EM I
NOTEBOOK

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POWER UNIT
ELECTRICAL ENGINEERING

EM 1

* Units:

- T = Tera
- G = Gega
- M = Mega
- K = Kilo
- m = mili
- μ = micro
- n = nano
- P = pico
- f = fimto

MKSC: mega/Kilo/ second/ cloumb

* Vectors and scalors:

- scalors : time/ charge/ current / potential / energy
  S C A V J

- Vectors : displcement / velocity / area / acceleration / length

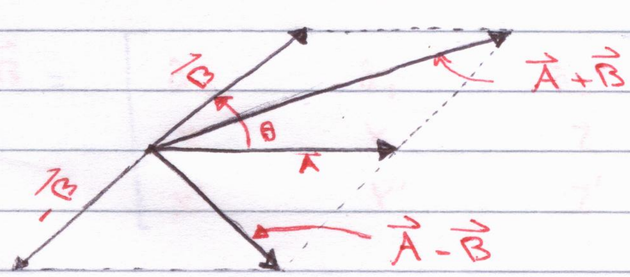
* $\vec{A} = |\vec{A}|\, \hat{a}_{\vec{A}}$

$|\vec{A}| \equiv$ Magnitude

$\hat{a}_{\vec{A}} \equiv$ unit vector (magnitude = 1; direction = in the direction of $\vec{A}$)

$\rightarrow \vec{a}_{\vec{A}} = \dfrac{\vec{A}}{|\vec{A}|}$

* $\vec{A} + \vec{B} = \vec{B} + \vec{A}$



* $c\vec{A}$ where c is a constant :

$c < 1$ $\quad c\vec{A}$

$c = 1$ $\quad \vec{A}$ $\qquad$ " the direction doesn't change "

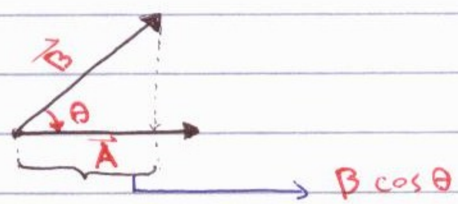
$c > 1$ $\quad c\vec{A}$

* Vector multiplication :

1) Dot product : $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta = \vec{B} \cdot \vec{A}$

2) Cross product : $\vec{A} \times \vec{B} = (|\vec{A}||\vec{B}| \sin\theta)\, \hat{a}_n$ (where $\hat{a}_n \perp \vec{A}$ & $\perp \vec{B}$)

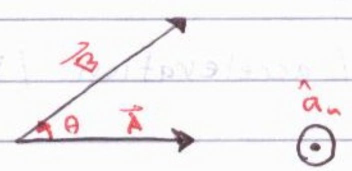
→



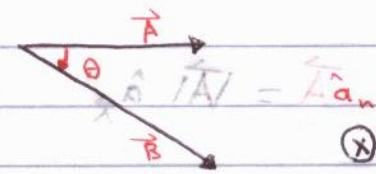
$A \cdot B = |\vec{A}||\vec{B}| \cos\theta$

(Dot)

→



Right hand Rule



$\hat{A}\, |\vec{A}| = \vec{A}\, \hat{a}_n$

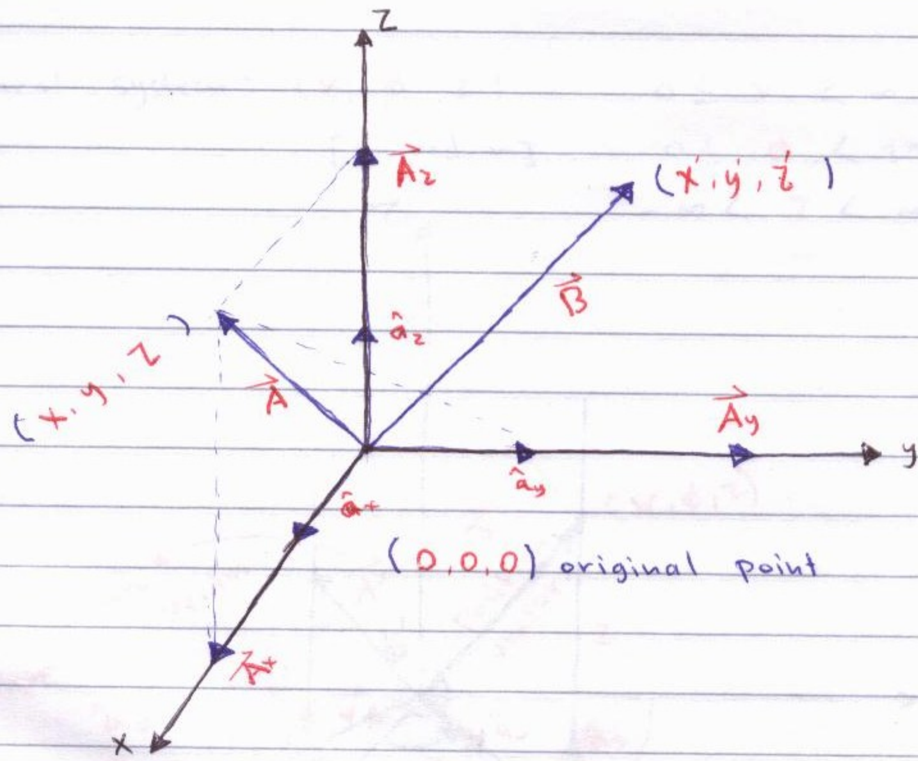
$\boxed{\text{conclution : } \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$

* Coordinate Systems :-

1) Cartesian (x, y, z) ... $-\infty < x < \infty$, $-\infty < y < \infty$, $-\infty < z < \infty$

2) Cylindrical (r, ϕ, z) ... $0 \le r < \infty$, $0 < \phi < 2\pi$, $-\infty < z < \infty$

3) Spherical (r, θ, ϕ) ... $0 \le r < \infty$, $0 \le \theta \le \pi$, $0 \le \phi < 2\pi$



$\longrightarrow \vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z = X\hat{a}_x + Y\hat{a}_y + Z\hat{a}_z$

$\longrightarrow \vec{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z = X'\hat{a}_x + Y'\hat{a}_y + Z'\hat{a}_z$

$\longrightarrow \vec{A} \mp \vec{B} = (X \mp X')\hat{a}_x + (Y \mp Y')\hat{a}_y + (Z \mp Z')\hat{a}_z$

$\longrightarrow \vec{A} \cdot \vec{B} = XX' + YY' + ZZ'$

$\longrightarrow \vec{A} \times \vec{B} = \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ X & Y & Z \\ X' & Y' & Z' \end{bmatrix}$

note :
$\hat{a}_x, \hat{a}_y, \hat{a}_z$ are fixed in direction

* $\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$ [meter]

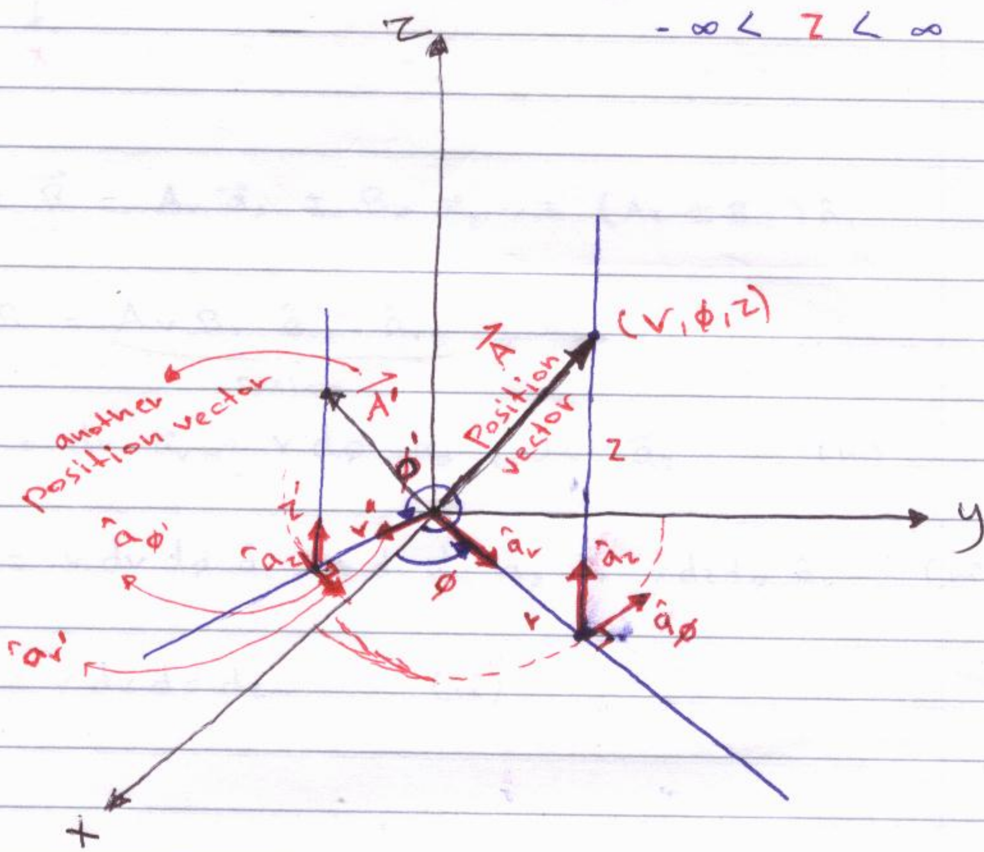
* $\vec{ds}$ (Area) $= dxdy\,\hat{a}_z + dxdz\,\hat{a}_y + dydz\,\hat{a}_x$ [meter^2]

* $dv = dxdydz$ [meter^3] "not vector"

→ Cylindrical system: (r, ϕ, z) $0 \le r < \infty$

[m, rad, m] $0 \le \phi < 2\pi$

$-\infty < z < \infty$



note:

$\hat{a}_z$ is fixed in it's direction

$\hat{a}_r$, $\hat{a}_\phi$ isn't fixed

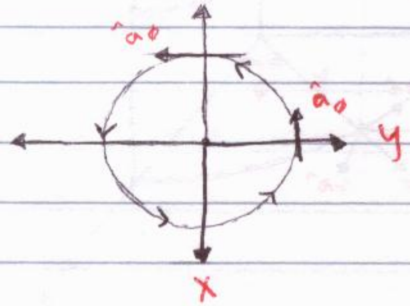
$\phi \to$ (x) تقاس بمرجعية

* Position vector :

$$\vec{A} = r\underbrace{\hat{a}_r}_{\text{changing}} + Z\underbrace{\hat{a}_z}_{\text{fixed}}$$

* General vector :

$$\vec{A}_{12} = A_{12}\,\hat{a}_r + A_{12}\phi\,\hat{a}_\phi + A_{12}Z\,\hat{a}_z$$



* $\hat{a}_\phi$ continusly changing

* r parallel to X-Y plane

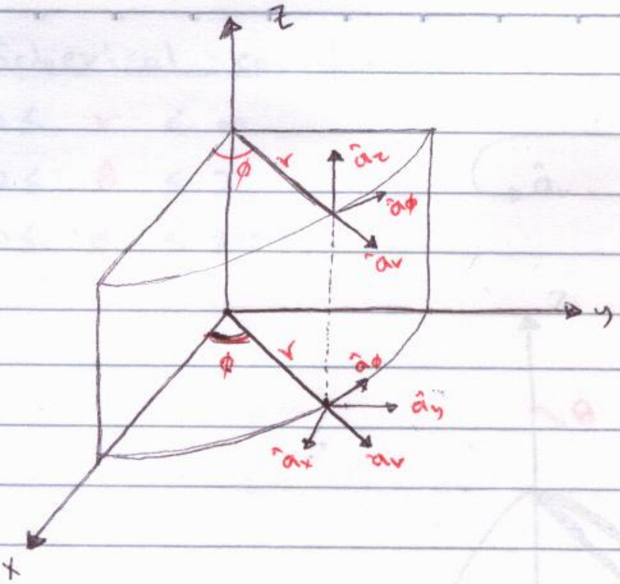
P.V $\vec{A} \pm \vec{B} = A_r\,\vec{a}_A \pm B_r\,\vec{a}_B \pm (A_z \pm B_z)\,\hat{a}_z$

P.V $\vec{A} \cdot B = \underbrace{A_r B_r\,\hat{a}_{rA} \cdot \hat{a}_{rB}}_{\text{scalar}} + A_z B_z$

$$\vec{dl} = dr\,\vec{a}_r + r\,d\phi\,\vec{a}_\phi + dz\,\vec{a}_z \qquad (m)$$

$$\vec{ds} = r\,dr\,d\phi\,\hat{a}_z + dr\,dz\,\hat{a}_\phi + r\,dz\,d\phi\,\hat{a}_r \qquad (m^2)$$

$$dv = r\,dr\,d\phi\,dz \qquad (m^3)$$



* $(x, y, z) \longleftrightarrow (r, \phi, z)$

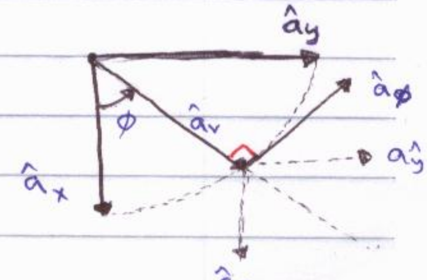
1) $x = r\cos\phi$
2) $y = r\sin\phi$
3) $r = \sqrt{x^2 + y^2}$
4) $\phi = \tan^{-1}(y/x)$
5) $z = z$

* $\hat{a}_x, \hat{a}_y, \hat{a}_z \longleftrightarrow \hat{a}_r, \hat{a}_\phi, \hat{a}_z$

1) $\hat{a}_r = \hat{a}_x\cos\phi + \hat{a}_y\sin\phi$
2) $\hat{a}_\phi = -\hat{a}_x\sin\phi + \hat{a}_y\cos\phi$
3) $\hat{a}_x = \hat{a}_r\cos\phi - \hat{a}_\phi\sin\phi$
4) $\hat{a}_y = \hat{a}_r\sin\phi + \hat{a}_\phi\cos\phi$
5) $\hat{a}_z = \hat{a}_z$

general

* $\hat{a}_r \cdot \hat{a}_r = 1$
* $\hat{a}_r \times \hat{a}_r = 0$
* $\hat{a}_i \cdot \hat{a}_i = 1$ where $i = r, \phi, z$
* $\hat{a}_i \times \hat{a}_j = \hat{a}_k$ where $i, j, k \equiv r, \phi, z$



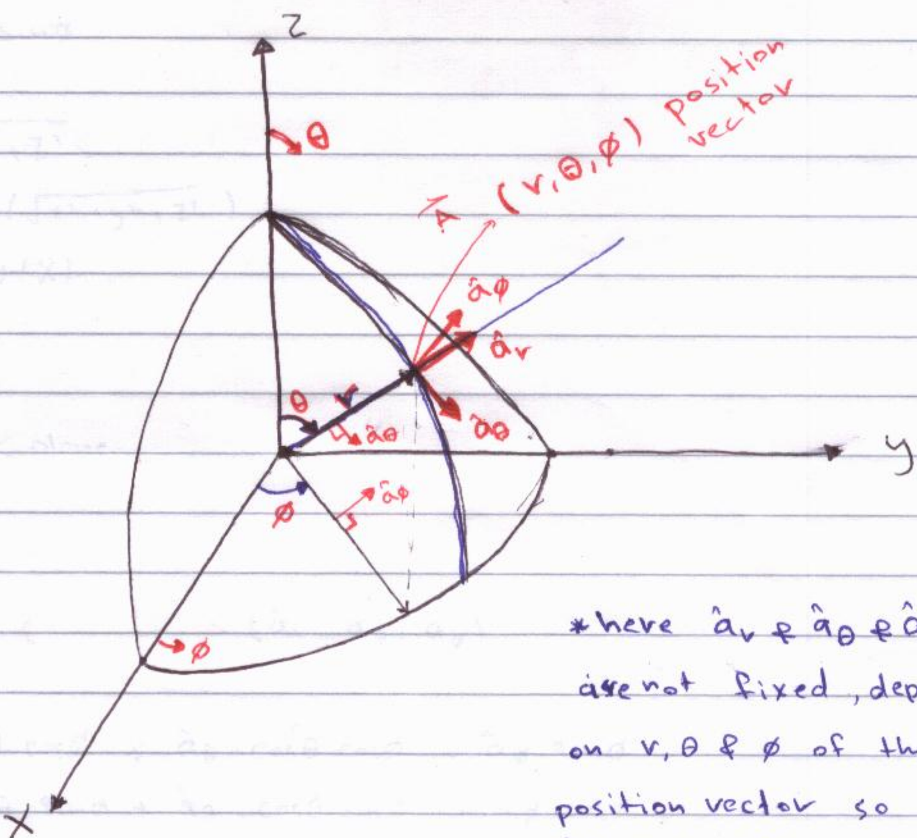
* Spherical coordinate:

$0 \le r < \infty$

$0 \le \theta \le \pi$

$0 \le \phi < 2\pi$

$\hat{a}_r \longrightarrow \hat{a}_\theta \longrightarrow \hat{a}_\phi$



*here $\hat{a}_r$ & $\hat{a}_\theta$ & $\hat{a}_\phi$ are not fixed, depends on r, θ & φ of the position vector so for $\vec{A}'$ that diffirs from $\vec{A}$ the direction of $\hat{a}_{r'}$, $\hat{a}_{\theta'}$ & $\hat{a}_{\phi'}$ will differ also from $\hat{a}_r$ & $\hat{a}_\theta$ & $\hat{a}_\phi$ of $\vec{A}$

- P.V $\quad \vec{op} = \vec{r} = r\,\hat{a}_r$

  $\vec{op'} = \vec{r}' = r'\,\hat{a}_{r'}$

- g.V $\quad \vec{PP'} = \vec{r} - \vec{r}' = A_r\,\hat{a}_r + A_\theta\,\hat{a}_\theta + A_\phi\,\hat{a}_\phi$

- $\vec{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$ (m)

- $\vec{ds} = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r + r\sin\theta\,dr\,d\phi\,\hat{a}_\theta + r\,dr\,d\theta\,\hat{a}_\phi$ (m^2)

- $dv = r^2\sin\theta\,dr\,d\theta\,d\phi$ (m^3)

* Convertion:-

i) $(x, y, z) \longleftrightarrow (r, \theta, \phi)$

1. $x = r \sin\theta \cos\phi$
2. $y = r \sin\theta \sin\phi$
3. $z = r \cos\theta$
4. $r = \sqrt{x^2 + y^2 + z^2}$
5. $\theta = \cos^{-1}(z / \sqrt{x^2+y^2+z^2})$
6. $\phi = \tan^{-1}(y/x)$

* $\hat{a}_\phi \perp \hat{a}_z$
* $\hat{a}_\phi \parallel$ X-Y plane

ii) $(\hat{a}_x, \hat{a}_y, \hat{a}_z) \longleftrightarrow (\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi)$

1. $\hat{a}_x = \hat{a}_r \sin\theta \cos\phi + \hat{a}_\theta \cos\theta \cos\phi - \hat{a}_\phi \sin\phi$
2. $\hat{a}_y = \hat{a}_r \sin\theta \sin\phi + \hat{a}_\theta \cos\theta \sin\phi + \hat{a}_\phi \cos\phi$
3. $\hat{a}_z = \hat{a}_r \cos\theta - \hat{a}_\theta \sin\theta$
4. $\hat{a}_r = \hat{a}_x \cos\phi \sin\theta + \hat{a}_y \sin\phi \sin\theta + \hat{a}_z \cos\theta$
5. $\hat{a}_\theta = \hat{a}_x \cos\phi \cos\theta + \hat{a}_y \sin\phi \cos\theta - \hat{a}_z \sin\theta$
6. $\hat{a}_\phi = -\hat{a}_x \sin\phi + \hat{a}_y \cos\phi$

* Electro static :-

→ Sources:

1) point charge q [C]
2) line charge ρ_L [C/m]
3) surface charge ρ_s [C/m²]
4) volume charge ρ_v [C/m³]

density

→ Resulting effects:

1) Force $\vec{F}_e$ [N]
2) electric field $\vec{E}$ [V/m]
3) potential V [V]
4) electric flux density $\vec{D}$ [C/m²]
   (Displacement vector)
5) energy W_e [J]
6) density w_e [J/m³]
7) power density $\vec{P}$ [w/m²]
   power P [w]
8) capacitor C [F]
9) medium charachtarstics
   → ϵ [F/m]
   → polarization (polarizability)

* Coloumb's law
* Gauss' law
* Divergence KVL

* Coloumbs law :

→ $\vec{F}_{12} = \frac{q_1 q_2}{4\pi r^2 \epsilon} \vec{a}_{r_{12}} = m\vec{a}$

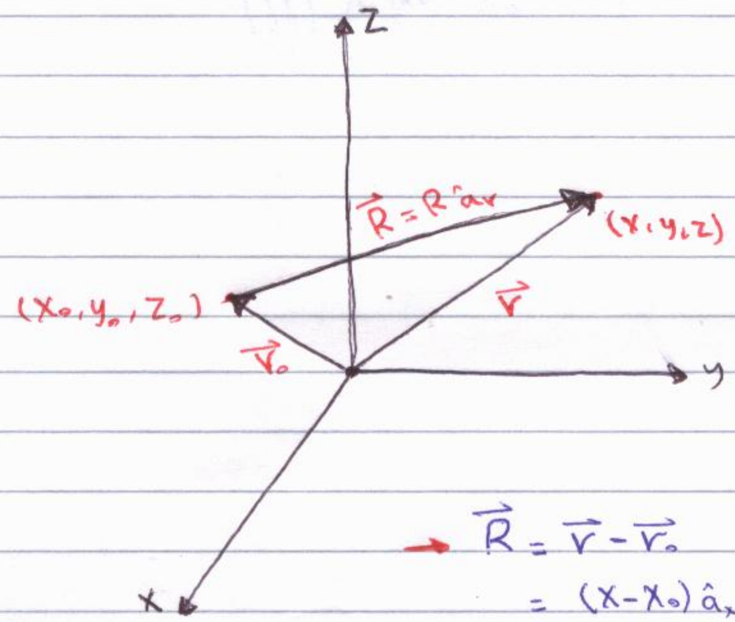
ϵ → medium permetivity [F/m]

(سماحية الوسط)

$q_1 \xrightarrow[r_{12}]{\vec{a}_{r_{12}}} q_2$

→ $\vec{F}_{12} = \underbrace{\iiint}_{V_1} \underbrace{\iiint}_{V_2} \frac{\rho_{v_1} \rho_{v_2}}{4\pi R_{12}^2 \epsilon} a_{r_{12}} \, dv_1 \, dv_2$ [N]

→ $\boxed{\vec{F} = q' \vec{E}}$ (C : V/m)



→ $\vec{R} = \vec{r} - \vec{r}_0$

$= (x - x_0)\hat{a}_x + (y - y_0)\hat{a}_y + (z - z_0)\hat{a}_z$

→ $\hat{a}_R = \frac{\vec{R}}{|\vec{R}|}$

$= \frac{(x - x_0)\hat{a}_x + (y - y_0)\hat{a}_y + (z - z_0)\hat{a}_z}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}}$

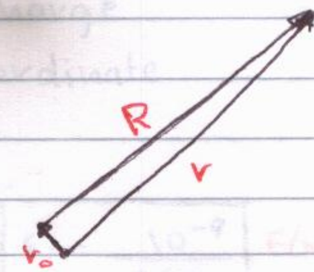
Note :

$|\vec{R}| = \sqrt{r^2 + r_0^2 - 2 r r_0 \cos\phi}$

if $r_0 \ll r \longleftrightarrow R \underset{\sim}{\parallel} r$

$|\vec{R}| \simeq [1 - 2\frac{r_0}{r}\cos\phi]^{1/2}$

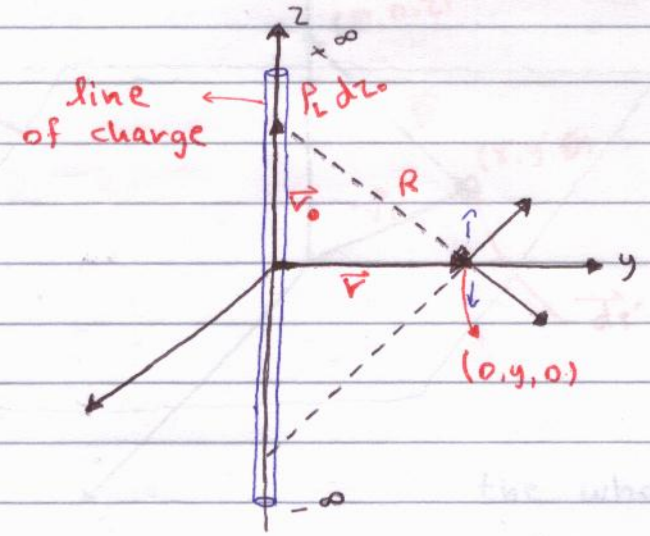
Taylor series



$$\vec{E} = \frac{1}{4\pi\epsilon} \begin{cases} \frac{q}{r^2}\,\vec{a}_r \\ \int_l \frac{\rho_l}{R^2}\,dl\,\vec{a}_R \\ \iint_s \frac{\rho_s\,ds}{R^2}\,\vec{a}_R \\ \iiint_V \frac{\rho_v\,dv}{R^2}\,\vec{a}_R \end{cases}$$

* Example:

Infinite line of charge having a line charge density ρ_L C/m, find $\vec{E}$ using cylindrical coordinate



$$\boxed{\epsilon_o = \frac{10^{-9}}{36\pi}} \ \text{F/m}$$

→ $\vec{E} \neq f(\phi) \neq f(z)$,,, only $f(r)$

→ $\vec{r} = y\,\hat{a}_y$,,, $\vec{r}_o = z_o\,\hat{a}_z$

→ $$\vec{E} = \frac{\rho_L}{4\pi\epsilon}\int \frac{dz_o}{R^2}\,\vec{a}_R$$

→ $\vec{R} = \vec{r} - \vec{r}_o = y\,\hat{a}_y - z_o\,\hat{a}_z$

→ $$\vec{a}_R = \frac{y\,\hat{a}_y - z_o\hat{a}_z}{\sqrt{y^2 + z_o^2}}$$

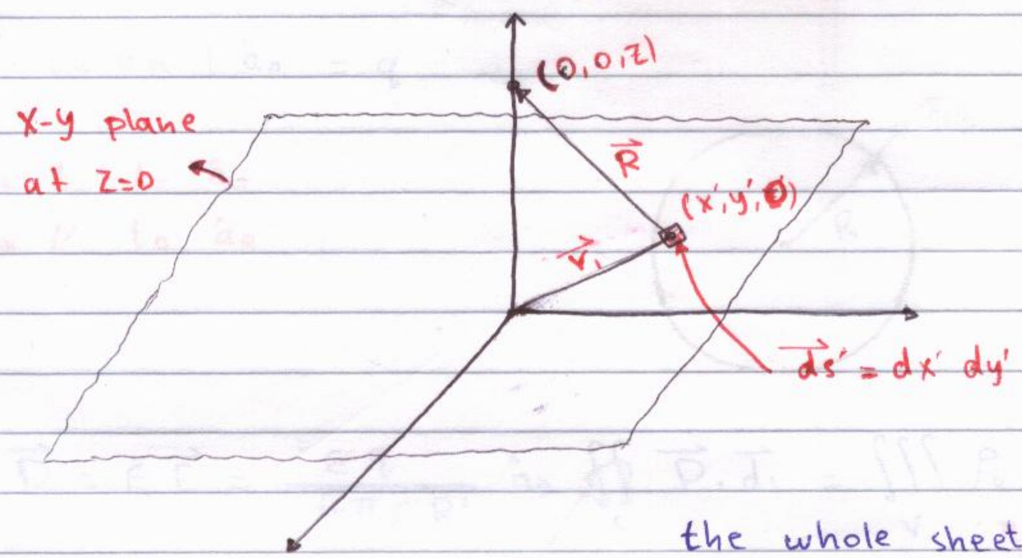
→ $$E = \frac{\rho_L}{4\pi\epsilon}\int_{-\infty}^{\infty} \frac{dz_o}{(y^2+z_o^2)^{3/2}}\,(y\,\hat{a}_y \cancel{- z_o\,\hat{a}_z}^{\,0})$$ (due to odd symmetry)

$$E_y = \frac{2\rho_L y}{4\pi\epsilon_o}\int_{0}^{\infty}\frac{dz_o}{(y^2+z_o^2)} = \boxed{\frac{\rho_L}{2\pi\epsilon_o y}} \ \text{V/m}$$

$$\boxed{E_r = \frac{\rho_L}{2\pi\epsilon r}} \ \text{V/m}$$

Example :

ρ_s (surface charge density) over ∞ sheet , find $\vec{E}$



the whole sheet is at
X-Y plane (Z=0)

→ $\vec{E} \neq f(x,y)$

→ $\vec{R} = -x'\,\hat{a}_x - y'\,\hat{a}_y + z\,\hat{a}_z$

→ $\hat{a}_R = \dfrac{-x'\,\hat{a}_x - y'\,\hat{a}_y + z\hat{a}_z}{\sqrt{x'^2 + y'^2 + z^2}}$

→ $\vec{E} = \iint \dfrac{\rho_s\, ds'}{4\pi\epsilon R^2}\,\hat{a}_R \xrightarrow[\text{symmetry}]{\text{from}} E_z = \dfrac{-\rho_s}{4\pi\epsilon}\iint \dfrac{z\, dx'\, dy'}{[x'^2 + y'^2 + z^2]^{3/2}}$

$x', y' \longrightarrow r', \phi'$

$dx'\, dy' \longrightarrow r'\, dr'\, d\phi'$

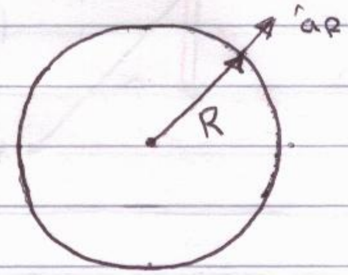
→ $\vec{E} = \displaystyle\int_0^{\infty}\int_0^{2\pi} \dfrac{r'\, d\phi'\, dr'}{[r'^2 + z^2]^{3/2}} = \boxed{\dfrac{\rho_s}{2\epsilon}}$ V/m

* Electric flux density (Displacement vector) $\vec{D}$:-

$$\vec{D}\ (\text{in } C/m^2) = \epsilon \vec{E} = \cancel{\epsilon} \frac{q}{4\pi \cancel{\epsilon} R^2}\, \hat{a}_R \ \ldots$$

$$\vec{D} \cdot (4\pi R^2)\, \hat{a}_R = q$$

- → $\perp$ to $\hat{a}_R$
- → $//$ to $\hat{a}_R$



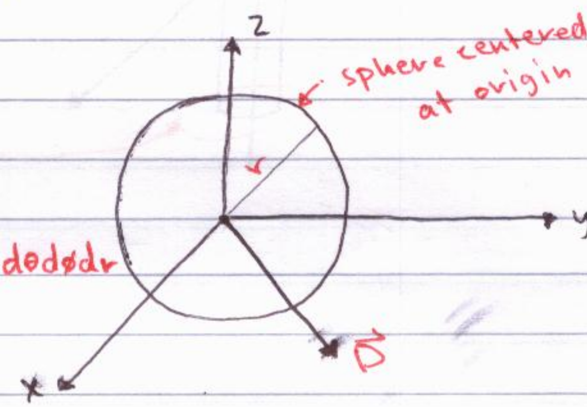
$$\vec{D} = \epsilon \vec{E} = \frac{\cancel{\epsilon} q}{4\pi \cancel{\epsilon} R^2}\, \hat{a}_R \qquad \underbrace{\oint\!\!\oint_s}_{\text{closed surface}} \vec{D} \cdot \vec{d_s} = \iiint_v \rho_v \, dv$$

closed surface

Gauss law

* Example :-

for point charge, find $D(x,y,z)$ & $D(r,\phi,z)$ & $D(r,\theta,\phi)$



→ $\vec{D} = f(r) \neq f(\theta) \neq f(\phi)$

→ $\iint \vec{D} \cdot \vec{d_s} = \iint D(r)\, \hat{a}_r\, \vec{d_s}$

↳ $r^2 \sin\theta \, d\theta d\phi dr$

→ $r^2 D_r(r) \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta \, d\theta \, d\phi = q$

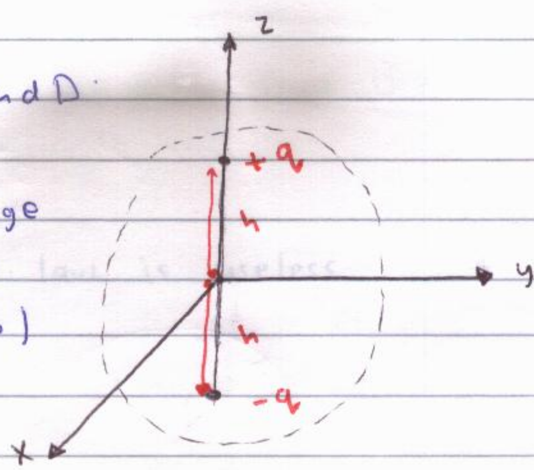
$$4\pi r^2 D_r = q \longrightarrow \boxed{D_r = \frac{q}{4\pi r^2}\, \hat{a}_r \ C/m^2}$$

example (2) :

→ 2 point charges $(+q)$ & $(-q)$, find D

$$\oiint \vec{D} \cdot \overrightarrow{ds} = \text{Zero}$$ (total charge inside the sphere = 0)

$\overrightarrow{ds} \rightarrow r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$



D ?

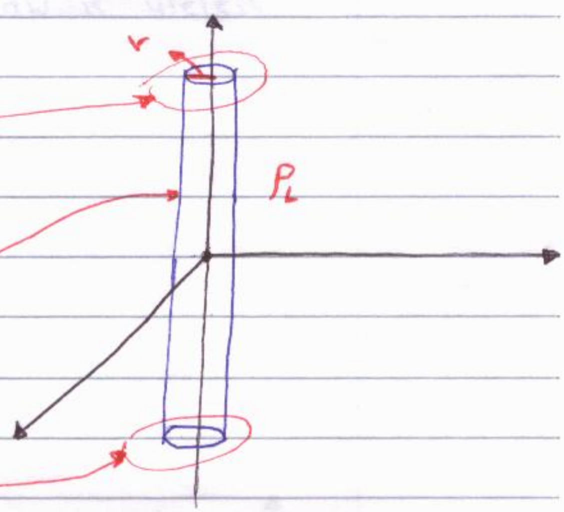
G.L is useless

Example (3) :

→ ∞ line of charge , find D ?

→ $\oiint \vec{D} \cdot \vec{ds} = \int_{-l}^{l} \rho_L \, dl$

$$\iint_u \xrightarrow{} \text{Zero} + \iint_L \xrightarrow{} \text{Zero} + \iint_s$$

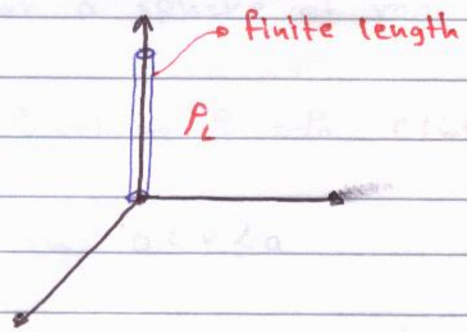


→ $ds = r \, d\phi \, dz \, \hat{a}_r$

→ $r D_r \int_{\phi=0}^{2\pi} \int_{-l}^{l} d\phi \, dz = 2 l \rho_L$

$$\boxed{D_r = \frac{\rho_L}{2\pi r}} \quad C/m^2$$

Example (4) :

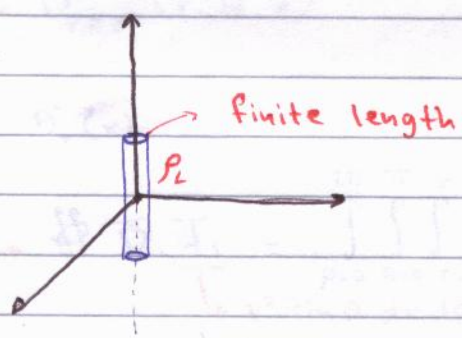


$D(r)$

$D(z)$

Gauss law is useless

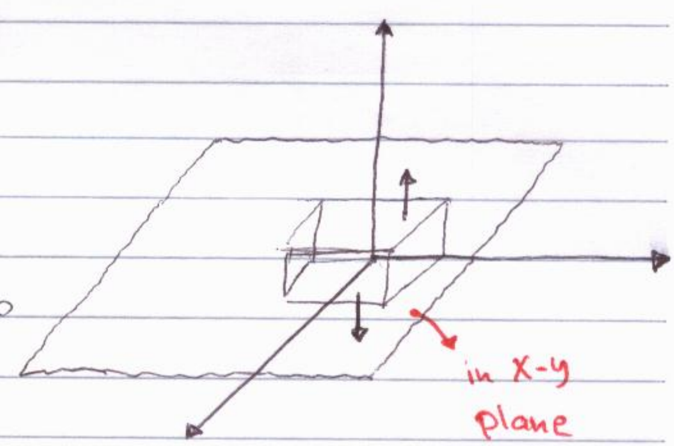
Example (5):



Gauss law is useless

Example (6) :

infinite sheet with ρ_s, find $\vec{D}$



→ $D(\cancel{X},\cancel{y},z)$ → parameters

$\cancel{X},\cancel{y},z$ → directions

→ $\oint\!\!\oint \vec{D}\cdot ds = \rho_s\, 2D_x\, 2D_y$

$\underbrace{\cancel{\iint}^{\,0}}_{\text{front}} + \underbrace{\cancel{\iint}^{\,0}}_{\text{back}} + \underbrace{\cancel{\iint}^{\,0}}_{\text{left}} + \underbrace{\cancel{\iint}^{\,0}}_{\text{right}}$

$$D_z\, \cancel{2D_x 2D_y} + D_z\, \cancel{2D_x 2D_y} = \rho_s\, \cancel{2D_x 2D_y}$$

$$\boxed{D_z = \frac{\rho_s}{2}} \quad C/m^2$$

Example (7):

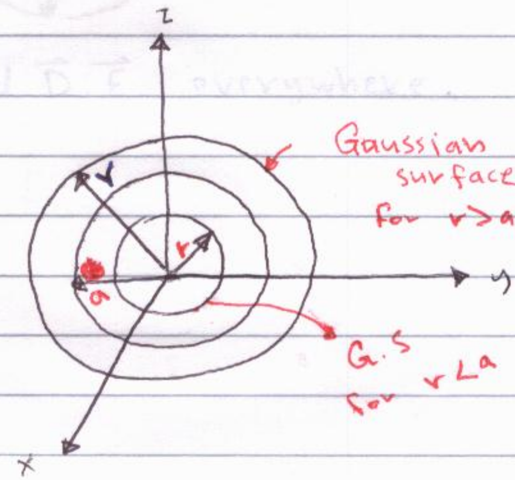
for a sphere of raduis a & $\rho_v(r,\theta,\phi)$ find $\vec{D}$?

→ $\rho_v(r) \Rightarrow \rho_v = \rho_0$ C/m²

→ in $0 \le r \le a$

→ D for $r < a$

D for $r > a$



→ $D_{r,\cancel{\theta},\cancel{\phi}}(r,\cancel{\theta},\cancel{\phi})$

$D_r(r)$

→ $\oint\oint \vec{D}\cdot\underbrace{\vec{d_s}}_{r^2 \sin\theta \, d\theta \, d\phi} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{r} \rho_v \, r^2 \sin\theta \, d\theta \, d\phi$

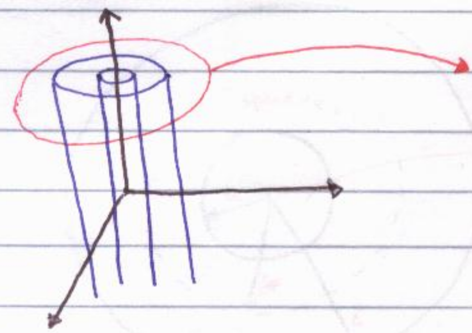
→ $\hat{a}_r = \frac{4}{3}\pi r^3 \rho_0$

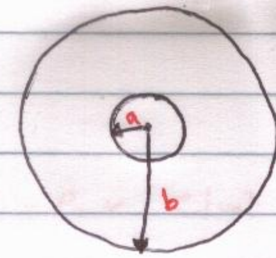
→ $D_r r^2 (4\pi) = \frac{4}{3}\pi r^3 \rho_0$

$D_r = \frac{r\rho_0}{3}$ C/m²

$= \boxed{\frac{a^3 \rho_0}{3r^2}}$ C/m²

Example (8) :





Find $\vec{D}, \vec{E}$ everywhere.

1) $0 \le r \le a$

→ $\oiint_S \vec{D} \cdot \vec{ds} = \iiint_V \rho_v \, dv$

→ $D_{r,\cancel{\phi},\cancel{z}}(r,\cancel{\phi},\cancel{z})$

$D_r \int_0^L \int_0^{2\pi} r \, d\phi \, dz = 0 \Rightarrow \underline{D_r = 0} \Leftrightarrow \underline{E_r = 0}$

$(D_r \, 2\pi r L = 0)$

2) $a < r < b$

→ $2\pi r \cancel{L} D_r = \int_0^L \rho_L \, dz = \int_0^{2\pi} \int_0^{l} \rho_s \, a \, d\phi \, dz = \rho_L \cancel{L}$

$D_r = \frac{\rho_L}{2\pi r}$ C/m^2 → $\vec{D} = D_r \, \hat{a}_r$

→ $E_r = \frac{\rho_L}{2\pi \epsilon r}$ V/m

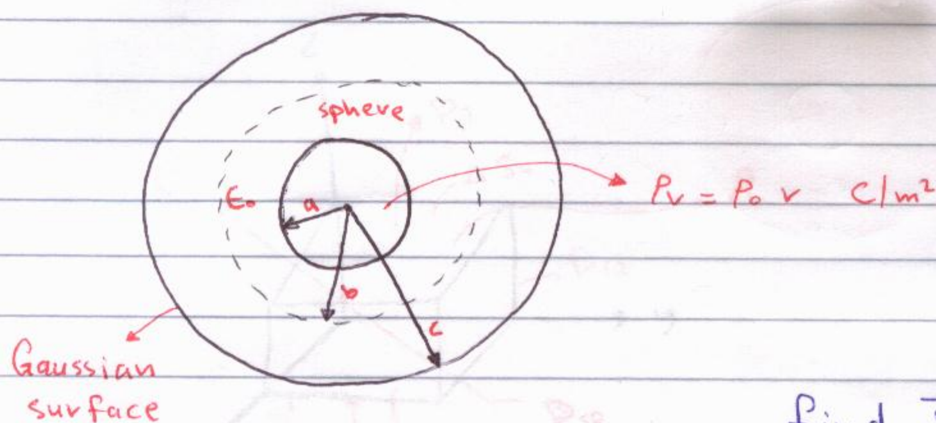
3) $r > b$

→ $\oiint \vec{D} \cdot \vec{ds} = 2\pi r L D_r = 0$

so $D_r = 0$

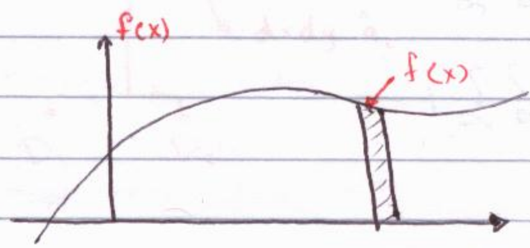
$E_r = 0$

H.W



find $\vec{D}, \vec{E}$ everywhere

* Taylor series:



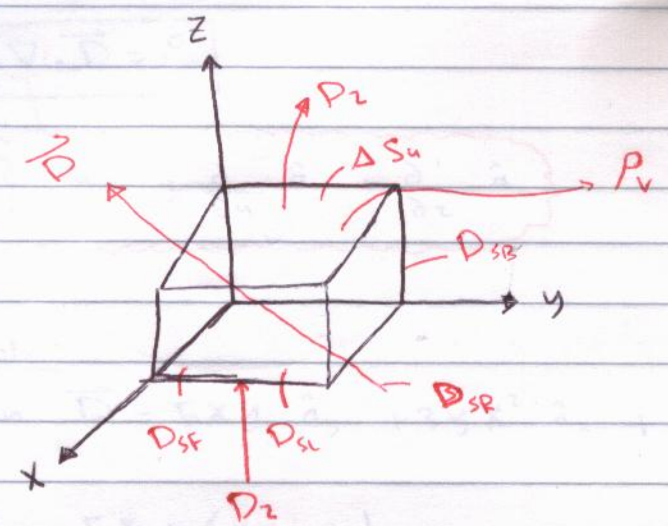
$$f(x_0) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x)\big|_{x=x_0}}{n!} (x-x_0)^n$$

Remember

$$\boxed{\begin{array}{l} 0! = 1 \\ 1! = 1 \end{array}}$$

$$= f(x_0) + f'(x_0)\,\Delta x + f''(x_0)\,\frac{\Delta x^2}{2} + \dots$$

$*\ \oiint \vec{D}\cdot\vec{ds} = \iiint_V \rho_v \, dv$



$$\iint_{D_{SLU}} + \iint_{D_{SLR}} + \iint_{D_{SFB}} \simeq \rho_v \,\Delta x \,\Delta y\, \Delta z$$

opposite Direction

because it's in z direction

$$\iint \vec{D}\cdot\vec{ds} = (-)\iint_{\Delta y\,\Delta x} D_z(0,x,y)\,\underbrace{dx\,dy}$$

$\vec{ds}$ → $dx\,dy\,\hat{a}_z$

D_z D_z'

$$+\iint_{\Delta y\,\Delta x} D_z'(D_z,x,y)\,dx\,dy$$

$$\simeq -D_z(0,x,y)\,\Delta x\,\Delta y + D_z'(D_z,x,y)\,\Delta x\,\Delta y$$

$$= \left[-D_z + D_z + \frac{\partial D_z}{\partial z}\Delta z + \frac{\partial^2 D_z}{\partial z^2}\frac{\Delta z^2}{2} + \dots\right]\Delta x\,\Delta y$$

$$\oiint_S \vec{D}\cdot\vec{ds} = \left[\frac{\partial D_z}{\partial z}\Delta z + \frac{\partial^2 D_z}{\partial z^2}\frac{\Delta z^2}{2} + \dots\right]\Delta x\,\Delta y$$

given this coordinate:

$$+\left[\frac{\partial D_x}{\partial x}\Delta x + \frac{\partial^2 D_x}{\partial x^2}\frac{\Delta x^2}{2} + \dots\right]\Delta z\,\Delta y$$

$\Delta x \to 0$

$$+\left[\frac{\partial D_y}{\partial y}\Delta y + \frac{\partial^2 D_y}{\partial y^2}\frac{\Delta y^2}{2} + \dots\right]\Delta x\,\Delta z$$

$\Delta y \to 0$

$\Delta z \to 0$

$$\simeq \rho_v \,\Delta x\,\Delta y\,\Delta z$$

cont.

* $\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_v$

→ $\boxed{\nabla \cdot \vec{D} = \rho_v}$

$\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$

Example:

Given $\vec{D} = 5XY\,\hat{a}_y + 3YX^2\,\hat{a}_x + Z\,\hat{a}_z$, find ρ_v

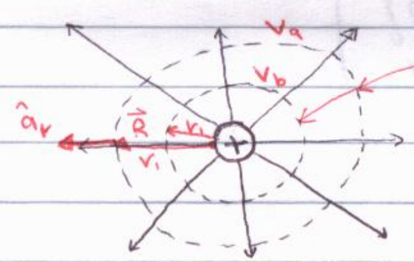
→ $\rho_v = 5X + 6YX + 1$

→ $\oiint_s \vec{D}\cdot\vec{ds} = \iiint_v \rho_v\, dv \longleftrightarrow \nabla\cdot\vec{D} = \rho_v$

* Potential (V) in volt: (كمية الشغل المبذول لنقل شحنة من نقطة إلى أخرى)

→ $V_{ab} = - \int \vec{E}\cdot\vec{dl}$ ⇛ "not a vector"

opposite direction



خطوط تساوي الجهد

→ $\boxed{V_{ab} = V_b - V_a}$

→ $V_{11} = - \oint \vec{E}\,\vec{dl} = 0$ KVL

$$* \; V_{ab} = -\int_a^b \frac{q}{4\pi\epsilon R^2} \, \hat{a}_R \cdot \vec{dl}$$

$$\rightarrow \; dl = dr \, \hat{a}_r + r \, d\phi \, \hat{a}_\phi + \cdots$$

$$\rightarrow \; V_{ab} = -\int_{r_1}^{r_2} \frac{q}{4\pi\epsilon r^2} \, dr$$

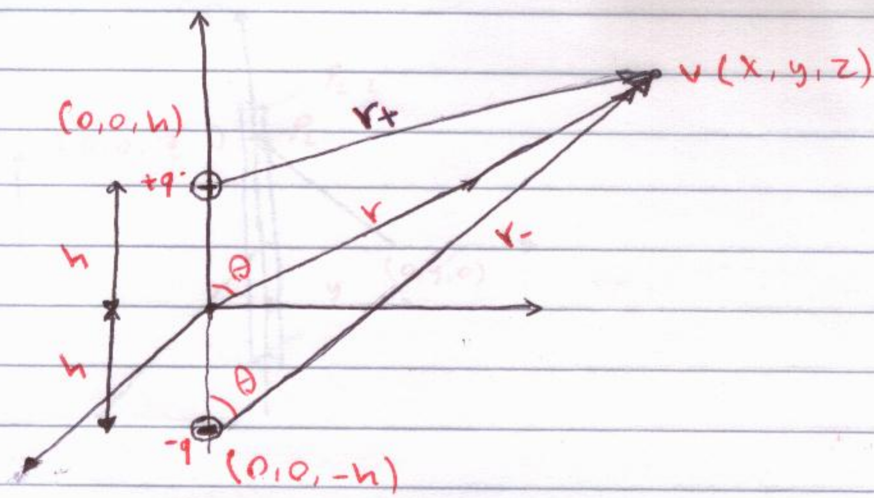
$$= \frac{q}{4\pi\epsilon} \left[\frac{1}{r_2} - \frac{1}{r_1} \right] \; (V)$$

$$\rightarrow \; \boxed{V = \frac{q}{4\pi\epsilon r}} \; (V)$$

$$\rightarrow \; V = \frac{1}{4\pi\epsilon} \begin{cases} \dfrac{q}{r} \\[2ex] \displaystyle\int_l \frac{\rho_l \, dl}{r} \\[2ex] \displaystyle\iint_s \frac{\rho_s \, ds}{r} \\[2ex] \displaystyle\iiint_v \frac{\rho_v \, dv}{r} \end{cases}$$

example:

* 2 poit charges $+q, -q$, find $V(x, y, z)$



* $r_\pm = \sqrt{x^2 + y^2 + (z \pm h)^2}$

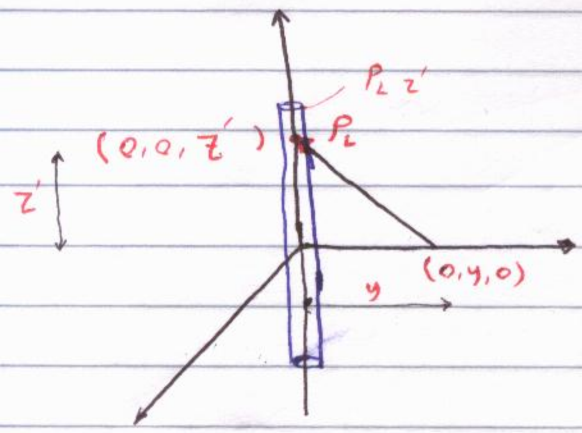
$$= \left[r^2 + h^2 \mp 2rh \cos\theta \right]^{1/2}$$

→ $V = V_+ + V_-$

$$= \frac{q}{4\pi\epsilon} \left[\frac{1}{\sqrt{x^2+y^2+(z-h)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+h)^2}} \right]$$

Example :-

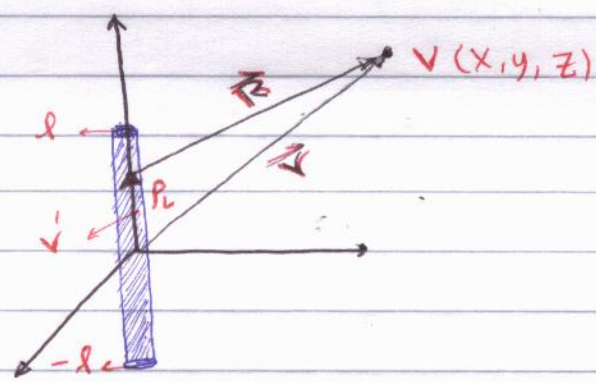
for infinite line of charge , find V ?



$\ast \rightarrow$ $V(r, \cancel{\phi}, \cancel{\phi})$

$$V_e = \int_{-\infty}^{\infty} \frac{\rho_L \, dz'}{4\pi\epsilon\sqrt{z'^2 + y^2}} \; \cdots = \infty$$

Example :-



find $V(x,y,z)$

$$\ast \; V(x,y,z) = \frac{\rho_L}{4\pi\epsilon} \int_{-\ell}^{\ell} \frac{dz'}{\sqrt{x^2 + y^2 + (z - z')}}$$

$$V = \frac{q}{4\pi\epsilon}\left[\frac{1}{r_+} - \frac{1}{r_-}\right]$$

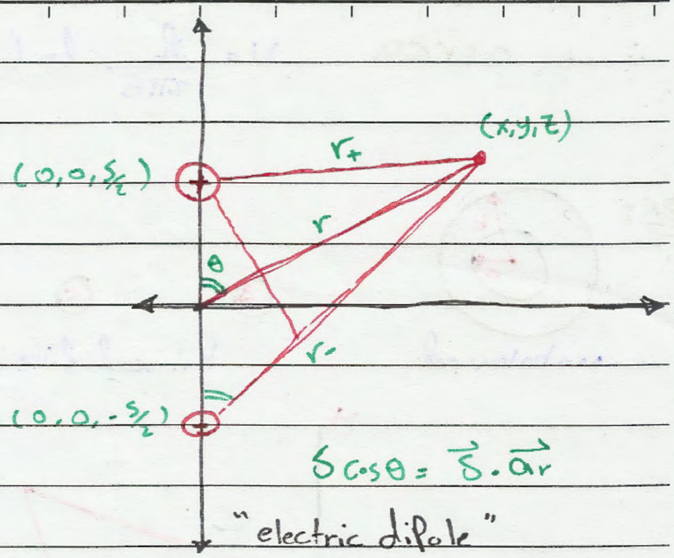
$$= \frac{q}{4\pi\epsilon}\left[\frac{r_- - r_+}{(r_+)(r_-)}\right]$$

$$\simeq \frac{q\, \delta \cos\theta}{4\pi\epsilon r^2} \longrightarrow \delta\cos\theta = q\vec{\delta}\,\vec{a_r}$$

$\equiv$ electric dipole moment

$$\equiv \vec{m_e} = q\vec{\delta}$$

$\oplus \leftarrow \ominus$



$\delta\cos\theta = \vec{\delta}\cdot\vec{a_r}$

"electric dipole"

$$r_\pm = r \mp \frac{\delta}{2}\cos\theta$$

$$V = \frac{q\, \delta \cos\theta}{4\pi\epsilon r^2}$$

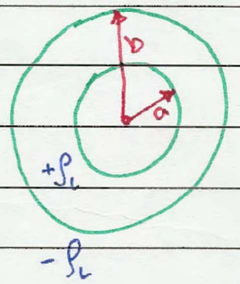
$$V = \frac{\vec{m_e}}{4\pi\epsilon_0 r^2}\cdot\vec{a_r} \qquad \boxed{r\cdot\vec{a_r} = \vec{r}}$$

$$\therefore\ V = \frac{\vec{m_e}\cdot\vec{r}}{4\pi\epsilon_0 r^3}$$

* if the dipole center is not at the origin then the equation become:

$$V = \frac{\vec{m_e}\cdot(r - r')}{4\pi\epsilon\,|r - r'|^3}$$

ex:



$$E_r = \begin{cases} \dfrac{\rho_L}{2\pi\epsilon r} & \text{for } a \le r \le b \\ 0 & \text{other wise} \end{cases}$$

zero , $r > b$

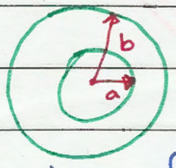
$$V = -\int_{\text{ref point}}^{\text{end point}} \vec{E}\cdot\vec{dl}$$

$$-\int_b^r \frac{\rho_L}{2\pi\epsilon r}\,dr = \frac{\rho_L}{2\pi\epsilon}\ln\left(\frac{b}{r}\right)$$

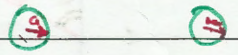
$$-\int_\infty^r E\cdot dl = \int_\infty^{b} 0 + \int_b^a + \int_a^r \vec{E}\cdot\vec{dl}$$

∴ → ocrca $\quad V = \dfrac{\rho_L}{2\pi\epsilon} \ln\left(\dfrac{b}{a}\right) \quad \underline{\underline{V}}$

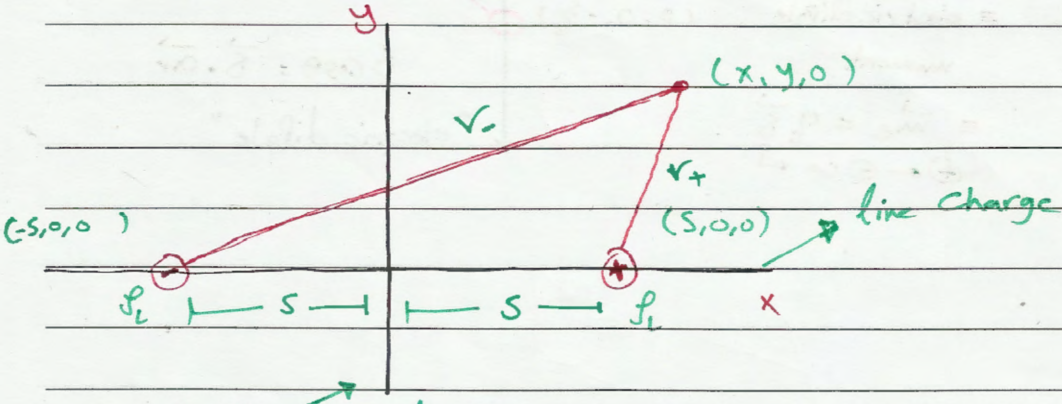
ex:



unbalanced



balanced line



⊛ ref Point (Surface)
$(V \simeq 0)$

$$V = \frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{s}{r_+}\right) - \frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{s}{r_-}\right)$$

$$= \frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{r_-}{r_+}\right)$$. equation of an equapotential Surface.

$$r_\pm = \sqrt{(x \mp s)^2 + y^2}$$

$$\frac{r_-}{r_+} = \sqrt{\frac{(x+s)^2 + y^2}{(x-s)^2 + y^2}} = e^{\frac{2\pi\epsilon V}{\rho_L}} = \gamma \quad \text{Constant}$$

$\Downarrow$

$$V = \frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{r_-}{r_+}\right)$$

$$e^{\frac{2\pi\epsilon}{\rho_L} * V} = e^{\ln\left(\frac{r_-}{r_+}\right)}$$

....

$$(x+s)^2+y^2 = \gamma^2\left[(x-s)^2+y^2\right]$$

$$x^2+2xs+s^2+y^2 = \gamma^2\left[x^2-2xs+s^2+y^2\right]$$

$$x^2+2xs+s^2+y^2 = \gamma^2x^2-2xs\gamma^2+s^2\gamma^2+y^2\gamma^2$$

$$x^2-\gamma^2x^2+2x\delta+2xs\gamma^2+\delta^2-\delta^2\gamma^2+y^2-y^2\gamma^2=0$$

$$x^2(1-\gamma^2)+2xs(1+\gamma^2)+\delta^2(1-\gamma^2)+y^2(1-\gamma^2)=0$$

$$x^2+2x\delta\left(\frac{1+\gamma^2}{1-\gamma^2}\right)+\delta^2+y^2=0$$

$$y^2+\left[x-\delta\frac{(\gamma^2+1)}{(\gamma^2-1)}\right]^2 = -\delta^2+\delta^2\left(\frac{\gamma^2+1}{\gamma^2-1}\right)^2$$

$$= \left(\frac{2\delta\gamma}{\gamma^2-1}\right)^2$$

→ Center at $\left(\delta\left(\frac{\gamma^2+1}{\gamma^2-1}\right), 0\right)$

→ rad $= \frac{2\delta\gamma}{\gamma^2-1} = r$

ex:

$$V = \frac{\rho_L}{2\pi\varepsilon}\ln\left(\frac{r_-}{r_+}\right)$$

$$r = \frac{2\gamma s}{\gamma^2-1}$$

$$\boxed{\frac{h}{r} = \frac{\gamma^2+1}{2\gamma}}$$

$$h = \delta\,\frac{\gamma^2+1}{\gamma^2-1}$$

$$\boxed{\gamma = \frac{r_-}{r_+} = e^{\frac{2\pi\varepsilon V}{\rho_L}}}$$

$$\gamma^2 - 2\gamma \frac{h}{r} + 1 = 0$$

$$\gamma = \frac{h}{r} + \sqrt{\left(\frac{h}{r}\right)^2 - 1}$$

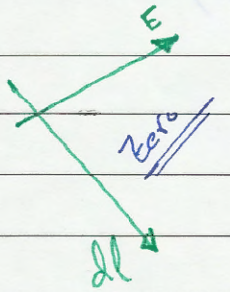
$$V = \frac{\rho_L}{2\pi\epsilon} \ln\left[\frac{h}{r} + \sqrt{\frac{h^2}{r^2} - 1}\right]$$

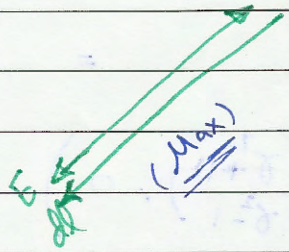
$$V_a = \frac{\rho_L}{\pi\epsilon} \ln\left[\frac{h}{r} + \sqrt{\frac{h^2}{r^2} - 1}\right] \text{ Volt}$$

Find $\underline{E}, D \Rightarrow \rho_c(\phi)$

نشتقه بالنسبة ل (ℓ)

$$V = -\int_a^b \vec{E} \cdot \vec{dl} \Rightarrow L \to \Delta L$$





$$\Delta V \simeq -\vec{E} \cdot \vec{\Delta L} \qquad \vec{E} \simeq -\frac{\Delta V}{\Delta L}$$

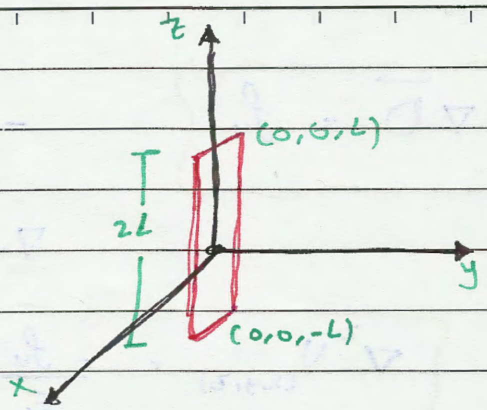
$$E_{x,y,z} \simeq \frac{-\Delta V}{\Delta x, y, z}$$

$$E_x = \frac{-dv}{dx} \qquad E_y = \frac{-dv}{dy}$$

$$E = -\nabla V = -\left[\frac{dv}{dx}\vec{a_x} + \frac{dv}{dy}\vec{a_y} + \frac{dv}{dz}\vec{a_z}\right]$$

$\vec{E}$ is the gradiant of $\underline{\underline{V}}$

ex 8



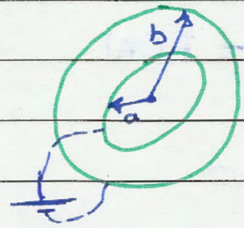
$$\nabla \rightarrow \frac{d}{dx}\vec{a}_x + \frac{d}{dy}\vec{a}_y + \frac{d}{dz}\vec{a}_z$$

$$\frac{d}{dr}\vec{a}_r + \frac{d}{r\,d\phi}\vec{a}_\phi + \frac{d}{dz}\vec{a}_z$$

$$\frac{d}{dr}\vec{a}_r + \frac{d}{r\,d\theta}\vec{a}_\theta + \frac{d}{r\sin\theta\,d\theta}\vec{a}_\theta$$

$$\rightarrow \boxed{E = -\nabla V}$$

ex: In the Coaxial Find $\vec{E}, \vec{D}, V, \rho_s$, assuming that the inner Conductor Voltage $= V_0$



$$V_{ab} = V_0 = \frac{\rho_L}{2\pi\epsilon}\ln\left(\frac{b}{a}\right)$$

$$\rho_{si} = \frac{\rho_L}{2\pi a} \quad ; \quad \rho_{so} = \frac{\rho_L}{2\pi b}$$

$a \le r \le b$

$$V(r) = \frac{\rho_L}{2\pi\epsilon}\ln\left(\frac{r}{a}\right)$$

$$V(r) = \frac{2\pi\epsilon_0 V_0}{2\pi\epsilon_0 \ln\left(\frac{b}{a}\right)}\ln\left(\frac{r}{a}\right)$$

$$V(r) = \frac{V_0}{\ln\left(\frac{b}{a}\right)}\ln\left(\frac{r}{a}\right)$$

$$\vec{E} = \frac{-V_0}{\ln\left(\frac{b}{a}\right)} \cdot \frac{a}{r} \cdot \frac{1}{a} = \frac{-V_0}{r\ln\left(\frac{b}{a}\right)} \quad V/m$$

(*) $D = \epsilon E$

$$\rho_{si} = \frac{\rho_L}{2\pi a}$$

$$\rho_{so} = \frac{\rho_L}{2\pi b}$$

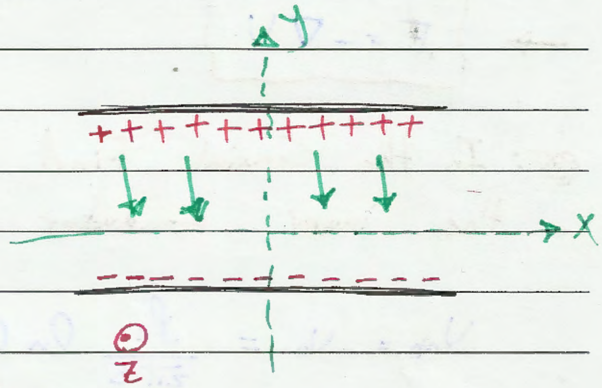
$$\boxed{\nabla \vec{D} = \rho_v} \qquad -\nabla V = \vec{E} = \frac{\vec{D}}{\epsilon}$$

$$\nabla . [\nabla V] = -\frac{1}{\epsilon}\, \nabla . D = -\frac{\rho_v}{\epsilon}$$

$$\boxed{\nabla^2 V_{(x,y,z)} = \frac{-\rho_v}{\epsilon}}$$

$$\boxed{\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}}$$

ex: find V, E, D, ρ_s



$V \neq f(x)$

$\neq f(z)$

$$\frac{dv^2}{dy} = 0 \qquad V = Ay + \cancel{B}$$

$V = 0$ since $y = 0$

$$\frac{dv}{dy} = A$$

$$V_0 = Ab \qquad A = \frac{V_0}{b}$$

$$E_y = \frac{V_0\,(y)}{b} \quad V/m$$

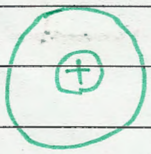
$$V = Ay + B$$

$$y = 0 \longrightarrow V = 0 \longrightarrow V = 0 + B = 0$$

$$y = d \longrightarrow V = V_0 = Ad \longrightarrow A = \frac{V_0}{d}$$

$$\boxed{V(y) = \frac{V_0}{d} y + 0}$$

* ch 5 : The material c/s ...



→ electric dipole

$\vec{m_e} = q\vec{l}$ Cm

N* of atoms (cluster of atoms) in a

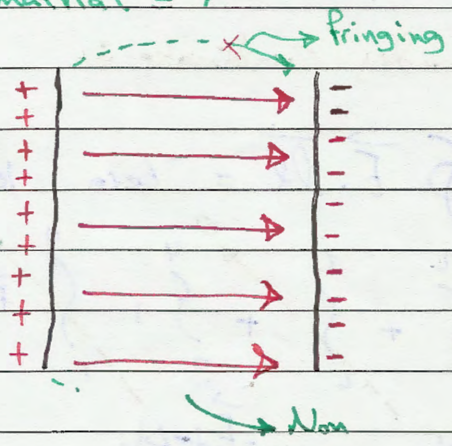
$N\vec{m_e} \Rightarrow \sum_{i=1}^{\infty} \vec{m_e}_i$ Cm

Value = ΔU

→ density : $\sum_{i=1}^{N} \frac{\vec{m_e}_i}{\Delta U}$ (c/m^2)

* dipole moment density :

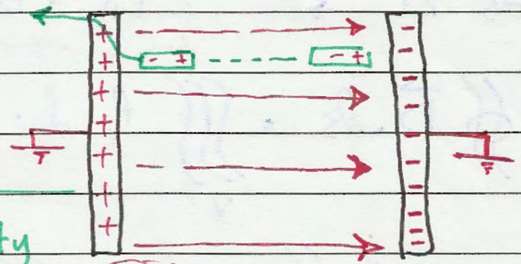
$\equiv \sum_{i=1}^{N} \frac{\vec{m_e}_i}{\Delta U} \Rightarrow$ Polarization of the matrial $= \vec{P}$



ϵ_0 (F/m) $= \frac{10^{-9}}{36\pi}$ F/m

ρ_{sF} : free charges c/m^2

$\vec{D} = \epsilon_0 \vec{E}$ for null Material



$\rho_{sP} \to$ Polarized density

$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E} = \epsilon_r \epsilon_0 \vec{E}$

↳ Permittivity of the Material

↳ Permittivity of the free Space (valume) or good Conducting

$\vec{a_s} \to$

$\vec{a_s'} \leftarrow$

$\therefore \vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_r \epsilon_0 \vec{E}$

$= \rho_{sF} \cdot \vec{a}_s$ (ناتج عن شحنات حرة)

$\rho_{sF} \pm ve$
$\rho_{sp} \mp ve$

remember

$\vec{P} = (\epsilon_r - 1)\epsilon_0 \vec{E}$

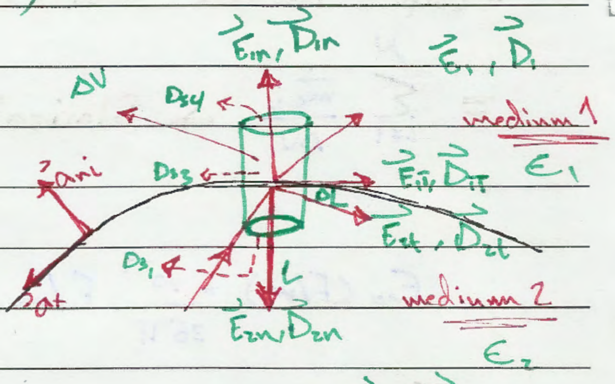
$= (\epsilon - \epsilon_0)\vec{E}$ (ناتج عن شحنات Bonded)

$= \rho_{sp} \ \vec{a}_s$

ϵ_r = relative Permitivity

$= \dfrac{\epsilon}{\epsilon_0} \geqslant 1 \qquad \vec{D} = \epsilon_r \epsilon_0 \vec{E}$

→ Remember $\epsilon_r = 1$ (لكل المواد الموصلة)



* Boundry Cordinate :

$\oint \vec{E} \cdot \vec{dl} = $ Zero KVL

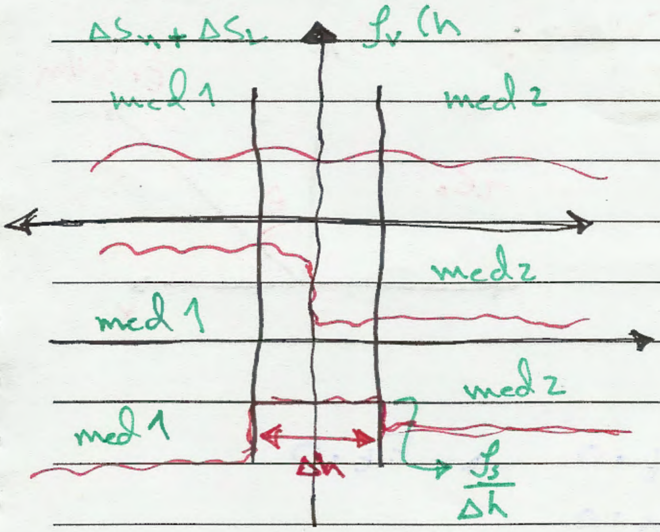
$\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \vec{E} \cdot \vec{dl} = 0$

$\vec{E}_{2t} \Delta L \qquad -\vec{E}_{1t} \Delta L$

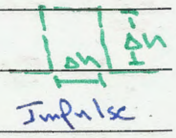
as $\Delta L \to 0 \qquad \boxed{E_{1t} = E_{2t}}$

(ii) $\oint\!\!\oint \vec{D} \cdot \vec{ds} = \iiint_V \rho_v \, dv = \iint_{\Delta S} ds \int_{\Delta h} \rho_v \, dh$

$\lim_{\Delta h \to 0} \left\{ \iint_{\Delta S_3} \vec{D} \cdot \vec{ds} + \iint \vec{D} \cdot \vec{ds} \right\} = \iint_{\Delta S} ds \int_{\Delta h} \rho_v \, dh$



(*) Area $= \frac{1}{\Delta h} \Delta h \, \rho_o = \rho_o$



$* \lim_{\Delta h \to 0} \int \rho_v \, dh = 0$

$* \lim_{\Delta h \to 0} \int \rho_v \, dh = 0$

$* \lim_{\Delta h \to 0} \int \rho_v \, dh = \rho_s$ (surface charge density)

$-D_{2n} \Delta S_L + D_{1n} \Delta S_u \simeq \begin{cases} \nearrow 0 \\ \searrow \rho_s \Delta S \end{cases}$ {إنكسار الضوء refraction}

as $\Delta S \to 0 \; \begin{Bmatrix} \Delta S_u \\ \Delta S_L \end{Bmatrix} \Rightarrow \Delta S \to 0$

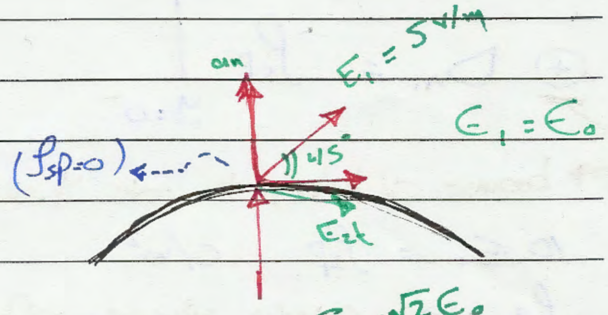
$D_{1z} - D_{2n} = \begin{cases} \nearrow \text{zero} \\ \searrow \rho_{sf} \end{cases}$

displacement vector

→ Normal electric flux density may contain discontinuety

$\therefore \; E_{1t} = E_{2t}$

$D_{1n} - D_{2n} = \rho_s \quad r_o$



ex: find $\vec{E}$, $\vec{D}$, ρ_{sf}, ρ_{sp}

$\therefore D_1 = 5 \epsilon_o \;\; c/m^2$

$E_{1t} = \frac{5}{\sqrt{2}} \;\; V/m \qquad D_{1t} = \frac{5}{\sqrt{2}} \epsilon_o \;\; c/m^2$

$E_{2n}, \; D_{2n} = \frac{5}{\sqrt{2}} \epsilon_o$

$\rho_{sp} = 0 \qquad \rho_{sf} = 0$

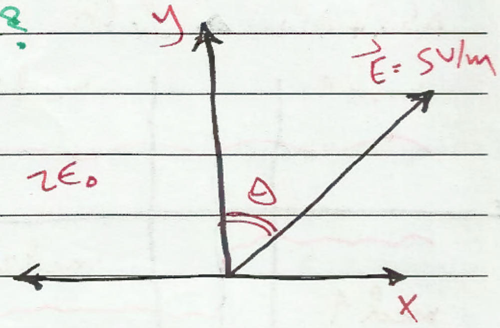
$* \; \rho_{sp} = (r_2 - 1) \epsilon_o$

$\therefore E_{2t} = \frac{5}{\sqrt{2}} \; V/m \Rightarrow D_{2t} = \frac{5}{\sqrt{2}} \epsilon_o$

ex: find $\vec{E}, \vec{D}, \rho_{sF}, \rho_{sP}$ every where?

$\Rightarrow$ good conducting Medium ϵ_0

find θ ??



Sol: for $y<0$:

$\vec{E} = 0 \Rightarrow$ conductor $\Rightarrow E_{2t} = 0 \Rightarrow E_{1t} = 0$

$D = 0$ $\quad E_{2n} = 0$

for $y>0$:

$E_{2t} = 0 \Rightarrow E_{1t} = 0 \Rightarrow \theta = 0$

$\therefore |E_y| = 5\ v/m$

$\vec{E} = 5\vec{a_y}\ v/m \Rightarrow D_y = 10\,\epsilon_0\ c/m$

$D_{n1} - D_{n2} = \rho_{sF}\Big|_{y=0^-}$

$\oplus\ D_{n1} = \rho_{sP}\Big|_{y=0^-}$

because a belong to the y axis

$10\,\epsilon_0 = \rho_{sP}\ \ c/m^2$

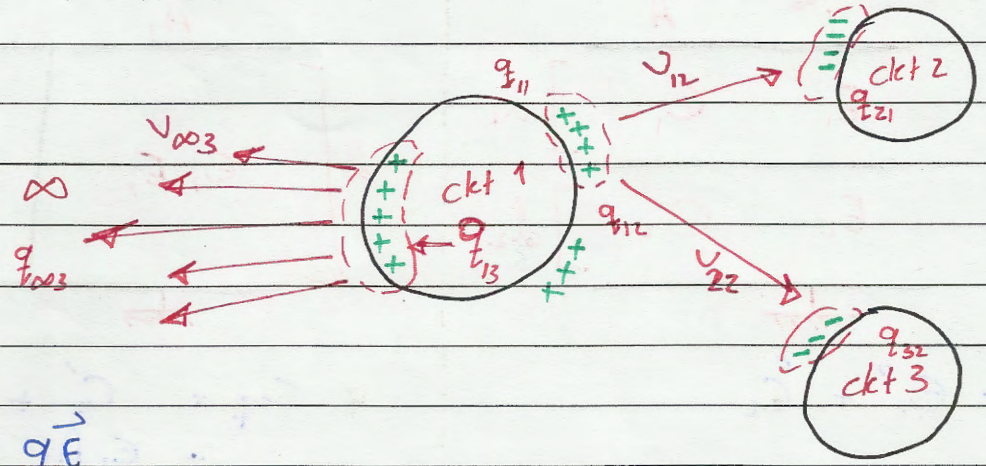
$\rho_{sf} = 0$ every where expect at $y = 0^+$

$\rho_{sP} = (\epsilon - \epsilon_0)\,E$

$= -(2\epsilon_0 - \epsilon_0)\,5$

$= -5\,\epsilon_0\ \ c/m^2$

ex:



$$m\vec{a} = \vec{F} = q\vec{E}$$

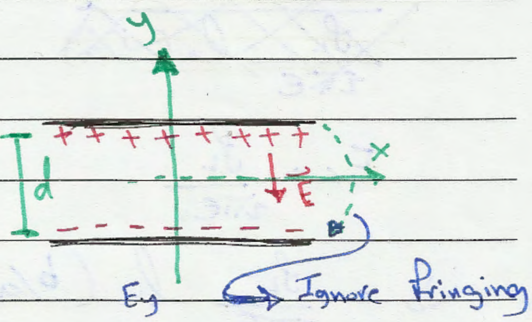
{electric energy} Stored in a Capacitor

where, Capacitance $\equiv C$ Farad (F)

$$C = q/V = \frac{\rho_s * \text{area}}{\frac{\rho_s}{\varepsilon} * \text{distance}} = \frac{\varepsilon * \text{Area}}{\text{distance}}$$

$$\therefore \quad C = \frac{q}{V} = \frac{\iint \rho_{sf}\, ds}{\int_L \vec{E}\cdot d\vec{l}}$$

ex: Good cond Plane Capacitor

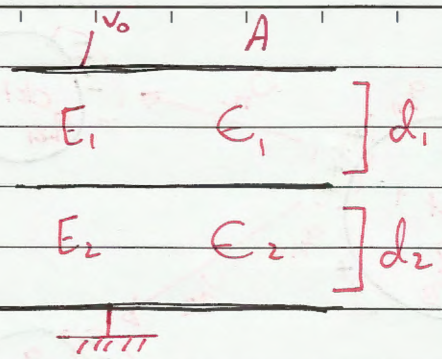


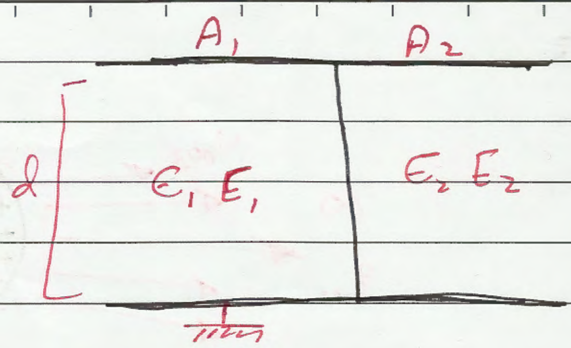
$$l * l = A$$

$$E_y = \frac{V_0}{d} \ \text{V/m}$$

$$C = \frac{\varepsilon_0 \cancel{\frac{V_0}{d}}\ \cancel{q}\ \cancel{\rho_s}\ \text{area}}{\cancel{V_0}}$$

ex :





$* \quad C_{eq} = C_1'' + ''C_2$

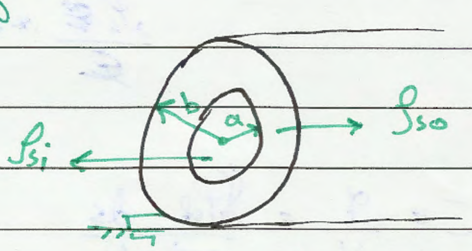
$$= \frac{C_1 A}{d_1} \; // \; \frac{C_2 A}{d_2}$$

$* \quad C_{eq} = C_1' + 'C_2$

$$= \frac{\epsilon_1 A_1}{d} + \frac{C_2 A_2}{d}$$

ex : Find the capacitance of coaxial where inner and outer radius = a, b resp F/m

$F \to \infty$



(لا تنسى الرسم)

$$\cancel{C = \frac{\oint \rho_L\, dl}{-\int E \cdot dl}}$$

$$D_r\, 2\pi r \cancel{l} = \rho_L \cancel{l}$$

$$\cancel{\frac{2\rho_L L}{\frac{\rho_L}{2\pi\epsilon} \ln(b/a)}}$$

$$D_r = \frac{\rho_L}{2\pi r} \quad c/m^2$$

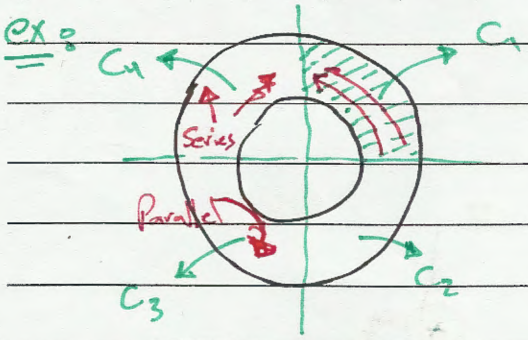
$$E_r = \frac{\rho_L}{2\pi\epsilon r}$$

$$V_0 = \frac{\rho_L}{2\pi\epsilon} \ln(b/a)$$

$$\therefore C/L = \frac{\cancel{Q}\ \rho_L \cancel{L}}{\cancel{V}}$$

$$\therefore C/\text{unit length} = \frac{4\pi\epsilon_0}{\ln(b/a)} = \frac{2\pi\epsilon}{\ln(b/a)}$$

ex:

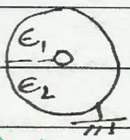


* الجهم نفس الشحنة , غير ذلك
يكون شابك على ∞

Parallel ← عندما تكون حركة دورانية تعكس خطوط العجلة

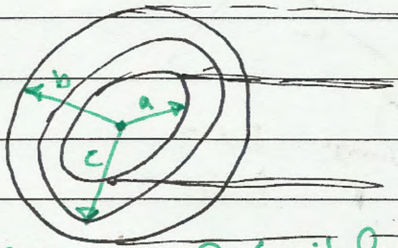
Series : ← عندما تكون إتجاه الحركة نحو الأعلى

ex:



$\Delta V = 10\ kV$

home work

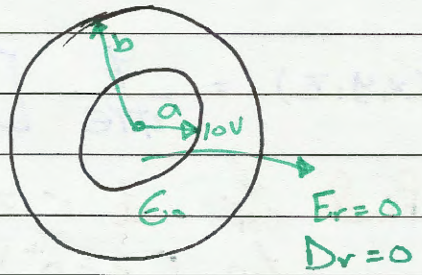


* find $\vec{E}, \vec{D}, \rho_{sf}, \rho_{sp}$ every where   C / unit length

ex: Capacitance of aspherical shell

$$\oiint \vec{D} \cdot ds = \iiint \rho_v \, dv \quad \hookrightarrow r^2 \sin\theta \, dr \, d\theta \, d\phi$$

" متوزع على مساحة صغيرة جدا



$$\oiint \vec{D} \cdot ds = \iint \rho_s \, a^2 \sin\theta \, d\theta \, d\phi$$

* Note that: we use " ρ_s " because almost all the Charges will be on the Surface :)

$$\iint D_r \, r^2 \sin\theta \, d\theta \, d\phi = \iint \rho_s \, a^2 \sin \, d\theta \, d\phi$$

$$D_r \, 4\pi r^2 = \rho_s \, a^2 \, 4\pi$$

$$D_r = \frac{\rho_s a^2}{r^2} \ C/m^2 \qquad E_r = \frac{\rho_s a^2}{r^2 \epsilon_o} \ V/m$$

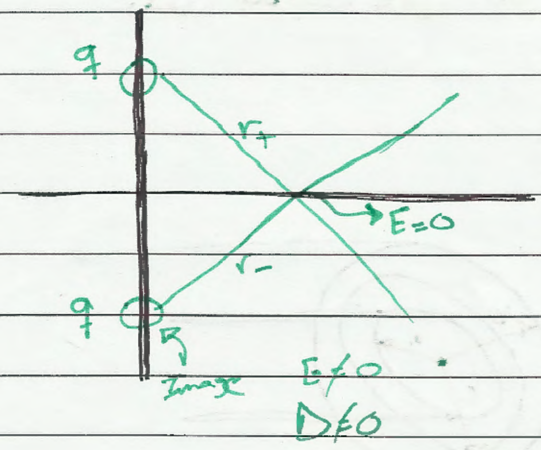
$$V_{10} = \frac{\rho_s a^2}{\epsilon_o} \left(\frac{1}{a} - \frac{1}{b} \right) \qquad \therefore \ C = 4\pi \epsilon_o a \simeq 10^{-10} \ af = 1 \ nf$$

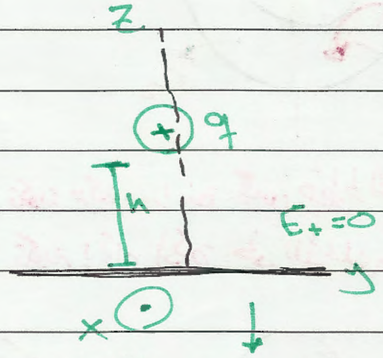
$$C = \frac{q}{V} = \frac{\rho_s 4\pi a^2 \epsilon_o}{\rho_s a^2} \cdot \frac{ab}{a-b} = \frac{4\pi \epsilon * ab}{a-b} \quad , \ V_{ab} = -\int_a^b \vec{E} \cdot \vec{dl} = \frac{-\rho_s}{2\pi\epsilon} \int dr$$

Five Apple

Image theory:

ex: find E, D, U every where





Plane mirror
good conducting
half space or
aluminium foil

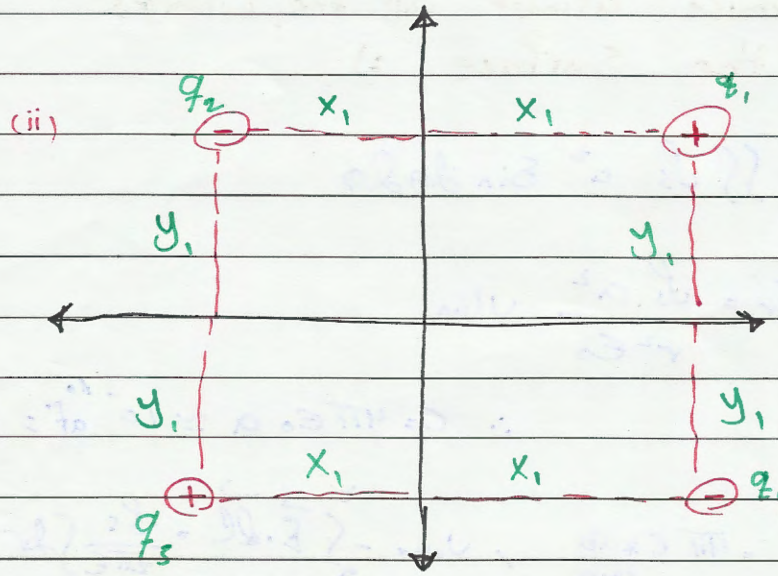
* $\vec{E} = 0 = \vec{D}$
* $V =$ constant

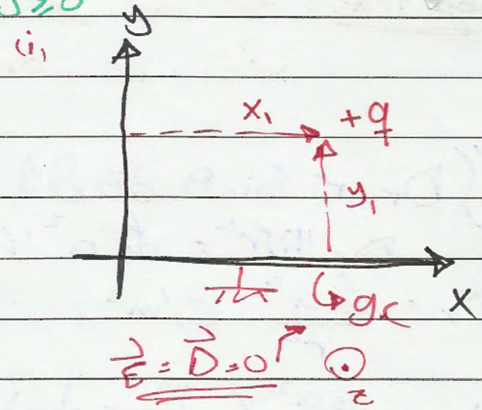
$$E(x,y,z) = \frac{q}{4\pi\epsilon}\left[\frac{\vec{r}_+}{r_+^3} - \frac{\vec{r}_-}{r_-^3}\right]$$

→ find ρ_{sf} at $\langle z = 0^- \rangle$ $2\epsilon_0$

ρ_{sp} $(z = 0^+)$

ex: find E, D, V every where $x \geq 0$ for all z
$y \geq 0$





Five Apple

Con

$V = V_1 + V_2 + V_3 + V_4$

$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$

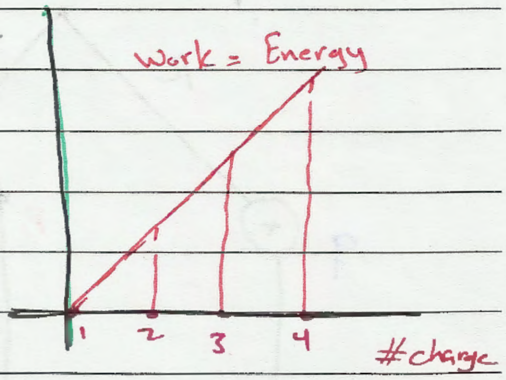
* electrostatic energy (energy density):

$$w_e = \frac{W_e}{\text{Volume}} = \frac{\frac{1}{2}\frac{\epsilon A}{\Delta d} U^2}{l * l * l} \quad l * l * E^2$$

$w_e = \frac{1}{2} \epsilon E^2 \quad J/m^2$

$\vec{F} \cdot \vec{L} = W \longrightarrow$ work

$q\vec{E}$

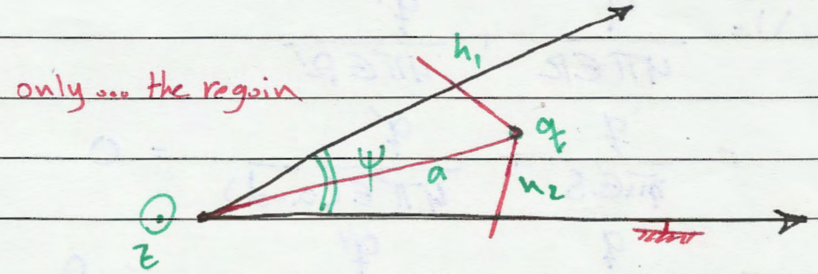


$\frac{1}{2} \sum_{i=1}^{N} Q_i N_i$ (total work)

$\int dw_e = \frac{1}{2} \int q\, dv$

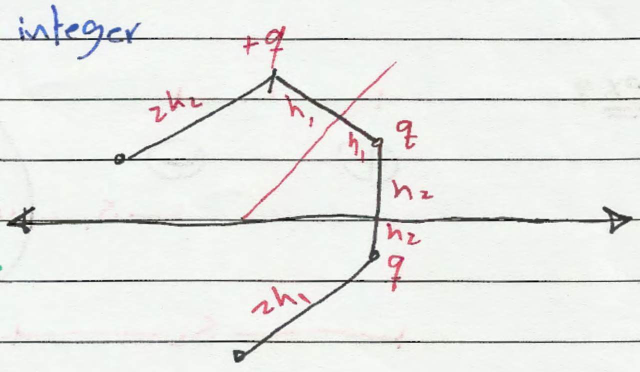
$w_e = \frac{1}{2} q U$

ex:



$E=0,\ D=0 \quad (0 \le \psi \le \psi)$

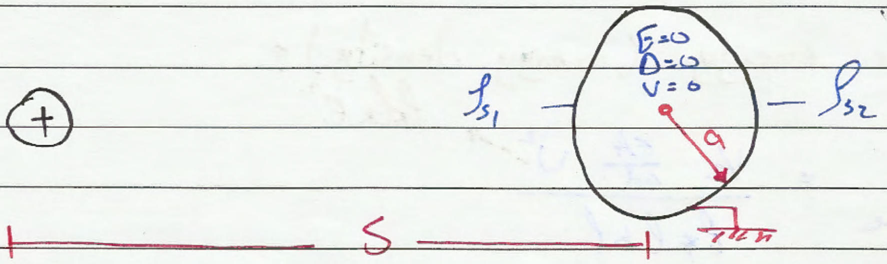
\# of Source $= \frac{360}{\psi_r}$ Should be integer



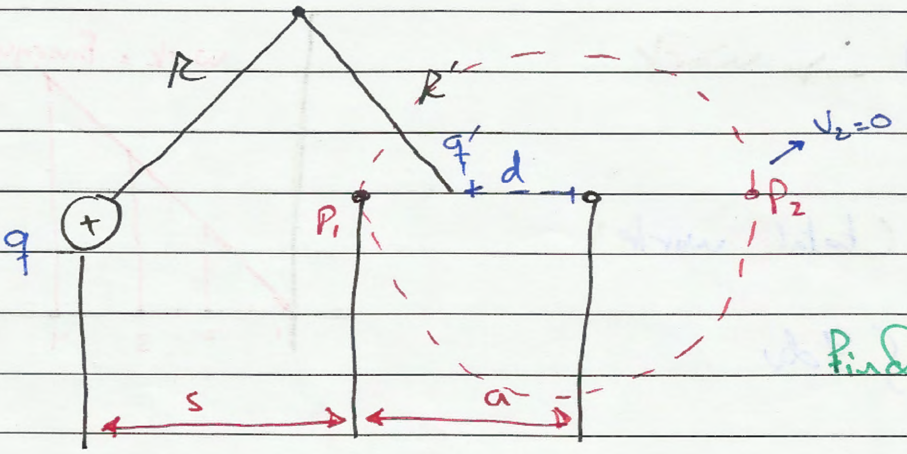
H.w: take $\psi_0 = 60°$ &
find E, D every where

* Image in non-Planner Surfaces

→ Point Charge infront of a Conducty sphere, we need E, D, V every where



when $r > a$:



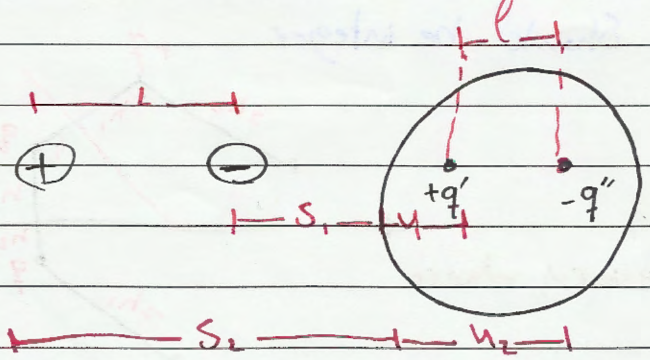
Find d, q'

$$V = \frac{q}{4\pi\epsilon R} + \frac{q'}{4\pi\epsilon R'}$$

$$= \frac{q}{4\pi\epsilon S} + \frac{q'}{4\pi\epsilon(a-d)} = 0$$

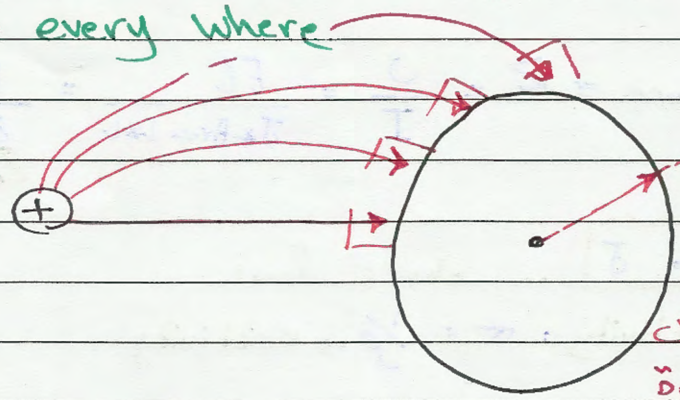
$$= \frac{q}{4\pi\epsilon(S+2a)} + \frac{q'}{4\pi\epsilon(d+a)} = 0$$

ex:



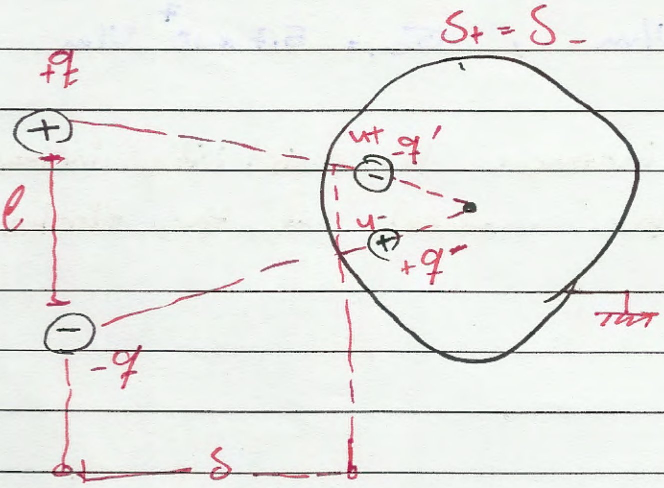
$l' < l$

ex: Point Charge infront of a conductiy Sphere Need $\vec{E}, \vec{D}$ and V every where



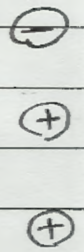
* البؤرة : نقطة تجمع الضوء بالعدسة المحدبة ويتشتت بالعدسة المقعرة

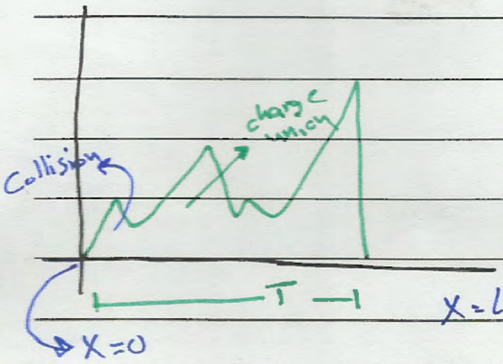
ex:



* Dc Current & the resistor

$I = \frac{q}{t} = \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$ (free q ($^{(+)}$ or/and $(-)$)) Present in Conductiy Meduim





$V_{average} = \frac{L}{T}$
drift Velocity

$x = Lt$

$\vec{v}_d = \mu \vec{E}$ → Mobility of the Charge inside the medium, mobility e^- is bigger than mobility of P

* thermal noise :

$$\text{Resistance} = R = \frac{V}{I} = \frac{EL}{J * \text{Area } L*L} = \frac{E}{JL} = \frac{\text{length}}{\sigma * \text{area}} = \rho\,\frac{\text{length}}{\text{area}}$$

$J \to A/m^2$ Current density

$\boxed{\sigma \vec{E} = J}$ ... ohm's law

$\hookrightarrow$ Conductivity : $\sigma = 1/\rho \to$ resistivity

$\boxed{J = \rho V_d}$ $\quad \sigma \to V/m$, $\sigma_{cu} = 5.7 * 10^7\ V/m$

* when the temprecture increases the resistivity increas
σ decrease due to the moment of the electrons.

* $\sigma \Longrightarrow$

Cu	$5.7 * 10^7$	
AL	10^7	
Au	10^7	
quarts	10^{-19}	V/m
Teflon	10^{-19}	V/m
Paper	10^{-10} .... 10^{-3} إذا نقع بالزيت	

$$\text{or } R = \frac{V}{I} = \frac{\int E . dl}{\iint J . ds} = \frac{E \int dL}{\sigma * (\text{Area}) \iint ds} = \frac{\int dL}{\sigma \iint ds}$$

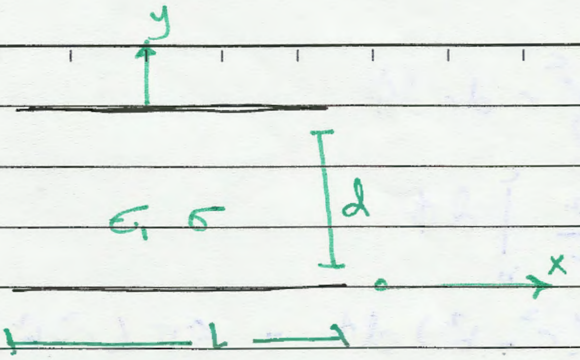
$V_d = \mu E$
$J = \sigma \vec{E}$
$\vec{J} = \rho V_d$

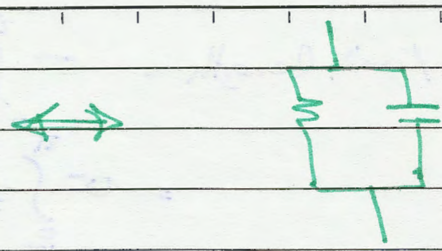
$\oiint J . ds = 0$ — kcl ckt

$\nabla . \vec{J} = 0$ — kcl mem

$$dR = \frac{dl}{\sigma\, dA} \Longleftrightarrow \boxed{dG = \frac{\sigma\, dA}{dl}}$$

ex:





$* \; d_3 R = \frac{dl}{\sigma dA} = \frac{dy}{\sigma dx dz}$

$$R = \frac{V}{I} = \frac{EL}{J \text{ area}} \qquad C = \frac{\epsilon A}{d}$$

$* \; d_2 R = \frac{1}{\sigma dx dz} \int_0^{d} dz$

$$= \frac{\cancel{E}L}{\sigma \cancel{E} \text{ area}} = \rho \frac{L}{A}$$

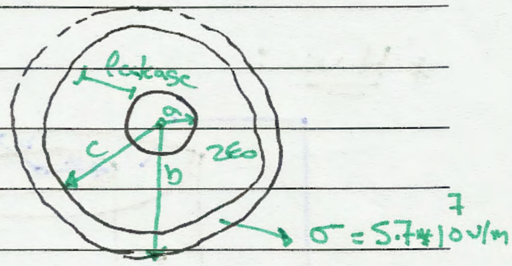
$$d_2 G = \frac{\sigma \, dx dy}{d}$$

$$G = \frac{\sigma}{d} \int_{-L/2}^{L/2}\int_{-L/2}^{L/2} dx dz = \frac{\sigma L^2}{d}$$

$$\boxed{\therefore R = \frac{d}{\sigma A}}$$

$\boxed{\begin{array}{c}\sigma, \rho \text{ for Current} \\ \epsilon \text{ for Charges}\end{array}}$

ex: find R_{in}, R_{out} ??



Sol: $dR_{in} = \frac{dl}{\sigma dA} = \frac{dz}{\sigma r dr d\phi}$

$$d_2 R_{in} / \text{unit length} = \frac{1}{\sigma r dr d\phi}$$

$$\iint d_2 G_{in} = \int_0^{2\pi}\int_0^{a} \sigma r \, dr \, d\phi = \cancel{2}\pi \sigma \frac{a^2}{\cancel{2}}$$

$$R_{in} / \text{unit length} = \frac{1}{\sigma_c \pi a^2} \; \Omega/m$$

$$= \frac{10^{-7}}{5.7 \pi a^2}$$

G_{outer} / unit length $= \sigma \int_0^{2\pi}\int_b^c r\,dr\,d\phi$

$= \sigma \int_0^{2\pi} \frac{r^2}{2}\Big|_b^c d\phi$

$= \frac{\sigma}{2}\int_0^{2\pi} (c^2-b^2)\,d\phi = 6\pi(c^2-b^2)\ \mho m$

$R = \frac{1}{\pi\sigma_c(c^2-b^2)}\ \Omega/m$

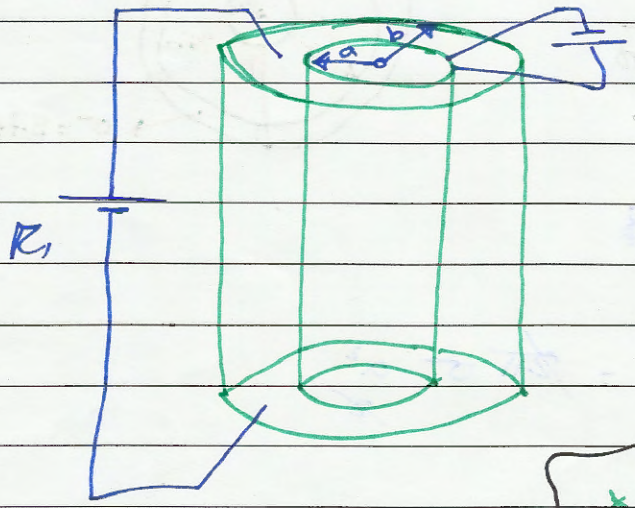
* total copper: $dR_{leakage} = \frac{dl}{\sigma dA} = \frac{dr}{\sigma\, dr\, d\phi\, dz}$

$d_2 R_L = \frac{1}{\sigma_d\, d\phi\, dz}\int_a^b \frac{dr}{r}\ \ln\left(\frac{b}{a}\right)$

$G = \frac{\sigma_0}{\ln(b/a)}\int_0^L\int_0^{2\pi} d\phi\, dz$

$= \frac{2\pi\sigma_d}{\ln(b/a)}$ $\qquad R_l = \frac{\ln(b/a)}{2\pi\sigma_d}$

* H.w :



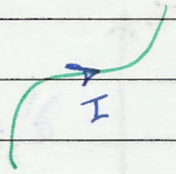
مهم جداً

* $\vec{E} \rightarrow$ electrostatic ينتج من
* $\vec{B}\ \vec{H} \rightarrow$ Current ينتج من

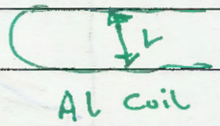
* Magneto Static :

Source :

line Current : $I \left(\frac{dq}{dt} \Rightarrow \frac{Q}{t} \right)$ A or C/s

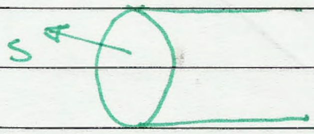


* linear Current density $\vec{K} = I / \vec{L}$ A/m



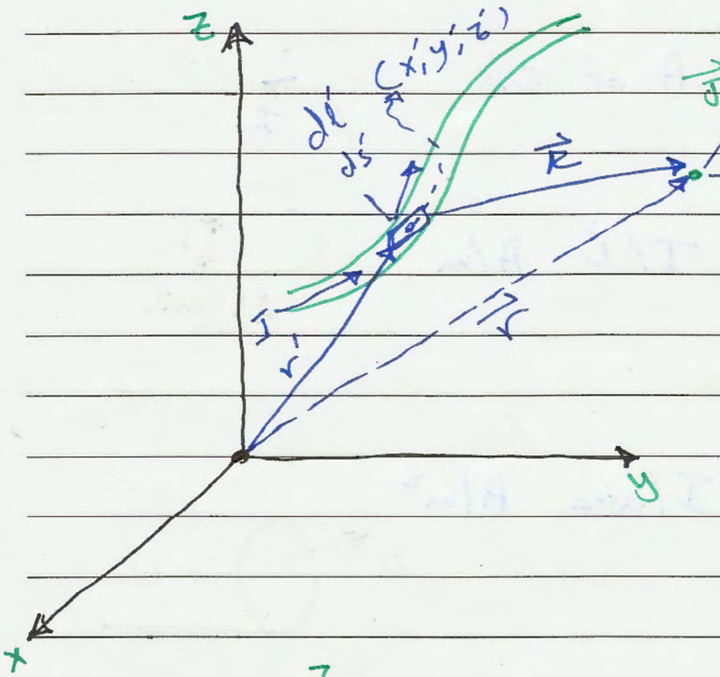
~~the~~ L, K نفس الإتجاه

* linear Surface desity $\vec{J} = I / \text{area}$ A/m^2



* F_m (N)
* $\vec{B}$ magnatic flux density
* $\vec{H}$ magnatic fild
* $\mho$ (NI) , N: # of turns (mag. Potential)
* $\vec{A}$ mag Vector Potential (wb/m)
* W_m (J)
* L (M) H
* Ψ mag. flux $\iint_{o} \vec{B} \cdot ds$ ($\oiint \vec{B} \cdot ds = 0$)

* Bio - Savart law : (we are talking about DC current)



$\vec{J}\, ds$

$\rightarrow ds\, \vec{a}_R$

* magnatic flux density

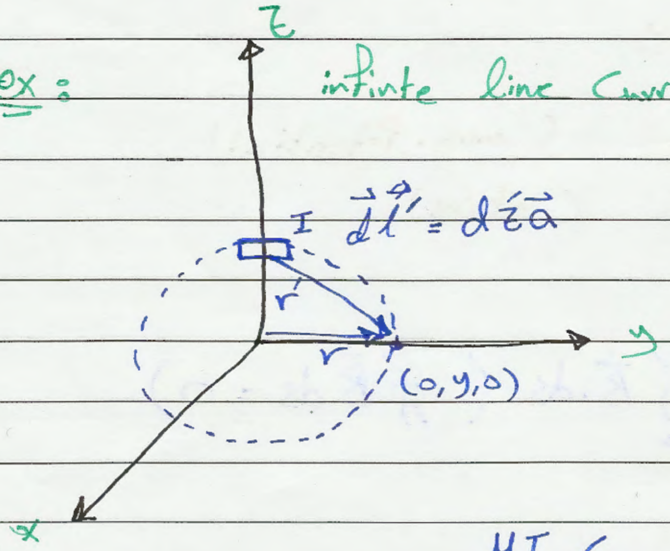
$$d\vec{B} = \iiint \frac{\mu \vec{J}\, dv}{4\pi R^2} * \vec{a}_r \ (wb/m^2)$$

μ → Medium Constant (for free space)

$= 4\pi \times 10^{-7}$ H/m

henry/m

ex : infinte line Current , $\vec{B}$??



$$\vec{B} = \int \frac{\mu I\, dz' \vec{a}_z}{4\pi R^2} * a_R$$

$$\vec{a}_R = \frac{\vec{R}}{|\vec{R}|}$$

$$= \frac{y \vec{a}_y - z' \vec{a}_z}{\sqrt{y^2 + z'^2}}$$

$$= \frac{\mu I}{4\pi} \int \frac{a_z\, (y a_y - \cancel{z' a_z})}{(y^2 + z'^2)^{3/2}}\, dz'$$

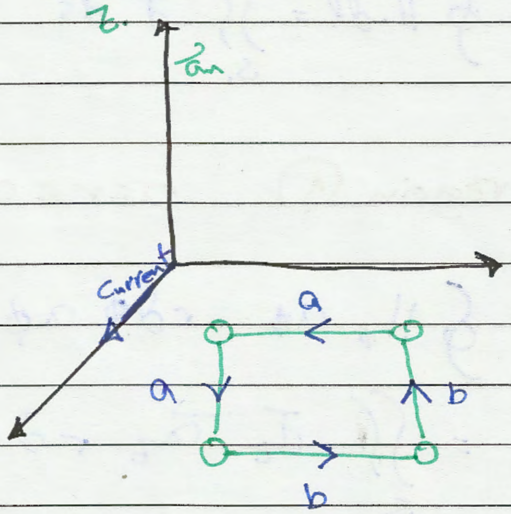
$$= -\frac{\mu I y}{4\pi} \int_{-\infty}^{\infty} \frac{dz'}{(y^2 + z'^2)^{3/2}} = \frac{-\mu I}{2\pi y} \quad wb/m^2$$

$$B_\phi = \frac{\mu I}{2\pi a} \quad \text{find it ...}$$

Five Apple

* Very Important examples :

ex : ∞ Sheet of current, find $\vec{B}$ and $\vec{H}$ of $(0,0,h)$



in the Plane xy at $z=0$

$\vec{k} = k\,\vec{a_y}$

$H \neq f(x,y)$

H has only y component

loop zz // to the yz-Plane at $x=0$
Symmetrically located x

$$\oint_L H.dl = \int_{l'} \vec{k}.dl' = \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \vec{H}.dl$$

$$= k_o \,\Delta L$$

$2H_y \,\Delta L = k_o \,\Delta L$ $\qquad$ * $H_y = \frac{k_o}{2}$

$\vec{a_n} \times \vec{H} = \vec{k}$ $\quad$ Flow of current $\vec{a_x}$

$\vec{a_n} \longrightarrow \vec{a_z} \qquad z>0$

$\vec{a_n} \longrightarrow -\vec{a_z} \qquad z<0$

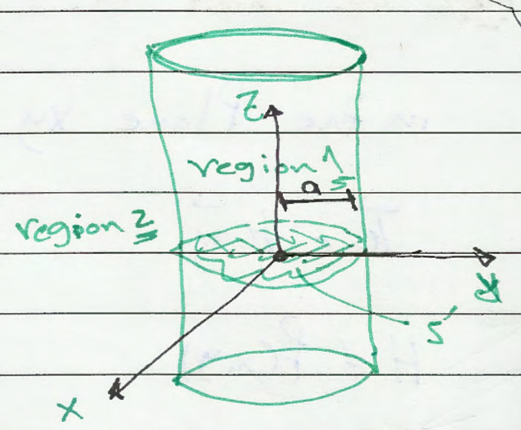
$\vec{a_n} \longrightarrow a_L \longrightarrow$ a Current

$\vec{a_z} \qquad \vec{a_y} \qquad \vec{a_x}$

Five Apple

ex: a good Conducting wire Copper Cylinder (∞) Carry Current I, Need $\vec{H}$ every where ??

$$\oint \vec{H}\cdot\vec{dl} = \iint_{S_1} \vec{J}\cdot\vec{ds}'$$



region ① $0 \le r \le a$

$$\oint H_\phi a_\phi \cdot r d\phi \, a_\phi$$

$$= \iint_{S_1} J_z \vec{a}_z \, r dr' \, d\phi' \, \vec{a}_z$$

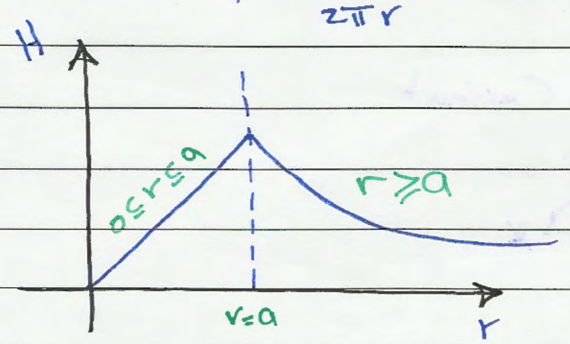
$$\text{or } H_\phi \, 2\pi = 2\pi \int_0^r \frac{I}{\pi a^2} r' dr'$$

$$= \frac{2\pi I}{\pi a^2}\left(\frac{r^2}{2}\right) = \frac{r^2 I}{a^2}$$

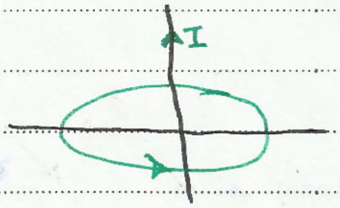
$$\boxed{H_\phi = \frac{Ir}{2\pi a^2} \text{ A/m}} \qquad 0 \le r \le a$$

region ② : $2\pi H_\phi r = I$

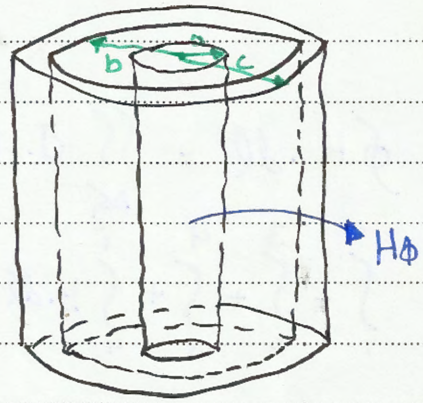
$$H_\phi = \frac{I}{2\pi r} \text{ A/m} \qquad r \ge a$$



ex: find $\vec{B}$ & $\vec{H}$ by bio law



ex: Coaxial Cable Carrying Current I:



* $\vec{H}\Big|_{r \geq c} = 0$

(Prev. ex.)

* $H_\phi = \dfrac{Ir}{2\pi a^2}$ $\qquad (\underbrace{a}_{\text{zero}} \leq r \leq a)$

* $H_\phi = \dfrac{I}{2\pi r}$ $\qquad (a \leq r \leq b)$

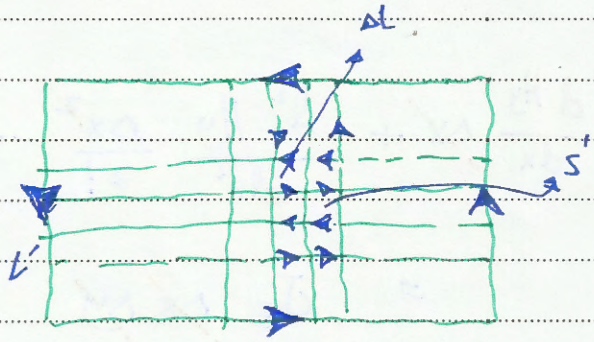
* $$\oint_c \vec{H} \cdot d\vec{l} = 2\pi r H_\phi = \iint_{S_1} J \cdot ds'$$

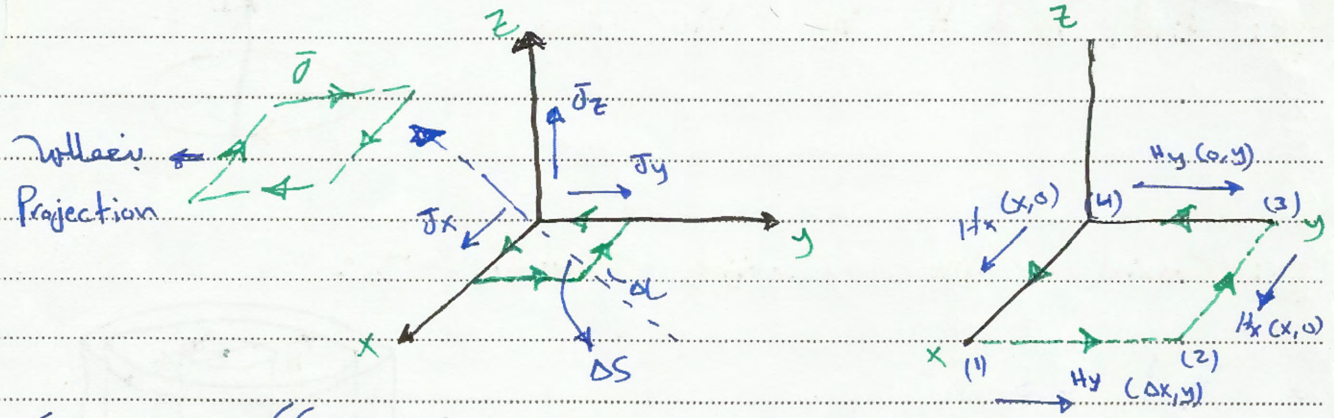
$$= I - \frac{-I}{\pi(c^2 - b^2)} \int_a^r \int_0^{2\pi} r' \, d\phi' \, dr'$$

The Curl: $(\nabla \times)$

$$\oint \vec{H} \cdot d\vec{l} = \iint_S \vec{J} \cdot d\vec{s}$$

$$\oint_{\Delta L} \vec{H} \cdot d\vec{l} = \iint_{\Delta S'} J \cdot ds'$$





$$\oint H \cdot dl = \iint_{\Delta S_1'} J \cdot ds'$$

$$\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 H \cdot dl \cong J_z \, \Delta x \, \Delta y$$

$$H_y' \, \Delta y - H_x' \, \Delta x - H_y \, \Delta y + H_x \, \Delta x \cong J_z \, \Delta x \, \Delta y$$

$$H'_{y\,(\Delta x, y)} = \sum \frac{H^{(n)}(o,y)}{n!} \Delta x^n$$

$$\therefore \left(H_y + \frac{\partial H_y}{dx} \Delta x + \Delta x^2 --- \right) \Delta y - \left(H_x + \frac{\partial H_x}{dy} \Delta y + \frac{\partial^2 H_x}{dy^2} \frac{\Delta y^2}{2!} \right) \Delta x$$

$$- H_y \, \Delta y + H_x \, \Delta x \simeq J_z \, \Delta x \, \Delta y$$

$$\left(\frac{\partial H_y}{dx} \cancel{\Delta x} + \frac{\partial^2 H_y}{dx^2} \frac{\Delta x^{\cancel{2}}}{2!} ---- \right) \cancel{\Delta y} - \left(\frac{dH_x}{dy} \cancel{\Delta y} + \frac{\partial^2 H_x}{dy} \frac{\Delta y^{\cancel{2}}}{2!} \right) \cancel{\Delta x}$$

$$= J_z \, \cancel{\Delta x} \, \cancel{\Delta y}$$

Subject:.................... / /

Taking the limit:

$$\Delta l \to 0 \quad \left\{ \begin{matrix} \Delta x \to 0 \\ \Delta y \to 0 \end{matrix} \right\} \quad \Delta s \to 0$$

$$\Rightarrow J_z = \left(\frac{dH_y}{dx} - \frac{dH_x}{dy} \right) = \Delta$$

$$J_y = \left(\frac{dH_x}{dz} - \frac{dH_z}{dx} \right) = \square$$

$$J_x = \left(\frac{dH_z}{dy} - \frac{dH_y}{dz} \right) = *$$

$$\therefore (\Delta)\, \vec{a}_z + (\square)\, \vec{a}_y + (*)\, \vec{a}_x = J_z \vec{a}_z + J_y \vec{a}_y + J_x \vec{a}_x$$

$$\rightarrow \boxed{\nabla \times \vec{H} = \vec{J}}$$

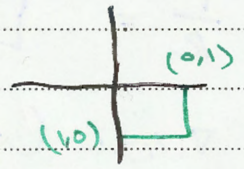
$$\vec{J} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ H_x & H_y & H_z \end{vmatrix}$$

$\nabla \times \vec{H} = 0$ → either $J \neq 0$
→ or $\vec{J} = 0$

ex: $\vec{H} = 5xy^2\, \vec{a}_x + 6zx\, \vec{a}_y + 3z\, \vec{a}_z$ A/m, find $\vec{J}$

$$\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 5xy^2 & 6zx & 3z \end{vmatrix} = \vec{a}_x (0 - 6x) - \vec{a}_y (0 - 0) + \vec{a}_z (6z - 10xy)$$

$$J = -6x\, \vec{a}_x + (6z - 10xy)\, \vec{a}_z \quad A/m^2$$



I?? blow, throw x-y Plane loop 1x1

$$\therefore I = \iint_{0\,0} \vec{J} \cdot \vec{ds}, \quad \vec{ds} \to dx\,dy\, \vec{a}_z$$

Subject:...................... / /

Magnatic Potential :

Scaler — Vector

$$\oint_L \vec{H}\cdot d\vec{l} = NI = \underset{A\text{-turn}}{\underline{U}}$$ magnatic Vector Potential $\vec{A}$ wb/m

→ the mag. vector Potential :

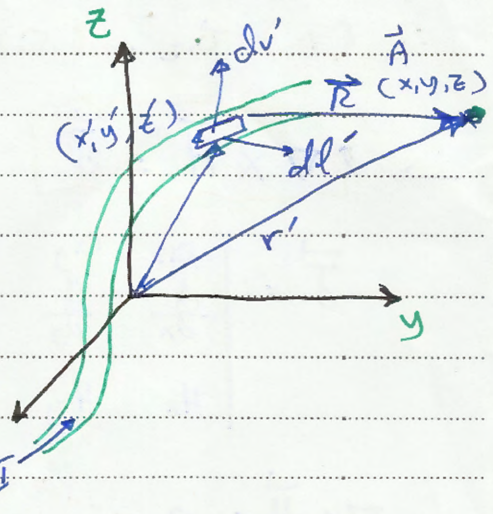
$$\vec{A} = \iiint \frac{\mu \vec{J}\, dv}{4\pi R} \quad \text{wb/m}$$

كثافة التيار
نفس الإتجاه

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$= \frac{\mu}{4\pi} \underset{(x,y,z)}{\Delta x} \iiint_{v'} \nabla\times\left[\frac{1}{R} J\,(x',y',z')\,dv'\right]$$

$$= \frac{\mu}{4\pi}\left[\nabla\left(\frac{1}{R}\times\vec{J} + \frac{1}{R}\nabla\times\vec{J}\,(x',y',z')\right)\right]$$



$$\vec{R} = (x-x')\vec{a}_x + (y-y')\vec{a}_y + (z-z')\vec{a}_z$$

$$\frac{1}{R} = \left[(x-x')^2 + (y-y')^2 + (z-z')\right]^{-1/2}$$

$$\nabla(x,y,z)\left(\frac{1}{R}\right) = \frac{-2}{3}\left[\ \ \right]^{-3/2}\left\{2(x-x')\vec{a}_x + 2(y-y')\vec{a}_y + 2(z-z')\vec{a}_z\right\} = \frac{\vec{R}}{R^3}$$

Subject: .... / /

$\rightarrow = \frac{\mu}{4\pi} \iiint J \times \frac{\vec{R}}{R^3} dv' \rightarrow \vec{B}$

$\vec{A}(x,y,z) = \frac{\mu}{4\pi} \iint_{S_1} \frac{k \, \vec{ds}}{R}$ wb/m $\Longleftrightarrow \nabla \times \vec{A} = \vec{B}$

$\int_{L_1} \frac{I \, dl}{R}$

$\iiint \frac{\vec{J} \, dv}{R}$

$\nabla_{(x,y,z)} \times \left(\underbrace{\frac{1}{\mu}}_{} \vec{B}_{(x,y,z)} \right) = J$ $\Rightarrow \boxed{\nabla \times \vec{H} = \vec{J}}$

→ for homo. media بنطلعها برا

$\boxed{\nabla \times B = \mu I}$

* isotropic : خصائص الوسط لا تتأثر بالإتجاه

* Homo : متماثلة لا تتغير من نقطة إلى أخرى

$\nabla \times [\nabla \times \vec{A}] = \mu I$

$\nabla(\nabla \cdot \vec{A}) - \nabla^2 (A) = \mu \vec{J}$

$I = \oiint J \cdot ds = 0$ → kcl in general ckt

$\nabla \cdot \vec{J} = 0$ → kcl (em) at a point

$\nabla \cdot \vec{A} = 0$

$\vec{A} \longleftrightarrow J$ $\nabla \cdot \bar{A} = 0$

$\nabla(\cancel{\nabla \cdot \vec{A}}^{\,0}) - \nabla^2(\vec{A}) = \mu \vec{J}$

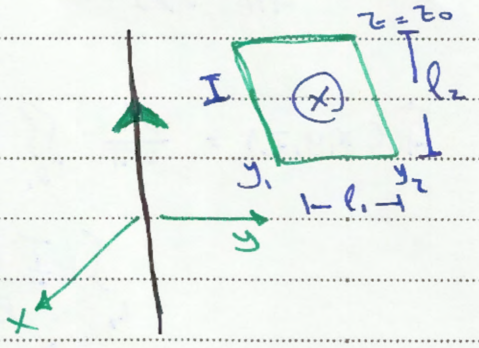
$\nabla^2(\vec{A}) = -\mu \vec{J}$ $\nabla^2 \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = -\mu \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix}$

S T A R S Notebook

Subject: .... / /

ex: Need the mag. flux linking the rectagular loop:

$$\oint \vec{H}\cdot d\vec{l} = I \qquad (d\vec{l} \to r\,d\phi\,d\phi)$$



$$2H_r\,H_\phi = I$$

$$H_\phi = \frac{I}{2\pi r}$$

$$\vec{B} = \frac{\mu I}{2\pi r}\,\vec{a}_\phi \quad wb/m^2$$

$$\vec{B} = -\frac{\mu I}{2\pi y}\,\vec{a}_x \quad wb/m^2$$

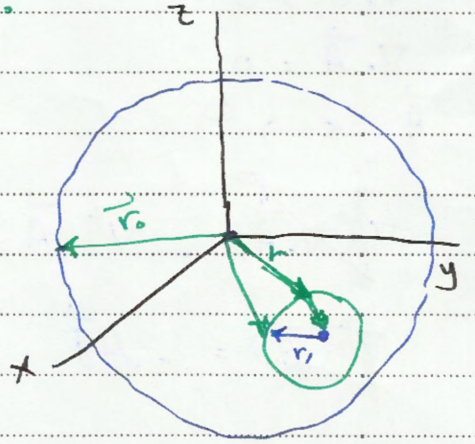
$$\Psi_{mag\ flux} = \iint_S \vec{B}\cdot d\vec{s} \qquad S \leftarrow \text{open}$$

$$\Psi_m = -\frac{\mu I}{2\pi}\iint \frac{a_x}{y}\,dy\,dz\,(-\vec{a}_x)$$

$$= \frac{\mu I L_2}{2\pi}\ln\left(\frac{y_2}{y_1}\right) = \frac{\mu I L_2}{2\pi}\ln\left(\frac{y_2}{y_1}\right) wb$$

بدل ← $\left(\frac{r_2}{r_1}\right)$ إذا استخدمت Cylinder *

ex: Find $\vec{B}$ and $\vec{H}$ inside the Cavity:



Subject:.......................   /   /

$$H_\phi\, a_\phi = \frac{J\vec{a}_z \times r\vec{a}_r}{2}$$

$$H_1 = \frac{J \times \vec{r}}{2} \quad A/m$$

$$B_1 = \frac{\mu \vec{J} \times \vec{r}}{2} \quad wb/m^2$$

$$H_2 = \frac{-\vec{J} \times \vec{r}_1}{2}$$

* inside the Cavity :

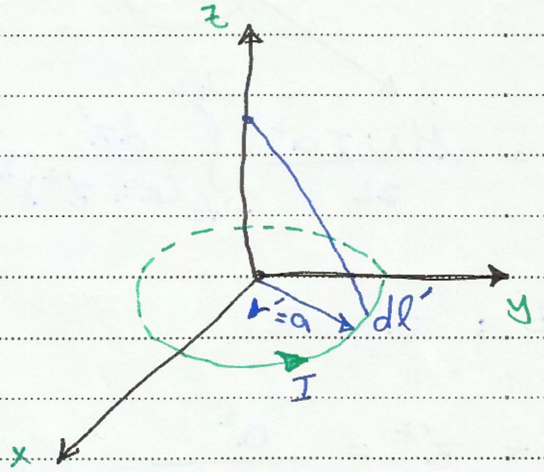
$$\vec{H} = \vec{H}_1 + \vec{H}_2$$

$$= \frac{\vec{J}}{2} \times (\overrightarrow{r - r'}) \quad y a_y$$

$$\vec{H} = y J \vec{a}_z \times \vec{a}_y$$

$$\vec{H}_x = -\frac{Jy}{2} \quad A/m$$

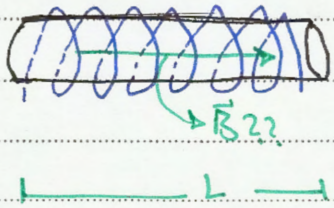
$$dB = \frac{\mu}{4\pi} \oint \frac{I \vec{dl} \times \vec{a}_R}{R^2}$$

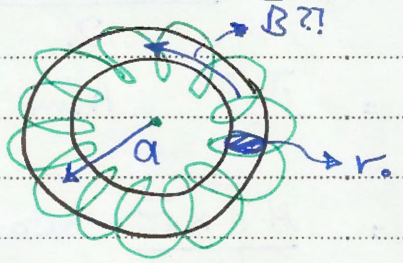


$$B_z = \frac{\mu I a^2}{2(a^2 + z^2)^{3/2}} \quad wb/m^2$$

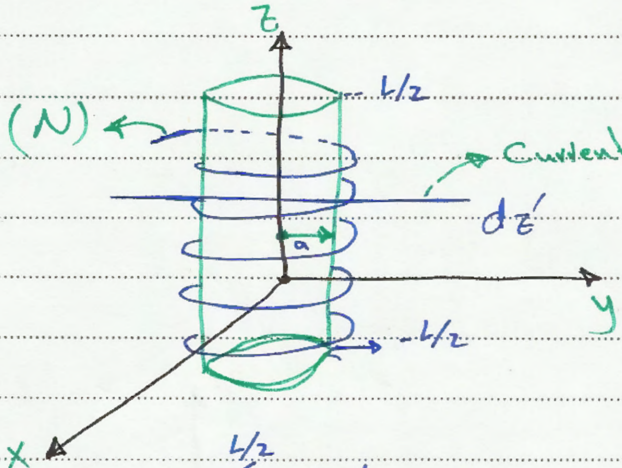
Subject:......................   /   /

* Solenoid :





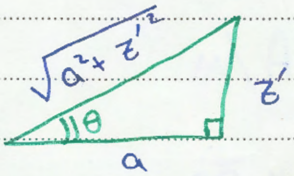
ex :



Current = $k\,dz'$

$k = \frac{NI}{L}$ line current density

$$B_z = \frac{\mu N I a^2}{2L} \int_{-L/2}^{L/2} \frac{dz'}{(a^2 + z'^2)^{3/2}}$$



$z' : a \tan\theta$

$$a^2 + z'^2 = \frac{a^2}{\cos^2\theta}$$

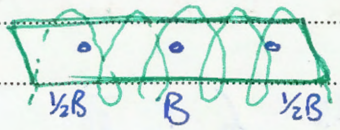
$$dz' = \frac{a\,d\theta}{\cos^2\theta}$$

$$B_z = \frac{\mu N I \cancel{a^2}}{2L} \int \frac{\cancel{a}}{\cancel{\cos^2\theta}} \frac{\cos^{\cancel{3}}\theta \, d\theta}{\cancel{a^3}}$$

$$B_z = \frac{\mu N I}{2L} \frac{z'}{\sqrt{a^2 + z'^2}} \Bigg|_{-L/2}^{L/2} = \frac{L}{\sqrt{a^2 + L^2/4}}$$

$a << L$

$B_z \simeq \mu \dfrac{NI}{L}$

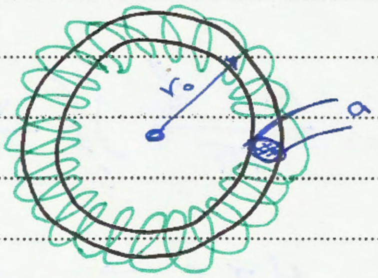


$H_z = \dfrac{NI}{L}$ ... at the Center

$H_z = \dfrac{NI}{2L}$ ... finite (على الأطراف)

* Toroidal Coil :

$H_\phi = \dfrac{NI}{2\pi r_0}$



$\Psi_m = \iint_S \vec{B} \cdot d\vec{s} = \dfrac{\mu NI}{2\pi r_0} \pi a^2$ wb

* يجب تقريب الحلقات لأنه إذا كانوا مبتعدة عن بعضها سينتج Fringing

ex : general loop Carrying Current I :

$\rightarrow \nabla \times \vec{H}$

$\oint \vec{H} \cdot d\vec{l} = \iint_S \vec{J} \cdot d\vec{s} = I$

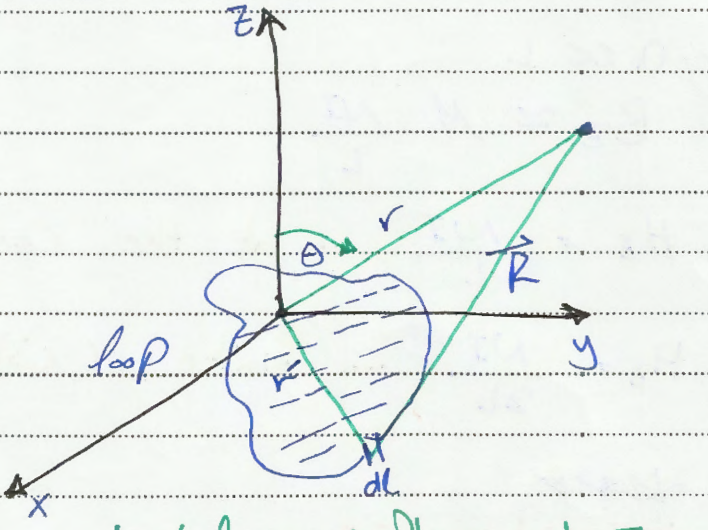
$\oint \vec{H} \cdot d\vec{l} = \iint_S \nabla \times \vec{H} \cdot d\vec{s} \Rightarrow$ Stoke's Theorem

Subject:...................... / /

ex: find $\vec{A}, \vec{B}$ and $\vec{H}$

$$\vec{A} = \frac{\mu I}{4\pi} \oint \frac{1}{R} \vec{dl}$$

$$= \frac{\mu I}{4\pi} \iint \underbrace{\nabla(1/R)}_{\left(\frac{\vec{R}}{R^3}\right)} \times \vec{ds}$$



$\vec{r} \parallel \vec{R}$

Located in xy Plane at z=0

area $\vec{S} = S' \vec{a_z}' = S' \vec{a_z}'$

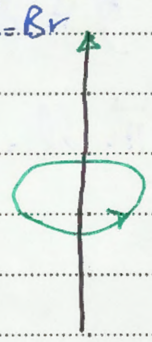
$$\simeq -\frac{\mu I}{4\pi} \iint_S \frac{\cancel{r}\, \vec{a_r}}{r^{3/2}} \times \vec{ds}'$$

$$\simeq -\frac{\mu I S' \vec{a_r}}{4\pi r^2} \times \cancel{a_r}\ a_z'$$

$$\simeq \frac{\mu (I S')' \times \vec{a_r}}{4\pi r^2}$$ → magnatic dipole moment $= \vec{m_m}$

$\vec{m_m} = \pi a^2 I = S I$

$$\vec{A}(\vec{r}) = \frac{\mu \vec{m_m} \times \vec{a_r}}{4\pi r^2} \qquad \vec{B} = \nabla \times \vec{A} \qquad \vec{B} = Br$$



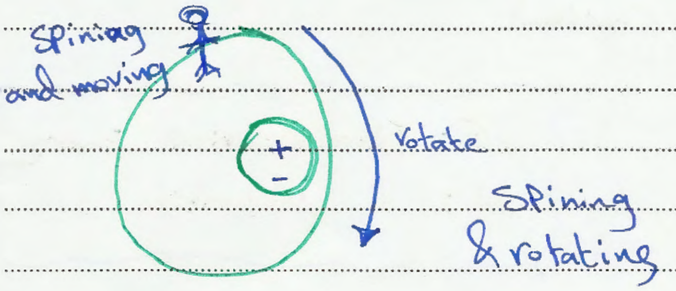
$$\left(V(\vec{\sigma}) = \frac{m_e \cdot a_r}{4\pi\epsilon r^2} \right) \xrightarrow{\text{Then}} E$$

m_e → q_L

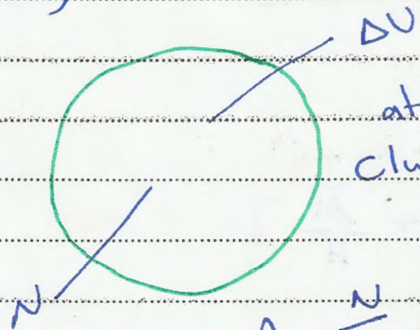
$$\& \quad \vec{B} = \frac{\mu\, m_m}{4\pi r^3} \left[2\cos\theta\, a_R + \sin\theta\, \vec{a_\theta} \right] \text{ wb/m}$$

Subject:....................... / /

* Magnetization and Material mag. c/s :



$\boxed{M_m = I S}$



atoms

cluster of atoms

Zero

$$A = \sum_{i=1}^{N} k \, \vec{m}_m \times \vec{a}_r$$

Non-mag ($\sum m_{m_i} = 0$) (all material expect (Fe, Co, Ni))

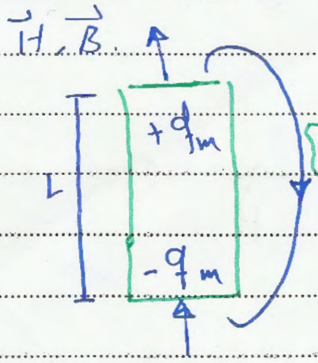
(Fe, Co, Ni) $\neq 0$ mag. me

$\vec{m}_c$ mag. moment density → for all material $\simeq 0$

for (Fe, Co, Ni) $= \sum_{i=1}^{N} \frac{m_{m_i}}{\Delta U}$ A/m

$= \underline{M}$ → magnitization Vector magnatic dipole

$\boxed{\vec{m}_m = I \vec{S}}$ ⇒



$\boxed{m_m = q_m L}$

$\vec{B} = \underline{\mu_0} \vec{H}$ → c/s of the Vacum with non-mag c/s

$$\vec{B} = \mu_0 [\underbrace{\vec{H}}_{\text{external current}} + \underbrace{\vec{M}}_{\text{Self - Current}}] = \mu \vec{H}$$

$$M = (\mu_r - 1)\vec{H}$$

ϵ: Store electrical Energy
μ: " Magnatic "

$$\mu \vec{H} = \mu_0 [\vec{H} + \vec{M}]$$

$$\mu_r = \frac{\mu}{\mu_0} \quad \mu_0 \rightarrow 4\pi \times 10^{-7}$$

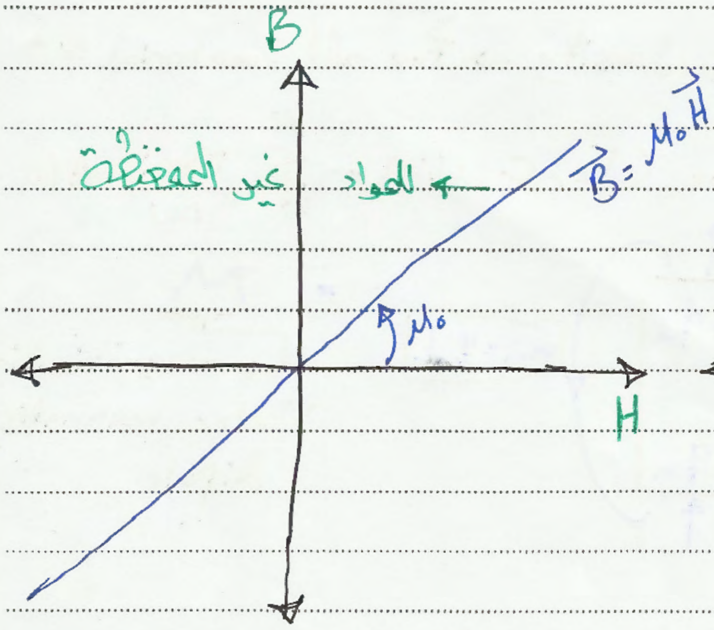
$$\mu = \mu_0 [1 + \frac{\vec{M}}{H}]$$

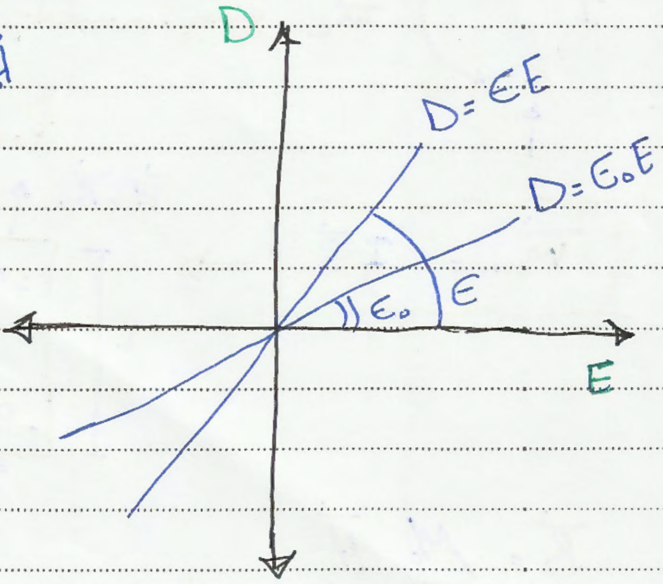
$$\mu = \mu_0 [1 + \underbrace{\frac{\vec{M}}{H}}_{\chi:\ \text{Susceptibility}}]$$

$M = 0$ ... for non magnatic Material

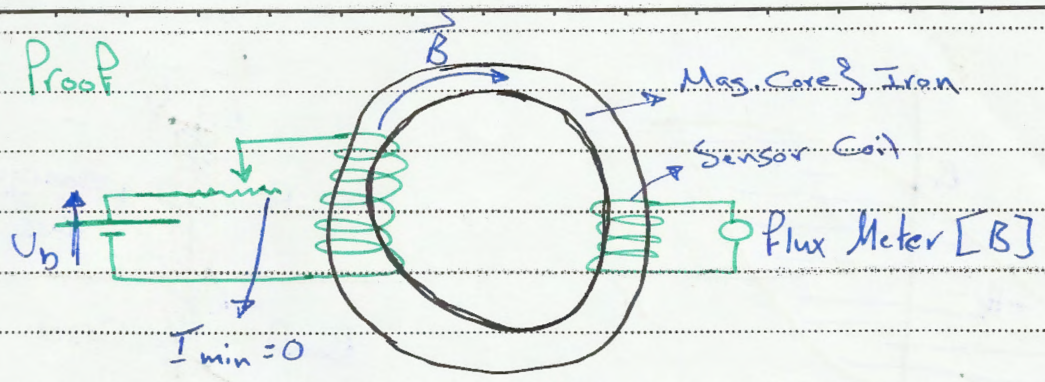
$\vec{M} \gg H$ ... for mag. Material

* B-H Curve (hysteresis loop):

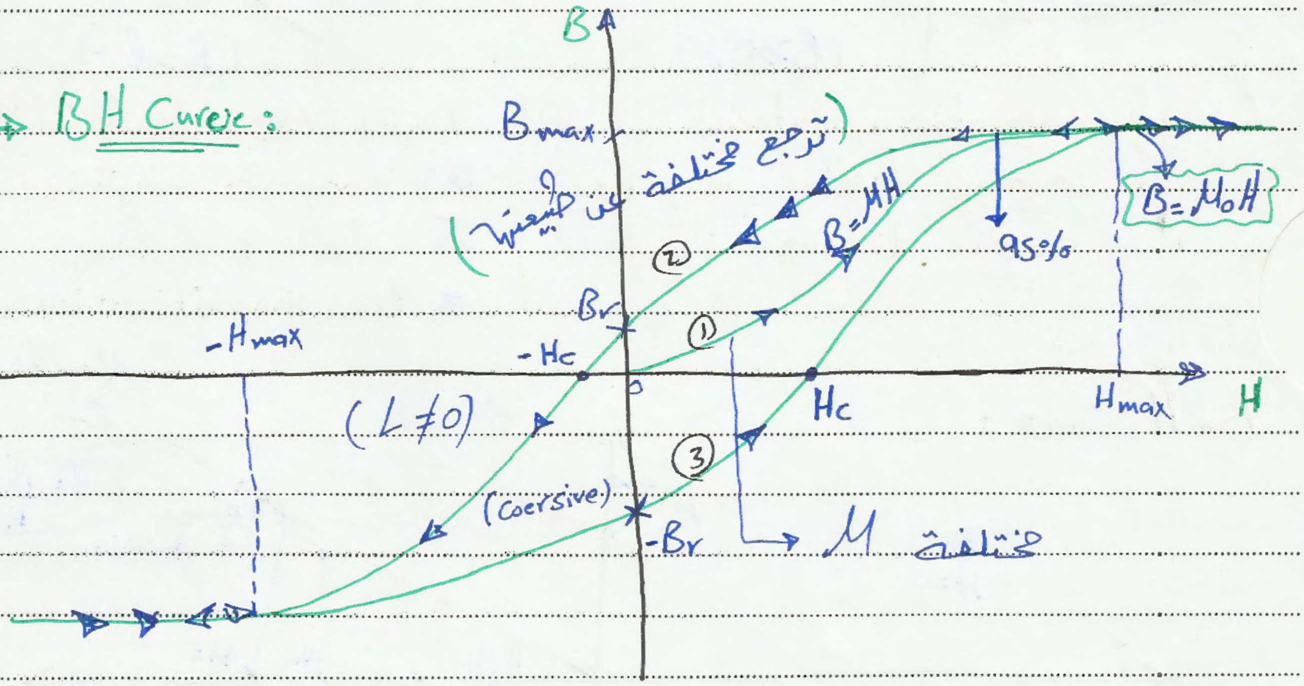




ex: Proof

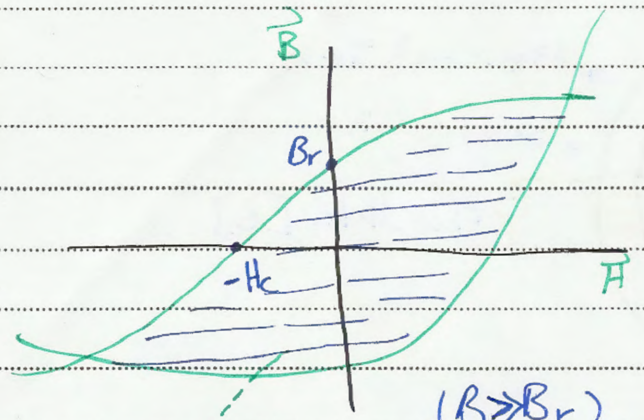


→ BH Curve:



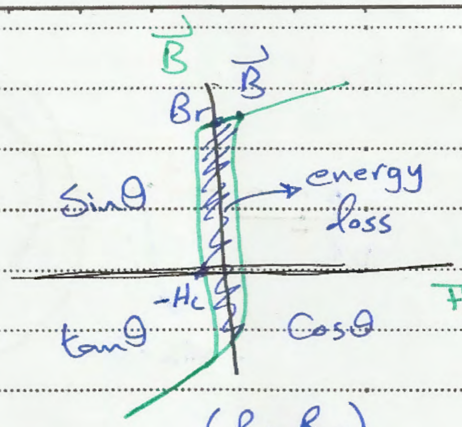
→ B_r (residual)

in ①: حتى تتكسر القطبية الداخلية (نقترب ثنائيات القطب) بالإتجاه الذي يجري فيه الـ $\overline{H}$



(hard magnatic Material)

→ energy lossing mag-tion and di-magation



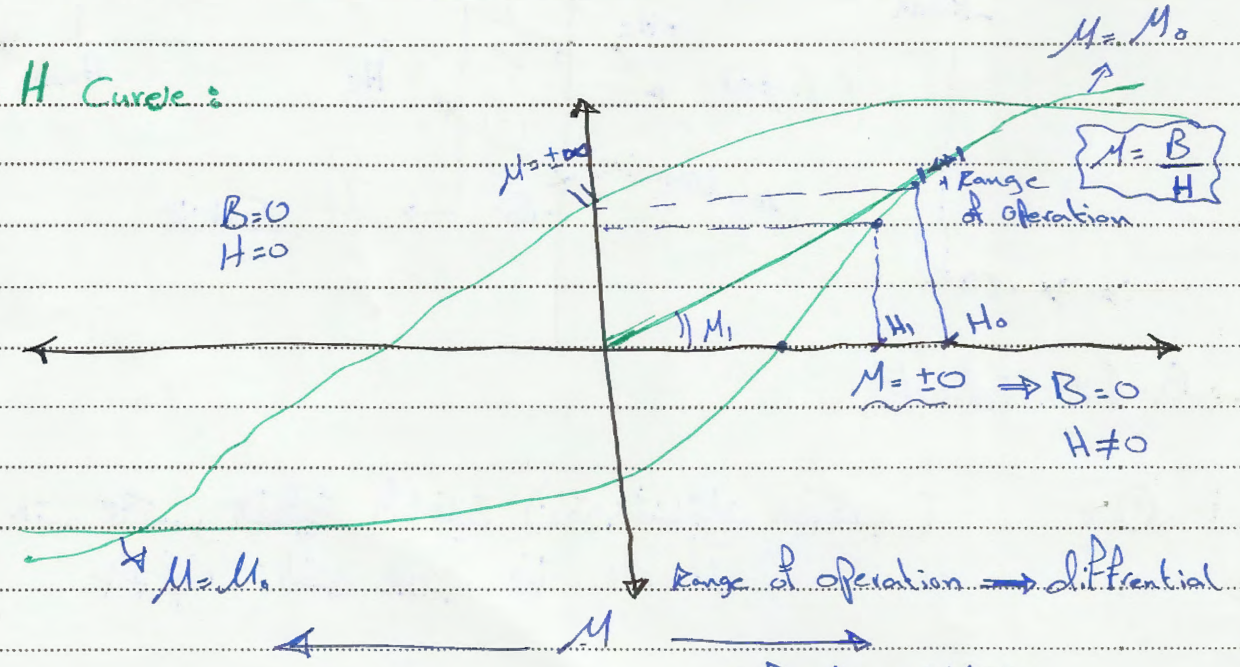
(Soft Mag. Material)

☒ hard to magnitize.

☒ " " lose

* the magnitization.

B-H Curve:

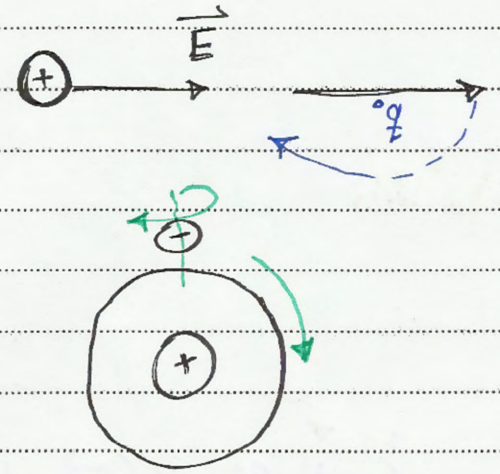


$\frac{\Delta B}{\Delta H}\Big|_{\text{at } H_1}$

* the Volume of the mag materials becomes Smaller than Before

Subject:...................... / /

* Boundry Conditions :



$$\oiint \vec{F}\cdot d\vec{s} = \iiint_V \nabla\cdot\vec{F}\, dv$$

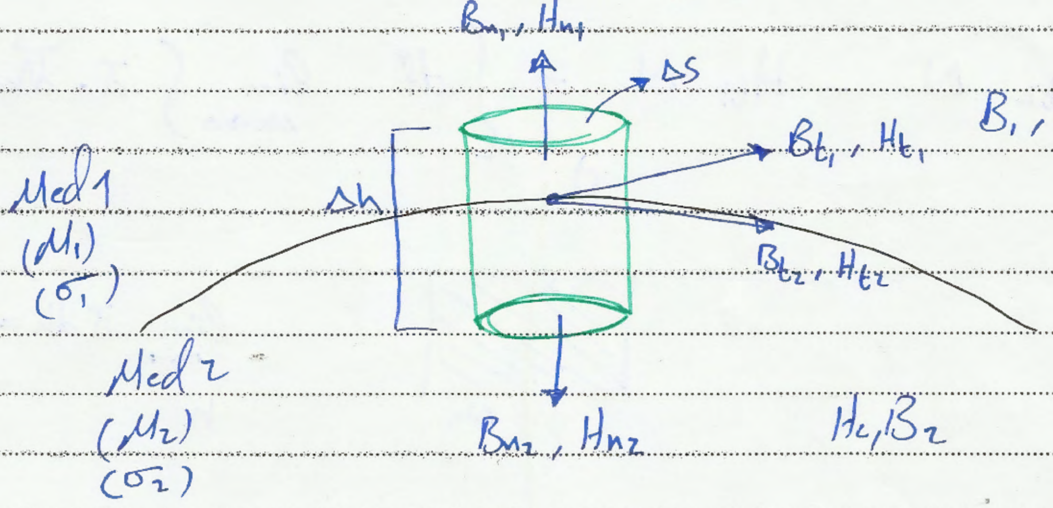
$$\oiint \vec{B}\, ds = \text{Zero} \Leftrightarrow \iiint \nabla\cdot\vec{B}\, dv$$

$$\nabla\cdot\vec{B} = 0$$

$$\boxed{\oiint \vec{F}\cdot d\vec{s} = \iiint_V \nabla\cdot\vec{F}\, dv}$$

$$\oint \vec{H}\cdot d\vec{l} = I = \int_t K\, dl' = \iint \vec{J}\cdot d\vec{s}'$$

$$\boxed{\vec{J} = \nabla\times H}$$



* Normal Component :

$$\oiint \vec{B}\cdot d\vec{s} = 0$$

$$\lim_{\Delta h\to 0}\left(\iint_{\Delta S_u} + \iint_{\Delta S_L} + \iint_{\Delta S_{side}}\right) B\cdot d\vec{s} = 0$$

$\rightarrow$

$B_{n_1} \Delta S_1 - B_{n_2} \Delta S_2 = 0$

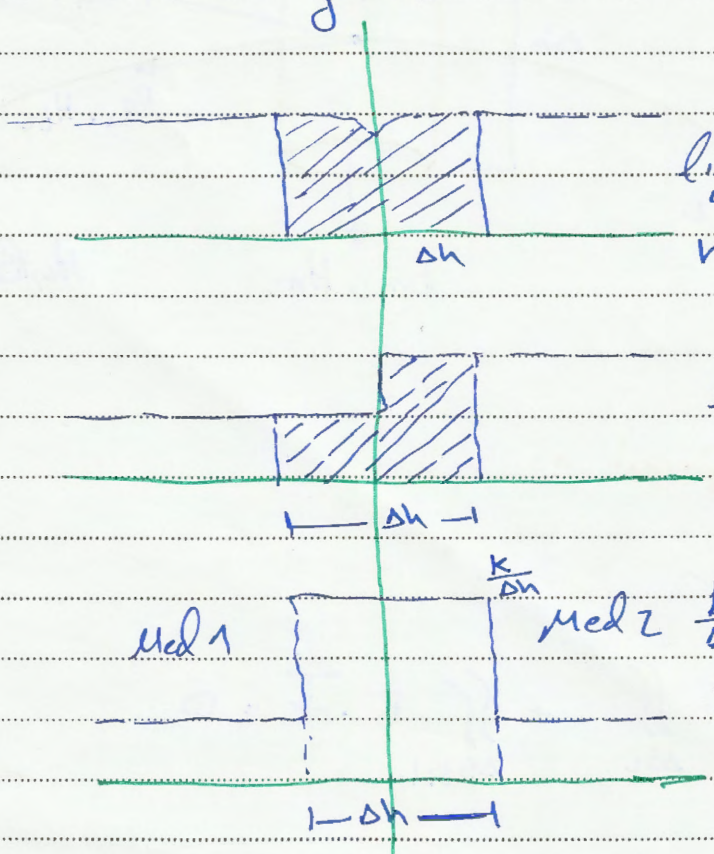
as ($\Delta S \to 0$)

$\boxed{B_{n_1} = B_{n_2}}$ .... دائماً وأنت مغمض

tangential Comp:

$$\oint_{\Delta h \to 0} H \cdot dl = \int_1^2 + \cancelto{0}{\int_2^3} + \int_3^4 + \cancelto{0}{\int_4^1} \vec{H} \cdot d\vec{l} = \iint \vec{J} \cdot d\vec{s}'$$

$$H_{t2} \Delta L - H_{t1} \Delta L = \int_{\Delta L} dl \lim_{\Delta h \to 0} \int J \cdot \vec{dh}$$



$\lim_{\Delta h \to 0} J\,dh = 0$

$\lim_{\Delta h \to 0} \int J\,dh = 0$

$\lim_{\Delta h \to 0} \int \frac{K}{\Delta h}\, dh = K$ (linear current density) $\left(\frac{A}{m}\right)$

Subject:....................... / /

$$H_{t2}\Delta L - H_{t1}\Delta L = \begin{cases} 0 \ \text{... Good conductivity} \\ K\Delta L \end{cases}$$

0 ... غالباً بوجود الـ Current في حالة الـ Good conductivity

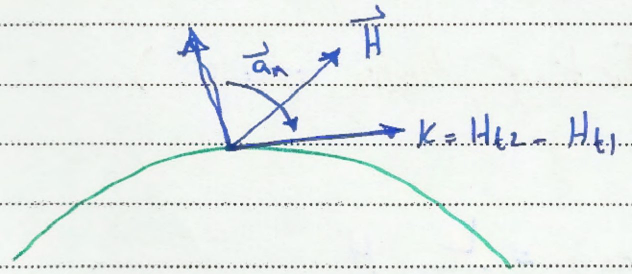
as $(\Delta L \to 0)$

$$H_{t2} - H_{t1} = \begin{cases} 0 \\ K \end{cases}$$

in E.S

$$E_{t1} = E_{t2} \longleftrightarrow B_{n1} = B_{n2}$$

$$D_{n1} - D_{n2} = \rho_s \longleftrightarrow H_{t2} - H_{t1} = K$$



$$\boxed{a_n \times \vec{H} = K}$$

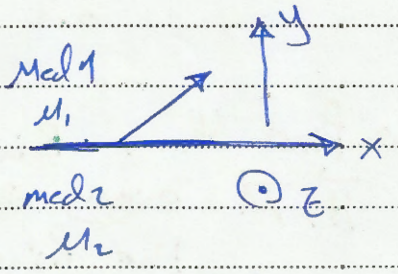
ex: find B & H every where and $\bar{M}$

$$\vec{B} = \mu_0[\vec{H} + \vec{M}] = \mu \vec{H}$$

$$= \mu_0 \mu_r \vec{H}$$

$$\therefore \mu = \mu_0\left[1 + \frac{\vec{M}}{H}\right]$$

$$\vec{H}(\mu_r - 1) = \vec{M}$$



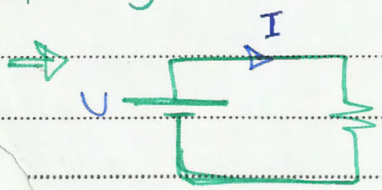
$$\boxed{\begin{aligned} \vec{B}_2 &= \vec{B}_{2n} + \vec{B}_{2t} \\ \vec{H}_2 &= \vec{H}_{2n} + \vec{H}_{2t} \end{aligned}}$$

$\overrightarrow{B_{1n}} = \overrightarrow{B_{2n}} = 6\, a_y\, \mu T$

$B_{1t} = 5\, a_x \mu_x$ $\quad H_{1t} = \dfrac{5\, a_x}{\mu_1}$ A/m

$H_{1t} = H_{2t}$ ... in this example (there's no current)

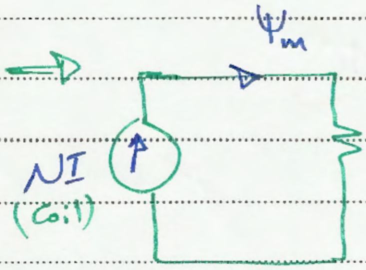
* Mag. ckt:



$\Rightarrow$ $R = \rho \frac{L}{A}$ ... electric ckt

o.L $(V = IR)$

* Short ckt: Perfect Conducting Media $\sigma = \infty$)

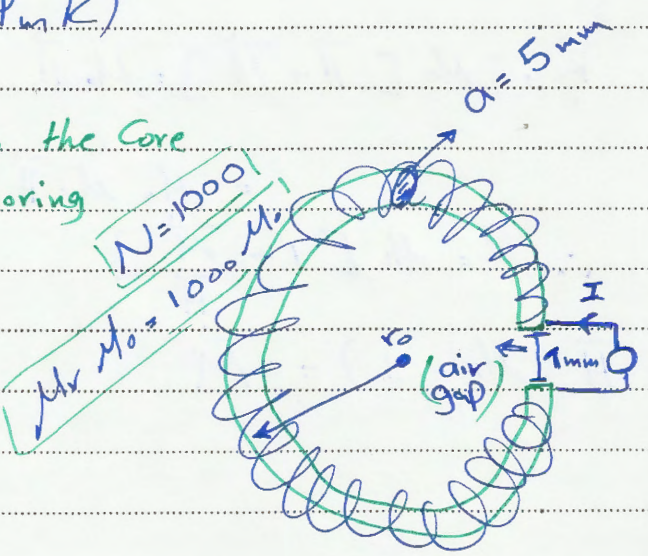


$\Rightarrow$ R (Reluctance) $= \dfrac{L}{\mu A}$ H

o.L $(NI = \Psi_m R)$

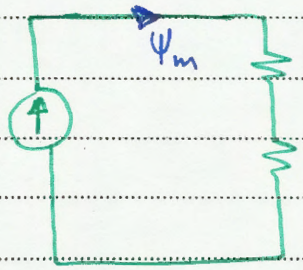
ex: find Ψ_m, $B_{in\,gap}$, B_o the core
$\vec{H}, \vec{M}$ every where (ignoring the fringing)

$\Rightarrow$ Circumfance mag mat
$= 2\pi r_0 - 1\text{mm}$
$= 1000$ mm



Subject: .... / /

→ equivelent Mag. ckt :



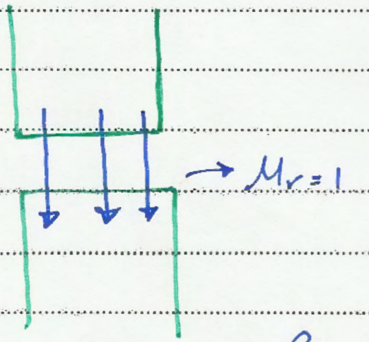
$$R_m = \frac{L}{\mu A} = \frac{1}{1000\,\mu_0 A} = \frac{1}{\mu_0 A} \ mH^{-1}$$

$$R_g = \frac{1\,mm}{\mu_0 A} = \frac{1}{\mu_0 A} \ mH^{-1}$$

$$\Longrightarrow (R_m = R = R_g)$$

$$\Psi_m = \frac{NI}{R_{eq}} = \frac{1000}{2R} \ wb$$

B_g = equal from B.C (B_n is constant) B_{core}



$$B_1 = B_g = B_2$$

$$H_1 \neq H_g$$

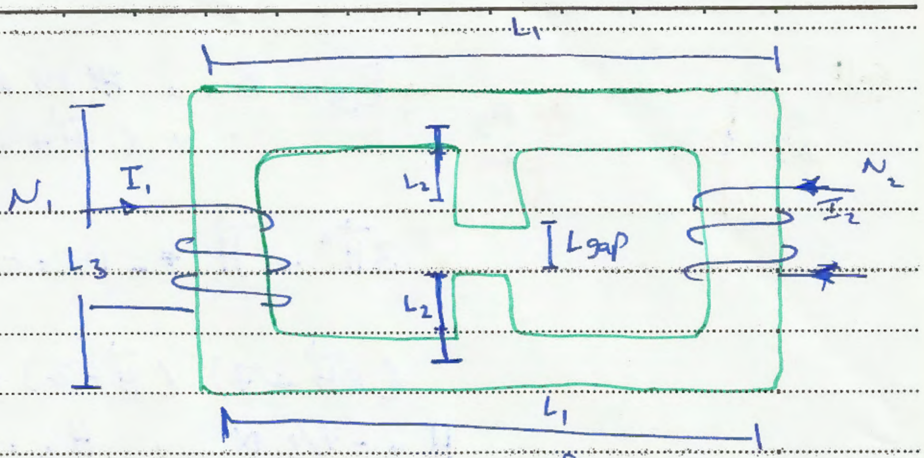
$$H_1 = H_2$$

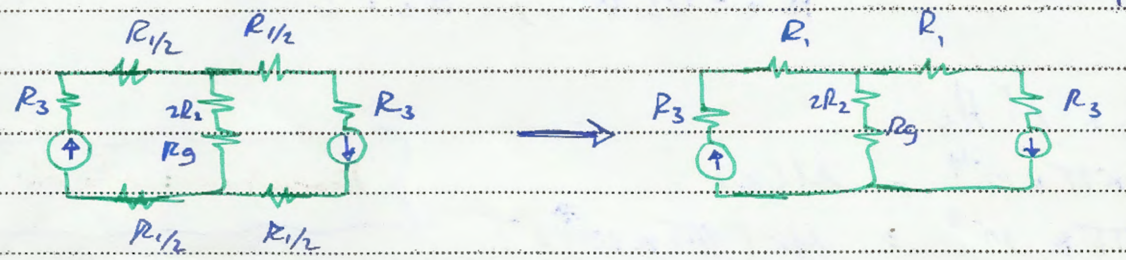
$$B_g = B_c = \frac{\Psi_m}{Area} = \frac{1000}{\frac{\cancel{A}\, 2 * 10^{-3}}{\mu_0 \cancel{A}}} = 0.2\,\pi \ wb$$

$$H_g = \frac{0.2\,\pi}{4\pi * 10^{-7} = \mu_0} \qquad H_c = \frac{B_g}{\mu_r \mu_0}$$

Subject: .... / /

ex: Find B_g:





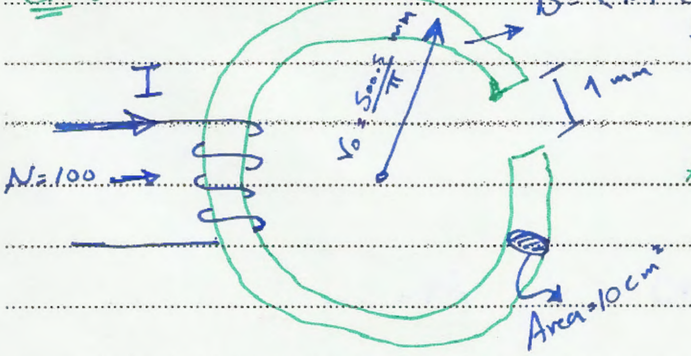
* Mesh analysis: $N_1 I_1 = \Psi_1 (R_3 + R_1 + 2R_2 + R_g) - \Psi_2 (2R_2 + R_g)$

* Mesh analysis: $N_2 I_2 = \Psi_2 [R_1 + 2R_2 + R_3 + R_g] - \Psi_1 [2R_2 + R_g]$

Find $\Psi_1, \Psi_2 \rightarrow \Psi_g = \Psi_1 + \Psi_2$

$$\boxed{B_g = \frac{\Psi_g}{area}} \qquad \boxed{H_g = \frac{B_g}{\mu_0}}$$

ex:



$B = (\vec{H} + 3\vec{H}^2)\pi * 10^{-4}$

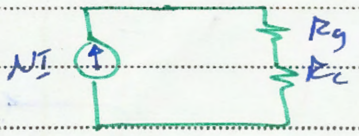
for $1 < H < 3$ A/m

$B_g = 14 * \pi * 10^{-4}$ wb/m

* Find I, H, M every where, μ_r?

Subject:...................... / /

Sol:



$$\rightarrow B_g = B_c = \cancel{\#}\, 14 * \pi * 10^{-4}$$
$$= (\vec{H} + 3\vec{H}^2) * \pi * 10^{-4}$$

$$\rightarrow 3\vec{H}^2 + \vec{H} - 14 = 0$$

$$(3\vec{H} + 7)(\vec{H} - 2) = 0$$

$$H = -7/3 \times \quad , \quad H_c = 2$$

$$B = \mu H_c$$

$$14 * \pi * 10^{-4} = \mu(2)$$

$$14\pi * 10^{-4} = \mu_r (4\pi * 10^{-7})^2$$

$$\boxed{\text{Remember} : \mu = \mu_r \mu_0}$$

$$\mu_r = (7/4) * 10^3$$

$$R_g = \frac{1m}{4\pi * 10^{-7}} = \frac{1 * 10^{-3}}{40\pi * 10^{-11}} = \frac{1}{40\pi} * 10^8$$

$$R_c = \frac{2\pi * 500.5 * 10^{-3}}{\mu \pi\, 10 * 10^{-4}} = \frac{1000.1 * 10^{-3}}{\mu_r \mu_0 * 10 * 10^{-4}}$$

$$100 I = B + A\left[\frac{l_c}{\mu_r \mu_0 A} + \frac{l_g}{\mu_r \mu_0 A}\right]$$

$$100 I = \frac{B}{\mu_0}\left[\frac{4}{7} * 10^{-7} + 10^{-3}\right]$$

$$\boxed{I = 0.055 \text{ A}}$$

$$M = H[1 - \mu_r]$$

$$M = 2[1 - 7/4 * 10^{-7}]$$

OR $B_c = \mu_0 [H + M]$

$$14\pi * 10^{-4} = 4\pi * 10^{-7}[2 + M] \rightarrow \boxed{M = \ldots}$$

Subject:....................... / /

* Mag. force, energy and storage :

$$\text{Inductor} \longrightarrow \text{Inductance} \equiv L = \frac{\Psi_m \overset{\text{linking}}{L}}{I} \; [H] \; [\text{Henry}]$$

$$\boxed{C = \frac{Q}{V}}$$

$$= \frac{B * \text{area}}{H * L}$$

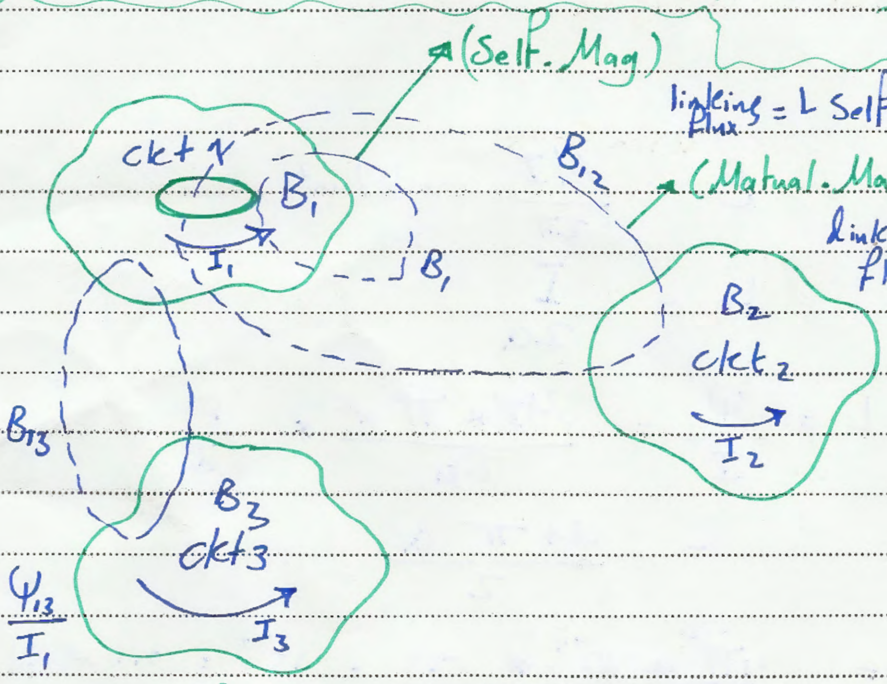
$$L = \frac{\mu H * A}{H L} = \underbrace{\frac{\mu A}{L}}_{\text{depends on Volume}} = R^{-1}$$

(Self. Mag)

linking flux = L Self $\rightarrow \boxed{L_1 = \frac{\Psi_1}{I}}$

(Mutual. Mag)

linking flux = $\boxed{L_{12} = \frac{\Psi_{12}}{I_1}}$



$$L_{13} = \frac{\Psi_{13}}{I_1}$$

* energy of the magnetostatic

$$W_m = \frac{1}{2} L I^2$$

$$= \frac{1}{2} \frac{\Psi_{mL}}{\cancel{I}} I^{\cancel{2}}$$

$$\boxed{\Psi_{mL} = B * \text{area}}$$

$$= \frac{1}{2} BH \text{ volume } J$$

$$W_m = \frac{1}{2} \iiint_V \underset{\mu H}{B} \cdot \vec{H} \, dv$$

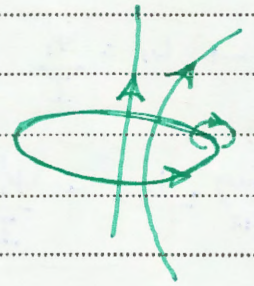
$$= \frac{\mu}{2} \iiint_V |\vec{H}|^2 \, dv$$

$$\boxed{w_m = \frac{W_m}{V}} \qquad \boxed{H = \frac{NI}{L} = K} \text{ as } V.I$$

$$W_m = \frac{1}{2} L I^2 = \frac{1}{2} \iiint_V \vec{B} \cdot \vec{H}\, dv$$

$$L = \frac{1}{I^2} \iiint_V B \cdot H\, dv \qquad H$$

ex : Find L of a Signal loop :



at the Center

$$B \sim \frac{\mu I}{2\pi} \quad wb/m^2$$

$$H = \frac{I}{2a}$$

$$L = \frac{\Psi}{I} = \frac{\mu \cancel{I} * \pi a^{\cancel{2}}}{2 \cancel{a}} * \frac{1}{\cancel{I}}$$

$$= \frac{\mu \pi a}{2}$$

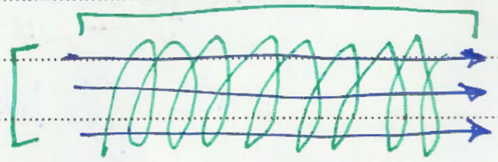
(a = 10cm)

$$L \simeq \frac{\mu \pi a}{2} = \frac{4\pi^2 * 10^{-7} * 0.1}{2} = 10^{-7} = 0.1 \mu H$$

ex Solenoid (10 ang)

$$\Psi_{mL} = (B * \text{area}) N = \frac{N^2 \mu A a^2 I}{L}$$

$$L = \frac{\Psi_{mL}}{I} = \frac{\mu N^2 \pi a^2}{L} \quad H$$



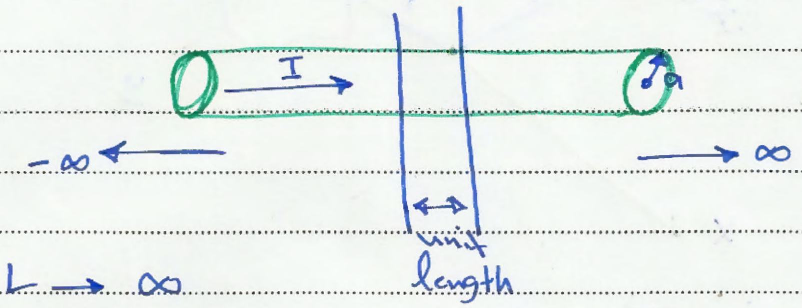
Subject:.................... / /

ex

Toroidal Coil

$$L = \frac{\mu N^2 \pi a^2 \cancel{I}}{2\pi r_o} \quad H$$

ex



$L \to \infty$

ex:

for unit length ??

$$H_1\, 2\pi r = \frac{I}{\pi a^2} \pi r^2$$

$$\Rightarrow H_1 = \frac{I r}{2\pi a^2}$$

$$\Rightarrow H_2 = \frac{I}{2\pi r}$$

$$\Rightarrow H_3 = \text{- - - etc}$$

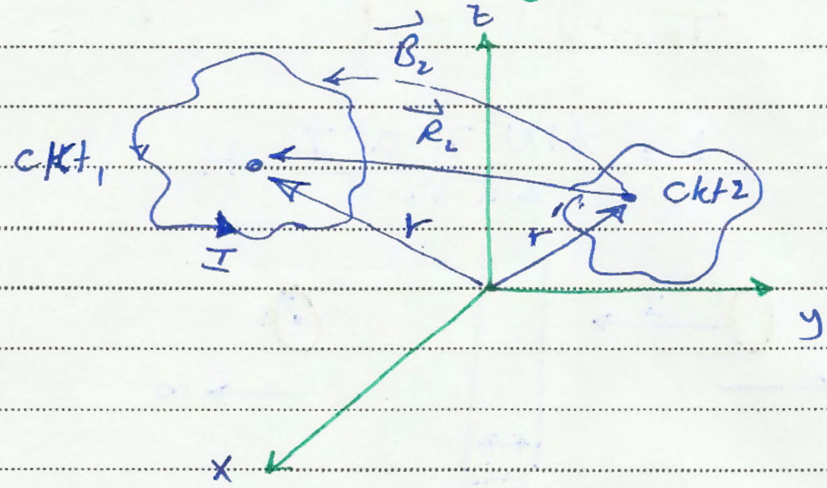
$$L_{total} = L_1 + L_2 + L_3$$

$$L_1 = L_{\substack{self \\ inner}} = \frac{1}{I^2} \iiint_v \vec{B} \cdot \vec{H}\, dv$$

$$= \frac{1}{\cancel{I^2}} \int_0^1 \int_0^{2\pi} \int_0^a \frac{\mu \cancel{I^2} r^2}{4\pi^2 a^4} r\, dr\, d\phi\, dz$$

$$= \frac{\mu}{6\pi}$$

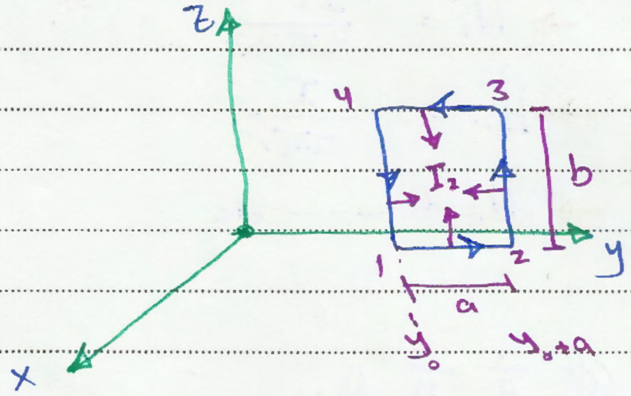
~~ex : go~~ * Forces in mag. ckts



$$\vec F_{21} = \iiint \vec J_1 * \vec B_2 \; dv$$

$$= \mu \iiint_{v_1} \iiint_{v_2} \vec J_1 \times \left(\frac{\vec J_2 \times \vec a_{R_2}}{4\pi R_2^2} \right) dv_1 \, dv_2$$

ex : an infinite line, Rect loop :



(مهم: الـ integral بالنسبة لـ I)

Sol : $B_{1x} = \dfrac{\mu_0 I_1}{2\pi y}$

$$F_{12} = \int_{y_0}^{y_0+a} I_2 \, dy \, a_y \times (-B_1 \vec a_x)$$

$$* \quad F_{12z} = \frac{\mu_0 I_2 I_1}{2\pi} \int_{y_0}^{y_0+a} \frac{dy}{y} = \frac{\mu_0 I_1 I_2}{2\pi} \ln\left(1+\frac{a}{y_0}\right) N$$

$$\boxed{F_{34z} = F_{12z}}$$

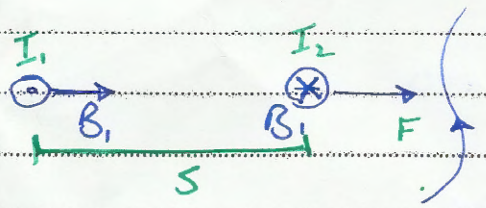
$$\boxed{F_{23y} = \frac{-\mu_0 I_1 I_2 b}{2\pi (y_0+a)} \; N}$$

$$\boxed{F_{23} = \int I_2 \, dz \, a_z \times (-B a_x}$$

$$L_m \propto \mu$$

$$\boxed{F_{41y} = \frac{\mu_0 I_1 I_2 b}{2\pi y} \; N}$$

ex: open wire transmission line



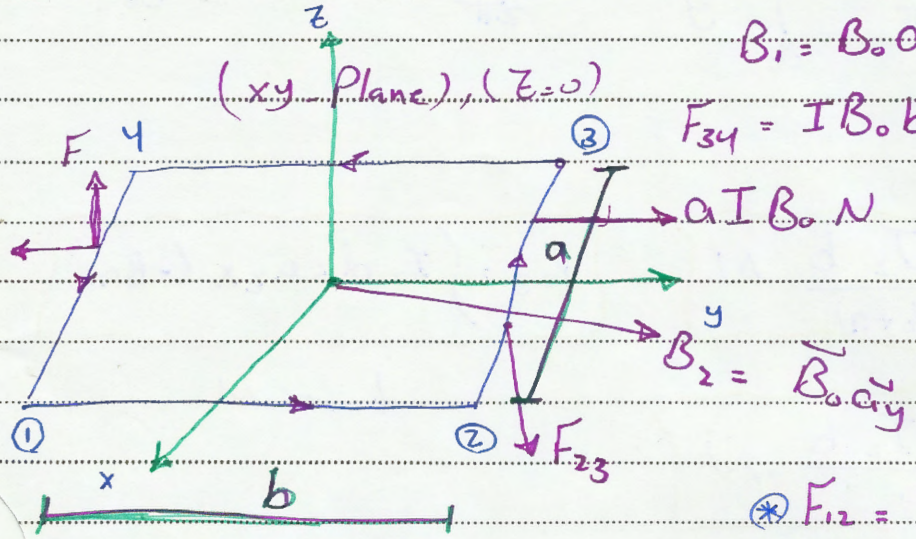
$$B_1 = \frac{\mu I_1}{2\pi S}$$

$$\boxed{F_{12} = \frac{\mu I_1 I_2}{2\pi S} \; N/m}$$

Attraction ...

Subject: .... / /

ex: Find the mag force on the loop for $\vec{B_1}$ and $\vec{B_2}$



(هنا في حالة وجود B_1 & B_2)

$\circledast\ F_{12} = I b\, a_y \times B_1\, a_z$

if we have B_2 only:

$\Rightarrow \vec{F_{12}} = I L * \vec{B} = F_{34} = 0$

$\Rightarrow \vec{F_{23}} = I a (-a_x) \times B_0\, a_y$

$= -I B_0\, a\, \vec{a_z}$

$\Rightarrow \vec{F_{41}} = I B_0\, a\, \vec{a_z}$

$\text{Torque} \equiv \vec{T} = I \underbrace{B_0}_{B_0 a_y} \underbrace{ab}_{\text{area}}$

$\boxed{T = \vec{m_m} \times \vec{B}}$

Subject:.................... / /

* Magnatic energy :

$W_m = F_m * \text{length}$

$\frac{dw_m}{dl} = F_{mag} = m\vec{a}$

$W_m = \frac{1}{2}\iiint_v B \cdot H \, dv = \frac{1}{2}\iiint \mu (H)^2 \, dv$

$= \frac{1}{2} L I^2 \quad J$

- Iteraction between electric and magnatic field and charged Practicle :

$\vec{F_e} = qE = ma' = m\frac{dv}{dt} = ma$

$F_m = q\vec{v} * B = m\vec{a}$

$F_e = qE = ma = m\frac{dv}{dt}$

$F_eL = qL'L = m\frac{dv}{dt}L = m\frac{L}{t}\,dv$

$qV = \frac{m}{2}(v_2^2 - v_1^2)$

if the initial Velocity = 0

$\frac{V_2^2 m}{2} = eV \Longrightarrow V_2 = \sqrt{\frac{2eV}{m}}$

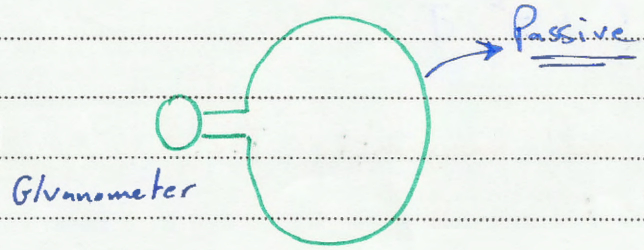
Subject:...................... / /

$\nabla \times E = -0$ $\boxed{I}$ KVL

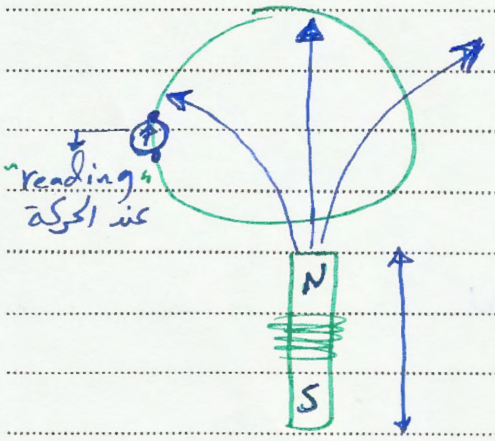
$\nabla \times \vec{H} = \vec{J}$ $\boxed{V}$

$\nabla \cdot J = 0$ KCL

* Farday's law :



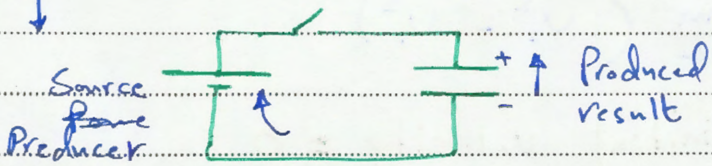
Reading = 0 $\oint E \cdot dl = 0$ KVL



→ if (B) increase then I c.w .

→ if (B) decreas then I c.c.w .

$$\oint E \cdot dl = -0 - \frac{d\Psi_m}{dt}$$ ← farady's law & KVL

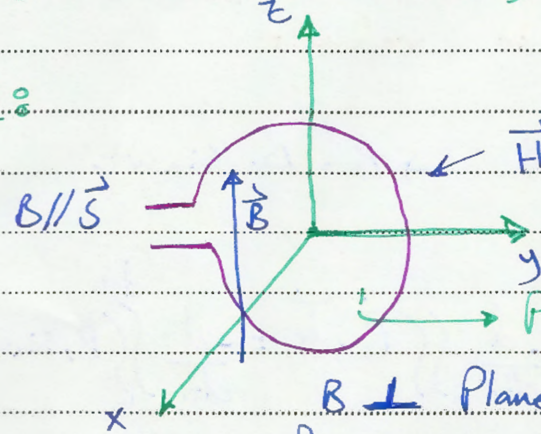


$$emf = \oint_L E \cdot dl = -0 - \frac{d}{dt} \iint \vec{B} \cdot \vec{ds}$$

Subject: .... / /

⊛ GKL or/and Faraday's law   1 mA/m

ex:



$\vec{H} = H_o \sin wt \, \vec{a_z}$
rad/s

$\boxed{w = 2\pi f}$

B ⊥ Plane of the area

$$emf = \oint_L \vec{E} \cdot \vec{dl} = V_r = \frac{-d}{dt} \iint \vec{B} \cdot \vec{ds}$$

$$= \frac{-d}{dt} \iint \mu H_o \sin wt \, \vec{a_z} \, ds \, \vec{a_z}$$

$$\therefore V_{r_{meaf}} = -\overset{10\ cm^2}{A} \, \underbrace{\mu}_{constant} \, \underbrace{w H_o}_{constant}$$

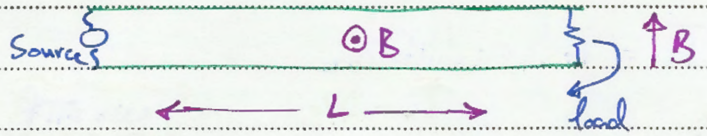
$$= 10 * 10^{-4} * 10^{-6} * 10^{7} * 10^{-3} \Longrightarrow 10 \ \mu V$$

* V_r is very small quantity (we can't make the area larger than $10\ cm^2$) ⟹ ① use $\mu = \mu_r \mu_o$.
② use N-turns.

$\vec{F} = m\vec{a} = m \frac{dv}{dt} = q\vec{E}$

$\vec{E} \cdot \vec{dl}$

ex : Tel. line , find noise ($V_{r\,noise}$) Suggest mean to reduce



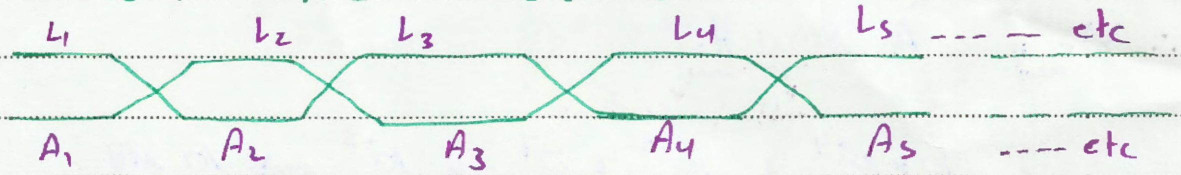
$\rightarrow B = B_0 \sin wt$

$$emf = V_{noise} = \oint \vec{E}\cdot\vec{dl} = -\frac{d}{dt}\iint \vec{B}\cdot\vec{ds} = -\frac{d}{dt}\int_0^d\int_0^L B_0 \sin wt\, dx\, dt$$

$$= -\frac{d}{dt}\left(B_0 \sin wt\; L d\right)$$

$$= -B_0 L d\, w \cos wt \quad V$$

* To reduce the answer :



$$V_{r\,noise} = \sum_{i=1}^{N} \Delta V_{n_i} = \begin{cases} \rightarrow 0 \\ \rightarrow \pm \Delta V_{n_i} \end{cases}$$

$$emf = \oint_L \vec{E}\cdot\vec{dl} = -0 - \frac{d}{dt}\iint_S \vec{B}\cdot\vec{ds}$$

B, S fixed $= 0$

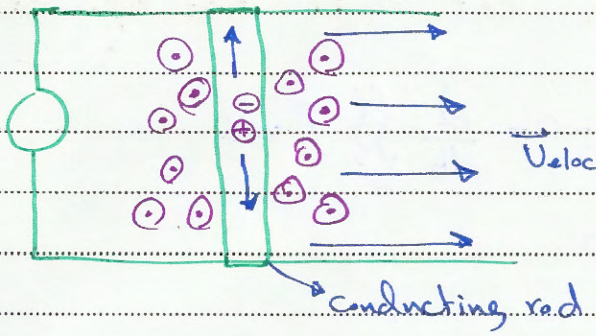
$\frac{d\,\Psi_m}{dt} = 0$

$\vec{B}$, S is fixed

$\vec{B}$ is fixed, $\cancel{S}$

Subject: .... / /

ex:



* $B = B_0 \vec{a_z}$

* $F = q\vec{v} * \vec{B}$

* $E\cancel{q} = \cancel{q}\,\vec{U} * \vec{B}$

* $E = \vec{U} * \vec{B}$

* $\oint_L \vec{E} \cdot \vec{dl} = \oint_L U * \vec{B}\, dl$

* M.E in integral ① ~~From~~ for :

$$emf = \oint E \cdot dl = -\frac{d}{dt} \iint_s B \cdot ds \quad --- ①$$

Generalized KVL

$$mmf \cong \oint \vec{H} \cdot dl = \iint \vec{J} \cdot ds + \iint \frac{d\vec{D}}{dt}\, ds \quad --- ②$$

General KCL

$$\boxed{J = \sigma E \quad , \quad B = \mu H \quad , \quad D = \epsilon E}$$

$$\boxed{\oiint D \cdot ds = \iiint \rho_v\, dv} \quad , \quad \boxed{\oiint B \cdot ds = 0}$$

S T A R S Notebook

Subject:...................... / /

Using divergence theorem let the first eq. stakes

$$\oint E.dl = \iint \nabla \times E . ds = -o- \frac{d}{dt} \iint_{s'} B.ds$$

$$L \longrightarrow a \longrightarrow o$$

$$\underset{S_1}{S} \longrightarrow \underset{\Delta S_1}{\Delta S} \longrightarrow o$$

$$\lim_{\Delta S_i \to 0} \nabla \times E . \cancel{\Delta S} = -\frac{d}{dt} B . \cancel{\Delta S}$$

$$\boxed{\nabla \times L = -\frac{d}{dt} B = -o- \frac{d}{dt} \vec{B}}$$

* Current : $I = \frac{q}{t} = \frac{CV}{t} = \frac{AD}{t}$ $\boxed{C = q/v}$

$$\vec{J} = \frac{I}{A} = \frac{D}{t} \quad \ldots \text{ displacment . Current}$$

$$\boxed{\vec{J} = \frac{d\vec{D}}{dt}} \qquad \oint H.dl = I_{total}$$

* (J_{cond}) conduction σE

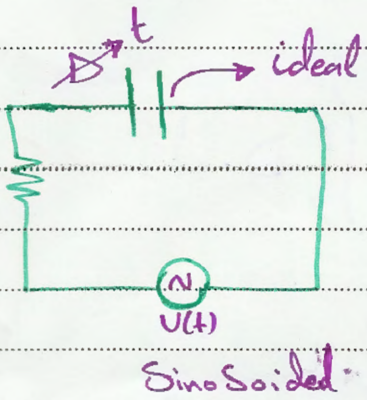
* Convention $(\bar{J}_{con}) = \frac{dq}{dt}$

* displacement $(\bar{J}_{disp}) = \frac{dD}{dt}$

Subject:........................ / /

J_{cond} → usually in Conducting medium with R.

J_{disp} → due to charging $\vec{D}$ not neccesarly due to Moving Charge.



→ ideal Vacum $\sigma \to 0$
No charge of Carrying Current ($J_{cond} = J_{conv} = 0$)

* Charge builds up on Plates but Keep on Charging due to Sino Soidal Source: (+ → − → + → −---)
So there is a $\vec{D}$ that changes with time

$$\oint H \cdot dl = I_{total} = I_{cod} + I_{conv} + I_{disp}$$

Conter clock wise

Note: if there is induction ⇒ there is no Convection in most Cases.

$$\iint_{S'} J_{cond} \cdot ds' + \iint_{S''} J_{conv} \cdot ds'' + \iint_{S'''} J_{disp} \cdot ds'''$$

$$= \iint_{S} \vec{J}_{Cond + Conv} \, \vec{ds} + \left(\frac{d}{dt} \iint_{S'''} D \cdot ds \right)$$ ⇒ 2^{nd} of max eq

Subject:..................... / /

mmf = $\oint H.dl$ (mag motive force)

* rewriting the equation in another form :

→ emf = $\oint_L E.dl = -0 - \frac{d}{dt}\iint B.ds$ --- ①

→ mmf = $\oint H.dl = \iint J.ds + \frac{d}{dt}\iint D.ds'$ --- ②

$\boxed{1}$ → 2 Casses : ⓐ $B \nearrow t$ and S fixed
ⓑ B fixed as $S \nearrow t$

$\boxed{2}$ → 2 Cases : ⓐ $D \nearrow t$ and S fixed
ⓑ D fixed as $S \nearrow t$ } .. not accepted

Prof. $+\frac{d}{dt}\iint \vec{D}.ds$ Since D is fixed
Mag. Material ... dielectric mat
$\mu = \mu_r \mu_o \sim 10^{-3}$ $\epsilon = \epsilon_o \epsilon_r \sim 10^{-11}$

→ it has to change (S) 10^8 times faster to generate J/S , and that not impossible in engineering ///

* in mmf : may be different Surface : for example a Resister in Parallel with Capacitor .

Subject:.................... / /

$$emf = \oint \vec{E} \cdot dl = -0 - \frac{d}{dt} \iint B \cdot ds$$

$$\oint \vec{E} \cdot dl = \iint_S \nabla \times \vec{E} \cdot ds = -0 - \frac{d}{dt} \iint_{S'} B \cdot ds' \quad (a)$$

* M.E of I. form

$$\boxed{\nabla \times \vec{E} = -0 - \frac{dB}{dt}} \quad \underline{kvl} \quad ---(1)$$

$$mmf = \oint H \cdot dl = \iint_{S'} J \cdot ds' + \frac{d}{dt} \iint_{S''} \frac{dD}{dt} ds'' \quad (b)$$

$$\iint_S \nabla \times H \, ds = \uparrow$$

$$(\Delta S \to 0, \Delta S' \to 0, \Delta S'' \to 0)$$

$$\nabla \times H \; \Delta S = J \Delta S' + \frac{dD}{dt} \Delta S''$$

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}} \quad -----(2)$$

Note: (1) & (2) ~~is~~ are a very important equations

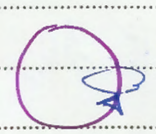
* M.E in differential:

* $emf = \nabla \times E = -0 - \frac{dB}{dt}$ ---(1

* $mmf = \nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$ ---(2

* $\vec{J} = \sigma \vec{E}$, $\vec{D} = \epsilon \vec{E}$, $\vec{B} = \mu \vec{H}$ ---(3

* $\nabla \cdot D = \rho_v$, $\nabla \cdot B = 0$ ---(4

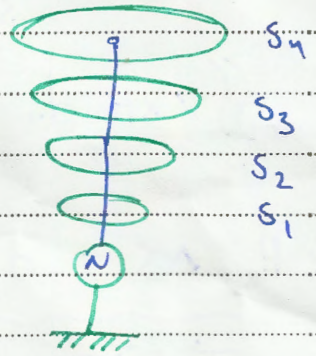

the flux = 0

$$\nabla\cdot[\nabla\times\vec{H}] = 0 = \nabla\cdot\vec{J} + \frac{d}{dt}\nabla\cdot\vec{D}$$

$$\nabla\cdot J = -\frac{d\rho}{dt}$$ ... GKCL at a point

$$\oiint \vec{J}\cdot\vec{ds} = -\frac{d}{dt}\iiint_V \rho_v\, dv$$ ... GKCL for a Surface

ex : Ampere's law, wire :



$$\oint H\cdot dl = \iint J\cdot ds$$

$$\oint_L H\cdot dl \neq 0$$

$$\iint_{S_1} \neq 0,\quad \iint_{S_2} \neq 0,\quad \iint_{S_3} \neq 0,\quad \iint_{S_4} \overset{= \text{zero}}{\cancel{\neq}\, 0}$$

Because it's outer of wire

$$\nabla\times\vec{H} = \sigma\vec{E} + \frac{d\vec{D}}{dt}$$

$$\nabla\cdot[\nabla\times H] = \nabla\cdot\left[\frac{\sigma}{\epsilon}\vec{D} + \frac{d\vec{D}}{dt}\right] = 0$$

$$= \frac{\sigma}{\epsilon}\rho + \frac{d\rho}{dt}$$

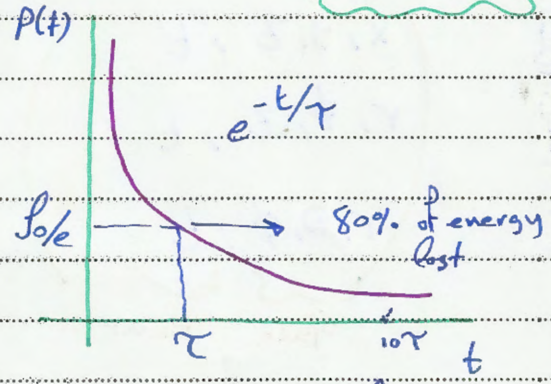
$$\boxed{\frac{d\rho}{dt} = \frac{-\sigma}{\epsilon}\, dt}$$

Subject:........................ / /

$$\therefore \ln \rho = \frac{-\sigma}{\epsilon} t + \underbrace{\ln \rho_0}_{\text{constant}}$$

$$\boxed{\therefore \ln(\rho/\rho_0) = \frac{-\sigma}{\epsilon} t} \qquad \boxed{t = \frac{\epsilon}{\sigma}}$$

$$\boxed{P(t) = \rho_0 \, e^{-t/\tau}} \qquad \boxed{\tau = \frac{\epsilon}{\sigma}}$$

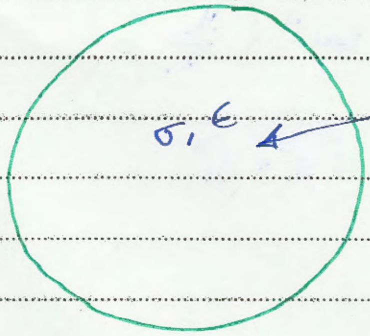


$$\epsilon = \frac{10^{-9}}{36\pi} = 10^{-11} \ F/m$$

$\tau \ll 1 \longrightarrow$ good conducting

$\tau \gg 1 \Longrightarrow$ dielectric (o.c.) $(\rho \to 0)$

σ_{copper}	τ	10τ
10^{7}	10^{-18} sec	10^{-17} sec
10^{-18}	10^{7} sec	10^{8} sec



(ρ_0 at $t=0$)

$$\boxed{\begin{gathered} \nabla \cdot J = -\frac{d\rho}{dt} \\ \oiint \vec{J} \cdot \vec{ds} = \frac{-d}{dt} \iiint_V \rho_v \, dv \end{gathered}}$$

$\longrightarrow$ continuity Relation

$$\boxed{\rho = \rho_0 \, e^{-t/\tau}} \longrightarrow \text{relaxation relation}$$

Subject:...................... / /

* Time harmonic Source and fields :

$$\sqrt{z} = (x^2+y^2)^{1/4} e^{j\theta/2} \quad , \underbrace{z = x + jy}_{\text{not continitions}}$$

$$f(t) = \sum_{i=1}^{\infty} F(w_i)$$

$$\begin{matrix} \vec{J} \\ \vec{E} \\ \vec{H} \\ \vec{D} \\ \vec{B} \end{matrix} \left(\begin{matrix} x, y, z, t \\ r, \phi, z, t \\ \underbrace{r, \theta, \phi}_{\text{space}}, \underbrace{t}_{\text{time}} \end{matrix} \right)$$

$$Re\left[e^{jwt}\right] = \cos wt = \frac{e^{jwt} + e^{-jwt}}{2}$$

$$Im\left[e^{jwt}\right] = \sin wt = \frac{e^{jwt} - e^{-jwt}}{2j}$$

$$\frac{d}{dt} e^{\pm jwt} = \pm jw\, e^{\pm jwt}$$

$$z_1 = x_1 + j y_1 = r_1 e^{j\theta_1} \qquad \theta_1 = \tan^{-1}\left(\frac{y_1}{x_1}\right)$$

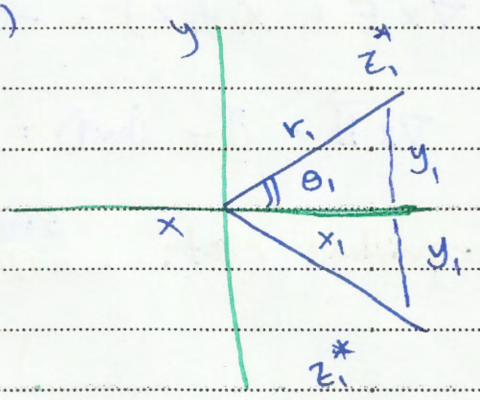
$$z_2 = x_2 + j y_2 = r_2 e^{j\theta_2} \qquad r_1 = \sqrt{x_1^2 + y_1^2}$$

Subject:...................... / /

$$z_1 \pm z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$$

$$z_1 z_2 = r_1 r_2 \, e^{j(\theta_1 + \theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$



$$\sqrt{z_1} = \sqrt{r_1} \, e^{\frac{j\theta_1}{2}}$$

$$\boxed{z_1^* = x_1 - jy_1 = r_1 e^{-j\theta}}$$: z_1^* is the image of z_1

$$(z_1)^\alpha = (r_1)^\alpha \, e^{j\theta_1 \alpha}$$

* M.E for harmonic Sources (Fields):

$$\nabla \times E(\vec{r})\, e^{j\omega t} = -\frac{d}{dt}\left[\vec{B}(\vec{r})\, e^{j\omega t}\right]$$

$$\nabla \times E(\vec{r})\cancel{e^{j\omega t}} = -j\omega B(\vec{r})\cancel{e^{j\omega t}}$$

$$\boxed{\nabla \times E = -j\omega \vec{B}} \quad \text{--- (1)}$$

$$\boxed{\nabla \times \vec{H} = \vec{J} + j\omega \vec{D}} \quad \text{--- (2)}$$

$$= \sigma E + j\omega \vec{D} = (\sigma + j\omega \epsilon)\vec{E}$$

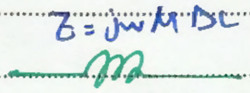
$$J = \sigma \vec{E}, \quad \vec{B} = \mu H, \quad \boxed{D = \epsilon E} \quad \text{--- (3)}$$

$$\nabla \cdot D = \rho_v, \quad \boxed{\nabla \cdot B = 0} \quad \text{--- (4)}$$

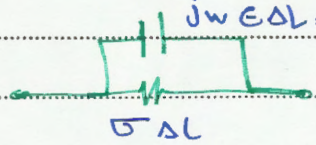
$\nabla \times E(\vec{r}\ x,y,z) = -0 - j\omega B(\vec{r}\ x,y,z) = -0 - j\omega\mu H$ --- ① *

$\nabla \times \vec{H} = J + j\omega D = \sigma E + j\omega\epsilon E = E(\sigma + j\omega t)$ --- ② *

equivlent ckt:



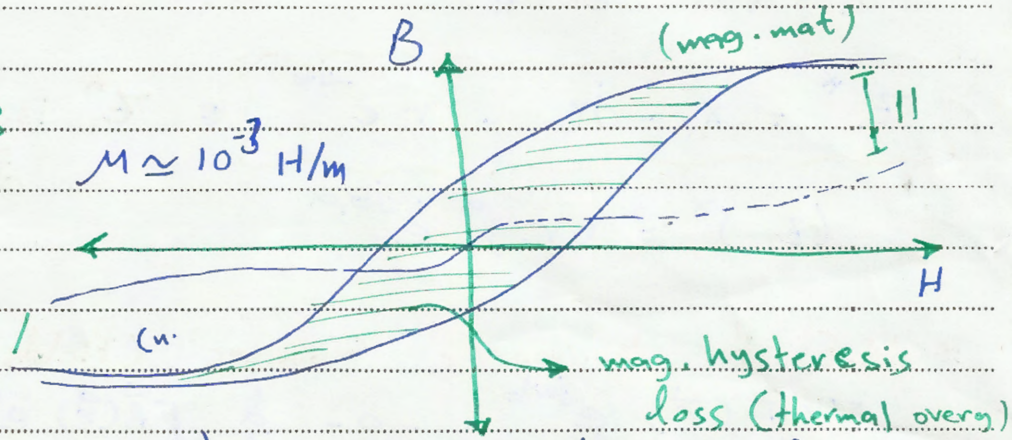
for --- ① *



for --- ② *

B.H Curve :

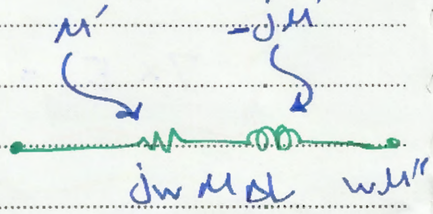
$\mu \simeq 10^{-3}$ H/m



$\nabla \times \vec{E} = -0 - j\omega\mu\vec{H} = 0 - j\omega(\mu' - j\mu'')\vec{H}$

$\boxed{\mu = \mu' - j\mu''}$

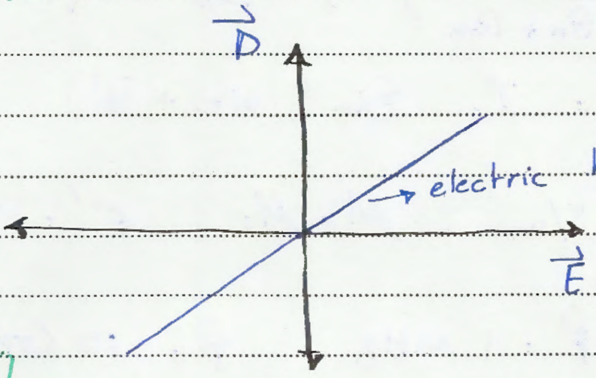
$= -[j\omega\mu' + \omega\mu'']\vec{H}$



*

* Hysteresis Concept :

Mag. dipole :

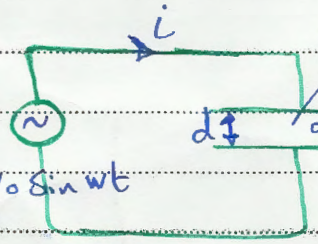


$\epsilon \sim 10^{-11}\ F/m$

$$\boxed{\epsilon = \epsilon' - j\epsilon''}$$

$$\nabla \times \vec{H} = [\sigma + w\epsilon'' + \frac{jw\epsilon'}{c}]\,E$$

ex :



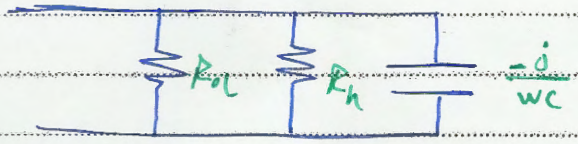
$\to R_{ohmic}$

$\to Am^2$

$\sigma, c, \mu_0, \epsilon = \epsilon' - j\epsilon''$

$\to R_{hysteresis}$

Find I, I_{rms} (Ignore fringing and inductance)



$$\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$$

$$\boxed{R_{ohmic} = \frac{d}{\sigma_c A}\ \Omega} \quad \boxed{R_h = \frac{d}{w\epsilon'' A}} \quad \boxed{C = \frac{\epsilon A}{d} = \frac{\epsilon' A}{d}}$$

$$Y = jwC + G_h + G_{oL}$$

$$I = VY = V_0[jwL + G_h + G_{oL}] = I_0 \angle \psi$$

Subject:...................... / /

$\Psi = \tan^{-1}\left(\frac{wc}{G_n + G_{on}}\right)$, $I_o = V_o\sqrt{(jwL)^2 + (G_n + G_{oL})}$

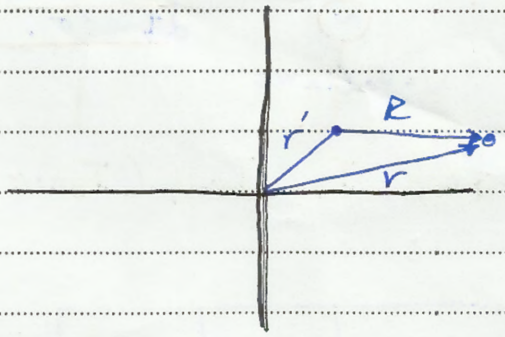
$\therefore \; i(t) = I_o \sin(wt + \Psi)$

Practice: $\sigma \sim 10^{-6}$ ℧/m $\mu = \mu_0$ $\epsilon' = 9\epsilon_0$ $\epsilon'' = 10^{-11}$ F/m

$f = 1$ MHz $f = 10$ GHz

Mag. Vector Potential : M.E :

$$A(\vec{r}) = \iiint_{V'} \frac{\mu J(\vec{r'})}{2\pi R} \, dv$$



$$B = \nabla \times A$$

$$\nabla \times E = -jwB = -jw[\nabla \times A]$$

$$\nabla \times [E + jwA] = 0$$

$$E + jwA = -\nabla V$$

$$\boxed{E = -jwA - \nabla V} \quad \text{---} \; (1)$$

$$\nabla \times H = J + jwD$$

$$\nabla \times \frac{B}{\mu} = J + jw\epsilon E$$

$$(\nabla \times B) = \mu J + jw\mu\epsilon E \rightarrow \boxed{\nabla \times [\nabla \times A] = \mu \vec{J} + jw\epsilon\mu\vec{E}}$$

Subject: .... / /

$$\nabla \times [\nabla \times A] = \nabla(\nabla \cdot A) - \nabla^2 A = \mu J + j\omega\mu\epsilon[-j\omega A - \nabla V]$$

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu \vec{J} + \underbrace{\omega^2\mu\epsilon}_{k^2 (\frac{rad}{m})^2} A - j\omega\mu\epsilon \nabla V$$

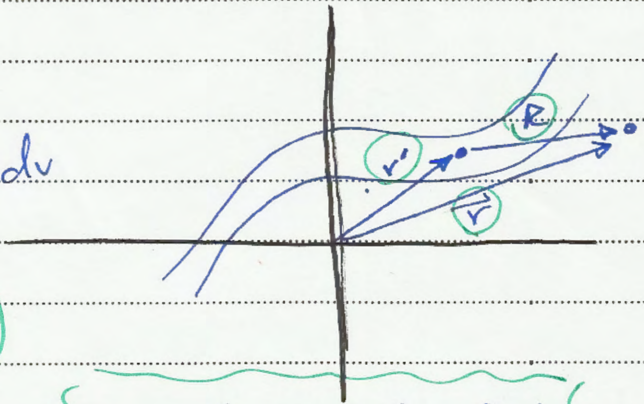
$\boxed{\nabla J = \frac{d\rho_v}{dt}}$ in time varing field

$\boxed{\nabla^2 A = \mu J} \Longrightarrow \nabla \cdot J = 0$

$$\boxed{\begin{aligned} \nabla \cdot A &= -j\omega\mu\epsilon V \\ \nabla \times A &= B \end{aligned}}$$

$\boxed{\nabla^2 A + k^2 A = -\mu \vec{J}}$ $\boxed{k = \omega^2\mu\epsilon}$

$$\vec{A}(r',t) = \frac{\mu}{4\pi} \iiint_{V_1} \frac{J}{R}(r',t')\,dv$$



(In time varing field)

$\boxed{e^{j\omega t'} = e^{j\omega(t - R/v)}}$

$$A(\vec{r}) = \frac{\mu}{4\pi} \iiint \frac{J(r')\,e^{-j\omega\frac{R}{v}}}{R}\,dv'$$

$\boxed{k = \omega/v}$

Subject:....................... / /

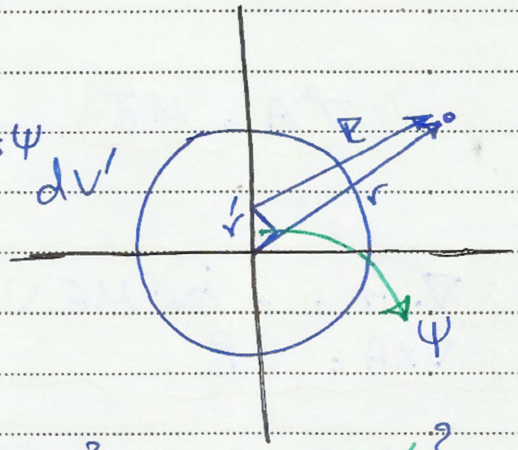
$$A(\vec{r}) = \frac{\mu}{4\pi} \iiint J(r') \, e^{-jkR} \, dv'$$

→ Retarted Field by $(\angle\psi = kR)$

$$E = -jwA - \frac{j}{w\mu\epsilon} \nabla(\nabla \cdot A)$$

$E \downarrow \mu \cdot E \downarrow H$

$$\therefore \vec{A} = \frac{\mu e^{-jkr}}{4\pi r} \iiint J(r') e^{jkr'\cos\psi} \, dv'$$



$$E \simeq -jw\vec{A}$$

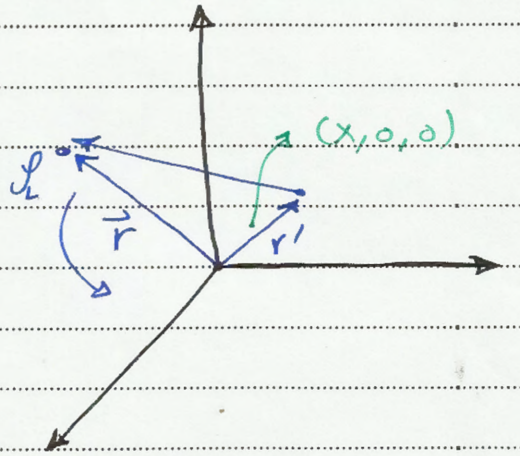
$\left\{ \begin{array}{c} \text{when } R \gg r' \\ R \parallel r \end{array} \right\}$

$$R \sim r - r'\cos\psi$$

Subject:......................   / /

* Past Papers :

→ Very long Conducting wire Carrying ρ_L along the X axis . Find E every where in Cartisian Cordinate ?



$$\vec{E} = \int \frac{\rho_L \, dl'}{4\pi \epsilon R^2} \vec{a_R} \qquad dl' \to dx'$$

$R = r - r'$

$= (x - x') a_x + y a_y + z a_z$

$$|R| = \sqrt{(x-x')^2 + y^2 + z^2}$$

$$\vec{E} = \frac{\rho_L}{4\pi \epsilon_0} \int \frac{dx' \, [(x-x')dx + y dy + z dz]}{[(x-x')^2 + y^2 + z^2]^{3/2}} \vec{a_R}$$

$$= \frac{\rho_L}{4\pi \epsilon_0} \int \frac{dx' \, [x a_x + y a_y + z a_z]}{[(x-x')^2 + y^2 + z^2]^{3/2}}$$