

$$u = -8 \ln(1+x^2)$$

$$V = \int \frac{16 \cancel{e^{-3x}} \cdot \cancel{e^{-3x}}}{(1+x^2)(\cancel{e^{-6x}})} = 16 \tan^{-1} x.$$

the general solution is

$$y = C_1 e^{-3x} + C_2 x e^{-3x} + 16 x e^{-3x} \tan^{-1} x + -8 e^{-3x} \ln(1+x^2)$$

Question (3) Solve $-y\,dx + (x + \sqrt{xy})\,dy = 0$

(homo)

$$ux = y$$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{y}{x + \sqrt{xy}} \begin{matrix} \div x \\ \div x \end{matrix}$$

$$u + x\frac{du}{dx} = \frac{u}{1 + \sqrt{u}}$$

$$x\frac{du}{dx} = \frac{u}{1+\sqrt{u}} - u$$

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$$-x\frac{du}{dx} = \frac{u^{3/2}}{1+\sqrt{u}}$$

$$-\frac{dx}{x} = \frac{1+\sqrt{u}}{u^{3/2}}\, du$$

↓

$$-\ln x + c = \int \frac{1+\sqrt{u}}{u^{3/2}}$$

$$-\ln \cancel{x+c=} \int u^{-3/2} + \frac{1}{u}\, du$$

$$-\ln x + c = \ln u - \frac{2}{\sqrt{u}}$$

$$\frac{c}{x} = u\, e^{\frac{-2}{\sqrt{u}}}$$

$$\frac{c}{x} = \frac{y}{x}\, e^{\frac{-2}{\sqrt{y/x}}}$$

Question 4

solve: $\frac{dy}{dx} - \frac{y}{x} = \frac{1}{y^2}$

$y' - \frac{y}{x} = \frac{1}{y^2}$

$* \, 3y^2$

$\boxed{u = y^3}$

$\frac{du}{dx} = 3y^2 \frac{dy}{dx}$

$\downarrow$

$\frac{du}{dx} - \frac{3u}{x} = 3$

$e^{\int p(x)} = e^{\int \frac{-3}{x}} = \frac{1}{x^3}$

$\int e^{-p(x)\,dx} = x^3$

$\int e^{\int p(x)dx} \cdot r(x) = \int \frac{1}{x^3} * 3 = \left(\frac{-3}{2x^2} + c \right)$

$y = e^{-\int p(x)} \cdot \int e^{\int p(x)} r(x)$

$$\boxed{y = \frac{-3x}{2} + cx^3}$$