(1) Solve the differential equation: $y'' - \frac{y'}{x} = -\frac{1}{2}x^{-1}(y')^3$, $x > 0$.

$y'' - \frac{y'}{x} = \frac{1}{2x}(y')^3$ $\boxed{y \text{ is missing}}$

$\boxed{y' = u \ , \ y'' = u'}$

$u' - \frac{1}{x}u = \frac{1}{2x}(u)^3$ --- Bernoulli

$g = u^{1-3} = u^{-2}$

$g' = -2u^{-3}u'$

$\Rightarrow \underbrace{-2u^{-3}u'}_{-2g'} + \frac{1}{x}\underbrace{2u^{-3}u}_{2g} = \frac{\cancel{2}u^{-3}\cdot u^{3}}{\cancel{2}x}$

$\therefore -2g' + \frac{2}{x}g = \frac{1}{x}$

2½

$\therefore 2g' - \frac{2}{x}g = -\frac{1}{x}$ $\quad \frac{2}{x} \to P(x)$, $-\frac{1}{x} \to Q(x)$

$$\boxed{g = e^{-\int P(x)dx}\left[\int e^{\int P(x)dx} Q(x)\,dx + C\right]}$$

$g = e^{\int \frac{2}{x}dx}\left[\int e^{\int -\frac{2}{x}dx} \cdot \frac{-1}{x}\,dx + C\right]$

$g = e^{2\ln x}\left[e^{\ln(x)^{-2}} \cdot \frac{-1}{x}\,dx + C\right]$

$g = x^2\left[\frac{1}{x^2} \cdot \frac{-1}{x}\,dx + C\right]$

$g = x^2\left[-\frac{1}{x^3}\,dx + C\right]$

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(2) If the integrating factor of the D.E: $(\cos 2y - \sin x)dx - 2f(x)\sin 2y\,dy = 0$, where $0 < x < \frac{\pi}{2}$, is $\mu(x) = \cos x$. Find $f(x)$, given that $f(0) = 0$.

$\underbrace{(\cos 2y - \sin x)}_{M}dx - \underbrace{2f(x)\sin 2y}_{N}\,dy$

$\frac{d\phi}{dx}$

$M_y = -2\sin 2y$, $N_x = 2f'(x)\sin 2y$

~~$\frac{M_y - N_x}{N}$~~

$\frac{M_y - N_x}{N} = \frac{-2\sin 2y - 2f'(x)\sin 2y}{2f(x)\sin 2y}$

$\therefore \frac{M_y - N_x}{N} = \frac{-2\cancel{\sin 2y}(1 + f'(x))}{2\cancel{\sin 2y}\,f(x)} = \frac{-(1 + f'(x))}{f(x)}$ $\boxed{\text{Pure in } x}$

$\mu(x) = e^{\int f(x)dx} = e^{\int \frac{1+f'(x)}{f(x)}dx} = \cos x$

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$\therefore e^{\int \frac{1+f'(x)}{f(x)}dx} = e^{\ln(\cos x)}$

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$(xe^x)^{-2}$ $(x^2e^{2x})^{-}$

(3) Find the general solution of: $y'' + 4y' + 4y = \dfrac{1}{x^2e^{2x}}$, $x > 0$

* find y_h $\quad y'' = r^2,\ y' = r,\ y = 1$

$r^2 + 4r + 4 = 0$

$(r+2)(r+2) = 0$

$\therefore \boxed{r = -2}$

$\therefore y_1 = e^{+rx}$, $y_2 = xy_1$

$y_1 = e^{-2x}$, $y_2 = xe^{-2x}$

$y_h = C_1 y_1 + C_2 y_2$

$y_h = C_1 e^{-2x} + C_2 x e^{-2x}$

* find y_p

$y'' + 4y' + 4y = x^{-2} e^{-2x}$

$y_p = (a_0 + a_1 x^{-1} + a_2 x^{-2}) e^{-2x}$ يوجد تشابه

$y_p = x^2(a_0 + a_1 x^{-1} + a_2 x^{-2}) e^{-2x}$

~~$y_p = (a_0 + a_1 x + a_2 x^{-2})$~~

$y_p = (a_0 x^2 + a_1 x + a_2) e^{-2x}$

$y'_p = (a_0 x^2 + a_1 x + a_2)(-2e^{-2x}) + e^{-2x}(2a_0 x + a_1)$

~~$y''_p = (a_0 x^2 + a_1 x + a_2)$~~

تكملة الحل خلف الورقة

(4) Let $y'' - 2y' + 2y = x^2 + \dfrac{x}{\csc^2 x}$. Determine a suitable form for the particular solution y_p if the method of undetermined coefficients is to be used.

$y_{p_1} = a_0 + a_1 x^1 + a_2 x^2$

$y_{p_2} = (a_0 + a_1 x) \cos 2x$

$r^2 - 2r + 2 = 0$

$(r-1)(r-1) = 0$

$r = 1$

$y_1 = e^x$, $y_2 = xe^x$

$\Rightarrow x^2 + x \sin^2 x$

$= x^2 + x\,\frac{1}{2}(1 - \cos 2x)$

$= \underbrace{x^2 + \tfrac{1}{2}x}_{y_{p_1}} - \underbrace{\tfrac{1}{2}x \cos 2x}_{y_{p_2}}$

$(r-1) + r$

$r^2 - r + r + 1$

(5) Find the second order differential equation whose general solution is $y = \underbrace{c_1x^2 + c_2x^{-2}}_{y_h} - \underbrace{x^3}_{y_p}$.

$y = C_1 e^{2t} + C_2 e^{-2t} - e^{3t}$

$r(r-1) + 8r + 4$

$r^2 - r + r + 4$

$r_1 = 2$, $r_2 = -2$

$(r - r_1)(r - r_2) = e^{3t}$

$(r-2)(r+2) = e^{3t}$

$r^2 - 4 = 0$

$r(r-2) + 2r - 4 = 0$

$y_p = e^{3t}$

$\dot{y}_p = 3e^{3t}$

$\ddot{y}_p = 9e^{3t}$

$r(r-2) + 2r - 4 = e^{3t}$

$\ddot{y} + 2\dot{y} - y = e^{3t}$

$9e^{3t} + 6e^{3t} - e^{3t} = e^{3t}$

$9x^3 + 6x^3 - x^3 = x^3$