Electromagnetic (I)

Homework 1

Q1) Describe the following surfaces with a drawing and determine the type of the coordinate system used:

a) $x=-5$
b) $\rho=3$
c) $\Phi=3\pi/2$
d) $r=2$ hollow
e) $\theta=60$

Q2) Describe the intersection of surfaces (1) and (2):

Surface (1)	Surface (2)
$\Phi=45$	$z=5$
$x=-2$	$z=3$
$\rho=5$	$\Phi=45$
$r=1$	$\theta=60$

$\tan\Phi = 1$

$\tan \tan^{-1} \frac{y}{x} = 1$ $y=x$

$z=5$

Q3) Verify that

(a) $\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) = 0 = \mathbf{B} \cdot (\mathbf{A} \times \mathbf{B})$

Q4) Convert the following points to **Cartesian** coordinates:

(a) $P1\ (5, 120°, 0)$
(b) $P3\ (10, 3\pi/4, \pi/2)$

Q5) Convert the following point to **Cylindrical** and **Spherical** coordinates:

(a) $P\ (1, -4, -3)$

Q6) Convert the following vector to **Cylindrical** and **Spherical** coordinates:

$\mathbf{Q} = y\ \mathbf{a_x} + x z\ \mathbf{a_y} + (x + y)\ \mathbf{a_z}$

Q7) Determine the gradient of the following scalar fields

(a) $V = e^{(2x + 3y)} \cos 5z$.
(b) $T = 5\rho\, e^{-2z} \sin \varphi$.
(c) $Q = (\sin\theta \sin\varphi) / r^3$

Q8) Find the divergence and curl of the following vectors:

(a) $\mathbf{A} = e^{xy}\ \mathbf{a_x} + \sin xy\ \mathbf{a_y} + \cos^2 xz\ \mathbf{a_z}$
(b) $\mathbf{B} = \rho\, z^2 \cos\varphi\ \mathbf{a_\rho} + z \sin^2\varphi\ \mathbf{a_z}$
(c) $\mathbf{C} = r\cos\theta\ \mathbf{a_r} - (1/r)\cos\theta\ \mathbf{a_\theta} + 2r^2 \sin\theta\ \mathbf{a_\varphi}$

Q9) Given the vector field
$\mathbf{R} = (2x^2y + yz)\ \mathbf{a_x} + (xy^2 - xz^3)\ \mathbf{a_y} + (c\ xyz - 2x^2 y^2)\ \mathbf{a_z}$, determine the value of c for **R** to be solenoidal.
(Note: solenoidal means the ~~gradient~~ divergence equal to zero)

Q10) If the vector field

$\mathbf{T} = (\alpha xy + \beta z^3)\,\mathbf{a}_x + (3x^2 - \gamma z)\,\mathbf{a}_y + (3xz^2 - y)\,\mathbf{a}_z$, is irrotational, determine α, β, and γ.

Find $\nabla \cdot \mathbf{T}$ at (2, -1, 0).

(Note: irrotational means the curl equal to zero)

or Conservative

Q11) Calculate the surface integral of the following:

(Verify your answer using the divergence theorem)

(A) Consider the conical surface S shown in figure 1.a. The cone has height h and base radius a. Evaluate the closed surface integral of the following vector fields:

(a) $\mathbf{F} = r\mathbf{a}_r$ (b) $\mathbf{F} = r\mathbf{a}_\theta$ (c) $\mathbf{F} = \cos\phi\,\mathbf{a}_\phi + r\mathbf{a}_\theta$

(B) Consider the closed cylindrical surface of height h and base radius a as shown in figure 1.b. Evaluate the closed surface integral of $\boldsymbol{F}$ over this surface if:

(a) $\mathbf{F} = \rho^2\mathbf{a}_\rho + \rho\sin\phi\,\mathbf{a}_\phi + \rho^2\sin\phi\,\mathbf{a}_z$ (b) $\mathbf{F} = x\mathbf{a}_x + z\mathbf{a}_z$

$\sqrt{a^2+h^2}$

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$\int_0 \int_0^{a} r\sin\alpha$

$\sin\alpha = \frac{a}{r}$

$r = a\csc\alpha$

$r\cos\alpha = a\cot\alpha$

$h = \cos\alpha\, r$

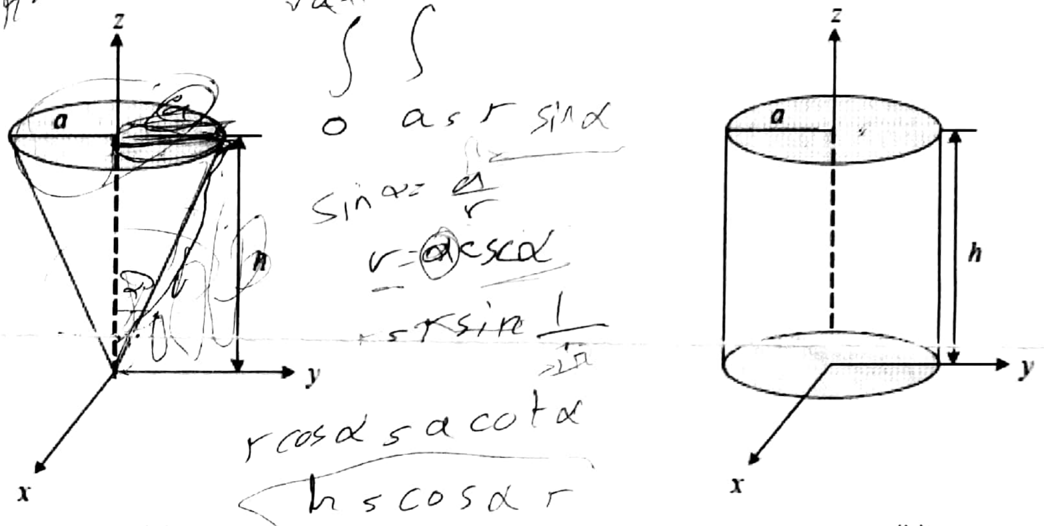
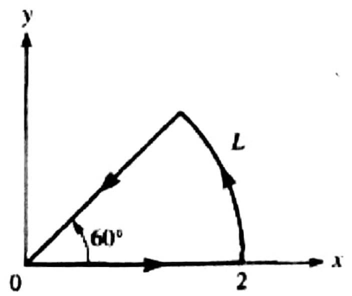


Figure 1

Q12) Calculate the circulation of $\mathbf{A} = \rho\cos\phi\,\mathbf{a}_\rho + \rho\sin\phi\,\mathbf{a}_\phi + z\sin\phi\,\mathbf{a}_z$ around the closed path L of the wedge defined by $0 \le \rho \le 2$, $0 \le \phi \le 60°$, $z = 0$ and shown in Figure 2

$\boxed{\tan\alpha = \frac{a}{h}}$



$$\int_0^{2\pi}\int_0^{\alpha}\int_0^{h\sec\alpha} (\nabla\cdot\vec{F})\, r^2\sin\theta\, dr\, d\theta\, d\phi$$

Figure 2