EE 251: Electromagnetics 1
First Exam (Fall 2016)
November 20th, 2016

10/30

Note that bold letters are vectors

Problem 1 (6 points)

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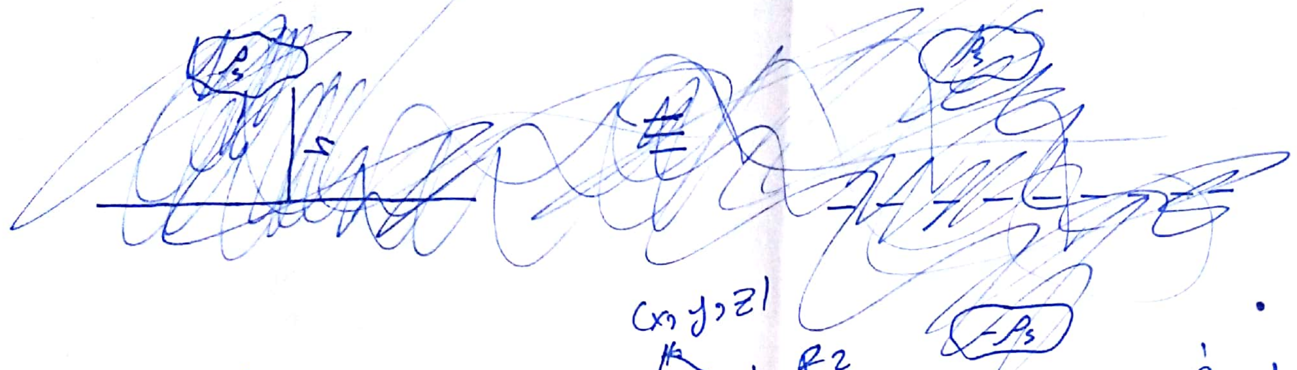
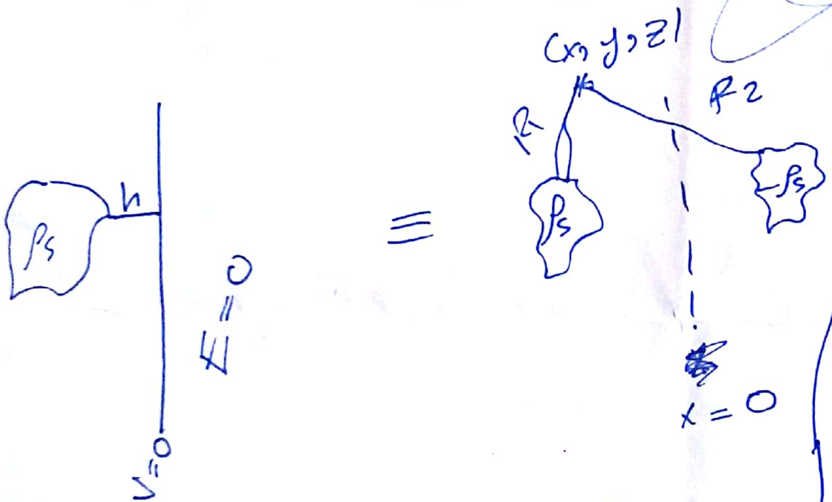
Consider a surface charge 'ρ_s' placed horizontally at a distance 'h' from a perfect grounded conducting plane of infinite extent. Find the induced charge per area on the conductor surface.





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on the conductor surface

$$E = \frac{-2hQ}{4\pi\epsilon_0}$$

$$Q = \frac{4\pi\epsilon_0}{-2h} E$$

$$R_1 = (x-h)^2 + y^2 + z^2$$

$$R_2 = (x+h)^2 + y^2 + z^2$$

$$E_{tot} = E_1 + E_2$$

$$= \frac{Q}{4\pi\epsilon_0 R_1^2}\hat{a}_r - \frac{Q}{4\pi\epsilon_0 R^2}\hat{a}_r$$

$$E = \begin{cases} \frac{Q}{4\pi\epsilon_0}\left[\frac{(x-h)^2 + y^2 + z^2}{((x-h)^2 + y^2 + z^2)^{3/2}} - \frac{(x+h)^2 + y^2 + z^2}{((x+h)^2 + y^2 + z^2)^{3/2}}\right] & x \geq 0 \\ \frac{-2hQ}{4\pi\epsilon_0} & x = 0 \end{cases}$$

Problem 2 (8 points)

A unit normal vector from region 2 ($\varepsilon = 2\varepsilon_o$) to region 1 ($\varepsilon = \varepsilon_o$) is $\mathbf{a}_{n12} = (6\mathbf{a}_x + 2\mathbf{a}_y - 3\mathbf{a}_z)/7$. If $\mathbf{D}_1 = 10\mathbf{a}_x + \mathbf{a}_y + 12\mathbf{a}_z$ A/m and $\mathbf{D}_2 = D_{2x}\mathbf{a}_x - 5\mathbf{a}_y + 4\mathbf{a}_z$ A/m, determine

(a) D_{2x}

(b) The surface charge density ρ_s on the interface

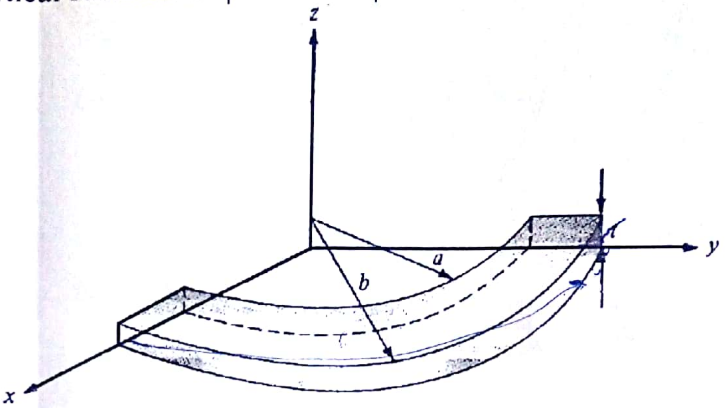
(c) The angles $\mathbf{E}_1$ and $\mathbf{E}_2$ make with the normal to the interface.

Problem 3: (8 points)

A metal bar of conductivity σ and dielectric constant of ε_r is bent to form a flat 90° sector of inner radius "a", outer radius "b", and thickness "t" as shown below. Find:

(a) the resistance of the bar between the vertical surfaces at $\phi = 0°$ and $\phi = 90°$

(b) the capacitance of the bar between the vertical surfaces at $\phi = 0°$ and $\phi = 90°$



$Q = \oint D \cdot ds$

$Q = \epsilon \oint E \; r dr d\phi$

$Q = \epsilon \int_0^{\frac{\pi}{2}} \int_b^a = \epsilon E \frac{\pi}{4} a^2 - b^2$

$E = \frac{Q}{\frac{\pi}{4} \epsilon a^2 - b^2} \hat{a}_\theta$

$V = -\int E \cdot dl = \int \frac{Q \; d\theta}{\frac{\pi}{4} \epsilon_0 a^2 - b^2} = \frac{Q \theta}{\frac{\pi}{4} \epsilon_0 a^2 - b^2}$

$C = \frac{Q}{V} = \frac{\frac{\pi}{4} \epsilon_0 a^2 - b^2}{\theta}$

$R = \frac{\epsilon C}{\sigma} = \frac{\frac{\pi}{4} \epsilon^2 a^2 - b^2}{\theta}$

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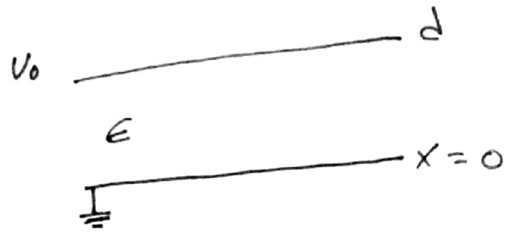
Problem 4: (8 points)

A parallel-plate capacitor has its plates at x = 0, d and the space between the plates is filled with an inhomogeneous material with permittivity $\varepsilon = \varepsilon_o(1 + x/d)$. If the plate at x = d is maintained at V_o while the plate at x = 0 is grounded, find:

(a) V and **E**

(b) **P**

(c) ρ_{ps} at x = 0, d



$$\nabla\cdot(\epsilon\cdot -\nabla V) = 0$$ (1)

$$\frac{\partial}{\partial x}(\epsilon\cdot -\nabla V) = 0$$

$$E = -\nabla V$$ (1)

$$\epsilon\cdot -\nabla V = A$$

$$\nabla V = -\frac{A}{\epsilon_0(1+\frac{x}{d})}$$

$$V =$$

(b) $P = \chi_e \epsilon_0 E$ (0.5)

(c) $\rho_{ps} = P\cdot \hat{a}_x$ (0.5)