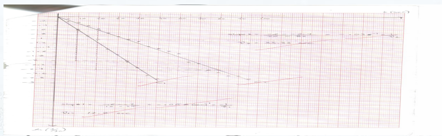


Contro Systems EE441 ordan
ept.

Student Name in Arabic

Answers should be written in ink. Exam Duration:70 min

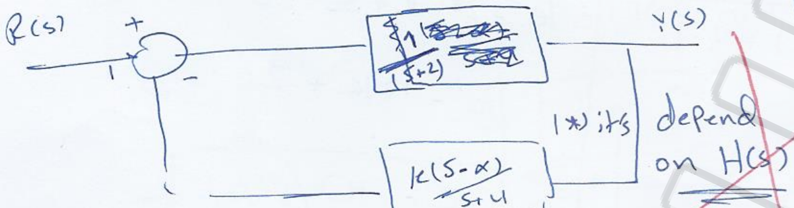
Sunday. 7/12/2014
Registration #: 0110494
Mark out of 30: 11

Question 1

a) A negative feedback system has $G(s) = \dfrac{1}{(s+2)}$ and $H(s) = \dfrac{K(s-\alpha)}{s+4}$; K and α are positive.

i) Comment on the open loop and closed loop system stability?

" closed loop "



1*) it's depend on H(s)

1*) closed loop → Control the accuracy speed and it's give us afeedback for that

open loop: it's not depend on H(s) like " closed loop "... and it's ~~less~~ accurate than closed loop ...

0

ii) Calculate K and α such that the closed loop system has $\zeta = 0.7$ and 2 seconds settling time.

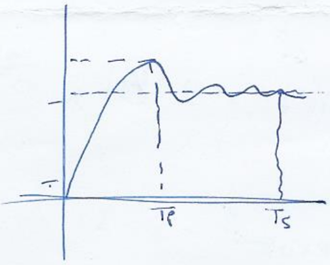
* $T_s = \frac{4}{w_n}$ → $w_n = \frac{4}{2} = 2$ rad/s

3

* $w_d = w_s\sqrt{1-\zeta^2} = 2\sqrt{1-(0.7)^2} = 1.428$

* $T_p = \frac{\pi}{w_d} = \frac{\pi}{1.428} = 2.199$

* $T_r = \frac{2.16\zeta + 0.6}{w_n} = \frac{2.16 * 0.7 + 0.6}{2} = 1.056$



~~$\frac{G(s)}{1+G(s)H(s)} = 0$~~

~~$1 + \frac{1}{s+2} * \frac{k(s-\alpha)}{s+2}$~~

$1 + G(s)H(s) = 0$

$1 + \frac{1}{s+2} * \frac{k(s-\alpha)}{s+4} = 0$

~~$\frac{(s+4)(s+2)(ks-k\alpha)}{(s+2)(s+4)} = 0$~~

$1 + \frac{k(s-\alpha)}{(s+2)(s+4)} = 0$

$\frac{(s+2)(s+4) + k(s-\alpha)}{(s+2)(s+4)} = 0$

(*) $2\zeta w_n = B$

$B = 2 * 0.7 * 2 = 2.8$

(*) $w_n = \sqrt{C}$

$C = w_n^2 = 4$

$(s+2)[(s+2)(s+4)] + (s+2)k(s-\alpha) = 0$

$As^2 + Bs + C = 0$

we Substitute B & C

$2K = 2.8 \Rightarrow \boxed{K = 1.4}$

$2\alpha = w_n^2 = 4 \Rightarrow \boxed{\alpha = 2}$

1.5

21

Determine K which results in $\boxed{-3+3j}$ as closed loop pole, hence determine the **factored closed loop transfer function** at that $\boxed{\text{value of K.}}$

$S = (-3+3j)$

$$\frac{-1}{(2s+K)} = \frac{1}{s^2+4s+8}$$

$$\frac{-1}{(2(-3+3j)+K)} = \frac{1}{(-3+3j)^2+4(-3+3j)+8}$$

$$\left|\frac{-1}{(2(-3+3j+K)}\right| = \left|\frac{1}{(-3+3j)^2+4(-3+3j)+8}\right|$$

$$\frac{1}{\sqrt{(K-6)^2+(6)^2}} = \frac{1}{\sqrt{(-4)^2+(-6)^2}}$$

$$\frac{1}{(K-6)^2+36} = \frac{1}{52}$$

$$(K-6)^2+36 = 52$$

$$K^2-2*6K+36+36 = 52$$

$$K^2-12K+20 = 0$$

$$\boxed{K = 10, 2}$$

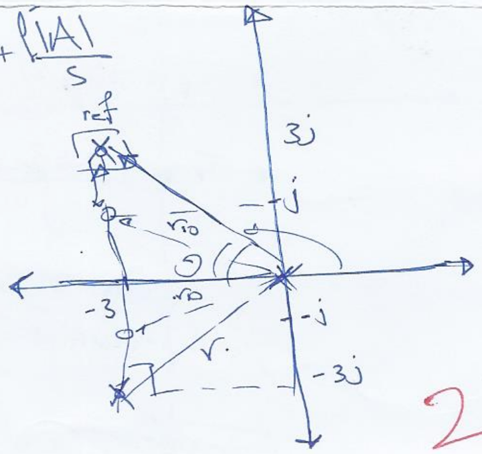
1

(*) Determine **graphically** the closed loop system time response **y(t)** due to a **unit step.**

unit step $\rightarrow$ u(t)

(*) $y(t) = 2|B| * e^{-3t} * (\cos(3t) + \cancel{\sin(3t)}) + \frac{|A|}{s}$

$(\cos(\beta k + \theta))$



Zeros : $[-3+3j]$

Poles = $0, -3 \pm 3j$

$$|A| = \frac{|\Sigma \text{zeros}|}{|\Sigma \text{poles}|}$$

$$|A| = \frac{10}{\sqrt{18}*\sqrt{18}} = \frac{10}{18}$$

2

Now we take $(-3+3j)$ as a reference :

$$|B| = \frac{\cancel{\sqrt{10}\sqrt{10}} \quad 1*1}{\sqrt{18} * 6} = \frac{1}{6\sqrt{18}}$$

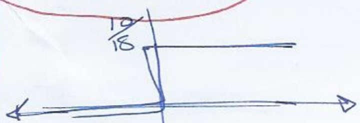
Now we compute θ

$$\theta = (180 - \tan^{-1}(\tfrac{3}{3})) + 90^\circ$$

$$\theta = 180 - 45 + 90^\circ = 225^\circ$$

y(t) for Unit Step function :

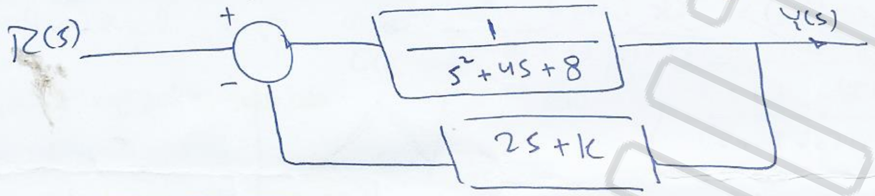
$$y(t) = \frac{10}{18}\, u(t)$$



Question 2

A negative feedback system has $G(s) = \dfrac{1}{s^2 + 4s + 8}$ and $H(s) = 2s + K$

Draw the block diagram representing the closed loop system with Y(s) as output and R(s) as set value.



Use appropriate error coefficients to calculate K to have a **steady state error 0f 0.1** due to a **unit step** set value. u(t) Kp

$\dfrac{1}{1+Kp} = e_{ss}$

$\dfrac{1}{1+Kp} = 0.1$

$1 = 0.1 + 0.1 Kp$

$0.1 Kp = 0.9$

$\boxed{Kp = 9}$

$\lim_{s \to 0} G(s) H(s) = Kp$

$\lim_{s \to 0} \dfrac{1}{s^2+4s+8} * (2s+K) = 9$

$\dfrac{1}{8} * K = 9$

$\therefore \boxed{K = 72}$

2

Obatain the characteristic equation for the closed loop system, rearrange the characteristic equation. Hence draw the root locus with K as the varying parameter.

C.E: $1 + G(s)H(s) = 0$

$1 + \dfrac{1}{s^2+4s+8} * (2s+K) = 0$

$\dfrac{s^2+4s+8 + 2s+K}{s^2+4s+8} = 0$

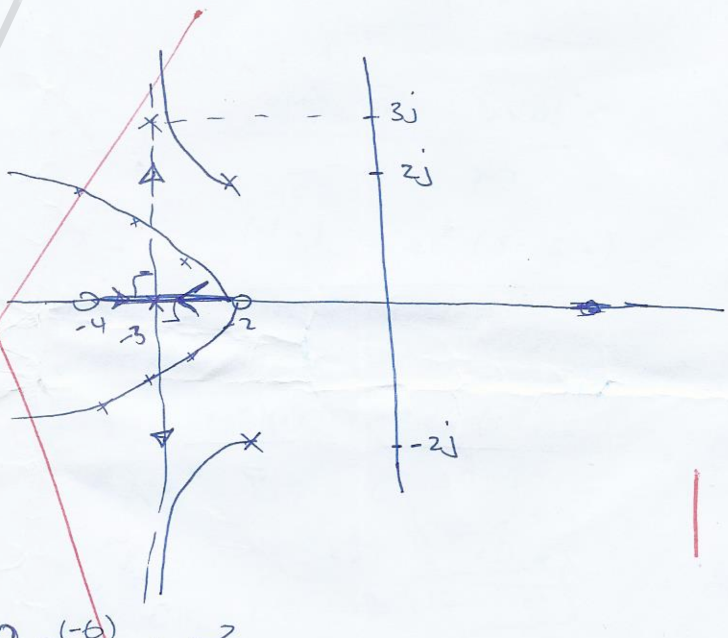
$\dfrac{s^2 + 6s + (8+K)}{s^2+4s+8} = 0$

0

when K=0 :

⇒ Zeros : −2, −4

⇒ Poles : $-2 \pm 2j$



we adjust the K to draw the diagram ...

$r_A = \dfrac{\sum \text{Poles} - \sum \text{Zeros}}{np - nz} = \dfrac{-4 - (-6)}{4 - 2} = -3$

$\theta_{1,3} = \dfrac{(2n+1)180^\circ}{np - nz} = \dfrac{(0+1)180^\circ}{2} = 90^\circ$

$\theta_2 = \dfrac{(3)180^\circ}{2} = 270^\circ = -90^\circ$

1

b)A particular feedback system has $1+G(s)H(s) = 1+\dfrac{K(s+1)}{s(s+2)(s^2+2s+5)} = 0$

i) Determine the values of K which results in $\boxed{2}$ and $\boxed{25}$ error coefficients to a unit ramp set value.

~~$K_v = \lim_{s\to 0} s\,G(s)H(s) = \dfrac{sK(s+1)}{s(s+2)(s^2+2s+5)} = 0$~~

$$\boxed{-\frac{1}{K} = \frac{s(s+1)}{s(s+2)(s^2+2s+5)}}$$

$1/s^2$

$\therefore$ $K = 20$, when $K_v = 2$

& $K = 250$, when $K_v = 250$

$K_v = \lim_{s\to 0} s\,G(s)H(s) = \lim_{s\to 0} \dfrac{sK(s+1)}{s(s+2)(s^2+2s+5)}$

$K_v = \frac{1}{10}K$

$e_{ss} = \dfrac{1}{K_v}$

when $K_v = 2$:

$\frac{1}{10}K = 2 \Rightarrow \boxed{K = 20}$

when $K_v = 25$:

$\frac{1}{10}K = 25 \Rightarrow \boxed{K = 250}$

1.5

ii) Sketch the root locus. Don't calculate the breakaway point(s).

$\boxed{G(s)H(s) = -1}$

$1 + G(s)H(s) = 0$

$$1 + \frac{K(s+1)}{s(s+2)(s^2+2s+5)} = 0$$

Zero = -1

Poles = $0, -2, -1 \pm 2j$

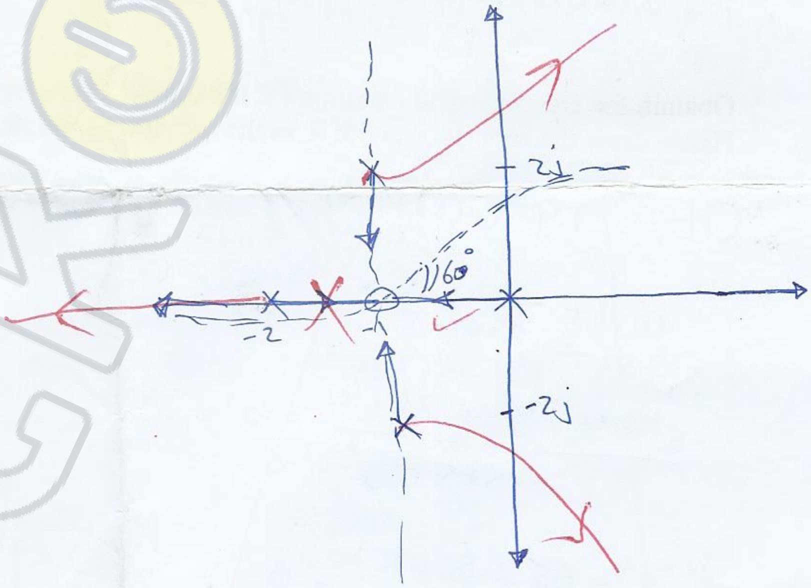
$$\sigma_A = \frac{\sum \text{Poles} - \sum \text{Zeros}}{n_p - n_z}$$

$$\sigma_A = \frac{(-2-2) - (-1)}{4-1} = \frac{-3}{3} = -1$$

$n = 0, 1$

$$\theta_1 = \frac{(2n+1)180^\circ}{n_p - n_z} = \frac{(1)180}{3} = 60^\circ, 180^\circ, 300^\circ$$

$$\theta_2 = \frac{3 * 180}{3} = 180^\circ$$



2