EE 251: Electromagnetics 1
First Exam (Fall 2016)
October 18th, 2016

10.5 / 20

Note that bold letters are vectors

$$\varepsilon_o = \frac{10^{-9}}{36\pi} F/m, \quad \nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}, \quad \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

3.5

Problem 1 (7 points)

A point charge 100 pC is located at (4, 1, -3) while the x-axis carries charge 2 nC/m. If the plane z = 3 also carries charge 5 nC/m², find **E** at (1, 1, 1).

For point charge

$$E_1 = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r$$

$R = (1,1,1) - (4,1,-3) = (-3,0,4)$

$|R| = \sqrt{-3^2+4^2} = 5$ 1

$\hat{a}_r = \frac{R}{|R|}$

$$E_1 = \frac{Q\,(r_1 - r_2)}{4\pi\epsilon_0 |r_1 - r_2|^3}$$

$$E_1 = \frac{100\cdot 10^{-12}\cdot(-3,0,4)}{4\pi\epsilon_0\cdot(5)^3} = -.0216\,\hat{a}_x - .028\,\hat{a}_z$$

+ 1

For line

$$E_2 = \frac{\rho_L}{2\pi\epsilon_0 \rho}\hat{a}_\rho$$

$\rho = \sqrt{x^2+y^2} = \sqrt{2}$ 0.5

$$E_2 = \frac{2\cdot 10^{-9}}{2\pi\epsilon_0\sqrt{2}} = 25.43\,\hat{a}_\rho$$

For plane

$$E_3 = \frac{\rho_s}{2\epsilon_0} = \frac{5\cdot 10^{-9}}{2\epsilon_0} = 282.48\,\hat{a}_y$$

1

5

Problem 2 (8 points)

In a certain region, the electric flux is given by

$$\mathbf{D} = 2\rho(z+1)\cos\phi\,\mathbf{a}_\rho - \rho(z+1)\sin\phi\,\mathbf{a}_\phi + \rho^2\cos\phi\,\mathbf{a}_z\ \mu C/m^2$$

(a) Find the charge density.

(b) Calculate the total charge enclosed by the volume 0 < ρ < 2, 0 < ϕ < π/2 and 0 < z < 4

(c) Confirm Gauss's law by finding the net flux through the surface of the volume defined in (b)

(a) $\rho_v = \nabla \cdot D$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D)\hat{a}_\rho + \frac{1}{\rho}\frac{\partial D}{\partial\phi}\hat{a}_\phi + \frac{\partial D}{\partial z}\hat{a}_z$$

$$= 4(z+1)\cos\phi\,\hat{a}_\rho - (z+1)\cos\phi\,\hat{a}_\phi + 0\,\hat{a}_z\ \mu C/m^3$$

1.5

(b) $Q_{enc} = \int_v \rho_v\,dv$

$$= \int_0^4\int_0^{\frac{\pi}{2}}\int_0^2 (4(z+1)\cos\phi - (z+1)\cos\phi)\cdot\rho\,d\rho\,d\phi\,dz$$

$$= 2\int_0^4\int_0^{\frac{\pi}{2}} 3(z+1)\cos\phi\,d\phi\,dz$$

$$= 6\int_0^4 (z+1)\,dz$$

$$= 6 \times 12 = 72\ \mu C$$

2

(c) on page 4-

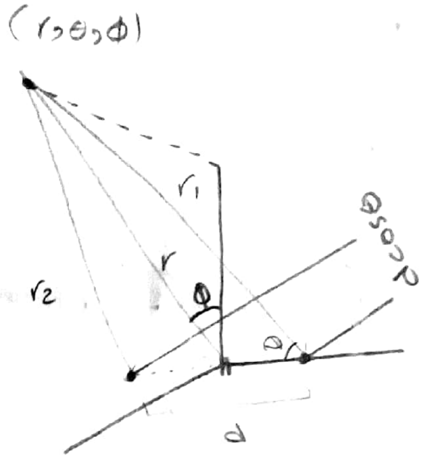
2

Problem 3: (5 points)

Point charges Q and -Q are located at (0, -d/2, 0) and (0, d/2, 0).

(a) Show that at point (r, θ, ϕ), where r >> d,

$$V = -\frac{Qd\sin\theta\sin\phi}{4\pi\varepsilon_o r^2}$$



(b) Find the corresponding **E** field.

(a) $V = \frac{Q}{4\pi\epsilon_o r}\hat{a}_r$

$V = \frac{Q(r_2 - r_1)}{4\pi\epsilon_o r|R_2 - R_1|}$

$r_2 - r_1 = d\cos\theta$, $\sqrt{r_2^2 + r_1^2} \approx r$

$V = \frac{Q\,d\cos\theta}{4\pi\epsilon_o r^2}$ $\quad d\cos\theta = d \cdot \hat{a}_r$

0.5

(b) $E = -\nabla V$

$= -\frac{Q d\sin\theta\sin\phi}{4\pi\epsilon_o r}\hat{a}_r \left(-\frac{Q d\cos\theta\sin\phi}{4\pi\epsilon_o r^2}\hat{a}_\theta \left(-\frac{Q d\sin\theta\cos\phi}{4\pi\epsilon_o r^2}\hat{a}_\phi\right.\right.$