EMI

Fall017

Dr. Yanal Fauori

By: Raneem Alshair

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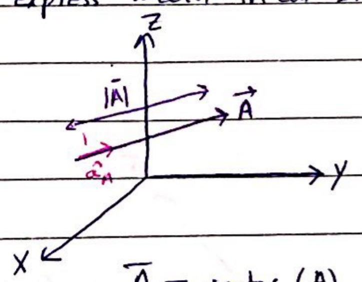
Chapter 1 Vector fields

To express avector in cartesian coordinates



$\bar{A}$ = vector (A)

$\bar{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$

A_x, A_y, A_z components of avector (magnitude)

$\hat{a}_x, \hat{a}_y, \hat{a}_z$ unit vector (direction)

Vector = Magnitude + direction

scalar = only a magnitude

$\bar{A} = (A_x, A_y, A_z)$

Magnitude of avector $A \rightarrow |\bar{A}|$ or A

$|\bar{A}| = A = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$

$\hat{a}_A$ unit vector in A Direction $\rightarrow \hat{a}_A = \frac{\bar{A}}{A} = \frac{(A_x, A_y, A_z)}{\sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}}$

→ operations on vectors :-

1. Addition

$\bar{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$

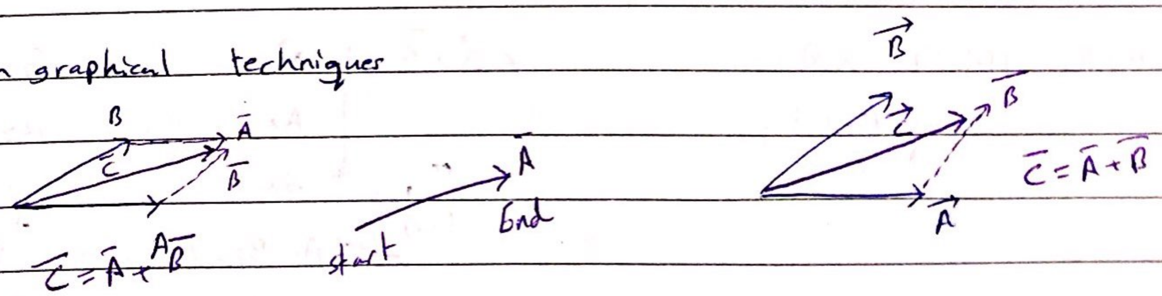
$\bar{B} = B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z$

$\bar{C} = \bar{A} + \bar{B}$

$= (A_x + B_x)\,\hat{a}_x + (A_y + B_y)\,\hat{a}_y + (A_z + B_z)\,\hat{a}_z$

$= C_x\,\hat{a}_x + C_y\,\hat{a}_y + C_z\,\hat{a}_z$

In graphical techniques

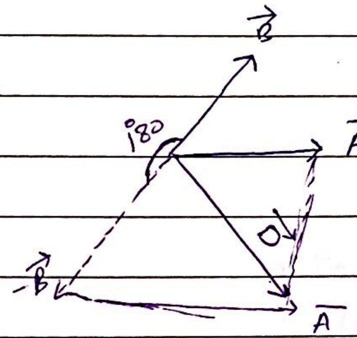


2 Substraction

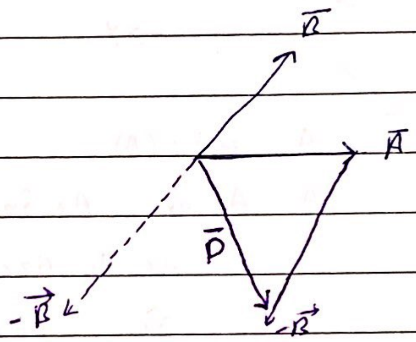
$\bar D = \bar A - \bar B$

$= (A_x - B_x)\,\hat a_x + (A_y - B_y)\,\hat a_y + (A_z - B_z)\,\hat a_z$

$= D_x\,\hat a_x + D_y\,\hat a_y + D_z\,\hat a_z$



$\vec D = \bar A - \bar B$
$= \bar A + -\vec B$



$\boxed{*}$ $\bar A + \bar B = \bar B + \bar A$

$\bar A + (\bar B + \bar C) = (\bar A + \bar B) + \bar C$

$K(\bar A + \bar B) = K\bar A + K\bar B$

3 Multiplacation

a) Dot product (scalar)

$\bar A \cdot \bar B = |A||B| \cos\theta_{AB}$

$\bar A \cdot \bar B = A_xB_x + A_yB_y + A_zB_z$

$\theta_{AB} = \cos^{-1} \dfrac{\bar A \cdot \bar B}{|A||B|}$

$* \; \hat a_x \cdot \hat a_x = 1 \cdot 1 \cdot \cos 0 = 1$

$* \; \hat a_x \cdot \hat a_y = 1 \cdot 1 \; \cos 90 = 0$

$\bar A \cdot \bar B = \bar B \cdot \bar A$

$\bar A \cdot \bar A = A^2 = A_x^2 + A_y^2 + A_z^2$

$\bar A \cdot (\bar B + \bar C) = \bar A \cdot \bar B + \bar A \cdot \bar C$

b) cross product (vector)

$|\bar A \times \bar B| = |A||B| \sin\theta_{AB}$

$\theta_{AB} = \sin^{-1}\left(\dfrac{|\bar A \times \bar B|}{AB}\right)$

$* \; \vec A \times \bar B = \begin{vmatrix} \hat a_x & \hat a_y & \hat a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

row column $(-1)^{1+1}$ $(-1)^{1+2}$

$= (+)(A_yB_z - A_zB_y)\,\hat a_x \,(-)(A_xB_z - A_zB_x)\,\hat a_y \,(+)(A_xB_y - A_yB_x)\,\hat a_z$

$(-1)^{1+3}$

$* \; |\hat a_x \times \hat a_x| = 1 \cdot 1 \cdot \sin 0^\circ = 0$

Vector Triple product

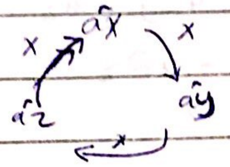
$\bar{A} \times (\bar{B} \times \bar{C}) \neq (\bar{A} \times \bar{B}) \times \bar{C}$

$|\hat{a}_x \times \hat{a}_x| = (1)(1) \sin\theta = 0$

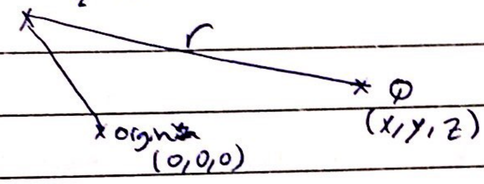
$|\hat{a}_y \times \hat{a}_z| = (1)(1) \sin 90 = 1$

$\hat{a}_x \times \hat{a}_z = -\hat{a}_y$

$a_y \times \hat{a}_z = \hat{a}_x$



*) Distance

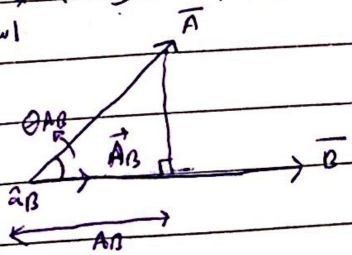


* $\vec{r}_{QP} = \vec{r}_P - \vec{r}_Q$
* $\vec{r}_{OP} = \vec{r}_P - \vec{r}_O = (x_2, y_2, z_2) - (0,0,0)$
* $\bar{r}_P = (x_2, y_2, z_2) = x_2\,\hat{a}_x + y_2\,\hat{a}_y + z_y\,\hat{a}_z$
* $\vec{r}_{QP} = \vec{r}_P - \vec{r}_Q = (x_2, y_2, z_2) - (x_1, y_1, z_1)$
  $= (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$
* $r_{QP} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
* $\hat{a}_{r_{QP}} = \dfrac{\vec{r}_{QP}}{r_{QP}}$
* $\vec{r}_{PQ} = -\vec{r}_{QP}$

*) component of a vector along another vector

$\bar{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$

x-axis جـ A بلاول



$\vec{A} \cdot \hat{a}_B = A \cos\theta_{AB}$
$= A_B$

~~AB~~ $\bar{A}_B$ along $\bar{B}$

along $\hat{a}_B$

$\cos\theta_{AB} = \dfrac{A_B}{A}$

$A_B = A \cos\theta_{AB}$

$A_B = \vec{A} \cdot \hat{a}_B \rightarrow$ scalar component of A along B

$\hat{a}_B = \dfrac{\bar{B}}{B}$

$\vec{A}_B = A_B\,\hat{a}_B$

$\vec{A}_B = (A \cdot \hat{a}_B)\,\hat{a}_B \rightarrow$ vector component of A long B example :- $\vec{A} = 3\hat{a}_x + 4\hat{a}_y + \hat{a}_z$

$\vec{B} = 2\hat{a}_y - 5\hat{a}_z$

find θ_{AB}, A_B, $\bar{A}_B$?

solution :-

$\bar{A} \cdot \bar{B} = AB\cos\theta_{AB}$ or $|\bar{A} \times \bar{B}| = AB\sin\theta_{AB}$

$|A| = \sqrt{(3)^2 + (4)^2 + (1)^2} = \sqrt{26}$

$|B| = \sqrt{(2)^2 + (5)^2} = \sqrt{29}$

$\vec{A} \cdot \vec{B} = 0 + 8 + -5 = 3$

$* \ \theta_{AB} = \cos^{-1}\left(\frac{3}{\sqrt{26}\sqrt{29}}\right) = 83.73°$

$* \ A_B = \bar{A} \cdot \hat{a}_B$

$\hat{a}_B = \frac{\vec{B}}{B} = \frac{(0, 2, -5)}{\sqrt{29}} \rightarrow \text{vector} = \frac{2}{\sqrt{29}}\hat{a}_y - \frac{5}{\sqrt{29}}\hat{a}_z$

$A_B = (3, 4, 1) \cdot \left(0, \frac{+2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\right) = 0 + \frac{8}{\sqrt{29}} + \frac{-5}{\sqrt{29}} = \frac{3}{\sqrt{29}}$ عنا اسقاط A w B
هذا طوله

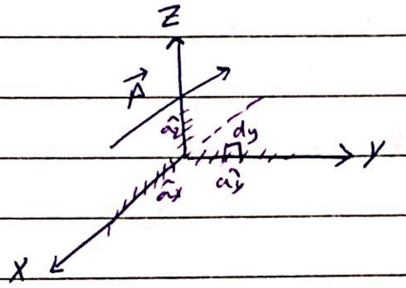
$* \ \bar{A}_B = \frac{3}{\sqrt{29}}\left(0, \frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\right)$

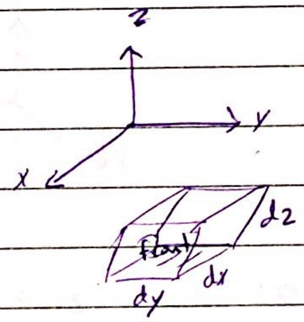
$= \frac{6}{\sqrt{29}}\hat{a}_y - \frac{15}{\sqrt{29}}\hat{a}_z$

$\boxed{\text{Chapter 2}}$ Coordinate system and Transformations

1) cartesian coordinates

$$\left.\begin{array}{l} \rightarrow -\infty < x < \infty \\ -\infty < y < \infty \\ -\infty < z < \infty \end{array}\right\} \begin{array}{l}\text{3D object} \\ \text{Infinite BOX}\end{array}$$





* Differential elements : dx, dy, dz
* Differential length : (vector)

$$\vec{dL} = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$$

* Differential normal surface area (vector)

$$\vec{ds}_{front} = dy\,dz\,\hat{a_x} \qquad \vec{ds}_{left} = -dx\,dz\,\hat{a_y}$$

$$\vec{ds}_{Back} = dy\,dz\,(-\hat{a_x}) \qquad \vec{ds}_{top} = dx\,dy\,\hat{a_z}$$

$$\vec{ds}_{bottom} = -dx\,dy\,\hat{a_z}$$

$$\vec{ds}_{right} = dx\,dz\,\hat{a_y}$$

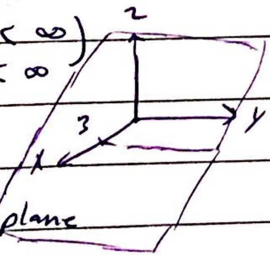
* Differential volume (scalar)

$$dv = dx\,dy\,dz$$

→ 2D surfaces (by fixing one variable)

$x = $ constant $\left(\begin{array}{l}-\infty < y < \infty \\ -\infty < z < \infty\end{array}\right)$

$x = 3$



$z =$ constant → inf plane // xy plane

$z = 0$ → inf plane along xy plane

inf plane // $\underline{\underline{yz}}$ plane

$x = 0$ → inf plane $\underline{\underline{\text{along}}}$ yz plane

$y =$ constant → inf plane // xz plane

$(y=0)$ → along

→ 1D segment (by fixing two variables)

$x = \text{constant}$ $\quad$ $z = \text{constant}$

$x = 5$ $\quad$ $z = -2$

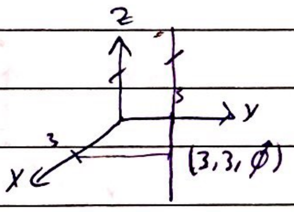
↓ inf ~~plane~~ plane $\quad$ ↓ inf plane

// yz plane $\quad$ // xy plane

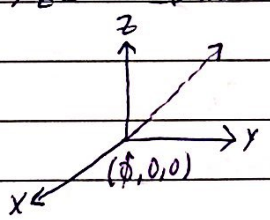
inf plane

// y-axis

ex: $x = 3$, $y = 3$ ⇒ inf line // z-axis



ex: $y = 0$, $z = 0$ ⇒ inf line along x-axis



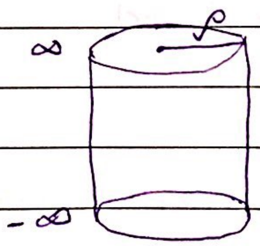
2) cylinderical coordinates

→ $0 \leq \rho < \infty$

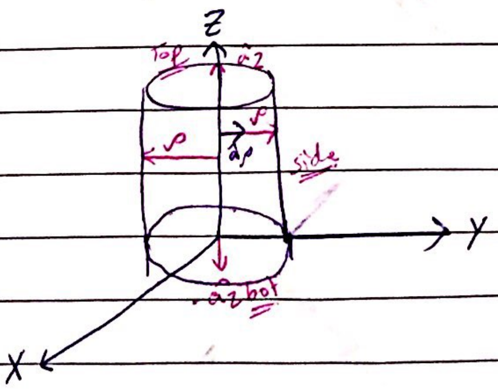
$0 \leq \phi \leq 2\pi$

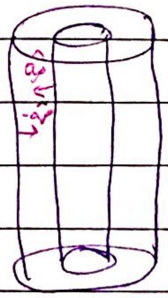
$-\infty < z < \infty$

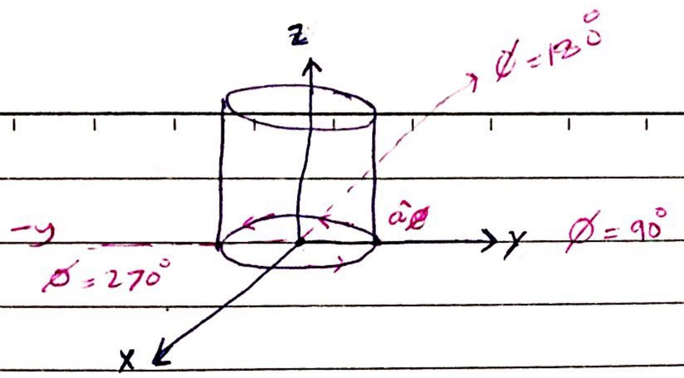
} 3D object inf cylinder

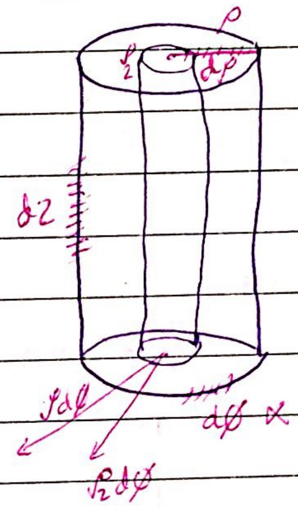


unit vectors: $\hat{a}_\rho$, $\hat{a}_\phi$, $\hat{a}_z$









⊛ Differential elements: $\underset{\text{line}}{d\rho}$, $\underset{\text{curvature}}{\rho d\phi}$, $\underset{\text{line}}{dz}$

⊛ $\vec{dl} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$

⊛ $\vec{ds}_{side} = \rho\, d\phi\, dz\, \hat{a}_\rho$

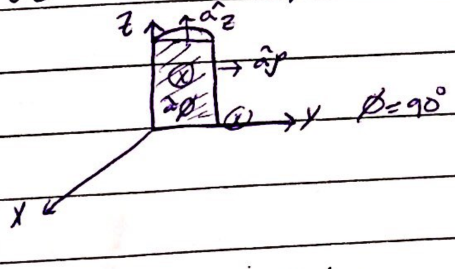
$$S = \int_0^{l} \int_0^{2\pi} \rho\, d\phi\, dz = 2\pi \rho l$$

$\vec{ds}_{top} = \rho\, d\rho\, d\phi\, \hat{a}_z$

$$S = \int_0^{2\pi} \int_0^{\rho} \rho\, d\rho\, d\phi = \frac{\rho^2}{2} 2\pi =$$

$\vec{ds}_{bot} = \rho\, d\rho\, d\phi\, (-\hat{a}_z)$

$\vec{ds} = d\rho\, dz\, \hat{a}_\phi$



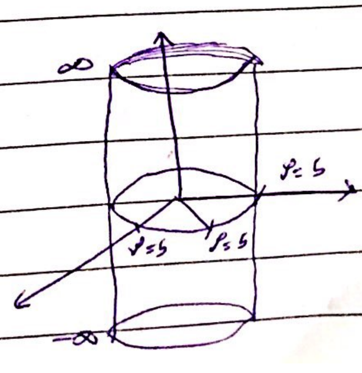
⊛ $dv = \rho\, d\rho\, d\phi\, dz$

→ 2D Surfaces

ρ = constant

$\rho = 5$ → inf hollow cylinder

if $\rho = 0$ → inf line along z-axis

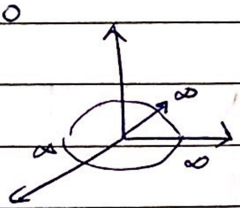


Z = constant

$0 \le \rho < \infty$

$0 \le \phi \le 2\pi$

z = 0

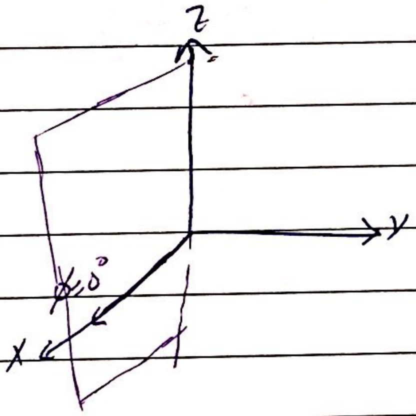


→ inf disk along xy plane

if z = -3 inf disk // xy plane

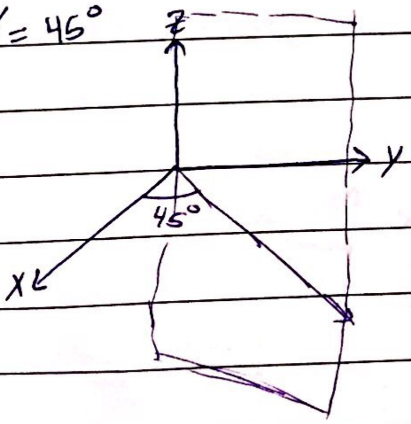
ϕ = constant

$\phi = 0°$ inf plane along xz-plane



$\phi = 90°$ → inf plane along yz plane

$\phi = 45°$

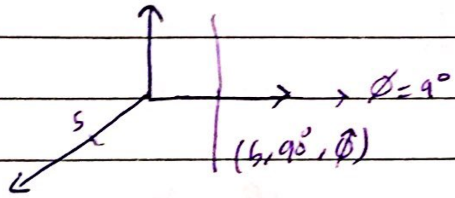


inf plane ⊥ xy plane

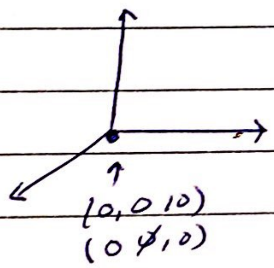
1D Segment

ρ and ϕ are constants

$\rho = 5$, $\phi = 9^\circ$ → inf line // z-axis except when $\rho = 0$ along z-axis

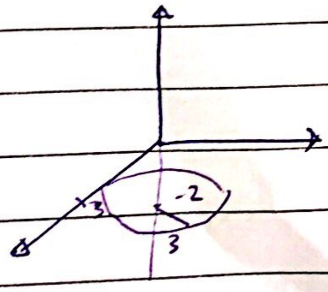


$\rho = 0$, $\phi = 0^\circ$ → z-axis / $\rho = 0$, $\phi = \frac{\pi}{2}$ → z-axis



⊛ ρ and Z are constant → circle   inf [illegible]

$\rho = 3$, $Z = -2$

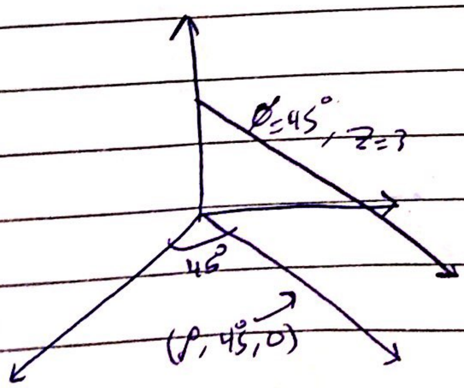


if $\rho = 0$, Z any # → point

⊛ ϕ, Z are constant

↳ semi-inf line (ray)

$\phi = 45^\circ$, $Z = 3$ → ray // xy plane



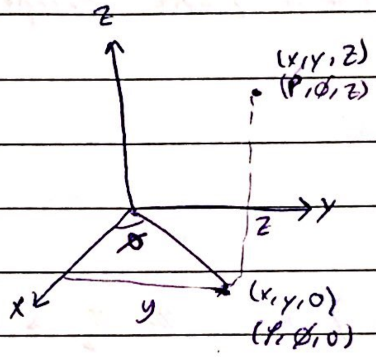
* $\phi = 0^\circ$, $Z = 2$ → ray // x-axis

$\phi = 90^\circ$, $Z = 0$ → ray along y-axis

* converting points between cyl - and cart

cart → cyl

$(x, y, z) \rightarrow (\rho, \phi, z)$?



$\rho = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$z = z$

$\sin\phi = \frac{y}{\rho}$

$\cos\phi = \frac{x}{\rho}$

cyl - cart

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z$

* convert vectors

$\bar{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$

$\bar{A} = A_\rho\,\hat{a}_\rho + A_\phi\,\hat{a}_\phi + A_z\,\hat{a}_z$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$A_\rho = A_x\cos\phi + \sin\phi\, A_y$, $\phi = \tan^{-1}\left(\frac{y}{x}\right)$

Transpose

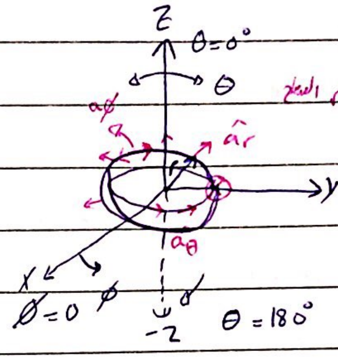
cyl → cart

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \quad = \quad \end{bmatrix} \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix}$$

3) spherical coordinates

→ $0 \le r \le \infty$
$0 \le \phi \le 2\pi$
$0 \le \theta \le \pi$
} 3D object
Inf. sphere

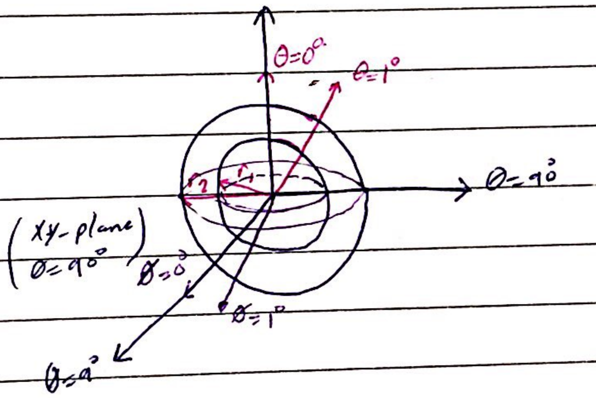


$\hat{a}_r$ طالع من سطح الكرة
$\hat{a}_\phi$ مماس من أي نقطة من السطح
$\hat{a}_\theta$

*) unit vectors : $\hat{a}_r$, $\hat{a}_\theta$, $\hat{a}_\phi$

*) Differential elements

dr , $r\,d\theta$, $r\sin\theta\,d\phi$



*) $\overline{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$

*) $\overline{ds}_{\text{Shell}} = r^2\sin\theta\,d\phi\,d\theta\ \hat{a}_r$

$$S = \int_0^{2\pi}\int_0^{\pi} r^2\sin\theta\,d\theta\,d\phi$$

$$S = r^2(2\pi)\left(-\cos\theta\Big|_0^{\pi}\right)$$

$$= \underline{\underline{4\pi r^2}}$$

$\overline{ds} = r\sin\theta\,dr\,d\phi\ \ \hat{a}_\theta$

$\overline{ds} = r\,dr\,d\theta\ \ \hat{a}_\phi$

*) $dv = r^2\sin\theta\,dr\,d\theta\,d\phi$

$$V = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$= \frac{4}{3}\pi r^3$$

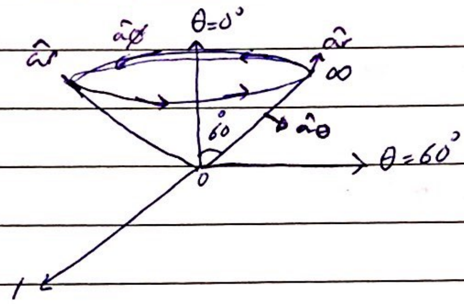
→ 2D Surfaces :

① r constant . hollow sphere

$r = 0 \rightarrow$ point

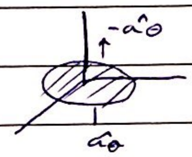
② θ = constant

$\theta = 60^\circ$ → inf hollow cone



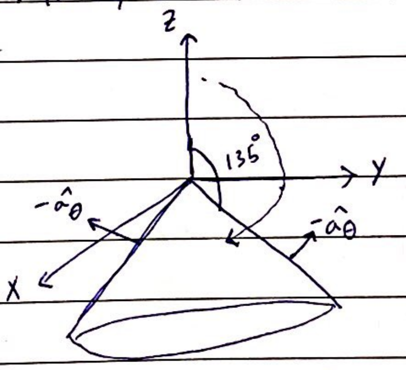
so we use $\boxed{\overline{ds} = r\sin\theta\, dr\, d\phi\ \hat{a}_\theta}$

$\theta = 90^\circ$ → inf disk



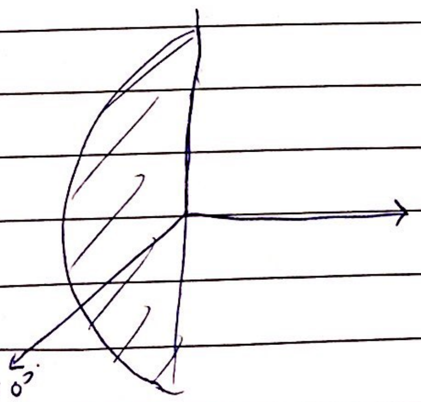
$\theta = 0^\circ$ → semi - inf. line
+ve Z-axis
(1D segment)

$\theta > 90^\circ$ → $\theta = 135^\circ$



③ ϕ constant

$\phi = 0^\circ$ → semi inf. Disk along XZ-plane



(1, 90°, 0°)
↑
x-y plane

→ 1D segment

① r, θ are constants → circle // xy plane

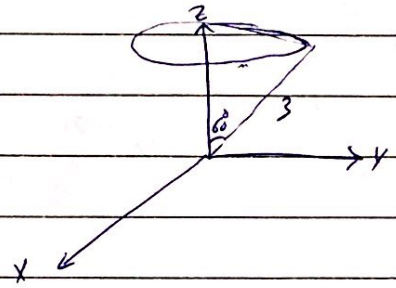
$r = 0$ → point ϕ any

$\theta = 0°$ → //

$\theta = 120°$ → //

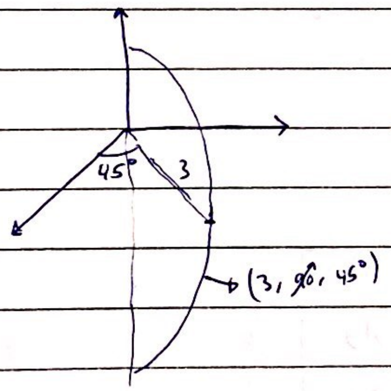
} special cases

$r = 3$, $\theta = 60°$ if r, $\theta = 90°$ → circle along xy plane



② r, ϕ are constants → half-circle

$r = 3$, $\phi = 45°$ → half circle ⊥ xy plane



③ ϕ, θ are constants

→ ray

→ semi-inf line

$\theta = 0°$, ϕ → +ve z-axis

$\theta = 180°$, ϕ → -ve z

$\theta = 90°$, $\phi = 0°$ → +ve x

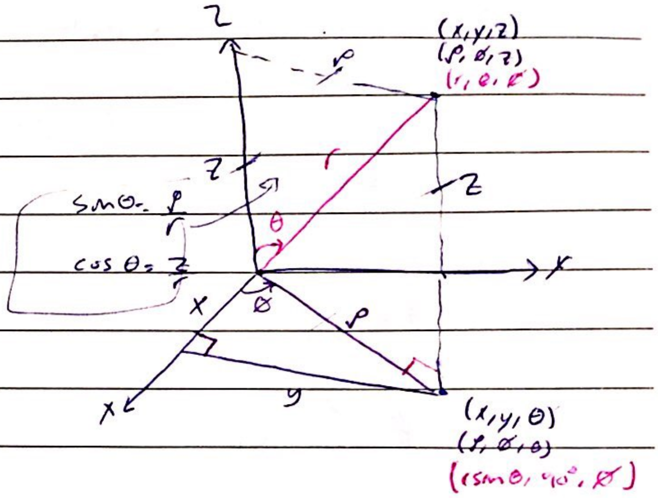
$\theta = 90°$, $\phi = 270°$ → -ve y

$\theta = 60°$, $\phi = 45°$

(*) converting between coordinates

→ points

* cart → sph	cyl → sph
(x, y, z)	(ρ, ϕ, z)
$r = \sqrt{x^2 + y^2 + z^2}$	$r = \sqrt{\rho^2 + z^2}$
$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$	$\theta = \tan^{-1}\left(\frac{\rho}{z}\right)$
$\phi = \tan^{-1}\left(\frac{y}{x}\right)$	$\phi = \phi$



sph → cyl	sph → cart
$\rho = r\sin\theta$	$x = r\sin\theta\cos\phi$
$\phi = \phi$	$y = r\sin\theta\sin\phi$
$z = r\cos\theta$	$z = r\cos\theta$

(*) vectors :

→ cart. → sph

$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$\bar{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$

$$\begin{array}{c} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{array} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \begin{array}{c} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{array}$$

↗ known

→ cyl → sph

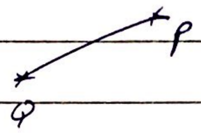
$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

← given

Chapter 3

[*] line Integral

$$\int_L \bar{A} \cdot \bar{dL}$$



$\bar{A} = (A_x, A_y, A_z)$

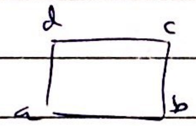
$\bar{dL} = (dx, dy, dz)$

$\bar{A} \cdot \bar{dL} = A_x dx + A_y dy + A_z dz$

$$\int_L \bar{A} \cdot d\bar{L} = \int_x A_x dx + \int_y A_y dy + \int_z A_z dz$$

[*] closed Integral

$$\oint_L \bar{A} \cdot \bar{dL}$$

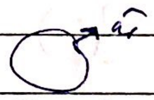


a - b - c - d - a

$$\oint_L \bar{A} \cdot d\bar{L} = \int_a^b \bar{A} \cdot d\bar{L} + \int_b^c \bar{A} \cdot \bar{dL} + \int_c^d \bar{A} \cdot \bar{dL} + \int_d^a \bar{A} \cdot \bar{dL}$$

[*] surface Integral

$$\int_s \bar{A} \cdot \bar{ds}$$

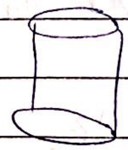


$$= \int_\phi \int_\theta A_r \, r^2 \sin\theta \, d\theta \, d\phi$$

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi$

[*] closed surface integral

$$\oint_s \bar{A} \bar{ds} = \int_s \bar{A} \bar{ds}$$



3 surface
integral

* Volume Integral

$$\int_v A\,dv = \int_z\int_y\int_x A\,dx\,dy\,dz$$

∇ :- Del operator (vector)

cart: $\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$

cyl: $\nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\hat{a}_\phi + \frac{\partial}{\partial z}\hat{a}_z$

sph: $\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{a}_\phi$

1) gradient (vector)

∇V = gradient of (V)

cart: $\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$

Example: find ∇V

cyl: $V = \rho^2 Z \cos 2\phi$

$\nabla V = \left(\frac{\partial V}{\partial \rho}, \frac{1}{\rho}\frac{\partial V}{\partial \phi}, \frac{\partial V}{\partial z}\right)$

$= 2\rho Z\cos 2\phi\,\hat{a}_\rho - 2\rho Z \sin 2\phi\,\hat{a}_\phi + \rho^2\cos 2\phi\,\hat{a}_z$

2) Divergence (scalar) — * solenoidal $\rightarrow \boxed{DIV = 0}$

$\nabla\cdot\bar{A} \equiv$ Divergence of $\bar{A}$

cart: $\nabla\cdot\bar{A} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot(A_x, A_y, A_z)$

✓ $\nabla\cdot\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

given:

in cyl $\nabla\cdot\bar{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$

in sph $\nabla\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$

$\underline{Ex:}$ $\bar{F} = \frac{1}{r^2}\cos\phi\,\hat{a_r} + r\sin\theta\cos\phi\,\hat{a_\theta} + \cos\theta\,\hat{a_\phi}$

$\nabla\cdot\bar{F} = 0 + 2\cos\theta\cos\phi + 0$

3) curl (vector)

$\circledast$ irrotational → curl = 0

$\nabla\times\bar{A} \equiv$ curl of $\bar{A}$

cart → $\nabla\times\bar{A} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$ ✓

* cyl → $\nabla\times\bar{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a_\rho} & \rho\hat{a_\phi} & \hat{a_z} \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$

given

* in sph → $\nabla\cdot\bar{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a_r} & r\hat{a_\theta} & r\sin\theta\,\hat{a_\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$

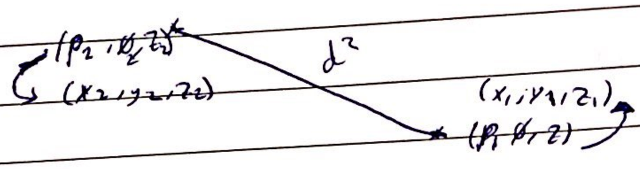
4) Laplacian

$\nabla\cdot\nabla V = \nabla^2 V$ laplacian of V

$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

ch. 6

distance :-



$$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2$$

in cyl:

$$d^2 = \rho_2^2 + \rho_1^2 - 2\rho_1\rho_2 \cos(\phi_2 - \phi_1) + (z_2-z_1)^2$$

Chapter 4

Electrostatic Fields

source of electrostatics

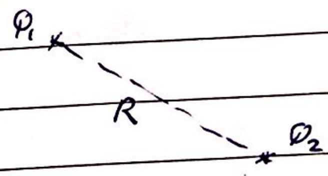
1) point charge : (Q) → unit in (C)
2) Line ~~chan~~ charge $\rho_L \to C/m$
3) surface charge $\rho_s \to C/m^2$
4) volume charge $\rho_v \to C/m^3$
5) electric dipole

Laws for electrostatics

1) coulomb's law (general case)
2) Gauss's Law (special case)

* coulomb's Law

study the force between two point charges



$F \propto \frac{Q_1 Q_2}{R^2}$ relation

$$F = \frac{k Q_1 Q_2}{R^2} \qquad k \text{ constant} \begin{cases} \text{media } (\epsilon) \\ \text{unit } \left(\frac{1}{4\pi}\right) \end{cases}$$

$$k = \frac{1}{4\pi\epsilon}, \quad \epsilon = \text{Permittivity } (F/m)$$

in ch. 4 → media will be free space

ϵ in free space $= \epsilon_0$

$\epsilon_0 \equiv$ free space premittivity

$$= \frac{10^{-9}}{36\pi} \; F/m = 8.858 * 10^{-12} \; F/m$$

$$k = \frac{1}{4\pi \frac{10^{-9}}{36\pi}} = 9 * 10^{9} \; m/F$$

$$\boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}}$$ Magnitude (N)

Force on Q_2 due to Q_1

$$\overline{F_{12}} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \hat{a}_{R_{12}} \quad \text{——} \; ①$$

$$\hat{a}_{R_{12}} = \frac{\overline{R_{12}}}{R_{12}}$$

$$\overline{F_{12}} = \frac{Q_1 Q_2 \overline{R_{12}}}{4\pi\epsilon_0 (R_{12})^3} \quad \text{——} \; ②$$

$$\overline{R_{12}} = \overline{r_2} - \overline{r_1}$$

$$\boxed{\overline{F_{12}} = \frac{Q_1 Q_2 (\overline{r_2} - \overline{r_1})}{4\pi\epsilon_0 |\overline{r_2} - \overline{r_1}|^3}} \quad ③$$

vector

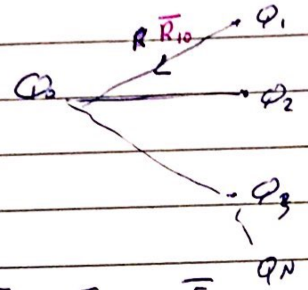
magnitude

$* \; \overline{F_{21}} = -\overline{F_{12}}$

$|\overline{F_{12}}|^* = |\overline{F_{21}}|$

$\hat{a}_{R12} = -\hat{a}_{R21}$

Force on (Q_0) due to N-charges



$$\bar{F} = \bar{F}_{10} + \bar{F}_{20} + \cdots \bar{F}_{N0}$$

$$= \frac{Q_0 Q_1 (\bar{r}_0 - \bar{r}_1)}{4\pi\epsilon_0 |\bar{r}_0 - \bar{r}_1|^3} + \frac{Q_0 Q_2 (\bar{r}_0 - \bar{r}_2)}{4\pi\epsilon_0 |\bar{r}_0 - \bar{r}_2|^3} + \cdots + \frac{Q_0 Q_N (\bar{r}_0 - \bar{r}_N)}{4\pi\epsilon_0 |\bar{r}_0 - \bar{r}_N|^3}$$

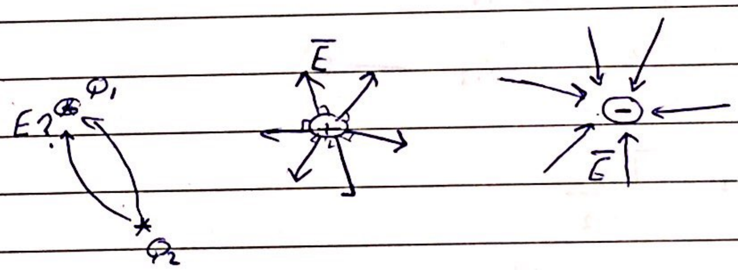
$$\boxed{F = \frac{Q_0}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_k (\bar{r}_0 - \bar{r}_k)}{|\bar{r}_0 - \bar{r}_k|^3}}$$

* Electric field Intinsity ($\bar{E}$)

$$\bar{E} = \frac{\bar{F}}{Q} \qquad \frac{N}{C} \text{ or } \frac{V}{m}$$

$\bar{E}$ field at Q_1

$$\bar{E} = \frac{\bar{F}}{Q_1}$$



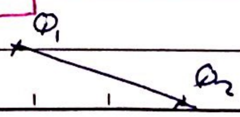
$$\bar{E} = \frac{Q_1}{4\pi\epsilon_0 (R_{12})^2} \hat{a}_{R_{12}} = \frac{\bar{F}_{12}}{Q_2}$$

$$\bar{E} = \frac{Q_1 (\bar{r}_2 - \bar{r}_1)}{4\pi\epsilon_0 |\bar{r}_2 - \bar{r}_1|^3}$$

For N charges:

$$\bar{E} = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_k (\bar{r}_0 - \bar{r}_k)}{|\bar{r} - \bar{r}_k|^3}$$

$$\boxed{\text{Distance } (r) = \text{field} - \text{source}}$$



F_{12}

source field

*Ex: point charges 1mc and 2mc located at (3, 2, -1) and (-1, -1, 4), Find $\bar{F}$ and $\bar{E}$ at 10 nc charge located at (0, 3, 1)?

Field

$$\bar{F} = \frac{1 \times 10^{-3} \times 10 \times 10^{-9} \left((0,3,1) - (3,2,-1)\right)}{4\pi \times \frac{10^{-9}}{36\pi} (\sqrt{14})^3}$$

$$\bar{F} = \frac{1 \times 10^{-3} \times 10 \times \cancel{10^{-9}} (-3\hat{a_x} + \hat{a_y} + 2\hat{a_z})}{\cancel{4\pi \times \frac{10^{-9}}{36\pi}} (\sqrt{14})^3} + \frac{(9) - 2 \times 10^{-3} \times 10 \times \cancel{10^{-9}} (1, 4, -3)}{\cancel{4\pi \times \frac{10^{-9}}{36\pi}} (26)^{3/2}}$$

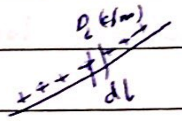
9

$$\bar{F} = -6.507\,\hat{a}_x - 3.817\,\hat{a_y} + 7.506\,\hat{a}_z \quad mN$$

$$\bar{E} = \frac{\bar{F}}{Q} = \frac{F}{10 \times 10^{-9}} = 10^8 F \Rightarrow \bar{E} = -650.7\hat{a_x} - 381.7\hat{a_y} + 780.6\hat{a_z} \quad M$$

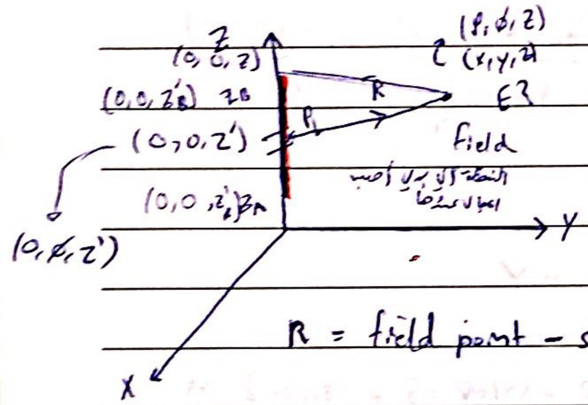
* converting charge Distribution :-

→ Line charge



$Q = \int_L \rho_L \, dL$, dL is scalar

* $\bar{E}$ - field due to a finit line along Z-axis



R = field point - source point

$\bar{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{a}_R$ for point charge

convention :-

- use dashes to represent the source point
- without " " " " field "

$\bar{E} = \int_L \frac{\rho_L \, dL}{4\pi\epsilon_0 R^2} \hat{a}_R$ for a line charge

$\hat{a}_R \to \frac{\bar{R}}{R}$

$dL = dz'$

$\boxed{\bar{R} = x\hat{a}_x + y\hat{a}_y + (z - z')\hat{a}_z}$ → Matrix

$R = x^2 + y^2 + (z - z')^2$

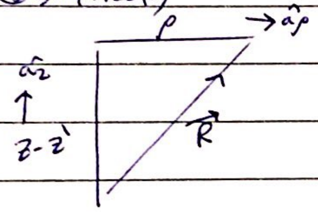
$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0} \int_{z'_A}^{z'_B} \frac{x\hat{a}_x + y\hat{a}_y + (z - z')\hat{a}_z}{\left[x^2 + y^2 + (z - z')^2\right]^{3/2}} \, dz'$$

since a line is a cylinder of ($\rho = 0$)

$\bar{R} = (\rho, \phi, z), \quad (0, \phi, z')$

$$\boxed{R = \rho \, \hat{a}_\rho + (z - z') \, \hat{a}_z} \quad ②$$

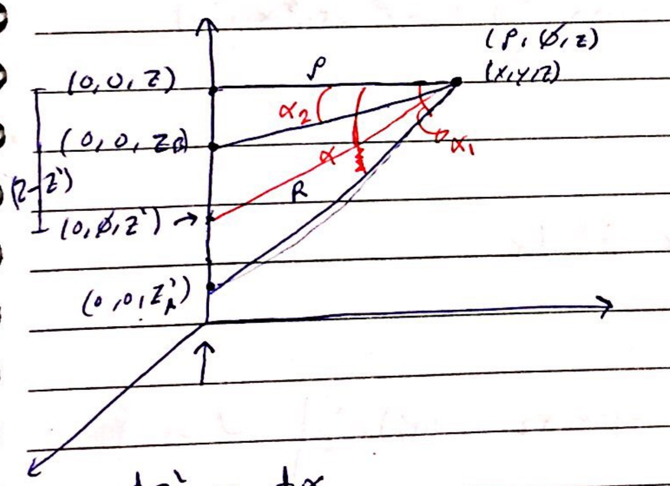
③ graphically



$|R| = \sqrt{\rho^2 + (z - z')^2} \longrightarrow$ in the next page

$$* \bar{E} = \frac{\rho_L}{4\pi\epsilon_0} \int_{z'_A}^{z'_B} \frac{\rho, 0, (z - z')}{[\rho^2 + (z - z')^2]^{3/2}} \, dz'$$

we can change the integral limits from $z'_A \longrightarrow z'_B$ to $\alpha_1 \longrightarrow \alpha_2$



$dz' = d\alpha$

$\sin \alpha = \frac{z - z'}{\sqrt{(\rho^2 + (z - z')^2)}} \longrightarrow z - z' = R \sin \alpha$

$\cos \alpha = \frac{\rho}{\sqrt{\rho^2 + (z - z')^2}} \longrightarrow z - z' = \rho \tan \alpha$

$\tan \alpha = \frac{z - z'}{\rho}$ ~~d/dα~~

$$+ \boxed{dz' = + \rho \sec^2 \alpha \, d\alpha}$$

$$R^2 = \rho^2 + (z-z')^2$$
$$= \rho^2 + \rho^2 \tan^2\alpha$$
$$= \rho^2 (1 + \tan^2\alpha)$$
$$= \rho^2 \left(1 + \frac{\sin^2\alpha}{\cos^2\alpha}\right)$$
$$= \rho^2 \left(\frac{\cos^2\alpha + \sin^2\alpha}{\cos^2\alpha}\right)$$

$$R^2 = \rho^2 \sec^2\alpha$$
$$R = \rho \sec\alpha$$
$$R^3 = \rho^3 \sec^3\alpha$$

$$\rho = R\cos\alpha$$
$$z - z' = R\sin\alpha$$

$$\bar{E} = \frac{P_L}{4\pi\epsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{\cancel{\rho}\, \cancel{\sec\alpha}\,(\cos\alpha\, \hat{a_\rho} + \sin\alpha\, \hat{a_z})\,(-\rho\, \cancel{\sec^2\alpha}\, d\alpha)}{\cancel{\rho^3}\, \cancel{\sec^3\alpha}}$$

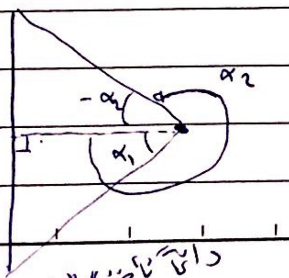
$$\bar{E} = \frac{-P_L}{4\pi\epsilon_0 \rho} \int_{\alpha_1}^{\alpha_2} (\cos\alpha\, \hat{a_\rho} + \sin\alpha\, \hat{a_z})\, d\alpha$$

$$\boxed{\bar{E} = \frac{-P_L}{4\pi\epsilon_0 \rho} \left[(\sin\alpha_2 - \sin\alpha_1)\, \hat{a_\rho} - (\cos\alpha_2 - \cos\alpha_1)\, \hat{a_z} \right] \ V/m}$$

for finit line along z-axis

$\boxed{*}$ for infinite line :- $z'_A \longrightarrow (0, 0, -\infty)$

$z'_B \longrightarrow (0, 0, \infty)$



$\alpha_1 = 90^\circ$

$\alpha_2 = -90$ or ~~270~~ 270

دائماً تأخذ الزاوية من Ref

for any infinite line

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho}\,\hat{a}_\rho} \text{ V/m}$$