EM2

Fall017

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EM2

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* Review EM1

* coordinate System

1. Cartesian coordinates $\langle x,y,z\rangle$, $d\vec{l}$, $d\vec{s}$, dv
$\nabla V, \nabla\cdot\vec{D}, \nabla\times\vec{E}, \nabla^2 V$

2. Cylindrical coordinates $\langle \rho,\phi,z\rangle$, $d\rho$, $\rho d\phi$, dz

3. Spherical coordinates $\langle r,\theta,\phi\rangle$, dr, $r d\theta$, $r\sin\theta d\phi$

* Electrostatics :-

Source: 1. Point charge (Q) in "C"
2. Continuous charge Dist → Line charge ρ_L C/m
→ Surface charge ρ_s C/m²
→ Volume charge ρ_v C/m³
3. Electric Dipole ⊕←⊖ $\vec{p} = Q\vec{d}$ C.m

method to find $\vec{E}$

1. $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2} a_r$ V/m , N/C

2. Gauss's law , $\oint \vec{D}\cdot d\vec{s} = Q_{enc} = \int_l \rho_L dl = \int_s \rho_s ds = \int_v \rho_v dv$

$= \int_v \nabla\cdot\vec{D} dv \Rightarrow \rho_v = \nabla\cdot\vec{D}$

3. Electric potential

$V = -\int_l \vec{E}\cdot dl \Rightarrow E = -\nabla V$

$\nabla\times E = \nabla\times(-\nabla V) \Rightarrow$ (2nd maxwell eq.)

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4. Boundary Value problems:

→ Poisson eq.:

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \quad \text{if homogenous}$$

$$\nabla \cdot (-\varepsilon \nabla V) = \rho_v \quad \text{if non-homogenous}$$

→ laplace eq.: [if $\rho_v = 0$]

$$\nabla^2 V = 0 \quad \text{for homogenous}$$

$$\nabla \cdot (-\varepsilon \nabla V) = 0 \quad \text{for non-homogenous}$$

5. *Method of Images*

↳ only for a Source is above a good conducting ground.

* Related Quantities:

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{F} = Q\vec{D}$$

$$Q = \int_s \varepsilon \vec{E} \cdot d\vec{s}$$

$$V = -\int_v \vec{E} \cdot d\vec{l}$$

$$I = \int_s \sigma \vec{E} \cdot d\vec{s} = \int_s \vec{J} \cdot d\vec{s} \quad , \vec{J} = \sigma \vec{E} \quad \text{(ohm's law)}$$

$$R = \frac{V}{I} = \frac{-\int_l E\, dl}{\int_s \sigma E\, ds} \quad , C = \frac{Q}{V} = \frac{\int_s \varepsilon \vec{E}\, ds}{-\int_l \vec{E} \cdot dl}$$

W_E

$$W_E = \frac{1}{2} \int_v \varepsilon E^2 dv = \frac{1}{2} C V^2 \quad \text{(Joule)}$$

$$W_E = \frac{1}{2} \varepsilon E^2 \quad J/m^3$$

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* Boundary conditions:

$\vec{E}_{1t} = \vec{E}_{2t}$, $D_{1n} - D_{2n} = \rho_s$

* Magnetostatics:

↳ Sources: - Permanent Magnet

- Charge moving with zero accelaration

- DC current following in a wire

Line current	Surface current	Volume current
$I d\vec{l}$	$\vec{K} ds$	$\vec{J} dv$

* method to find $\vec{H}$

1. Biot-Savart's law (for line current $H = \int_l \frac{I d\vec{l} \times \vec{R}}{4\pi R^3}$)

2. Ampere's law $\oint_l \vec{H} \cdot d\vec{l} = I_{enc}$

3. Mag. Potential → Scalar Potential , $\vec{H} = -\nabla V_m$, V_m

↳ Vector potential , $\nabla \times \vec{A} = \mu \vec{H} = \vec{B}$

4. B.V.P

$\nabla^2 \vec{A} = -\mu \vec{J}$ (vector Poisson's eq.)

↳ if $\vec{J} = 0 \Rightarrow \nabla^2 A = 0$

5. method of Images

↳ a Source is covered by a ferro magnetic matterial ($\mu_r \ggg 1$)

↳ hysterisis loop

* Related Quantities:

$\vec{B} = \mu \vec{H}$

$\Psi = \int_s B \cdot ds$ | $W_m = \frac{1}{2} L I^2 = \frac{1}{2} \int_v \mu H^2 dv$

$\lambda = \Psi \times N$ | $\vec{M} = (\mu_r - 1)\vec{H}$

$L = \frac{N\Psi}{I}$ | $K_d = M \times \vec{a}_n$, $J_M = \nabla \times \vec{M}$

* Boundary:

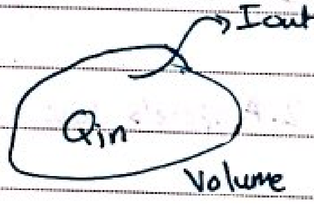
$\vec{B}_{1n} = \vec{B}_{2n}$, $\oint_s \vec{B} \cdot d\vec{s} = 0$ "4th Maxwell eq."

$\nabla \cdot \vec{B} = 0$, $H_{1t} - H_{2t} = K$, $\oint_l \vec{H} \cdot d\vec{l} = I = \int_s \vec{J} \cdot d\vec{s}$

$\int_s \nabla \times \vec{H} \cdot d\vec{s} = \int_s \vec{J} \cdot d\vec{s}$ $\nabla \times \vec{H} = \vec{J}$ $\Leftrightarrow$ "3rd Maxwell eq.s"

* Continuity eq. & Relaxation time :-

$I = -\frac{d\,Q_{in}}{dt} = \oint_s \vec{J} \cdot d\vec{s}$



$-\frac{d}{dt} \int_v \rho_v \, dv$, $Q_{in} = \int_v \rho_v \, dv$

$\oint_s \vec{J} \cdot d\vec{s} = \int_v \nabla \cdot \vec{J} \, dv$; by Divergence theorem

$\nabla \cdot \vec{J} = -\frac{d\rho_v}{dt}$ continuity eq.

Relaxation time (T_r) , $T_r = \frac{\epsilon}{\sigma} = RC$

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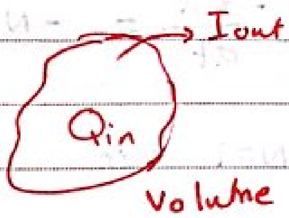
* continuity eq. & Relaxation time

$$I = -\frac{dQ_{in}}{dt} = \oint_S \vec{J}\cdot d\vec{s}$$



$$-\frac{d}{dt}\int_V \rho_v \, dv, \quad Q_{in} = \int_V \rho_v \, dv$$

$$\oint_S \vec{J}\cdot d\vec{s} = \int_V \nabla\cdot\vec{J}\, dv$$

Divergence theorem

$$\nabla\cdot J = -\frac{d\rho_v}{dt}$$

Relaxation time (T_r)

$$\boxed{T_r = \frac{\varepsilon}{\sigma} = RC}$$

* Chapter 9 : Maxwell's eq. in time Varying fields

$$\vec{E}(x,y,z\,;t) = \vec{E}_s(x,y,z)$$

s → phasor

$$\vec{E} = \mathrm{Re}\{E_s e^{j\omega t}\}$$

$e^{j\omega t}$ ⇑ harmonic (Periodic) ⇓ Sinusoidal

* Faraday's law

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\Psi}{dt}$$

for $N=1$ turn

$$V_{emf} = -\frac{d\Psi}{dt}$$

$$V_{emf} = \oint_l \vec{E}\cdot d l = -\frac{d\Psi}{dt}$$

$$\oint_l \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B}\cdot ds$$

$\downarrow$

Stoke's theorem

$$\Psi = \int_s \vec{B}\cdot ds$$

$$\int_s \nabla \times E \, ds = -\frac{d}{dt}\int_s \vec{B}\cdot \vec{ds}$$

$$\boxed{\nabla \times E = -\frac{d\vec{B}}{dt}}$$

Displacement Current (I_d)

$$I_d = \int_s \vec{J_d}\cdot\vec{ds}$$

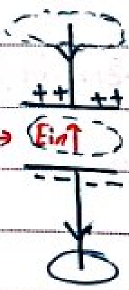
J_d: Displacement current density (A/m^2)

→ added by maxwell's to validate Ampere's law for time varying fields

In Static fields

$$I = \oint_l \vec{H}\cdot d\vec{l} = \int_s \vec{J}\cdot\vec{ds}$$

Dielectric $\rho_s, \rho_v, \rho_{pv}, \rho_{ps}$ →



$\vec{E}\downarrow$

$$\boxed{\vec{E} = -\nabla V}$$

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Taking divergence for both sides of

$$\nabla \cdot \vec{J} = \nabla \cdot (\nabla \times \vec{H}) = 0$$

$$\nabla \cdot \vec{J} \neq 0 = \frac{-d\rho_v}{dt}.$$

$$\boxed{\nabla \cdot \vec{J}_d = \frac{d\rho_v}{dt}}$$

$$\nabla \cdot \vec{J}_d = \frac{d}{dt}(\nabla \cdot \vec{D})$$

$$\nabla \cdot \vec{J} + \nabla \cdot \vec{J}_d = 0$$

$$\Rightarrow \boxed{\vec{J}_d = \frac{d\vec{D}}{dt}}$$

$$\nabla \cdot J_d = -\nabla \cdot J$$

$$\nabla \times \vec{H} = \vec{J} + \vec{J}_d$$

$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$ — 3rd maxwell's eq. in time varying

$$I = \oint_l \vec{H} \cdot dl = \int_s \vec{J} \cdot ds + \int_s \vec{J}_d \cdot ds$$

$$\boxed{\oint_l H \cdot dl = \int_s \left(\vec{J} + \frac{d\vec{D}}{dt}\right) \cdot \vec{ds}}$$

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* Time Varying Potential

In Static fields

~~$V = \int \frac{\rho_v \, dv}{4\pi\epsilon R}$~~

$$V = \int_v \frac{\rho_v \, dv}{4\pi\epsilon R} \quad , \quad \vec{A} = \int \frac{\mu \vec{J}}{4\pi R} \, dv$$

mag. Vector Potential

$$\nabla \cdot \vec{B} = 0 \quad , \quad \boxed{\vec{B} = \nabla \times \vec{A}} \quad (1)$$

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\nabla \times \vec{E} = -\frac{d}{dt}(\nabla \times \vec{A})$$

$$\nabla \times \left(\vec{E} + \frac{d\vec{A}}{dt}\right) = 0$$

$$\nabla \times (-\nabla V) = 0$$

$$\boxed{\vec{E} + \frac{d\vec{A}}{dt} = -\nabla V} \quad (2)$$

Also

$$\nabla \cdot \vec{D} = \rho_v \Rightarrow \vec{D} = \varepsilon \vec{E}$$

$$\nabla \cdot \varepsilon \vec{E} = \rho_v \Rightarrow$$

$$\boxed{\nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon}} \quad (*)$$

Taking the $\nabla .$ for eq. (2)

$$\boxed{\nabla \cdot \vec{E} + \frac{d}{dt} \nabla \cdot \vec{A} = -\nabla^2 V}$$

Sub (*)

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$$\boxed{\nabla^2 V + \frac{d}{dt}(\nabla\cdot\vec{A}) = -\frac{\rho_v}{\varepsilon}} \quad (3)$$

from (1)

Taking the curl for (1)

$$\nabla\times\nabla\times\vec{A} = \nabla\times\vec{B}$$

but $\left(\nabla\times\vec{H} = \vec{J} + \frac{d\vec{D}}{dt}\right)$, $\vec{B} = \mu\vec{H}$

$$\nabla\times\vec{B} = \mu\vec{J} + \mu\frac{d\vec{D}}{dt}$$

$$\nabla\times\nabla\times\vec{A} = \mu\vec{J} + \mu\frac{d\vec{D}}{dt}$$

In general

$$\nabla\times\nabla\times\vec{A} = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$$

$$\boxed{\nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A} = \mu\vec{J} + \mu\epsilon\frac{d\vec{E}}{dt}} \quad (4)$$

Sub eq. (2) in (4)

$$\vec{E} = -\nabla V - \frac{d\vec{A}}{dt}$$

$$\nabla(\nabla\cdot\vec{A}) - \nabla^2 A = \mu\vec{J} + \mu\epsilon\frac{d}{dt}\left(-\nabla V - \frac{d\vec{A}}{dt}\right)$$

$$\nabla(\nabla\cdot\vec{A}) - \nabla^2 A = \mu\vec{J} - \mu\epsilon\,\nabla\frac{dv}{dt} - \mu\epsilon\,\frac{d^2A}{dt^2}$$

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we can write

$$\cancel{\nabla}(\nabla\cdot\vec{A}) = -\mu\epsilon\,\cancel{\nabla}\frac{dv}{dt}$$

$$\boxed{\nabla\cdot\vec{A} = -\mu\epsilon\frac{dv}{dt}} \text{ — } 5a$$

$$\boxed{\nabla^2 V + \frac{d}{dt}(\nabla\cdot\vec{A}) = -\frac{\rho_v}{\varepsilon}} \text{ — } 3$$

$$-\nabla^2\vec{A} = \mu\vec{J} - \mu\varepsilon\frac{d^2\vec{A}}{dt^2}$$

$$\boxed{\nabla^2\vec{A} - \mu\varepsilon\frac{d^2\vec{A}}{dt^2} = -\mu\vec{J}} \text{ — } 5.b$$

Sub 5a in 3

$$\nabla^2 V + \frac{d}{dt}\left(-\mu\varepsilon\frac{dv}{dt}\right) = -\frac{\rho_v}{\varepsilon}$$

$$\boxed{\nabla^2 V - \mu\varepsilon\frac{d^2 V}{dt^2} = -\frac{\rho_v}{\varepsilon}}$$

wave equations

smile for life

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~~$E = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8$ m/s~~

Chapter 10: Electromagnetic wave propagation

* In this chapter, wave propagation will be considered in the following media

1) Free Space ($\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$)
2) lossless media ($\sigma = 0$, $\varepsilon = \varepsilon_0 \varepsilon_r$, $\mu = \mu_0 \mu_r$)
3) Lossy media ($\sigma \neq 0$, $\varepsilon = \varepsilon_0 \varepsilon_r$, $\mu = \mu_0 \mu_r$) "General"
4) Good conductor ($\sigma \simeq \infty$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0 \mu_r$)

Wave equations:

$$\nabla^2 V - \mu\varepsilon \frac{d^2 V}{dt^2} = -\frac{\rho_v}{\varepsilon}$$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{d^2 A}{dt^2} = -\mu J$$

* We need to solve the wave equation in the Lossy media "the general case"

$$\nabla^2 V - \mu\varepsilon \frac{d^2 V}{dt^2} = -\frac{\rho_v}{\varepsilon}$$

Some Simplification & Assumptions

- Source free Region ($\rho_v = 0$, $\vec{J} = 0$)

$$\nabla^2 V - \mu\varepsilon \frac{d^2 V}{dt^2} = 0$$

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* In the cartesian Coordinates:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

- Assume wave propagation in the z-direction

$$\frac{\partial^2 V}{\partial z^2} - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = 0$$

$$\frac{\partial^2 V}{\partial t^2} - \frac{1}{\mu\varepsilon}\frac{\partial^2 V}{\partial z^2} = 0 \quad , V = Ed$$

$$\frac{\partial^2 E}{\partial t^2} - \frac{1}{\mu\varepsilon}\frac{\partial^2 E}{\partial z^2} = 0$$ "divide all by (d)"

* $E(x,y,z;t)$

$E(x,y,z;t) = Re\left(E_s(x,y,z)\, e^{j\omega t}\right)$

convert Into phasors

$$\frac{\partial^2 E_s}{\partial t^2} - \frac{1}{\mu\varepsilon}\frac{\partial^2 E_s}{\partial z^2} = 0$$

$\frac{1}{\mu\varepsilon} = u^2$ $\quad u = \frac{1}{\sqrt{\mu\varepsilon}}$ " velocity of the wave"

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$$(j\omega)^2 E_s - u^2 \frac{d E_s}{dz^2} = 0$$

$$-\omega^2 E_s - u^2 \frac{d E_s}{dz^2} = 0 \qquad \div(-u^2)$$

$$\frac{d^2 E_s}{dz^2} + \frac{\omega^2}{u^2} E_s = 0 \qquad \frac{\omega^2}{u^2} = \beta^2$$

β = Phase constant (rad/m)

* $\omega \Rightarrow$ rad/s
* $u \Rightarrow$ m/s

$$\frac{d^2 E_s}{dz^2} + \beta^2 E_s = 0$$

2 solutions

$m^2 + \beta^2 = 0 \qquad m = \pm j\beta$ "imaginary"

$$\left.\begin{aligned} E_s &= A\cos\beta z \\ E_s &= \beta \sin\beta z \\ E_s &= A\cos\beta z + \beta\sin\beta z \end{aligned}\right\} \text{Solution}$$

$$E = jA\sin(\omega t - \beta z) + \beta\cos(\omega t - \beta z)$$

* If we take the Imaginary part

$E = A\sin(\omega t - \beta z)$

amplitude of the wave

phase arguments in radian

$$\sin(\theta \pm \pi/2) = \pm\cos\theta$$

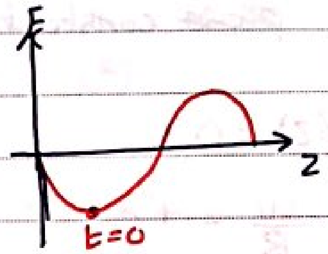
$$\sin(\theta \pm \pi) = -\sin\theta$$

$$\cos(\theta \pm \pi/2) = \mp\sin\theta$$

$$\cos(\theta \pm \pi) = -\cos\theta$$

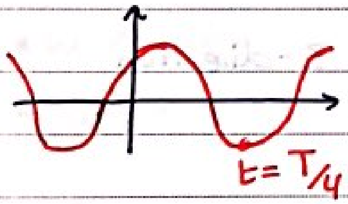
~~Electromagnetic wave Propagation:~~

$$E = A\sin(\omega t - \beta z)$$



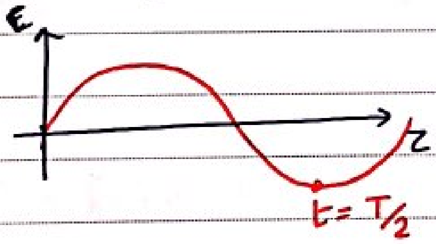
at $t=0 \Rightarrow A\sin(-\beta z)$

at $t=\frac{T}{4} \Rightarrow A\sin(90^\circ - \beta z)$



looks like $\cos(\beta z)$

at $t=\frac{T}{2} \Rightarrow A\sin(180-\beta z)$



the locus moves toward the z-direction

$$\beta = \frac{\omega}{u} \rightarrow \frac{2\pi/f}{\lambda/f} \Rightarrow \beta = 2\pi/\lambda$$

Notes:

$\omega t + \beta z$: wave travels in the $-ve$ z-direction

$\omega t - \beta y$: wave travels in the $+ve$ y-direction

$\sin(-\theta) = -\sin\theta = \sin(\theta \pm \pi)$

$\cos(-\theta) = \cos(\theta)$

No.

β = phase constant (rad/m)

Assuming ~~phase~~ constant phase

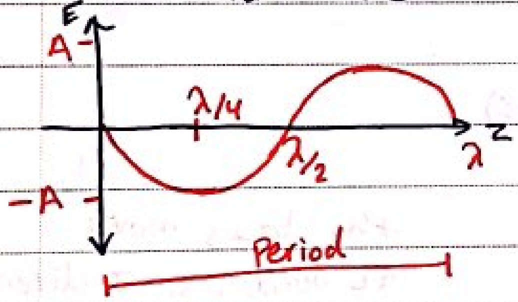
$(\omega t - \beta z) = 0$

$$\frac{dz}{dt} = \frac{\omega}{\beta} = +u$$

$E = A\sin(\omega t + \beta z)$

the wave Propagates in the -ve z-direction

at constant time

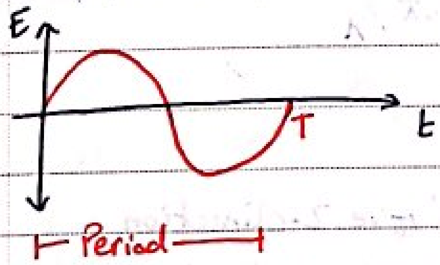


Period in free Space = λ

λ = wave length

at constant z (z=0)

$E = A\sin(\omega t)$



$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8 \text{ m/s}$$

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Ex: Given $\vec{E} = 50\cos(10^8 t + \beta x)\,\hat{a}_y$ V/m

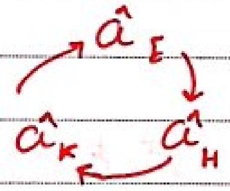
In free space $(\epsilon = \epsilon_0,\ \mu = \mu_0,\ \sigma = 0)$

find: a) Direction of wave propagation

b) β & time the wave takes to travel a distance $\lambda/2$

c) Sketch the wave at $t = 0, T/4, T/2$

a) $\hat{a}_k$ = Unit vector in the wave propagation direction



$\hat{a}_k = -\hat{a}_x$

$\hat{a}_H = \hat{a}_k \times \hat{a}_E = -\hat{a}_z$

b) $\beta = \frac{\omega}{u} = \frac{\omega}{c} = \frac{10^8}{3\times10^8} = \frac{1}{3}$ rad/m $= 0.333$ rad/m

$\lambda = \frac{c}{f} = \frac{c}{\omega/2\pi} = \frac{3\times10^8}{10^8/2\pi} = 6\pi$ m

$\beta = \frac{2\pi}{\lambda}$

$\lambda_1 = \frac{\lambda}{2} = 3\pi$ m , $t_1 = \frac{\lambda_1}{c} = \frac{3\pi}{3\times10^8} = 31.42$ ns

$T = \frac{2\pi}{\omega} = \frac{2\pi}{10^8}$, $t_1 = T/2$

~~at t=0~~

No.

c) at $t=0$

$\vec{E} = 50 \cos(\beta x)\, \hat{a}_y$

at $t = T/4$

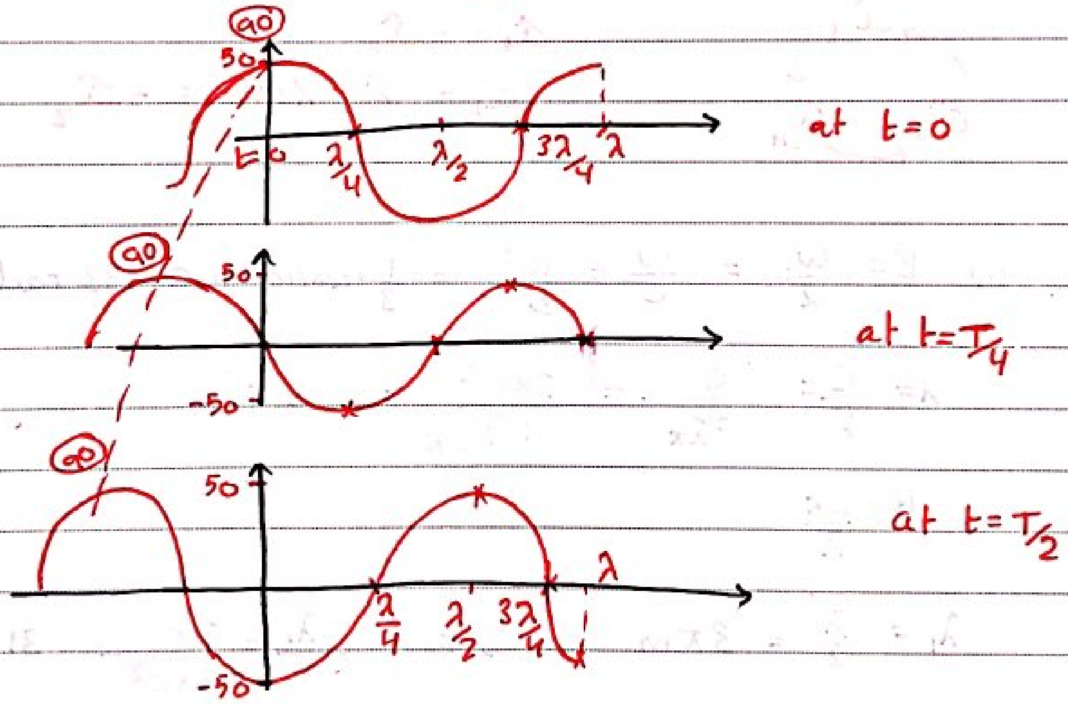
$\vec{E} = 50 \cos\left(\frac{2\pi}{T} \frac{T}{4} + \beta x\right)$

$= 50 \cos\left(\beta x + \frac{\pi}{2}\right) = -50 \sin(\beta x)$

at $t = T/2$

$\vec{E} = 50 \cos\left(\frac{2\pi}{T} \times \frac{T}{2} + \beta x\right)$

$= 50 \cos(\beta x + 180) = -50 \cos(\beta x)$



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* Wave Propagation In lossy direction ~~($\sigma \neq 0$)~~ $\begin{pmatrix} \sigma \neq 0 \\ \varepsilon = \varepsilon_0 \varepsilon_r \\ \mu = \mu_0 \mu_r \end{pmatrix}$

$\rightarrow$ Source free region ($\rho_v = 0$, $\vec{J} = 0$)

ϵ is homogen-ous isotropic linear

$\nabla \cdot \vec{E}_s = 0$ — ⓐ ($\nabla \cdot \epsilon E_s = 0$)

$\nabla \cdot \vec{H}_s = 0$ — ⓑ

$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s$ — ⓒ

$\nabla \times \vec{H}_s = (\sigma + j\omega\epsilon)\vec{E}_s$ — ⓓ

Taking the curl of eq. (C) ($\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot A) - \nabla^2 A$

$\nabla \times \nabla \times \vec{E}_s = -j\omega\mu \; \nabla \times \vec{H}_s$

$\nabla(\nabla \cdot \vec{E}_s) - \nabla^2 \vec{E}_s = -j\omega\mu(\sigma + j\omega\epsilon)\vec{E}_s$

$\nabla^2 E_s = +j\omega\mu(\sigma + j\omega\epsilon)\vec{E}_s$

let $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

$\boxed{\nabla^2 \vec{E}_s - \gamma^2 \vec{E}_s = 0}$ $(\nabla^2 E_s - \nabla^2 E_s) = 0$

wave eq.

Helmholtz's equation

$\sigma = 0$

$\beta \Rightarrow$ real

$m^2 - \gamma^2 = 0$

$m = \pm \gamma$

γ: Propagation constant

it is complex

Sol. ~~$f(\omega t - \gamma z)$~~

$\vec{E} = A\, e^{j(\omega t - \gamma z)} + B\, e^{j(\omega t + \gamma z)}$

forward Backward

No.

If you take curl of (d)

$\nabla^2 \vec{H}_s - \gamma^2 \vec{H}_s = 0$

γ: depend on the $\epsilon, \mu, \sigma, \omega$

$\gamma = \alpha + j\beta \rightarrow$ complex

$\gamma^2 = \alpha^2 + 2j\beta\alpha - \beta^2$, $\boxed{|\gamma^2| = \alpha^2 + \beta^2}$ — ①

$= \sqrt{(\alpha^2 - \beta^2)^2 + (2j\beta\alpha)^2}$

$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$ $= \alpha^2 + \beta^2$

$\boxed{\gamma^2 = j\omega\mu\sigma - \omega^2\mu\epsilon}$ — ②

Solve (1) & (2) to find α, β

$|\gamma^2| = \sqrt{\omega^2\mu^2\sigma^2 + \omega^4\mu^2\epsilon^2} = \omega\mu\sqrt{\sigma^2 + \omega^2\epsilon^2}$

~~equating the real part~~

$\cancel{\alpha^2 - \beta^2 = -\omega^2\mu\epsilon}$

equating the real part

$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon \Rightarrow \boxed{\beta^2 = \alpha^2 + \omega^2\mu\epsilon}$

$\alpha^2 + \beta^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\epsilon^2}$

$\alpha^2 + \alpha^2 + \omega^2\mu\epsilon = \omega\mu\sqrt{\sigma^2 + \omega^2\epsilon^2}$

$2\alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\epsilon^2} - \omega^2\mu\epsilon$

$2\alpha^2 = \omega\mu\sqrt{\omega^2\epsilon^2\left(\frac{\sigma^2}{\omega^2\epsilon^2}\right)} - \omega^2\mu\epsilon$

smile for life

$$\alpha^2 = \frac{\omega^2 \mu \epsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\epsilon}\right)^2 + 1} - 1\right]$$

$$\alpha^2 = \frac{\omega^2 \mu \epsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} - 1\right]$$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} - 1\right]}}$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} + 1\right)}}$$

*Waves in Lossy media

$$\begin{cases} \nabla^2 \vec{E}_s - \gamma^2 \vec{E}_s = 0 \\ \nabla^2 \vec{H}_s - \gamma^2 \vec{H}_s = 0 \end{cases}$$

↳ helmholtz's eq.

$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$ ← maxwell's eq.

$\gamma = \alpha + j\beta$ complex

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} - 1\right]}$$

&

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} - 1\right]}$$

Solution:

$$E_{xs} = \underbrace{E_0 e^{-\gamma z}}_{\text{forward/+ve z}} + \underbrace{E_0 e^{\gamma z}}_{\text{backward/-ve z}}$$

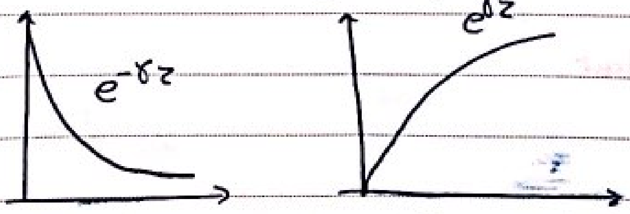
↳ assume E is in x-direction

$\vec{E}(z,t) = Re\{E_{xs} e^{j\omega t}\}$

↑ instantaneous form

$\vec{E}(z,t) = Re\{E_0 e^{-\gamma z} e^{j\omega t} \vec{a}_x\}$

only +ve z



$z \to \infty \Rightarrow E \to 0$ ✓ $z \to \infty \Rightarrow E \to \infty$ X

$\vec{E}$ must vanish at infinity

No.

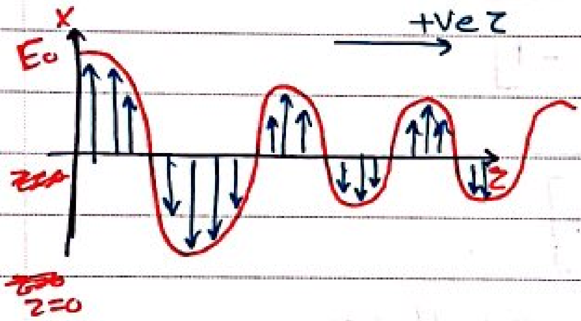
$$E(z,t) = Re\{E_0 e^{-\gamma z} e^{j\omega t} \vec{a}_x\}$$

$$= Re\{E_0 e^{j\omega t} e^{-(\alpha + j\beta)z} \vec{a}_x\}$$

$$\vec{E}(z,t) = Re\{E_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \vec{a}_x\}$$

$$\boxed{\vec{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\ \hat{a}_x}$$

In Lossy media ($\sigma \neq 0$, $\varepsilon = \varepsilon_0 \varepsilon_r$, $\mu = \mu_0 \mu_r$)



The direction of $\vec{E}$ is called the polarization of the wave.

$\alpha \equiv$ Attenuation constant (Neper/m) or (Np/m) or (dB/m)

In free space $\alpha = 0$, $\beta = \omega\sqrt{\mu\varepsilon}$

$$1NP = 20\log_{10} e$$

$$1Np = 8.686dB$$

$\beta \equiv$ phase constant (rad/m)

$\gamma \equiv$ Propagation constant

$\gamma = \alpha + j\beta$

No.

$\vec{H}$ can be found from Maxwell's eq.

$$\vec{H}(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z)$$

$$\hat{a}_H = \hat{a}_k \times \hat{a}_E$$
$$= \hat{a}_z \times \hat{a}_x = \hat{a}_y$$

$H_0 = \frac{E_0}{\eta}$ ohm's law

$\eta \equiv$ Intrinsic Impedance (ممانعة جوهرية) $[\Omega]$ (إيتا)

Impedance (ممانعة عامة)

$I = \frac{V}{Z}$

$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$ from Maxwell's eq.

$$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s \quad , \quad \nabla \times \vec{H}_s = (\sigma + j\omega\varepsilon)\vec{E}_s$$

$$\vec{H}_s = \frac{-1}{j\omega\mu} \nabla \times \vec{E}_s$$

$$\vec{H}_s = \frac{-1}{j\omega\mu} \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ d/dx & d/dy & d/dz \\ E_0 e^{-\gamma z} & 0 & 0 \end{vmatrix}$$

$$= \frac{+1}{j\omega\mu} \left[E_0 \gamma e^{-\gamma z} \hat{a}_y \right] =$$

$$\vec{H}_s = H_0 e^{-\gamma z} \hat{a}_y$$

smile for life

No.

$$\frac{\gamma E_o}{j\omega\mu} = H_o$$

$$\frac{E_o}{H_o} = \frac{j\omega\mu}{\gamma} = \eta$$

$$= \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma+j\omega\varepsilon)}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}}$$ complex

$$\eta = |\eta| \angle \theta_\eta$$

$$\eta^2 = \frac{j\omega\mu}{\sigma+j\omega\varepsilon} * \frac{\sigma - j\omega\varepsilon}{\sigma - j\omega\varepsilon}$$

$$\cancel{= j\omega\mu\sigma}$$

$$= \frac{j\omega\mu\sigma + \omega^2\mu\varepsilon}{\sigma^2 + \omega^2\varepsilon^2}$$

$$\eta^2 = \frac{\omega^2\mu\varepsilon}{\sigma^2+\omega^2\varepsilon^2} + j\frac{\omega\mu\sigma}{\sigma^2+\omega^2\varepsilon^2}$$

$$|\eta^2| = \sqrt{\frac{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2}{(\sigma^2+\omega^2\varepsilon^2)^2}}$$

$$= \sqrt{\frac{\omega^2\mu^2\cancel{(\omega^2\varepsilon^2+\sigma^2)}}{(\sigma^2+\omega^2\varepsilon^2)^2}}$$

$$|\eta^2| = \frac{\omega\mu}{\sqrt{\sigma^2+\omega^2\varepsilon^2}}$$

$$= \sqrt{\frac{\omega^2\mu^2/\omega^2\varepsilon^2}{\left(\frac{\sigma}{\omega\varepsilon}\right)^2+1}}$$

No.

$$|\gamma^2| = \frac{\mu/\varepsilon}{\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2+1}}$$

$$\boxed{|\gamma| = \frac{\sqrt{\mu/\varepsilon}}{\left[\left(\frac{\sigma}{\omega\varepsilon}\right)^2+1\right]^{1/4}}}$$

$$\cancel{\theta_\gamma = \frac{1}{2}\tan^{-1}\left(\frac{\omega^2\mu\varepsilon}{\omega\mu\sigma}\right)}$$

$$\theta_\gamma = \frac{1}{2}\tan^{-1}\left(\frac{\omega\mu\sigma}{\omega^2\mu\varepsilon}\right) \Rightarrow \boxed{2\theta_\gamma = \tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

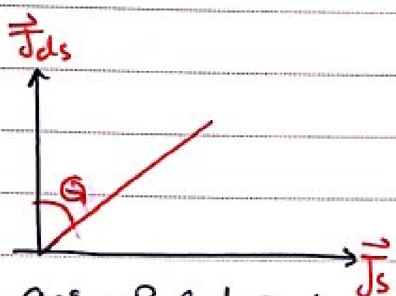
$$\theta = \tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$$

↳ loss tangent

$$\boxed{\gamma = \frac{\sqrt{\mu/\varepsilon}}{\left[\left(\frac{\sigma}{\omega\varepsilon}\right)^2+1\right]^{1/4}} \angle \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

Loss tangent : $\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$

$$\frac{\sigma}{\omega\varepsilon} = \frac{|\sigma \vec{E}_s|}{|j\omega\varepsilon \vec{E}_s|} = \frac{\vec{J}_s}{\vec{J}_{ds}}$$



If $\theta = 90° \rightarrow$ Perfect Conductor

If $\theta = 0° \rightarrow$ lossless or free space

$$0 \leq \theta \leq 90°$$

$$0 \leq \theta_\gamma \leq 45°$$

lossless free space

Perfect conductor

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| \angle \theta_\eta$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$

$$\theta = 2\theta_\eta = \tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right) \equiv \text{Loss tangent angle}$$

$$\text{loss tangent} = \frac{\sigma}{\omega\varepsilon} = \frac{\vec{J}_s}{\vec{J}_{ds}}$$

for perfect dielectric $\frac{\sigma}{\omega\varepsilon} <<< 1$
or $\sigma <<< \omega\varepsilon$
very low loss tangent
θ is very small $\approx 0°$

for perfect conductors $\frac{\sigma}{\omega\varepsilon} >>> 1$

$\sigma >>> \omega\varepsilon$

$\theta \approx 90°$ or $\theta_\eta \approx 45°$

$$\nabla \times \vec{H} = \vec{J} + \vec{J}_d$$
$$= \sigma\vec{E} + \frac{d\vec{D}}{dt}$$
$$= \sigma\vec{E} + \varepsilon\frac{d\vec{E}}{dt}$$

$$\nabla \times \vec{H}_s = (\sigma + j\omega\varepsilon)\vec{E}_s$$

$$\nabla \times \vec{H}_s = j\omega\varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)\vec{E}_s$$

No.

$$\nabla \times \vec{H}_s = j\omega \varepsilon_c \vec{E}_s$$

$$\varepsilon_c = \left(1 - j\frac{\sigma}{\omega\varepsilon}\right)\varepsilon$$

$\varepsilon_c \equiv$ Complex permittivity

$$\varepsilon_c = \varepsilon' - j\varepsilon''$$

$$\varepsilon' = \varepsilon \quad , \quad \varepsilon'' = \frac{\sigma}{\omega}$$

$$\tan\theta = \frac{\sigma}{\omega\varepsilon} = \frac{\varepsilon''}{\varepsilon'}$$

* Wave Propagation in lossless media:
 ($\sigma = 0$, $\varepsilon = \varepsilon_0 \varepsilon_r$, $\mu = \mu_0 \mu_r$

$\alpha = 0$ (No attenuation)

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{u} \quad ; \quad u = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\eta = \sqrt{\frac{\mu}{\varepsilon}} \angle 0^\circ$ real part

$$\lambda = \frac{2\pi}{\beta} = \frac{u}{f}$$

$\vec{E}$ & $\vec{H}$ are in phase

$\angle\theta_\eta = 0^\circ$

$\gamma = j\beta$

No.

* wave Propagation in free Space
  ($\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$)

$\alpha = 0$ (No attenuation)

$\beta = \frac{\omega}{C} = \omega\sqrt{\mu_0\varepsilon_0}$, $u = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = C$

$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \approx 377\,\Omega = \eta_0$

$\lambda = \frac{2\pi}{\beta} = \frac{C}{f}$

$\angle\theta_\eta = 0°$ $\vec{E}$ & $\vec{H}$ are in phase.

i.e

$\vec{E} = E_0 \cos(\omega t - \beta z)\,\hat{a}_x$ V/m

$\cancel{\vec{H} = H_0 \cos(\omega t - \beta z)\,\hat{a}_y}$

$\vec{H} = +H_0 \cos(\omega t - \beta z)\,\hat{a}_y$

$\hat{a}_H = \hat{a}_k \times \hat{a}_E$
$= \hat{a}_z \times \hat{a}_x$
$= \hat{a}_y$

$\vec{H} = +\frac{E_0}{\eta_0} \cos(\omega t - \beta z)\,\hat{a}_y$

wave $= \vec{E} + \vec{H}$

electric field + magnetic field

Uniform Plane wave (UPW)

No.

Since there are no component if $\vec{E}$ & $\vec{H}$ along $\hat{a}_k$
The wave is called as Transverse electromagnetic
(TEM)

* E_z , H_z $\times$ E_x, E_y H_x, H_y $\hat{a}_k = \hat{a}_z$
* Coaxial cable ⇒ TEM "2 conductors"

* Wave Propagation in perfect conductors
$(\sigma \approx \infty,\ \varepsilon = \varepsilon_0,\ \mu = \mu_0 \mu_r)$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

for lossy

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

* $\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}}$ >>> 1 ⇒ High attenuation

* $\boxed{\alpha = \beta}$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}, \quad |\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$

$$\tan(2\theta_\eta) = \frac{\sigma}{\omega\varepsilon}$$

$\cancel{|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1+\sigma/\omega\varepsilon}}}$ $$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{\sigma/\omega\varepsilon}} = \sqrt{\frac{\mu\omega\varepsilon}{\sigma\varepsilon}}$$

No.

$$\boxed{\begin{array}{l} |\eta| = \sqrt{\frac{\mu\omega}{\sigma}} \\ \theta_\eta = 45^\circ \end{array}}$$

$\vec{E} = E_0 \, e^{-\alpha z} \cos(\omega t - \beta z) \; \hat{a}_x \quad V/m$

$\vec{H} = \frac{E_0}{\eta} \, e^{-\alpha z} \cos(\omega t - \beta z) \, \hat{a}_y \quad A/m$

$\vec{H} = \frac{E_0}{|\eta| \, e^{j\theta_\eta}} \, e^{-\alpha z} \cos(\omega t - \beta z) \, \hat{a}_y \quad A/m$

$= \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \underbrace{45^\circ}_{\theta_\eta}) \, \hat{a}_y \quad A/m$

$\vec{E}$ leads $\vec{H}$ by 45°

smile for life

*Propagation in good conductors ($\sigma \gg \omega\varepsilon$), $\varepsilon = \varepsilon_0$, $\mu = \mu_0\mu_r$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{\omega^2}{\sqrt{\frac{\omega\mu\sigma}{2}}}} = \sqrt{\frac{2\omega}{\mu\sigma}} = \frac{c}{\sqrt{\mu_r}}$$

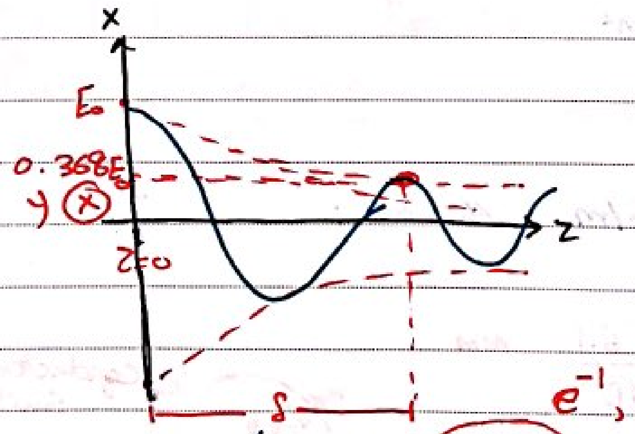
$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ \ \Omega$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$$

$$\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x \quad V/m$$

$$\vec{H} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ)\, \hat{a}_y \quad A/m$$

$\vec{E}$ leads $\vec{H}$ by 45° ; V leads I by 45° => Inductive



$\sigma \simeq \infty$

e^{-1}, τ

when E_0 drop to 36.8% of it's maximum

=> The distance inside the conductor ~~is~~ called the Skin depth. (δ) in meter.
delta

No.

$\cancel{E_0 e^{-\alpha z} = E_0 e^{-1}}$

when $z = \delta$

$$E_0 e^{-\alpha\delta} = E_0 e^{-1}$$

$$\delta = \frac{1}{\alpha}$$

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad m}$$

$$\eta = \frac{\sqrt{2}}{\sigma\delta} e^{j\pi/4}$$

$$\eta = \frac{\sqrt{2}}{\sigma\delta} \times \frac{1}{\sqrt{2}}(1+j) \Rightarrow \boxed{\eta = \frac{1+j}{\sigma\delta} \ \Omega} \quad \text{complex}$$

The real part called $R_s \equiv$ Surface resistance

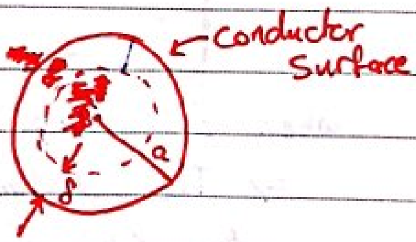
$$R_s = \frac{1}{\sigma\delta} \ (\Omega/m^2)$$

$$= \sqrt{\frac{\pi f \mu}{\sigma}} \ \Omega/m^2$$

i.e: for copper

$\sigma = 5.8 \times 10^7$ S/m , $\mu = \mu_0$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{66.1}{\sqrt{f}} \ mm$$

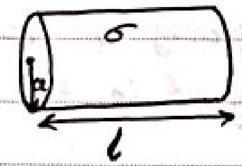


f(Hz)	δ (mm)
10	20.8 mm
60	8.6 mm
100	6.6 mm
500	2.99
10^{10}	6.6×10^{-4}

No.

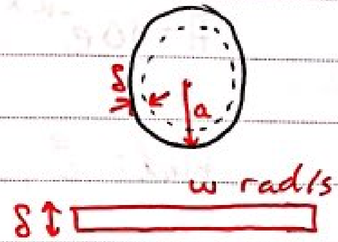
Resistance:

$$R_{DC} = \frac{l}{\sigma S}$$



$$S = \pi a^2$$

$$R_{ac} = \frac{R_s l}{w} = \frac{l}{\sigma \delta w}$$



$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = \frac{1}{\sigma \delta}$$

for a wire of radius = a $\Rightarrow w = 2\pi a$

$$\frac{R_{ac}}{R_{DC}} = \frac{\frac{\cancel{l}}{\cancel{\sigma}\, \delta\, 2\cancel{\pi} a}}{\frac{\cancel{l}}{\cancel{\sigma}\, \pi a^2}} = \frac{a}{2\delta}$$

- for $f = 0$ $R_{ac} = R_{DC}$
- as $f \uparrow$ $R_{ac} > R_{DC}$
- for low frequency $\Rightarrow \frac{R_{ac}}{R_{DC}} \approx 1$

No.

Ex: A lossy dielectric has $\eta = 200\angle 30°\ \Omega$ at a certain frequency, for the same frequency the plane wave propagated in the ~~mossy~~ lossy media has

$$\vec{H} = 10e^{-\alpha x}\cos(\omega t - \underbrace{\tfrac{1}{2}}_{\beta}x)\ \hat{a}_y \quad A/m$$

Find: $\vec{E}$, α, δ, Polarization

$$(\hat{a}_E = \hat{a}_H \times \hat{a}_k)$$
$$= \hat{a}_y \times \hat{a}_x$$
$$\hat{a}_E = -\hat{a}_z$$

$$\vec{E} = E_o e^{-\alpha x}\cos(\omega t - \tfrac{1}{2}x)\ (-\hat{a}_z) \quad V/m$$

$$E_o = H_o\,\eta$$
$$= 10 \times 200\angle 30°$$
$$= 2000\, e^{j\pi/6}$$

$$\vec{E} = 2000 e^{j\pi/6} e^{-\alpha x}\cos(\omega t - \tfrac{1}{2}x)\ (-\hat{a}_z) \quad V/m$$
$$\vec{E} = 2000\, e^{-\alpha x}\cos(\omega t - \tfrac{1}{2}x + 30°)\ (-\hat{a}_z) \quad V/m$$

α

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2+1} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2+1} + 1\right]} = \frac{1}{2}$$

$$\frac{\alpha}{\beta} = \sqrt{\frac{\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2+1} - 1}{\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2+1} + 1}}$$

No.

$$2\underbrace{\theta_\gamma}_{30^\circ} = \tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$$

$$\frac{\sigma}{\omega\varepsilon} = \tan 60^\circ$$

$$\frac{\sigma}{\omega\varepsilon} = \sqrt{3}$$

$$\boxed{\frac{\alpha}{\beta} = \sqrt{\frac{1}{3}}} \Rightarrow \boxed{\cancel{\alpha = \frac{1}{\sqrt{6}}}} \quad \boxed{\alpha = \frac{1}{2\sqrt{3}} = 0.2887\,Np/m}$$

$$\boxed{\delta = \frac{1}{\alpha} = 3.464\ mm}$$

$\sigma = 0$

* Ex. In a lossless medium that has a $\eta = 60\pi\ \Omega$, $\mu_r = 1$

$\vec{H} = -0.1\cos(\omega t - z)\,\hat{a}_x + 0.5\sin(\omega t - z)\,\hat{a}_y$ A/m

calculate $\varepsilon_r, \omega, \vec{E}$??

lossless $\Rightarrow \angle 0^\circ$

η real in lossless

ε_r

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \eta_0 \times \frac{1}{\sqrt{\varepsilon_r}} = 60\pi$$

$$\sqrt{\varepsilon_r} = 2 \rightarrow \boxed{\varepsilon_r = 4}$$

ω

$$\beta = \omega\sqrt{\mu\varepsilon} = 1$$

$$\omega\sqrt{\mu_0\varepsilon_0\mu_r\varepsilon_r} = 1$$

$$(2)\frac{\omega}{c} = 1 \longrightarrow \boxed{\omega = 1.5\times10^{8}\ \text{rad/s}}$$

$\vec{E}$

$$\vec{H} = \vec{H_1} + \vec{H_2}$$

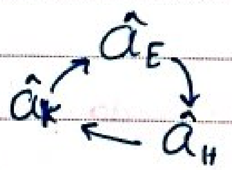
$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\vec{E_1} = \vec{E}_{10}\cos(\omega t - z)\,\hat{a}_{E_1}$$

$$\vec{E_2} = E_{20}\sin(\omega t - z)\,\hat{a}_{E_2}$$

$$E_{10} = \eta H_{10} = 6\pi\ \text{V/m} \quad ; \quad E_{20} = \eta H_{20} = 30\pi\ \text{V/m}$$

$$\vec{E} = 6\pi\cos(\omega t - z)\,\hat{a}_y + 30\pi\sin(\omega t - z)\,\hat{a}_x$$



$$\hat{a}_{E_1} = \hat{a}_{H_1}\times\hat{a}_{k_1} = -\hat{a}_x\times\hat{a}_z = \hat{a}_y$$

$$\hat{a}_{E_2} = \hat{a}_{H_2}\times\hat{a}_{k_2} = \hat{a}_y\times\hat{a}_z = \hat{a}_x$$

or Method 2 "Solved in book"

$$\nabla \times \vec{H} = \cancel{\sigma \vec{E}}^{\,0} + \varepsilon \frac{d\vec{E}}{dt}$$

$$\vec{E} = \frac{1}{\varepsilon} \int \nabla \times \vec{H} \, dt$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ d/dx & d/dy & d/dz \\ H_x & H_y & 0 \end{vmatrix} = \hat{a}_x\left(-\frac{dH_y}{dz}\right) - \hat{a}_y\left(-\frac{dH_x}{dz}\right) + \hat{a}_z\left(\cancel{\frac{dH_y}{dx}}^{\,0} - \frac{dH_x}{dy}\right)$$

$$=$$

$$\vec{E} = \frac{1}{\varepsilon} \int \nabla \times \vec{H} \, dt$$

Ex. A uniform plane wave $\vec{E} = E_0 \cos(\omega t - \beta z)\hat{a}_x$ is incident on a good conductor at $z=0$.
Find the current density in the conductor.

- Since no phase shift or attenuation → ~~lossless~~ $\alpha = 0$ → lossless, free space
$e^{j\omega t}$, $0.1E_0$

$\vec{J} = \sigma \vec{E}$, $\nabla^2 \vec{E_s} - \gamma^2 \vec{E_s} = 0$

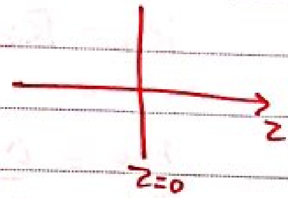
$\nabla^2 \vec{J_s} - \gamma^2 \vec{J_s} = 0$

$\vec{J_s} = Ae^{-\gamma z} + \cancel{Be^{\gamma z}}$ → in +ve or −ve z-direction
⇒ good conductor @ $z=0$

(B) must be zero to have a vanishing field at infinity
$\vec{J_s} = Ae^{-\gamma z}$

No.

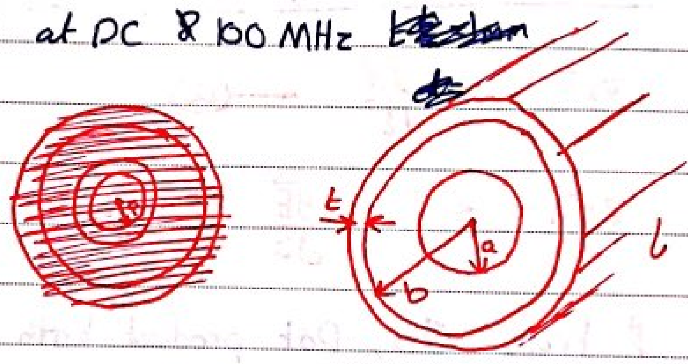
$\gamma = \alpha + j\beta \Rightarrow \underbrace{\alpha = \beta = \frac{1}{\delta}}_{\text{Good conductor}}$



$\gamma = \frac{1+j}{\delta} \Rightarrow \boxed{\vec{J}_s = J_{so}\, e^{-(1+j)z/\delta} \quad A/m^2}$

* **Ex** for a copper coaxial cable $a = 2mm$ $t = 1mm$
$\sigma = 5.8 \times 10^7$ S/m $b = 6mm$ $l = 2m$

find the resistance at DC & 100 MHz



$R = \underset{\text{inner}}{R_i} + \underset{\text{outer}}{R_o}$

at DC → $R_{dc} = \frac{l}{\sigma A}$

$R_{i_{dc}} = \frac{l}{\sigma S} = \frac{l}{\sigma \pi a^2} = 2.744$ m-Ω

$R_{o_{dc}} = \frac{l}{\sigma S} = \frac{l}{\sigma \pi((b+t)^2 - b^2)} = 0.8429$ m-Ω

$R_{dc} = R_{i_{dc}} + R_{o_{dc}} = 3.587$ m-Ω

at $f = 100$ MHz

$R_{iac} = \frac{R_s l}{w} = \frac{l}{\sigma \delta 2\pi a} = \frac{l}{2\pi a}\sqrt{\frac{\pi f \mu}{\sigma}} = 0.41\,\Omega$

$\delta = 6.6\,\mu m \ll t$

$R_{oac} = \frac{R_s l}{w} = 0.1384\,\Omega$

$w = 2\pi b$

No.

$R_{ac} = R_{iac} + R_{idc} = 0.5484\ \Omega$

$\frac{R_{ac}}{R_{dc}} = \frac{0.5484}{0.003587} \approx 150$ times.

* Power & Poynting vector: Pointing

$\nabla \times \vec{E} = -\mu \frac{d\vec{H}}{dt}$ — (a)

$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{d\vec{E}}{dt}$ — (b)

from (b), Dot product both sides with $\vec{E}$

$\vec{E} \cdot \nabla \times \vec{H} = \sigma E^2 + \vec{E} \cdot \varepsilon \frac{d\vec{E}}{dt}$

In general: $\nabla \cdot (A \times B) = \vec{B} \cdot \nabla \times \vec{A} - \vec{A} \cdot \nabla \times \vec{B}$

let $\vec{A} = H$, $\vec{B} = \vec{E}$

$\vec{H} \cdot \nabla \times \vec{E} + \nabla \cdot (\vec{H} \times \vec{E}) = \vec{E} \cdot \nabla \times \vec{H}$

$= \sigma E^2 + \vec{E} \cdot \varepsilon \frac{d\vec{E}}{dt}$ — (1)

$\vec{H} \cdot \nabla \times \vec{E} = \vec{H} \cdot \left(-\mu \frac{d\vec{H}}{dt}\right) = -\frac{\mu}{2} \frac{d\vec{H} \cdot \vec{H}}{dt}$

$\frac{dH^2}{dt} = 2H \frac{dH}{dt}$

$= -\frac{\mu}{2} \frac{dH^2}{dt}$ — sub in (1)

No.

$$-\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \nabla\cdot(\vec{E}\times\vec{H}) = \sigma E^2 + \frac{\varepsilon}{2}\frac{\partial E^2}{\partial t}$$

rearrange :

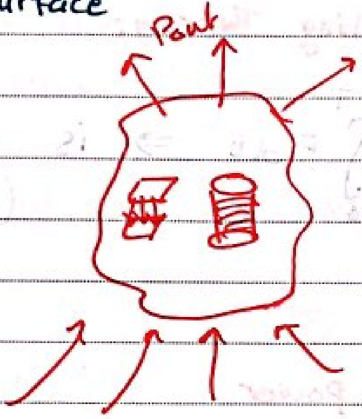
$$\nabla\cdot(\vec{E}\times\vec{H}) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) - \sigma E^2$$

$$\int \nabla\cdot(\vec{E}\times\vec{H})\,dv = -\frac{\partial}{\partial t}\int_v \left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) dv - \int \sigma E^2\,dv$$

$$\boxed{\oint_s \vec{E}\times\vec{H}\cdot d\vec{s} = - - - - - - - -}$$ Poynting theorem

* Poynting Theorem:

$$\underbrace{\oint_s \vec{E}\times\vec{H}\cdot ds}_{\text{Total Power Leaving the surface}} = -\frac{d}{dt}\int_v \underbrace{\left(\tfrac{1}{2}\varepsilon E^2 + \tfrac{1}{2}\mu H^2\right)dv}_{\text{Stored energy in C \& L}} - \underbrace{\int \sigma E^2 dv}_{\text{Conduction (ohmic) losses}}$$



$$\sigma E^2 = \sigma E\cdot E$$
$$= J\mathcal{E}$$
$$= IV$$

$\vec{P} = \vec{E}\times\vec{H}$ in (w/m^2)

↳ Poynting vector in $\hat{a}_k$ air

$$\frac{V}{m}\cdot\frac{A}{m} = \frac{VA}{m^2} = \frac{w}{m^2}$$

* for alossy media

$$\vec{E} = E_o\, e^{-\alpha z}\cos(\omega t - \beta z)\,\hat{a}_x$$

$$\vec{H} = \frac{E_o}{|\eta|}\, e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\,\hat{a}_y$$

$$\vec{P} = \vec{E}\times\vec{H} = \frac{E_o^2}{|\eta|}\, e^{-2\alpha z}\times\frac{1}{2}\left[\cos(2\omega t - 2\beta z - \theta_\eta) + \cos\theta_\eta\right]\hat{a}_z$$

No.

$$\vec{P}(z,t) = \vec{P} = \frac{E_o^2}{2|\eta|} e^{-2\alpha z}\left[\cos\theta_\eta + \cos(2\omega t - 2\beta z - \theta_\eta)\right]$$

is time Variant

-Time average poynting theorem:

$$\vec{S}_{avg} = \vec{P}_{avg} = \frac{1}{T}\int_0^T P\,dt \Rightarrow \text{is time Invariant}$$

(w/m^2)

Total time average power

$$P_{avg} = \int_s \vec{P}_{avg} \cdot d\vec{s} = \int_s S_{avg} \cdot d\vec{s}$$

space

$$\vec{P} = \vec{P}(z,t) = \vec{E} \times \vec{H}$$

$$\vec{P}_{avg} = \vec{S}_{avg} = \frac{1}{T}\int_T \vec{P}(z,t)\,dt$$) Proof Problem 10.28

$$= \frac{1}{2} Re(\vec{E}_s \times \vec{H}_s^*)$$

$$P_{avg} = \int_s \vec{P}_{avg} \cdot d\vec{s} = \int_s \vec{S}_{avg} \cdot d\vec{s}$$

$$\vec{P}_{avg} = \frac{E_o^2}{2|\eta|} e^{-2\kappa z} \cos\theta_\eta \quad w/m^2$$

smile for life

No.

* **Ex.** In a non-magnetic medium

$$\vec{E} = 4\sin(2\pi\times10^7 t - 0.8x)\,\hat{a}_z \text{ V/m}$$

Find:

a) ε_r, η

b) Time average power carried by the wave

c) Total Power crossing 100 cm^2 of plane $2x+y=5$

a)

* Since $\beta \neq \frac{\omega}{c}$ it is a lossless medium.

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

$$\cancel{0.8 = 2\times10^7 \sqrt{}}$$

$$0.8 = \frac{2\pi\times10^7}{3\times10^8}\sqrt{\varepsilon_r} \Rightarrow \boxed{\varepsilon_r = 14.59}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r}} = \frac{\eta_o}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{14.95}}$$

$$\eta = 10\pi^2\ \Omega$$
$$= 98.7\ \Omega$$

b) $\vec{P}(x,t) = \vec{E}\times\vec{H}$

$$= \frac{E_o^2}{\eta}\sin^2(\omega t - 0.8x)\,\hat{a}_x$$

$$= \frac{16}{98.7}\sin^2(\omega t - 0.8x)\,\hat{a}_x \text{ W/m}^2$$

$$\vec{P}_{ave} = \vec{S}_{avg} = \frac{1}{T}\int_0^T \frac{16}{98.7}\times\frac{1}{2}\left(1 - \overset{0}{\cancel{\cos(2\omega t - 1.6x)}}\right)dt\ \hat{a}_x$$

$$= \frac{16}{98.7}\times\frac{1}{2}\times T\times\frac{1}{T} = 81\ \hat{a}_x \text{ mW/m}^2$$

No.

c) $P_{ave} = \int_s \vec{P}_{ave} \cdot d\vec{s}$; $d\vec{s} = ds\, \hat{a}_n$

$= \vec{P}_{ave} * . S\, \hat{a}_n$

$\hat{a}_n = \frac{\nabla f}{|\nabla f|}$, $f = 2x + y - 85$

$\hat{a}_n = \frac{\langle 2, 1, 0 \rangle}{\sqrt{5}} = \frac{2}{\sqrt{5}} \hat{a}_x + \frac{1}{\sqrt{5}} \hat{a}_y$

~~$P_{ave} = 81 \times 10^{-3}$~~

$P_{ave} = 81 \times 10^{-3} * 100 \times 10^{-4} * \frac{2}{\sqrt{5}} * (\hat{a}_x \cdot \hat{a}_x)$

$\boxed{P_{ave} = 724.5\ \mu W}$

* Electromagnetic wave polarization:

- Types of Polarization (at a fixed Point in Space)
Time variant

1) linearly polarized wave (LPW)

$\vec{E} = E_1 e^{j(\omega t - ky)} \hat{a}_x$ V/m $\longrightarrow$ Horizontal LPW

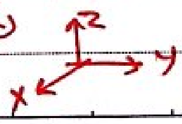
at $y = 0$

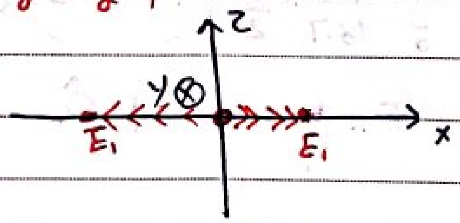
$\vec{E} = E_1 e^{j\omega t} \hat{a}_x$ V/m

$E_x = E_1 e^{j\omega t} = E_1 (\cos \omega t + j \sin \omega t)$

Taking the Imaginary part:

$E_x = E_1 \sin \omega t$





at $\omega t = 0$

at $\omega t = \pi/2 \rightarrow E_x = E_1$

at $\omega t = \frac{3\pi}{2} \rightarrow E_x = -E_1$

sm:)e for life

No.

for $\vec{E} = E_z e^{j(\omega t - ky)} \hat{a}_z$ V/m

at $y=0$

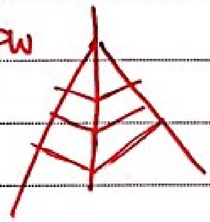
$E_z = E_z e^{j\omega t}$

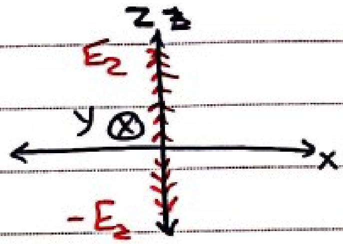
Take $\cancel{E_z}$ $E_z = E_z \sin\omega t$

Vertically LPW

(V LPW)

$L = 250$ m





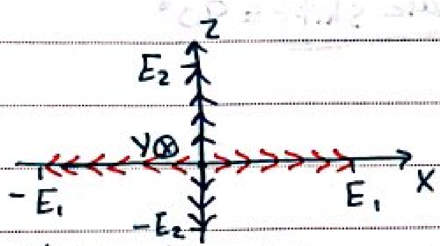
*lec 10
10/10

No.

1) *linear polarization

$\vec{E} = E_1 e^{j(\omega t - ky)} \hat{a}_x$ V/m → (H)LPW
Horizontal

$\vec{E} = E_2 e^{j(\omega t - ky)} \hat{a}_z$ V/m → (V)LPW
Vertical   Linear Polarized wave



$y=0$ (fixed location in Space)

$\omega t = 0, \frac{\pi}{2}, \ldots$

uniform plane wave

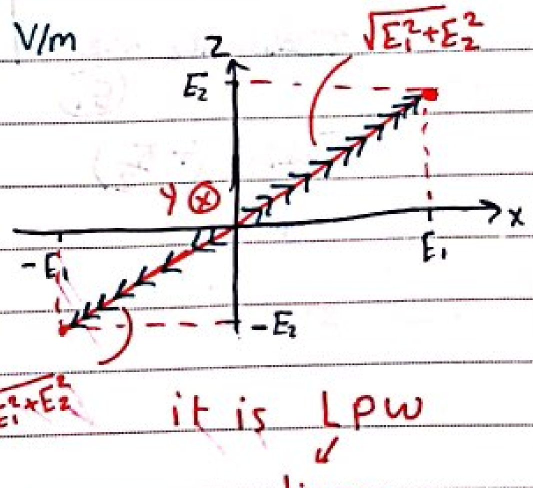
* for a UPW has:
$\vec{E} = (E_1 \hat{a}_x + E_2 \hat{a}_z) e^{j(\omega t - ky)}$ V/m

$\hat{a}_k = +\hat{a}_y$

at $y=0$
$\vec{E} = (E_1 \hat{a}_x + E_2 \hat{a}_z) e^{j\omega t}$ V/m

Take: $\vec{E} = (E_1 \hat{a}_x + E_2 \hat{a}_z) \sin \omega t$ V/m

at $\omega t = 0$	at $\omega t = \pi/2$
$E_x = 0$	$E_x = E_1$; $E_z = E_2$
$E_z = 0$	$\vec{E} = E_1 \hat{a}_x + E_2 \hat{a}_z$
$\vec{E} = 0$	$\lvert\vec{E}\rvert = \sqrt{E_1^2 + E_2^2}$



it is LPW
linear

No.

2) Circularly Polarized wave (CPW)

For a UPW has:

$\vec{E} = \underbrace{E_1}_{\text{Same}} e^{j\underline{(\omega t - kz)}} \hat{a}_x + \underbrace{E_1}_{} e^{j\underbrace{(\omega t - kz \pm \pi/2)}_{}} \hat{a}_y$ V/m

Same because radius — must have phase shift = 90°

[Same magnitude (E_1)
Phase Shift = 90°]

$\hat{a}_k = \hat{a}_z$

@ $z=0$

$\cancel{\vec{E} = E_1 e^{j\omega t} \hat{a}_x +}$

$\vec{E} = E_1 \sin \omega t \, \hat{a}_x + E_1 \sin(\omega t \pm \pi/2) \, \hat{a}_y$ V/m

$\vec{E} = E_1 \sin \omega t \, \hat{a}_x \pm E_1 \cos(\omega t) \, \hat{a}_y$

* for $\mp \pi/2$

@ $\omega t = 0$

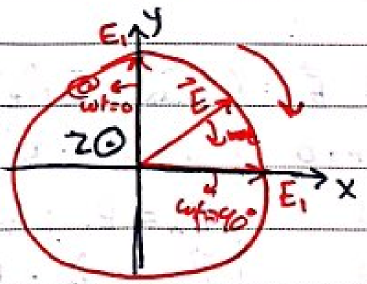
$\vec{E} = E_1 \hat{a}_y$

Taking the +ve E_1

Taking $(+\frac{\pi}{2})$

@ $\omega t = \pi/2$

$\vec{E} = E_1 \hat{a}_x$



CW - CPW $(+\pi/2)$

$\boxed{\text{LHCPW}}$

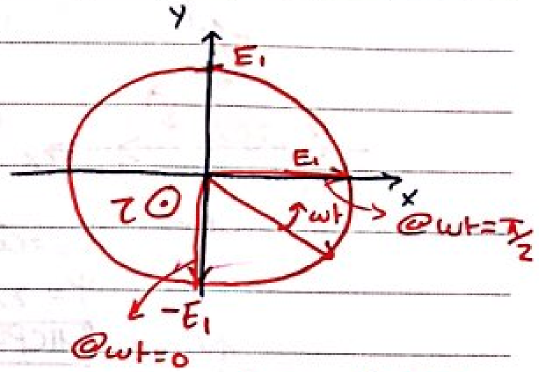
* for $-\frac{\pi}{2}$

$\omega t = 0$

$\vec{E} = -E_1\,\hat{a}_y$

$\omega t = \pi/2$

$\vec{E} = E_1\,\hat{a}_x$



CCW − CPW ($-\pi/2$)

$\boxed{\text{RH CPW}}$

* for the general case

$$\vec{E} = E_1\left(e^{\mp jkz}\,\hat{a}_x + e^{\mp j(kz \pm \pi/2)}\,\hat{a}_y\right)e^{j\omega t} \quad \text{V/m}$$

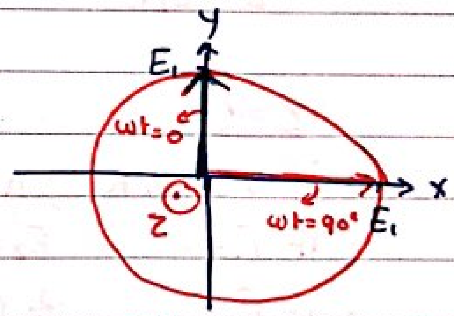
4-Cases

$\hat{a}_k = +\hat{a}_z$ → $+\pi/2$, → $-\pi/2$

$\hat{a}_k = -\hat{a}_z$ → $+\pi/2$, → $-\pi/2$

* at $z = 0$

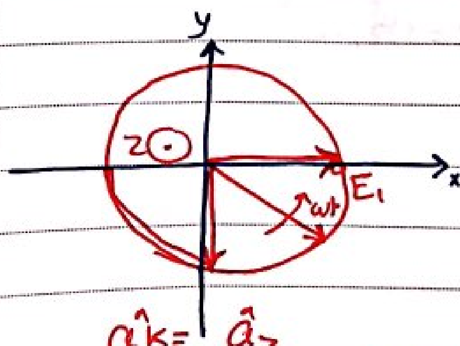
$\vec{E} = E_1 \sin\omega t\,\hat{a}_x \pm E_1 \cos\omega t\,\hat{a}_y$



$\hat{a}_k = \hat{a}_z$

$\psi = \pi/2$

$\boxed{\text{LHCPW}}$

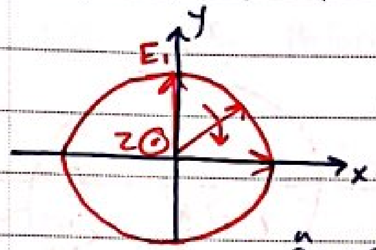


$\hat{a}_k = \hat{a}_z$

$\psi = -\pi/2$

$\boxed{\text{RHCPW}}$

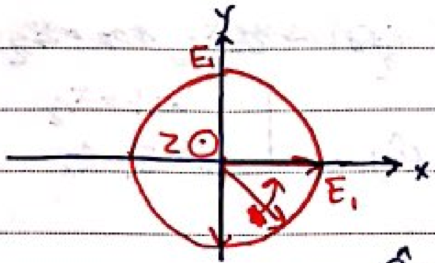
No.



$\hat{a}_k = \hat{a}_z$

$\psi = \pi/2$

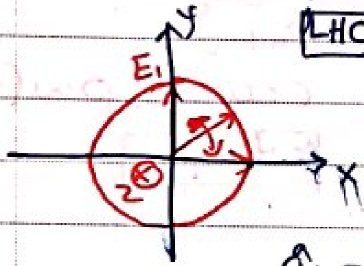
$\boxed{\text{LHCPW}}$



$\hat{a}_k = \hat{a}_z$

$\psi = -\pi/2$

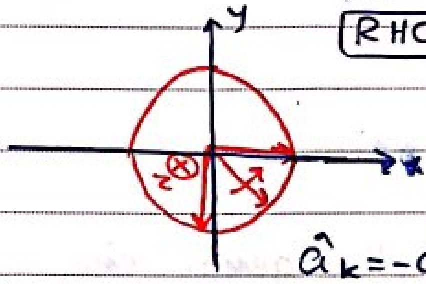
$\boxed{\text{RHCPW}}$



$\hat{a}_k = -\hat{a}_z$

$\psi = \pi/2$

$\boxed{\text{RHCPW}}$



$\hat{a}_k = -\hat{a}_z$

$\psi = -\pi/2$

$\boxed{\text{LHCPW}}$

3) Elliptically Polarized wave (EPW)

for

$$\vec{E} = E_1 e^{j(\omega t - kz)}\hat{a}_x + E_2 e^{j(\omega t - kz \pm \psi)}\hat{a}_y$$

* Notes:

- if $\psi = n\pi$; $n = 0, 1, 2..$ $\rightarrow$ LPW
- if $E_1 = 0$ or $E_2 = 0$, $\psi \neq 0$ $\rightarrow$ LPW
- if $E_1 = E_2$, $\psi = \pm(2n-1)\cdot\frac{\pi}{2}$, $n = 1, 2, \ldots$
  it is a CPW
- otherwise, it is an EPW.

No.

* If $E_1 \neq E_2$ & $\psi = \pi/2 \rightarrow$ EPW

$$\vec{E} = E_1 e^{j\omega t} \hat{a}_x + E_2 e^{j(\omega t \pm \pi/2)} \hat{a}_y \quad (\text{at } z=0)$$

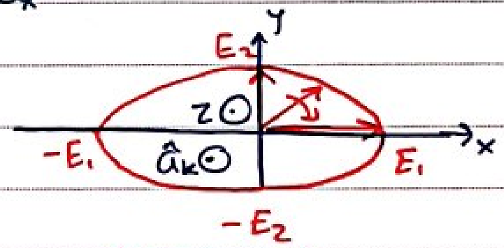
$$\vec{E}_1 = E_1 \sin \omega t \, \hat{a}_x \pm E_2 \cos(\omega t) \, \hat{a}_y \quad \text{V/m}$$

$+\pi/2$

at $\omega t = 0 \rightarrow \vec{E} = E_2 \hat{a}_y$

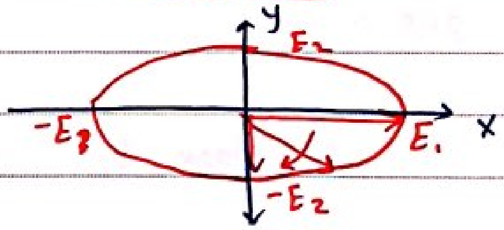
at $\omega t = \pi/2 \rightarrow \vec{E} = E_1 \hat{a}_x$

If $E_1 > E_2$



$\boxed{\text{LHEPW}}$

$-\pi/2$



$\boxed{\text{RHEPW}}$

$$\text{Axial Ratio} = \frac{\text{Major axis}}{\text{Minor axis}}$$

$$AR = \frac{E_1}{E_2}$$

$$1 \leq AR < \infty$$

CPW   LPW

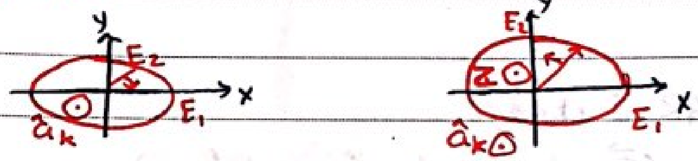
smile for life

If $E_1 > E_2$

*for

$$\vec{E} = E_1 e^{j(\omega t - kz)}\,\hat{a}_x + E_2 e^{j(\omega t - kz \pm \pi/2)}\,\hat{a}_y \quad V/m$$

at $z=0$



LHEPW
$\psi = +\frac{\pi}{2}$
$\tau = 0$

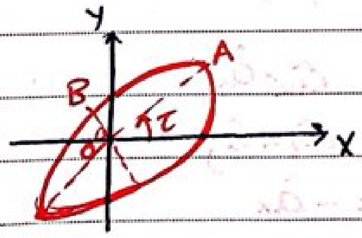
RHEPW
$\psi = -\frac{\pi}{2}$
$\tau = 0$

$$AR = \frac{\text{major axis}}{\text{minor axis}} \quad ; \quad AR = \frac{E_1}{E_2}$$

CPW $(E_1 = E_2)$ $\rightarrow 1 \le AR \le \infty$ ← $E_2 = 0$ (LPW)

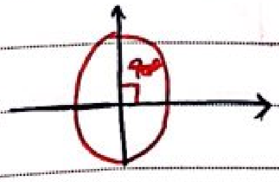
if $\psi \neq (2n-1)\frac{\pi}{2}$, $n = 1, 2, \ldots$ 0-3dB CPW

The ellipse will be tilted by angle (τ)



$$AR = \frac{OA}{OB}$$

$$\tau = \frac{1}{2}\tan^{-1}\left[\frac{2E_{xo}E_{yo}}{E_{xo}^2 - E_{yo}^2}\cos(\psi)\right]$$



$\tau = 90°$

No.

Ex. the $\vec{E}$-field for a UPW in freespace is:

$$\vec{E} = (3\hat{a}_x + j4\hat{a}_y)\, e^{-j\frac{1}{2}\pi z}$$

Find:

a) f and $\vec{H}$

b) Polarization, AR, τ if any

c) Find $\vec{P}_{ave}$ or $(\vec{S}_{ave})$ and the total Power crossing a $2*2\ m^2$ plane along xy

$$j = e^{j\pi/2} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}$$
$$= 0 + j1$$

a) $\beta = \frac{\omega}{c}$ ← freespace $\Rightarrow 0.5\pi = \frac{2\pi f}{c}$

$$f = \frac{3}{4} \times 10^8 \Rightarrow \boxed{f = 75\,MHz}$$

$$\vec{H} = \vec{H}_1 + \vec{H}_2$$

$\vec{a}_{H_1} = \vec{a}_k \times \hat{a}_{E_1} = a_z \times \hat{a}_x = +\hat{a}_y$; $\hat{a}_{H_2} = \hat{a}_k \times \hat{a}_{E_2} = \hat{a}_z \times \hat{a}_y = -\hat{a}_x$

$H_{o1} = \frac{3}{\eta_o}$; $H_{o2} = \frac{4}{\eta_o}$

$$\boxed{\vec{H} = \frac{(-j4\,\hat{a}_x + 3\,\hat{a}_y)}{120\pi}\, e^{-j\pi/2\, z} \quad A/m}$$

No.

c) $\vec{S}_{ave} = \frac{1}{T}\int_0^T P(z,t)\,dt$

$= \frac{1}{2} Re(E_s \times \vec{H}_s^*)$

$= \frac{1}{2} Re\left\{(3\hat{a}_x + j4\hat{a}_y) \times \left(\frac{j4\hat{a}_x + 3\hat{a}_y}{120\pi}\right)\right\}$

$= \frac{1}{240\pi} Re\{(9\hat{a}_z + 16\hat{a}_z\}$

$$\boxed{\vec{S}_{ave} = \frac{25}{240\pi}\hat{a}_z \quad w/m^2}$$

$P_{ave} = \int_S S_{ave} * d\vec{s}$

$= \vec{S}_{ave} * S$

$= \frac{25}{240\pi} * 4 \Rightarrow \boxed{P_{ave} = 0.417w}$

b) Polarization

@ $z=0$

$\vec{E} = (3\hat{a}_x + j4\hat{a}_y)e^{j\omega t}$

$\vec{E} = Re\{E_s e^{j\omega t}\}$

$\vec{E} = 3\sin\omega t + 4\cos\omega t$

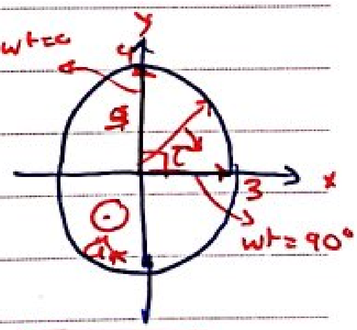
or $+4\sin(\omega t + 90°)$

at $\omega t = 0$

$\vec{E} = 4\hat{a}_y$

at $\omega t = 90$

$\vec{E} = 3\hat{a}_x$



$\boxed{LH\ EPW}$

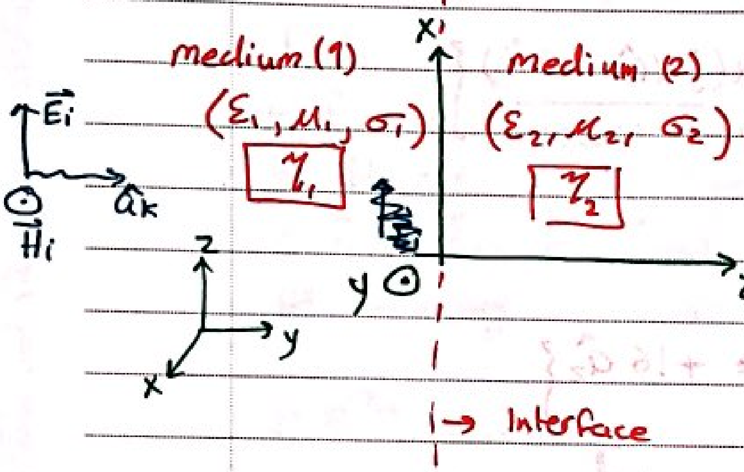
$\boxed{\tau = 90°}$

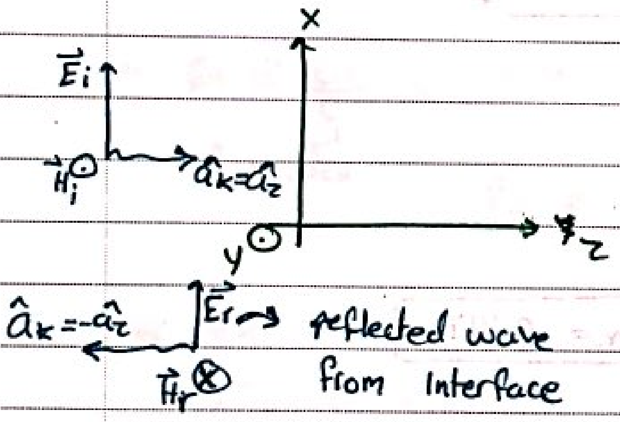
$\boxed{AR = \frac{4}{3}}$

No.

*Reflection of a plane wave at Normal ~~incidents~~ Incidence :

* Things that happen to waves
  - Reflection
  - Refraction
  - Deflection
  - Defraction





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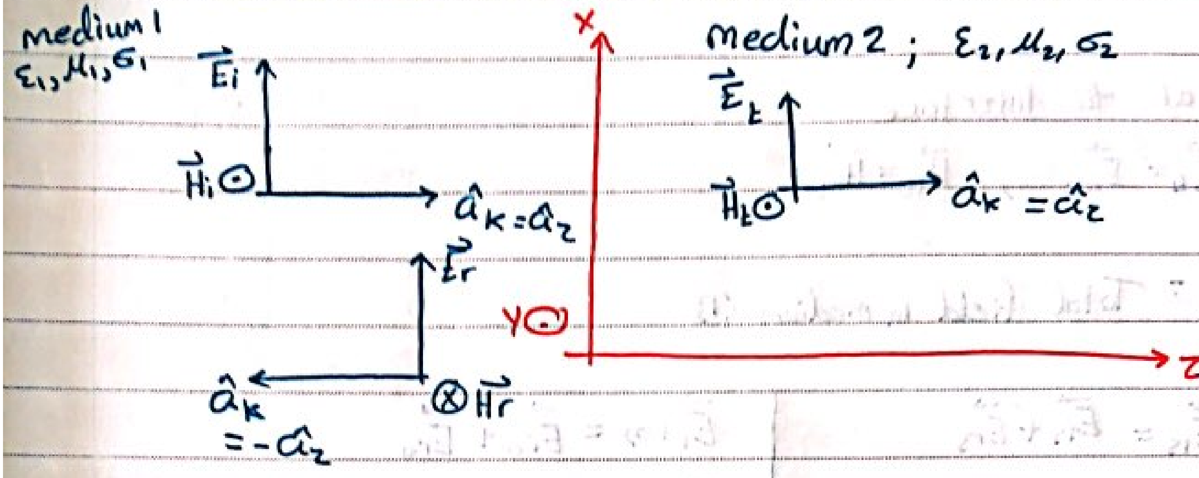
No.

* Reflection of EM wave of normal Incidence.



* The Incident waves

$$\vec{E}_{is} = E_{io}\, e^{-\gamma_1 z}\, \hat{a}_x \quad V/m$$

$$\vec{H}_{is} = \frac{E_{io}}{\eta_1}\, e^{-\gamma_1 z}\, \hat{a}_y \quad A/m$$

* The Reflected waves

$$\vec{E}_{rs} = E_{ro}\, e^{-\gamma_1 z}\, \hat{a}_x \quad V/m$$

$$\vec{H}_{rs} = -\frac{E_{ro}}{\eta_1}\, e^{\gamma_1 z}\, \hat{a}_y \quad A/m$$

* The transmitted waves

$$\vec{E}_{ts} = E_{to}\, e^{-\gamma_2 z}\, \hat{a}_x \quad V/m$$

$$\vec{H}_{ts} = \frac{E_{to}}{\eta_2}\, e^{-\gamma_2 z}\, \hat{a}_y \quad A/m$$

No.

* Boundary Conditions:

- at the Interface

$\vec{E}_{1t} = \vec{E}_{2t}$, $\vec{H}_{1t} = \vec{H}_{2t}$

E_1 = Total field in medium (1)

$\vec{E}_{1s} = \vec{E}_{is} + \vec{E}_{rs}$

$\vec{E}_{2s} = \vec{E}_{ts}$

$\vec{E}_1(0) = \vec{E}_{io} + \vec{E}_{ro}$

$E_2(0) = E_{t}(0)$

$E_{1t} = E_{2t} \Rightarrow \boxed{\vec{E}_{io} + \vec{E}_{ro} = \vec{E}_{to}}$ — (1)

- at $z=0$

$\vec{H}_{1t} = \vec{H}_{2t}$

$\boxed{\vec{H}_{io} + \vec{H}_{ro} = \vec{H}_{to}}$

$\boxed{\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}}$ — (2)

Solve (1) & (2) to find E_{ro} & E_{to}

Take eq (2) $\Rightarrow E_{to} = \frac{\eta_2}{\eta_1}(E_{io} - E_{ro})$

Sub in (1)

$\boxed{E_{ro} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{io}}$ $\boxed{E_{to} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{io}}$

No.

* Define Reflection coefficient (Γ)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \Rightarrow \boxed{E_{ro} = \Gamma E_{io}} \longrightarrow \Gamma = \frac{E_{ro}}{E_{io}}$$

$$0 \leq |\Gamma| \leq 1$$

No Reflection — All the wave is reflected

* Transmission Coefficient (τ)

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} \Rightarrow \boxed{E_{to} = \tau E_{io}} \rightarrow \tau = \frac{E_{to}}{E_{io}}$$

$$\boxed{1 + \Gamma = \tau} \rightarrow 1 + \underbrace{\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}_{\Gamma} = \frac{2\eta_2}{\eta_2 + \eta_1} \rightarrow \tau$$

Γ, τ Dimensionless, and can be complex if η is complex
[lossless, free space $\equiv$ Real / lossy, Good Conductor $\equiv$ complex]

*Special case:
medium(1) is lossless ($\sigma_1 \approx 0$)
medium(2) is Perfect Conductor ($\sigma_2 \to \infty$)

$\eta_2 = 0 \rightarrow \Gamma = -1$	$\vec{E}_2 = 0$	at $z=0$
$\tau = 0$	$\vec{E}_1 = \vec{E}_i + \vec{E}_r$	$E_{io} = E_{io} + \Gamma E_{io}$
	$= \vec{E}_i + \Gamma \vec{E}_i$	$E_1 \Rightarrow$ Standing wave

No.

The standing wave in medium(1)

$\vec{E}_{1s} = \vec{E}_{is} + \vec{E}_{rs}$

$= (E_{io} e^{-\gamma_1 z} + E_{ro} e^{\gamma_1 z})\, \hat{a}_x$, $E_{ro} = \Gamma E_{io}$

$= -1\, E_{io}$

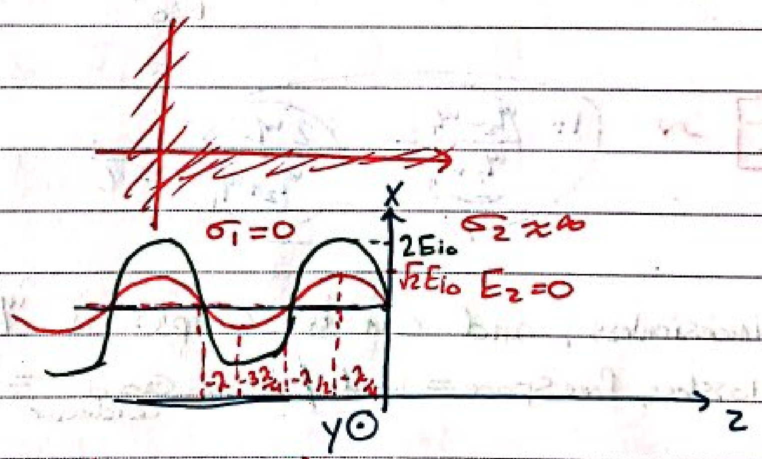
$= E_{io} (e^{-\gamma_1 z} - e^{\gamma_1 z})\, \hat{a}_x$

$= -E_{io} (e^{\gamma_1 z} - e^{-\gamma_1 z})\, \hat{a}_x$; $\sin\phi = \dfrac{e^{j\phi} - e^{-j\phi}}{2j}$

$= -E_{io}(2j) \sin\beta_1 z\, \hat{a}_x$, $\gamma_1 = j\beta_1$

$\vec{E}_1 = Re\{E_{1s}\, e^{j\omega t}\}$

$\vec{E}_1 = -2j E_{io} \sin\beta_1 z \sin\omega t$



at $t=0 \longrightarrow \vec{E}_1 = 0$

$t = T/8 \longrightarrow E_1 = \dfrac{-2jE_{io}}{\sqrt{2}} \sin\beta_1 z$ → z here is negative

$t = T/4$ → $E_1 = -2jE_{io} \sin\beta_1 z$

$t = 3T/8$

$t = T/2$

$\vec{E}$ is only changing in magnitude (Amplitude) ; Not moving

⇒ Standing wave.

~~The maximum occurs~~

No.

The maximum Occurs:

- $E_{1max} = 2E_{i0}$ @ $\sin \beta_1 z = 1$; $\beta_1 z = (2n+1)\pi/2$

$$\boxed{z_{max} = -\frac{(2n+1)\pi}{2\beta_1}}$$

- z_{min} , $\beta_1 = n\pi$, $n = 0, 1, 2 \ldots$

$$\boxed{z_{min} = \frac{-n\pi}{\beta_1}}$$

*Special case:

Medium (1) is lossless ($\sigma = 0$)
and medium(2) is perfect conductor ($\sigma \to \infty$)

$\eta_2 = 0$, $\Gamma = -1$, $\tau = 0 \Rightarrow \gamma_1 = j\beta_1$
$\alpha_1 = 0$

$\vec{E}_{1s} = \vec{E}_{is} + \vec{E}_{rs}$, $\vec{E}_2 = 0$

$= E_{io} e^{-j\beta_1 z} \hat{a}_x + E_{ro} e^{j\beta_1 z} \hat{a}_x$

$= E_{io} (e^{-j\beta_1 z} - e^{j\beta_1 z}) \hat{a}_x$

$= -E_{io} (e^{j\beta_1 z} - e^{-j\beta_1 z}) \hat{a}_x$

$\vec{E}_{1s} = -2j E_{io} \sin\beta_1 z \; \hat{a}_x$

$\vec{E}_1 = Re\{\vec{E}_{1s} e^{j\omega t}\}$

$= Re\{E_{io}(-2j)\sin\beta_1 z (\cos\omega t + j\sin\omega t)\}\hat{a}_x$

$$\boxed{\vec{E}_1 = 2E_{io} \sin\beta_1 z \sin\omega t \; \hat{a}_x}$$

$-z_{max}\,\beta_1 = (2n+1)\frac{\pi}{2}$

$z_{max} = \frac{-(2n+1)\pi}{2\beta_1}$, $n = 1, 2, 3 \ldots$

$-z_{min}\,\beta_1 = n\pi$

$z_{min} = \frac{-n\pi}{\beta_1}$, $n = 0, 1, 2 \ldots$

No.

*Special case

lossless medium (1) has (η_1) ; $\sigma=0$

lossless medium (2) has (η_2) ; $\sigma=0$

A) $\eta_2 < \eta_1$, $\eta_2 \neq 0$

$\Gamma < 1$

$$z_{max} = \frac{-(2n+1)\pi}{2\beta_1} \quad ; n=1,2,3\cdots$$

$$z_{min} = \frac{-n\pi}{\beta_1} \quad ; n=0,1,2\cdots$$

max E-field $\rightarrow |\vec{E}_{is}| + |\vec{E}_{rs}|$

min E-field $\rightarrow |\vec{E}_{is}| - |\vec{E}_{rs}|$

Standing wave ratio (S)

$$S = \frac{E_{max}}{E_{min}} = \frac{H_{max}}{H_{min}}$$

$$S = \frac{|E_{is}| + |E_{rs}|}{|E_{is}| - |E_{rs}|} = \frac{\cancel{|E_{is}|}(1+|\Gamma|)}{\cancel{|E_{is}|}(1-|\Gamma|)}$$

$$\boxed{S = \frac{1+|\Gamma|}{1-|\Gamma|}} \Rightarrow \boxed{|\Gamma| = \frac{S-1}{S+1}}$$

magnitude only

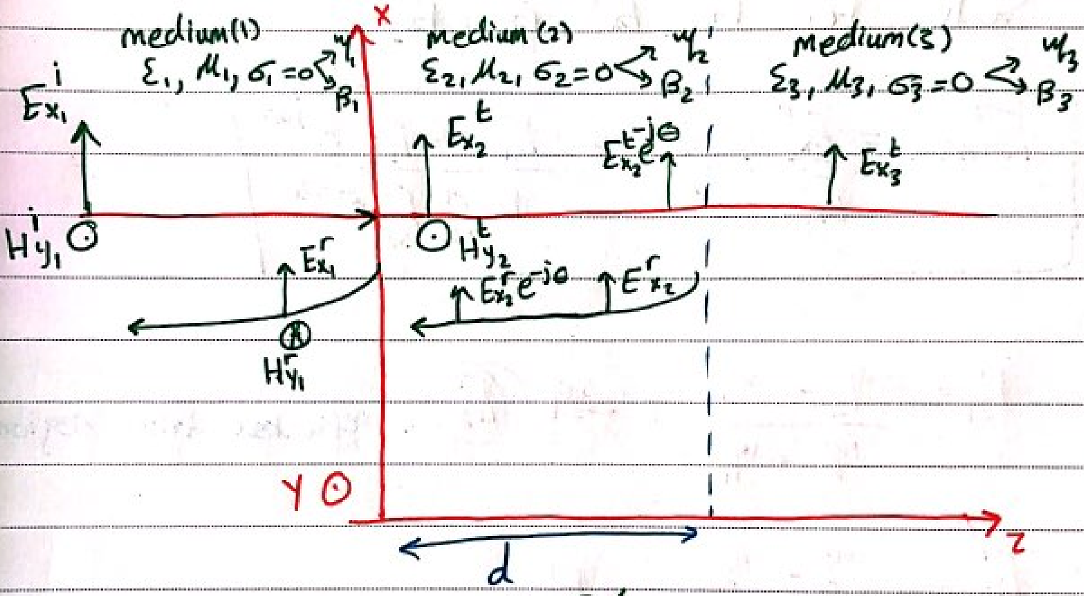
No.

$0 \leq |\Gamma| \leq 1$

$1 \leq S < \infty$

$S_{dB} = 20 \log_{10} S$

* normal Incidence on multiple mediums



$\theta = \beta d$ electrical length

Total fields in each region

$E_{1t} = E_{2t}$ @ $z=0$

$E^i_{x_1} + E^r_{x_1} = E^t_{x_2} + E^r_{x_2} e^{-j\theta}$ —①

$H_{1t} = H_{2t}$ @ $z=0$

$\frac{E^i_{x_1}}{\eta_1} - \frac{E^r_{x_1}}{\eta_1} = \frac{E^t_{x_2}}{\eta_2} - \frac{E^r_{x_2} e^{-j\theta}}{\eta_2}$ —②

smile for life

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No.

$E_{1t} = E_{2t}$ @ $z=d$

$$E_{x_2}^{t} e^{-j\theta} + E_{x_2}^{r} = E_{x_3}^{t} \quad --- (3)$$

$H_{1t} = H_{2t}$ @ $z=d$

$$\frac{E_{x_2}^{t} e^{-j\theta}}{\eta_2} - \frac{E_{x_2}^{r}}{\eta_2} = \frac{E_{x_3}^{t}}{\eta_3} \quad --- (4)$$

Solve for $E_{x_1}^{r}, E_{x_2}^{t}, E_{x_2}^{r}, E_{x_3}^{t}$

$$\boxed{\Gamma_{overall} = \frac{E_{x_1}^{r}}{E_{x_1}^{i}} = \frac{E_{x_1}^{r}}{1} = \frac{\Gamma_{21} + \Gamma_{32} e^{-j2\theta}}{1 + \Gamma_{21}\Gamma_{32} e^{-j2\theta}}}$$

$$\Gamma_{21} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -\Gamma_{12}$$ → reflected from region I

$$\Gamma_{32} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} = -\Gamma_{23}$$

$$\tau_{21} = \frac{2\eta_1}{\eta_1 + \eta_2}, \quad \tau_{12} = \frac{2\eta_2}{\eta_1 + \eta_2}, \quad \tau_{23} = \frac{2\eta_3}{\eta_2 + \eta_3}$$

$$\tau_{overall} = \frac{E_{x_3}^{t}}{E_{x_1}^{i}} = \frac{E_{x_3}^{t}}{1}$$

Shielding effectiveness

$$= \frac{\tau_{12}\tau_{23} e^{-j2\theta}}{1 + \Gamma_{21}\Gamma_{32} e^{-j2\theta}}$$

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No.

Ex: In free space $(z \le 0)$ a plane wave with

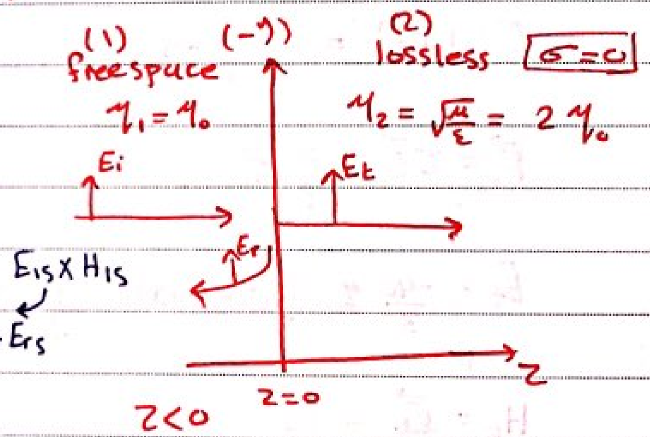
$$\vec{H}_i = 10\cos(10^8 t - \beta z)\,\hat{a}_x \text{ mA/m}$$

is incident normally on a lossless medium $(\varepsilon = 2\varepsilon_0, \mu = 8\mu_0)$ in region $z \geq 0$

Determine: $\vec{H}_r$, $\vec{E}_r$, $\vec{H}_t$ and $\vec{E}_t$

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 2\eta_0$$

$$\beta_1 = \frac{\omega}{c} = \frac{10^8}{3\times10^8} = \frac{1}{3} \text{ rad/m}$$



$E_i = ?$

$E_{io} = \eta_0 H_{io} = 10\,\eta_0$

$$\hat{a}_{E_1} = \hat{a}_{H_1} \times \hat{a}_{k_1}$$
$$= \hat{a}_x \times \hat{a}_z = -\hat{a}_y$$

$$E_r = -10\,\eta_0 \cos(10^8 t - \tfrac{1}{3}z)\,\hat{a}_y \quad \text{mV/m}$$

$$\vec{E}_r = \Gamma \vec{E}_i \quad , \quad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\eta_0 - \eta_0}{2\eta_0 + \eta_0} = \frac{1}{3}$$

$$\vec{E}_r = \frac{1}{3}\vec{E}_i \quad , \quad E_{ro} = \frac{E_{io}}{3} = -\frac{10\eta_0}{3} \quad , \quad E_{ro} = E_{io}|\Gamma|$$

smile

No.

$$\vec{E}_r = \frac{-10\,\eta_0}{3} \cos(10^8 t + \tfrac{1}{3} z)\,\hat{a}_y \quad mV/m$$

$$\vec{H}_r = \frac{E_r}{\eta_0} \Rightarrow \vec{H}_r = \frac{-10}{3} \cos(10^8 t + \tfrac{1}{3} z)\,\hat{a}_x \quad mA/m$$

$$\beta_2 = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = 4\beta_1 = \frac{4}{3}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{4\eta_0}{3\eta_0} = \frac{4}{3}$$

$$\tau = 1 + \Gamma = 1 + \frac{1}{3} = \frac{4}{3}$$

$$\vec{E}_t = \tau \vec{E}_i = \frac{4}{3}\vec{E}_i$$

$$E_t = \frac{-40}{3}\eta_0 \cos(10^8 t - \tfrac{4}{3} z)\,\hat{a}_y \quad mV/m$$

$$H_t = \frac{\vec{E}_t}{\eta_2} = \frac{\vec{E}_t}{2\eta_0}$$

$$\hat{a}_H = \hat{a}_K \times \hat{a}_E = \hat{a}_z \times -\hat{a}_y = \hat{a}_x$$

$$\boxed{\vec{H}_t = \frac{20}{3} \cos(10^8 t - \tfrac{4}{3} z)\,\hat{a}_x \quad mA/m}$$

$$H_{i0} + H_{r0} = H_{t0}$$

$$10 - \frac{10}{3} = \frac{20}{3}$$

No.

$$H_i \to E_i \to E_r \qquad E_t$$
$$\qquad\qquad\quad \downarrow \qquad\quad \downarrow$$
$$\qquad\qquad\quad H_r \qquad\ H_t$$

$E_i \to E_r$, $E_i \to E_t$

$H_i \to H_r$, $H_i \to H_t$

$$\frac{H_r}{H_i} = -\Gamma \quad , \quad \Gamma = \frac{E_r}{E_i} \quad ; \quad \boxed{H_r = -\Gamma H_i}$$

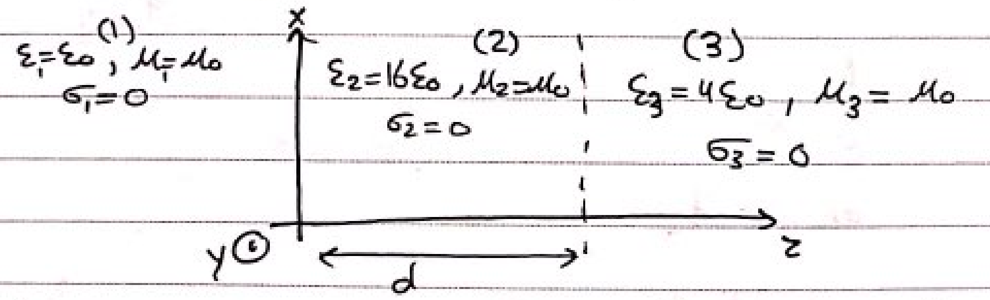
$$\frac{E_t}{E_i} = \tau \quad ,$$

$E_t = \tau E_i$

$E_t = \eta_2 H_t = \tau(\eta_1 H_i) \rightarrow \boxed{\tau = \frac{\eta_2}{\eta_1} * \frac{H_t}{H_i}} \Rightarrow \boxed{H_t = \frac{\eta_1}{\eta_2} \tau H_i}$

---

* Ex: If a Signal is incident normally from medium (1)



to mediums (2) & (3) as shown in figure, find ~~Γoverall~~ $\Gamma_{overall}$, $\tau_{overall}$, S.E for $d = \frac{\lambda}{2}$ then find the characteristics of medium (2) to have $\Gamma_{overall} = 0$ at $d = \frac{\lambda}{4}$

matching

smile

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$$\eta_2 = \eta_0 \sqrt{\frac{1}{\varepsilon_{r_2}}} = 30\pi\ \Omega$$

$$\eta_3 = \eta_0 * \sqrt{\frac{1}{\varepsilon_{r_3}}} = 60\pi\ \Omega$$

$$\Gamma_{overall} = \frac{\Gamma_{21} + \Gamma_{32}\, e^{-j2\theta}}{1 + \Gamma_{21}\Gamma_{32}\, e^{-j2\theta}}$$

$$\Gamma_{21} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.6$$

$$\Gamma_{32} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} = \frac{1}{3}$$

$$\tau_{overall} = \frac{\tau_{12}\,\tau_{23}\, e^{-j2\theta}}{1 + \Gamma_{21}\Gamma_{32}\, e^{-j2\theta}}$$

$$\tau_{12} = \frac{2\eta_2}{\eta_1 + \eta_2} = 0.4 \Rightarrow (1 + \Gamma_{21})$$

$$\tau_{23} = \frac{2\eta_3}{\eta_2 + \eta_3} = \frac{4}{3} \Rightarrow (1 + \Gamma_{32})$$

$$S.E = 20 \log (\tau_{overall})$$

$e^{-j2\theta}$, $\theta = \beta d$

at $d = \lambda/2$

$$e^{-j2\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2}\right)} = e^{-j2\pi} = \cos 2\pi - j\sin 2\pi$$
$$= 1$$

$$\boxed{\Gamma_{overall} = -0.33}$$
$$\boxed{\tau_{overall} = 0.67}$$

*3 mediums are better for transmission than 2 mediums.

No.

to have $\Gamma_{overall} = 0$

$$\Gamma_{21} + \Gamma_{32} e^{-j2\theta} = 0$$

at $d = \lambda/4$

$$e^{-j2\theta} = e^{-j2\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right)} = e^{-j\pi}$$

$$= \cos\pi \mp j \overset{0}{\cancel{\sin\pi}}$$

$$= -1$$

$$\boxed{\Gamma_{21} = \Gamma_{32}} \Rightarrow \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \Rightarrow \boxed{\eta_2 = \sqrt{\eta_1 \eta_3}}$$

this confirms matching

$$\boxed{\eta_2 = 60\pi * \sqrt{2} \ \Omega}$$

for non-magnetic

$\mu_2 = \mu_0$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = 60\pi\sqrt{2} \Rightarrow \boxed{\varepsilon_{r_2} = 2}$$

$$\boxed{S.E = 20\log \tau_{overall}}$$

Shielding efficiveness

26/10

No.

* 10.7 Reflection of a plane wave at oblique Incidence

A UPW takes the general form of:

$$\vec{E}(r,t) = E_0 \cos(k.r - \omega t) = \mathrm{Re}\{E_0 e^{j(k.r-\omega t)}\}$$

$\vec{r} = \langle x, y, z \rangle$ Position (radius) vector

$\vec{k} = \langle k_x, k_y, k_z \rangle$ Propagation vector or wave number vector

k Same as β for lossless medium

→ we will consider lossless mediums:

[for lossy medium just replace ε by ε_c]

$K^2 = k_x^2 + k_y^2 + k_z^2 = \beta^2 = \omega^2 \mu \varepsilon$ "Dispersion relation"

For maxwell's equations reduce to

$$K.E = 0 \qquad K \times E = \omega \mu \vec{H}$$

$$K.H = 0 \qquad K \times H = -\omega \varepsilon \vec{E}$$

$$k.r = k_x x + k_y y + k_z z$$

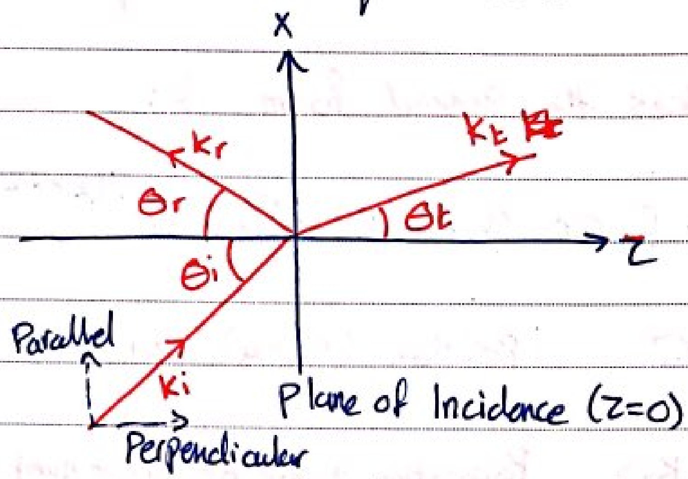
$$K \times E = \omega \mu H \longrightarrow H = \frac{1}{\omega \mu} K \times E \quad , \quad K = \omega \sqrt{\mu \varepsilon}\, \hat{a}_k$$

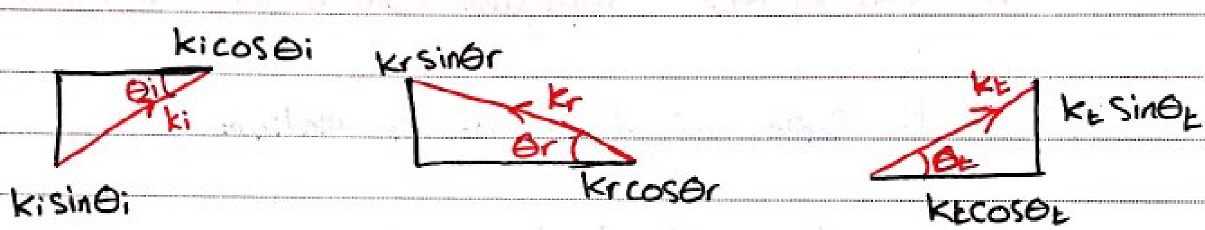
$$\vec{H} = \sqrt{\frac{\varepsilon}{\mu}}\, \hat{a}_k \times \vec{E}$$

$$\vec{H} = \frac{\hat{a}_k \times \vec{E}}{\eta}$$

No.

Illustration of oblique incidence





$$E(r,t) = E_o \cos(k.r - \omega t)$$

$$E_i = E_{io} \cos(k_{ix} X + k_{iy} Y + k_{iz} Z - \omega_i t)$$

$$E_r = E_{ro} \cos(k_{rx} X + k_{ry} Y + k_{rz} Z - \omega_r t)$$

$$E_t = E_{to} \cos(K_{tx} X + K_{ty} Y + K_{tz} Z - \omega_t t)$$

@ Z=0 (Interface)

$\leftrightarrow$ $E_{tangential}$ is Continuous $\boxed{E_i(0) + E_r(0) = E_t(0)}$

No.

* Conditions to Satisfy this boundary condition

$E_{io} + E_{ro} = E_{to}$

1. $\omega_i = \omega_r = \omega_t$
2. $k_{ix} = k_{rx} = k_{tx} = k_x$
3. $k_{iy} = k_{ry} = k_{ty} = k_y$

$k_i \sin\theta_i = k_r \sin\theta_r$; $\theta_i = \theta_r$

$k_i \sin\theta_i = k_t \sin\theta_t$; $\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_i}{k_t}$

$k_i = \beta_i = \omega\sqrt{\mu_1\varepsilon_1} \longrightarrow k_r = \omega\sqrt{\mu_1\varepsilon_1}$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_i}{k_t} = \frac{\omega\sqrt{\mu_1\varepsilon_1}}{\omega\sqrt{\mu_2\varepsilon_2}} = \frac{u_2}{u_1}$$

$$\frac{u_2}{u_1} = \frac{n_1}{n_2}$$

* Snell's law for refraction $n_1 \sin\theta_i = n_2 \sin\theta_t$

* Snell's law for reflection $\theta_i = \theta_r$

* Refraction Index of the medium n

$$n_1 = \frac{c}{u_1} = c\sqrt{\mu_1\varepsilon_1} = \sqrt{\mu_{r_1}\varepsilon_{r_1}}$$

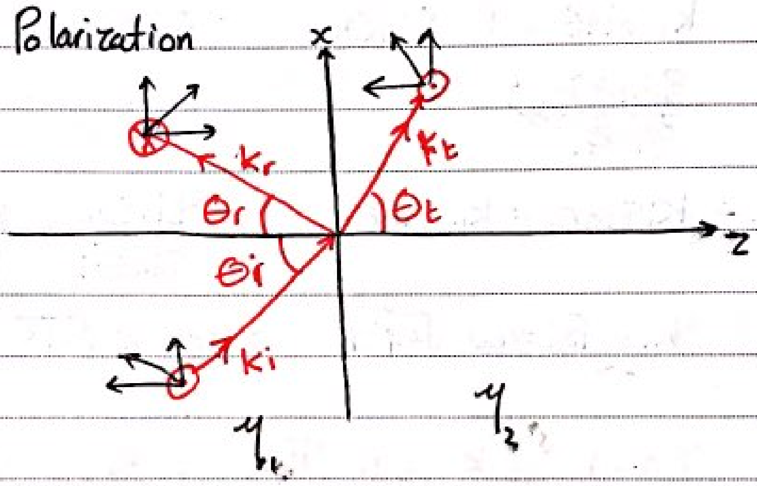
$$n_2 = \frac{c}{u_2} = c\sqrt{\mu_2\varepsilon_2} = \sqrt{\mu_{r_2}\varepsilon_{r_2}}$$

No.

*Two Cases will be considered based on $\vec{E}$-field direction either

1. $\vec{E}$ is ~~par~~ parallel to the interface
2. $\vec{E}$ is perpendicular to the interface

A. Parallel Polarization



*Perpendicular Component of E only reflected

*The Incident fields

$$\vec{E_{is}} = E_{io}(\cos\theta_i\,\hat{a}_x - \sin\theta_i\,\hat{a}_z)\,e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\ \text{V/m}$$

$$\vec{H_{is}} = \frac{E_{io}}{\eta_1}\,e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\,\hat{a}_y\ \text{A/m}$$

*The reflected fields

$$E_{rs} = E_{ro}(\cos\theta_r\,\hat{a}_x + \sin\theta_r\,\hat{a}_z)\,e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}\ \text{V/m}$$

$$H_{rs} = -\frac{E_{ro}}{\eta_1}\,e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}\,\hat{a}_y\ \text{A/m}$$

No.

* The transmitted fields:

$$\vec{E}_{ts} = E_{to}\,(\cos\theta_t\,\hat{a}_x - \sin\theta_t\,\hat{a}_z)\,e^{-\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad V/m$$

$$\vec{H}_{ts} = \frac{E_{to}}{\eta_2}\,e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\,\hat{a}_y \quad A/m$$

*Boundary Conditions at $z=0$
[tangent comp. must be cont.]

$\theta_i = \theta_r$, $k_{ix} = k_{rx}$

* $E_i(0) + E_r(0) = E_t(0)$

$$\boxed{[E_i(0) + E_r(0)]\cos\theta_i = E_t(0)\cos\theta_t} \text{ — } ①$$

* $H_i(0) + H_r(0) = H_t(0)$

$$\boxed{\frac{E_{io} - E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}} \text{ — } ②$$

By Solving ① of ②

(~~Reflection~~ Γ, τ Coefficients for parallel case)

$$\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{E_{ro}}{E_{io}}$$

$$\tau_{\parallel} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{E_{to}}{E_{io}}$$

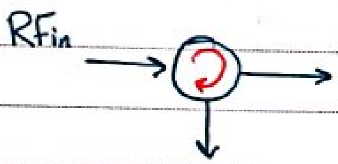
$$\boxed{1 + \Gamma_{\parallel} = \tau_{\parallel}\,\frac{\cos\theta_t}{\cos\theta_i}}$$

No.

* $\Gamma_{\parallel}$ & $\tau_{\parallel}$ are called Fresnel's eq for parallel polarization.

For $\Gamma_{\parallel}=0$ → No reflection if ($\theta_i = \theta_{B\parallel}$)
→ $E_{ro}=0$
→ Total Transmission ($\tau_{\parallel}=1$)

"Circulator" يستخدم حتى لا ترجع الموجة المنعكسة للمصدر



(To prevent Standing wave)

$\eta_2 \cos\theta_t = \eta_1 \cos\theta_i$ if $\varepsilon_{r_2} > \varepsilon_{r_1} \rightarrow \theta_t$ bigger
$\varepsilon_{r_2} < \varepsilon_{r_1} \rightarrow \theta_i$ bigger

$\theta_{B\parallel} \equiv$ Brewster angle "Polarizing angle"

$$\eta_2^2(1-\sin^2\theta_t) = \eta_1^2(1-\sin^2\theta_{B\parallel})$$

$$\sin^2\theta_{B\parallel} = \frac{1-\frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}}{1-\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2}, \quad \sin\theta_t = \frac{u_2}{u_1}\sin\theta_{B\parallel}$$

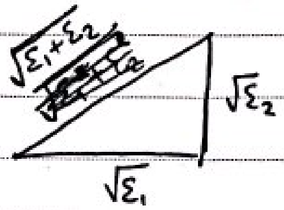
For non magnetic materials ($\mu_1=\mu_2=\mu_0$)

$$\sin^2\theta_{B\parallel} = \frac{1}{1+\frac{\varepsilon_1}{\varepsilon_2}} \quad \text{or} \quad \frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}$$

$$\boxed{\sin\theta_{B\parallel} = \sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}+\varepsilon_{r_2}}}}$$

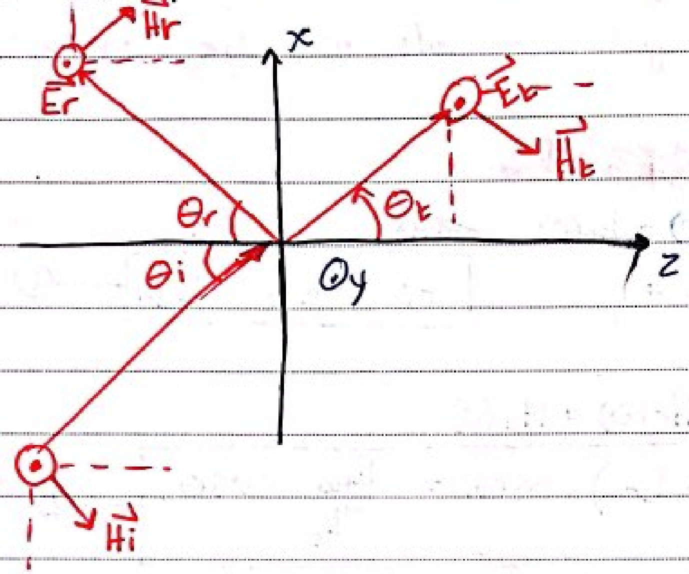
$\theta_{B\parallel}$ may be DNE (Does not exist)

Other way



$\tan\theta_{B\parallel} = \sqrt{\dfrac{\varepsilon_1}{\varepsilon_2}}$, for nonmagnetic μ_0 only

B. Perpendicular Polarization



* parallel component of $\vec{E}$ only reflected

Incident

$$\vec{E}_i = E_{io} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \hat{a}_y \quad V/m$$

$$\vec{H}_i = \frac{E_{io}}{\eta_1}(-\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z) e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad A/m$$

reflected

$$\vec{E}_r = E_{ro} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \hat{a}_y \quad V/m$$

$$\vec{H}_r = \frac{E_{ro}}{\eta_1}(\cos\theta_r \hat{a}_x + \sin\theta_r \hat{a}_z) e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \quad A/m$$

transmitted

$$\vec{E}_t = E_{to} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \hat{a}_y \quad V/m$$

$$\vec{H}_t = \frac{E_{to}}{\eta_2}(-\cos\theta_t \hat{a}_x + \sin\theta_t \hat{a}_z) e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

No.

*Boundary Conditions at $z=0$

[tangent components must be continuous]

* $E_i(0) + E_r(0) = E_t(0)$

$$\boxed{E_{io} + E_{ro} = E_{to}} \quad (1)$$

* $H_i(0) + H_r(0) = H_t(0)$

$$\boxed{\frac{1}{\eta_1}(E_{io} - E_{ro})\cos\theta_i = \frac{E_{to}}{\eta_2}\cos\theta_t} \quad (2)$$

By solving (1) of (2)

(Γ, τ for Perpendicular case)

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{E_{ro}}{E_{io}}$$

$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{E_{to}}{E_{io}}$$

$$\boxed{1 + \Gamma_\perp = \tau_\perp}$$

$\Gamma_\perp$, $\tau_\perp$ are called Fresnel's eq. for Perpendicular Polarization

No.

For no reflection $\Gamma_\perp = 0$, $\tau_\perp = 1$

$\theta_i = \theta_{B\perp} \rightarrow$ Brewster angle

$$\boxed{\eta_2 \cos\theta_{B\perp} = \eta_1 \cos\theta_t}$$

$$\eta_2^2 (1 - \sin^2\theta_{B\perp}) = \eta_1^2 (1 - \sin^2\theta_t)$$

$$\sin^2\theta_{B\perp} = \frac{1 - \frac{\mu_1 \varepsilon_2}{\mu_2 \varepsilon_1}}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2}$$

for nonmagnetic $(\mu_1 = \mu_2 = \mu_0)$

$\sin^2\theta_{B\perp} \rightarrow \infty$

$\theta_{B\perp}$ DNE (Does not Exist)

for $\mu_1 \neq \mu_2$ & $\varepsilon_1 = \varepsilon_2$

$$\sin\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1 + \mu_2}} \quad , \quad \tan\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1}}$$

Ex 10.10 : An EM wave travels in free space with

$$\vec{E}_s = 100 e^{j(0.866y + 0.5z)} \hat{a}_x \ \text{V/m}$$

Determine:

a) ω & λ

b) The magnetic field component

c) The time average Power in the wave.

No.

Sol:

$$E_s = E_0 e^{jk.r} = E_0 e^{j(k_x x + k_y y + k_z z)} \hat{a}_x$$

$k_x = 0$; $k_y = 0.866$; $k_z = 0.5 \Rightarrow |k| = \sqrt{k_x^2 + k_y^2 + k_z^2} = 1$ rad/m

$\vec{k} = 0.866\,\hat{a}_y + 0.5\,\hat{a}_z$

a) $|k| = \beta = 1$ rad/m $= \underbrace{\frac{\omega}{c}}_{\text{free space}} = \frac{2\pi}{\lambda}$

$$\boxed{\omega = \beta c = 3\times10^8 \text{ rad/s}}$$

$$\boxed{\lambda = \frac{2\pi}{\beta} = 6.283\text{m}}$$

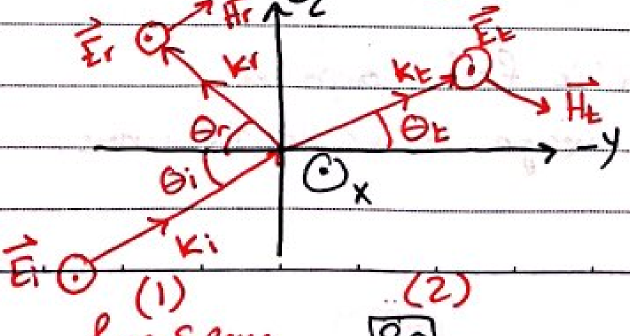
b) $\vec{H_s} = \frac{1}{\mu\omega}\,\vec{k}\times\vec{E_s} = \frac{(0.866\,\hat{a}_y + 0.5\,\hat{a}_z)}{4\pi\times10^{-7}\times3\times10^8} \times 100\,\hat{a}_x e^{jk.r}$

$$\boxed{\vec{H_s} = (1.33\,\hat{a}_y - 2.3\,\hat{a}_z)\,e^{j(0.866y + 0.5z)} \quad \text{mA/m}}$$

c) $P_{ave} = \frac{1}{2}\text{Re}\{\vec{E_s}\times\vec{H_s}^*\} = \frac{E_0^2}{2\eta}\,\hat{a}_k$

$= \frac{(100)^2}{2(120\pi)}(0.866\,\hat{a}_y + 0.5\,\hat{a}_z)$

$$\boxed{P_{ave} = 11.49\,\hat{a}_y + 6.631\,\hat{a}_z \text{ W/m}^2}$$

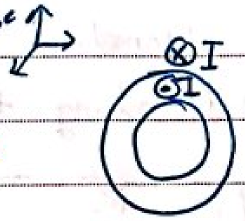


smile for life

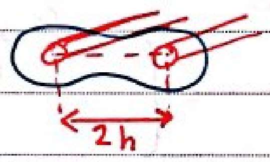
Ch11: Transmission lines

- One conductor T.L (Ch.12)
  (Not TEM)

- Two conductor T.L → (TEM)
  "Coaxial cable up to 300 MHz"



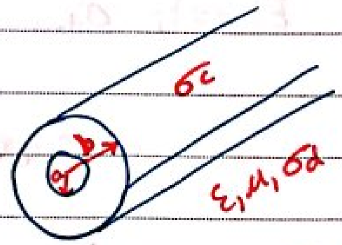
- Twin-wire T.L



- Planer T.L
  - → Microstrip line
  - → Strip line
  - → fin line
  - → Slot line

*T.L Parameters
$\{R/l,\ G/l,\ C/l,\ L/l\}$

For coaxial cable:



$$\frac{R_{ac}}{l} = \frac{1}{2\pi\sigma_c\delta}\left(\frac{1}{a}+\frac{1}{b}\right)\ \Omega/m = R_d$$

$$\frac{C}{l} = \frac{2\pi\varepsilon}{\ln(b/a)}\quad F/m$$

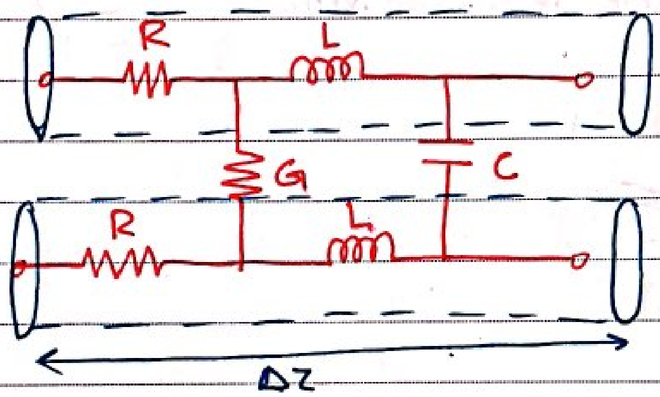
$$G = \frac{1}{R_d} = \frac{2\pi\sigma_d}{\ln(b/a)}\ S/m,\quad \frac{L_{ext}}{l} = \frac{\mu}{2\pi}\ln(b/a)\quad H/m$$

No.

Lin is negligable at high frequencies

$jX_L = j\omega L$, $L_{in} = \frac{X_L}{\omega}$

*These parameters are not ~~lumbed~~ (discrete) lumbed but it's distributed along the line

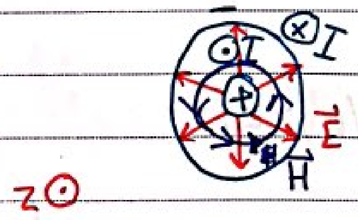


G, C Shunt Elements

(General two conductor T.L)

- $R_dC = T_r = \frac{\varepsilon}{\sigma}$, $\frac{C}{G} = \frac{\varepsilon}{\sigma}$, $L_{ext}C = \mu\varepsilon$

$\vec{E} \times \vec{H} = P_{in}\, \hat{a}_k$



$I - I = 0$ (No $\vec{H}$ outside cable)

*T.L equations:-

$V = -\int_l \vec{E} \cdot dl$, $I = \oint_l \vec{H} \cdot dl$

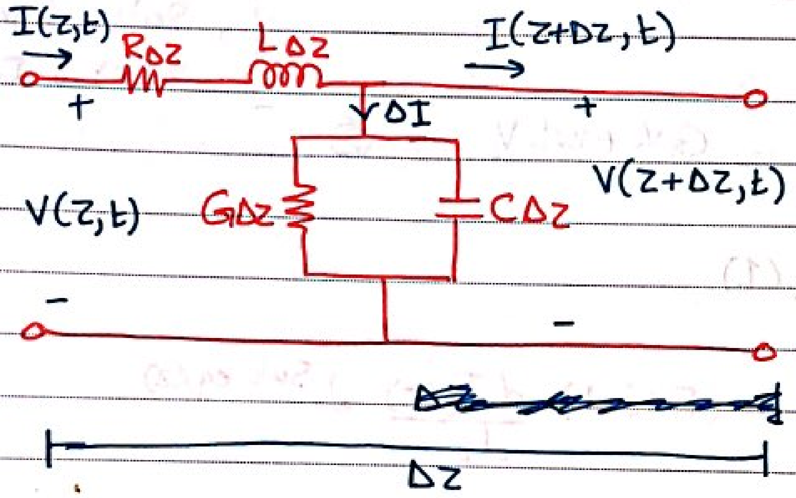
No.

Approximate eq. ckt for T.L :-

→ ~~Ls eq~~ L-eq ckt (easier) ~~HS~~

→ T- eq ckt

→ π-eq ckt } 1L3



Solve using ckt theory :- (KVL, KCL)
V I

KVL eq.

$$-V(z,t) + R\Delta z\, I(z,t) + L\Delta z \frac{dI(z,t)}{dt} + V(z+\Delta z,t) = 0$$

$$-\frac{(V(z+\Delta z,t) - V(z,t))}{\Delta z} = R\,I(z,t) + L\frac{dI(z,t)}{dt}$$

$$\boxed{-\frac{dV(z,t)}{dz} = R\,I(z,t) + L\frac{dI(z,t)}{dt}} \quad ①$$

KCL eq.

$$\boxed{-\frac{dI(z,t)}{dt} = G\,V(z,t) + C\frac{dV(z,t)}{dt}} \quad ②$$

No.

If V, I are harmonic $\quad \frac{d(e^{j\omega t})}{dt} = j\omega e^{j\omega t}$

$$v(z,t) = Re\{V_s e^{j\omega t}\}$$

$$-\frac{dV_s(z)}{dz} = RI_s + j\omega L I_s \quad — (1)$$

$$-\frac{dI_s(z)}{dz} = GV_s + j\omega L V_s \quad — (2)$$

→ Solve for $V_s(z)$ & $I_s(z)$

Derive eq (1)

$$\frac{d^2V_s(z)}{dz} = (R+j\omega L)\frac{dI_s(z)}{dz}$$ Sub eq(2)

$$\frac{d^2V_s(z)}{dz} = (R+j\omega L)(G+j\omega L)\, V_s(z)$$

$$\frac{d^2V_s(z)}{dz^2} - \gamma^2 V_s(z) = 0$$ Voltage Equation

$$\frac{d^2I_s(z)}{dz^2} - \gamma^2 I_s(z) = 0$$ Current Equation

$$V(z) = \underbrace{V_0^+ e^{-\gamma z}}_{\substack{\text{forward} \\ +ve}} + \underbrace{V_0^- e^{\gamma z}}_{\substack{\text{Backward} \\ -ve}}, \quad I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

No.

$$V(z,t) = \mathrm{Re}\{V_s(z)\, e^{j\omega t}\}$$

$$V(z,t) = V_0^+ \cos(\omega t - \gamma z) + V_0^- \cos(\omega t + \gamma z)$$

$\lambda = \frac{2\pi}{\beta}$, $\beta = \frac{\omega}{u}$

attenuation constant

$\gamma = \alpha + j\beta \rightarrow$ Propagation constant

$1\,Np = 8.686\ dB$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

*Circuit theory (V,I)

$$\frac{-d^2 V_s(z)}{dz^2} - \gamma^2 V_s(z) = 0$$

$$\frac{-d^2 I_s(z)}{dt} - \gamma^2 I_s(z) = 0$$

Sol.

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$$I_s(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}$$

$$V(z,t) = \Re e\{V_s(z) e^{j\omega t}\}$$

$$\frac{-dV_s(z)}{dz} = (R+j\omega L) I_s(z)$$

$$*\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta$$

Characterestic Impedance (Z_o) in Ω

$\underset{\text{in T.L}}{Z_o} \equiv \eta$ in waves

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R+j\omega L}{\gamma} = \frac{\gamma}{G+j\omega C} = \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$\boxed{Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}}} \ \Omega = R_o + jX_o$$

$$Y_o = \frac{1}{Z_o} = G + j\beta$$ Charatterestic admittance ($\mho$)

* The Previous (γ) and (Z_o) it is for the lossy line.

*** Special cases:**

1) lossless line $(R=0 \; , \; G=0)$

$$\sigma_c = \infty \qquad \sigma_d = 0$$

$$\gamma = j\omega\sqrt{LC} = \alpha + j\beta$$

$$\boxed{\alpha = 0} \; , \; \beta = \omega\sqrt{LC}$$

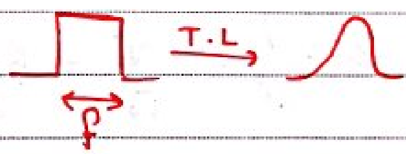
$$\boxed{L_{ext} C = \mu \varepsilon}$$

$$Z_o = \sqrt{\frac{L}{C}} = R_o \; ; \; X_o = 0$$

$$\lambda = \frac{2\pi}{\beta} \; , \; v_p = U = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

2) Distortionless line

Non Distortion line



$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

$$\gamma = \sqrt{RG}\sqrt{\left(1 + \frac{j\omega L}{R}\right)^2}$$

$$\gamma = \sqrt{RG}\left(1 + \frac{j\omega L}{R}\right) \Rightarrow \alpha = \sqrt{RG} \neq 0 \; , \; \beta = \sqrt{\frac{\omega^2 L^2 RG}{R}}$$

No.

$$\beta = \omega \sqrt{\frac{L^2 G}{R}} \quad , \frac{L}{R} = \frac{C}{G}$$

$$\beta = \omega \sqrt{\frac{LGC}{G}}$$

$$\boxed{\beta = \omega \sqrt{LC}} \quad rad/m$$

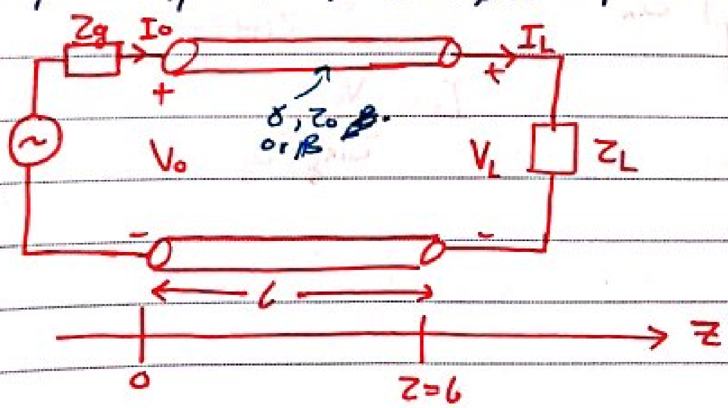
$$\alpha = \sqrt{RG} \neq 0 \quad ; \alpha \text{ is not a function of } \omega.$$

$$Z_o = \sqrt{\frac{R(1+\frac{j\omega L}{R})}{G(1+\frac{j\omega C}{G})}}$$

$$Z_o = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_o \, , \quad X_o = 0$$

$$\lambda = \frac{2\pi}{\beta} \, , \quad u = \frac{\omega}{\beta}$$

* Input Impedance, SWR and Power:



$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$$I_s(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}$$

$$= \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}$$

at Generator $(z=0)$

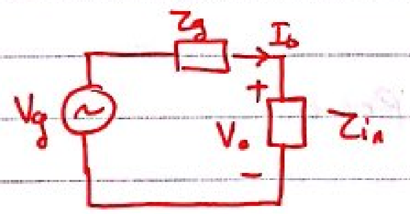
$$V_s(0) = V_o = V_o^+ + V_o^- \quad \text{——} ①$$

$$I_s(0) = I_o = \frac{V_o^+}{Z_o} - \frac{V_o^-}{Z_o} \quad \text{——} ②$$

Solve for V_o^+ & V_o^-

$$(2) \Rightarrow I_o Z_o = V_o^+ - V_o^-$$

$$\boxed{\begin{aligned} V_o^+ &= \tfrac{1}{2}(V_o + I_o Z_o) \\ V_o^- &= \tfrac{1}{2}(V_o - I_o Z_o) \end{aligned}}$$



$$V_o = V_g \frac{Z_{in}}{Z_{in}+Z_g}$$

$$I_o = \frac{V_g}{Z_{in}+Z_g}$$

~~at~~

No.

at the load end $(z=l)$

$$V_s(l) = V_L = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} \quad \text{—}(1)$$

$$I_s(l) = \frac{V_0^+ e^{-\gamma l} - V_0^- e^{\gamma l}}{Z_0} \quad \text{—}(2)$$

Solve for V_0^+ & V_0^-

$$V_0^+ = \frac{1}{2}(V_L + I_L Z_0) e^{\gamma l}$$

$$V_0^- = \frac{1}{2}(V_L - I_L Z_0) e^{-\gamma l}$$

$$\boxed{Z_L = \frac{V_L}{I_L}}$$

Input Impedance @ $z=0$

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{V_0}{I_0} = Z_0 \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}$$

Input Impedance @ $z=l$ (load end)

$$Z_{in} = Z_0 \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}, \quad V_0^+ = \frac{1}{2}(V_L + I_L Z_0) e^{\gamma l}$$

$$V_0^- = \frac{1}{2}(V_L - I_L Z_0) e^{-\gamma l}$$

$$Z_{in} = Z_0 \left(\frac{(V_L + I_L Z_0) e^{\gamma l} + (V_L - I_L Z_0) e^{-\gamma l}}{(V_L + I_L Z_0) e^{\gamma l} - (V_L - I_L Z_0) e^{-\gamma l}} \right) \text{“} \div I_L \text{”}$$

No.

$$Z_{in} = Z_0 \frac{(Z_L + Z_0)e^{\gamma l} + (Z_L - Z_0)e^{-\gamma l}}{(Z_L + Z_0)e^{\gamma l} - (Z_L - Z_0)e^{-\gamma l}}$$

~~take~~

take Z_L & Z_0 as a common factor.

$$Z_{in} = \frac{Z_L(e^{\gamma l} + e^{-\gamma l}) + Z_0(e^{\gamma l} - e^{-\gamma l})}{Z_L(e^{\gamma l} - e^{-\gamma l}) + Z_0(e^{\gamma l} + e^{-\gamma l})}$$

Divide by $(e^{\gamma l} + e^{-\gamma l})$

$$\boxed{Z_{in} = Z_0 \left(\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right)}$$

for lossy line.

Recall:

$$\cosh \gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2}$$

$$\sinh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2}$$

$$\tanh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}}$$

for lossless $\rightarrow \gamma = j\beta$

$$\tanh \gamma l = \tanh j\beta l$$
$$= j \tan \beta l$$

$$\boxed{Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}}$$

***Reflection coefficient (Γ):**

***if $Z_0 \neq Z_L$:**

$$\Gamma_L = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}} = \frac{V_L - I_L Z_0}{V_L + I_L Z_0}$$

$$V_s(l) = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l}$$

$$V_0^- = \tfrac{1}{2}(V_L - I_L Z_0) e^{-\gamma l}$$

$$V_0^+ = \tfrac{1}{2}(V_L + I_L Z_0) e^{\gamma l}$$

divide by I_L

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}} = |\Gamma| \angle \theta_r \quad , \quad 0 \leq |\Gamma_L| \leq 1$$

***current Reflection coefficient:**

$$\Gamma_{iL} = -\Gamma_L$$

l: Distance from load to line.

- Γ at any distance

$z = l - l'$

$$\Gamma = \frac{V_0^- e^{\gamma(l-l')}}{V_0^+ e^{-\gamma(l-l')}} = \frac{V_0^- e^{\gamma l} e^{-\gamma l'}}{V_0^+ e^{-\gamma l} e^{\gamma l'}} = \Gamma_L e^{-2\gamma l'}$$

***Standing wave Ratio: (SWR)**

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}$$

$$S = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \quad , \text{ divide by } V_0^+$$

No.

$$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \qquad 1 \le S \le \infty$$

- $|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} * \frac{V_{min}}{V_{min}} = S Z_0$

$|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} * \frac{V_{max}}{V_{max}} = \frac{Z_0}{S}$

*Power:

$V_{rms} = \frac{V_{Peak}}{\sqrt{2}}, \ I_{rms} = \frac{I_{Peak}}{\sqrt{2}}$

$$P_{ave} = \frac{1}{2} Re\{V_s(l)\ I_L^*(l)\} = Re\{V_{rms}\ I_{rms}^*\}$$

lossless:

$$V_s(l) = V_0^+ e^{-j\beta l} + V_0^- e^{j\beta l}$$

$$I_s(l) = \frac{V_0^+}{Z_0} e^{-j\beta l} - \frac{V_0^-}{Z_0} e^{j\beta l}$$

$$\rightarrow P_{ave} = \frac{1}{2} Re\left\{\left(V_0^+ e^{-j\beta l} + V_0^- e^{j\beta l}\right) * \left(\frac{V_0^+}{Z_0} e^{j\beta l} - \frac{V_0^-}{Z_0} e^{-j\beta l}\right)\right\} ; \ V_0^- = |\Gamma| V_0^+$$

$$P_{ave} = \frac{1}{2} Re\left\{\frac{V_0^{+2}}{Z_0}\left(e^{-j\beta l} + |\Gamma| e^{j\beta l}\right)\left(e^{j\beta l} - |\Gamma|^* e^{-j\beta l}\right)\right\}$$

$$P_{ave} = \frac{1}{2} Re\left\{\frac{V_0^{+2}}{Z_0}\left(1 - |\Gamma|^2 + |\Gamma| e^{2j\beta l} - |\Gamma|^* e^{-2j\beta l}\right)\right\}$$

$$P_{ave} = \frac{1}{2} \frac{V_0^{+2}}{Z_0}\left(1 - |\Gamma|^2\right)$$

$$P_{ave} = P_t = P_i - P_r = \frac{V_0^{+2}}{2Z_0} - |\Gamma|^2 \frac{V_0^{+2}}{2Z_0}$$ for lossless

No.

For M.P.T $|\Gamma| = 0 \Rightarrow P_r = 0$

$$\boxed{P_t = \frac{{V_0^+}^2}{2Z_0}}$$

*Special cases:

A) if the load is short circuit

$Z_L = 0$

$\Gamma_L = -1 = 1\angle 180°$

$S = \infty$

$$\boxed{Z_{in(sc)} = Z_0 \frac{\cancel{Z_L} + jZ_0 \tan\beta l}{\cancel{Z_0} + j\cancel{Z_L} \tan\beta l}}$$

$$\boxed{Z_{in(sc)} = jZ_0 \tan\beta l} \rightarrow \text{Pure Imaginary}$$

βl: electrical length in (rad) or (degrees)

B) Load is an open circuit

$Z_L = \infty$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_L} - \frac{Z_0}{Z_L}}{\frac{Z_L}{Z_L} + \frac{Z_0}{Z_L}}$$

$$\boxed{Z_{in(oc)} = \frac{Z_0}{j\tan\beta l} = -jZ_0 \cot\beta l}$$

↳ Pure Imaginary

$\Gamma_L = 1 = 1\angle 0°$

$S = \infty$

$$\boxed{\sqrt{Z_{in_{sc}} Z_{in_{oc}}} = Z_0}$$

C) Matched load (for M.P.T) max Power Transfer

$Z_L = Z_0$

$\Gamma = 0$, $S = 1$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$\boxed{Z_{in} = Z_0} \quad / \quad Z_{in} = \sqrt{Z_L Z_0}$$

No.

Ex: A certain T.L operates at $w = 10^6$ rad/s has $\alpha = 8$ dB/m $\beta = 1$ rad/m & $Z_o = 60 + j40\ \Omega$ and length of (2m) is connected to a load of $20 + j50\ \Omega$, and a Source of $10\angle 0°$ V & $Z_g = 40\ \Omega$ Find:

a) Z_{in}

b) The Sending end Current (I_o)

c) The current at the middle of the line (1m)

Sol: a) $Z_{in} = Z_o \dfrac{Z_L + Z_o \tanh\gamma l}{Z_o + Z_L \tanh\gamma l}$, $\alpha = \dfrac{8}{8.686} = 0.921$ Np/m

$\gamma = 0.921 + j1$

$\gamma l = 2\gamma = 1.84 + j2$

$\tanh(1.84 + j2) = 1.033 - j0.03924$

$\Rightarrow Z_{in} = 60.25 + j38.79 = 71.66 \angle 32.77°\ \Omega$

$\tanh(x \pm jy) = \dfrac{\sinh(x)}{\cosh(2x) + \cos(2y)} \pm j \dfrac{\sin(2y)}{\cosh(2x) + \cos(2y)}$

b) $I_o = \dfrac{V_g}{Z_g + Z_{in}} = 93.03 \angle -21.15°$ mA

c) $V_o^+ = \frac{1}{2}(V_o + I_o Z_o) = 6.687 \angle 12.08°$ V

$V_o^- = \frac{1}{2}(V_o - I_o Z_o) = 0.0518 \angle 260°$ V

$V_o = V_g \dfrac{Z_{in}}{Z_{in} + Z_g} = 6.667 \angle 11.82°$

$I(z = 1m) = \dfrac{V_o^+}{Z_o} e^{-\gamma(1)} - \dfrac{V_o^-}{Z_o} e^{\gamma(1)}$, $\gamma = 0.921 + j1$ /m

$I = 35.1 \angle 281°$ mA

smile for life

* Smith chart

→ use normalized Value of Impedances

$$Z_L \rightarrow z_L = \frac{Z_L}{Z_0}$$

$$\Gamma_2 = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| \angle \theta_\Gamma \quad , z_L = \frac{1+\Gamma}{1-\Gamma}$$

$$\Gamma_2 = \frac{z_L - 1}{z_L + 1} = \Gamma_r + \Gamma_i$$

$$z_L = r + jx = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i} \text{ + conjugate}$$

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$x = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

rearrange:

circles equation

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 \pm r}\right)^2$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

No.

r-circle has radius $\frac{1}{1+r}$

and center $\left(\frac{r}{1+r}, 0\right)$

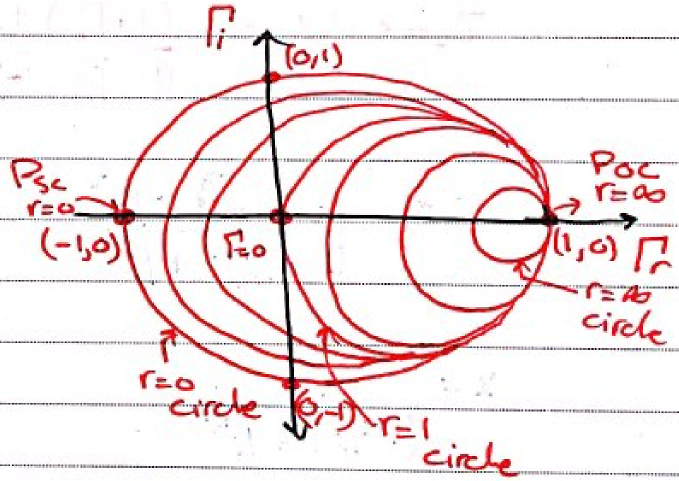
$\rightarrow (\Gamma_r, \Gamma_i)$

x-circle has radius $\frac{1}{x}$

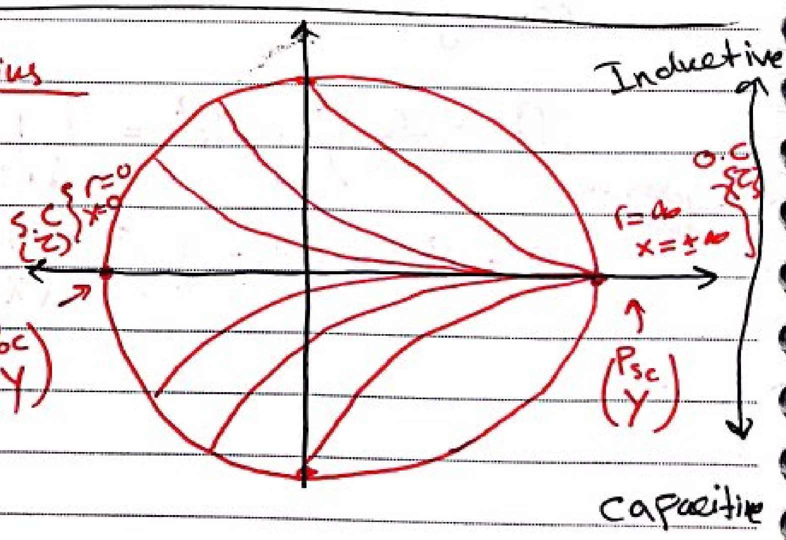
and center $\left(1, \frac{1}{x}\right)$

for ex.

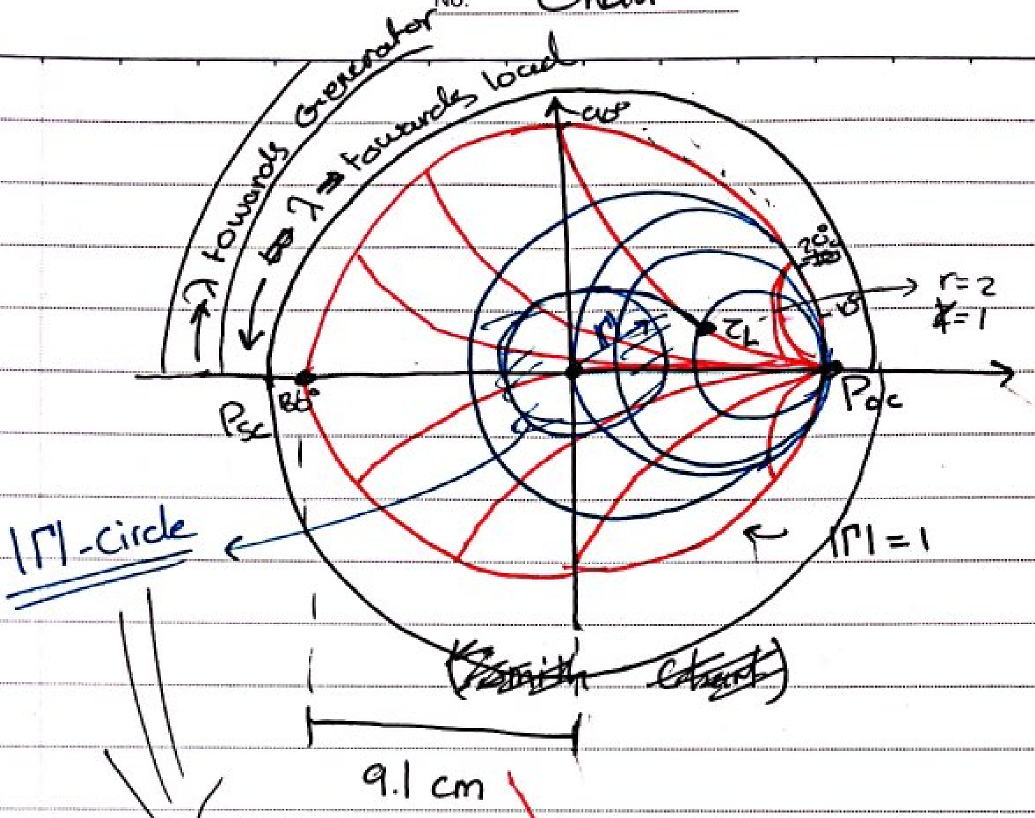
	center	radius
$r=0$	$(0,0)$	1
$r=1$	$(\frac{1}{2},0)$	$\frac{1}{2}$
$r=2$	$(\frac{2}{3},0)$	$\frac{1}{2}$
⋮		
$r=\infty$	$(1,0)$	0



	center	radius
$x=0$	$(1,\infty)$	∞
$x=1$	$(1,1)$	1
$x=\pm2$	$(1,\pm\frac{1}{2})$	$\frac{1}{2}$
⋮		
$x=\pm\infty$	$(1,0)$	0 $\binom{P_{oc}}{Y}$

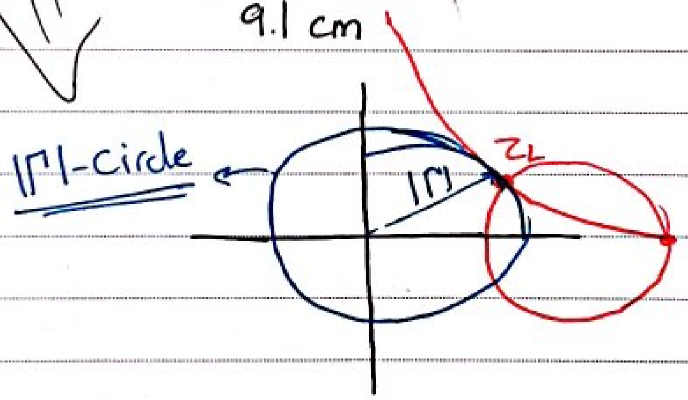


Smith Chart



$\Gamma_r = 1$

$\Gamma_i = 0$



9.1 cm → $|\Gamma| = 1$

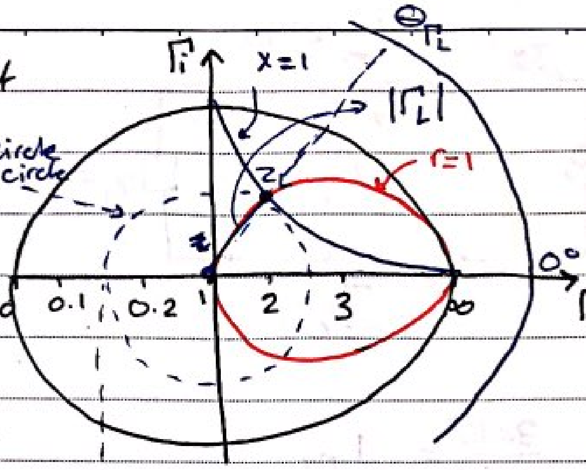
No.

*Smith chart

$Z = r + jx$

i.e: $Z_L = 100 + j100\,\Omega$

$Z_0 = 100\,\Omega$



Inductive $\begin{bmatrix} r + jx \\ G - jB \end{bmatrix}$

capacitive $\begin{bmatrix} r - jx \\ G + jB \end{bmatrix}$

$z_L = \frac{Z_L}{Z_0} = 1 + j1$ $\quad \Gamma$ —— $|\Gamma|$ —— 5cm

$0 < \gamma < 1$, $Z_{in,Min}$ $\quad$ $1 < \gamma < \infty$ $\quad Z_{in,max}$

$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$; $0 < |\Gamma| < 1$

5cm → ?

7.2cm → 1

$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$

→ Intersection of S-circle with $+\Gamma_r$ axis.

$Y = \frac{1}{Z_L} = \frac{1}{1+j1} = \frac{1}{1\angle 45^\circ} = 1\angle -45^\circ$

If we are working in Impedance $r + jx$

" " " " " admittance $G \pm jB$

$\tau = 1 + \Gamma$

$0 < \tau < 2$

$z_1 = 2 + j$

$z_2 = 1 - j0.5$

I am assuming Z-chart

S, $|\Gamma|$-circles (from origin)

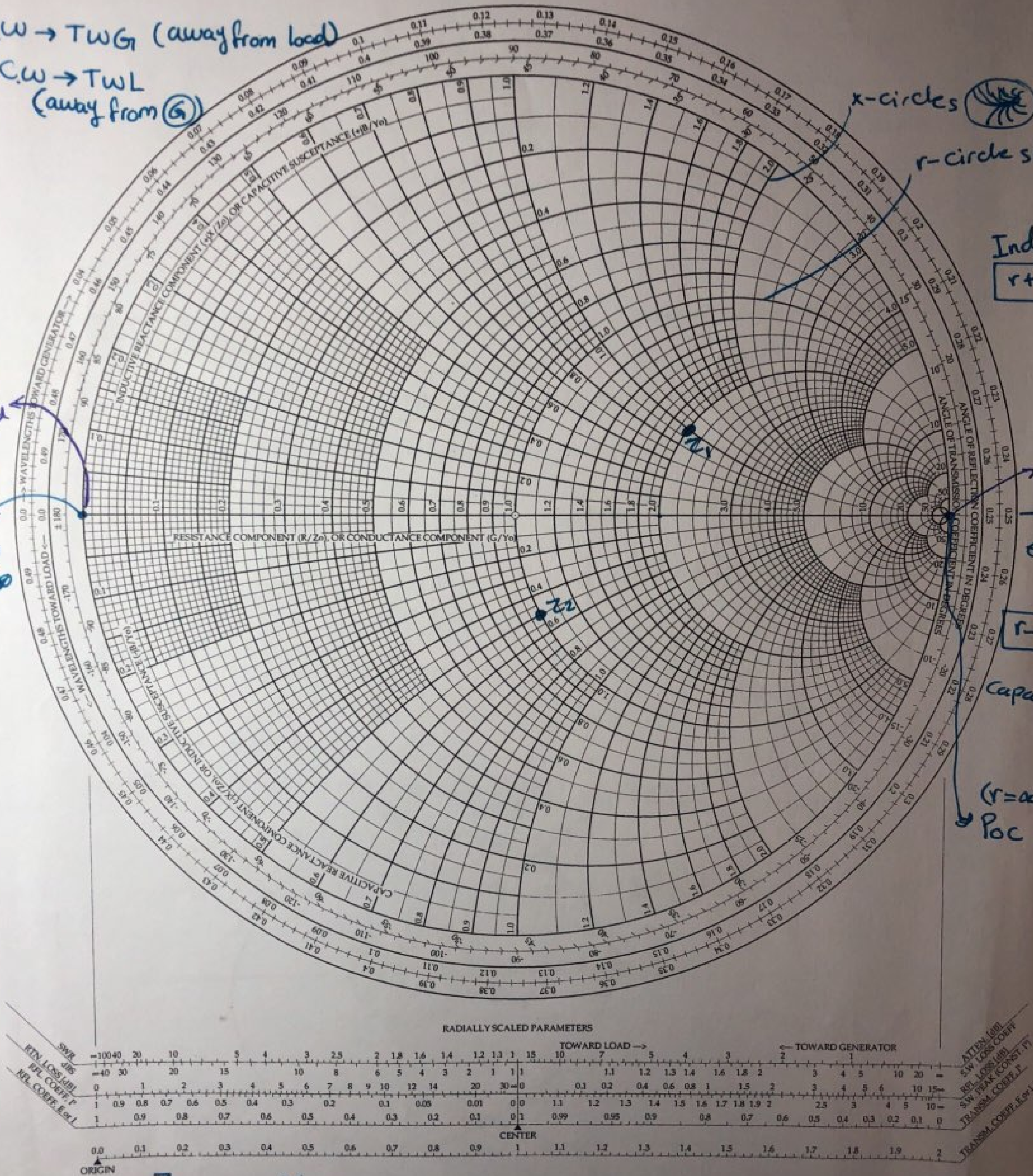
The Complete Smith Chart

Black Magic Design

$\lambda = 720°$

C.W → TWG (away from load)

C.C.W → TWL (away from G)



Z Y

$r+jx \leftrightarrow g+jb$

$P_{oc} \leftrightarrow P_{sc}$

$P_{sc} \leftrightarrow P_{oc}$

Ex NB

$Z_0 = 100$

$Z_L = 100 + j100 \rightarrow 1 + j1$

$l = \frac{3 \times 10^{-2}}{1} \times \lambda = .03\lambda$

$j\beta = 1 \quad f = 300\,MHz$

$u = c \quad , l = 3cm$

$\lambda f = c$

$\lambda = \frac{3 \times 10^8}{3 \times 10^8} = 1m$

8-22-16

find S, Γ, Z_{in} at gen.

$S = 2.6$

$\Gamma = \frac{3.4}{8.9} \angle 64°$

$= 0.382 \angle 64°$

$Z_{in} = z_{in} \times Z_0$

$= 150 + j110\,\Omega$

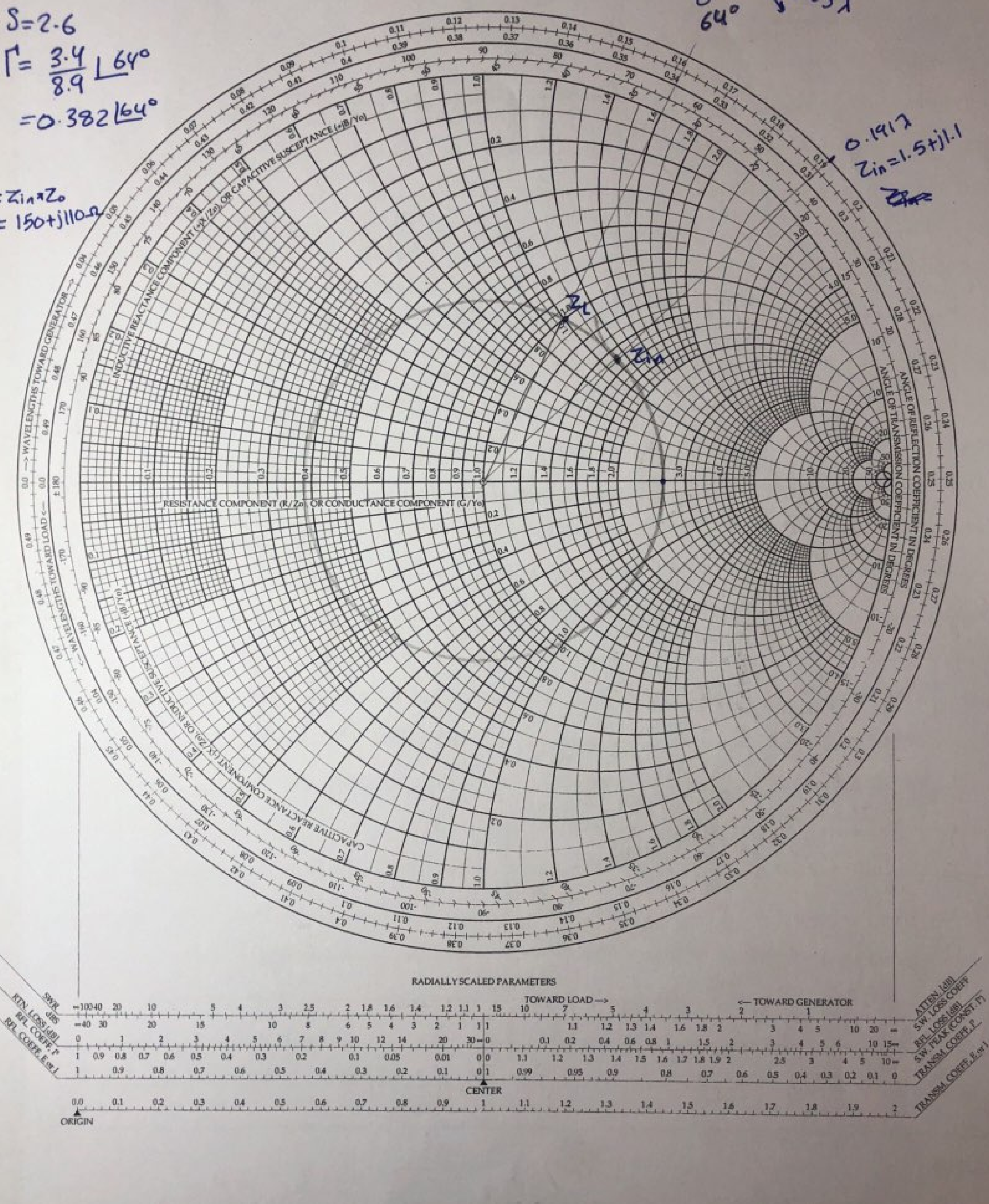
The Complete Smith Chart

Black Magic Design

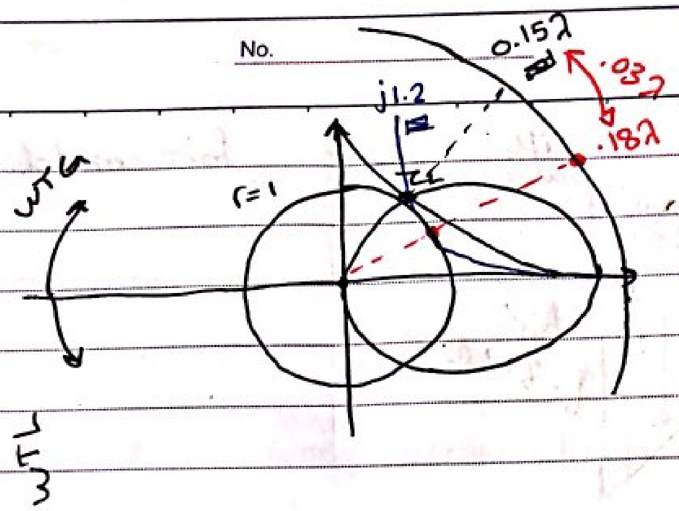
0.161 64° $\searrow$ 0.03λ

0.191λ

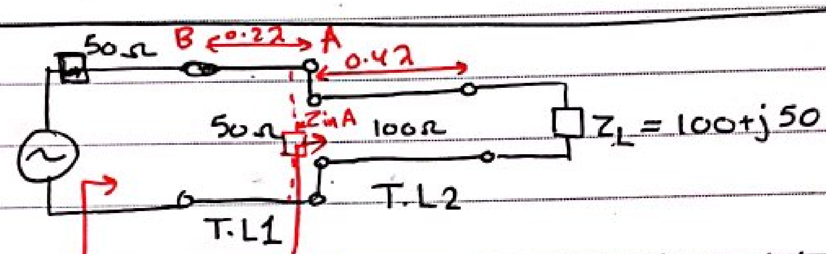
$z_{in} = 1.5 + j1.1$



No.



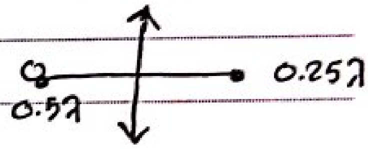
<u>Ex2.</u>



$Z_{inB} =$ $\quad$ $Z_{inA} =$ $\quad$ Z_L

1 revolution $= 360° = \frac{\lambda}{2}$

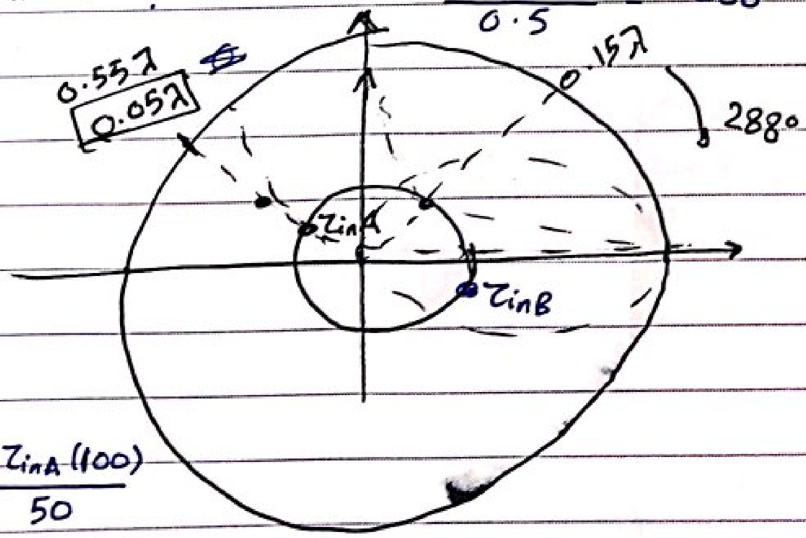
$\lambda = 720°$



$$z_L = \frac{100+j50}{100} = 1+j0.5$$

$0.5\lambda \rightarrow \cancel{360°}\ 360°$

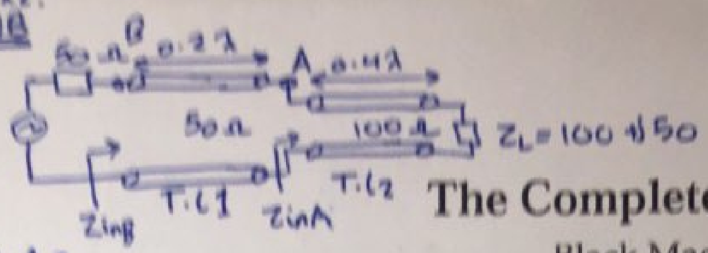
$0.4\lambda \rightarrow\ ? \Rightarrow 0.4\lambda \rightarrow \frac{0.4(360)}{0.5} = 288°$



$$Z_{inA} = \frac{z_{inA}(100)}{50}$$

$$Z_{inA} = \frac{65+j20}{50} \quad ; \quad z_{inA} = 1.3+j0.4\ \Omega$$

Ex2.
NB



The Complete Smith Chart

Black Magic Design

0.145λ → 0.4λ

75°

1) Z_{inA} =

$Z_L = 1 + j0.5$

$Z_{inA} = 0.65 + j0.175$

$Z_{inA} = 65 + j17.5\,\Omega$

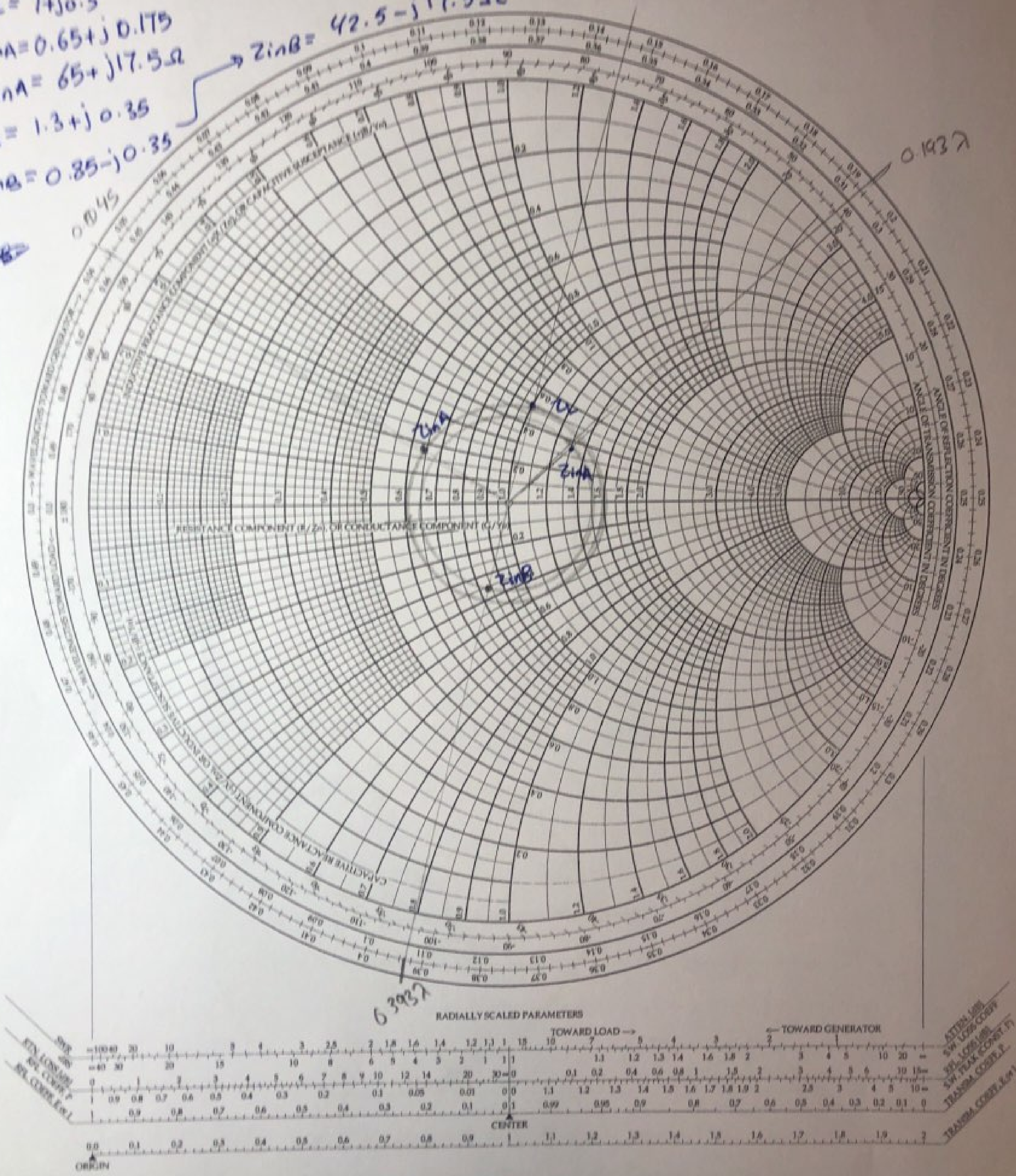
$Z_{inA}' = 1.3 + j0.35$

$Z_{inB} = 0.85 - j0.35$

→ $Z_{inB} = 42.5 - j17.5\,\Omega$

0.045

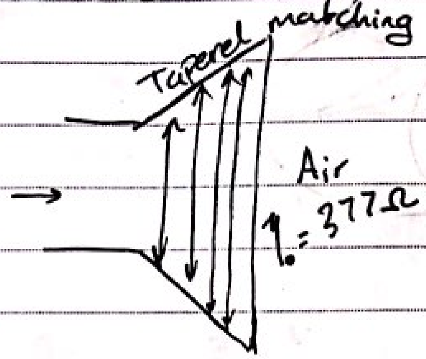
Z_{inB}



Scanned by CamScanner

No.

We use Smith Chart for matching purposes



* Application on T.L

A) Quarter wave length transformer

$\lambda/4$ Tr

Only applied for ~~[illegible]~~ Purely resistive loads

$Z_L = R_L$

$l = \frac{\lambda}{4}$ if T.L is lossless

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l}$$

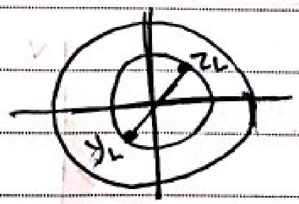
① $\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} = 90^\circ$

$$\boxed{Z_{in} = \frac{Z_o'^2}{Z_L}}$$

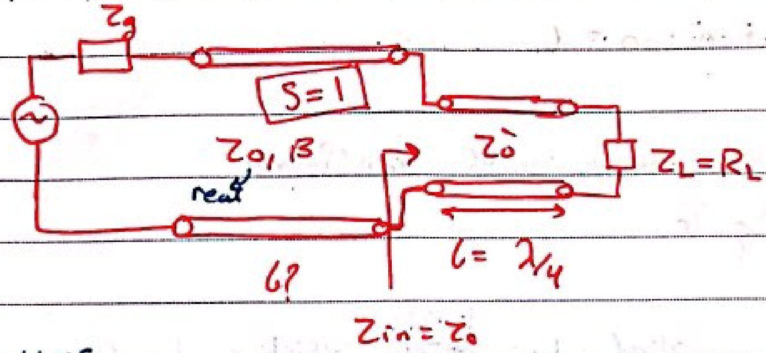
$$\boxed{Z_o' = \sqrt{Z_{in} Z_L}}$$

$$\boxed{\begin{array}{c} Z_{in} = \frac{1}{Z_L} \\ Z_{in} = Y_L \end{array}}$$

Normalized



No.



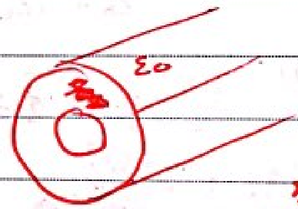
Air T.L ($u = c$)

if $Z_0 = 50\,\Omega$ real

$Z_L = 100\,\Omega$ real

$f = 3\times10^8$ Hz

Design $\frac{\lambda}{4}$ Tr.



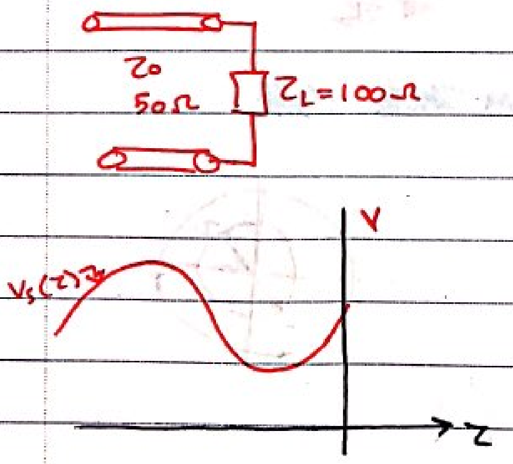
$Z_0' = \sqrt{50(100)} \cong 70\,\Omega$

$l = \lambda/4$, $\lambda = \frac{c}{f} = 1\,m$

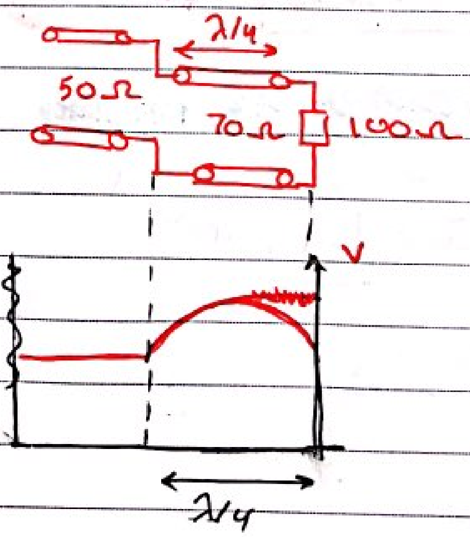
$l = 0.25\,m$

$50\,\Omega$

$75 + j37.5\,\Omega$

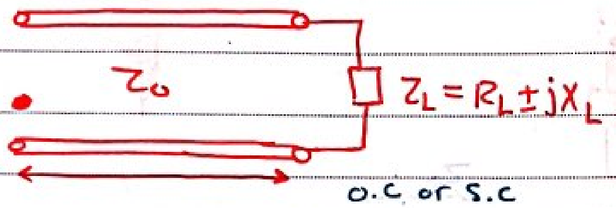


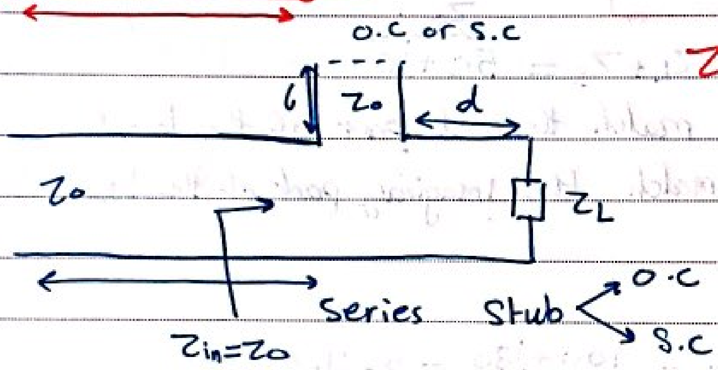
S ≠ 1



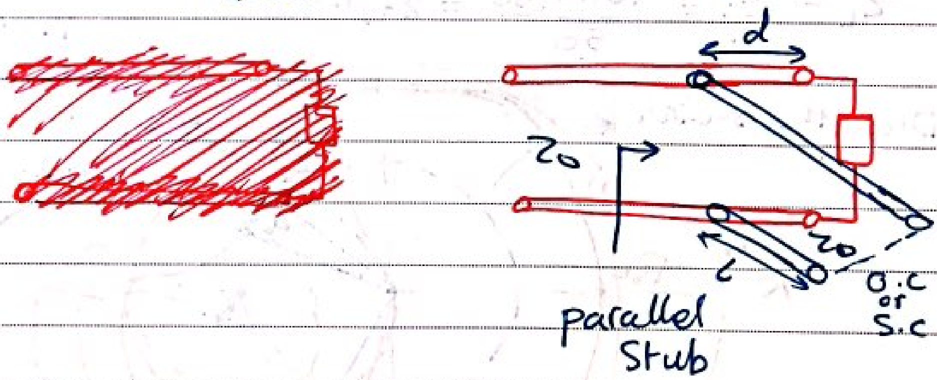
B) Matching using Stubs:

1) Single Stub matching (Tuner)





Z-Smith chart
Z_L



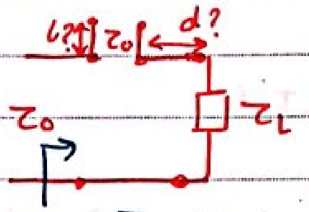
Y-Smith Chart
$Z_L \rightarrow Y_L$

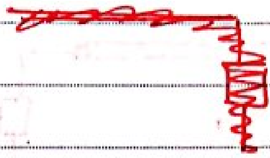
most common $\boxed{\text{Parallel S.C}}$

No.

Ex. Match a load 100+j80 to a 50-Ω T.L using a single stub tuner:

A) Series o.c stub





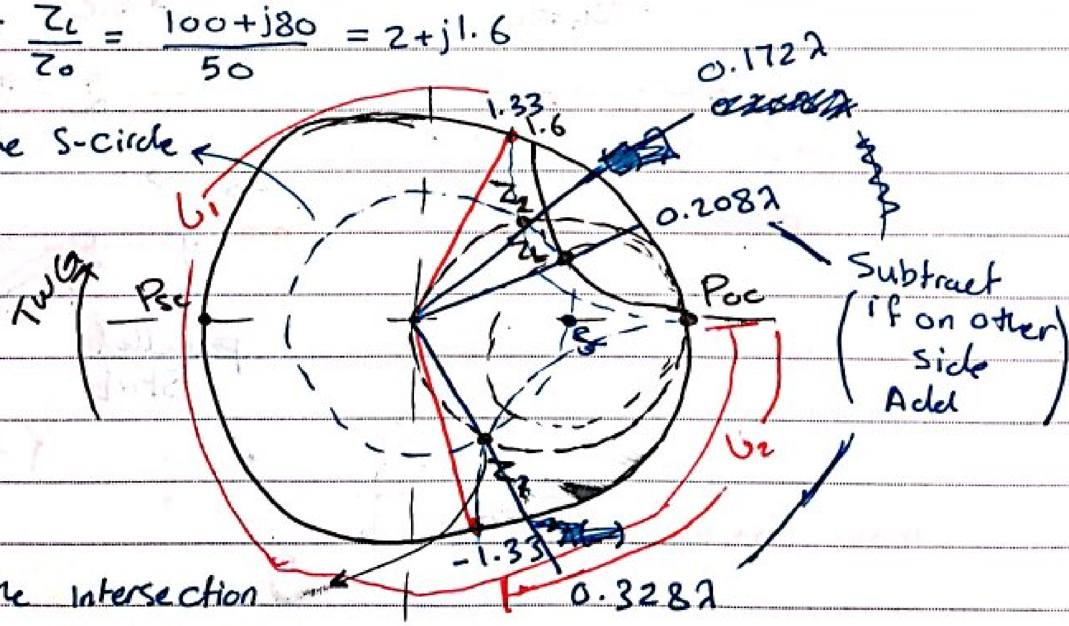
$$Z_{in} = Z_L + Z_s = \overbrace{50+j0}^{Z_0}$$

(d) to match the real part of the load

(l) to match the Imaginary part of the load.

① $z_L = \frac{Z_L}{Z_0} = \frac{100+j80}{50} = 2+j1.6$

② Draw the S-circle



③ Find the intersection between S-circle & r=1 circle.

(two - solution)

$z_1 = 1 - j1.33$

$z_2 = 1 + j1.33$

No.

④ The distance between Z_L & Z_1 is (d_1)

" " " Z_L & Z_2 is (d_2)

$$d_1 = 0.328 - 0.208 = 0.12\lambda$$

$$d_2 = (0.5 - 0.208) + 0.172 = 0.463\lambda$$

⑤ $Z_{s_1} = +j1.33$ So that we get $1+j0$

$Z_{s_2} = -j1.33$

C_1: distance from o.c point $= 0.398\lambda$ → from Z_{s_1} to o.c pt.

$C_2 = 0.102\lambda$ → from Z_{s_2} to o.c pt.

always

$C_1 + C_2 = 0.5\lambda$

Ex NB p106

$z_L = 100 + j80\,\Omega$, $Z_0 = 50\,\Omega$

A) Series stub tuner

or

$z_L = 2 + j1.6\,\Omega$

$z_1 = 1 + j1.3$

$z_2 = 1 - j1.3$

$d_2 = (0.5 - (0.208 - 0.172))\lambda$

$d_2 = 0.464\lambda$

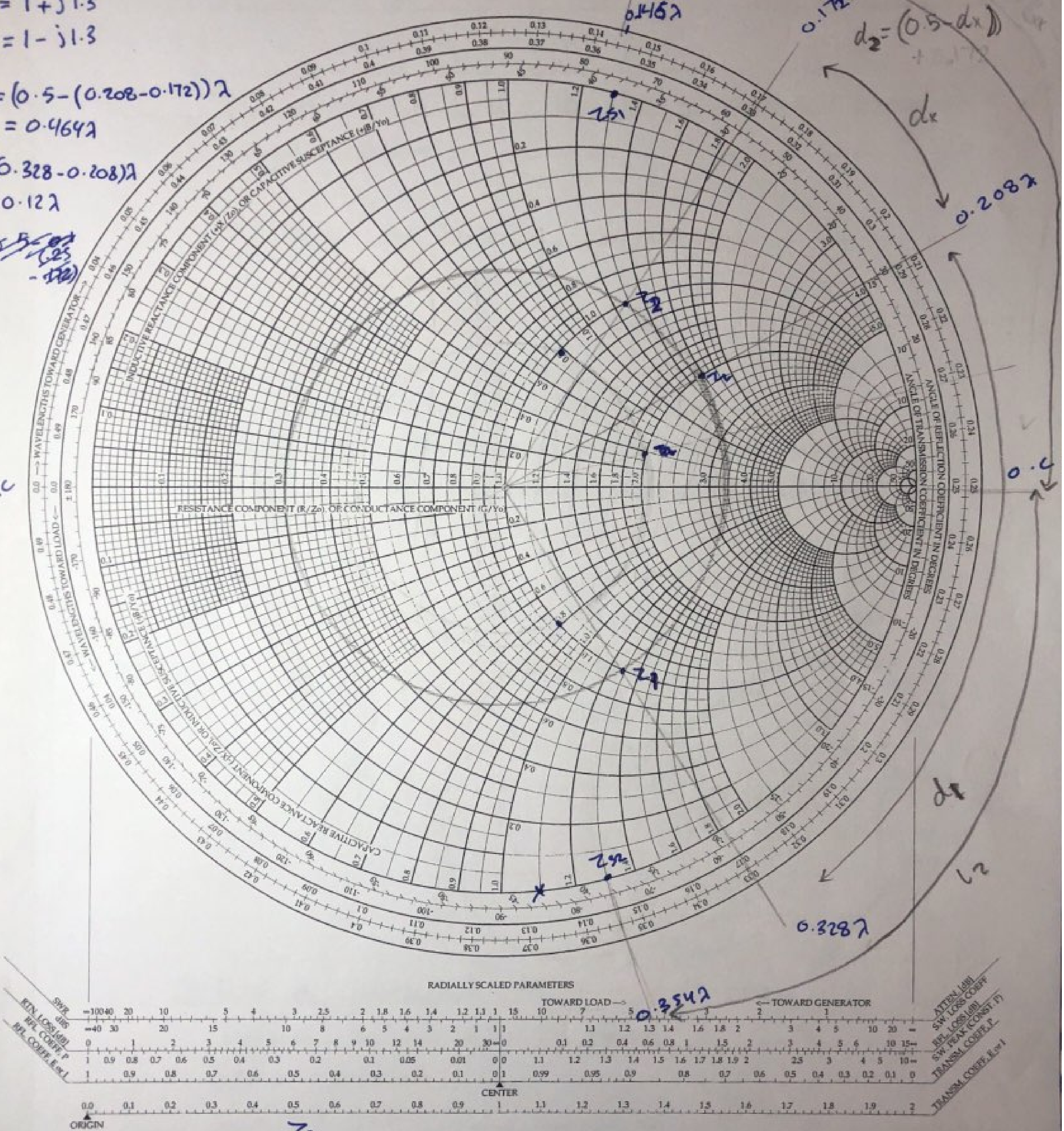
$d_1 = (0.328 - 0.208)\lambda$

$= 0.12\lambda$

$l_1 = 0.5 - l_x$

The Complete Smith Chart

Black Magic Design



$l_1 = 0.5 - (0.25 - \overset{z_{s1}}{0.146})$

$= 0.396\lambda$

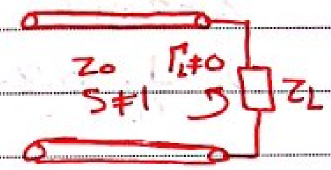
$\boxed{l_1 + l_2 = 0.5\lambda}$

$l_2 = \cancel{0.25}\ \overset{z_{s2}}{0.354} - 0.25$

$= 0.104\lambda$

*Ex. Match a load $Z_L = 100+j80\,\Omega$ to a $50\,\Omega$ T.L

A) Single Series Open-Circuit Stub



$z_L = 2 + j1.6$

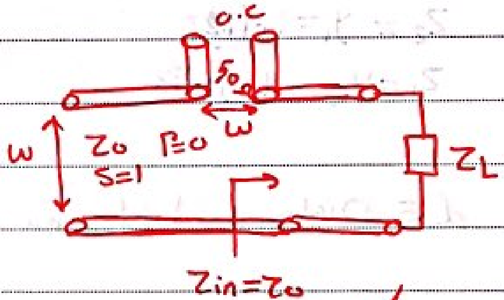
$d_1 = 0.12\lambda$

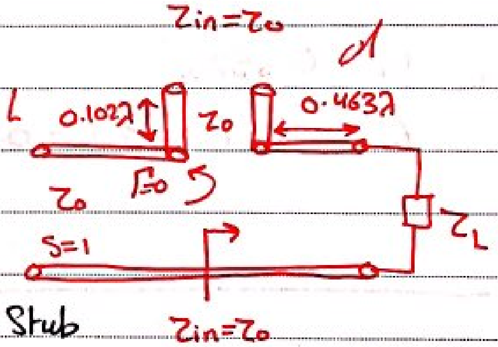
$d_2 = 0.463\lambda$

$l_1 = 0.398\lambda$

$l_2 = 0.102\lambda$

$l_1 + l_2 = 0.5\lambda$





B) Single Series Short-Circuit Stub

$z_L =$

$z_1 =$

$z_2 =$

$d_1 = 0.12\lambda$

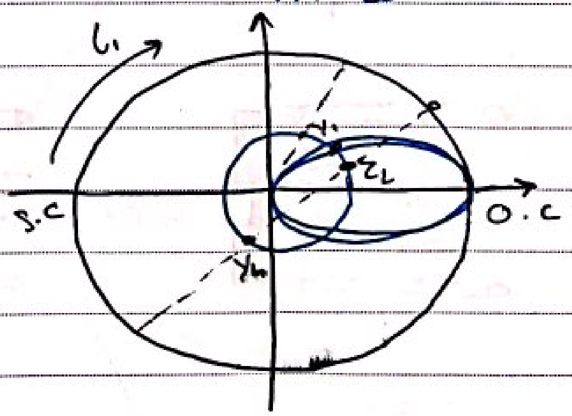
$d_2 =$

$l_1 = 0.148\lambda$

$l_2 = 0.352\lambda$

* Always the difference in length between open & short is $(\lambda/4) = 180°$

$\lambda \to 720°$



Ex P108 $Z_L = 100 + j80$

$Z_0 = 50$

S.C Stub

$Z_L' = 2 + j1.6$

$Z_1 = 1 + j1.3$

$Z_2 = 1 - j1.3$

$Z_{s1} = 1.3j \rightarrow l_1 =$

$Z_{s2} = -1.3j \rightarrow l_2 =$

$d_2 = 0.5 - (0.208 - 0.172)$

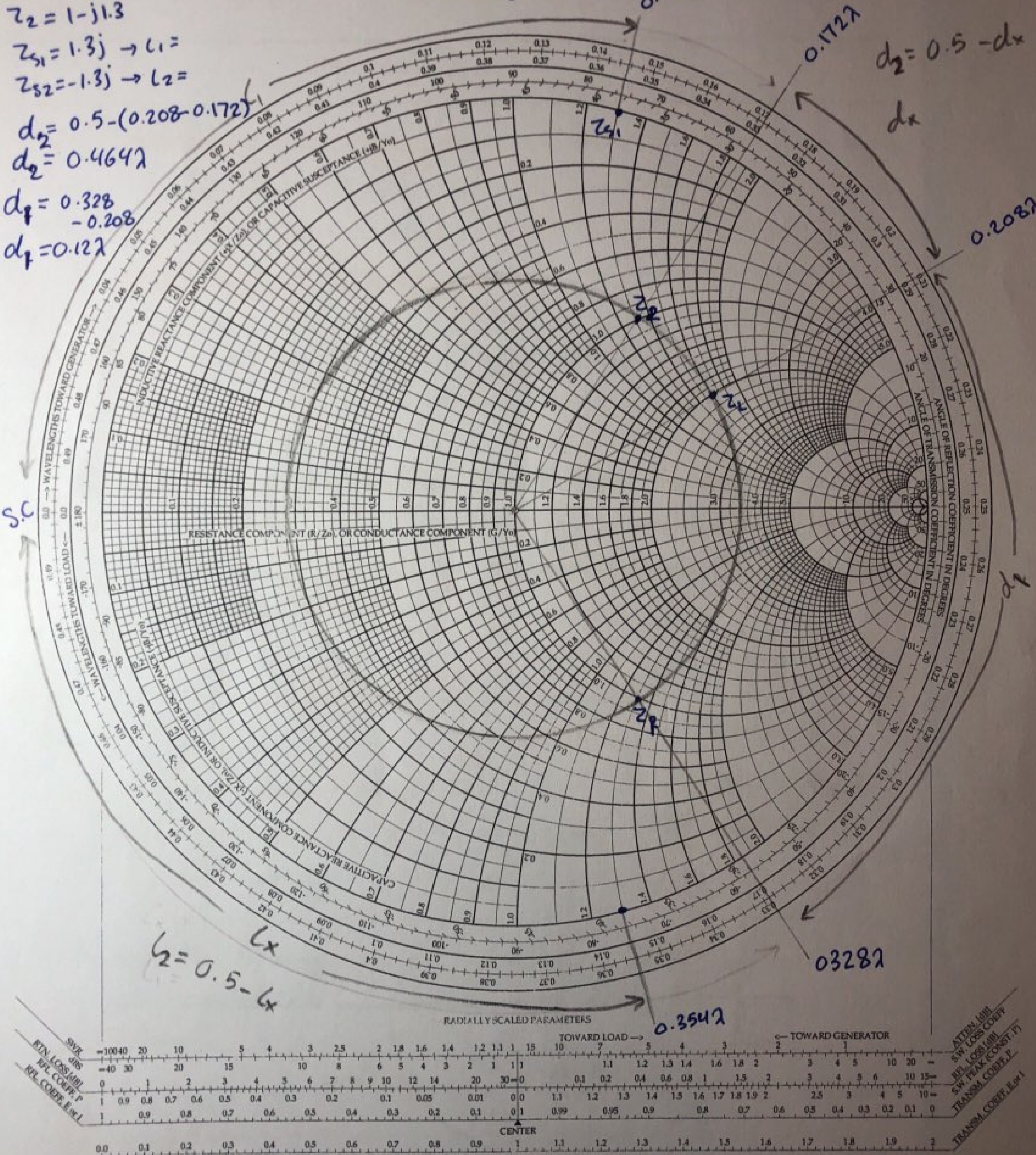
$d_2 = 0.464\lambda$

$d_1 = 0.328 - 0.208$

$d_1 = 0.12\lambda$

The Complete Smith Chart

Black Magic Design



$l_1 = 0.146\lambda$

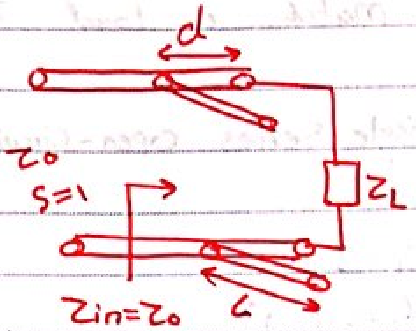
~~$l_2 = 0.5 - 0.354\lambda$~~

$l_2 = 0.354\lambda$

$\boxed{l_1 + l_2 = 0.5\lambda}$

No.

c) Parallel O.C Stub



$Z_L = 2 + j1.6$

$y_L = \frac{1}{Z_L} = 0.3 - j0.24\ \lambda$

$Z_2 = y_1 = 1 + j1.33$

$Z_1 = y_2 = 1 - j1.33$

$d_1 \equiv$ Distance between y_L & $y_1 \equiv 0.214\lambda$

$l_1 = 0.352\lambda$

$l_2 = 0.5 - 0.352\lambda = 0.148\lambda$ | if S.C

$l_1 = 0.102\lambda$

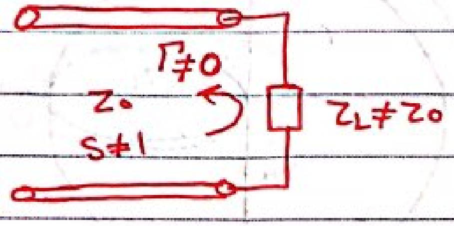
$l_2 = 0.398\lambda$

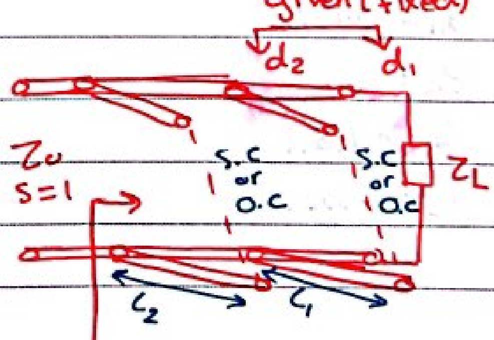
---

*Double Stub tuner:

The distance (d) from the load is fixed

Use for parallel Stub is more common





$Y_{in} = Y_0 + j0$

$Y_{in} = 1 + j0 \Rightarrow Z_{in} = 50\,\Omega = Z_0$

Ex: P109 Parallel O.C stub $\underline{\underline{Y}}$

$Z_L = 100 + j80$

$Z_0 = 50$

$Z_L' = 2 + j1.6 \rightarrow Y_L' = 0.3 - j0.245$

$Y_1 = Z_2 = 1 + 1.3j$

$Y_2 = Z_1 = 1 - 1.3j$

$Y_{s_1} = -1.3j$

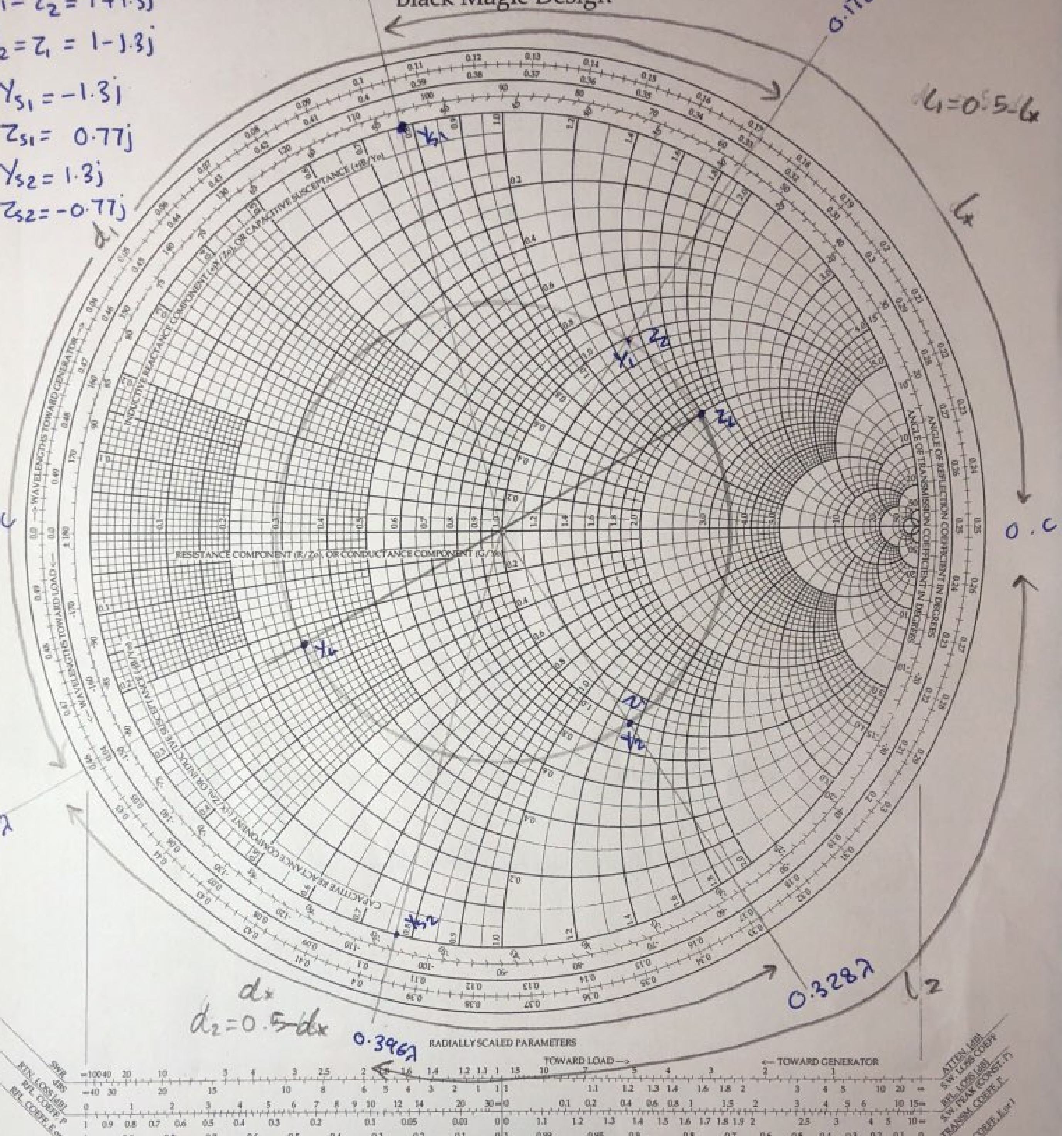
$Z_{s_1} = 0.77j$

$Y_{s_2} = 1.3j$

$Z_{s_2} = -0.77j$

The Complete Smith Chart

Black Magic Design



$d_1 = 0.172 + 0.5 - 0.46 = 0.212\lambda$ | $l_1 = 0.5 - (0.25 - 0.104) = 0.354\lambda$

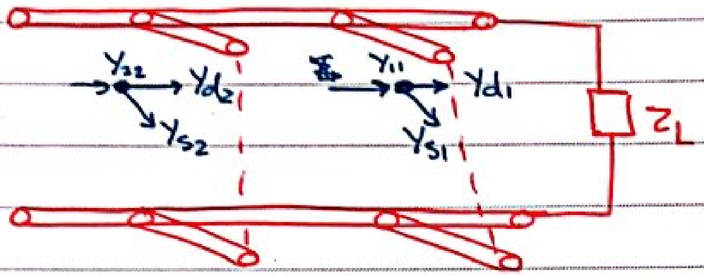
$d_2 = 0.5 - (0.46 - 0.328) = 0.368\lambda$ | $l_2 = 0.396 - 0.25 = 0.146\lambda$

<u>if S.C</u>

$l_1 = 0.104\lambda$ "d_1, d_2 must stay the same"

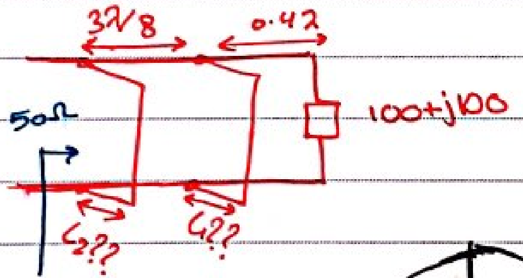
$l_2 = 0.396\lambda$

No.



$\boxed{Y_{22} = Yd_2 + Y_{S2}}$ $\boxed{Y_{11} = Yd_1 + Y_{S1}}$

Ex. Design a double Stub Shunt tuner to match $Z_L = 60 - j80\,\Omega$ to a $50\,\Omega$ T.L, The stubs are open-circuited and Separated by $\frac{5\lambda}{8}$ and the distance between the first Stub and the load is 0.4λ



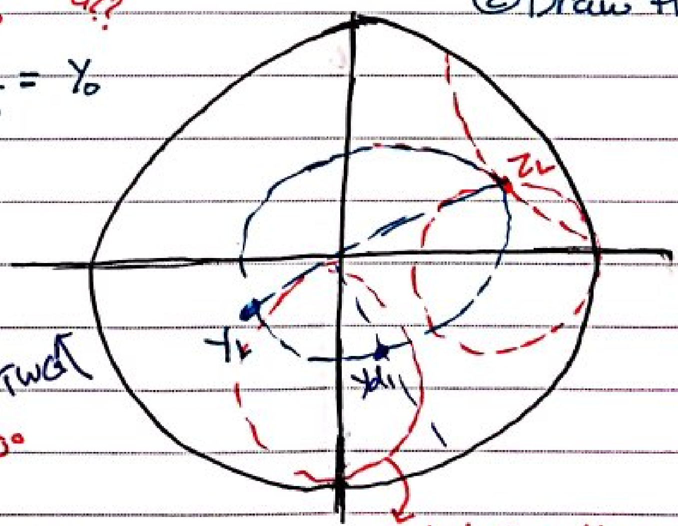
$Y_{in} = \frac{1}{Z_0} = Y_o$

① $Z_L = 2 + j2$

$Y_L = 0.25 - j0.25$

② Draw the SWR

③ Draw the Spacing Circle (rotated 1+jb circle).



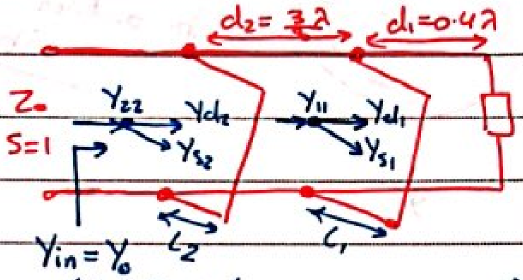
$\frac{3\lambda}{8} = \frac{3}{8}(720) = 270°$

④ move Y_L (0.4λ) TWG

$\lambda_{Y_L} = 0.458\lambda$
$+0.4\lambda$

$\lambda_{yd_1} = 0.858\lambda > 0.5\lambda$
$= 0.358\lambda$

Ex. match $Z_L = 100+j100$ to a 50Ω T.L using double Shunt Short-circuited Stub if $d_1 = 0.4\lambda$ & $d_2 = \frac{3\lambda}{8}$



(1+j0) circle

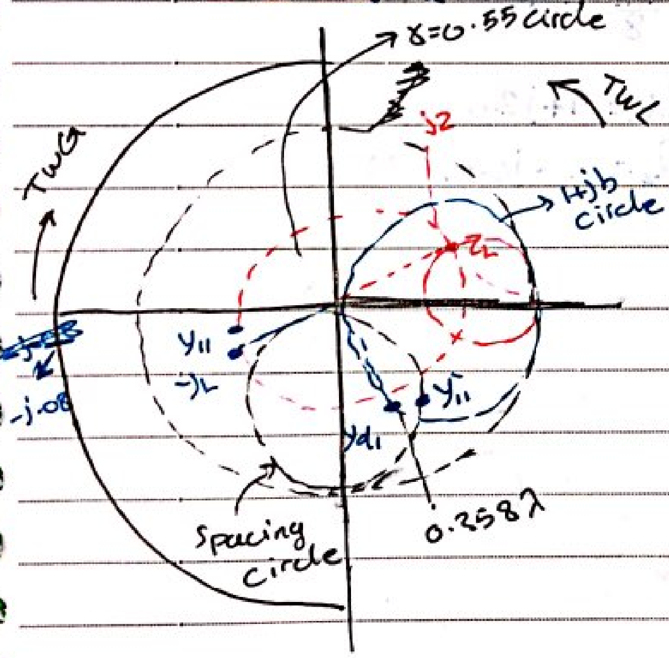
$Y_{11} = Y_{d_1} + Y_{s_1}$

$Y_{22} = Y_{d_2} + Y_{s_2}$

1) $Z_L = \frac{Z_L}{Z_o} = 2+j2$

2) Draw SWR circle & locate Y_L

$Y_L = \frac{1}{Z_L} = 0.25 - j0.25$



3) Draw the Spacing circle rotated (1+jb) circle

4) Move Y_L on TWG Scale a distance $d_1 = 0.4\lambda$ to find Y_{d_1}

$\Rightarrow Y_{d_1}$ located at $0.458 + 0.4 = 0.858\lambda$

$0.858\lambda > 0.5\lambda \Rightarrow 0.358\lambda$

~~$Y_{d_1} = 0.55 - 0.358$~~

$Y_{d_1} = 0.55 - j1.08$

5) Find the Intersection between the spacing circle & the r-circle of Y_{d_1}

You have two Intersections

$\Rightarrow Y_{11} = 0.55 - j0.11$

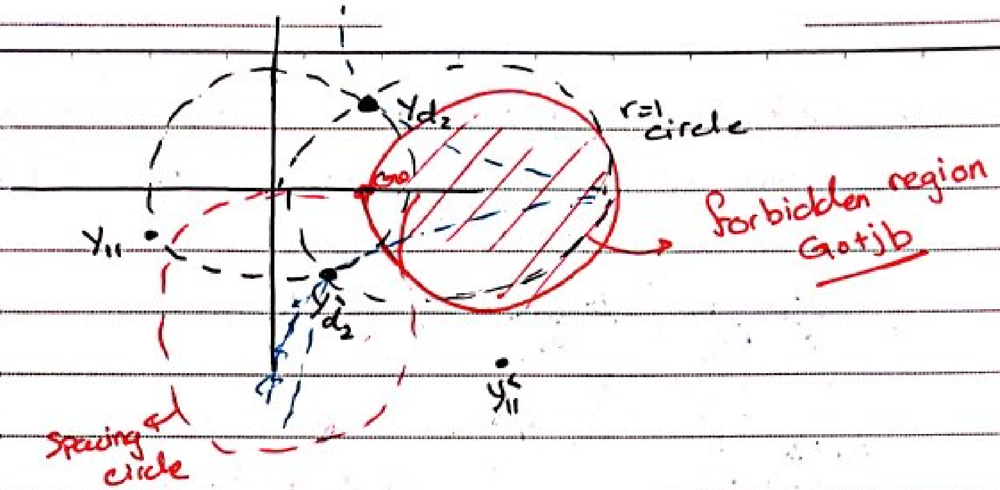
$Y_{11}' = 0.55 - j1.88$

6) $Y_{s_1} = Y_{11} - Y_{d_1} = +j0.97$

$Y_{s_1}' = Y_{11}' - Y_{d_1} = -j0.08$??

7) find $l_1 = 0.373\lambda$

$l_1' = 0.225\lambda$??



8) Rotate Y_{11} or Y'_{11} & find the Intersection with $(1+jb)$ circle $\frac{3\lambda}{8} = 270°$

$Y_{d_2} = 1+j2.6$

$Y'_{d_2} = 1-j0.61$??

$Y_{s_2} = Y_{22} - Y_{d_2}$
$= 1+j0 - (1-j0.61)$
$= +j0.61$

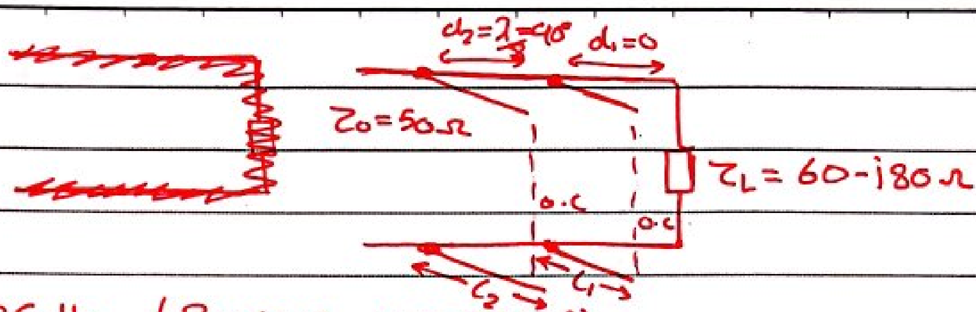
$Y'_{s_2} = -j2.6$??

9) find $l_2 = 0.337\lambda$
$l'_2 = 0.065\lambda$??

$l_1 = 0.373\lambda$

$l_2 = 0.337\lambda$

Ex:



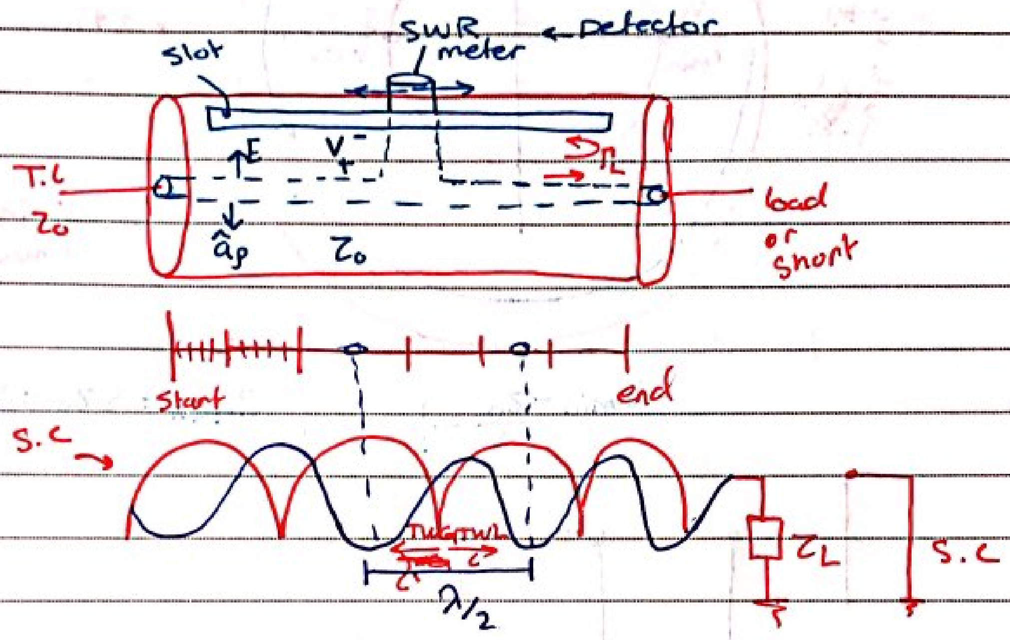
f = 2 GHz ($R_L = 60\,\Omega$, $C = 0.995\,pF$)

Ans.

$l_1 = 0.146\lambda$	$l_1' = 0.482\lambda$
$l_2 = 0.204\lambda$	$l_2' = 0.35\lambda$

* Impedance Measurement using Slotted line

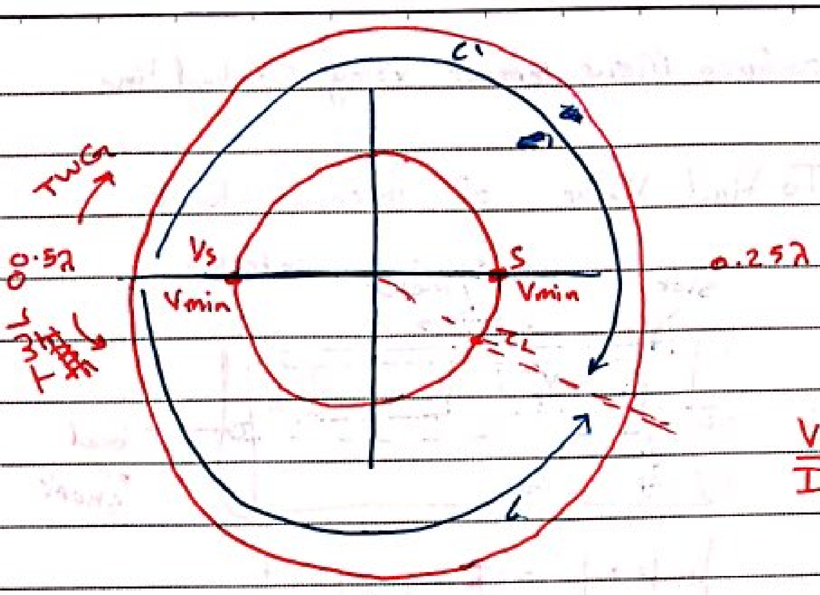
- To find Value of unknown load.



lossless & air filled Slotted line

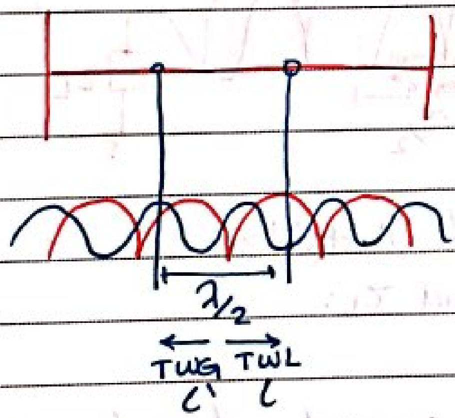
* Procedure to find Z_L:

1) Connect the unknown load and read the SWR from the detector
2) Notice the minimum values of the voltage with the load connected.
3) Replace the load with a short-circuit and notice the voltage minimum and select one of them as reference
4) measure the length (l) between the ref. and the first minimum of the Z_L.



$\frac{V_{min}}{I_{max}} = Z_{min}$

$Z_{min} = \frac{Z_0}{S} = \frac{1}{S}$, $Z_{max} = SZ_0 = S$



*Ex : Unknown load connected to a slotted line air ~~line~~ line has $\boxed{S=2}$ and minima are found at 11cm, 19cm, ... on the Scale.
when the load is replaced with a S.C, the minima are located at 16 cm, 24cm.
If $Z_o = 50\,\Omega$, find λ, f and Z_L

1) $\lambda/2 = 19 - 11 = 8\text{cm}$ → $\boxed{\lambda = 16\text{cm}}$

$\lambda/2 = 24 - 16 = 8\text{cm}$

2) $f = \frac{u}{\lambda} = \frac{c}{\lambda} = \frac{3\times10^8}{16\times10^{-2}} = 1.875\ \text{GHz}$

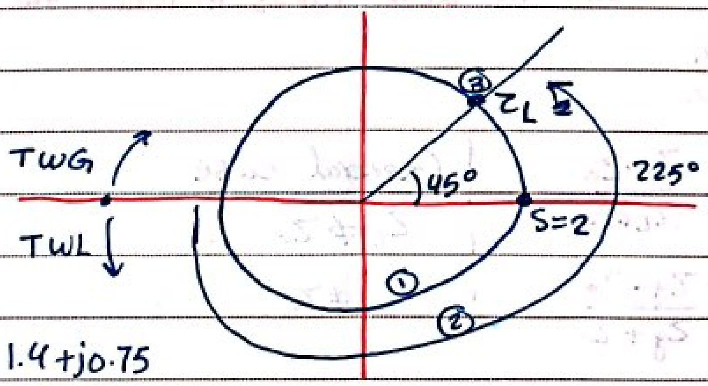
3) Z_L

$16-11 = 5\text{cm} = l$ (TWL)

$19-16 = 3\text{cm} = l'$ (TWG)

$l = \frac{5}{\cancel{16\lambda}}\lambda = \frac{5}{16}\lambda = 0.3125\lambda$

$= 0.3125(720°) = 225°$



① Draw S-circle
② move l TWL
③ Z_L intersection

$Z_L = 1.4 + j0.75$

$Z_L = 70 + j37.5$

Ex: Slotted line, air

Ex 11.6

NB $S = 2$, min 11cm, 19cm (load)

min 16cm, 24cm (S.C)

$Z_0 = 50\,\Omega$

λ, f, Z_L ??

① $\lambda f = u$

$\lambda/2 = 19 - 11 = 8\text{cm}$

$\lambda/2 = 24 - 16 = 8\text{cm}$

$\boxed{\lambda = 16\text{cm}}$

② $f = \frac{u}{\lambda}$

$= \frac{3 \times 10^8}{16 \times 10^{-2}}$

$\boxed{f = 1.875\text{GHz}}$

③ Z_L

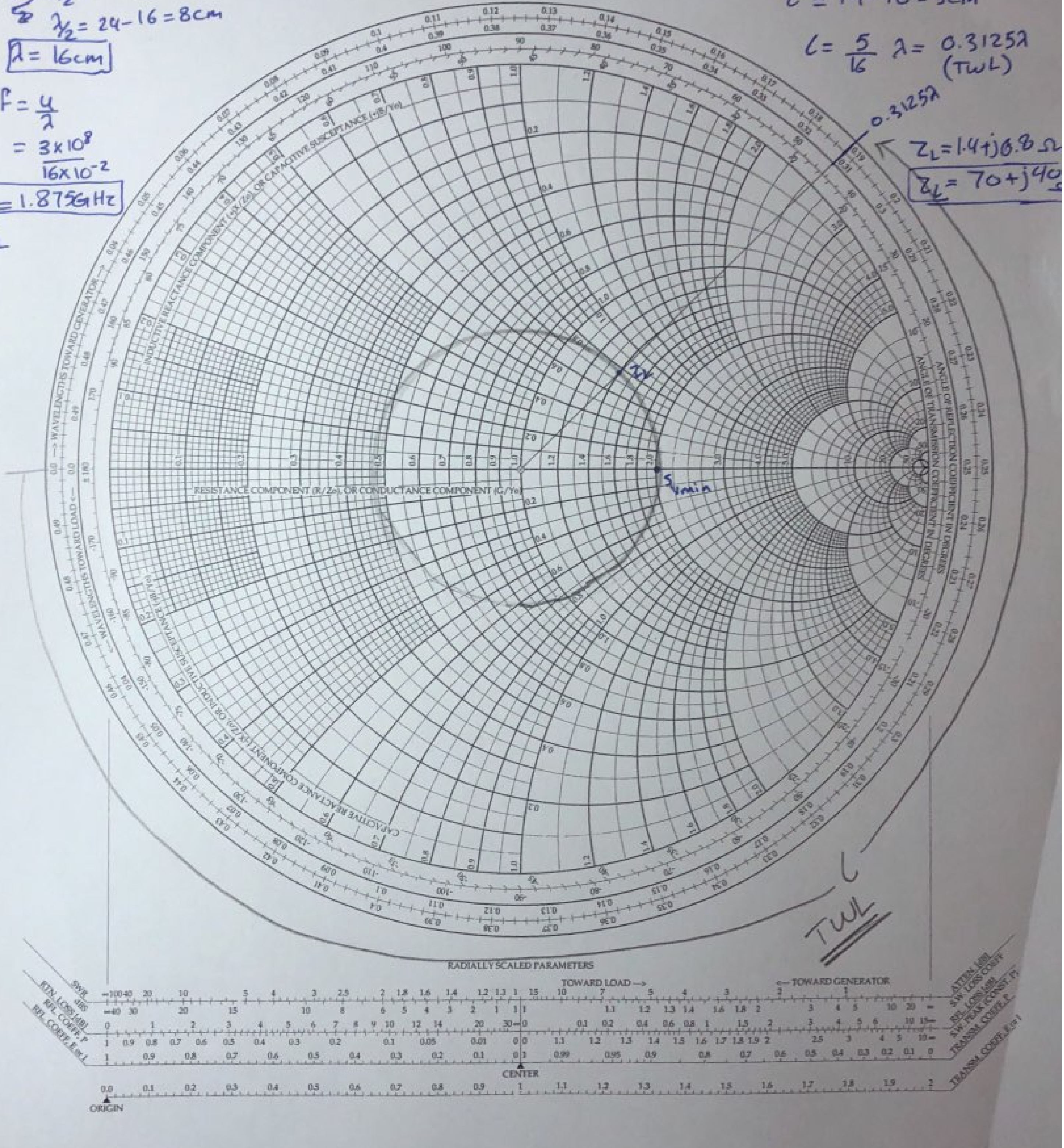
$l = 16 - 11 = 5\text{cm}$

$l' = 19 - 16 = 3\text{cm}$

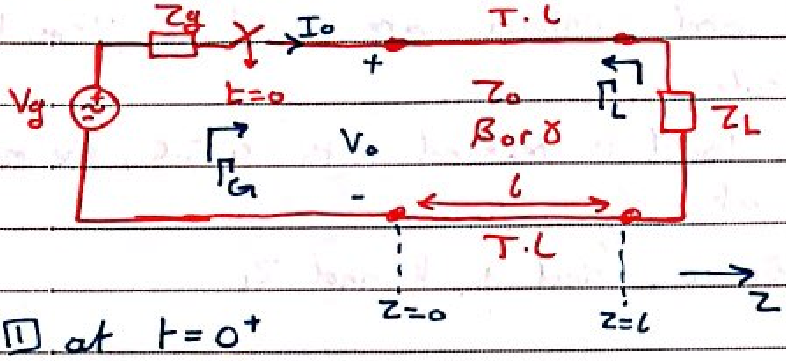
$l = \frac{5}{16}\lambda = 0.3125\lambda$ (TWL)

The Complete Smith Chart

Black Magic Design



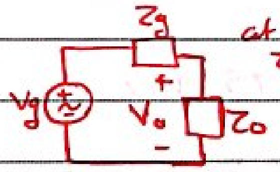
* Transients on T.L



* u given
or L,C given
or μ,ε given

[1] at $t = 0^+$

$$I(z,t) = I(\overset{\text{space}}{0}, \overset{\text{time}}{0^+}) = I_o$$



at $t=0^+$
$z=0$

$$I_o = \frac{V_g}{Z_g + Z_o}$$

[2] at the end of the line

$$I(l, \infty) = I_\infty$$

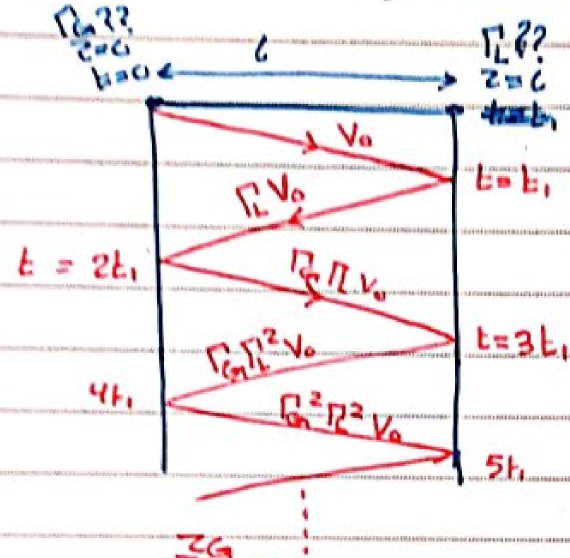
t_1 = transient time

(The time the wave takes to reach the load).

$$t_1 = \frac{l}{u}$$

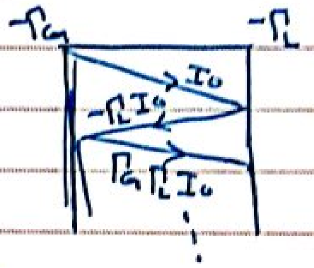
$\Gamma_L = \dfrac{Z_L - Z_o}{Z_L + Z_o}$	General case
	$Z_L \neq Z_o$
$\Gamma_G = \dfrac{Z_g - Z_o}{Z_g + Z_o}$	$Z_G \neq Z_o$

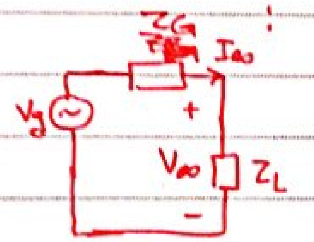
Lattice diagram
or Bounce diagram
or space-time diagram



Voltage lattice diagram
or current lattice diagram

for current

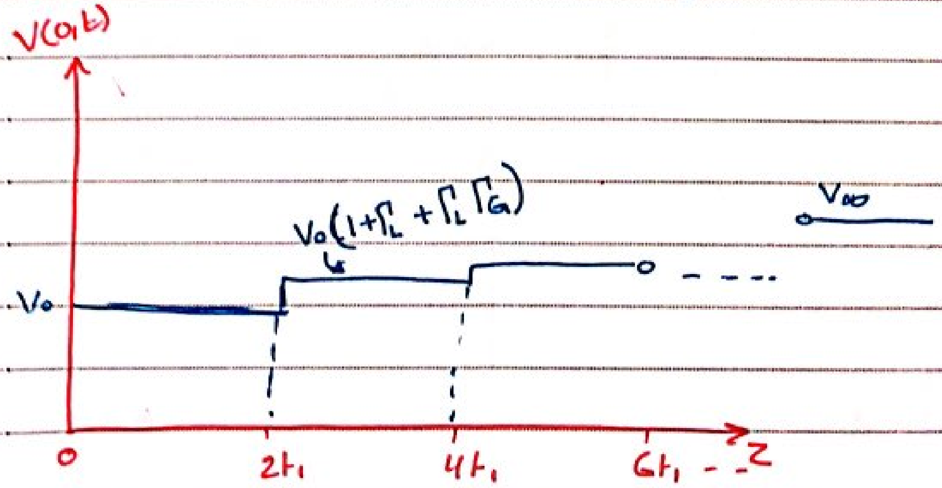




$$V(l, \infty) = V_\infty$$

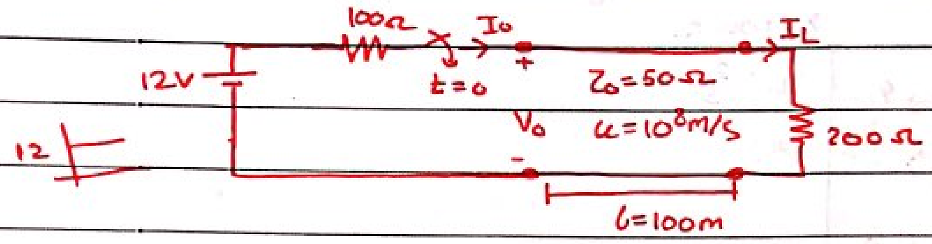
$$I_\infty = \frac{V_g}{Z_G + Z_L}$$

$$V_\infty = \frac{V_g \, Z_L}{Z_G + Z_L}$$



V(l,t)
I(l,t)
I(0,t)

<u>Ex.</u> For ~~the~~ T.L shown



a) Sketch the voltage at the load ~~and~~ and generator ends for $0 < t < 6\mu s$

b) Sketch the current " " " " " " for $0 < t < 6\mu s$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{3}{5}$$

$$\Gamma_G = \frac{Z_G - Z_0}{Z_G + Z_0} = \frac{1}{3}$$

$$t_1 = \frac{l}{u} = \frac{100}{10^8} = 1\mu s$$

$$V_0 = V_g \frac{Z_0}{Z_0 + Z_g} = 4V$$

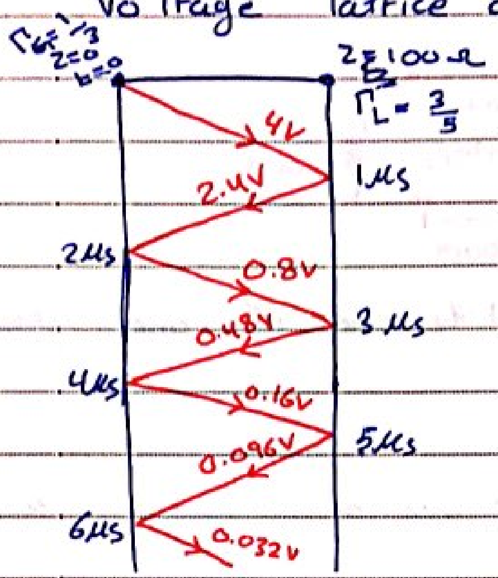
$$V_\infty = V_g \frac{Z_L}{Z_G + Z_L} = 8V$$

$$I_0 = \frac{V_0}{Z_0} = 80mA$$

$$I_\infty = \frac{V_\infty}{Z_L} = 40mA$$

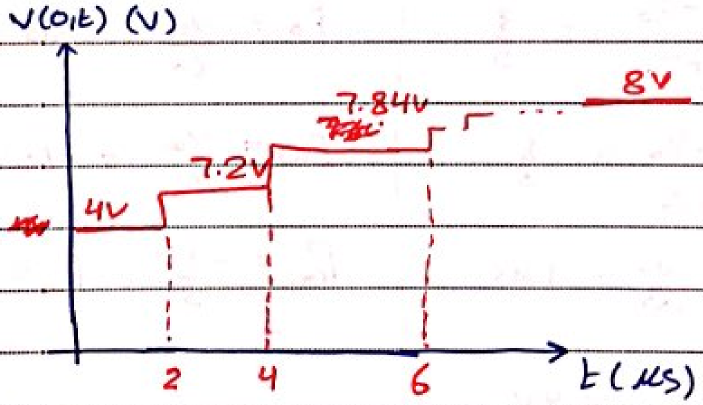
$$\Gamma_{Gi} = -\frac{1}{3}, \quad \Gamma_{Li} = -\frac{3}{5}$$

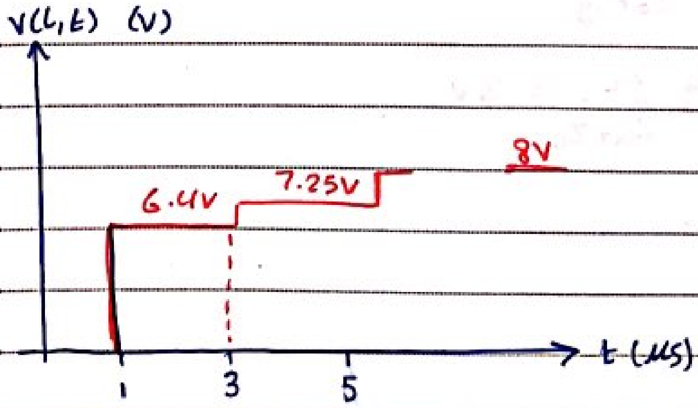
Voltage lattice diagram

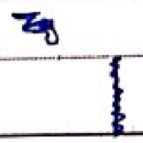


space

a) $V(0,t)$ → generator end







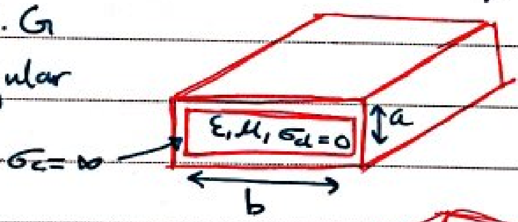
Ch.12 wave guides:

It is a one conductor T.L

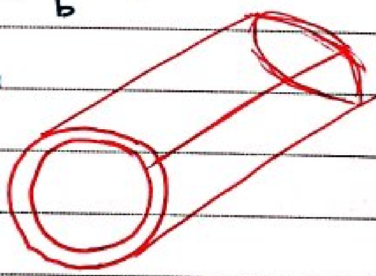
2- Conductor T.L usually not valid for $f > 3\,GHz$
(begining of microwave bands 3-300 GHz)
→ Valid for DC ($f=0$)

* W-G (wave guide) can't be used for DC
each W-G has a starting freq. based on it's dimension called the dominant freq.

R.W.G
Rectangular



lossless R.W.G

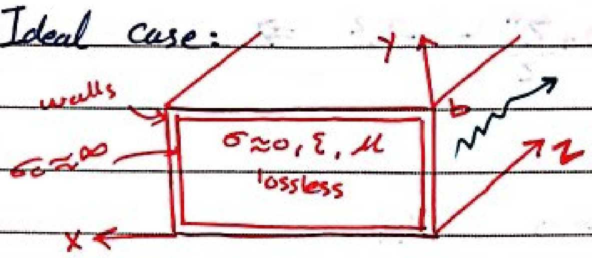


C.W.G

many Shapes of W.Gs

*Rectangular waveguide (RWG)

$ Ideal case:



~~[illegible]~~ works like a H.P.F

from Helmholtz's eq.

$$\nabla^2 \vec{E_s} + k^2 \vec{E_s} = 0$$

& $$\nabla^2 \vec{H_s} + k^2 \vec{H_s} = 0$$

$$\vec{E_s} = (E_{xs}, E_{ys}, E_{zs})$$
$$\vec{H_s} = (H_{xs}, H_{ys}, H_{zs})$$

Taking the wave propagates in the +ve z-direction.

$$\frac{d^2 E_{zs}}{dx^2} + \frac{d^2 E_{zs}}{dy^2} + \frac{d^2 E_{zs}}{dz^2} + k^2 E_{zs} = 0 \quad — ①$$

using separation of variables

let $E_{zs} = X(x)\, Y(y)\, Z(z)$ — ②

Sub ② in ①

$$X'' Y Z + X Y'' Z + X Y Z'' + k^2 X Y Z = 0$$

divide by XYZ

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2 \Rightarrow$$

$$\frac{X''}{X} + k_x^2 = 0 \quad -(a)$$

$$\frac{Y''}{Y} + k_y^2 = 0 \quad -(b)$$

$$\frac{Z''}{Z} - \gamma^2 = 0 \quad -(c)$$

$X'' + k_x^2 X = 0$, let $m = \frac{d}{dx}$

$(m^2 + k_x^2) X = 0 \Rightarrow m = \pm j k_x$

Imaginary

Since the roots are Imaginary

$X(x) = C_1 \cos k_x x + C_2 \sin k_x x$

$Y'' + k_y^2 Y = 0$

Sol.

$Y(y) = C_3 \cos(k_y y) + C_4 \sin(k_y y)$

$Z'' - \gamma^2 Z = 0$

$m^2 - \gamma^2 = 0 \Rightarrow m = \pm\gamma$ Real

Sol.

$Z(z) = C_5 e^{-\gamma z} + C_6 e^{\gamma z}$

Since we assume $\hat{a}_k = +\hat{a}_z$

$\boxed{C_6 = 0}$ must be zero

to avoid having $E_{zs} \Rightarrow \infty$ as $z \to \infty$

$$E_{zs} = (C_1 \cos k_x x + C_2 \sin k_x x)(C_3 \cos k_y y + C_4 \sin k_y y)\, C_5 e^{-\gamma z}$$

$$\boxed{E_{zs} = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin(k_y y))\, e^{-\gamma z}}$$

$$E_{zs} = X(x)\, Y(y)\, Z(z)$$

by same method (solve $\nabla^2 \vec{H}_s + k^2 \vec{H}_s = 0$)

$$\vec{H}_{zs} = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin B_y y)\, e^{-\gamma z}$$

To find other components : E_{xs}, E_{ys}, ~~E_{zs}~~ H_{xs}, H_{ys}

Use Maxwell's eq.

$$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s \quad \& \quad \nabla \times \vec{H}_s = j\omega\varepsilon \vec{E}_s$$

$$\begin{vmatrix} a_x & a_y & a_z \\ d/dx & d/dy & d/dz \\ E_{xs} & E_{ys} & E_{zs} \end{vmatrix} = -j\omega\mu \begin{bmatrix} H_{xs} \\ H_{ys} \\ H_{zs} \end{bmatrix}$$

$$\frac{dE_{zs}}{dy} - \frac{dE_{ys}}{dz} = -j\omega\mu H_{xs} \quad \text{--- (3.a)}$$

$$\frac{dE_{xs}}{dz} - \frac{dE_{zs}}{dx} = -j\omega\mu H_{ys} \quad \text{--- (3.b)}$$

$$\frac{dE_{ys}}{dx} - \frac{dE_{xs}}{dy} = -j\omega\mu H_{zs} \quad \text{--- (3.c)}$$

(3.d)

(3.e)

(3.f)

Solve $\nabla \times H_s = j\omega\varepsilon E_s$

$$\frac{dH_{zs}}{dy} - \frac{dH_{ys}}{dz} = j\omega\varepsilon E_{xs} \quad (3.d)$$

$$\frac{dH_{xs}}{dz} - \frac{dH_{zs}}{dx} = j\omega\varepsilon E_{ys} \quad (3.e)$$

$$\frac{dH_{ys}}{dx} - \frac{dH_{xs}}{dy} = j\omega\varepsilon E_{zs} \quad (3.f)$$

Solving eq. (3.d) with eq. (3.b)
To find E_{xs}

$$j\omega\varepsilon E_{xs} = \frac{dH_{zs}}{dy} + \frac{1}{j\omega\mu}\left(\frac{d^2E_{xs}}{dz^2} - \frac{d^2E_{zs}}{dx\,dz}\right)$$

$$E_{xs} \propto e^{-\gamma z} = E_{xs} = (\;)(\;)e^{-\gamma z}$$

$$\frac{dE_{xs}}{dz} = -\gamma E_{zs}$$

$$\frac{d^2E_{zs}}{d^2z} = \gamma^2 E_{zs}$$

$$\frac{dE_{zs}}{dz^2} = -\gamma E_{zs}$$

$$j\omega\varepsilon E_{xs} = \frac{dH_{zs}}{dy} + \frac{1}{j\omega\mu}\left(\gamma^2 E_{xs} + \gamma\frac{dE_{zs}}{dx}\right)$$

$$E_{xs}\left(j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right) = \frac{dH_{zs}}{dy} + \frac{\gamma}{j\omega\mu}\frac{dE_{zs}}{dx}$$

$$-\frac{1}{j\omega\mu}(\omega^2\mu\varepsilon + \gamma^2) = \dots\dots$$

let $\boxed{h^2 = \omega^2\mu\varepsilon + \gamma^2}$

$$\frac{-\gamma^2}{j\omega\mu} E_{xs} = \frac{dH_{zs}}{dy} + \frac{\gamma}{j\omega\mu}\frac{dE_{zs}}{dx}$$

$$\boxed{E_{xs} = \frac{-j\omega\mu}{h^2}\frac{dH_{zs}}{dy} - \frac{\gamma}{h^2}\frac{dE_{zs}}{dx}}$$

Using E_{zs} & H_{zs}

$$\cancel{E_{xs}} \quad E_{xs} = -\frac{\gamma}{h^2}(-A_1 k_x \sin k_x x + A_2 k_x \cos k_x y)(y)\, e^{-\gamma z}$$

$$-\frac{j\omega\mu}{h^2}(-A_3 k_y \sin k_y y + A_4 k_y \cos k_x y)(x)\, e^{-\gamma z}$$

$$E_{ys} = -\frac{\gamma}{h^2} \cdot \frac{dE_{zs}}{dy} + \frac{j\omega\mu}{h^2} \frac{dH_{zs}}{dx}$$

$$H_{xs} = \frac{j\omega\varepsilon}{h^2} \cdot \frac{dE_{zs}}{dy} - \frac{\gamma}{h^2} \frac{dH_{zs}}{dx}$$

$$H_{ys} = -\frac{j\omega\varepsilon}{h^2} \cdot \frac{dE_{zs}}{dx} - \frac{\gamma}{h^2} \frac{dH_{zs}}{dy}$$

** $h^2 = \gamma^2 + k^2 = k_x^2 + k_y^2 \Rightarrow k^2 = k_x^2 + k_y^2 - \gamma^2$

There are four cases

① $E_{zs} = 0$, $H_{zs} = 0 \Rightarrow$ TEM

All field components equal to zero, so R.W.G can't support TEM

② $E_{zs} = 0$, $H_{zs} \neq 0 \Rightarrow$ TE mode

$E_x, E_y, H_x, H_y, H_z \neq 0$

③ $E_{zs} \neq 0$, $H_{zs} = 0 \Rightarrow$ TM mode

$E_x, E_y, E_z, H_x, H_y \neq 0$

④ $E_{zs} \neq 0$, $H_{zs} \neq 0 \Rightarrow$ HE mode or EH mode

(Hybrid mode) $\longrightarrow$ optical fiber

6- components exist.

** For TEM mode $(h=0)$

$h^2 = 0 \Rightarrow \gamma^2 + k^2 = 0$

$\gamma^2 = -k^2$

$\gamma = \pm jk = \pm j\beta \rightarrow \boxed{\alpha = 0}$

* TM modes : ($E_{zs} \neq 0$, $H_{zs} = 0$) ($\vec{H} \perp \hat{a}_k$)

find $E_{zs}, E_{xs}, E_{ys}, H_{xs}, H_{ys}$?

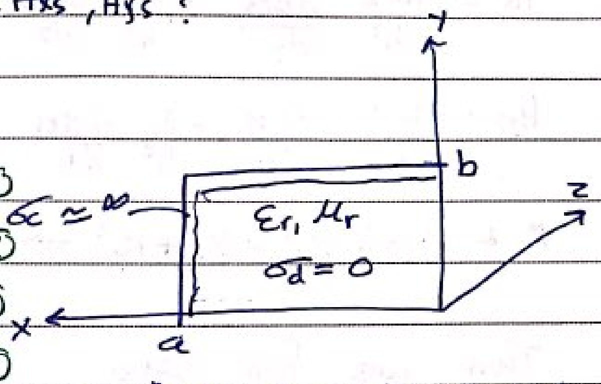
B.C.S are :

$E_{zs} = 0$ at $x = 0$ — ①

$E_{zs} = 0$ at $x = a$ — ②

$E_{zs} = 0$ at $y = 0$ — ③

$E_{zs} = 0$ at $y = b$ — ④



→ Because walls perfect conductors

* Applying ① :

$E_{zs} = A_1 \cos k_x(0) = 0 \Rightarrow \boxed{A_1 = 0}$

* from ③ :

$E_{zs} = A_3 \Rightarrow \boxed{A_3 = 0}$

* from ② :

$0 = A_2 \sin k_x a$, $A_1 \neq 0$

trivial solution

if $A_2 = 0$

$\Rightarrow k_x a = m\pi$, $m = 1, 2, 3, \ldots$

$k_x = \frac{m\pi}{a}$ at $x = a$

* from ④ :

$0 = A_4 \sin k_y b$, $A_4 \neq 0$

$\Rightarrow \sin(k_y b) = 0$

$k_y b = n\pi$, $\cancel{k_y = \frac{n\pi}{b}}$ $k_y = \frac{n\pi}{b}$, $n = 1, 2, 3, \ldots$

$$\boxed{E_{zs} = E_o \sin\left(\left(\frac{m\pi}{a}\right)x\right) \sin\left(\left(\frac{n\pi}{b}\right) y\right) e^{-\gamma z}}$$

$E_o = A_z A_x$

$m \neq 0, n \neq 0$ for TM mode

$\rightarrow k = 0$ if $m = 0, n = 0 \rightarrow E_{zs} = 0$ not TM

$$E_{xs} = \frac{-\gamma E_o}{h^2}\left(\frac{m\pi}{a}\right) \cos\left(\left(\frac{m\pi}{a}\right)x\right) \sin\left(\left(\frac{n\pi}{a}\right)y\right) e^{-\gamma z}$$

$$E_{ys} = \frac{-\gamma E_o}{h^2}\left(\frac{n\pi}{b}\right) \sin\left(\left(\frac{m\pi}{a}\right)x\right) \cos\left(\left(\frac{n\pi}{b}\right)y\right) e^{-\gamma z}$$

$$H_{xs} = \frac{j\omega\varepsilon E_o}{h^2}\left(\frac{n\pi}{b}\right) \sin\left(\left(\frac{m\pi}{a}\right)x\right) \cos\left(\left(\frac{n\pi}{b}\right)y\right) e^{-\gamma z}$$

$$H_{ys} = \frac{-j\omega\varepsilon E_o}{h^2}\left(\frac{m\pi}{a}\right) \cos\left(\left(\frac{m\pi}{a}\right)x\right) \sin\left(\left(\frac{n\pi}{b}\right)y\right) e^{-\gamma z}$$

$H_{zs} = 0$

TM_{mn} mode ; $m, n \neq 0$

TM_{11} : dominant mode

$$\gamma^2 = h^2 - k^2$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}$$

* 3 cases:

1) cut-off $(k^2 = h^2)$

$\gamma = 0 = \alpha + j\beta \rightarrow \alpha = 0 \ , \beta = 0$

$k^2 = \omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = 2\pi f_c$

$\Rightarrow f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$ Hz

$\Rightarrow f_{c_{11}} = \frac{\pi}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} \Rightarrow f_c = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$f_c = \frac{u'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$; $u' = \frac{1}{\sqrt{\mu\varepsilon}}$ Same as TEM

2) Evanescent mode

$k^2 < h^2 \longrightarrow \omega^2 \mu \varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$\gamma = \alpha + j\beta$ = Two real Solution

$\beta = 0 \rightarrow$ no propagation

$f < f_c$

3) Propagation mode

$k^2 > h^2 \longrightarrow \omega^2 \mu \varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

γ is Imaginary

$\gamma = \alpha + j\beta = 0$

$\beta = \sqrt{k^2 - h^2}$

$= \sqrt{\omega^2 \mu \varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \equiv$ phase constant rad/m

$$\beta = \sqrt{\omega^2\mu\varepsilon - (2\pi f_c)^2\mu\varepsilon * (\omega^2/\omega^2)}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega^2\mu\varepsilon\frac{f_c^2}{f^2}} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2} \Rightarrow \beta' = \omega\sqrt{\mu\varepsilon} \text{ as in TEM}$$

28/11

$f_c \equiv$ cutoff frequency $(\alpha = 0, \beta = 0 \Rightarrow \gamma = 0)$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad [Hz] \quad , u' = \frac{1}{\sqrt{\mu\varepsilon}}$$

for $f < f_c \Rightarrow$ Evanescent mode

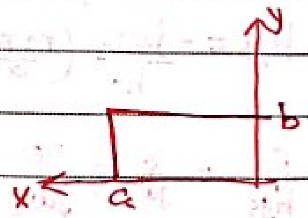
$f > f_c \Rightarrow$ Propagation

$$\lambda_c = \frac{u'}{f_c} \equiv \text{cutoff wavelength}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \quad [m]$$

For $TM_{11} \rightarrow$ lowest mode (dominant mode for TM modes)

$$f_{c_{TM_{11}}} = \frac{u'}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$



$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$h_{TEM} = 0 \qquad \begin{matrix} a \to \infty \\ b \to \infty \end{matrix}$$

$$\beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2} \equiv \text{phase constant (rad/m)}$$

$\beta' = \omega\sqrt{\mu\varepsilon}$ like TEM

$\rightarrow \gamma = j\beta = j\beta' \sqrt{1-\left(\frac{f_c}{f}\right)^2}$ (for propagation)

$\rightarrow \gamma = \alpha = \beta' \sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$ (for evanescent)

* Phase Velocity:

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\beta' \sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\boxed{v_p = \frac{u'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}} \quad \text{or} \quad v_p' = \frac{u'}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

* Intrinsic Impedance : η_{TM} (Ω)

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

$\hat{a}_E$ $\hat{a}_k$ $\hat{a}_H$

$$E_{xs} = \frac{-\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_{ys} = \frac{-j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$\frac{E_{xs}}{H_{ys}} = \frac{\gamma}{j\omega\varepsilon} = \frac{j\beta}{j\omega\varepsilon} = \frac{\beta}{\omega\varepsilon}$$

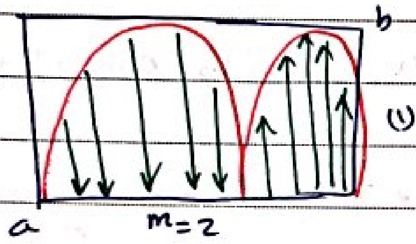
$$\boxed{\eta_{TM} = \frac{\beta}{\omega\varepsilon}}$$

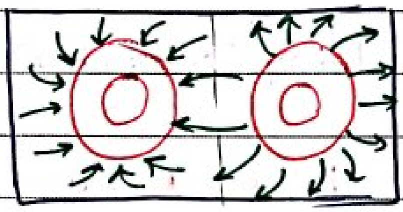
$$\eta_{TM} = \frac{\beta'}{\omega\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} \;;$$

$$\boxed{\eta_{TM} = \eta' \sqrt{1-\left(\frac{f_c}{f}\right)^2}} \quad , \quad \eta' \equiv TEM = \sqrt{\mu/\varepsilon}$$

m = no of half cycles in x-dir.
$n \equiv$ no. of half cycles in y-dir. (both are Integers)
variation

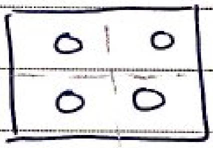
For TM_{21} mode $\Rightarrow m=2$, $n=1$





TM_{21}

Front



TM_{22}

* Transverse Electric (TE) mode:

$E_z = 0$

$E_x, E_y, H_x, H_y, H_z \neq 0$

B.C.S

$E_{xs} = 0$ at $y=0$

$E_{xs} = 0$ at $y=b$

$E_{ys} = 0$ at $x=0$

$E_{ys} = 0$ at $x=a$

$$E_{xs} = \frac{-\gamma}{h^2} \cdot \frac{dE_{zs}}{dx} - \frac{j\omega\mu}{h^2} \cdot \frac{dH_{zs}}{dy}$$

$$E_{ys} = \frac{-\gamma}{h^2} \cdot \frac{dE_{zs}}{dy} - \frac{j\omega\mu}{h^2} \cdot \frac{dH_{zs}}{dx}$$

$\Rightarrow \dfrac{\partial H_{zs}}{\partial y} = 0$ at $y=0, y=b$

$\Rightarrow \dfrac{\partial H_{zs}}{\partial x} = 0$ at $x=0, x=a$

* apply the B.C.S on H_{zs}

$$H_{zs} = \left[\left((B_1 \cos(k_x x) + B_2 \sin(k_x x)\right) + \left(B_3 \cos(k_y y) + B_4 \sin(k_y y)\right)\right] e^{-\gamma z}$$

$B_2 = 0$, $B_4 = 0$ $\qquad k_x = \dfrac{m\pi}{a}$, $k_y = \dfrac{n\pi}{b}$.

$B_1 B_3 = H_0$

$$\boxed{H_{zs} = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}}$$

$m = 0,1,2\ldots$, but m & $n \neq 0$
$n = 0,1,2\ldots$ at the same time

* Other field components:

$$E_{xs} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_{ys} = \frac{-j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_{xs} = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_{ys} = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$\gamma = j\beta$ (for Propagation)

$m=0$, $n\neq 0$ $\Rightarrow (H_z, E_x, H_y)$

$n=0$, $m\neq 0$ $\Rightarrow (H_z, H_x, E_y)$

* The lowest mode TE is either TE_{10} or TE_{01}, but the Standard is to have a W.G with $a>b$

$\frac{1}{a} < \frac{1}{b} \longrightarrow m=1, n=0 \longrightarrow TE_{10}$

TE_{10} is the lowest mode for TE

$f_{c_{TE10}} < f_{c_{TM11}}$

$TE_{10} \equiv$ dominant mode for all W.G

$TE_{10} \rightarrow TE_{01} \rightarrow TM_{11} \rightarrow TE_{11}$

* Continue TE mode:

TE_{10} is the dominant mode if $(a>b)$

$$f_{c_{TE10}} = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{u'}{2a} \quad , \quad u' = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda_{c_{TE10}} = 2a = \frac{u'}{f_{c\,TE10}}$$

β, u_p, λ ~~as~~ same as TM mode

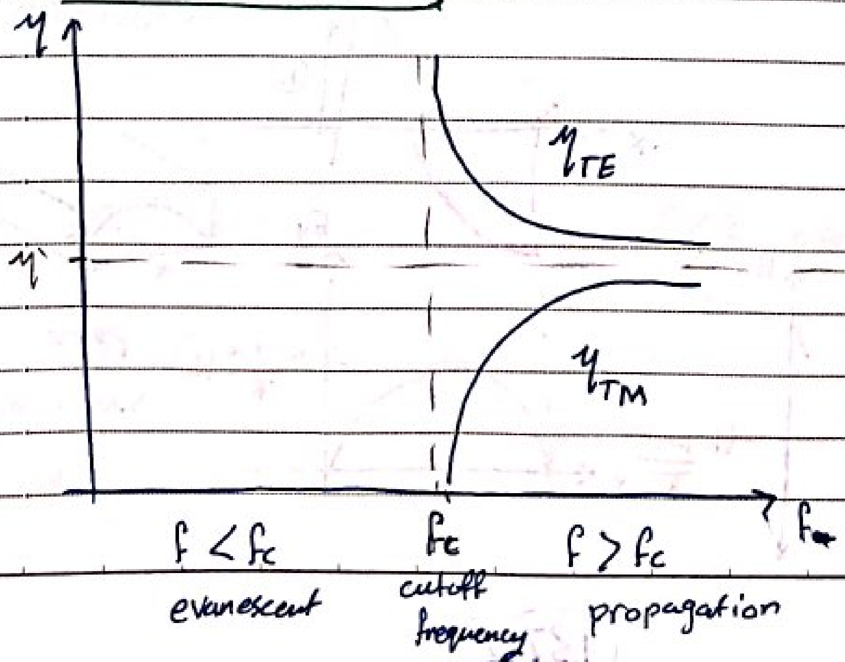
$$\eta_{TE} \neq \eta_{TM}$$

$$\eta = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

$$\boxed{\eta_{TE} = \frac{\eta'}{\sqrt{1-(f_c/f)^2}}} \quad , \quad \boxed{\eta_{TM} = \eta'\sqrt{1-(f_c/f)^2}}$$

$\eta' = \sqrt{\mu/\varepsilon}$ as in TEM

$$\boxed{\eta_{TE} * \eta_{TM} = \eta'^2}$$



* The field patterns for TE_{10} $(E_z=0)$

E_x, E_y, H_x, H_y, H_z , $m=1$, $n=0$

$$H_{zs} = H_0 \cos\left(\frac{\pi x}{a}\right) e^{-j\beta z} \quad , \gamma = j\beta$$

In time domain (Introduce ~~practice~~ $e^{j\omega t}$)

$$H_z = Re\{H_{zs}\, e^{j\omega t}\}$$

$$= H_0 \cos\left(\frac{\pi x}{a}\right) \cos(\omega t - \beta z)$$

+ve - direction

Other fields:

$$E_y = \frac{\omega \mu \varepsilon}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z)$$

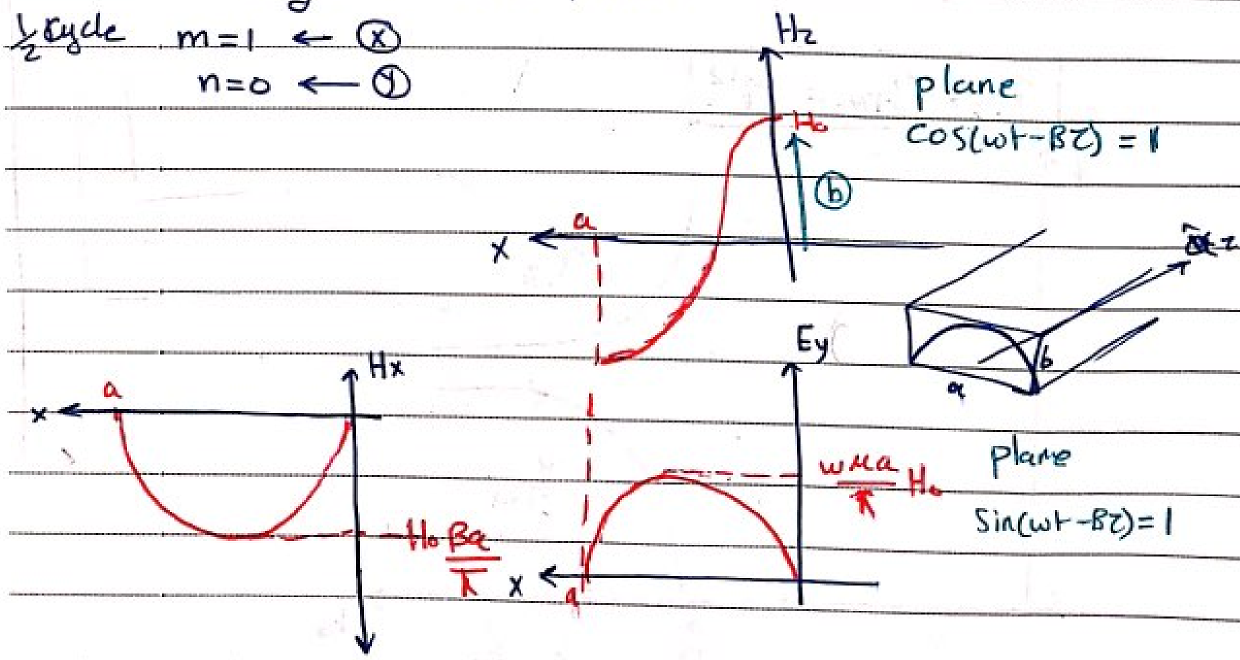
$$H_x = -\frac{\beta a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z)$$

$E_z, H_y, E_x = 0$

Drawing: for TE_{10} patterns

$\frac{1}{2}$ cycle $m=1 \leftarrow (x)$

$n=0 \leftarrow (y)$



Ex: In a RWG for which $a = 1.5\,cm$, $b = 0.8\,cm$, $\sigma_d = 0$

$\mu = \mu_0$, $\varepsilon = 4\varepsilon_0$

$$H_x = 2\sin\left(\frac{\pi x}{a}\right).\cos\left(\frac{3\pi y}{b}\right)\sin(\pi \times 10^{11} t - \beta z)\ A/m$$

Determine:

a) Mode of propagation

b) f_c

c) β

d) γ

e) η

f) other fields for this mode

a) either TM_{13} or TE_{13}

b) $f_c = \frac{\omega}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{3}{b}\right)^2}$

$\omega = \frac{c}{2}$

$f_{c_{13}} = 28.57\ GHz$, for TE_{13} & TM_{13}

c) $\beta = \beta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$, $\beta' = \omega\sqrt{\mu\varepsilon}$

$f = \frac{\omega}{2\pi} = 50\,GHz$ $(f > f_c)$

$\beta = 1718.81\ rad/m$

$f < f_c$ evanescent

$f = f_c$ cut off

$f > f_c$ propagation

d) $\gamma = j\beta$

e) $\eta_{TM_{13}} = \eta' \sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{377}{\sqrt{\varepsilon_r}} \sqrt{1-\left(\frac{28.57}{50}\right)^2}$

$= 154.7\,\Omega$

for $\eta_{TE13} = 229.69\,\Omega$

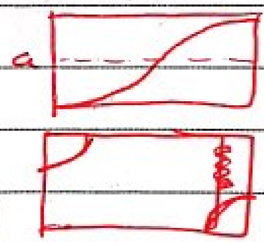
$\frac{120\pi}{2} = \sqrt{\eta_{TM13}\,\eta_{TE13}} = \eta'$

f) Since $TM_{13} \rightarrow H_z = 0$

E_x, E_y, E_z, H_y, ($H_x \rightarrow$ given)

$H_{ys} = \frac{-\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right)\cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$

$m=1$, $n=3$, $\gamma = j\beta$



In time domain

$H_y = \frac{\omega\varepsilon x}{h^2 a}\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{3\pi y}{b}\right)$ $\downarrow$ $\sin(\omega t - \beta z)$

$h^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{3\pi}{b}\right)^2$

$H_x = -\frac{\omega\varepsilon}{x^2}\frac{3\pi}{b}E_o \sin\left(\frac{\pi x}{a}\right)\cdot\cos\left(\frac{3\pi y}{b}\right)\sin(\omega t - \beta z)$

(2) $\rightarrow$ from Question

* Wave propagation in the waveguide:

for TE_{10} mode

There are E_y, H_x, H_z exist

$$E_{ys} = \frac{-j\omega\mu a}{\pi} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

from euler's identity

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$E_{ys} = \frac{-\omega\mu a}{2\pi}\left(e^{j\frac{\pi x}{a}} - e^{-j\frac{\pi x}{a}}\right) e^{-j\beta z}$$

$$= \frac{\omega\mu a}{2\pi}\left(e^{-j\beta z} e^{-j\frac{\pi x}{a}} - e^{j\frac{\pi x}{a}} e^{-j\beta z}\right)$$

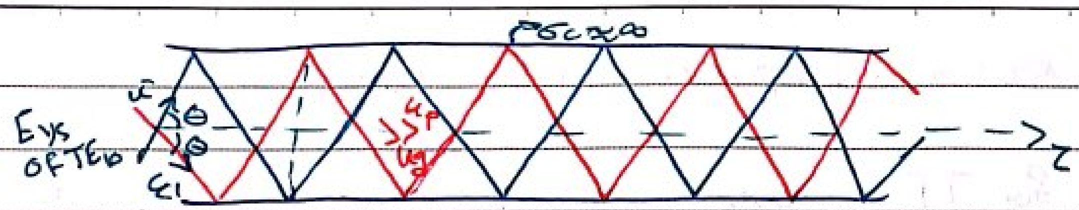
$$E_{ys} = \underbrace{\frac{\omega\mu a}{2\pi}}_{E_o}\left(\underbrace{e^{-j\beta\left(z + \frac{\pi x}{\beta a}\right)}}_{(1)} - \underbrace{e^{-j\beta\left(z - \frac{\pi x}{\beta a}\right)}}_{(2)}\right)$$

phase shift

↳ contains two waves:

The first term represent a wave travelling in the +ve z-direction with $a = \tan^{-1}\left(\frac{\pi}{\beta a}\right)$ with the z-axis.

The 2^{nd} term makes an angle of $a = \tan^{-1}\left(\frac{-\pi}{\beta a}\right) = -\theta$ with the z-axis.



There are 3 types of Velocity

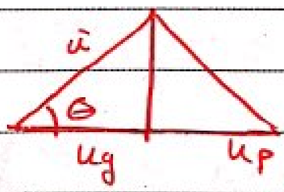
1) $u' = \frac{1}{\sqrt{\mu\epsilon}}$ is the (TEM) medium velocity.

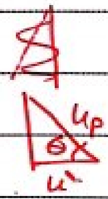
2) $u_p = \frac{u'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\omega}{\beta} = \frac{u'}{\cos\theta}$

↳ The loci of the phase constant

3) Group Velocity (u_g)

$u_g = \frac{d\omega}{d\beta} = u'\cos\theta$





$\cos\theta = \frac{u'}{u_p} = \frac{u_g}{u'}$

$u' = \frac{1}{\sqrt{\mu\epsilon}}$

$u_p = \frac{u'}{\cos\theta} = \frac{u'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$ ← phase velocity

$u_g = u'\cos\theta = u'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$u_p \geqslant u' \rightarrow$ may be larger than (c)

$u_g \leqslant u' \rightarrow$ always $\leqslant c$

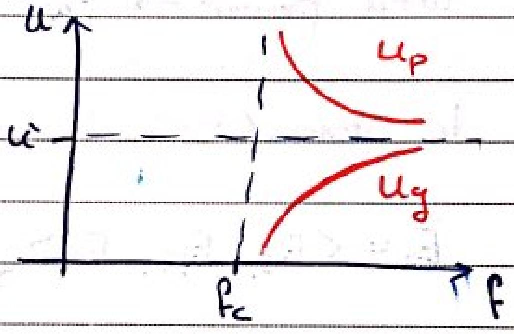
↳ Packet (Energy) Velocity

The wave length along the axis of the guide is

$\lambda_g \equiv$ waveguide wavelength

$$\lambda_g = \frac{\lambda'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \quad , \quad \lambda' = \frac{u'}{f} \text{ (TEM)}$$

$$\boxed{u_p u_g = u'^2}$$



Ex: An air-filled R.W.G with $a = 8.636\,cm$, $b = 4.318\,cm$, is fed by a 4GHz carrier from a Coaxial cable.

Determine if the dominant mode will be propagated, If so Calculate the U_p & U_g.

TE_{10} $(a > b)$

$$f_{c_{10}} = \frac{u'}{2}\sqrt{\left(\frac{\pi}{a}\right)^2}$$

$$= \frac{u'}{2a} = \frac{c}{2a} = 1.7375\,GHz < 4\,GHz$$

$$\cos\theta = \sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$u_p = \frac{u'}{\cos\theta} = 3.33\times10^8\ m/s > c$$

$$u_g = u'\cos\theta = 2.702\times10^8\ m/s < c$$

*Power Transmission and attenuation:

$$\vec{P}_{ave} = \frac{1}{2} Re\{\vec{E}_s \times \vec{H}_s^*\}$$

↳ Average poynting vector in (W/m^2)

Total power → $\vec{P}_{ave} = \int_s P_{ave} \cdot d\vec{s}$ in (W)

In general:

$\vec{E}_s = \langle E_{xs}, E_{ys}, E_{zs} \rangle$

$\vec{H}_s^* = \langle H_{xs}^*, H_{ys}^*, H_{zs}^* \rangle$

E_{zs} & H_{zs} one of them is zero

$$\vec{P}_{ave} = \frac{1}{2} Re\{\underbrace{E_{xs} H_{ys}^*}_{\vec{a}_x \times \vec{a}_y} - \underbrace{E_{ys} H_{xs}^*}_{-\vec{a}_y \times \vec{a}_x}\} \hat{a}_z$$

$\eta = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$

$H_{ys}^* = \frac{E_{xs}^*}{\eta}$

$$\vec{P}_{ave} = \frac{1}{2\eta} (|E_{xs}|^2 + |E_{ys}|^2) \hat{a}_z$$

for both TE & TM modes

↳ only chose η_{TE} or η_{TM}

$$P_{ave} = \int \vec{P}_{ave} \cdot d\vec{s}$$

$$= \int_{y=0}^{b} \int_{x=0}^{a} \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta} dx\, dy$$

* Attenuation in W.G ($\sigma_d \neq 0$, $\sigma_c \approx \infty$) $\rightarrow$ lossy W.G

by replacing ε by ε_c , $\varepsilon_c = \varepsilon' - j\varepsilon'' = \varepsilon - j\frac{\sigma_d}{\omega}$

In general, the power in the W.G is in form
$P_{ave} = P_o e^{-2\alpha z}$, if $\alpha = 0$

* $\alpha = \alpha_c + \alpha_d$

α_c: attenuation in the W.G walls ($\sigma_c \neq \infty$)

α_d: " " " " dielectric medium ($\sigma_d \neq 0$)

Starting with σ_d

$$\gamma = \sqrt{h^2 - k^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} = \alpha_d + j\beta_d$$

$$\varepsilon_c = \varepsilon - j\frac{\sigma_d}{\omega}$$

$$\alpha_d^2 - \beta_d^2 + 2j\alpha_d\beta_d = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon + j\omega\mu\sigma_d$$

Equating:

* real : $\alpha_d^2 - \beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$; $\alpha_d > \beta_d$

$\Rightarrow \alpha_d^2 \ll \beta_d^2$

$\Rightarrow \alpha_d^2 - \beta_d^2 \approx -\beta_d^2$

$$\beta_d = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$\beta_d = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$; Same as β for lossless

Imaginary:

$$2\,\alpha_d \beta_d = \omega \mu \sigma_d$$

$$\boxed{\alpha_d = \frac{\omega \mu \sigma_d}{2\beta_d}} \quad ; \text{ sub } \beta_d$$

sub. β_d

$$\boxed{\alpha_d = \frac{\sigma_d \eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}} \quad , \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

[Np/m]

Consider α_c:

For E_{10} $(a>b)$

$E_{ys}, H_{xs}, H_{zs} \neq 0$

$$E_{ys} = \frac{\omega \mu a}{\pi} H_o \sin\left(\frac{\pi x}{a}\right) e^{-\gamma z}$$

$$H_{xs} = \frac{-\beta a}{\pi} H_o \sin\left(\frac{\pi x}{a}\right) e^{-\gamma z}$$

$$H_{zs} = H_o \cos\left(\frac{\pi x}{a}\right) e^{-\gamma z}$$

for E_y:

$$P_{ave} = \int_0^b \int_0^a \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\, \eta_{TE_{10}}}\, dx\, dy$$

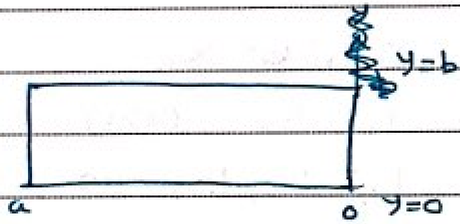
$$P_{ave} = \int_0^b \int_0^a \frac{\omega^2 \mu^2 a^2}{2 \pi^2 \eta} H_o^2 \sin^2\left(\frac{\pi x}{a}\right) dx\,dy$$

$$= \frac{\omega^2 \mu^2 a^3 H_o^2 b}{4 \pi^2 \eta_{TE10}} \quad \text{watt}$$

Power loss (P_L)

$$P_L = P_L\Big|_{x=0} + P_L\Big|_{x=a} + P_L\Big|_{y=0} + P_L\Big|_{y=b}$$

$$= 2\left(P_L\Big|_{x=0} + P_L\Big|_{y=0}\right)$$



$$P_{ave} = P_o e^{-2\gamma z}$$

For the wall $y=0$

$$P_L\Big|_{y=0} = \frac{1}{2} Re\left\{ \eta_c \int_0^a |H_{xs}|^2 + |H_{zs}|^2 \, dx \right\}\Bigg|_{y=0}$$

$$Re\{\eta_c\} = R_s = \frac{1}{\sigma_c \delta} = \sqrt{\frac{\pi f \mu}{\sigma_c}}$$

Skin depth $= \frac{1}{\alpha_c}$

$$P_L\Big|_{y=0} = \frac{1}{2} R_s \left\{ \int_0^a \frac{\beta^2 a^2}{\pi^2} H_o^2 \sin^2\left(\frac{\pi x}{a}\right) dx + \int_0^a H_o^2 \cos^2\left(\frac{\pi x}{a}\right) dx \right\}$$

$$P_L\Big|_{y=0} = \frac{R_s}{2}\left(\frac{\beta^2 a^2 H_0^2}{\pi^2}\left(\frac{a}{2}\right) + H_0^2\left(\frac{a}{2}\right)\right)$$

$$\boxed{P_L\Big|_{y=0} = \frac{R_s\, a H_0^2}{4}\left(1+\frac{\beta^2 a^2}{\pi^2}\right)}$$

Same at $y=b$

$$P_L\Big|_{x=0} = \frac{1}{2}\,\mathrm{Re}\left\{\eta_c \int_0^b |H_{zs}|^2\, dy\right\}$$

$$= \frac{1}{2} R_s H_0^2 \left(\frac{b}{2}\right)$$

$$\boxed{P_L\Big|_{x=0} = \frac{R_s\, b\, H_0^2}{4}}$$

Total loss

$$\boxed{P_L = \frac{R_s H_0^2}{2}\left(b + a\left(1+\frac{\beta^2 a^2}{\pi^2}\right)\right)}$$

* Conservation of Power :

$$P_L = \frac{-dP_{ave}}{dz} = +2\alpha_c P_{ave}$$

$$\alpha_L = +\frac{P_L}{2P_{ave}}$$

$$\alpha_c = \frac{2R_s}{b\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(\frac{1}{2} + \frac{b}{a}\left(\frac{f_c}{f}\right)^2\right)$$

only for TE_{10}

~~general α_c~~

General α_c:

$$\alpha_{c_{TM}} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \cdot \frac{\left(\frac{b}{a}\right)^3 m^2 + n^2}{\left(\frac{b}{a}\right)^2 m^2 + n^2}$$

$$\alpha_{c_{TE_{mn}}} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \left[1 + \frac{b}{a}\left(\frac{f_c}{f}\right)^2 + \frac{\frac{b}{a}\left(\frac{b}{a}m^2+n^2\right)}{\frac{b^2}{a^2}m^2+n^2}\left(1-\left(\frac{f_c}{f}\right)^2\right)\right]$$

$$\alpha = \alpha_c + \alpha_d$$

Ex: Air filled R.W.G

$a = 4cm$, $b = 2cm$

dominant mode (TE_{10})

$P_{ave} = 2\,mW$

$f = 10\,GHz$

Find E_o ??

$$E_{xs} = 0 \;,\; E_{ys} = -jE_o \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

$$E_o = \frac{\omega \mu a}{\pi} H_o$$

$$f_c = \frac{u'}{2a} = 3.75\,GHz$$

$$\eta_{TE_{10}} = 406.7\,\Omega \Leftarrow \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 40$$

$$P_{ave} = \int_0^b \int_0^a \frac{E_o^2}{2\eta} \sin^2\left(\frac{\pi x}{a}\right) dx\,dy$$

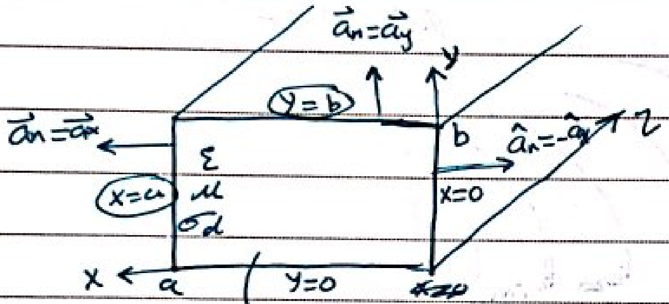
$$= \frac{E_o^2\, ab}{4\eta}$$

$$E_o^2 = 4067$$

$$\boxed{E_o = 63.77 \text{ V/m}}$$

* waveguide current:

$$\phi \begin{cases} \vec{K} = \vec{H} \times \vec{a}_n & \text{A/m} \\ \rho_s = \vec{D} \cdot \vec{a}_n & \text{C/m}^2 \end{cases} , \quad \vec{D} = \varepsilon \vec{E}$$



TE modes < E, H

TM modes < E, H

$TE_{10} \rightarrow E_y, H_x, H_z$

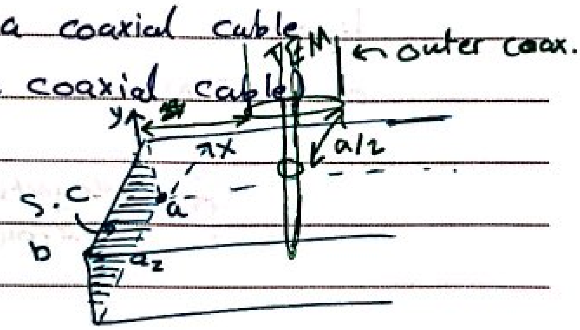
x=0

k=a

* wave guide Excitation:

How to insert a signal inside a waveguide.

* most used technique is by a coaxial cable
(inner conductor of the coaxial cable)
(Probe) → TEM



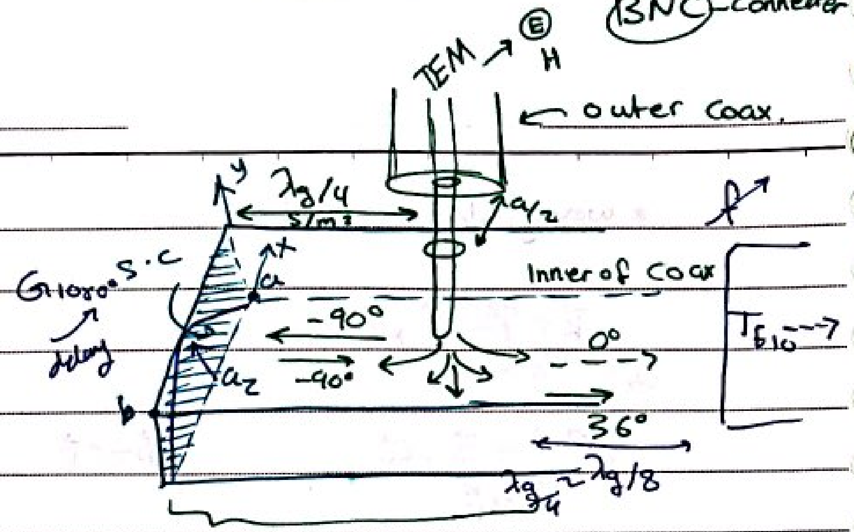
for TE_{10} Mode

$a \rightarrow x$

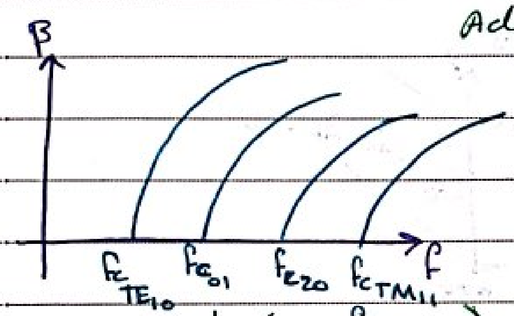
$b \rightarrow y$

$$E_{y_{TE10}} = \frac{\omega \mu a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z)$$

$$\frac{\pi x}{a} = \frac{\pi}{2} \rightarrow \boxed{x = \frac{a}{2}}$$



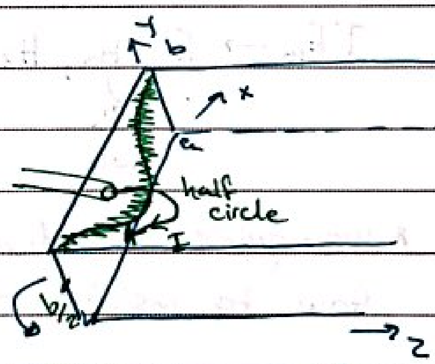
Adapter



non degeneret mode

$f_{c_{TE_{11}}}$ → degeneret mode (2 modes with same f_c)

To excite TM_{11} mode



$E_z =$
$E_x =$
$E_y =$

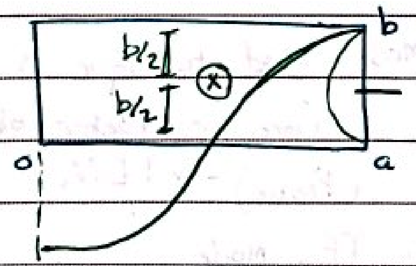
⊕ ⊖ dipole

⊕ Gnd monopole

$H_x = \sin \rightarrow$ max at $y = b$

$H_y =$

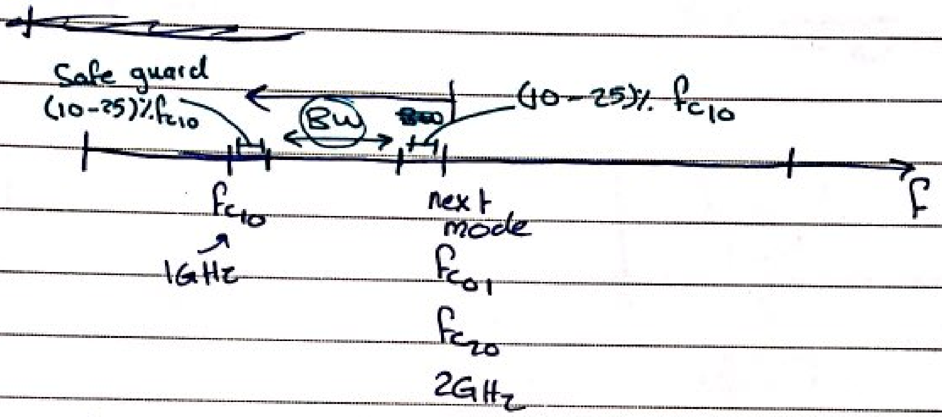
$H_z = 0$ (TM)

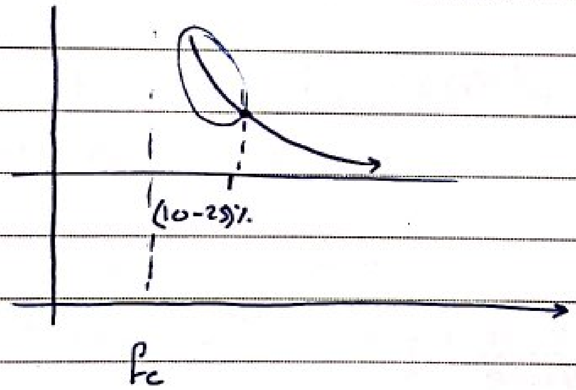


I Magnetic monopole

Design Problem:

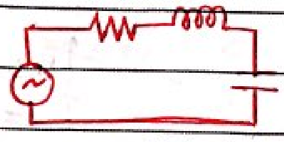
find a, b such as only TE_{10} mode exist with a certain BW and take a safe margin of 10% of $f_{c_{10}}$ mode.





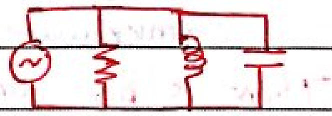
12/12

* Wave guide Resonators



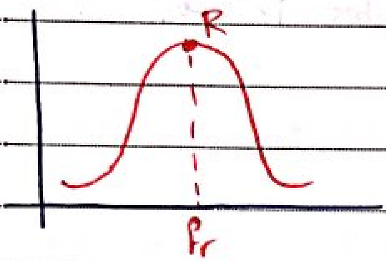
Series Resonant

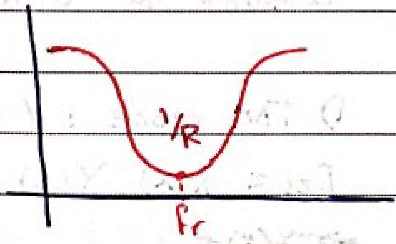
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$



Parallel Resonant

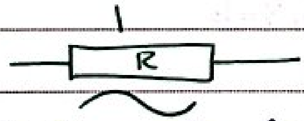
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$





* At high freq. ($f \geq 100MHz$)

Size of $R, L, C \approx \lambda$

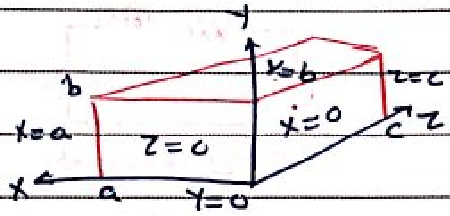


at $f = 30GHz \longrightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{300\times10^8} = 1cm$

↳ must be $\gg 10\lambda \Rightarrow \lambda \ll \frac{l}{10}$

* wave guide Resonators used for $f \gg 100\,MHz$
i.e Klystron tube (Amp)
- microwave oven

There is no propagation
There is a Standing wave



assume the wave propagates in $\hat{a}_z$

1) TM mode : ($H_z = 0$)

$E_{zs} = X(x)\, Y(y)\, Z(z)$

~~$\Rightarrow X(x) =$~~

$\Rightarrow X(x) = C_1 \cos k_x x + C_2 \sin k_x x$

$\Rightarrow Y(y) = C_3 \cos k_y y + C_4 \sin k_y y$

$\Rightarrow Z(z) = C_5 \cos k_z z + C_6 \sin k_z z$

$E_{zs} = 0$, $x = 0$, $x = a$ $y = 0$, $y = b$

$\left.\begin{array}{l} E_{xs} = 0 \\ E_{ys} = 0 \end{array}\right\}$ at $z = 0$, $z = c$

from $E_{zs} = 0$ at $x = 0 \Rightarrow C_1 = 0$

$E_{zs} = 0$ at $y = 0 \Rightarrow C_3 = 0$

$E_{zs} = 0$ at $x = a \rightarrow \sin k_x a = 0 \Rightarrow k_x = \dfrac{m\pi}{a}$, $m = 1, 2, 3 \ldots$

$E_{zs} = 0$ at $y = b \rightarrow k_y = \dfrac{n\pi}{b}$, $n = 1, 2, 3 \ldots$

From E_{zs} & Maxwell's equation $\nabla \times \vec{H} = j\omega\varepsilon \vec{E}$

$$j\omega\varepsilon E_x = \cancel{\frac{\partial H_{zs}}{\partial y}}^{\,0} + \frac{1}{j\omega\varepsilon}\left(\frac{\partial^2 E_{xs}}{\partial z^2} - \frac{\partial^2 E_{zs}}{\partial z \partial x}\right)$$

$$j\omega\varepsilon E_y = \cancel{\frac{\partial H_{zs}}{\partial x}}^{\,0} - \frac{1}{j\omega\varepsilon}\left(\frac{\partial^2 E_{zs}}{\partial y \partial z} - \frac{\partial^2 E_{ys}}{\partial z^2}\right)$$

applying $E_x = 0$ at $z = 0$

$\rightarrow \frac{\partial E_{zs}}{\partial z} = 0$ at $z = 0,\ z = c$

$$Z'(z) = -C_5 k_z \sin k_z z + C_6 k_z \cos k_z z$$

$$Z'(0) = 0 \Rightarrow C_6 = 0$$

$$Z'(c) = 0 \Rightarrow \sin k_z z = 0 \longrightarrow \boxed{k_z = \frac{p\pi}{c}}\ ,\ p = 0, 1, 2, 3 \cdots$$

lowest TM mode is TM_{110} (when we see 3 num ⇒ cavity not T.L)

$* \; k^2 = k_x^2 + k_y^2 + k_z^2 = \beta^2$ (if lossless)

$$\Rightarrow \beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2} = \omega\sqrt{\mu\varepsilon} \quad \text{rad/m}$$

At resonance (f_r)

$$\beta = \frac{\omega_r}{u'} = \frac{2\pi f_r}{u'}$$

$$\rightarrow f_r = \frac{u'}{2\pi}\beta$$

$$\boxed{f_r = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}$$

$$\boxed{\lambda_r = \frac{u'}{f_r} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}}$$

$$\boxed{E_{zs} = E_0 \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi x}{b}\right) \cos\left(\frac{p\pi z}{c}\right)}$$ where $E_0 = C_2 C_4 C_6$

$E_x, E_y, H_x, H_y \rightarrow$ find them using maxwell's eq. just like what we did before

2) for TE mode $(E_z = 0)$

$$H_{zs} = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y) * (B_5 \cos k_z z + B_6 \sin k_z z)$$

with B.C:

$H_{zs} = 0$ at $z = 0$, $z = c$

$\frac{dH_{zs}}{dx} = 0$ at $x = 0$, $x = a$

$\frac{dH_{zs}}{dy} = 0$ at $y = 0$, $y = b$

$B_2 = 0$, $B_4 = 0$, $B_5 = 0$

$H_0 = B_1 B_3 B_6$, $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$, $k_z = \frac{p\pi}{c}$

$$\boxed{H_{zs} = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right)}$$

$\left.\begin{array}{l} m = 0, 1, 2 \text{ —} \\ n = 0, 1, 2 \text{ —} \\ p = 1, 2, 3 \text{ —} \end{array}\right\}$ $m = 0$ & $n = 0$ × (TEM)

TE_{001}
TM_{100}
TM_{010}
مش موجودين

If $a > b > c \rightarrow \frac{1}{a} < \frac{1}{b} > \frac{1}{c}$

Dominant mode is $\boxed{TE_{101}}$ $\quad f_{r_{TE_{101}}} < f_{r_{TM_{110}}}$

* Quality factor (Q):

Q represents the amount of losses in energy stored in a

$$Q = 2\pi \frac{\text{Time average energy stored}}{\text{Energy loss per cycle}}$$

$$Q = 2\pi \frac{W}{P_L T} \quad , T = \frac{1}{f}$$

$$\boxed{Q = \omega \frac{W}{P_L}} \quad , W = W_E + W_m$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$= \left(\tfrac{1}{2}\varepsilon E^2 + \tfrac{1}{2}\mu H^2\right) \text{Volume}$$

abc

For TE_{101} mode

$$Q_{TE101} = \frac{(a^2+c^2)\,abc}{\delta\,[2b\,(a^3+c^3) + ac\,(a^2+c^2)]}$$

$$\delta = \frac{1}{\sqrt{\pi f_{101} \mu_0 \sigma_c}} \quad \text{if non-mag.}$$

In Circuits

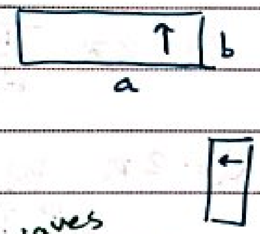
ωL or $\frac{1}{\omega C}$

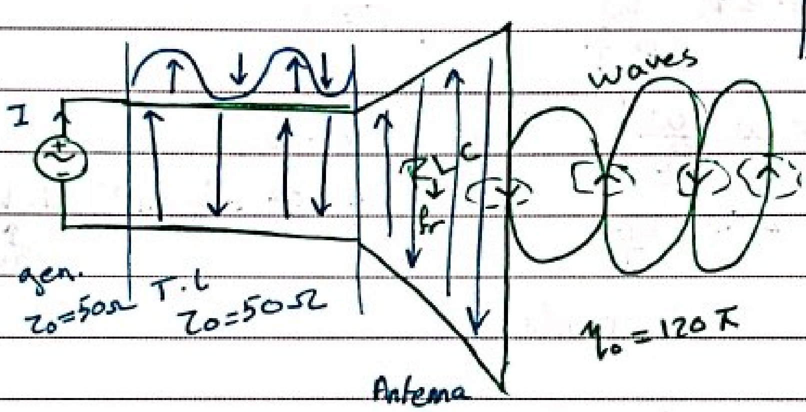
Series RLC $\quad Q = \frac{f_r}{BW} = \frac{X}{R} = \frac{\omega_r L}{R}$

Parallel RLC $\quad$ // $\quad$ // $\quad$ // $\quad = \frac{1}{\omega_r C R}$

Ch13: Antennas

Sources: charge moving with accelaration to have a time-varying fields.

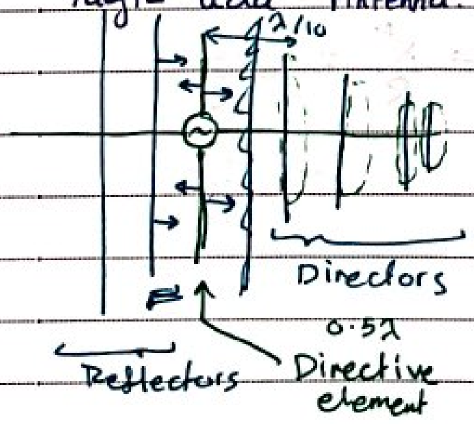




1) matching between T.L and medium
2) efficient radiation

6. Typical Antennas

Yagi-uda Antenna:



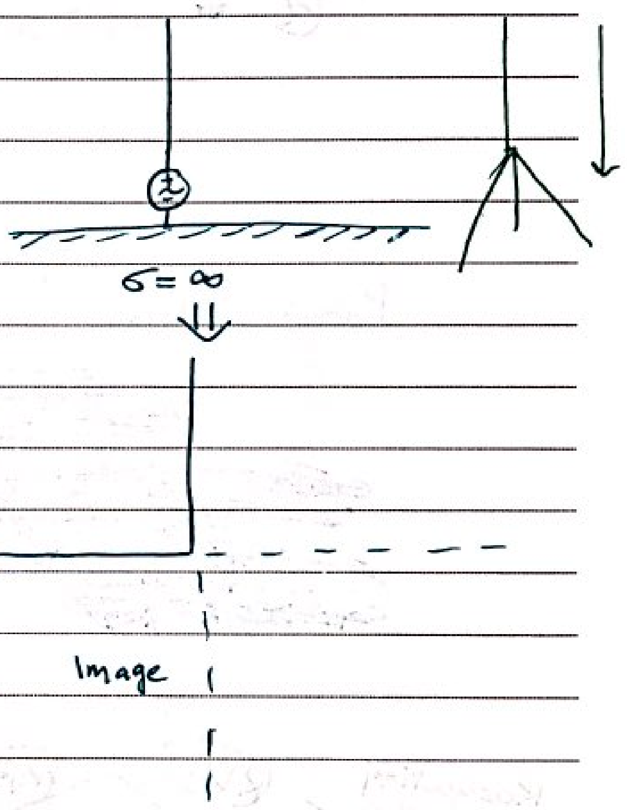
$$\boxed{\lambda = \frac{c}{f}}$$

~~wire~~

wire - Antenna

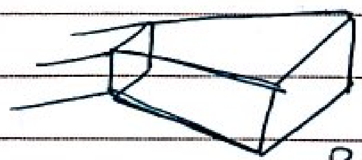
1) Dipole Antenna

2) monopole

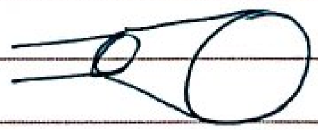


loop - Antenna

Horn Antenna

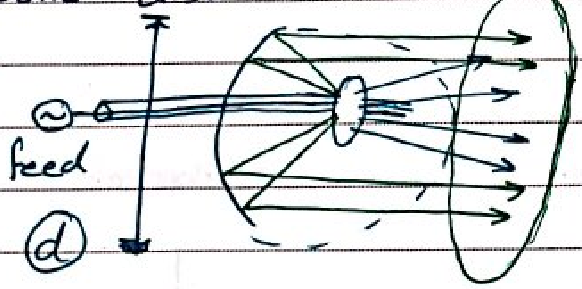


~~Pyramidical~~ Pyramidical Horn

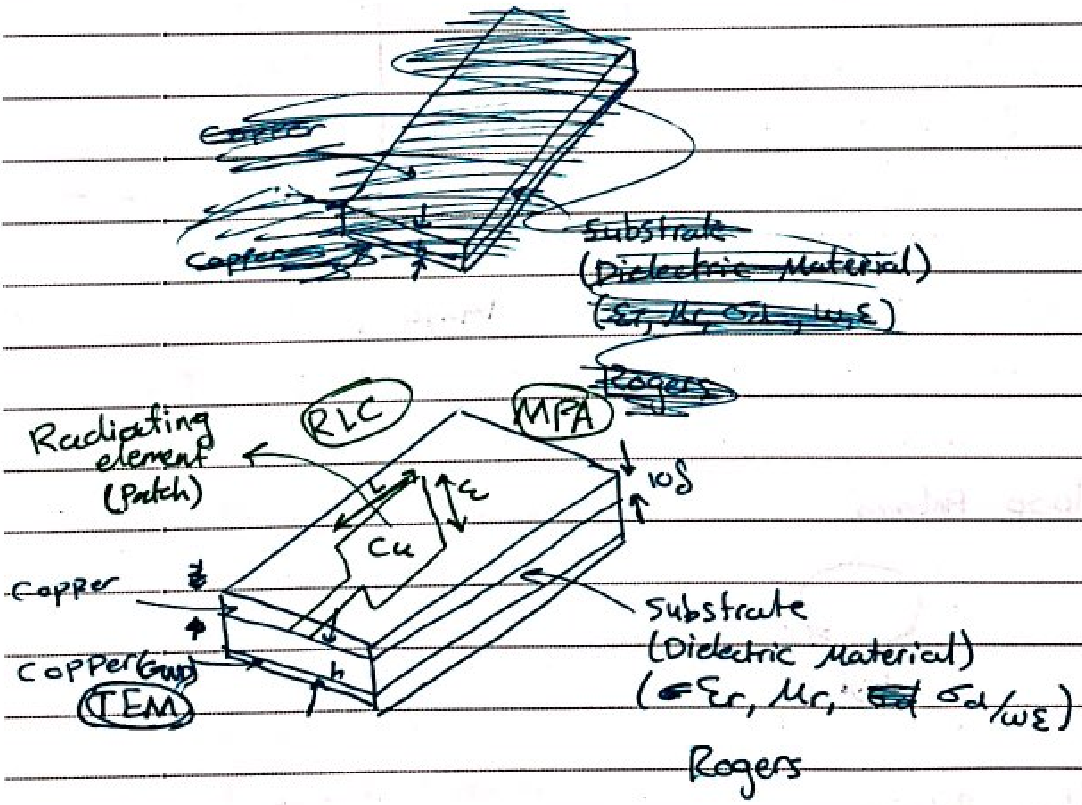


Cronical Horn

Probolic dish antenna



Planer Antenna

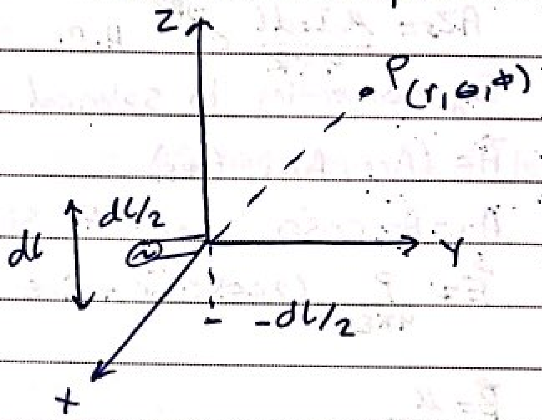


wire Antenna:

Hertzian dipole

$l = dL$

Infinitesmal dipole



Assume $I = I_o$ (uniform current)

From ch.9

$$\nabla^2 A - \mu\varepsilon \frac{dA^2}{dt^2} = -\mu \vec{J}$$

Solution: $\vec{A} = \int_l \frac{\mu [I] d\vec{l}}{4\pi R}$

$[I] = I_o(x, y, z, t)$

$\vec{A} = Az_s$

$\vec{B} = B_\phi \, \hat{a}_\phi$

$\vec{B} = \nabla \times \vec{A}$

$$Az = \frac{\mu I_o dl}{4\pi R} \cos\omega\left(t - \frac{R}{u}\right)$$

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$Az = Re\{Az_s\, e^{j\omega t}\}$

$AZ = Re\{AZ_s\, e^{j\omega t}\}$

$AZ_s = \frac{\mu I_o dl}{4\pi R} e^{j\omega t} e^{-j\omega R/u}$ $\qquad \frac{\omega}{u} = \beta$

$AZ_s = \frac{\mu I_o dl}{4\pi R} e^{-j\beta R}$ A.m

By converting to spherical coordinates:

$\vec{A} = (A_{rs}, A_{\theta s}, A_{\phi s})$

$A_{rs} = A_{zs}\cos\theta$, $A_{\theta s} = A_{zs}\sin\theta$, $A_{\phi s} = 0$

$\vec{E} = \frac{\rho}{4\pi\varepsilon_o r^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta)$

$\vec{B} = \frac{\mu}{4\pi r^3}$

$\vec{A}_s = (A_{zs}\cos\theta, A_{zs}\sin\theta, 0)$; $\vec{B}_s = \mu\vec{H}_s - \nabla\times\vec{A}_s$

$$\vec{H}_s = \frac{1}{\mu}\nabla\times\vec{A}_s = \frac{1}{\mu}\cdot\frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_z\cos\theta & A_z\sin\theta & 0 \end{vmatrix}$$

$H_{rs} = 0$, $H_{\phi s} = 0$.

$$H_{\phi s} = \frac{I_o dl}{4\pi}\sin\theta\left(\frac{j\beta}{R} + \frac{1}{R^2}\right)e^{-j\beta R}$$

$\nabla\times\vec{E}_s = -j\omega\mu\vec{H}_s$, $\boxed{\nabla\times\vec{H}_s = j\omega\varepsilon\vec{E}_s}$

$$E_{rs} = \frac{\eta I_o dl}{2\pi}\cos\theta\left(\frac{1}{R^2} - \frac{j}{\beta R^2}\right)e^{-j\beta R}$$

$$E_{\theta s} = \frac{\eta I_o dl}{4\pi}\sin\theta\left(\frac{j\beta}{R} + \frac{1}{R^2} - \frac{j}{\beta R^3}\right)e^{-j\beta R}$$

$E_{\phi s} = 0$

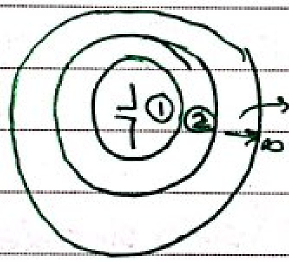
fields $H_{\phi s}$, E_{rs}, $E_{\phi s}$

Term $(1/R^3) \Rightarrow$ electrostatic term for electrical dipole

Term $(1/R^2) \Rightarrow$ Inductive part

Term $(1/R) \Rightarrow$ far- field term or radiation - field term

$R \ggg dl$ (dipole)



In the far-field region:

$$H_{\phi s} = \frac{j I_o dl \beta}{4\pi R} \sin\theta \, e^{-j\beta R}$$

$$E_{\theta s} = j\frac{\eta I_o dl \beta}{4\pi R} \sin\theta \, e^{-j\beta R} = \eta H_\phi$$

E_θ, H_ϕ

$H_\phi \perp E_\theta$, $\eta = \frac{E_\theta}{H_\phi}$

The Inter face between near and far fields is estimated by

$$\boxed{R = \frac{2d^2}{\lambda}}$$

, d: is the largest diameter of the antenna

* Power:

The time average power density

$$\vec{P}_{ave} = \frac{1}{2} Re\{\vec{E}_s \times \vec{H}_s^*\} = \frac{1}{2} Re\{E_{\theta s} \times H_{\phi s}^*\} \hat{a}_r$$

$$= \frac{1}{2} \eta |H_{\phi s}|^2 \hat{a}_r$$

TEA

The total Power

$$P_{ave} = \int_s \vec{P}_{ave} \cdot d\vec{s} = \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{2} \frac{I_o^2 \, dl^2 \beta^2}{16\pi^2 \beta^2} \sin^2\theta \; r^2 \sin\theta \, d\theta \, d\phi$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta = \int_0^{\pi} (1-\cos^2\theta) \sin\theta \, d\theta$$

let $z = \cos\theta$
$dz = \sin\theta \, d\theta$

$$= \frac{4}{3}$$

$$P_{ave} = \frac{I_o^2 \pi \eta}{3} \left(\frac{dl}{\lambda}\right)^2 = P_{rad}$$

$P_{rad} \equiv$ radiation power from the Hertzian dipole.

peak

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$$

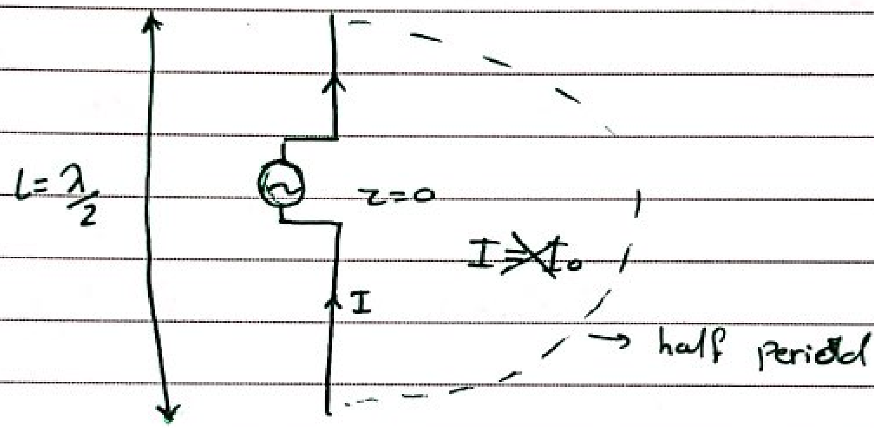
$$R_{rad} = \frac{2 P_{rad}}{I_o^2}$$

if $\eta = \eta_o$ (free space) $\Rightarrow$ $\boxed{R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2}$

if $dl = \frac{\lambda}{20}$

$R_{rad} \cong 2\pi$

* $l = \frac{\lambda}{2}$ (half wavelength dipole)



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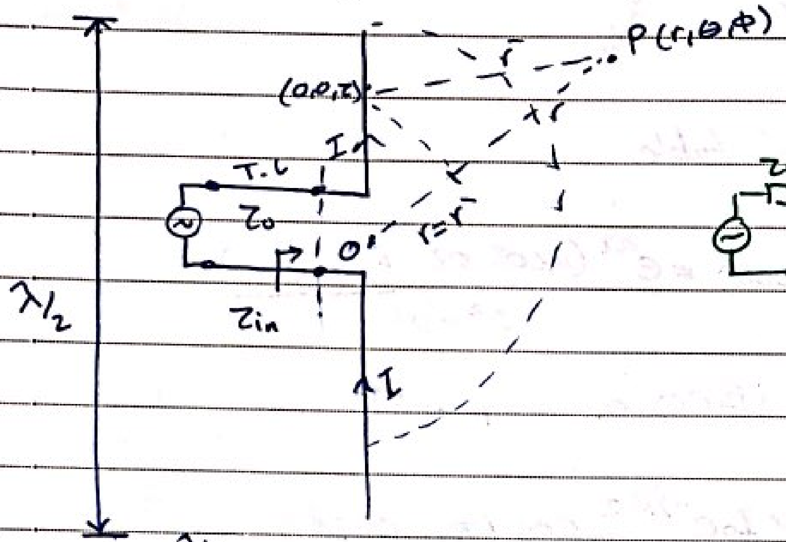
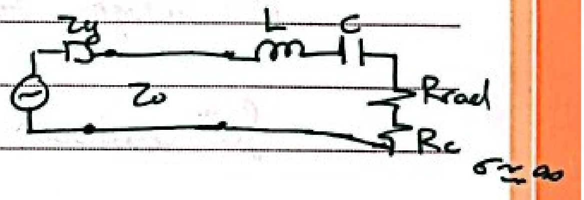
* Half wave length dipole antenna





$$A = \int_{\lambda/4}^{\lambda/2} \frac{\mu [I]}{4\pi r} \vec{dl} \qquad dl = dz\,\hat{a}_z$$

$$dA_{zs} = \frac{\mu [I] dz}{4\pi r^2}$$

$$[I_s] = I_o \cos\beta z$$

$$dA_{zs} = \frac{\mu I_o \cos\beta z \, dz}{4\pi r`} e^{-j\beta r`}$$

$$A_{zs} = \frac{\mu I_o dl\, e^{-j\beta r}}{4\pi r}$$ Hertzian dipole

$r - r` = z\cos\theta$ , $r` = r - z\cos\theta$

Approximation

In denominator $r` \simeq r$ (magnitude only)

$P \ggg \frac{2d^2}{\lambda}$ far field

In numerator → phase $r` = r - z\cos\theta$

$$dA_{zs} = \frac{\mu I_o \cos\beta z \, dz}{4\pi r} e^{-j\beta(r - z\cos\theta)}$$

$$A_{zs} = \frac{\mu I_o}{4\pi r} e^{-j\beta r} \int_{-\lambda/4}^{\lambda/4} \cos \beta z \, e^{j\beta z \cos\theta} dz$$

From Integral table

$$\int e^{az} \cos \beta z dz = \frac{e^{az}(a \cos bz + b \sin bz)}{a^2+b^2}$$

Let $a = j\beta \cos a$, $b = \beta$

$$A_{zs} = \frac{\mu I_o e^{-j\beta z} \cos(\frac{\pi}{2} \cos\theta)}{2\pi r \beta \sin^2\theta}$$

By using $\vec{B}_s = \mu \vec{H}_s = \nabla \times \vec{A}_s$

Far-field

$H_{rs} = 0$

$H_{\theta s} = 0$

$$H_{\phi s} = \frac{j I_o e^{-j\beta r} \cos(\frac{\pi}{2}\cos\theta)}{2\pi r \sin\theta}$$

Using $\nabla \times \vec{H}_s = j\omega\varepsilon \vec{E}_s$

$E_{rs} = 0$

$E_{\theta s} = \eta H_{\phi s}$

$E_{\phi s} = 0$

~~$\vec{P}_{ave} = \frac{1}{2}$~~

$$\vec{P}_{ave} = \frac{1}{2} Re\{\vec{E}_{\theta s} \times \vec{H}^*_{\phi s}\}$$

$$= \frac{1}{2} \eta |H^*_{\phi s}|^2 \ a_r$$

$$P_{ave} = \int_s \vec{P}_{ave} \cdot d\vec{s}$$

$$= \frac{\eta}{2} \int_0^{2\pi}\int_0^{\pi} \frac{I_o^2}{4\pi^2 r^2 \sin^2\theta} \cos^2(\frac{\pi}{2}\cos\theta)\, r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{\eta I_o^2}{4\pi} \int_0^{\pi} \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta} d\theta \quad ; \quad \cos\theta = 1 + \frac{\theta^2}{2!} \mp \frac{\theta^4}{4!} + \cdots$$

~~cos~~

$$\boxed{P_{rad} = P_{ave} = 36.56 \, I_o^2 \ W}$$

$$= \frac{1}{2} I_o^2 R_{ad}$$

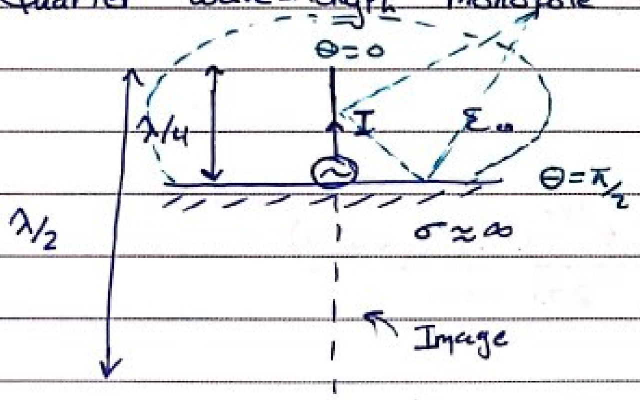
$R_{ad} \approx 73 \, \Omega$ coaxial cable $\begin{cases} 50 + j0 \, \Omega \\ 75 + j\,42.5 \, \Omega \end{cases}$

$Z_{in} = R_{in} + jX_{in}$

$l = 0.5\lambda \rightarrow X_{in} = 42.5 \, \Omega$

$l = 0.485\lambda \rightarrow X_{in} = 0 \, \Omega$

* Quarter wave-length monopole antenna:



Same fields of $\lambda/2$ dipole antenna

$H_{\phi s}, E_{\theta s}$

$P_{rad} = \frac{1}{2} P_{rad}\big|_{\lambda/2}$

$P_{rad} = 18.28\, I_o^2$ w

$R_{rad} = 36.5\,\Omega$

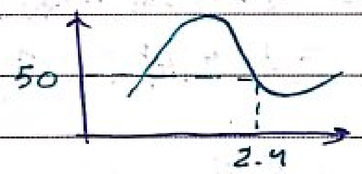
$Z_{in} = R_{in} + jX_{in}$, $R_{in} = R_{rad}$ if $\sigma_c \approx \infty$

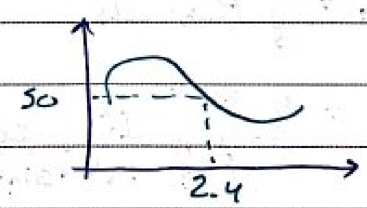
~~$l = 0.25\lambda \rightarrow X_{in} = 42.5\,\Omega$~~

~~$l = 0.485\lambda \rightarrow X_{in} = 0$~~

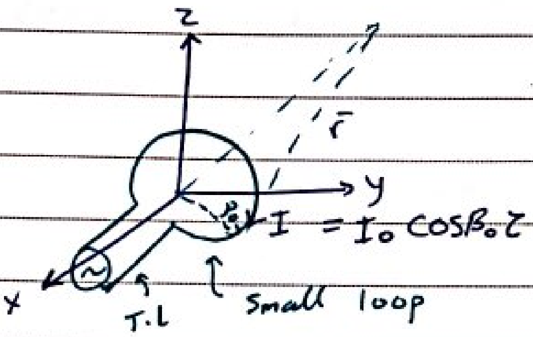
$l = 0.25\lambda \rightarrow X_{in} = 21.25\,\Omega$

$l = \frac{0.485}{2}\lambda \rightarrow X_{in} = 0\,\Omega$





* Small loop antenna (Directional finder D.F)



$\rho_o << r$

it is like a magnetic dipole

$r - r` = z\cos\theta$

$r = r`$

$$A = \oint_s \frac{\mu [I] dl}{4\pi r}$$

$$I = I_o \cos\omega t` = I_o \cos\omega\left(t - \frac{r}{u}\right)$$

$$= I_o \cos(\omega t - \beta r)$$

$$= Re\{I_o e^{j(\omega t - \beta r)}\}$$

$$I_s = I_o e^{-j\beta r} \quad , r = r`$$

From ch.8

$$\vec{A} = \frac{\mu I \overbrace{\pi \rho_o^2}^{S}}{4\pi r^2} \sin\theta \, \hat{a}_r \quad \text{for DC current } (\beta = 0)$$

$$A_{\phi s} = \frac{\mu I_o S}{4\pi r^2} (1 + j\beta r) e^{-j\beta r} \sin\theta$$

for N-turns loop

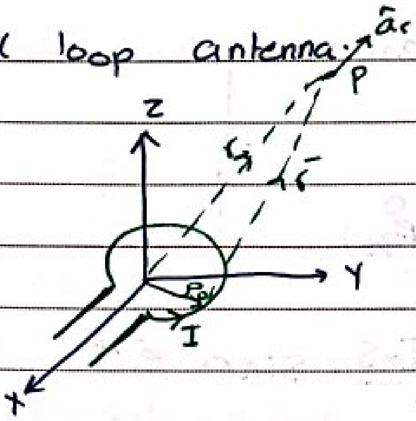
$S = N\pi \rho_o^2$

for Square loop of edge (a)

$g = a^2$ $\qquad g = N a^2$ turn

* The small loop antenna

$P_o <<< r$



$I = I_o \cos(\omega t - \beta \xi)$

$I_s = I_o e^{-j\beta r}$

$$A_{\phi s} = \frac{\mu I_o S}{4\pi r^2} \sin\theta \, (1 + j\beta r) e^{-j\beta r}$$

$: r \simeq r'$

$\vec{B}_s = \mu \vec{H}_s = \nabla \times \vec{A}_s$

$\nabla \times \vec{H}_s = j\omega \varepsilon \vec{E}_s$

fields in all region

but in far-field region

$\frac{1}{r^2}, \frac{1}{r^3}$ will be neglected

$$H_{\theta s} = \frac{-E_{\phi s}}{\eta} \quad (\hat{a}_\phi \times \hat{a}_\theta = -\hat{a}_r)$$

$$E_{\phi s} = \frac{(\omega)\, \mu \, I_o \, S \, (\beta)}{4\pi r} \sin\theta \, e^{-j\beta r}$$

if $\beta = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$, $f = \frac{u}{\lambda}$

$u = \frac{1}{\sqrt{\mu\varepsilon}}$, $\eta = \sqrt{\mu/\varepsilon}$

For wire antenna

$H_{\phi s}, E_{\theta s}$

$\vec{A}_s = (A_{rs}, A_{\theta s}, \overset{0}{A_{\phi s}})$

$$E_{\phi s} = \frac{\eta \pi I_o S}{r \lambda^2} \sin\theta \, e^{-j\beta r}$$

$$H_{\theta s} = \frac{-E_{\phi s}}{\eta}$$

If free space $\rightarrow \eta_o = 120\pi$

$$\vec{P}_{ave} = \frac{1}{2} Re\left\{ \frac{|E_{\phi s}|^2}{\eta} \right\} \hat{a}_r \quad W/m^2$$

$$P_{rad} = \int_s \vec{P}_{ave} \cdot d\vec{s}$$

$\hookrightarrow r \sin\theta \, d\theta \, d\phi$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad} \quad (W)$$

$$R_{rad} = \frac{320\pi^2 S^2}{\lambda^4} \, \Omega$$

* Antenna characterestics:

A) Radiation Pattern:

→ Field Pattern → $|E_s|$ V.s θ, V.s ϕ

→ $|E_s|$ V.s θ → E-plane (vertical plane)

→ $|E_s|$ V.s ϕ → H-plane (Horizontal plane)

→ Power Pattern ($\vec{P}_{ave}$) V.s θ, V.s ϕ

For Hertzian dipole:

$$E_{\theta s} = \eta H_{\phi s} = \frac{j\eta I_o \beta dl}{4\pi r} \sin\theta \, e^{-j\beta r}$$

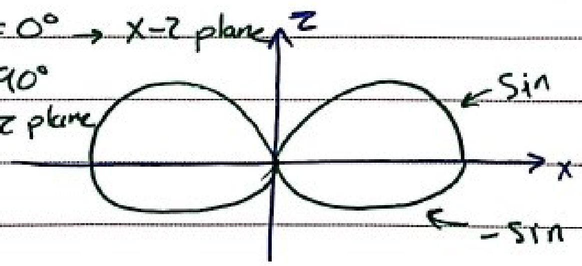
Normalized $E_{\theta s}$ → largest value is (1)

$$|E_{\theta s}| = \frac{E_{\theta s}}{E_{\theta s\,maximum}} = f(\theta) = |\sin\theta|$$

E-plane → $|E_{\theta s}|$ VS θ

$\phi = 0°$ → x-z plane

$\phi = 90°$ → yz plane



~~H-plane~~

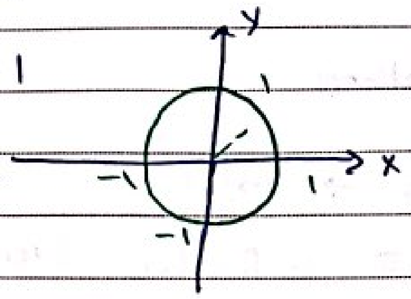
H-plane → $|E_{\theta s}|$ V.s ϕ at a certain (θ)

$|E_{\theta s}|$ is independent of ϕ

~~$\theta = \frac{\pi}{2}$ → x-y plane~~

$\Theta = \frac{\pi}{2} \rightarrow$ x-y plane

$|E_{\theta s}| = 1$



Adding E-plane and H-plane you will get a 3D pattern called Radiation pattern

$\Theta = 0°$, $\Theta = 180° \rightarrow E = 0 \rightarrow$ Null

$\Theta = \frac{\pi}{2} \rightarrow$ Max Radiation

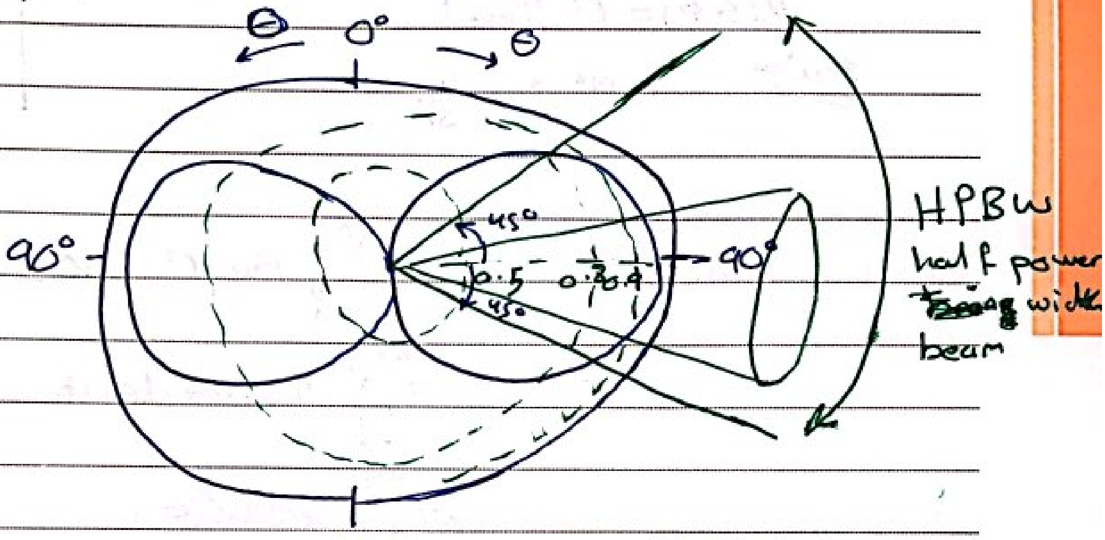
$\Theta = \pi/2 \rightarrow$ XY plane $\perp$ Z-axis
$\perp$ dipole location

Broad side Pattern (omni-directional)

Isotropic Antenna $\rightarrow$ it has same radiation in all (θ, ϕ)
↳ Not physical

Normalized polar plot

$r > \frac{2d^2}{\lambda}$

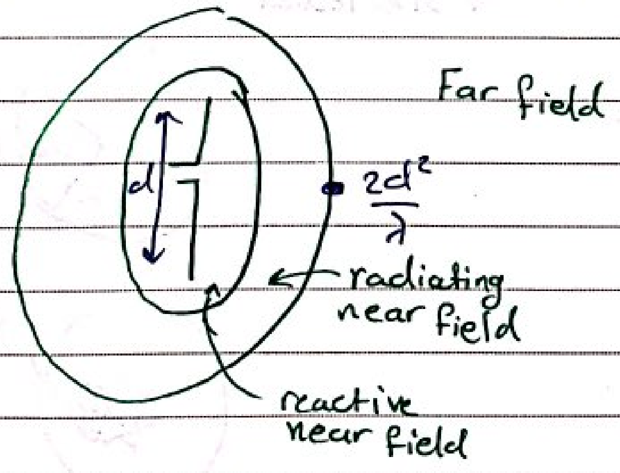


FNBW = 180°

↳ first null beam width

$P_{ave} = f^2(\theta) = \sin^2\theta$

HPBW ↑ , more people , less power to people



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B) radiation Intensity : $U(\theta,\phi)$

$U(\theta,\phi) = r^2 \, |\vec{P}_{ave}|$

Scalar

$m^2 \frac{W}{m^2} = \frac{W}{Sr}$

$\vec{P}$

$\vec{P}_{ave}$

$P_{ave} = P_{rad} = \int_s P_{ave} \cdot ds$

$P_{rad} = \int_s P_{ave} \cdot ds = \int_0^{2\pi}\int_0^{\pi} P_{ave} \, r^2 \sin\theta \, d\theta \, d\phi$

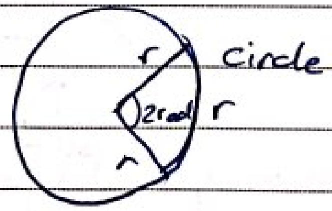
$= \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi$

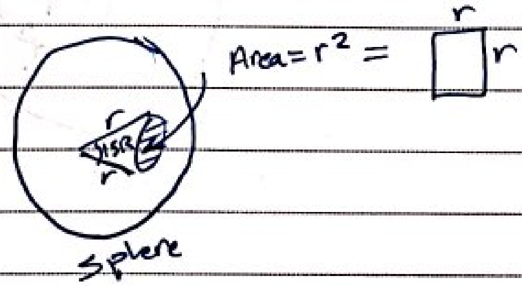
$= \int_0^{2\pi}\int_0^{\pi} U \, d\Omega$

$d\Omega = \sin\theta \, d\theta \, d\phi \Rightarrow$ differential solid angle

$\Omega = 4\pi \, (Sr)$

↳ Steriodian





$\boxed{P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \, d\Omega}$, $U = r^2 \, P_{ave}$

for isotropic antenna (constant for all (θ,ϕ))

$U = U_0 = U_{ave}$

$P_{rad} = U_0 4\pi$

$$\boxed{U_{ave} = U_0 = \frac{P_{rad}}{4\pi}}$$

Average radiation intensity

or " " " for Isotropic antenna

C) Directivity → $D(\theta,\phi)$

Directive gain → $G_d(\theta,\phi)$

$$D = \frac{U(\theta,\phi)}{U_{ave}} = \frac{4\pi U(\theta,\phi)}{P_{rad}}$$

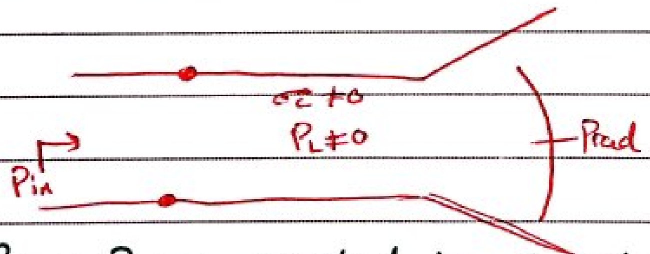
Maximum Directivity (D_{max})

$$D_o = D_{max} = \frac{4\pi U_{max}}{P_{rad}} \quad , \quad D_o\big|_{dB} = 10 \log_{10} D_o$$

D) Gain $G(\theta,\phi)$

Power gain $G_p(\theta,\phi)$

* Gain accounts for losses in the antenna Sin $R_L \neq 0$ because $\sigma_c \neq \infty$, there is (P_L)



P_{in} : Power accepted by the antenna

$$P_{in} = P_L + P_{rad}$$
$$= \frac{1}{2} I_o^2 (R_L + R_{rad})$$

$$P_L = P_{in} - P_{rad}$$

$$\boxed{G(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{in}}}$$

E) Radiation efficiency (η_r)

$$\eta_r = \frac{G}{D} = \frac{G_p}{G_d}$$, most antennas have $\eta_r \approx 95\%$

$$\eta_r = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad}+R_L}$$, $G(dB) = 10\log_{10}(G)$

*Ex. Show that the directivity for a hertzian dipole is

$D = 1.5 \sin^2\theta \Rightarrow D_o = 1.5$

$D = 1$ for Isotropic antenna (lowest value for D)

$$D = \frac{4\pi\, u(\theta,\phi)}{P_{rad}} = \frac{4\pi r^2 P_{ave}}{P_{rad}}$$

$$P_{ave} = \frac{1}{2\eta}|H_{\phi s}|^2 = \frac{1}{2}\,\frac{4 I_o^2 B^2 \phi l^2}{16\pi^2 r^2}\sin^2\theta$$

$u(\theta,\phi) = \sin^2\theta$, $|E_s| = f(\theta) = |\sin\theta|$

$P_{ave} = f^2(\theta)$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} u(\theta,\phi)\, d\Omega, \quad \iint \underbrace{P_{ave}\, r^2}_{u} \underbrace{\sin\theta\, d\theta\, d\phi}_{du}$$

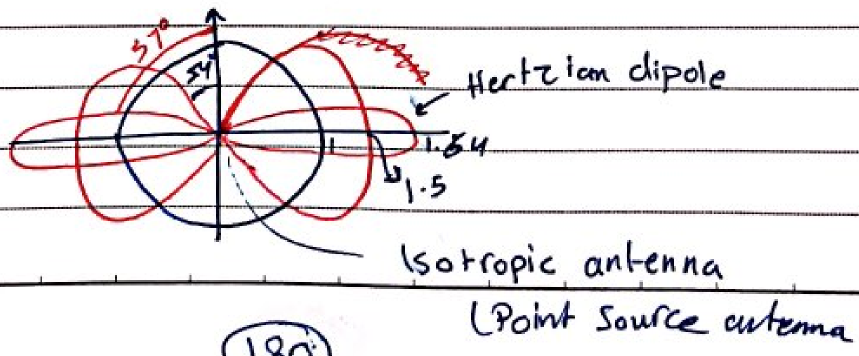
$$P_{rad} = 2\pi \int_0^{\pi} \sin^3\theta\, d\theta$$

$$= 2\pi\left(\frac{4}{3}\right)$$

$$D = \frac{4\pi \sin^2\theta}{2\pi\,(4/3)} = 1.5\sin^2\theta$$

$D = 1.5\sin^2\theta_o = 1$

$\theta_o = \sin^{-1}\left(\frac{1}{1.5}\right)^{1/2} \Longrightarrow \theta_o = 54°$



for $\lambda/2$ dipole

$D = 1.64 \; \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$

$D_o = 1.64 \rightarrow \Theta_o = 57°$

HPBW: Half Power Beam width

$\Theta = \Theta_h$

for Hertzian dipole

$U(\theta, \phi) = \sin^2\theta$

$\sin^2\Theta_h = 0.5\, U_{max}$

$\Theta_h =$

$HPBW = 2\Theta_h$

FNBW

$\Theta = \Theta_n$

$\sin^2\Theta_n = 0$

$\Theta = 0, 180°$

$FNBW = 180°$

Ex: For a certain antenna:

$$U(\theta,\phi) = 2\sin\theta \sin^3\theta$$

$$0 \leq \theta \leq \pi, \quad \cancel{\theta}\ 0 \leq \phi \leq \pi$$

and $U(\theta,\phi) = 0$, else where

Find D_o

$$D_o = \frac{U_{max}}{U_o} = \frac{4\pi U_{max}}{P_{rad}}$$

$$P_{rad} = \int U \, d\Omega$$

$$U_{max} = 2$$

$$P_{rad} = \int_0^{\pi}\int_0^{\pi} 2\sin\theta \sin^3\phi \sin^2\theta \, d\theta \, d\phi$$

$$P_{rad} = 2\int_0^{\pi}\int_0^{\pi} \sin^3\theta \sin^3\phi \, d\theta \, d\phi = \frac{32}{9}$$

$$D_o = \frac{8\pi(9)}{\cancel{2\pi}\,32}$$

$$D_o = \frac{9\pi}{4}$$

$$D_{o\,dB} = 10\log_{10}\left(\frac{9\pi}{4}\right) \text{ dB}$$

Ex. Determine $|E|$ at a distance of 10 km in free space from an antenna having $D = 5\,dB$ and $P_{rad} = 20\,kW$

$$5 = 10 \log_{10} D_o \Rightarrow D_o = 10^{0.5} = 3.162$$

$$D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi r^2 |P_{ave}|}{P_{rad}}$$

$$P_{ave} = \frac{1}{2}\eta_o |E_s|^2 \qquad \boxed{|E_s| = 0.1948\ V/m}$$

Ex. A magnetic field strength of $5\,\mu A/m$ is required at a point in $\theta = \pi/2$, 2 km from an antenna in air, ($\sigma_c \simeq \infty$) how much power must be transmitted by the antenna if

a) Hertzian dipole of $l = \lambda/25$

b) $\lambda/2$ dipole

c) $\lambda/4$ monopole

d) small loop for N=10 and $\rho_o = \lambda/20$

a) Hertzian dipole

$$|H_{\phi s}| = \frac{I_o \beta dl}{4\pi r} \sin\theta \quad , \quad \beta\, dl = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{25}$$

$$5\times10^{-4} = \frac{I_o \frac{2\pi}{25} (1)}{4\pi(2\times10^{3})} \Rightarrow I_o = 0.5A$$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad} = 40\pi^2 \left(\frac{dl}{\lambda}\right)^2 I_o^2 = 158\, mW$$

b) for a $\lambda/2$ dipole

$$|H_{\phi s}| = \frac{I_o \cos(\frac{\pi}{2}\cos\theta)}{2\pi r \sin\theta}$$

$I_o = 20\pi\, mA$

$P_{rad} = \frac{1}{2} I_o^2 R_{rad} \rightarrow 73\Omega$

$= 144\, mW$

c) for a $\lambda/4$ monopole

$P_{rad} = 72\, mW$

d) for a loop antenna

$$|H_{\phi s}| = \frac{\pi I_o S}{r \lambda^2} \sin\theta$$

$S = N(\pi \rho_o^2)$

$I_o = 40.53\, mA$

$R_{rad} = \frac{320\pi^4 S^2}{\lambda^2} = 192.3\Omega$

$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$

$= 158\, mW$