Spring017

POWERUNIT

COMMUNICATIONS2

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Binary Source

Information A   Information B

$H_A = -\log_2(P_A)$ Bit. , Information of A

$H_B = -\log_2(P_B)$ Bit

Avg. of the source $H = -P_A(\log_2 P_A) + - P_B(\log_2 P_B)$

$$\boxed{H = -\sum_i P_i \log(P_i)} \sim \boxed{\text{Entropy}}$$

Digital Source
$\to I = \{a_i \ , \ i = 0, 1, 2, \dots, I-1\}$, This source have I elements .a_i

$\to P = \{P_i \ , \ P_r(a_i)\}$ each element have prob. P_i

$C \xrightarrow{\text{sending}} C'$, we need a measure of mutual info. between C and C' because C' have errors.

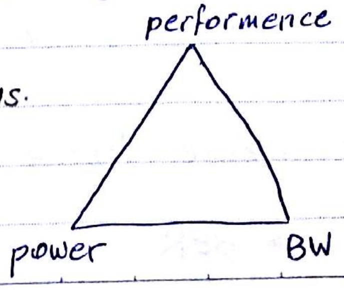
Tx info.   Rx info.

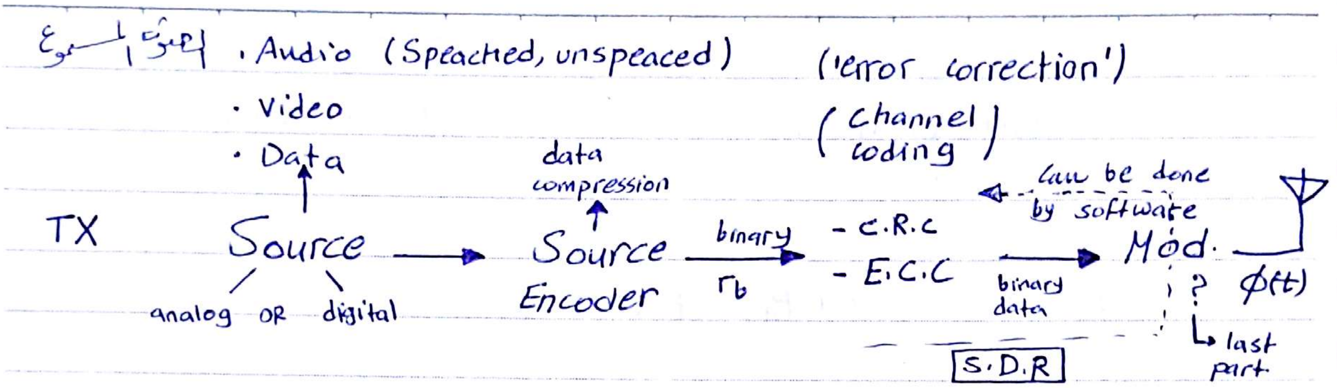
$$\boxed{\overbrace{\text{Capacity}}^{I(C,C')} = B_{CH} * \log_2(1+SNR)}$$ shannon's law

- performence triangle

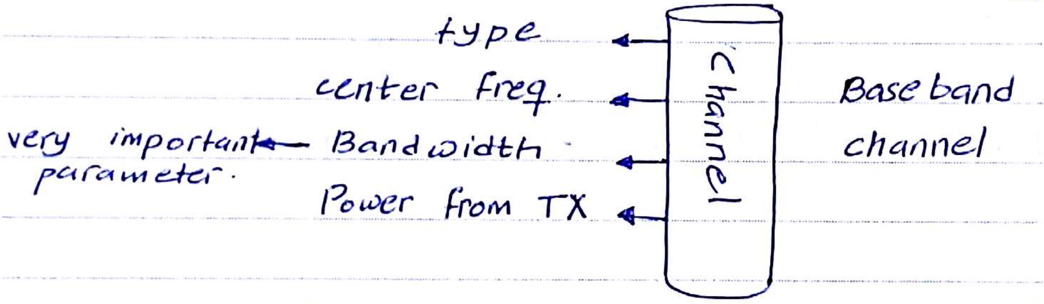
performence ≥ performence min

communication sys. triangle

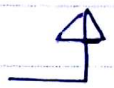




$\phi(t)$: physical waveform. "energy signal"
- time limited
- bounded



BW $\propto$ r_b (linear proportional)



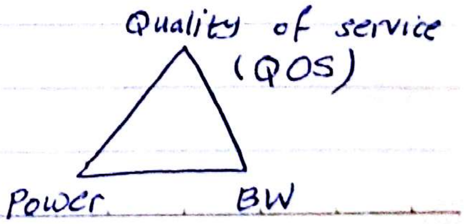
RX

- AWGN valid for wired channel. <u>only</u>.

- Deep fading phenomena is the worst case in comm. system.

- non linearity → Image freq.

- Recall performence triangle :



QOS → BER

- Power ↓ , interference ↓
- : data arrive
- ال detection لل error لا يمكن لتحسين ال performence
- Toll Quality : الصوت مسموع ونستطيع تمييز صوت المتحدث
- Source encoder : Generate a binary sequence that represent the physical source under acceptable quality at min data rate.
  , required bit rate to satisfy best quality.

التعبير عن ال signal
ب min. data rate
و best quality

- Data rate (ADC)
  - Sampling bit rate : $r = f_s \cdot m$
    m : bits per sample.

Voice (toll quality)

$$r_b = 8 * 8 = \boxed{64\ Kbps} \rightarrow PCM_{64}$$

$$\times 32 = 2\,Mbps = E1$$

30 voice

2 control

- Encryption $\equiv$ E.C.C

- Modulation: mapping between logical binary data into physical waveform ($\phi(t)$)

  - $\phi(t)$: Square integrable signal

  - the data will be changed after signaling interval within the interval.

  - S.D.R : Software..

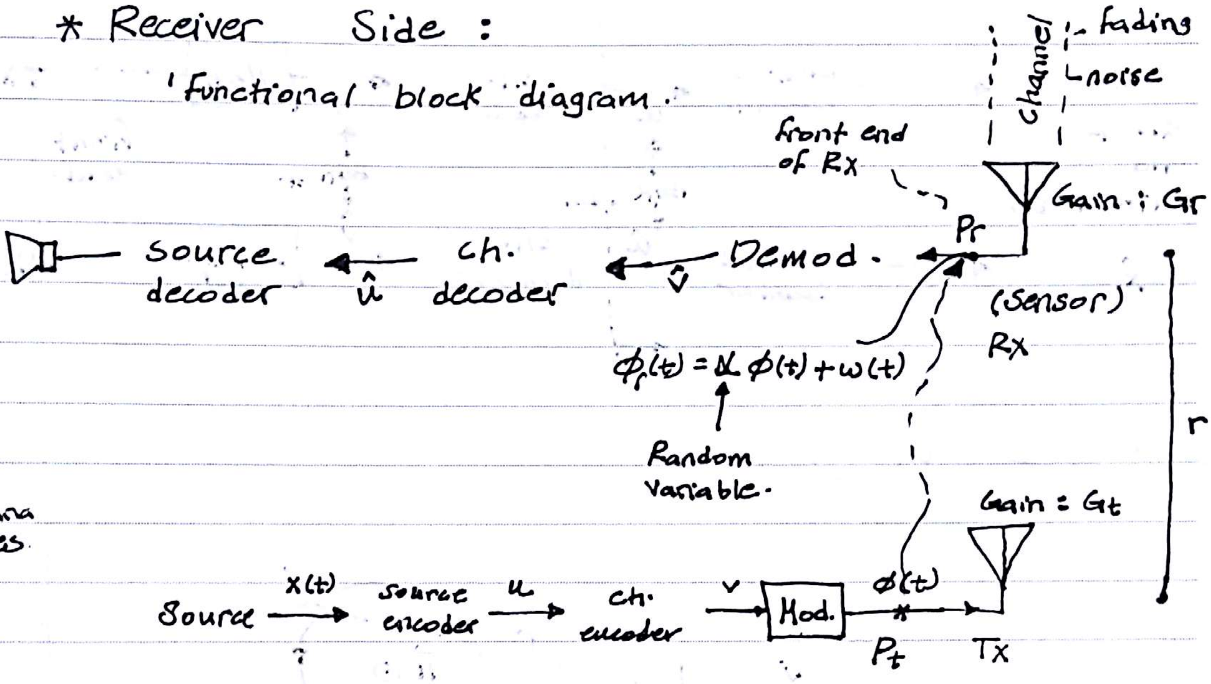
  - last part of Mod. : Mixer..

Radio : 10^0 pW
power received.

lec No. 6/2/2017

* Receiver Side :

'Functional' block diagram.



- Antenna types.

Pt : front end of Tx

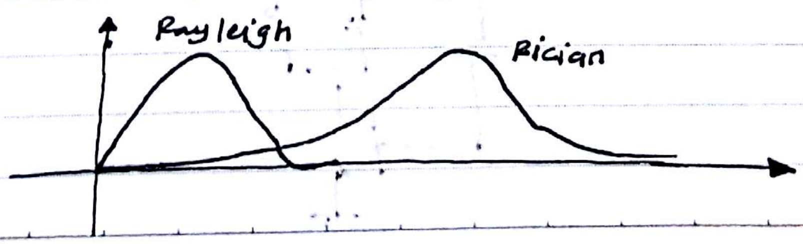
$$* \quad P_r = P_t + G_t + G_r - 10\,\gamma\,\log(r) \qquad \text{unit : dBw}$$

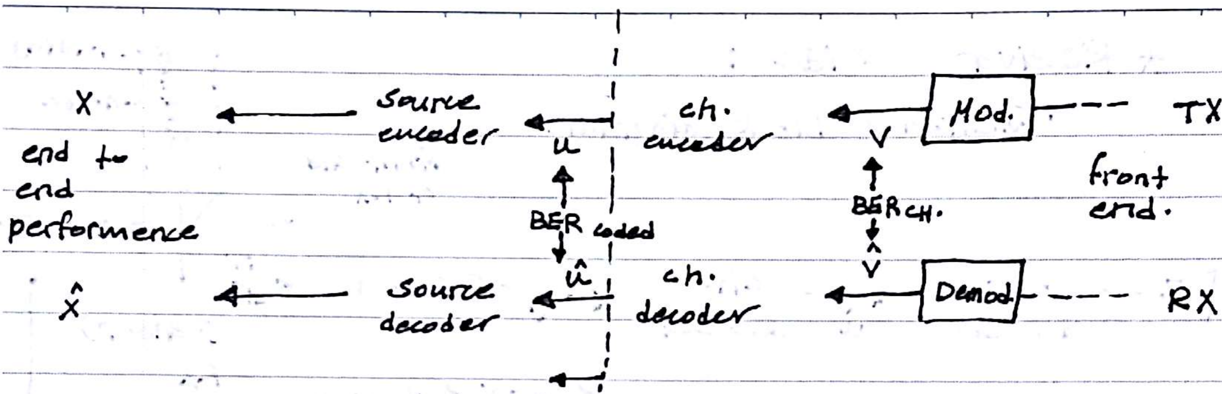
$$2 \leq \gamma \leq 4$$

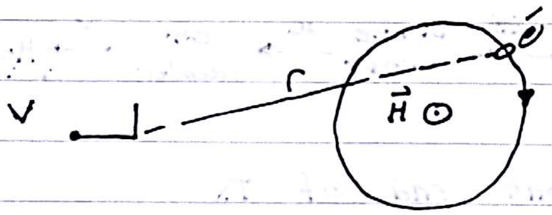
* Usually cellular phones received $P_r = -110$ dBm

$$P_r = -110 \text{ dBm}$$
$$= 10^{\frac{-110}{10}} = 10^{-11} \text{ mW} = 10^{-14} \text{ W} = 0.01 \text{ pW}$$

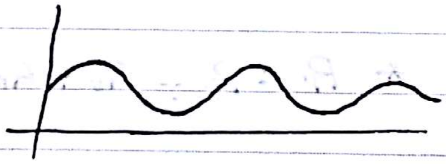
* wrost case : Rayleigh

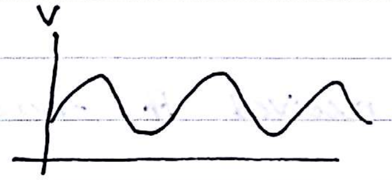


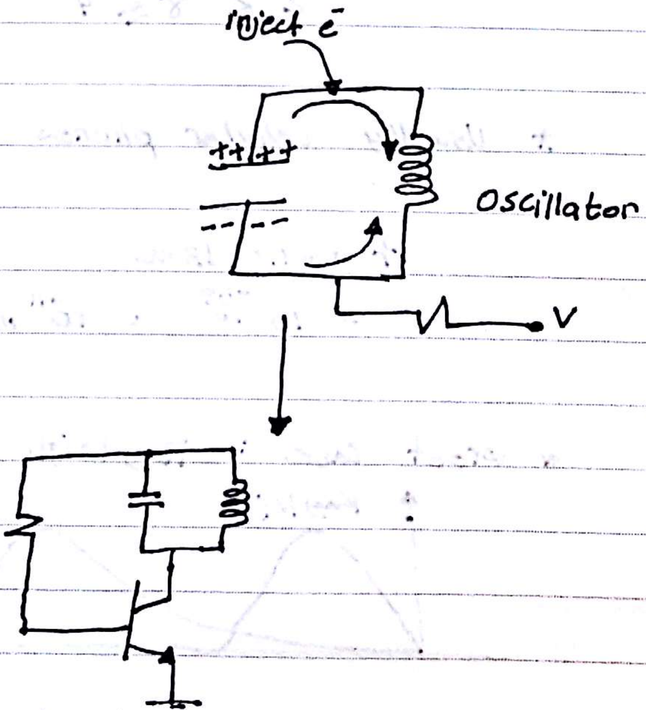




V (Voltage) $= \frac{K\bar{e}}{r}$







- power signal carrys no information. (Periodic signal)

∴ Energy " carrys information.

- Orthogonality principle : Distance between the signals

★ Orthogonality : Separate signals at RX.

Wave form

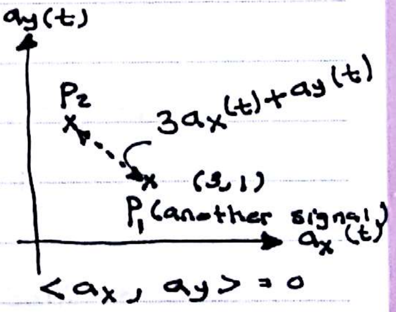
- $< s_1(t), s_2(t) > = 0$

inner product between two wave form equal ZERO.
↳ projection between 2 signal

$$= \int_T s_1(t)\, s_2^*(t)\, dt$$

Vector

- $a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$



$< a_x , a_y > = 0$

Axis : orthonormal (have the same unit)

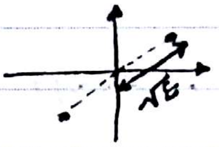
- Correlation coeff. $= \rho_{12} = \dfrac{< s_1(t), s_2(t) >}{\sqrt{E_1} \cdot \sqrt{E_2}}$

- $E_i = \int_T |s_i|^2 \, dt$

normalized $-1 \leq \rho \leq 1$. very high distance have very hi -ve ρ

No. Ortho. + Distance

- Anti-podal signals have corr. coeff. = -1.
  تميز ال signals عن بعض



- ASK not used in communication system.

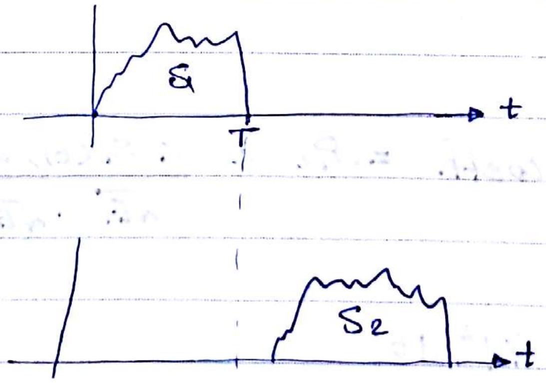
Lec. 8 / 2 / 2017 :

* Non overlaped signal in any domain are orthogonal.

- Domains of orthogonality :
  - ✓ - Time
  - ✓ - freq.
  - - Space : cellular based on space orthogonality.
  - ✓ - code : sin & cos with same freq.
  - - polarization.

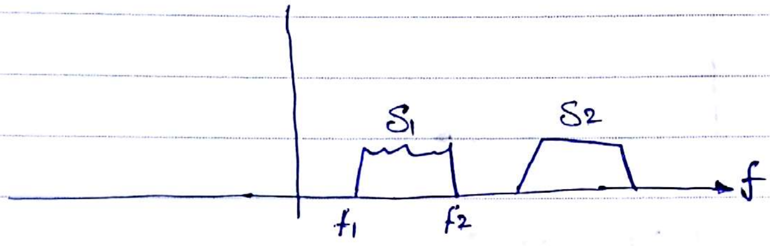
✓ Used for modulation.

- Time orthogonality : Not over laped in time means orthogonal.



S_1 & S_2 are orthogonal.

- freq. orthogonality :



S_1 & S_2 are orthogonal (No overlap)

- in time domain S_1 & S_2 are overlaped.

- Limited freq. BW = unlimited time.

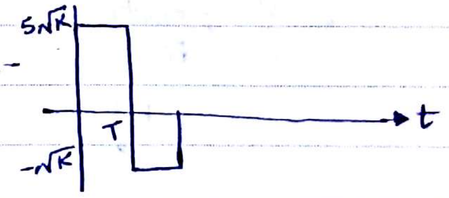
- ax1 , ax2

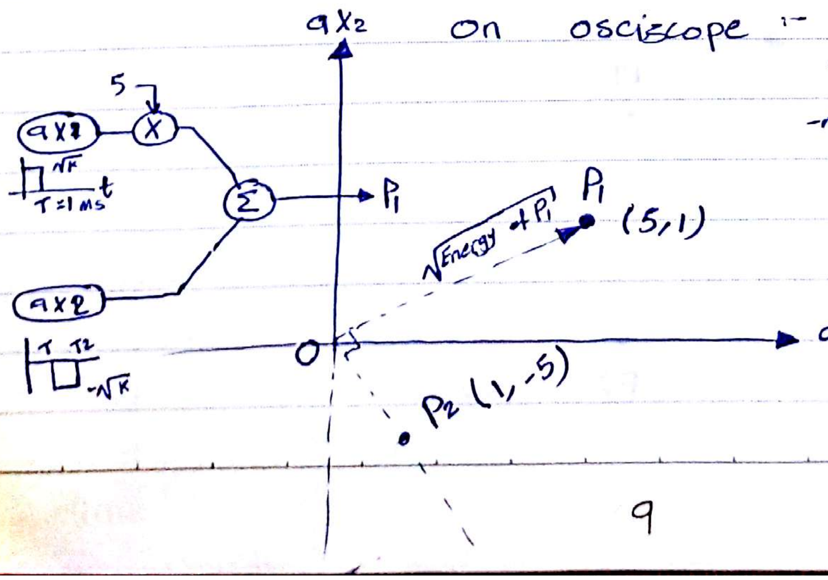
$\langle ax1, ax2 \rangle = 0$

$\| ax1 \| = \| ax2 \| = 1$

$\int |S|^2 dt = \int |S|^2 df$; Parseval's Theorem.

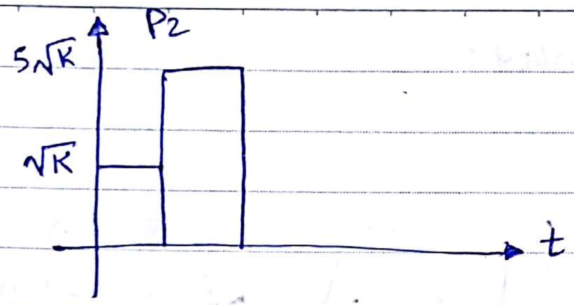
on osciscope :-

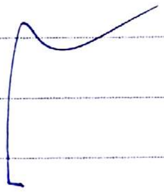




any point at any line ⊥ to OP gives P orthogonal to P1

1) Orth. , 2) Dist, 3) BER.





- Eqlidian Distance :

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

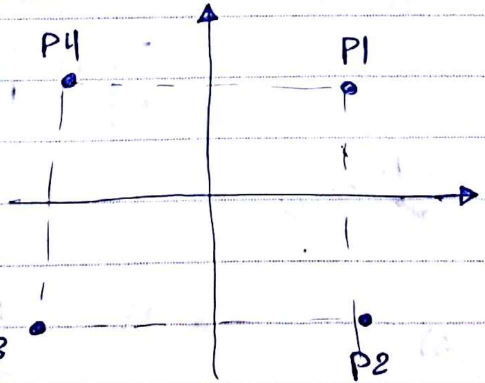
for the last ex :

$d_{12} = \sqrt{(5-1)^2+(1+5)^2} = \sqrt{52}$ avg. unit: volt.

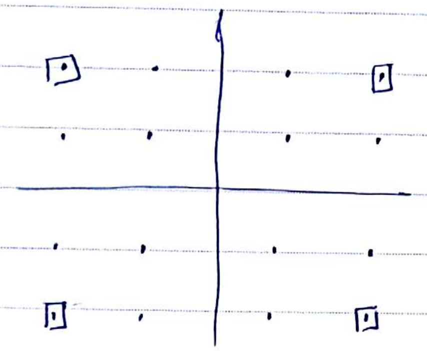
* Constillation ≡ Space representation ≡ lates.

- The first goal of designing a constillation is to have max. eqlidian distance amoung constillation point with min. avg. energy

- 4 points : in 2 dim.



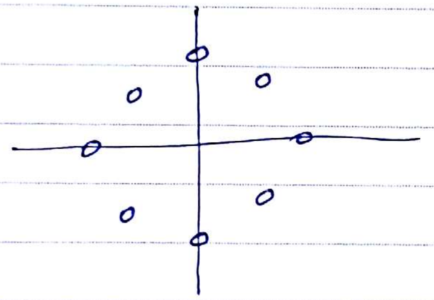
- QAM constillation.

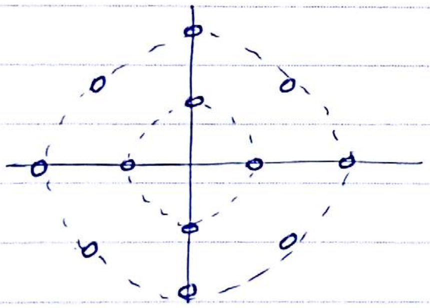


$e \in N^2$

farest point.

to reduce energy we can remove some points with keeping the same features.





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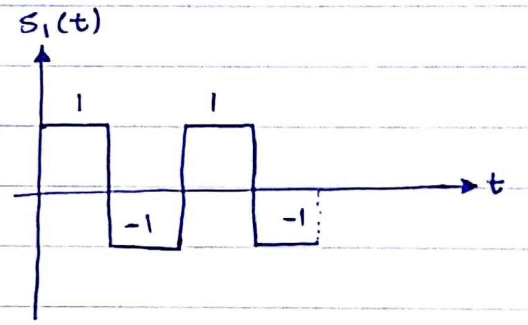
Ex : vectors $S_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ $S_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$, S_1 and S_2 are orthogonal vectors.

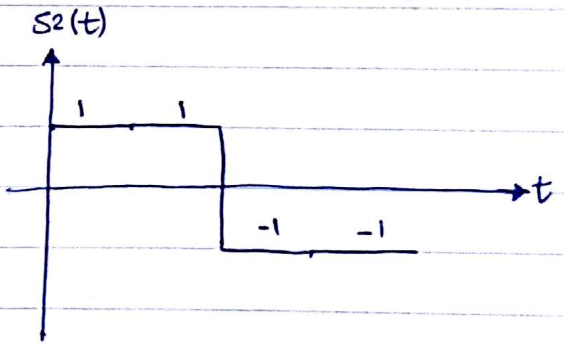
we know that 2 vectors are orthogonal

$$\langle S_1 , S_2 \rangle = S_1^T \, S_2 = 0$$

waveform



$$\int_T S_1(t) \cdot S_2^*(t) \, dt = 0$$



* Generation of orthogonal signals

1- Gram schmidt

2- Transformer: 1- Fourier transform. (DFT) — Discrete fourier transform

تعني أن F.T حل وله تمثيل سريع ← F.F.T ✓ : faster fourier trans.

2 - Discrete cosine trn. Re(FFT) , 3 - Dis. sine trn. Im(FFT)

Read Note ← $\boxed{Y_{N\times1} = A_{N\times N} \, \underbrace{X_{N\times1}}}$ → N sample، الا طول معين

4 - W.H.T

5- K.L.T لجميع أنواع { Signals.

$\underline{H}$ complex conjugate transpose

$$\boxed{A^{-1} = A^{H}} \quad \text{, if } A \text{ is unitary transform matrix.}$$

$$A_{FFT} = \left[e^{-jnk\frac{2\pi}{N}} \right] \quad , n \rightarrow \text{Row} \quad , k \rightarrow \text{col.}$$

$$= \begin{array}{c|ccccc} & 0 & 1 & 2 & \cdots & \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & e^{-j\frac{2\pi}{N}} & \cdots & & \\ 2 & \vdots & & \ddots & & \\ \vdots & 1 & & & \ddots & \end{array} * \boxed{\frac{1}{\sqrt{N}}} \quad \text{, orthonormal matrix.}$$

Scaled matrix. ← حتى ما يصير gain أو loss. بنقسم على ال $\sqrt{N} = \det.(A)$

~~~~~~~~~~~~

Not In exam

$$H_{2\times2} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \longrightarrow H_{22} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

without scaling

orthonormal matrix

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}$$

- Digital transformation.
- matrix size is power of $\underline{\underline{2}}$.

~~~~~~~~~~~~

$$Y_{K\times1} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \end{bmatrix} \begin{array}{l} \text{DC} \\ \text{1st harmonic} \\ \text{2nd} \quad '' \\ \vdots \end{array}$$

↑ freq. ↓

- كل ما مشينا داخل ال vector بنقطع ال signal ع freq. أعلى

- ↓ Time resolution ↑ $\propto$ ↓ (freq. resolution) ↑ **NOT** period.

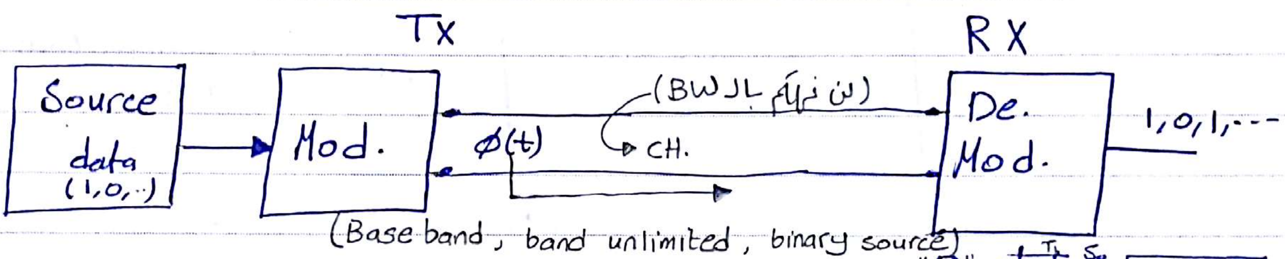
المسافة بين كل sample بال freq. domain.

No.

- **Base band Modulation :** "signal centered around zero Hz".

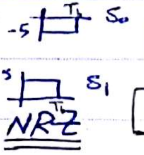
* Used for wired communication.

* 1- Band unlimited channels : ال BW للـ ch. كبير جداً
وال signal بطيئة بالنسبه للـ ch. (ال BW أكبر بكثير من سرعة ال data)



(Base band , band unlimited , binary source)

- Binary transmission data "0" OR "1"

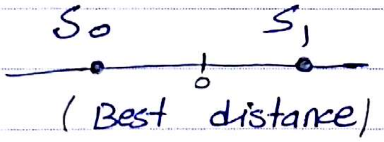


$\boxed{S_0 = -S_1}$

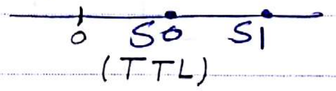
$\boxed{\text{anti-podal}}$

أفضل طريقة للتمييز ال 0 وال 1

- T_b : تمثل سرعة ال data



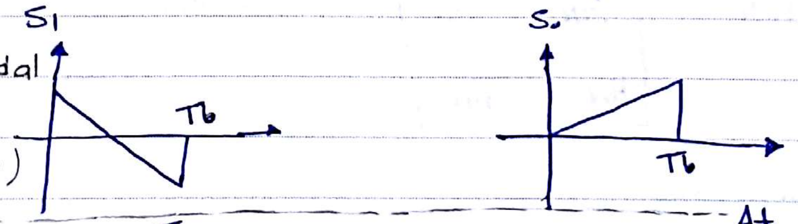
(Best distance)



(TTL)

Not anti-podal

(بنربط نستخدمهم)



At RX side

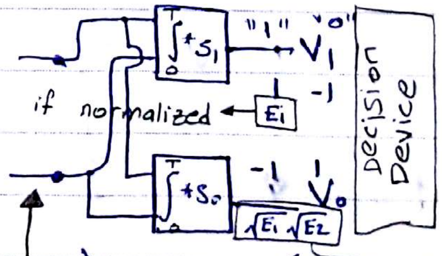
$$(\text{Projection})\ \boxed{V_1} = \int_0^{T_b} \phi(t) . S_1(t)\, dt$$

S_1

Result of projection of the received signal on S_1

$$V_0 = \int_0^{T_b} \phi(t) . S_0(t)\, dt$$

S_0



نقسم ال signal على قيمة ال ϕ(t)

coherent RX (synch.)

if not normalized

* V_1 , V_0 : Decision variables.

* Decision Device : وظيفته معرفة هل ال data 0 أو ال 1 من خلال V_1 , V_0

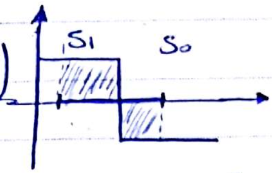
طريقة عمل ال Decision → if $V_1 = 1$, $V_0 = -1$ then data $= 1$ و $V_1 = -1$, $V_0 = 1$, data $= 0$

smile...

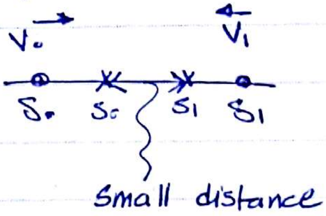
Scanned by CamScanner

* inter-symbol interference (SYNC)

(تداخل ال bit الاولى مع ال bit الثانية)



* due to mismatching we might have multiple reflections and this will cause inter-symbol interference.



* To solve inter-symbol interference we need matching (EH2). The device must be matched to solve this problem

No. Lec. 15/2/2017

- Base band TX :

- Binary
  - Band unlimited
  - Band limited

- Non Binary
  - BW unlimited
  - BW limited

Rev : Independent

X Y , 2 Random process
(time variables)

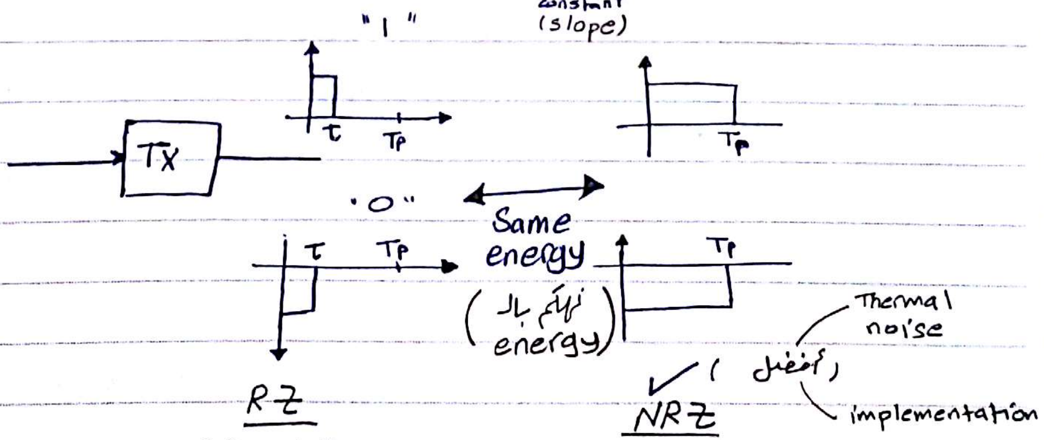
$\triangleq f_{XY}(x,y) = f_X(x) \cdot f_Y(y)$

خصائص ال data يالي بدي أرسلها :-

Data :

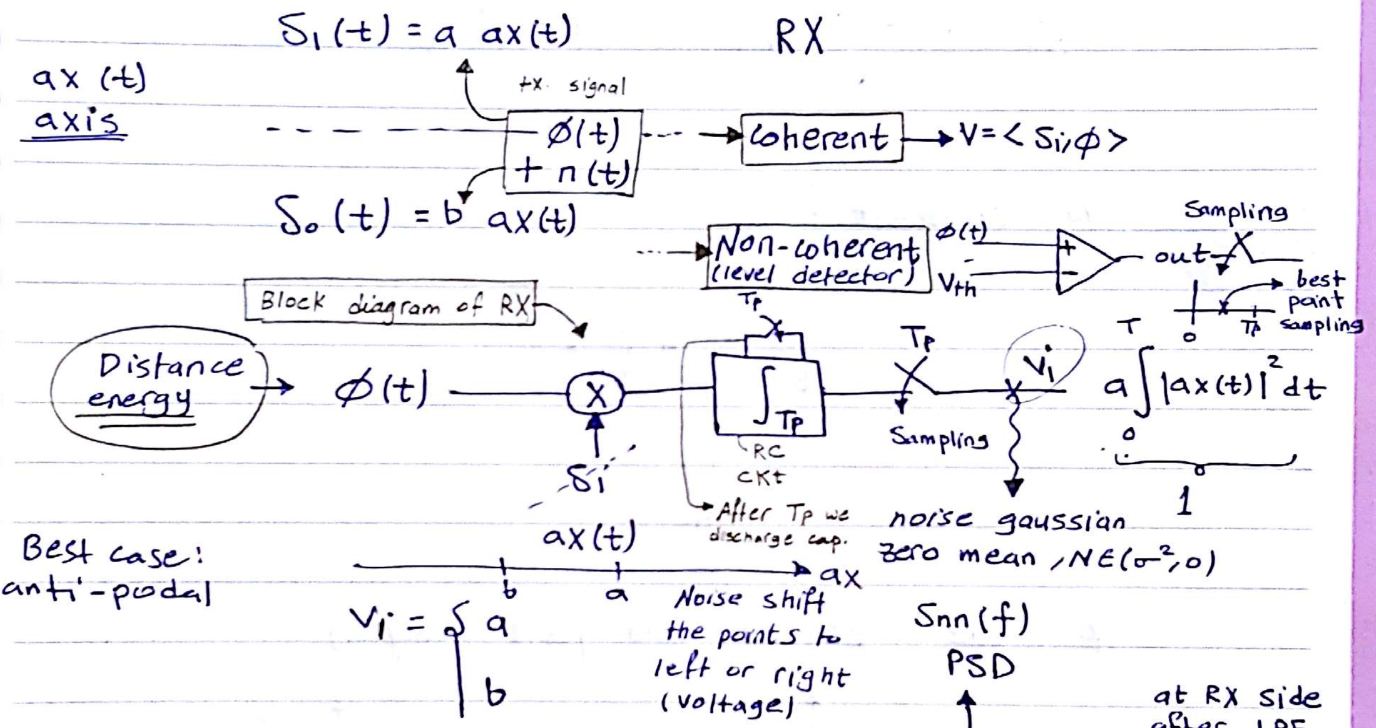
- Serial bits
- bit rate $r_b = \frac{1}{T_b}$
- Independent
  ↳ correlation
    ↳ $\boxed{\text{uncorrelated}}$
    $E\{X,Y\} = 0$
    $\boxed{R_{xx}(\tau) = \delta(\tau)}$
- Equally likelly
  $P_r(0) = P_r(1)$

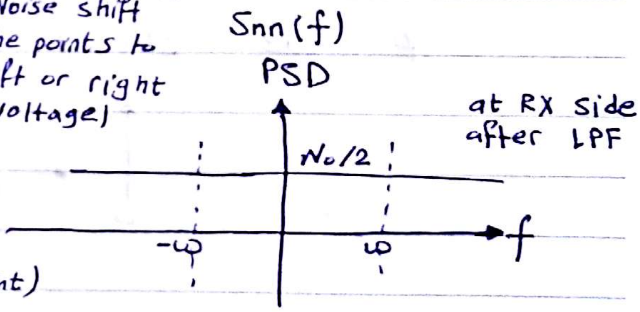
Δ mod. the simplest dependancy case : $y = \underset{\text{constant (slope)}}{a}\overset{\text{R.V}}{X} + \underset{\text{RV}}{b}$ (linear dependancy)



مشكلات ال RZ [عالية Voltage
قليل جداً time sync.

* Binary Band unlimited :





power of noise $= \boxed{\sigma_n^2 = N_o \cdot \omega}$

after RX at (V_i point)

N_o : W/Hz.

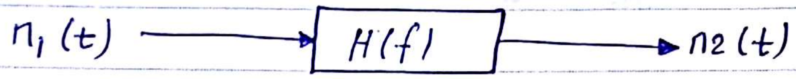
$$\underbrace{\sigma^2}_{\text{AC power}} + \underbrace{\mu^2}_{\text{DC power}}$$

$V_{rms} = \sigma$

$DC = \mu$

$$\boxed{R_{nn}(0) = \sigma^2 + \mu^2}$$ Total power

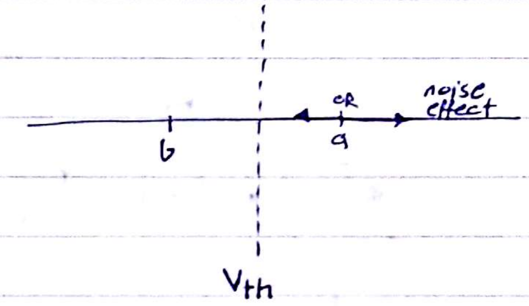
* Noise equv. bandwidth :-



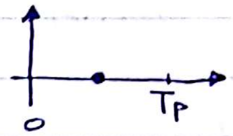
$$S_{nn}(\omega) = \int S_{n_1 n_2}(f) \ |H(f)|^2 \ df$$

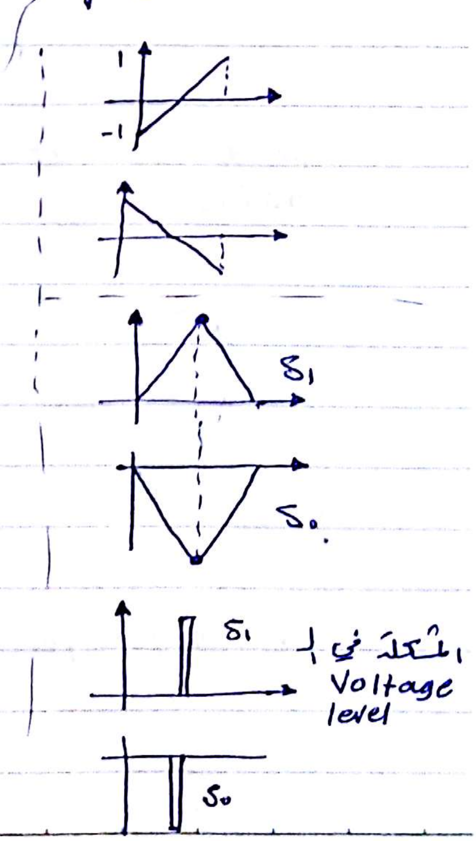
Distance
Voltage.

Non-coherent :



if we choose mid. point sampling





المشكلة في ال Voltage level

- Hard Decision : we take one sample.

- Soft Decision : Multi - sample decision.

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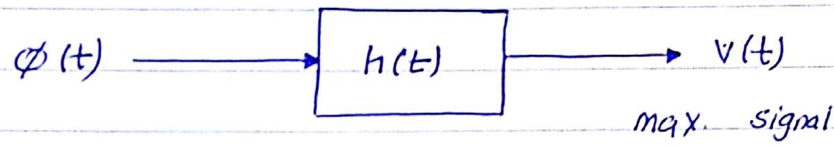


max. signal to noise ratio $(\frac{S}{N})$

$$\phi_r(t) = S_i(t) + n(t)$$

$$\rightsquigarrow = S(t) + n(t)$$

to find : SNR

- when only signal $s(t)$:

$$v(t) = s(t) * h(t)$$

$$E_v = \int |v(t)|^2 dt = \int |V(f)|^2 df = |s(t)|^2$$

- when noise only :

$$\text{output power} = \int \underbrace{S_{nn}(f)\ \ |H(f)|^2}_{\text{'two positive}} df$$

$$= \int A(f) \cdot B(f)\ df > 0$$

$$SNR = \frac{(B.W) \int |S(f)|^2 df}{\int S_{nn}(f)\ |H(f)|^2 df} \quad , B.W = \frac{1}{T} \quad , S_{nn}(f) = N_o/2$$

$$= \frac{B.W \int |S(f)|^2\ df}{N_o/2 \int |H(f)|^2 df}$$

$$\boxed{h(t) = s^*(T-t)}$$

No.

$$v(t) = s(t) * h(t)$$

$$v(t) = \int s(\tau) \cdot h(t-\tau)\, d\tau \quad \sim \boxed{I}$$

subs $h(t-\tau) = s^*(T-t+\tau)$ in $\boxed{I}$

$$v(t) = \int s(\tau)\, s^*(T-t+\tau)\, d\tau \quad , \quad T-t = u$$

$$= \int s(\tau)\, s^*(u+\tau)\, d\tau \quad = V(u)$$ Auto 'correlation

at $t=T$, $v(t) = \int_0^T s(\tau)\, s^*(\tau)\, d\tau = E_s$

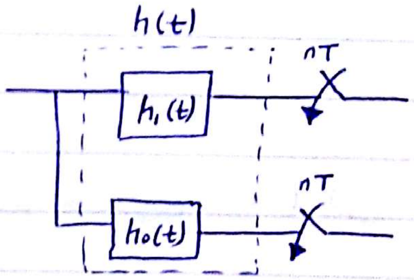
when $t = T$ then matched filter output = correlation between $s(t)$ and filter.

$$S_1(t) \longrightarrow h_1(t) = S_1^*(T-t)$$

$$S_0(t) \longrightarrow h_0(t) = S_0^*(T-t)$$

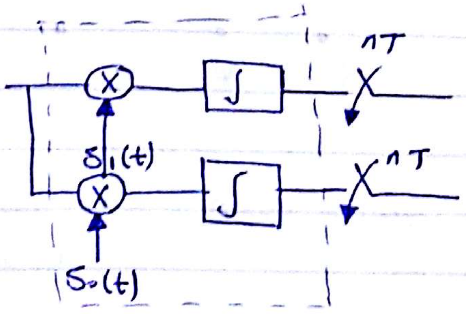
for binary tx.

<u>Matched filter</u> receiver : $\phi(t)$

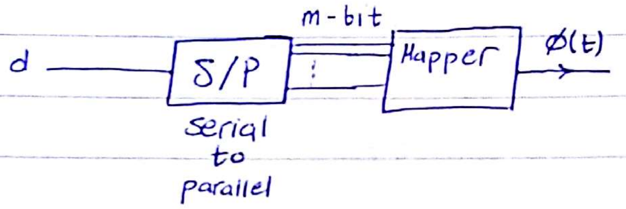


optimal receiver
we can get
(coherent receiver)

Correlator receiver : $\phi(t)$



- for non-binary (BW unlimited)



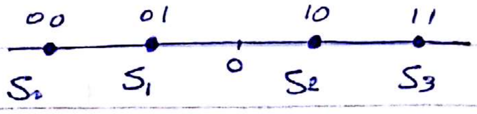
- # of combination : $2^m = M$
(signal to represent)
the data

- $\phi(t) = \{S_i(t);\ i = 0, 1, 2, 3, .., M-1\}$

Ex: Assume $m = 2$ then $M = 4$, we can draw table

		symbols
0	0	S_0
0	1	S_1
1	0	S_2
1	1	S_3

$\boxed{1-}$ 1-D for simplicity :



gray code : adjacent symbol differ in only 1 bit.

1 0
0 0
- - - - - - -
0 0
1 0

Ex: Gray code : Using to min error in bits

- 2bit : 10
  00
  01
  11

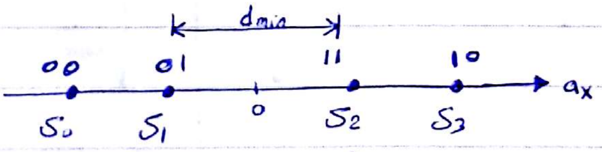
- 111
  110
  100
  101

  001
  000
  010
  011

1-D after gray coding :

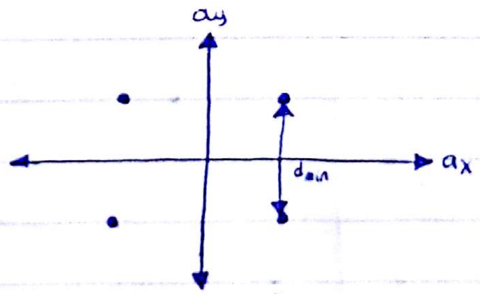
0 0	S_0	only 1-bit error
0 1	S_1	
1 1	S_2	
1 0	S_3	

we need only 1 signal



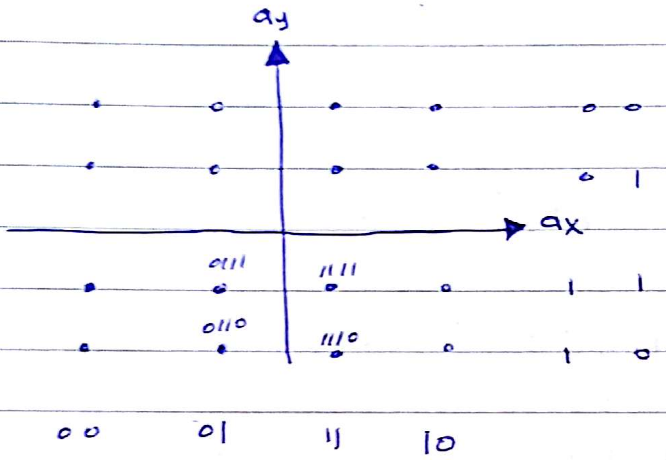
Avg. Energy = $\frac{2.5}{2} d_{min}^2 T$

$\boxed{2-}$ 2-D :



Avg. Energy = $\frac{d_{min}^2 T}{2}$

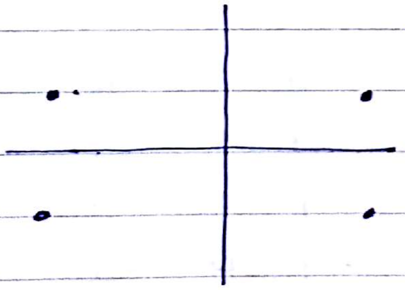
$\boxed{3}$- m = 4 bit

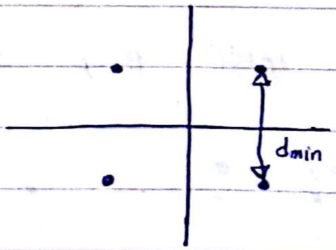


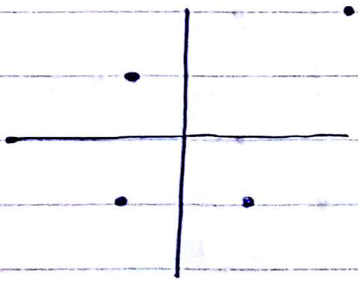
lec 22/2/2017

Q: $V = \sum_{i=1}^{I} \alpha_i^2$, optimal α_i's ?

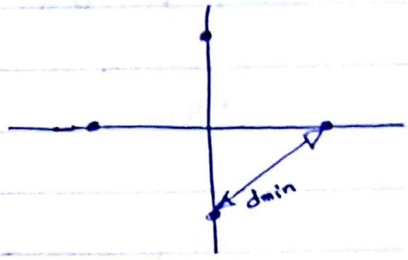
Sol.: $\alpha_i = \alpha_j \quad \forall \; i, j$







- easy to implement.



* Assume non-binary , $m = 4$

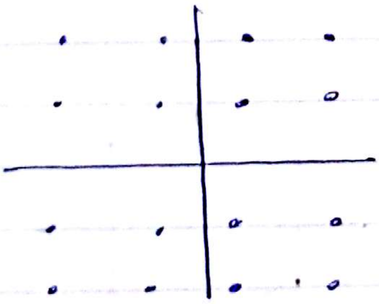
$M = 16$, for one axis : (1-D)

$S_0 = \alpha_0 \, a x(t)$

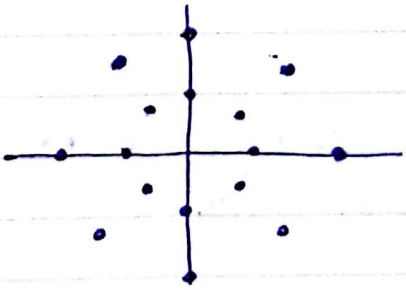
⋮

$S_{15} = \alpha_{15} \, a x(t)$

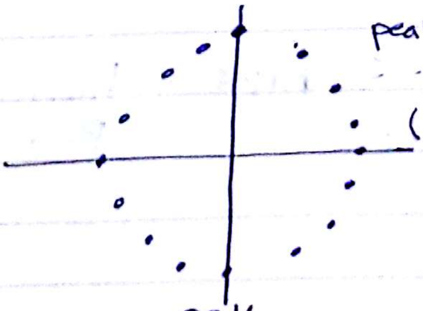
for more than one axis : (2-D)



QAM



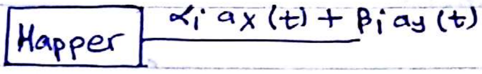
APSK

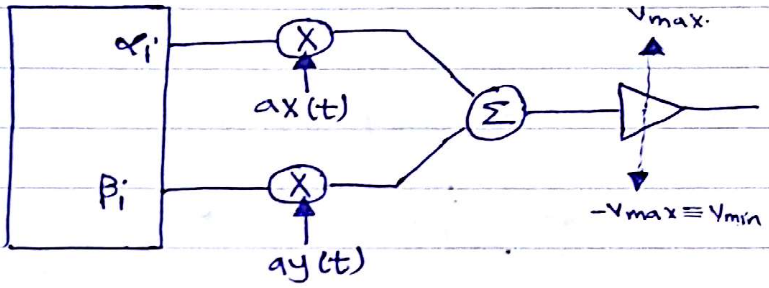


peak avg Power ratio (PAPR) =1

PSK

- from practical point of view :-

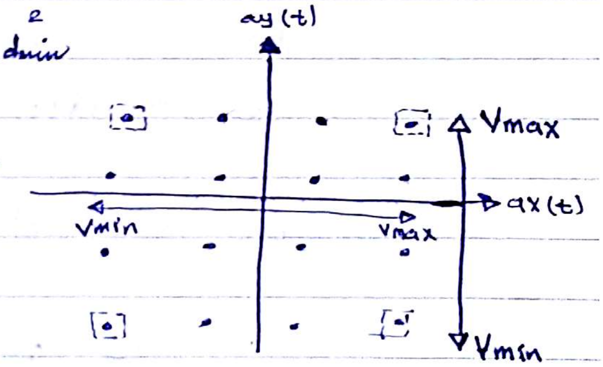




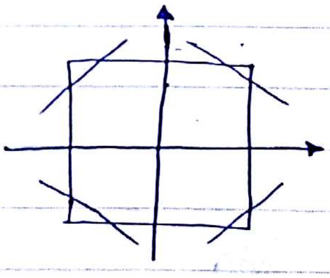
- $E_{av.} = \left(\frac{1}{2} + 2 * \frac{10}{4} + \frac{18}{4}\right) d_{min}^2$

$= \frac{10}{4} d_{min}^2 = 2.5 d_{min}^2$

- $peak = \frac{18}{4} = 4.5 d_{min}^2$



- to reduce peak to avg. power ratio



- PAPR → $PAPR = \frac{P_{max.}}{P_{avg.}} \geq 1$

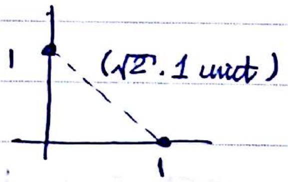
(peak avg. power ratio)

* Multi-dimensional modulation (3-D or more):

N-D mod.
(N axis)

EX: $m=2$, $M=4$, $N=4$

0 0	S_0	$ax_0(t)$
0 1	S_1	$ax_1(t)$
11 ~~1 0~~	S_3	$ax_3(t)$
10 ~~1 1~~	S_2	$ax_2(t)$



• Performence of modulation : (BER) :

$$BER \triangleq Pr(\underset{event}{error}) = P_1 Pr(event1) + P_2 Pr(event2)$$

event 1 : Tx "1" ⟶ Rx "0"

$Pr(0/1)$

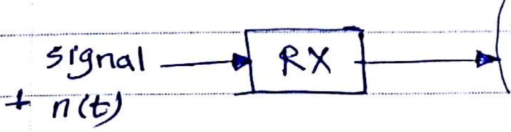
OR

event 2 : TX "0" ⟶ Rx "1"

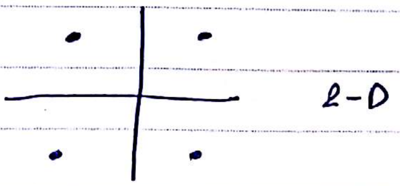
$Pr(1/0)$

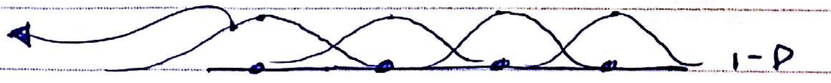
$$BER = \frac{1}{2} Pr(event1) + \frac{1}{2} Pr(event2)$$ 'Equally likely

In general : $BER = Pr(0/1) P_1 + \underbrace{Pr(1/0) P_0}_{Pr(1,0) \text{ joint.}}$



(receiving zero given that one was transmitted)

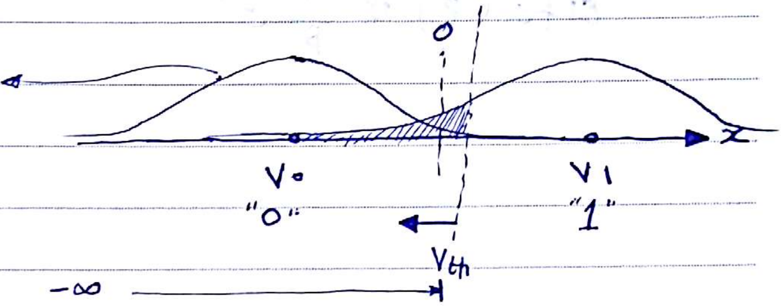




$f(V_r/S_0)$

(pdf fun.) if noise is AWGN

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2\sigma^2}(x+V_0)^2}$$



$$V_{th.} = \frac{V_1 + V_0}{2}$$

$$BER = \int_{-\infty}^{V_{th}} f(x/1)\,dx + \int_{V_{th}}^{\infty} f(x/0)\,dx$$

$$= \int_{-\infty}^{V_{th}} f_x(x/1)\,dx = \int_{V_{th}}^{\infty} f_x(x/0)\,dx$$

general ex : $V_{th} = 0$, $V_1 = -V_0 = \frac{d_{min}}{2}$

$$\longrightarrow BER = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-1}{2\sigma^2}\left(x - \frac{d_{min}}{2}\right)^2} dx$$

$$Q(\alpha) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\alpha}{\sqrt{2}}\right) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-t^2/2}\,dt.$$

$$BER = \int_{0}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-1}{2\sigma^2}\left(x + \frac{d_{min}}{2}\right)^2} dx = \frac{1}{\sqrt{2\pi}} \int_{-\frac{d_{min}}{2\sigma}}^{\infty} e^{-t^2/2}\,dt$$

let $x = \frac{t - d_{min}/2}{\sigma} \rightarrow dx = \frac{dt}{\sigma}$

$$BER = Q\left(\frac{-d_{min}}{2\sigma}\right) = 1 - Q\left(\frac{d_{min}}{2\sigma}\right)$$

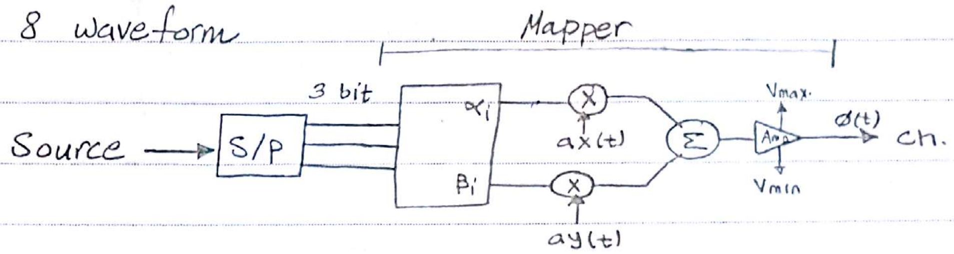
$$\boxed{BER = \frac{1}{2}\,\mathrm{erfc}\left(\frac{-d_{min}}{2\sqrt{2}\,\sigma}\right)}$$

$$\mathrm{erfc}(\beta) \approx \frac{1}{\beta} e^{-\beta^2}$$

Ex1: 2-dim. baseband mod. : (3 bit)
OR 8-PSK

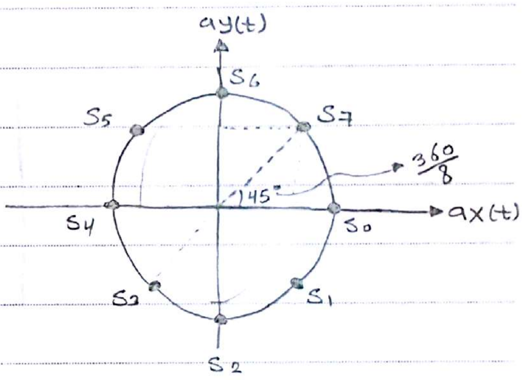
Sol. 8 waveform

TX :



Using gray code :

1 1 0	S_0
1 0 0	S_1
1 0 1	S_2
1 1 1	S_3
0 1 1	S_4
0 0 1	S_5
0 0 0	S_6
0 1 0	S_7



* Assume 1-unit circle

→ find α_i, β_i for :

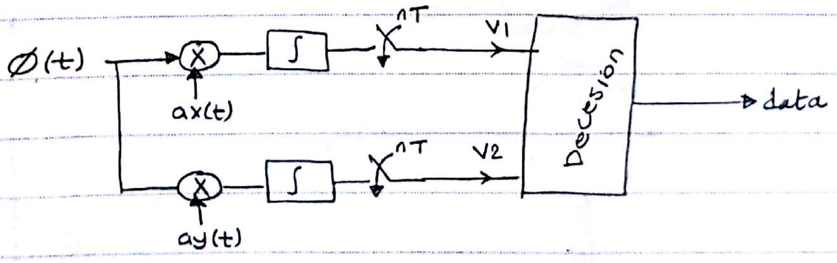
1- S_0 : $\alpha_i = 1$, $\beta_i = 0$

2- S_7 : $\alpha_i = \beta_i = 1/\sqrt{2}$

3- S_6 : $\alpha_i = 0$, $\beta_i = 1$

4- S_3 ; $\alpha_i = \beta_i = -1/\sqrt{2}$

RX :



EX2 : Try it for 16-QAM :

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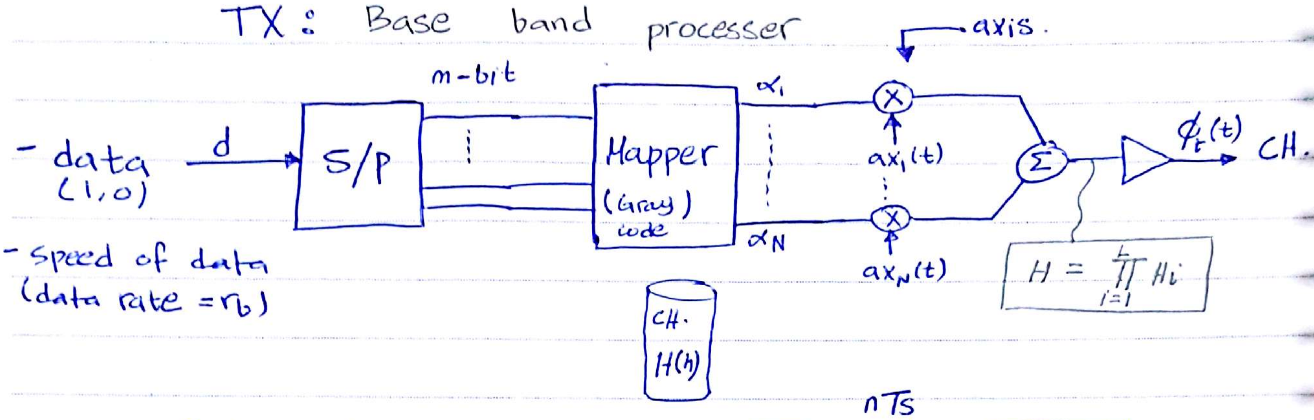
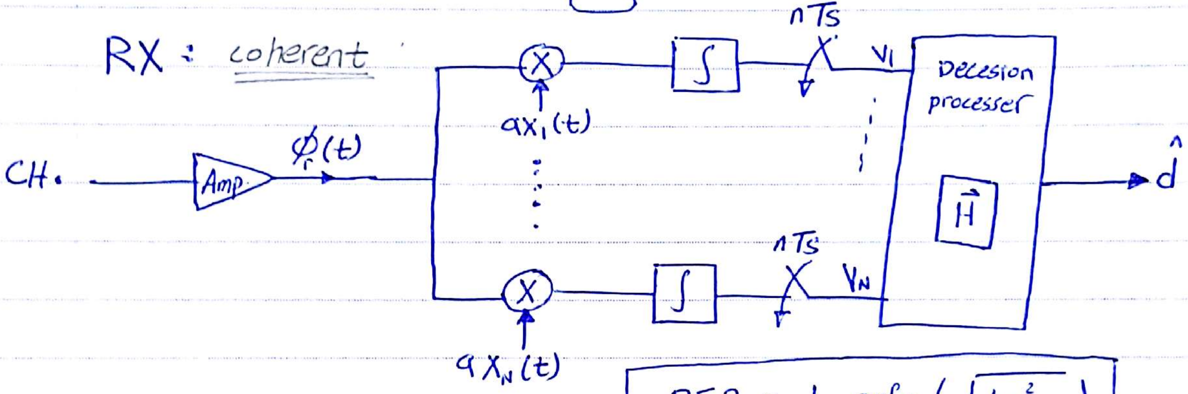
TX : Base band processer



RX : coherent

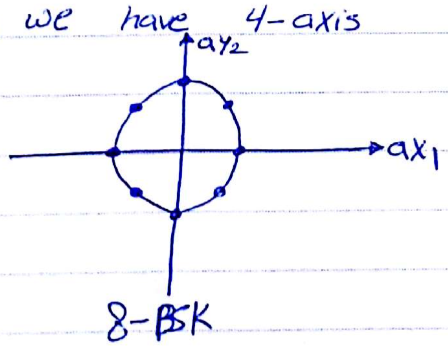


$$\boxed{BER = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{d_{min}^2}{8\,\sigma_m^2}}\right)}$$

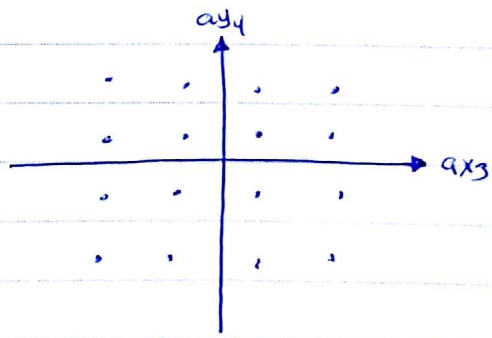
$$\boxed{V_i = \alpha_i + n_i}$$

Ex : Hybrid modulation (8-PSK , 16-QAM)

Sol: we have 4-axis



8-PSK



16-QAM

we take m = 7 bits at same TX & RX figure.

V_i In general :

- $V_i = h_i \alpha_1 + n_i$

in vector form : $\boxed{\vec{V} = H\vec{\alpha} + \vec{n}}$, H : ch. matrix.

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} h_1 & & \emptyset \\ & \ddots & \\ \emptyset & & h_N \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix}$$

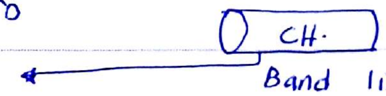
$$\boxed{\vec{Y} = H\vec{x} + \vec{n}}$$

OR fading ch.

Rec. • for band limited ch. the ch. matrix is not diagonal.

Rec. •

$$* \quad H = \begin{bmatrix} h_1 & h_{12} & \cdots \\ h_{21} & \ddots & \\ & & h_N \end{bmatrix} \rightarrow \text{Not zero}$$



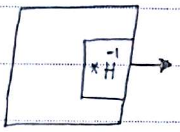
Band limitted (inter-symbol interference)

- we can separate the H matrix into multiple matrices each represent phenomena like : fading , miss synch.

Rec. • We can apply post process on received data to equalize channel, reduce effect of Fading.

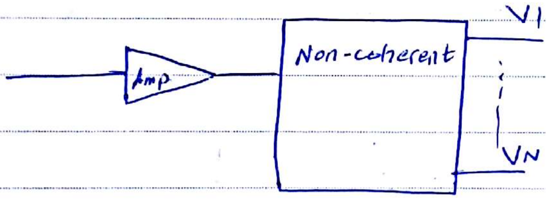
SVD • $\boxed{H^{-1}V = \vec{\alpha} + H^{-1}n}$ المشكلة في هذه العلاقة إنه قد تكون ،
ال H ما الها inverse.

• TX → Mapper



نضرب ب H^{-1} داخل ال mapper
بدل ما أضرب على جهاز ال RX.

• RX : non-coherent



$$\boxed{BER = \frac{1}{2} e^{-\frac{d_{min}^2}{16 \sigma_n^2}}}$$

• Non-coherent has $\underline{\underline{3 dB}}$ less in performence
تعني بضعف

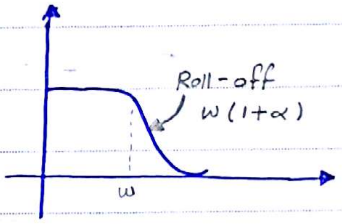
• if BW of axis ($\phi(t)$) (signal)
>
BW of channel
⟶ inter-symbol interference. (ISI) $\xrightarrow{\text{cont.}}$

.. Only sinc(ηt) function will work

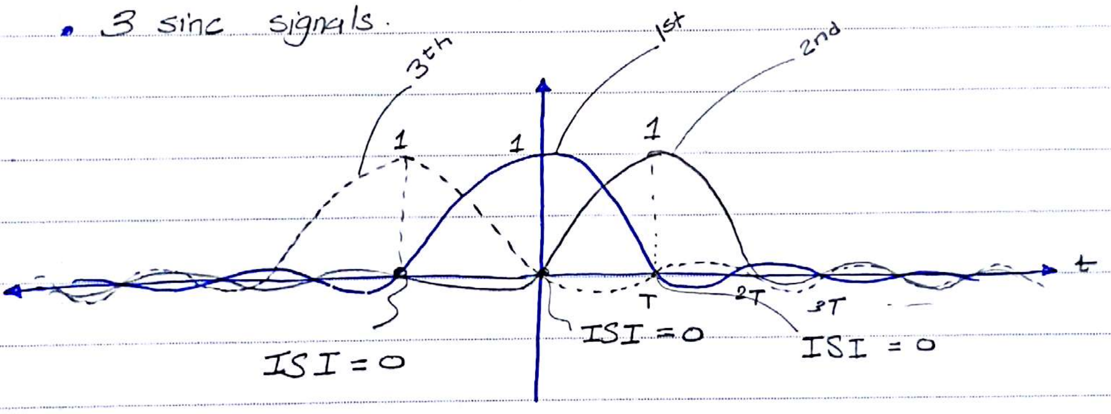
→ practical solution : Raised cosine (RC_α)

→ sinc : can't implement it because not causal

$$RC_\alpha = \text{sinc}\,(\eta t) * g(t)$$



- 3 sinc signals .



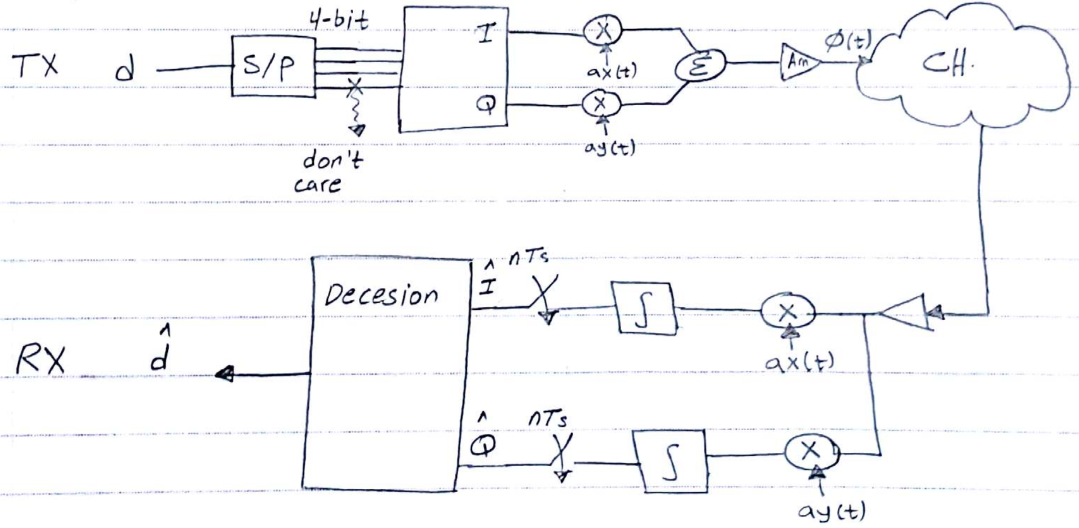
No. lec 1/3/2017

Quiz 1:

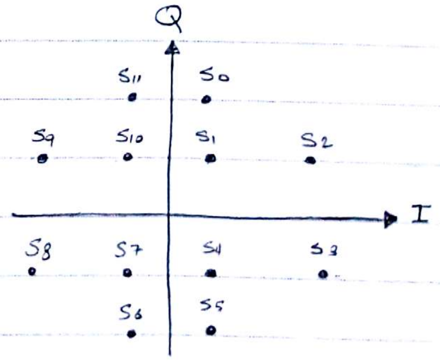
Q: Design the TX & RX of a 12-ary (QAM cons.) 2-dim. base band system.

Sol.

TX & RX :



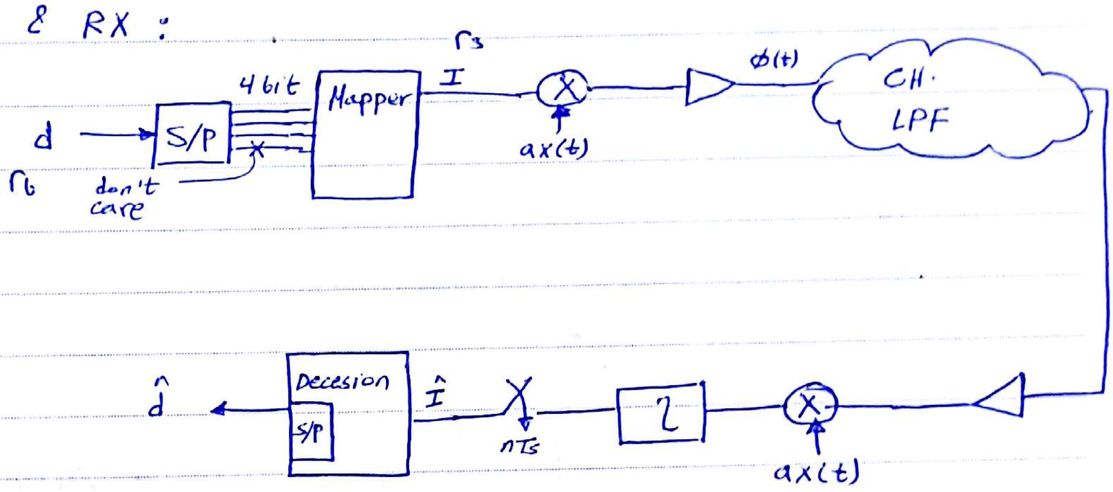
$m = \log_2 12 \simeq 3 < m < 4$ so we will not send 4-bit,

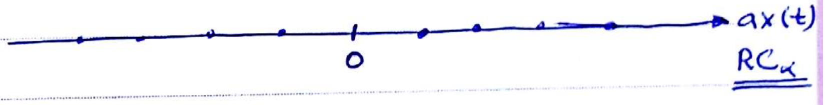


for last Q : (Pulse amplitude mod.) (1-dim.)

if $BW_{CH.} = 100$ KHZ : means ch. → LPF

TX & RX :



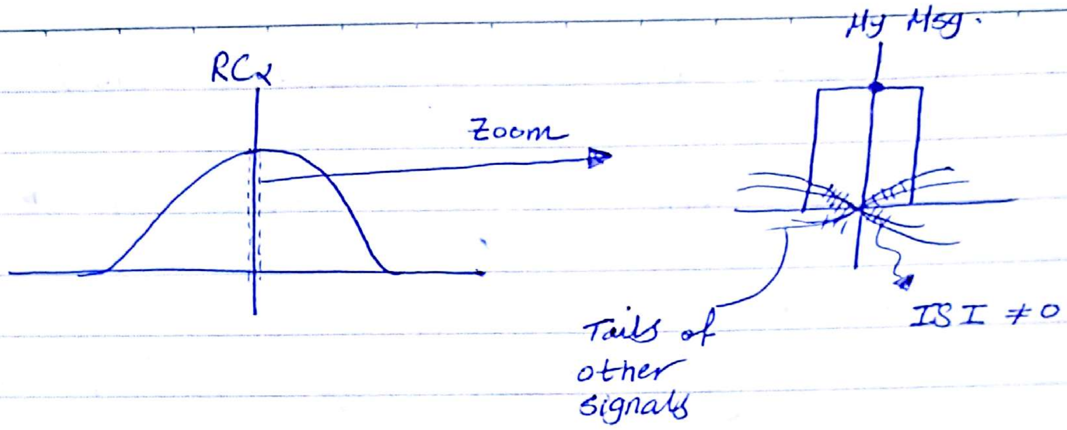


$$BW_{CH} = 100\,KHz = w(1+\alpha) = \frac{r_b}{2m}(1+\alpha).$$

$$w = \frac{1}{2T_s} \quad , \quad \frac{1}{T_s} = r_b = \frac{r_b}{m}$$

let $\alpha = 0.2$, $\frac{r_b}{m} = \frac{200K}{1.2} = 166$ K symbol/s

- The dimen of the cons. (H) is a design factor that can we change to Tx. any bit rate.

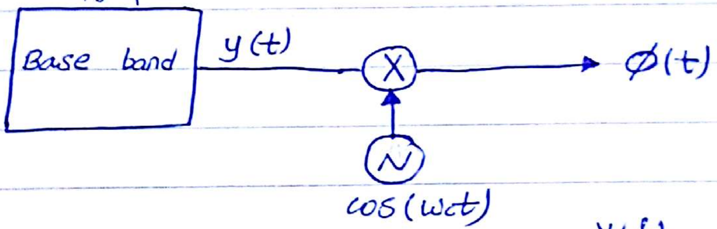


* * *

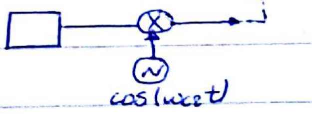
..o Band Pass modulation :

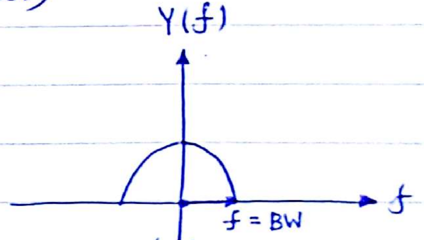
(ch. → env.)

BW+freq.

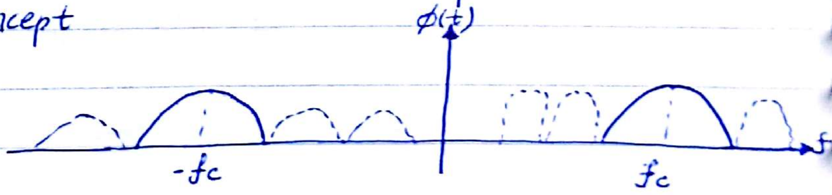


• for other signal





• Using FDM concept to make signals orthogonal.



Rec. •

No.

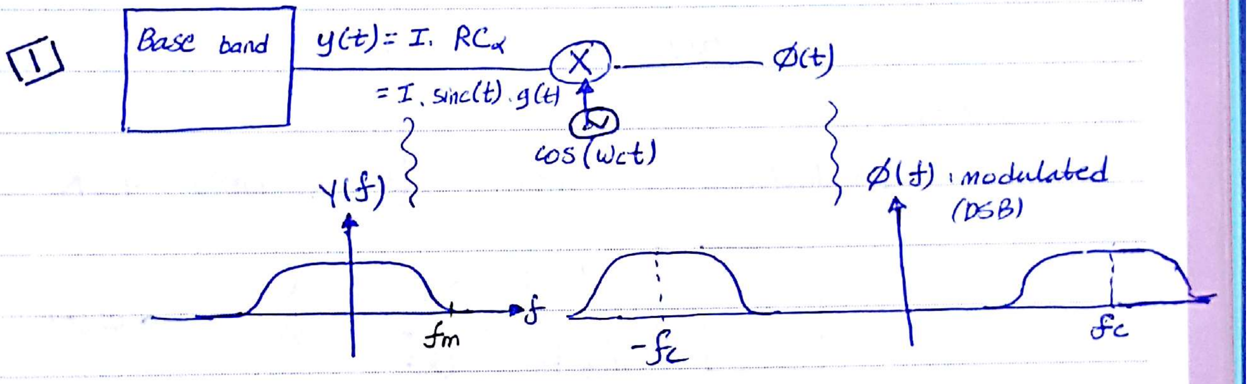
- Image station :

extra data.

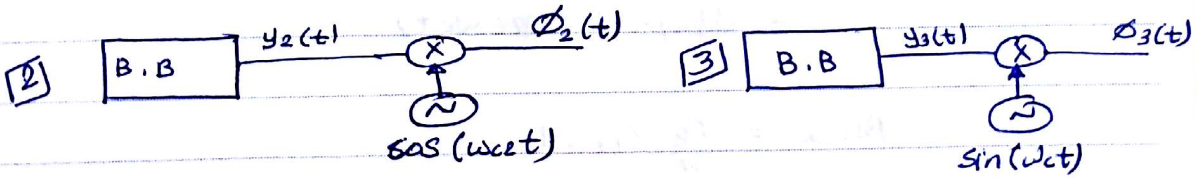
$$\boxed{f_I = \frac{\mp n}{m} f_{If} + \frac{f_{Station}}{m}}$$, m, n : integers $< N$

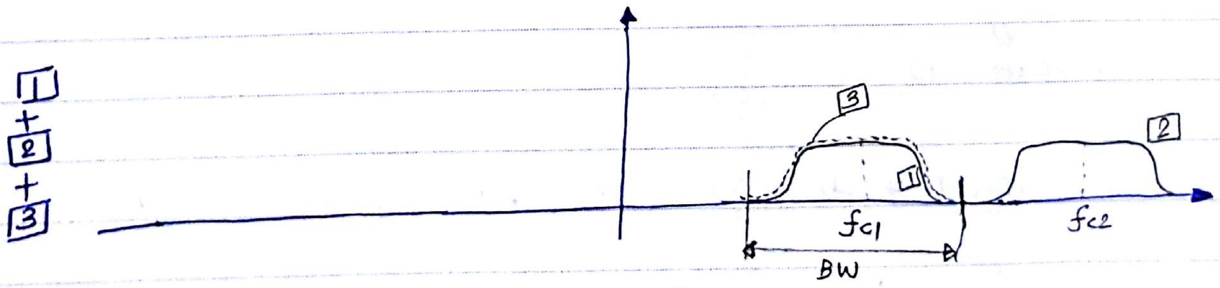
N: order of non-linearity.

-



for 3 signals :





- sin, cos with same freq : orthogonal $f_c \gg f_m$

- $BW = 2 f_m = 2 * \frac{r_b}{2m}(1+\alpha) = \frac{r_b}{m}(1+\alpha)$

Mod. • 1-D :

Pulse shapping function.

$\phi(t) = y(t) \cdot RC_\alpha(t) \cdot \cos(w_c t)$

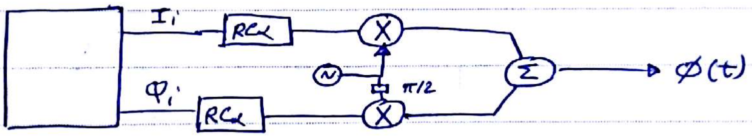
$\boxed{\phi(t) = A_i\, p(t) \cos(w_c t)}$ $\boxed{ASK}$

A's

$BW_{ch.} = \frac{r_b}{m}(1+\alpha)$

• 2-D :

Quadrature modulation : $\boxed{PSK,\ QAM,\ APSK, \ldots}$



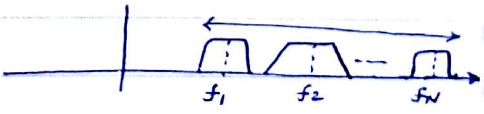
$\phi(t) = I_i\, p(t) \cos(w_c t)$
$\quad + Q_i\, p(t) \sin(w_c t)$

$BW_{CH} = \frac{r_b}{m}(1+\alpha)$

• Multi-D $\overset{(N)}{}$:

$\phi(t) = A_i\, p(t) \cos(w_j t)$, $j = 1, 2, 3, \ldots, \underline{\underline{N}}$

$\boxed{FSK}$



$BW_{CH.} = N \frac{r_b}{m}(1+\alpha)$

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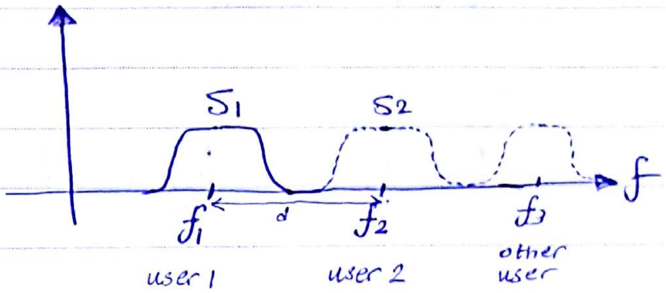
Assume :

$p(t)\ \cos(\omega_{c1} t)$



OR $p(t)\ \cos(\omega_{c2} t)$

$p(t)\ \cos(\omega_{c1} t - \frac{\pi}{2}) = p(t)\ \sin(\omega_{c1} t)$

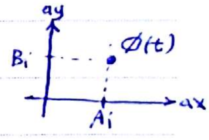
* إذا حصل تداخل بين S_1 و S_2 يسمى

(Adjacent channel interference)

for 1-D : ASK $\quad \phi(t) = \underline{A_i}\ \underline{p(t)}\ \underline{\cos(\omega_c t)}$

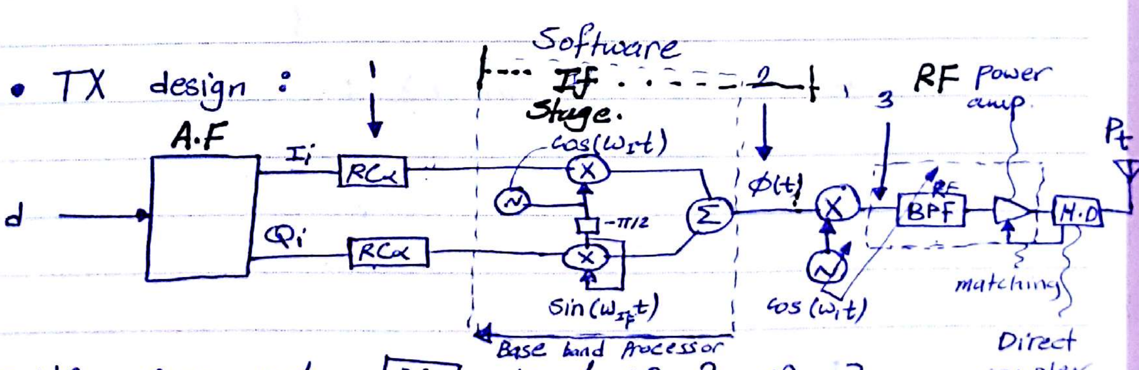
for 2-D : QAM $\quad \phi(t) = A_i\ \underbrace{p(t)\ \cos(\omega_c t)}_{ax}$

$+ B_i\ \underbrace{p(t)\ \sin(\omega_c t)}_{ay}$



* Quadrature modulator :

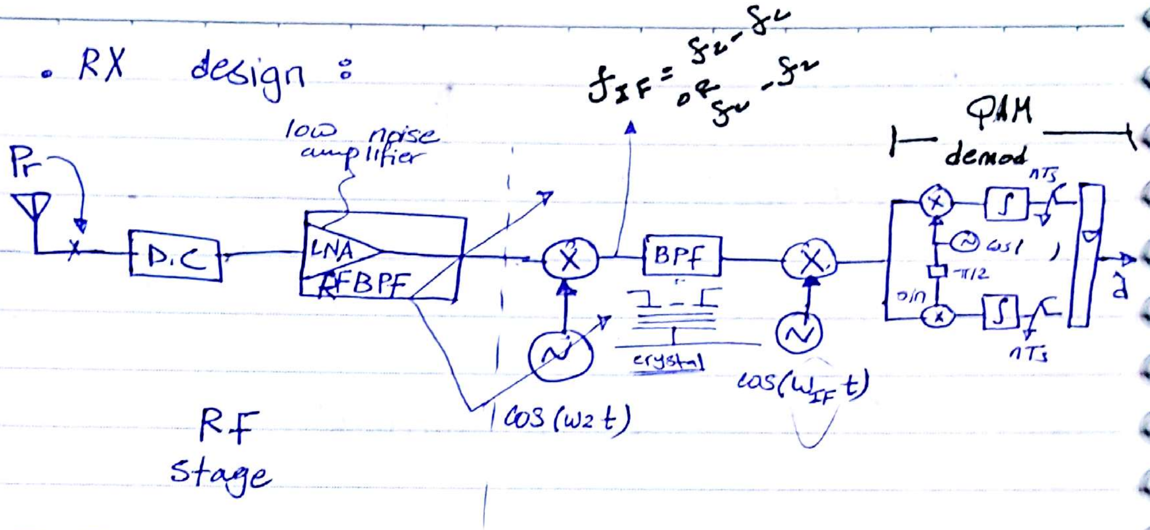
- TX design :

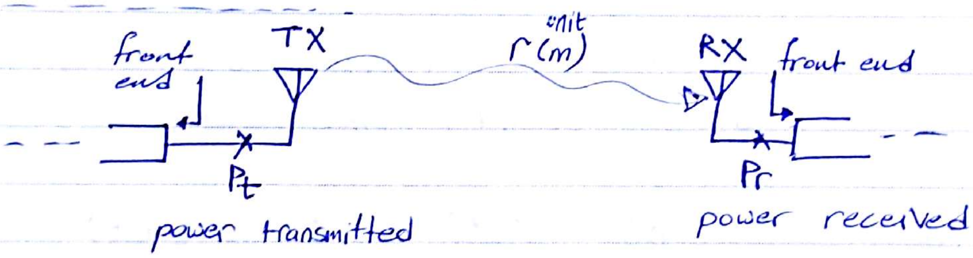


- we can put $\boxed{RC_\alpha}$ in 1 OR 2 OR 3

- $f_c = f_1 \mp f_{If}$

• RX design :





$P_r = K P_t \, r^{-\gamma}$, r : distance between TX & RX

, $2 \leq \gamma \leq 4$

P in front of antenna $= \frac{|E|^2}{\eta}$ ⇒ Power density

* Before d in TX :

No.

* أهم نقطة أحسب عندها الـ power لل tx. signal
بعد الـ power amp.

- We limit SNR at the If stage using:

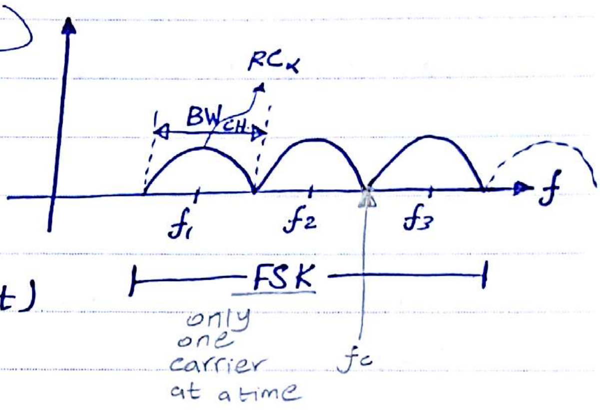
* If BPF

* RF BPF at RX: reject other station that might be

- SAW filter: Service acosting wave filter (RF BPF @ Tx & RX)

No. Lec. 13/3/2017.

- Multi-dimensional mod.:



- $\phi(t) = \sum_{i=1}^{L} A_i^2 \, p(t) \cos(\omega_i t)$

A_i : information

- $BW_{CH.} = \frac{r_b}{\underbrace{\log_2(M)}_{m}} (1+\alpha)$, Narrow band

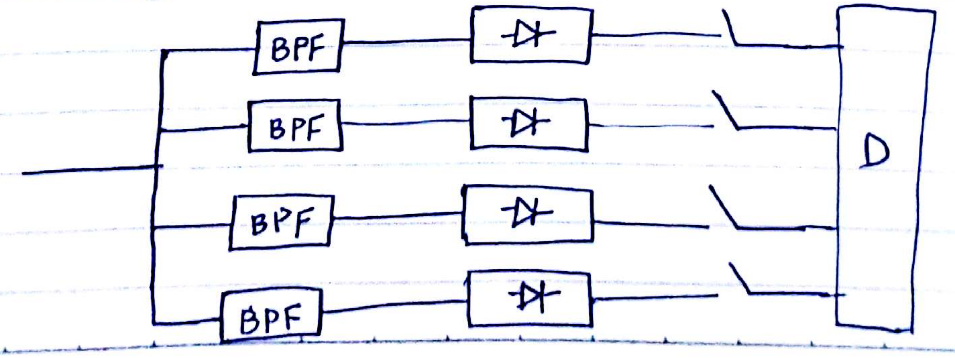
- $BW_{FSK} = L \cdot \frac{r_b}{\log_2 M} (1+\alpha)$, Wide band , $L = \#$ of

- Assume :

0 0	$A_1 = 1$, other A's $= 0$
0 1	$A_2 = 1$
1 0	$A_3 = 1$
1 1	$A_4 = 1$

إذا وصلنا عل RX 00
تكون قيمة أول signal = 1
والباقي 0
(نختار ال signal الأولى)

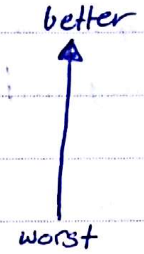
RX design



- Wide band mod. is immune against fading more than PSK and ASK

- FSK is the best among fading

- ✓ FSK
  - ✓ PSK
  - ✓ QAM
  - X ASK



w.r.t <u>fading</u>

Ex:

↳ Multi-carrier (ASK)

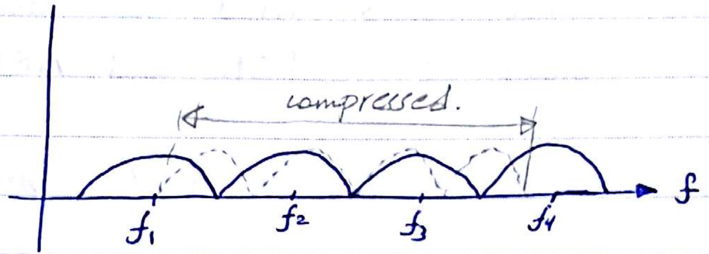
	A1	A2	A3	A4
0000	-1	-1	-1	-1
0001	1	-1	-1	-1
⋮				
1111	1	1	1	1

Not in Ex. * Multi-carrier (PSK) using sin & cos.

(A) carry 2 bit

A = 1 or -1

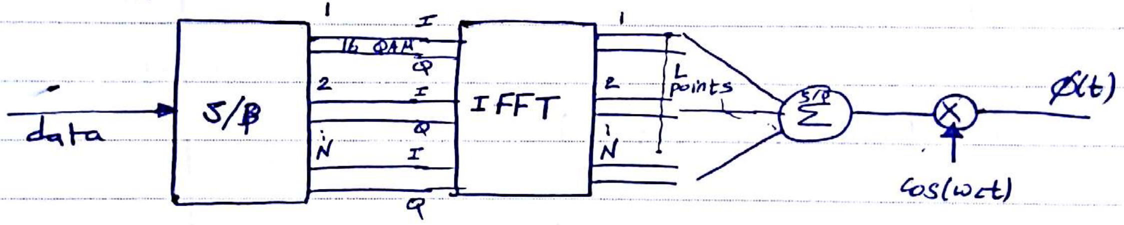
* Communication mangement include :-
  1- Resource allocation.
  2- Power control.



* if we compress the spectrum :
we will have non orthogonal signals (problem).

Now, we need to make all signal with compressd spectrum (less BW) orthogonal (OFDM)

2-D mod. signal in time domain



- each symbol 4 bit

GB: Gaurd band , Δf: freq. resolution

No.

مشاكل ال OFDM

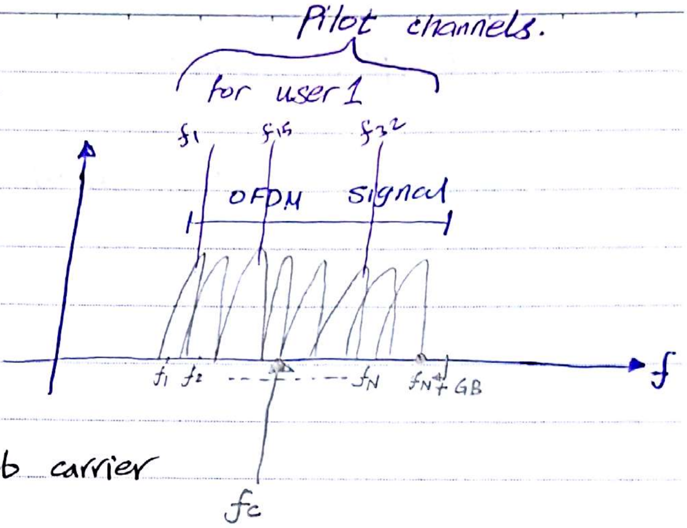
- Doppler shifts: motion of

$$\Delta f = \frac{v}{c} \cos(\theta)$$

- نستطيع معالجة ال orth. من خلال ال G.B. بسرعات قليلة

No. lec. 15/3/2017

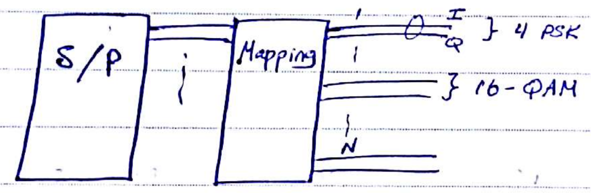
OFDM :



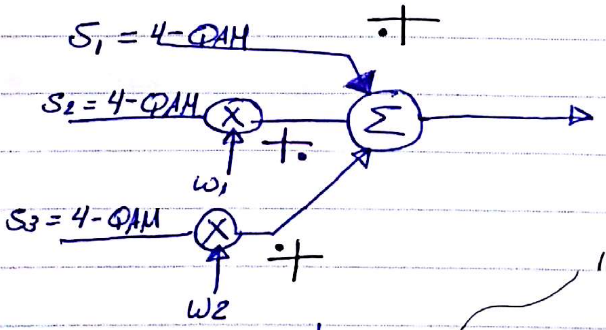
- f_1, f_2: sub carrier

  - Δf = 10.7 KHZ for Wi-Max
    ↳ 2048 sub carrier

•



- Adaptive Modulation :

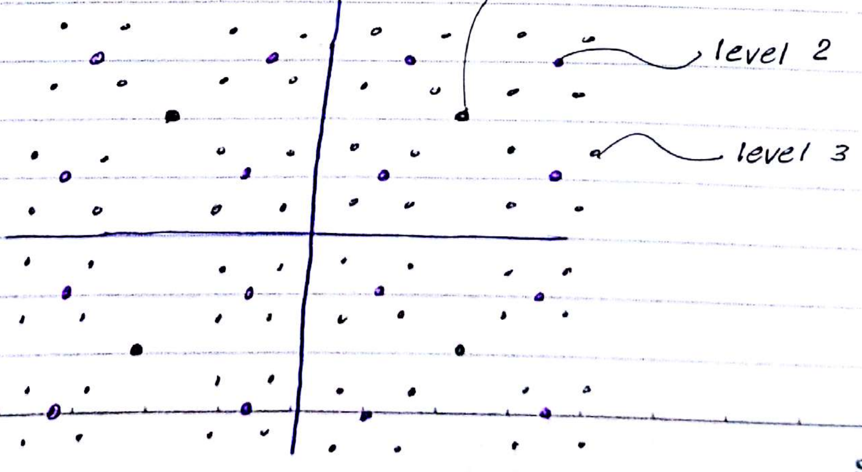


to select the point

1- starts from level 1 (S_1)

2- select point for level 2 from S_2

3- select 3rd point for level 3 from S_3



- if and only if OFDM works fine (no loss of orthogonality) we will get the following:

1- Convert freq selective fading ch. into freq. non-selective fading ch.

2- Comm. mangement is simple (resourse allocation) this will lead to adaptive/variable rate comm. (BW on demand).

3- Can be used for multiple access system.
(can be used by multi-user at same freq.)

4- simple to implement.

∴ $\boxed{1}$ Recall: $\vec{Y} = H\vec{x} + \vec{n}$

<u>for selective</u>

$$H = \begin{bmatrix} H_{11} & H_{12} & \cdots \\ \vdots & & \\ \vdots & & H_{NN} \end{bmatrix}, \quad H: \text{sub matrix}$$

$$H_{ij} = [h_{ij}]$$

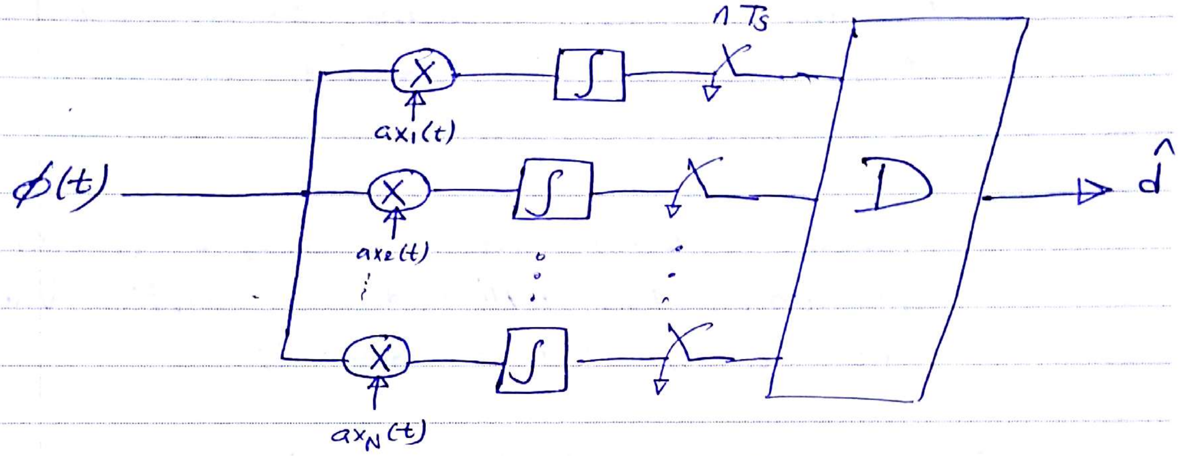
$$H_{ij} = \begin{bmatrix} h_1 & & 0 \\ h_2 & \ddots & \\ \vdots & & \\ h_L & \ddots & \\ & 0 & \end{bmatrix}$$

rank = L (not invertable)

(easy)
<u>for non-selective</u>

H_{ij} has inverse

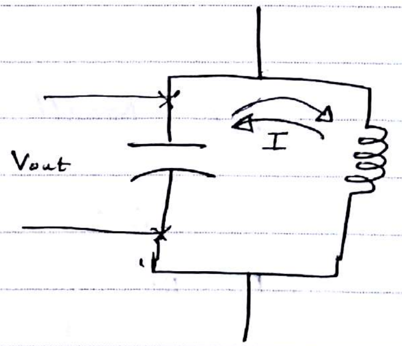
@ RX : (synch.) ≡ co-herent.

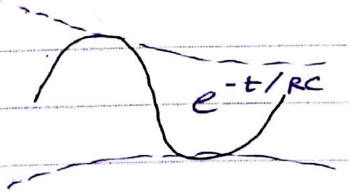


$* \ ax_i(t) = p(t) \cdot \cos(\omega_i t)$

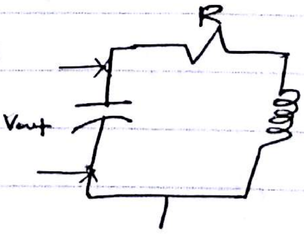
** Oscillators :

- V_{out} ⇒ second order (sinusoidal)



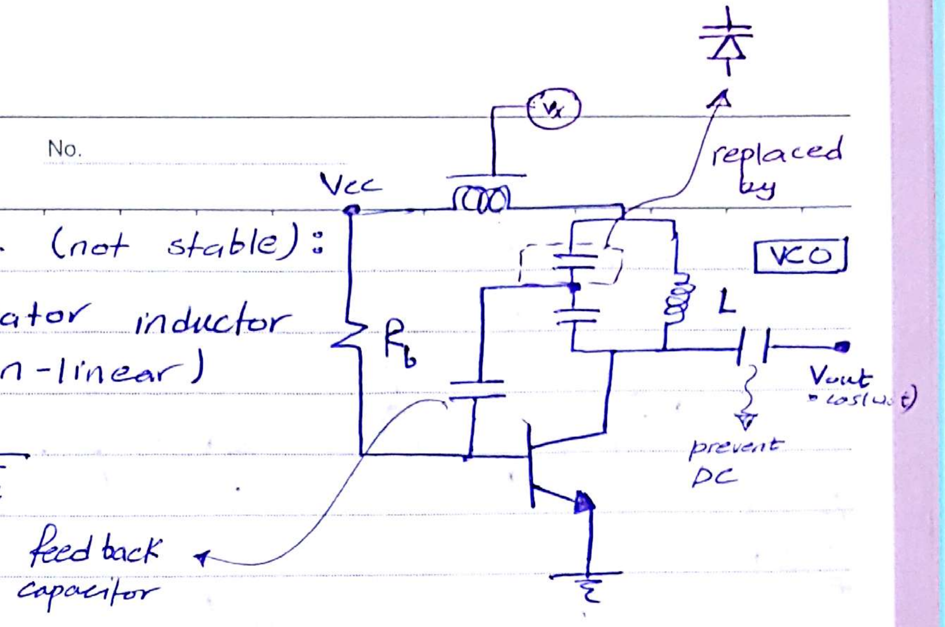


- effects of : (adding R)
  - * heat loss



- To remove 'R'

No.



* Simplest Osc. (not stable):

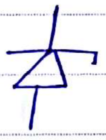
·  : isolator inductor (non-linear)

· $f_o = \frac{1}{2\pi\sqrt{LC_{eq}}}$

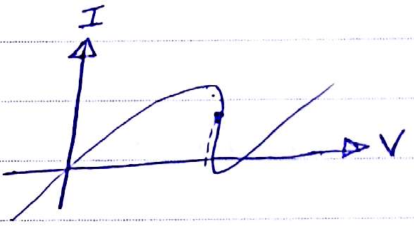
* * *

* extra component

1 –



tunnel diode

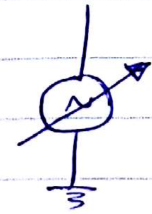


* * *

2 –

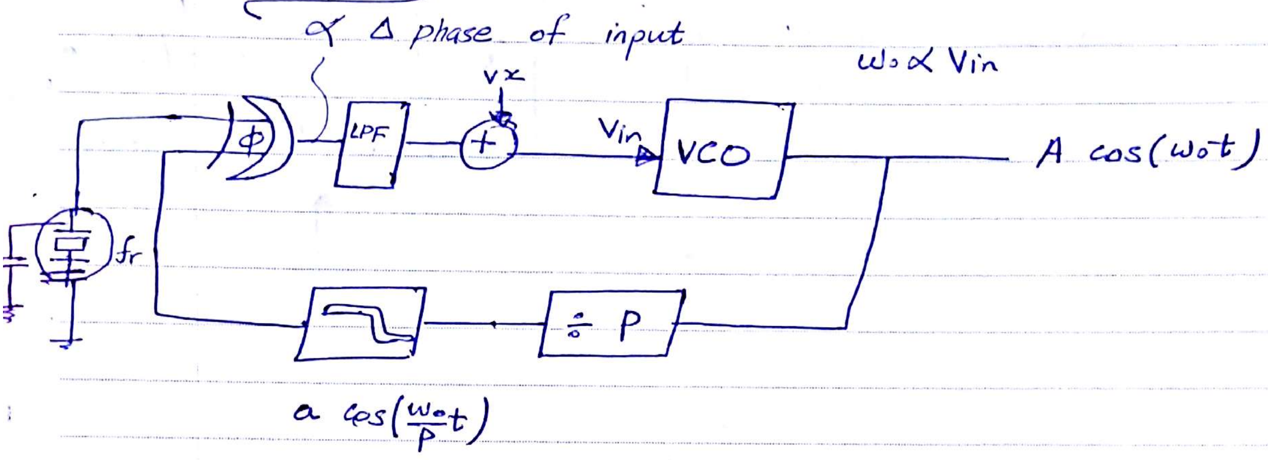
* * *

3 –

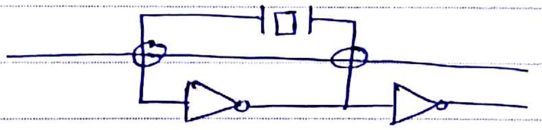


* * *

∘ PLL :



digital osc. : (∿ → ⊓)



* @ lock state :

$$\boxed{f_o = P \cdot f_r}$$

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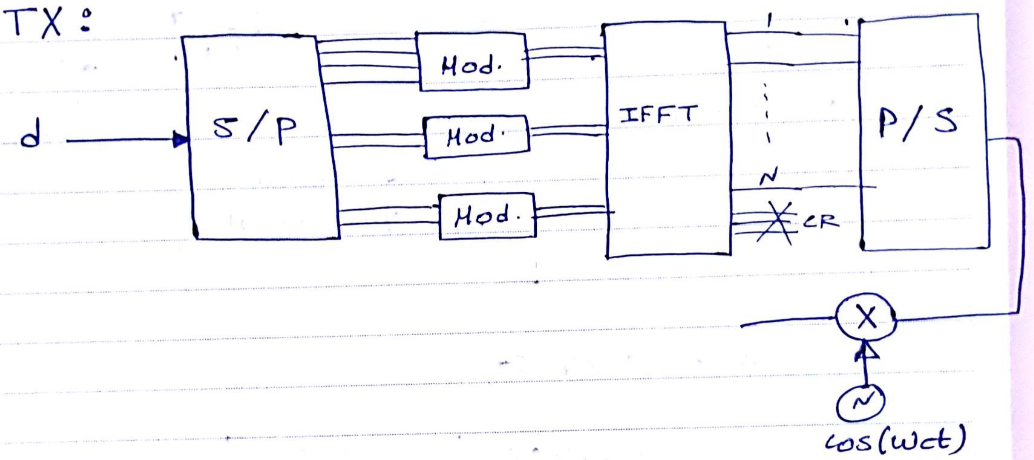
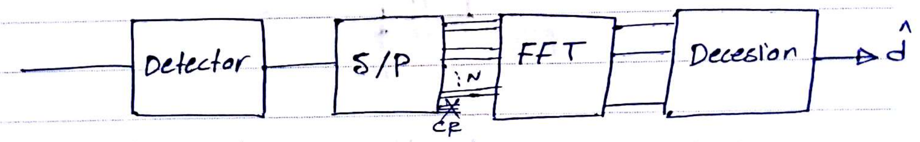
Quiz 2 : Draw the block diagram of OFDM transmitter , assume N=8

Sol.:

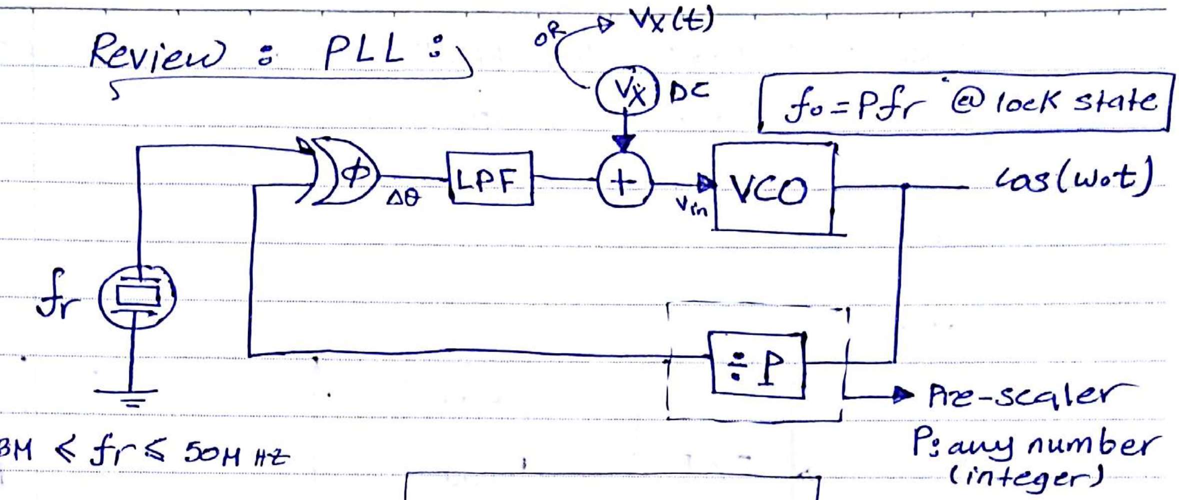
TX :



RX :

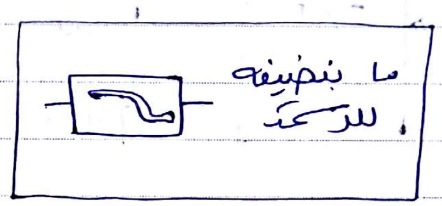


Review : PLL :

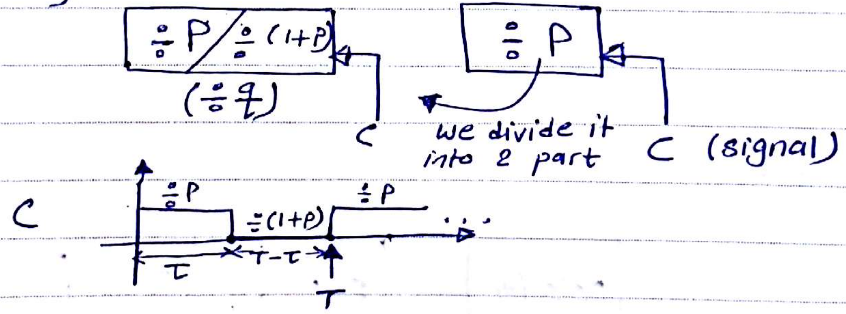


$f_o = Pf_r$ @ lock state

8M ≤ fr ≤ 50MHz



* Pre-Scaler :



- we divide by P or 1+P depends on C :

  0 → τ ÷P

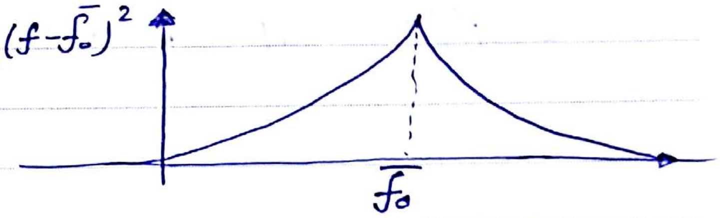
  τ → T ÷(1+P)

$$q = \frac{\tau P + (T-\tau)(1+P)}{T} \quad , \; q \text{ avg.}$$

- we can divide by 5 or 15 but the standard is P, 1+P

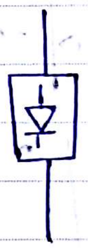
- f_0 / ω_0 changes with temp. (Thermal noise).
  - → f_0 is Random variable (Gaussian R.V)

$$\underset{(pdf)}{f(f_0)} = \frac{1}{\sqrt{2\pi}\,\sigma_f}\, e^{-\frac{1}{2\sigma_f^2}(f-\bar{f_0})^2}$$

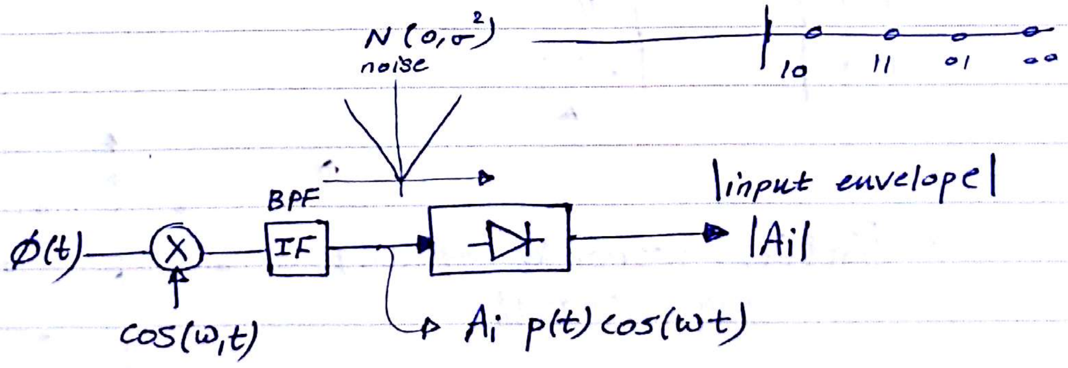


- Co-herent RX very difficult to implement.

- **ASK :**

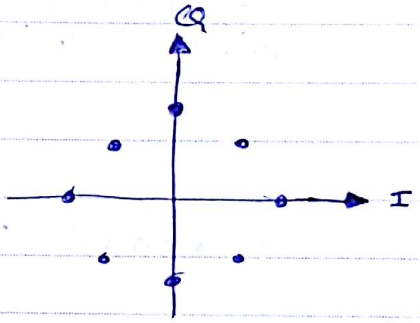


for single sided cons.

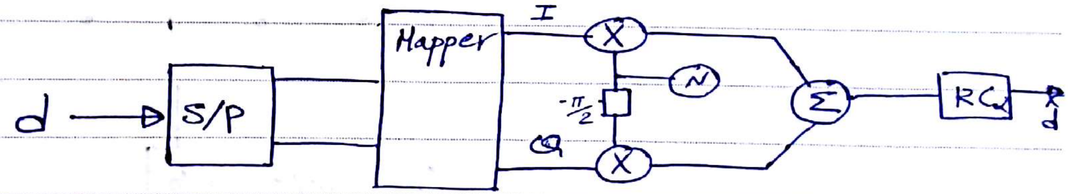


Note : symbols in non-coherent RX will have different BER due to nature device.

- Differential Modulation :



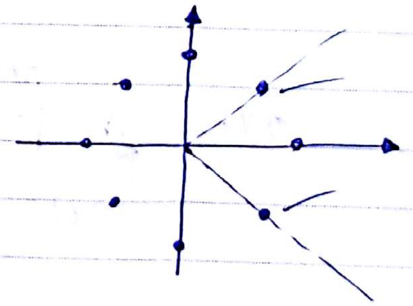
TX :)



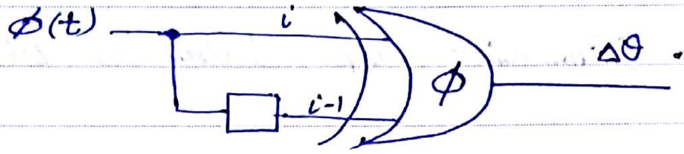
e.g.

data	I	Q	$\Delta\theta$	$\theta i = \theta_{i-1} + \Delta\theta$
0 0			e.g. $0 / \pi/4$	$\pi/4$
0 1			$\pi/2 / \pi/2$	$\pi/4 + \pi/2$
1 1			$\pi / -\pi/2$	
1 0			$3\pi/2 /$	

× Partial response signaling (with memory)



RX :)



* full response signaling (without memory)

e.g.

data	I	Q	$\Delta\theta$
0 0			
0 1			
1 1			
1 0			

- In fading condition we can't control partial response

$\longrightarrow$ Sol. : Transmit full response

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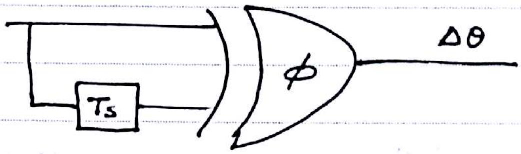
we don't need coherent RX

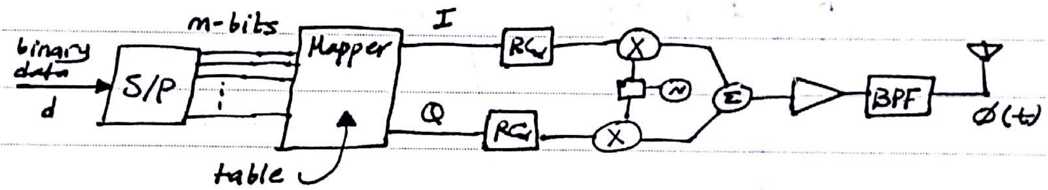
- **DPSK :**

→ Transmitt the data in Δθ's

RX .. D-coherent RX



TX ..



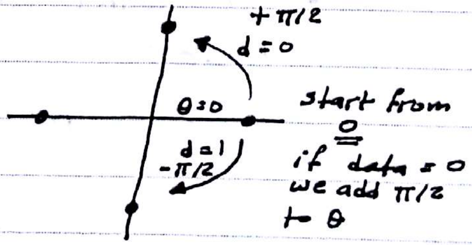
$$\phi(t) = I_i\, p(t) \cos(\omega_c t) + Q_i\, p(t) \sin(\omega_c t)$$

$$= A\, p(t) \cos(\omega_c t + \theta_i) = A \angle \theta_i$$

Ex 1 :

d	Δθ
0	$\pi/2$
1	$-\pi/2$

$\pi/2$ - DPSK

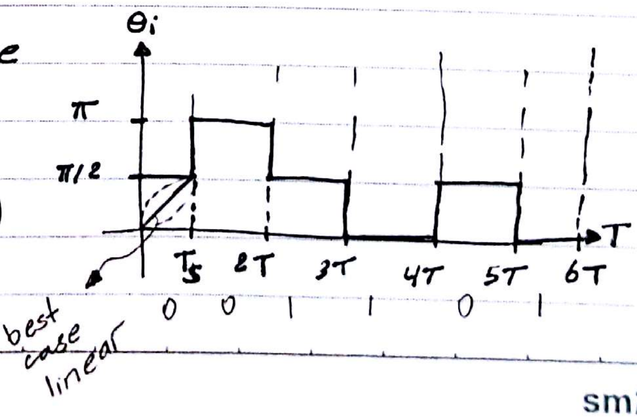


start from 0
if data = 0 we add π/2 to θ

(بنطبق هنا الكلام عند كل نقطه بتنتج معنا)

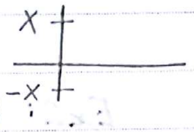
Draw phase tree
d = 001101

$BW \propto 2(f_m + \Delta f)$

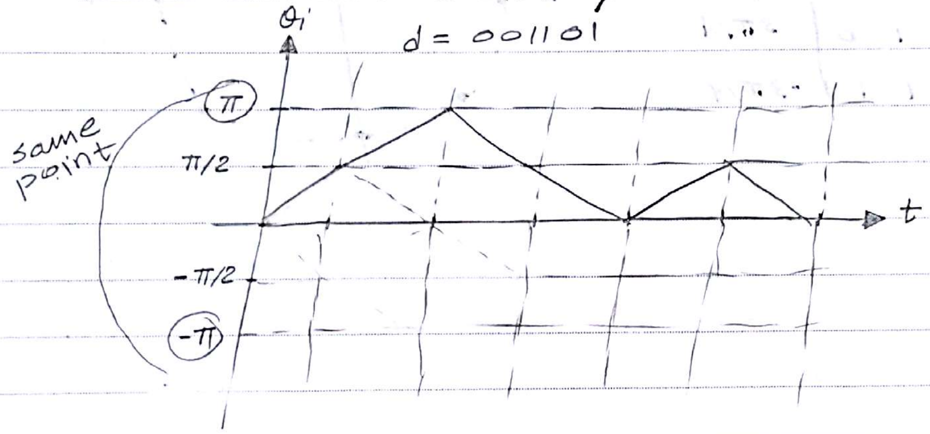


* Best way to perform phase change is : the straight line, and edges will be smooth by the RC$_\alpha$ filter

⊛ Phase tree : cylindrical draw (x, -x : the same point)



⊛ All cases of phase tree : (Trilles diagram)

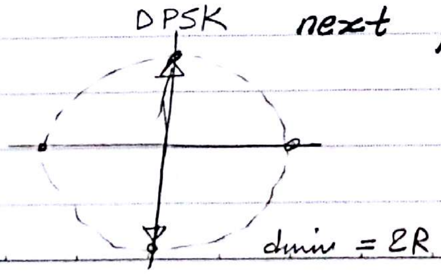


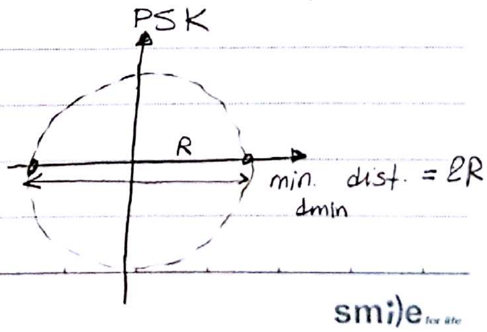
# of points on circle = $\frac{2\pi}{\Delta\theta}$

⊛

$$A\,\underbrace{p(t)}_{RC_\alpha,\ \text{Gaussian Filter}}\cos(\omega_c t+\theta_i) \xrightarrow[\text{it as}]{\text{we can write}} A\cos(\omega_c t+p(t)\,\theta_i)$$

⊛ Min. distance : min. distance between all posible next points



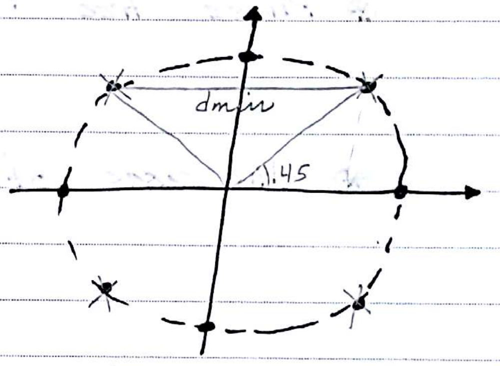


⊛ GMSK .
MSK .
GSM

Ex2:

Non binary

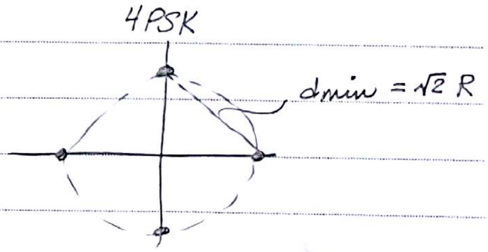
d	$\Delta\theta$
0 0	$\pi/4$
0 1	$-\pi/4$
1 0	$3\pi/4$
1 1	$-3\pi/4$

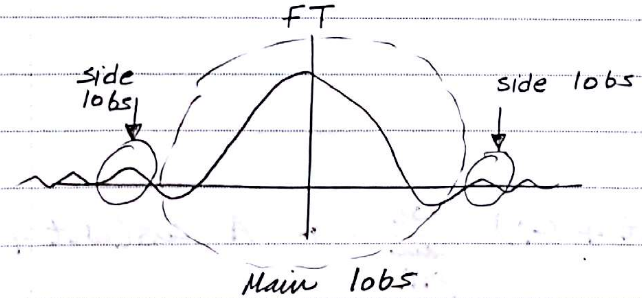


$\pi/4$ - QDPSK
Best BW

$d_{min} = \sqrt{2}\,R$

∘ $\Delta\theta$ ↑ , side lobs ↑

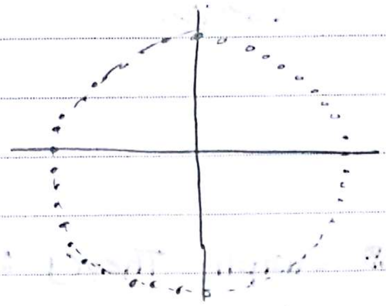




• IS 54 : American cellular standard .
/ 95 .

Ex. 3 :

d	Δθ
0	0.1π
1	-0.1π



20-points ← $\frac{2\pi}{0.1\pi}$

change Δθ

d	Δθ
0	$\frac{47}{197}\pi$
1	$\frac{-17}{37}\pi$

\# of points : ∞ phases

$d_{min} = \min(\Delta(\Delta\theta))$

* min. of the min.

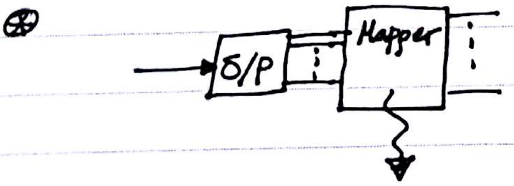
Continuous phase :

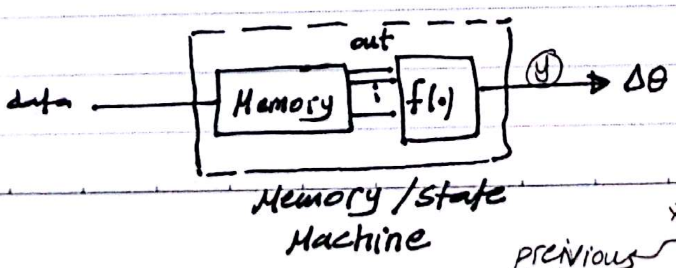
$\boxed{\text{CPFSK}}$ } DPSK

or multi-h PSK

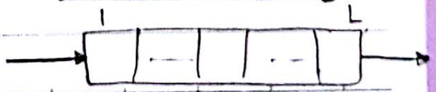
Mid

• all types before are partial response signaling.





Any memory machine have serial shift reg.



\# of states = 2^L

previous data ≡ memory content.

length-L

⊛ Memory machine (device) = have memory stores the prv. data and fun. operates on the prv. and current data

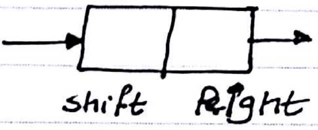
⊛ Graph Theory :

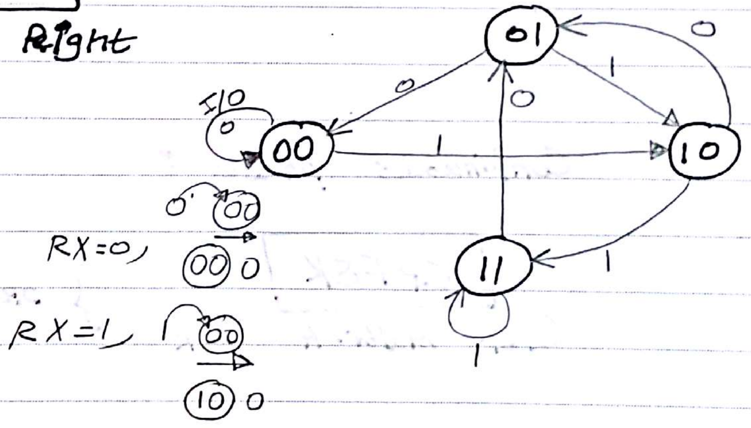
State → Node

Transition → arrow

State diagram

—— -





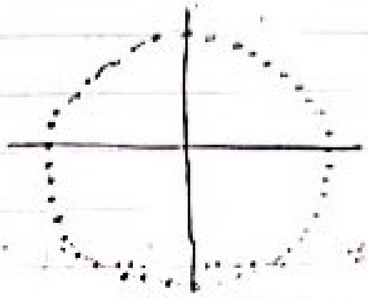
* Maison's Rule

No.

Ex. 3 :

d	$\Delta\theta$
0	0.1π
1	-0.1π

change $\Delta\theta$



20 - points ← $\frac{2\pi}{0.1\pi}$

d	$\Delta\theta$
0	$\frac{47}{197}\pi$
1	$\frac{-17}{37}\pi$

$d_{min} = \min(\Delta(\Delta\theta))$

← min. of the min.

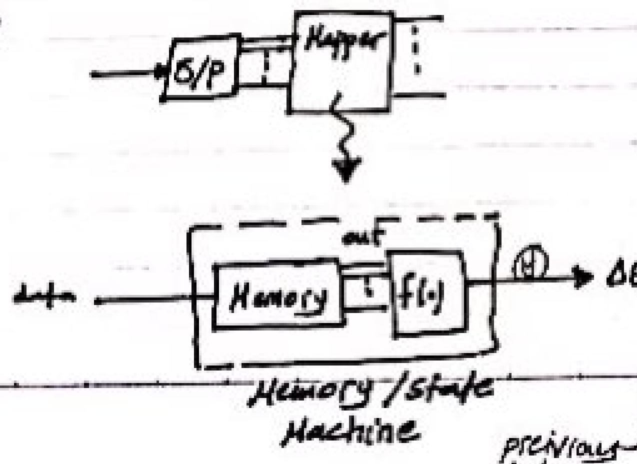
# of points ≈ ∞ phases

Continuous phase :

$\boxed{\text{CPFSK}}$ or multi-h PSK } DRSK

Mid

• all types before are partial response signaling .



Any memory machine have serial shift reg.

# of states = 2^L smile...

previous data ≡ memory content .

⊛ Memory machine : have memory stores the prv. data and fun. operates on the prv. and current data
(device)

length-L

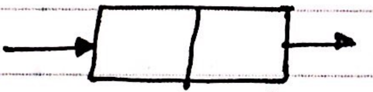
⊛ Graph Theory :

State diagram

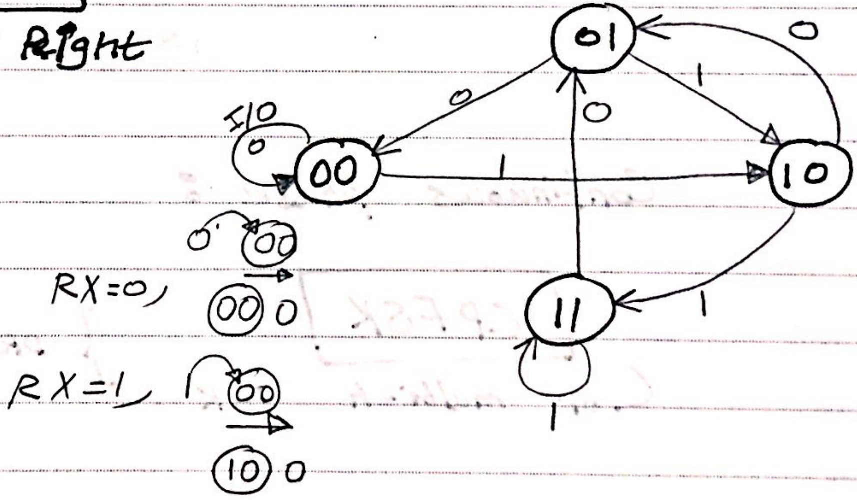
state → Node

Transition → arrow

— -



shift Right



* Maison's Rule

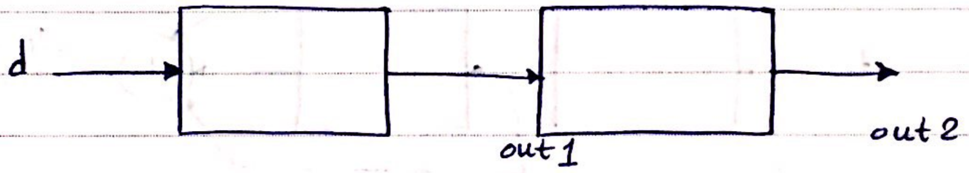
No. lec. 29/3/2017

• State / Memory machine :

Serial shift reg. → state ≡ content of memory cells

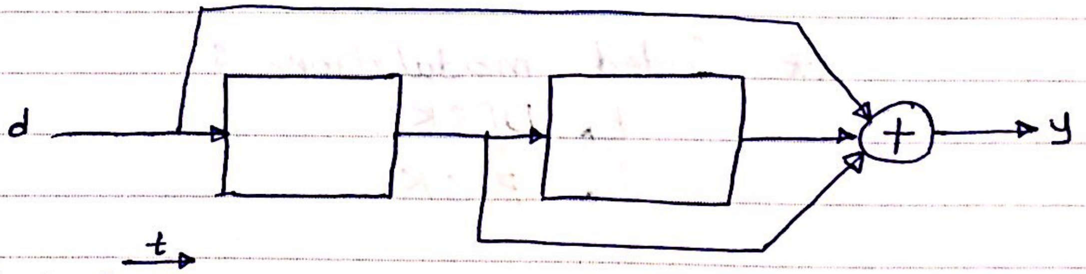
output = g (input , state)

↑ Linear Function (Reversable)



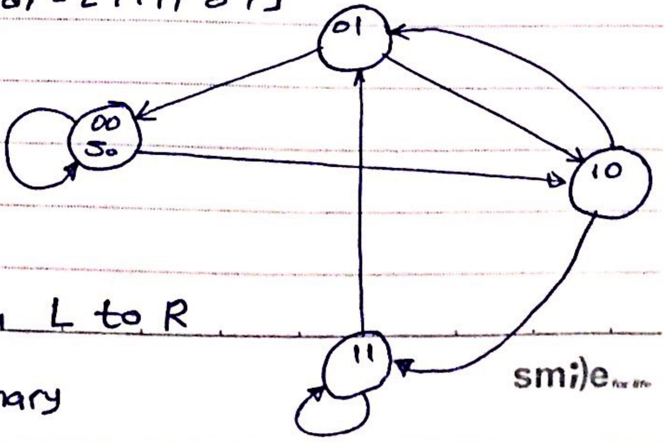
Non-linear : we can't reverse the output to get the input

we can make more than one fun. in the same machine



1) d = [1 0 1 1 0 1] , 2) d_1 = [1 1 1 1 0 1]

→ state diagram :



current state → next state

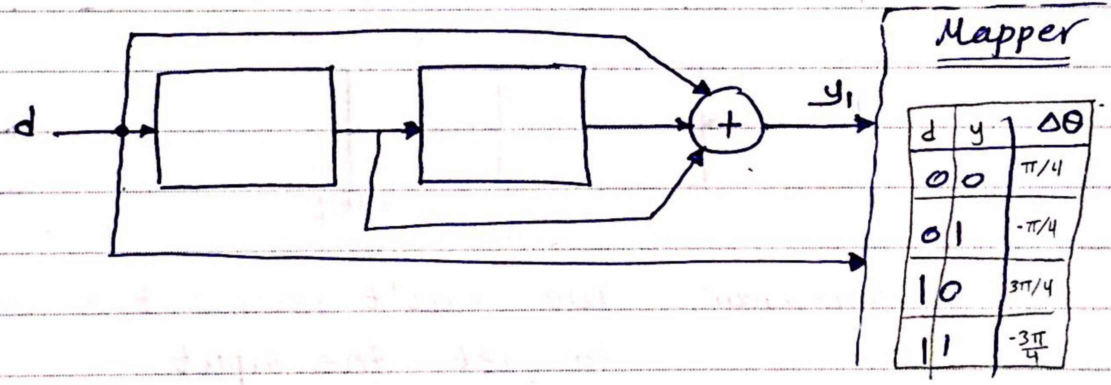
** Time seq. goes from L to R

Read as binary digits ** No significant for binary

** State / Memory machine is a partial response signaling

** Correlation is independent in the output of the state machine through the fun. $\underline{\underline{g}}$

we send data d with extra data y, y used for check if the out is true or not.



d	y	$\Delta\theta$
0	0	$\pi/4$
0	1	$-\pi/4$
1	0	$3\pi/4$
1	1	$-\frac{3\pi}{4}$

** Coded modulation :

- ↳ DPSK
- ↳ CPFSK

FSK ↓ least energy

PSK low cost

Review: Any PSK modulation tx signal:

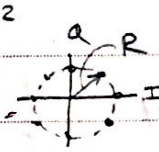
$$\phi(t) = A\, p(t)\, \cos(\omega_c t + \theta_i)$$

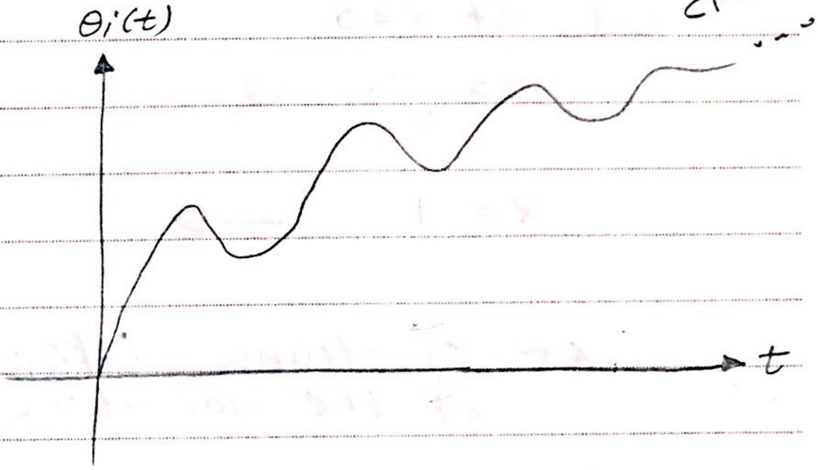
OR

$$\phi(t) = I\, p(t)\, \cos(\omega_c t + \theta_i) + Q\, p(t)\, \sin(\omega_c t + \theta_i)$$

$$I^2 + Q^2 = R^2$$

R → radius of the circle





Ex: Coded modulation, bandpass Tx, BW=10KHz @ 1 MHz carrier. Need to tx. 1 Mbps

Use $\boxed{\text{Set}}$.

Sol., we need to find m:

$$BW = \frac{r_b}{m}(1+\alpha)$$

Set $\alpha = 0$ ← DON'T USE ASSUME

$$m * 10K = 1M$$

$$m = \frac{1M}{10K} = 100 \text{ bits}$$

→ Set $\alpha = 0.1$

m = 110 bits

Cont. Ex →

cont. Ex : BW = 10KHz , @ 1 MHz carrier
need to tx. 30 Kbps

Sol., set $\alpha = 0$

$m = \frac{30K}{10K} = 3$

$\alpha = -1 \longrightarrow$

take $m = 4$

$\alpha = \frac{1}{3} = 0.333$

** spectrum efficiency of the modulation $\boxed{\eta}$, unit : b/s/Hz

η → represent BW in comm. sys. triangle.

η → critical

→ To compare 2 modulation type we use η

* Direct Sequence spread spectrum :

$a_x(t) = C_x(t)$, $a_y(t) = C_y(t)$, $a_x \perp a_y$

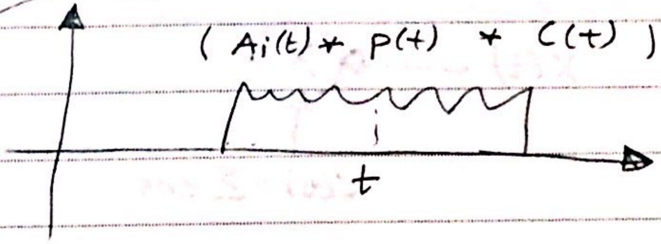
$\phi_1(t) = I_i \, p(t) \, C_x(t) + Q_i \, p(t) \, C_y(t)$

$\phi(t) = \phi_1(t) \cdot \cos(\omega_c t)$

Ex : $ax(t) = C(t)$

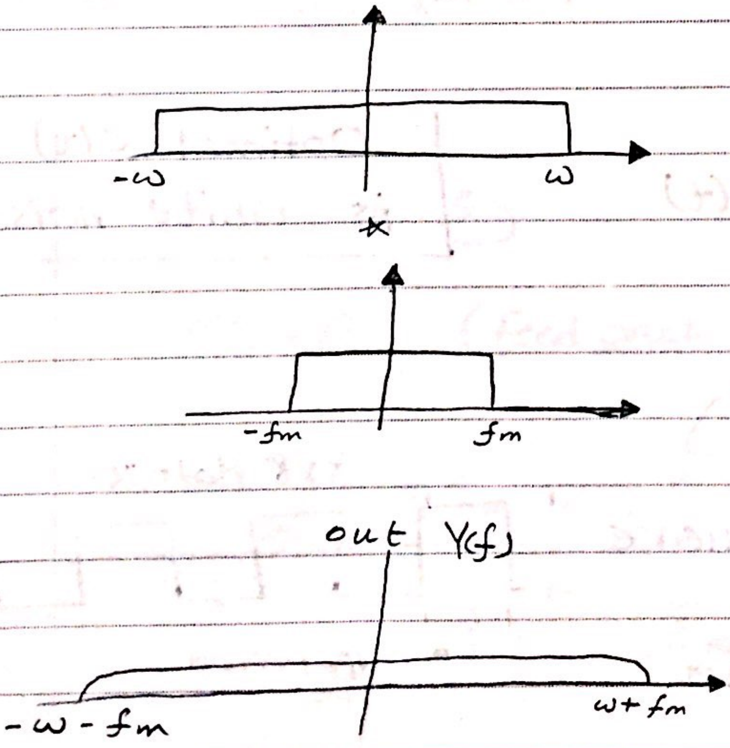
$\phi_1(t) = A_i\ C(t)\ \cancel{p(t)}$

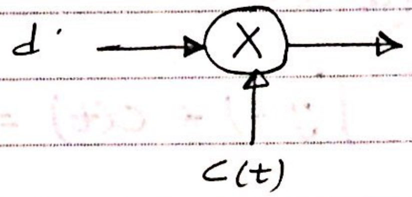
$\phi(t) = A_i\ C(t)\ \cancel{p(t)}\ \cos(\omega_c t)$



Cont. • Direct seq. spread spectrum : (DS.SS)

if $BW_{c(t)} \gg f_b \longrightarrow$ DS.SS

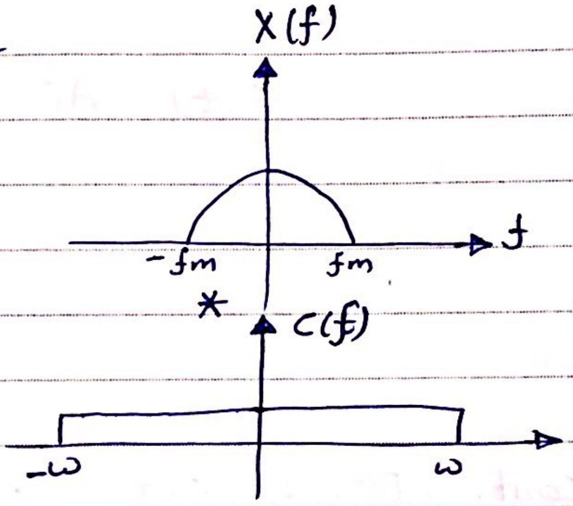


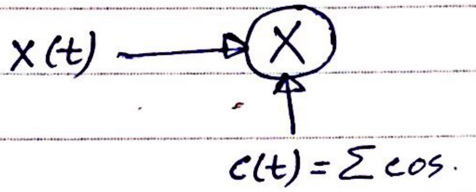


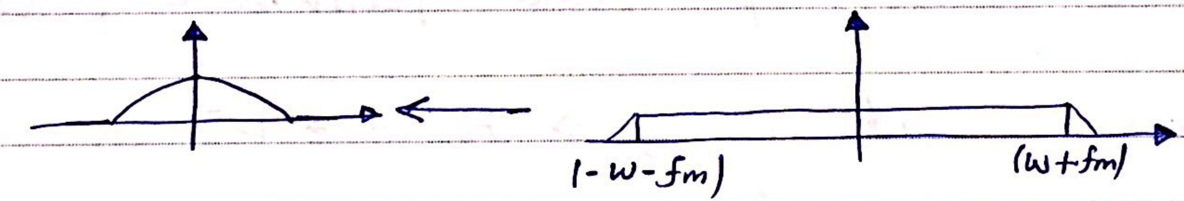
No. lec. 3/4/2017

• Spread spectrum :

$x(t) \longrightarrow BW = f_m$







@ Rx

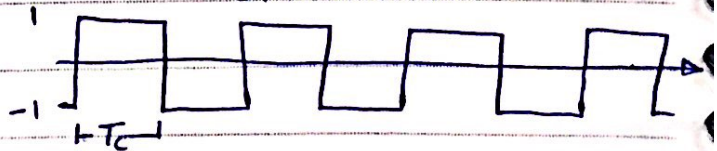
$\int y(t) * c(t) = x(t)$

$$\boxed{\text{Optimal } c(t) \text{ is white noise}}$$

PN sequences :

1- Hadmard sequence

8x8 Matrix.



→ used in civilian application.

✓ 2- m-sequence (Maximal length sequence)
between civilium and military

3- Gold codes (security)

→ How to generate random number between 0 & 1:

1) take 2 primary number

2) rem$\left(\frac{37}{17}\right)$ = 0.17447...
reminder

3) from the ans. take another 2 primary number

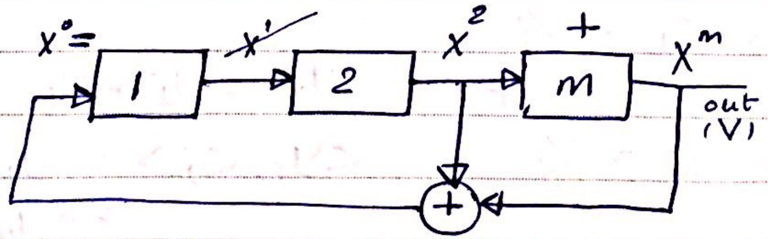
4) rem$\left(\frac{47}{17}\right)$ = 0.----

⋮

- m-sequence:

FBSR (Feed Back Shift Register)

we have
m-reg.

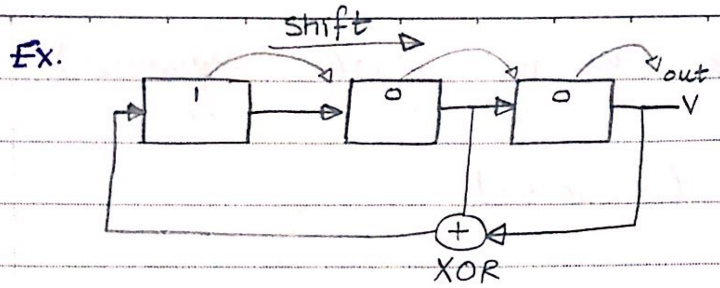


راح نوخذ الإتجاه
من اليسار إلى اليمين

x^1
مش موجوده
لأنه ما في
أي عملية
على الخط
إلي بعد
الـ reg1

$$x^0 = x^2 + x^3$$ if $m = 3$

Ex.



بيرجع يتكرر

$\boxed{m=3}$

m_1	m_2	m_3	V
1	0	0	0
0	1	0	0
1	0	1	1
1	1	0	0
1	1	1	1
0	1	1	1
0	0	1	1
1	0	0	0

بنوقف هون ما في داعي نكمل →

$$V = m\text{-seq.} = \underset{\longleftrightarrow N}{[0010111]}$$

length of m-seq. $\boxed{N = 2^m - 1}$

$$N = 2^3 - 1 = 7$$

shift V1

$$V_1 = [0010111]$$

$$V_2 = [1001011]$$

$$\sum V_1 \cdot V_2 = -1$$

$$\frac{\langle V_i , V_j \rangle}{N} = \frac{-1}{N}$$

m for GPS

$N = 10M * 365 * 24 * 60 * 60$

$\log_2 N = m$

* security ≡ size

* $X_j \times C_j = Y_j$

$+$

$X_K \times C_K = Y_K$

$y_r = y_j + y_K$

$\langle y_r , C_j \rangle = \langle y_j , C_j \rangle + \langle y_K , C_j \rangle$

$$y_r = \sum_{n=1}^{N} y_n \leftarrow \text{CDMA}$$

$$= X_j + \underbrace{\left(\frac{-1}{N} X_K\right)}_{MAI}$$

→ MAI : Multiple access interference

→ CDMA : code divison multiple axis -

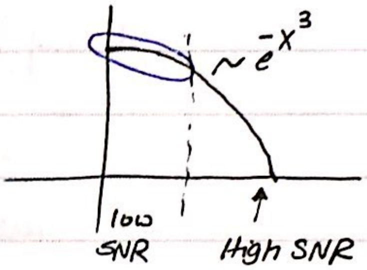
Review : PN seq. : - Hadmard
- m-seq.
- Gold code

$$y_{total} = \sum_{i=1}^{N} y_i + noise$$

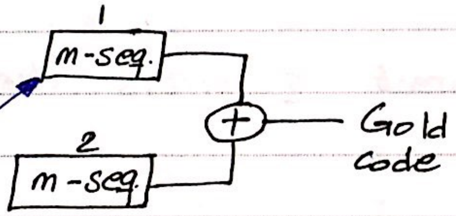
$$< y_{total} , C_K > = y_K + \underline{\underline{MAI}}_{\simeq \text{ Gaussian}}$$

without fading

→ Gold code :



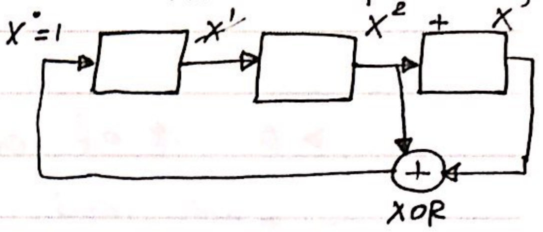
'BER'



$\underline{1 + x^2 + x^3} = g_1(x)$

receprocal function of $g_1(x)$

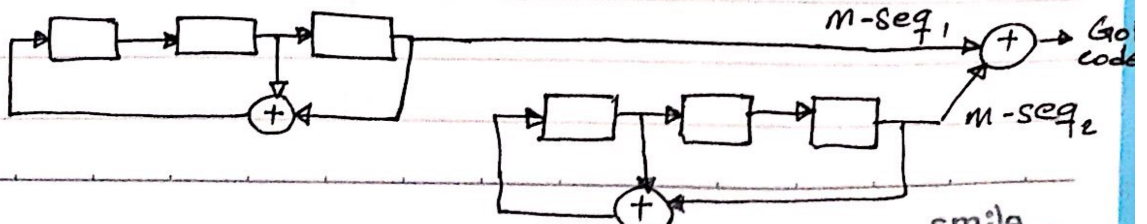
Generate m-seq :



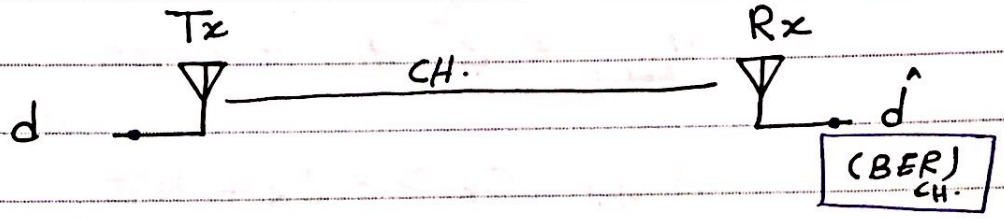
$g_2(x) = x^m \cdot g(x^{-1})$
$= x^3 (1 + x^{-2} + x^{-3})$
$= x^3 + x + 1$

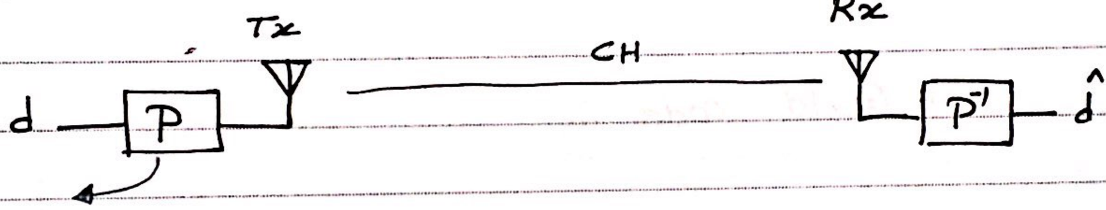
XOR last 2 bit

length of Gold code $\boxed{N_T = (2^m - 1)^2}$



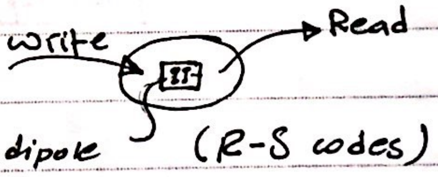
• Coding Theory :





* If we can correct $\underline{\underline{t}}$ errors then BER_{coded} : $\simeq (BER_{CH.})^{t+1}$

* Hard disk : fading channel.



→ t : # of errors we can correct.

→ we need to increase $\underline{\underline{t}}$

** Code discovered not derive.

* fields :

label (colors, number, ... etc)

$\mathcal{C} = \{0, 1, \alpha's\}$

binary يعني طرفين أو ثنائي مش نظام ال ١ وال ٠

→ with two binary operators :

+ Addition

* Multiplication

→ $\mathcal{C} = \{a, b, c, d\}$

$a + b = c$

$a * b = d$

identity for + → ZERO

" " * → One

* Binary Field :- (GF (2)) , Gola field

Gallelio

$\mathcal{C} = \{0, 1\}$

Addition

+	0	1
0	0	1
1	1	0

XOR

Multiplication

*	0	1
0	0	0
1	0	1

AND

→ GF (10) , Decimal

→ Extended field : GF(2^m)

$\boxed{N = 2^m - 1}$ non zero element.

?. → if $m = 3$ then we have 3-bit field.

Two forms to represent extended field

- vector form :

$V_1 = [1\ 0\ 1]$

$V_2 = [1\,1\,0\,0\,1\,1\,0\,1]$

- polynomial form :

$V_1 = 1 + 0 \cdot x^1 + 1 \cdot x^2$

$V_1 = 1 + x^2$

$V_2 = 1 + x + x^4 + x^5 + x^7$

→ if we add n zeros @ Right side of V_1

$V_1 = [1\ 0\ 1\ \ 0\ 0\ 0\ 0\ 0]$

V_1 in polynomial the same as $[1\ 0\ 1]$

* Any field is equv. to a binary extended field.

Any " " " " any field

* We don't have subtraction in this field because we use +ve numbers

No.

Operations :

1- Addition : (without carry)

Ex : $V_1 = [1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0]$ + In vector form

$V_2 = [1\ 1\ 0\ 0\ 1\ 1\ 0\ 1]$

$V_3 = [0\ 1\ 1\ 0\ 1\ 1\ 0\ 1]$

the same result

$V_1 = 1 + X^2$ in polynomial form

$V_2 = 1 + X + X^4 + X^5 + X^7$

$V_3 = X + X^2 + X^4 + X^5 + X^7$

2- Multiplication :

Ex : $V_1 = 1 + X^2$ *

$V_2 = 1 + X + X^4 + X^5 + X^7$

المتشابهات تنعدم

$V_3 = 1 + X + X^4 + X^5 + \cancel{X^7} + X^2 + X^3 + X^6 + \cancel{X^7} + X^9$

$V_3 = 1 + X + X^2 + X^3 + X^4 + X^5 + X^6 + X^9$

3- Division : from last Ex : $\frac{V_3}{V_2} = V_1$

$$\begin{array}{r|l} & X^2 + 1 \\ \hline 1 + X + X^4 + X^5 + X^7 & X^9 + X^6 + X^5 + X^4 + X^3 + X^2 + X + 1 \\ & X^9 + X^7 + X^6 + X^3 + X^2 \\ \hline & X^7 + X^5 + X^4 + X + 1 \\ & X^7 + X^5 + X^4 + X + 1 \\ \hline & 0 \end{array}$$

NO SUBTRACTION

No.

* Chinese remainder theorem :)

for a prime polynomial g(X) with order $\underline{\underline{m}}$

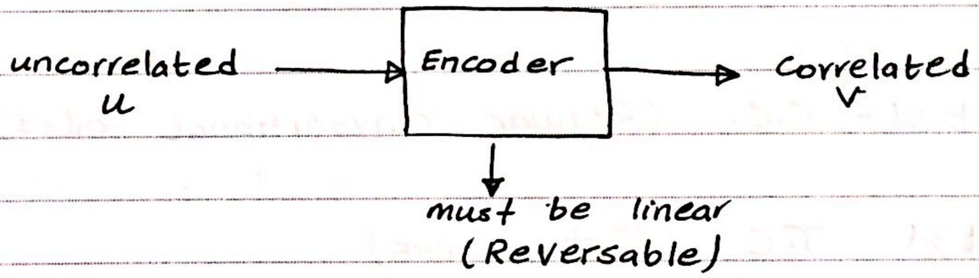
No polynomial with order less than N divides g(X)

No. Lec. 10/4/2017.

* Binary field Arithmetic :

$+$ Addition
$*$ Multiplication
$\div$ Division

* Coding Theory :



* Error correction codes :
- Error detection codes
- " correction codes

* Code :

1- LBC (linear block codes)

2- C.C (convolution code)

1- LBC :

t=1 - Hamming code

t=1 - Cyclic codes

t≥1 - BCH codes → G.C : Golle codes
→ Reed - Solomon code

t≥1 - LDPC codes (low density parity check)

Best performence

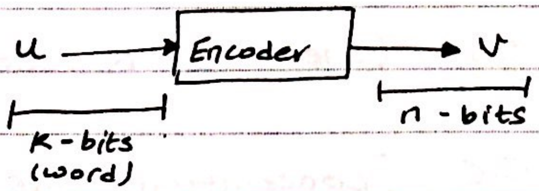
2- C.C :

t≥1 - C.C. (Regular convolutional codes)

t≥1 - T.C (Turbo codes)

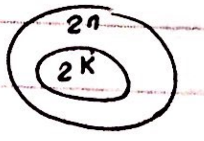
* TCM : Trillis coded modulation
→ C.C + modulation

** LBC :-



2^K possible inputs 2^K code words

→ LBC is to select 2^K code words from possible 2^n combinations, such that all 2^K code words have max. difference (hamming distance).



LBC → Hamming Distance :

Max. distance between 2 codes.

$d^H = 2$

$$\begin{array}{r} 10\ 11\ 01 \\ 11\ 01\ 01 \\ \hline 0\underline{1}\ \underline{1}0\ 00 \end{array} \oplus$$

i.e. Hamming distance = $\underbrace{\text{weight}}_{\text{sum.}} (V_1 \underset{XOR}{\oplus} V_2)$

Ex : $n=3$, $K=2$

u	v
0 0	0 0 0
0 1	0 0 1
1 0	0 1 0
1 1	0 1 1
	1 0 0
	1 0 1
	1 1 0
	1 1 1

→ To represent codes we need min. of n-dimentional space

Ex: $K=1$, $n=3$

choose any 2 codes have max. distance

u

0

1

v

$\boxed{000}$

001

010

011

100

101

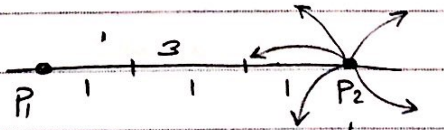
110

$\boxed{111}$

$$\begin{array}{r} 000 \\ 111 \\ \hline 111 \end{array} \oplus$$

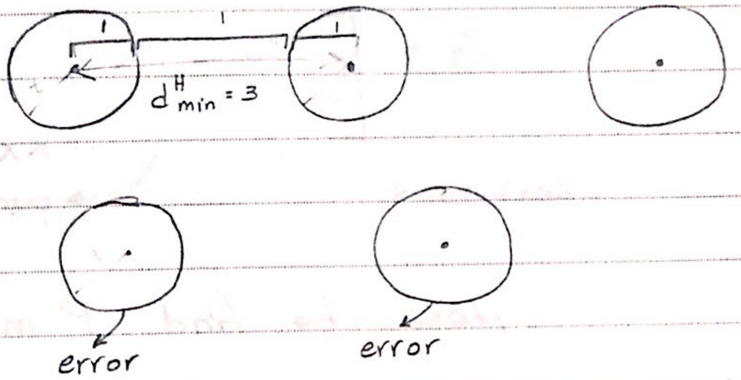
$$d^H = 3$$

Higher BW $\boxed{r_b' = \underbrace{\left(\frac{n}{K}\right)}_{\text{BW expantion factor}} r_b}$, $\frac{K}{n}$: Code Rate



IF only one error occures every 3-bits through the channel, we are 100% sure we can fix it.

- If we want to correct one error, we need a d^H_{min} of $\underline{\underline{3}}$. 1 unit for each error & still 1 unit distance = 1 + 1 + 1



(sphere packing problem)

$$t = \left\lfloor \frac{d^H_{min} - 1}{2} \right\rfloor_{floor}$$

* To design a good code we need min. BW expantion factor (max. code rate) with max. $\underline{\underline{d^H_{min}}}$

* Hamming code :-

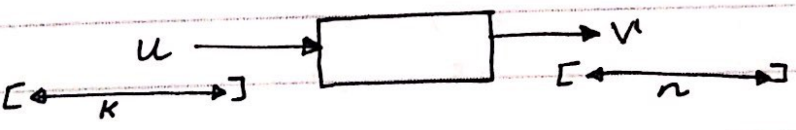
min. max problem :>
Mod ← min. energy , max. distance
coding ← min n , max. d^H_{min}

$K = 4$, $n = 7$

$d^H_{min} = 3 \longrightarrow t = 1$

$n = 2^{\overset{\text{\# of digits}}{m}} - 1$

$n - \underset{t=1}{t}m = K$



$$\boxed{\vec{v} = \vec{u}\ \underbrace{G}_{\text{binary matrix}}}$$

No.

$G = \left[\underset{\text{Identity}}{I} \;\vdots\; \underset{\text{parity}}{P} \right]_{K \times n}$

How to find P matrix :-

Ex: $n = 7$, $m = 3$ $GF(2^m) \Rightarrow g(x) = 1 + x + x^3$

	X_n	Rem.		
1	0	0	0	0
2	1	1	0	0
	x	0	1	0
	x^2	0	0	1
	x^3	1	1	0
	x^4	0	1	1
	x^5	1	1	1
n	x^6	1	0	1

Rows 1, x, x^2: Identity matrix (NOT for G)

Rows x^3 to x^6: → P

For $m = 3 \rightarrow n = 7$

$x^n + 1 = (\ )(\ )..$

$$\rightarrow x^7 + 1 = (1 + x)\underbrace{(1 + x + x^3)(1 + x^2 + x^3)}_{\text{prime polynomials.}}$$

$\frac{x^5}{g(x)} \rightarrow$

$$\begin{array}{r|l} & x^2 + 1 \\ \hline 1 + x + x^3 & x^5 \\ & x^5 + x^3 + x^2 \\ \hline & x^3 + x^2 \\ & x^3 + x + 1 \\ \hline & x^2 + x + 1 \end{array}$$

Rem → باقي قسمة X_n على $g(x)$

$$G \equiv \text{Generator matrix} = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

No. lec. 12/4/2017

Quiz : Derive the P matrix using $g(x) = 1 + x^2 + x^3$.

Sol.) -

other method for division.

$g(x) = 0$

$1 + x^2 = x^3$

$x + x^3 = x^4$

$1 + x + x^2 = x^4$

$x + x^2 + x^3 = x^5$

$1 + x = x^5$

No.

$$G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]_{k \times n}$$

$\vec{u} = [1\ 0\ 1\ 1]$

$\vec{v} = \vec{u}\, G = [\underbrace{1\ 0\ 1\ 1}_{u}\ \ 0\ 0\ 0]$

@ RX : $H = [I \mid P^T]_{n-k \times n}$

$$H = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

$\underbrace{\quad\quad\quad\quad}_{P^T}$

→ at RX we need to find syndrom :

$$\boxed{s = v H^T}$$

If and only if $s = [0\ 0\ 0]$ → No errors.

if $s \neq 0$ → errors exist. → position when s^T = col. of H.

smile for life

to find S :

$S = V H^T$

$$H = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

$V = [1\ 0\ 1\ 1\ 0\ 0\ 0]$

if error happens in bit 4 :

$$\vec{v} = [1\ 0\ \underset{\uparrow}{1}\ 1\ 0\ 0\ 0]$$

$e = [0001\ 000]$, error in position $\underline{4}$

from H take col. number 4

$$\begin{array}{ccc|cccc} & & & & & \text{col 6} & \\ & & & & & \downarrow & \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ & & & \uparrow & & & \\ & & & \text{position 4} = \text{col 4} & & & \end{array}$$

$s = [1\ 0\ 1]$

$\hat{u} = [1\ 1\ 0\ \underline{\underline{1}}]$

if $e = [0000\ 010]$, error in position $\underline{\underline{6}}$
$(e(x) = x^3)$

$s = [1\ 1\ 0]$

$\hat{u} = [\qquad\qquad]$

$\hat{v} = [1\ 0\ 1\ 1\ 0\ 1\ 0]$

* Detect $d_{min}^{H} - 1$ errors :

(n, k)

$g(x)$ is a prime in $GF(2^m)$
with order $\underline{\underline{m}}$

* prime polynomial in a field can spaned the whole field.

$(X^n+1) \rightarrow (X^7+1) = (1+x)(1+x+x^3)(1+x^2+x^3)$

* To select g(x) from tables using value of m (M≡order of g(x))

Ex: $m=6$

1 $(0,1,6) = 1+X+X^6$ order 6 ✓

9 $(0,2,3) = 1+X^2+X^3$ order 3 X

7 $(0,3,6) = 1+X^3+X^6$ order 6 ✓

$\phi_i(x) = \phi_{i.2^l}(X)$, l : integer

$1+X+X^m$

OR except $M=5,8,10$

$1+X^{m-1}+X^m$

No.

@RX : if $m = 9 \rightarrow n = 511 \rightarrow K = 502$

$$\vec{v} = \vec{u}\,G$$

$$\text{Code rate} = \frac{K}{n} = \frac{502}{511}$$

No. lec. 17/4/2017

Quiz : For the shown code if the data is $u = [1\ 0\ 0\ 0]$.

$$G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

1- find the coded data

2- If an error in the 4th position of the coded data, show how to correct it.

Sol.

- Cyclic codes :-

data $\rightarrow u(X)$.

size of $v(X)$ = n-bits.

$$u(X) * g(X) = v(X)$$

OR $$\frac{V(X)}{g(X)} = u(X)$$

- $g(X)$ is prime factor of (X^n+1)

$$\begin{cases} n = 2^m - 1 \\ n - K = m \end{cases}$$

- $\frac{X^i}{g(X)} = a(X) + \frac{r(X)}{g(X)}$, $r(X) \neq 0$, $i < n$

$$GF(2^m) \begin{cases} 0 & \longrightarrow \text{no errors} \\ X^0 = 1 & \longrightarrow e(X) = 1 \\ X^1 & \longrightarrow e(X) = X \\ X^2 & \\ \vdots & \quad \vdots \\ X^{n-1} & \longrightarrow e(X) = X^{n-1} \end{cases}$$

$$\left(\hat{V}(X) = V(X) + e(X) \right) \div g(X)$$

$$\boxed{\frac{\hat{V}(X)}{g(X)} = u(X) + a(X) + \frac{r(X)}{g(X)}}$$

Ex : $m=3$, $g(x)=1+x+x^3$

(syndrom table)

$e(x)$	$a(x)$	$r(x)$
1	0	1
x	0	x
x^2	0	x^2
x^3	1	$1+x$
x^4	x	$x+x^2$
x^5	$1+x^2$	$1+x+x^2$
x^6	$1+x+x^2$	$1+x^2$

$$\frac{e(x)}{g(x)} \rightarrow \underset{1}{a(x)} + \underset{2}{r(x)}$$

$$\boxed{x} \rightarrow 1 \equiv a(x)$$

$$1+x+x^3 \;\big)\; x^4$$

$$x^4+x^2+x$$

$$\boxed{x^2+x} \rightarrow 2 \equiv r(x)$$

@ RX : Decoding

find $\frac{\hat{v}(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$

if $r(x)=\Phi \rightarrow u(x)=q(x)$

if $r(x) \neq \Phi \rightarrow$ from syndrom table

find $a(x)$

$u(x)=q(x)+a(x)$

for $u(x)=1+x$

$v(x) = 1+x+x^3+x+x^2+x^4 = 1+x^2+x^3+x^4$

in ch. $e(x)=x^3$ error in position 3

$\hat{v}(x) = 1+x^2+x^4 \Rightarrow [1\,0\,1\,0\,1\,0\,0]$

$$\begin{array}{r|l} & x \longrightarrow q(x) \\ \hline 1+x+x^3 & x^4+x^2+1 \\ & x^4+x^2+x \\ \hline & 1+x \longrightarrow r(x) \end{array}$$

from table $r(x)=1+x$

$\rightarrow a(x)=1$

$\rightarrow u(x)=q(x)+a(x)$

$= x+1$

if $m=5$

$n=31$

$K=26$

cyclic codes

$\underline{\underline{t=1}}$

* Ex: if $BER_{CH}=0.1$

we need $BER=10^{-4}$

$10^{-4}=(10^{-1})^{t+1}$

$\underline{\underline{t=3}}$, every $\frac{1}{BER_{CH.}} = 10$ bits.

Ex: $BER_{CH.}=10^{-2}$ = 1 error in 100 bits

we need 10^{-8} $\boxed{3}$ errors can be corrected in 100 bits

$\underline{\underline{t=3}}$

we divide 100 to 3 packets

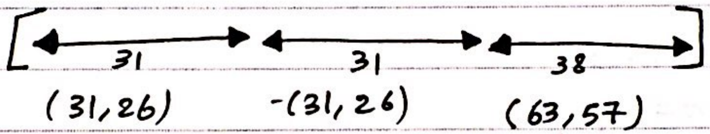
$$\left[\underset{31}{\overset{1}{\longleftrightarrow}} + \underset{31}{\overset{1}{\longleftrightarrow}} + \underset{38}{\overset{1}{\longleftrightarrow}}\right]$$

(31, 26) (31, 26) (63, 57)

→ shortened codes :-

any (n, K) code can be shortened by l to $(n-l , K-l)$

* Shortened code ①



(63,57) → using shortened code → (38,32)

code rate in shortened code less than code rate without using shortened code.

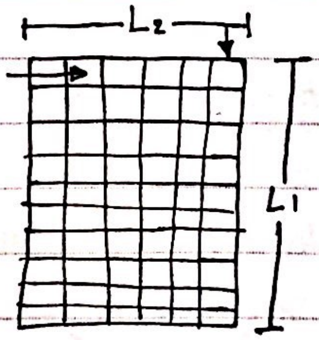
$$\frac{K}{n} > \frac{K-l}{n-l}$$

Scramble ② the coded packet (طريقة تهدف الحماية)

* Interleaving :

convert burrest of errors into random errors

write rows.
read cols.



write in col.
read rows

الـ error مجمع في مكان واحد

* fading ch. produce burrest of errors therefore interleaving must be used

* Error correcting code are better in correcting random errors

- BCH codes :

$$g(x) = LCM\ (g_i(x),\ i = 1, 2, \dots, 2t)$$

$$\boxed{\begin{matrix} n = 2^m - 1 \\ n - K = mt \end{matrix}}$$

Ex : $t=3$, $m=7$

find $g(x)$ from table.

1 (0, 3, 7)
2 (0, 1, 2, 3, 7)
5 (0, 2, 3, 4, 7)

$$g(x) = (1+x^3+x^7) * (1+x+x^2+x^3+x^7) * (1+x^2+x^3+x^4+x^7)$$

CRC (error detection) No. lec. 19/4/2017

- CRC: (cyclic redandency check):

Error detection codes

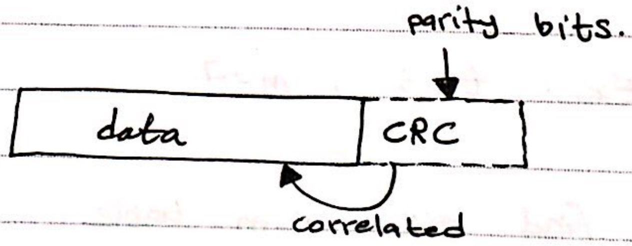
syndrum: يدل على
مكان وجود
ال error

→ IP protocols: - TCP -UDP

→ if we use UDP protocol (streaming) to transmit data (audio, video) and we have missing part of the data. To correct missing data we use: (Interpolation.)

→

parity bits.



LBC

t-errors

$$\underset{BCH}{g}(x) = LCM(\Phi_i ; i=1,2,\cdots 2t)$$

$$\Phi_i = \Phi_{i \cdot 2^{L}}$$

No.

BCH: let m = size of packet

conditions of packet.

- $n = 2^m - 1$, $d_{min.} = 2t+1$
- $n - K = t * m$

Ex: $m = 4$, let $t = 2$

from table :- 1 (0, 1, 4)   3 (0, 1, 2, 3, 4)

$$\underset{BCH}{g(X)} = (1+X+X^4)(1+X+X^2+X^3+X^4)$$

$$= 1+\cancel{X}+\cancel{X^2}+\cancel{X^3}+\cancel{X^4}+\cancel{X}+\cancel{X^2}+\cancel{X^3}+\cancel{X^4}+X^5+X^4+X^6+X^6+X^7$$
$$+X^8$$

$$= 1+X^4+X^6+X^7+X^8$$

$$\boxed{d^H_{min} = 5}$$

detect $d^H_{min} - 1 = \underline{\underline{4}}$ errors every $n = 15$ bits.

$$\underset{CRC}{g(X)} = (1+X)(\underset{BCH}{g(X)})$$, $1+X$: even , prime $= \underline{\underline{2}}$

$$= (1+X)(1+X^4+X^6+X^7+X^8)$$

$$= 1+X^4+X^6+\cancel{X^7}+\cancel{X^8}+X+X^5+\cancel{X^7}+\cancel{X^8}+X^9$$

detect $d^H_{min} \Rightarrow$ 5 errors.

@Tx : Sending packet

- number of bits = K
- max. order = K-1

u(X) → data

$$\frac{u(X)}{g_{CRC}(X)} = a(X) + \frac{r(X)}{g_{CRC}(X)}$$

order of rem. = mt

r(X)

$$[\longleftrightarrow]$$

\# of bits mt + 1

$$\underbrace{V}_{n+1\ \text{bits}} = [\ \overset{\text{data}}{\underbrace{u}_{K\ \text{bits} = n-mt}}\ /\ \overset{\text{CRC}}{\underbrace{r}_{mt+1\ \text{bits}}}\ ]$$

@RX : received packet contain u(X) & r(X).

$$\frac{\hat{u}(X)}{g_{CRC}(X)} = \hat{a}(X) + \frac{\hat{r}(X)}{g_{CRC}(X)}$$

Same Rem.

if $\hat{r}(X) = r(X)$ → No errors

else → errors exist.

* Some of $\overset{g(x)_{CRC}}{\underline{GCRC}}$ generated polynomial are better than others in extra features:

→ extra features:

Therefore we find those good CRC polynomial as standard polynomial.

Ex: $g(x)_{CRC} = (1+x)(1+x+x^7)$ ← given this $g(x)_{CRC}$

$= 1+x+x^7+x+x^2+x^8$

$= 1+x^2+x^7+x^{\boxed{8}} \longrightarrow mt+1$

can be used to detect 3 bit from 128 bit $\left\{ \begin{array}{l} n = 2^m - 1 \\ n-K = mt \end{array} \right.$ $\longrightarrow m = 7, t = 1, \overset{H}{d_{min}} = 3$

$mt+1 = 8$

$mt = 7$

$\underset{m}{7} \cdot \underset{t}{1} = 7$

$n = 2^7 - 1 = 127$

$K = 120$ bits

$V = [120 ; 8] \Rightarrow 128$ bits

- we can use CRC generater polynomial for the data length of $\underline{\underline{K}}$ and below

- This

Ex : $L_p = 244$ bits (packet length)

Find the $\underset{CRC}{g(x)}$ that detect 3 errors.

$$244 \overset{?}{\leq} n+1$$

$$n = 255 = 2^m - 1 \longrightarrow m = 8$$

$$\underset{CRC}{g(x)} = (1+x)\, g(x)$$

$$g(x) = (0,1,3,7,8) = 1 + x + x^3 + x^7 + x^8$$

$$d^H_{min} = 3 \longrightarrow t = 1$$

$K = 255 - 8 = 247$ bits

$L_p = 244$

$$V = [\overset{data}{235} \,\vdots\, \overset{CRC}{9 \text{ bits}}]$$

Ex : $L_p = 244$ bits

errors = 5

$$2t + 1 = 5 \longrightarrow t = 2$$

$$2^m - 1 = 255 \longrightarrow m = 8$$

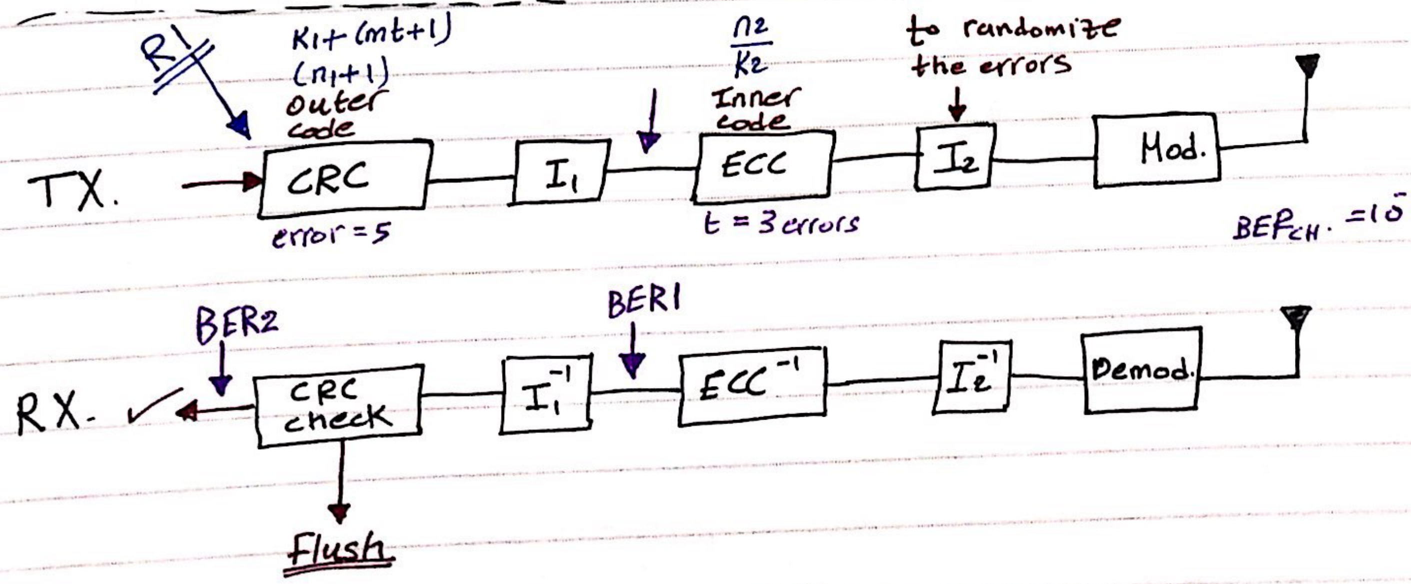
$$n - K = mt = 16$$

$K = 239$ bits <u>max.</u>

$$\underset{CRC}{g(x)} = (1+x)\,[\,(0,1,3,7,8)\,(0,1,3,4,8)\,]$$

$$= (1+X)(1+X+X^3+X^7+X^8)(1+X+X^3+X^4+X^8)$$

$$V = [\,\overset{244-17}{227} \;\vdots\; 17\,]$$



Ex : ↑ $BER_2 = (BER_1)^{5+1} = (10^{-8})^6 = 10^{-48} \cong 0$

$BER_1 = (BER_{CH.})^{t+1} = (10^{-2})^4 = 10^{-8}$

No.

R1 : $BW = \frac{K + mt + 1}{K} = \left(1 + \frac{mt+1}{K}\right)$

$BW_{expansion} = \left(1 + \frac{mt+1}{K}\right) \cdot \left(\frac{n2}{K_2}\right)$

lec. 24/4/2017

Q : a packet of length 220 bits, design the CRC code to detect 7 errors.
"find the length of parity bits & data bits"

Sol.

$m = 8$

$$\underset{CRC}{g(x)} = \overset{1}{(1+x)}\ \overset{8}{g_1}\ \overset{8}{g_3}\ \overset{8}{g_5} \longrightarrow \text{parity}$$

parity bits = 25

data bits = 195 → 220 − 25

- LDPC codes :
(low density parity check), not used in hi-speed data.

$$v = uG \qquad G = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & P \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & \end{array}\right]$$

@ RX : $s = vH^T$

G not square matrix, to find inverse of the G matrix :

$$\boxed{G^{-1} = [G \cdot G^H]^{-1} \cdot G^H}$$

→ If G not full rank then we add rows and cols to make it full rank (we add dual space H) if $G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}_{3\times 8}$

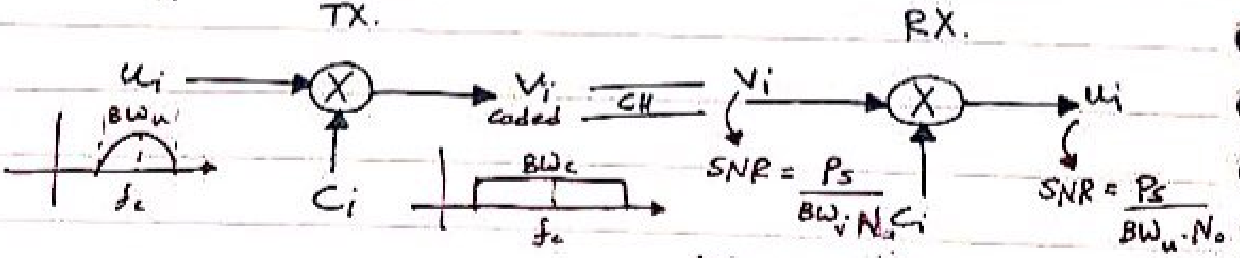
* Sparse matrix ⇒ عدد ال 1 قليل

→ @ RX we use sum-product algorithm

BER near shannon's limit.

- Spread spectrum as a code :

DS-SS :



$BW_v = BW_c + BW_u$

$Noise = BW_v * N_o$

* Coding gain (CG) :

if $BER_{uncoded}(SNR) = 10^{-3}$.

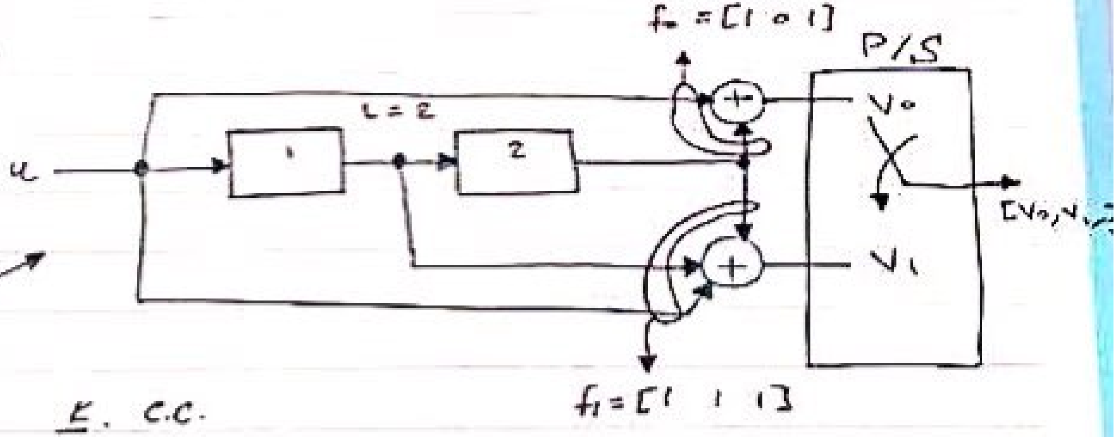
$$\boxed{CG = \frac{P_s / BW_u \cdot N_o}{P_s / BW_v \cdot N_o} = \frac{BW_v}{BW_u}}$$

$BER_{coded}(SNR) = (10^{-3})^{t+1} = BER_{uncoded}(SNR * CG)$

• Convolutional Codes :

$$\vec{V} = \vec{f}(\vec{u}, \text{device})$$

Ex :



rate $\frac{k}{n}$. C.C.

$\boxed{\text{rate} : \frac{\overset{k}{1}}{\underset{n}{2}} . \underset{L=2}{CC}}$

(n, k, L) CC

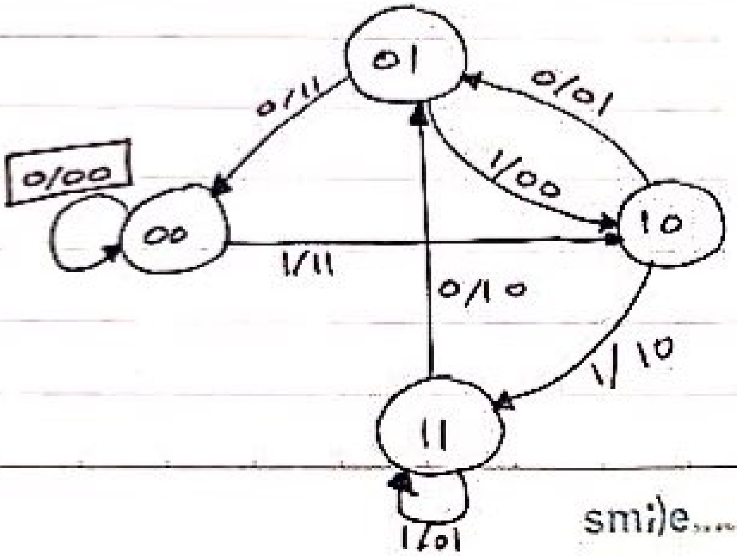
(2, 1, 2) CC

If we have only f_0 & f_1

$f_0 = [1\ \ 0\ \ 1]$

$f_1 = [1\ \ 1\ \ 1]$

ممنوع أي loop غيرها يكون له weight
الا يكون 0



smile...

Scanned by CamScanner

- Good convolutional code :)

1 -

2 - No loop other than

نطلع من أي states ونرجعلها بعدين ما نعرف → Min. error event :

In the prev. state diagram we have 5 ones

→ $d_{free} = 5$

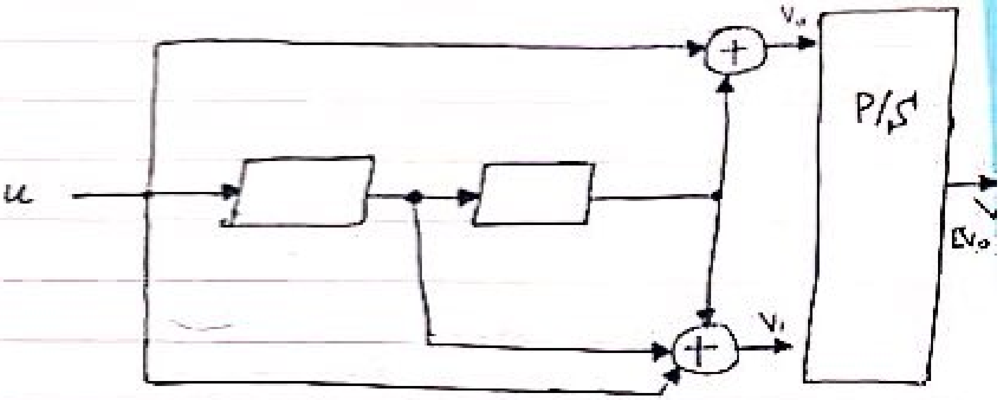
$t = 2$ (num. of transition * n)

↳ every $3*2=\underline{\underline{6}}$ bits lec. 26/4/2017

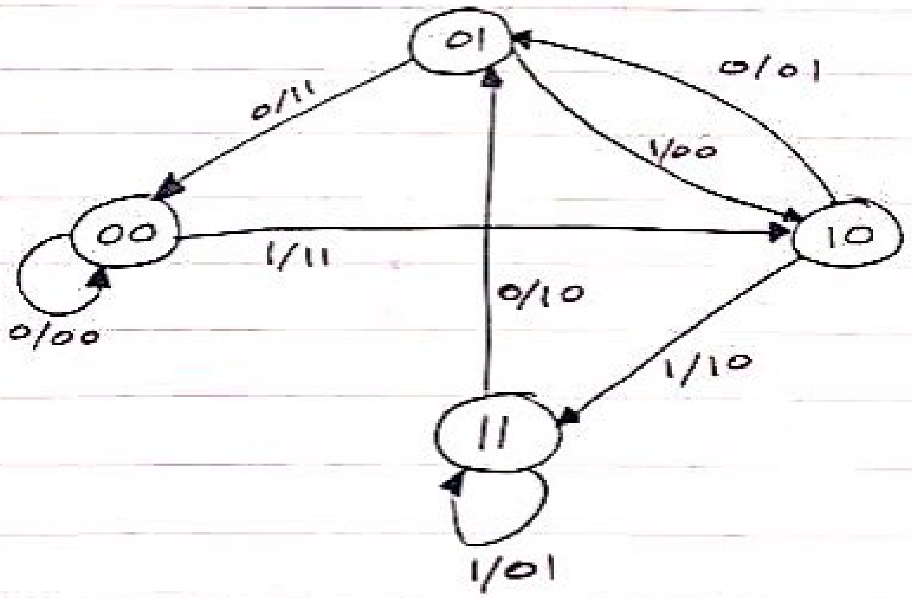
Convolutional codes :

- Ex : $\overset{n}{(2},\overset{k}{1},\overset{L}{2})$ C.C convolutional code

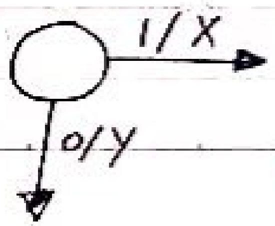
best code $f_0 = [1 \quad 0 \quad 1]$ $f_1 = [1 \quad 1 \quad 1]$



There is no loop with zero state except 00/0



→ مجموع ال in وال out لازم يساوي X



→ لازم تكون المسافة بين X و Y أكبر ما يمكن

good ← وإذا كانت أكبر ما يمكن smile....

bad ← " " قليلة

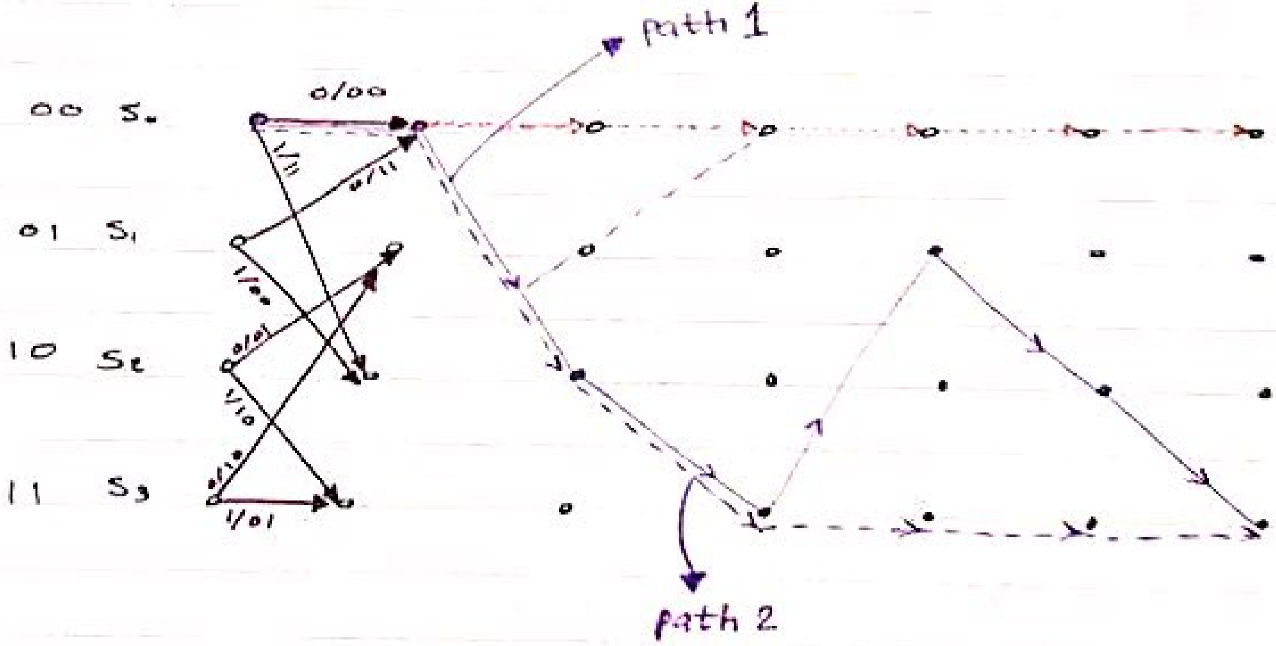
L → R

if $u = [0\ 11\ 0\ 11]$

$v = [00\ 11\ 10\ 10\ 00\ 10]$

starting
from
0/00
state

- **Tree / Trellis diagram :**



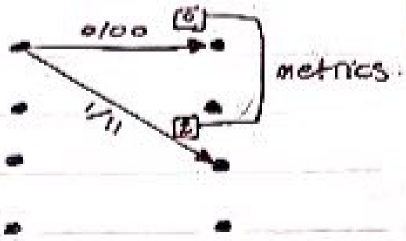
$u_e = [0\ 11\ \underline{\underline{1}}\ 11]$

$\boxed{d_{free} = 5}$ for any two path (min)

لكن مش شرط يكون <u>5</u>

* Convolutional code Decoding Viterbi Algorithm (VA) :

- At any node compute the branch metrics.
- Accumilate all branch metrics in all possible paths.



- Path with min. summation metrics called survivor path.
  → Sort path on summation metrics.
- Correct path with min. sum. metrics.

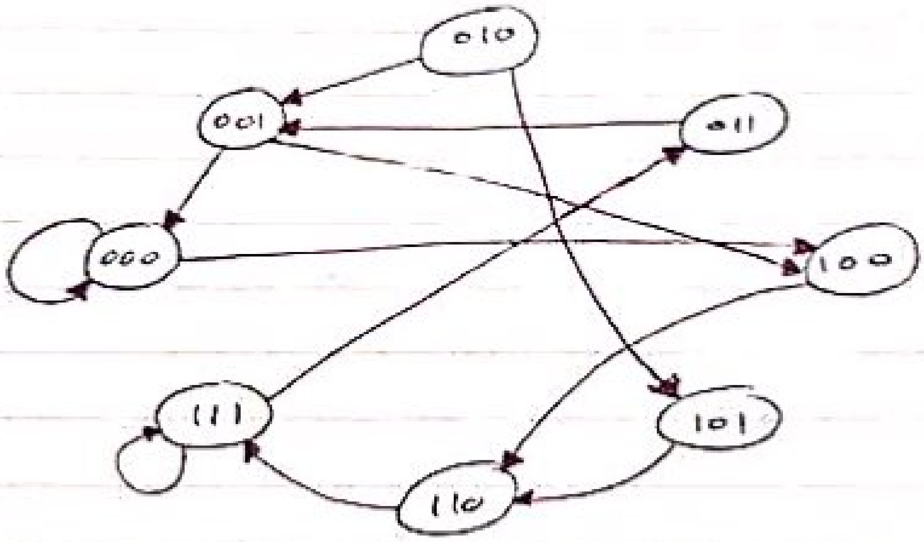
ال non correct path تفضل ال metric تبعه بتزداد

كل ما قل ال code rate كان ال performance أفضل

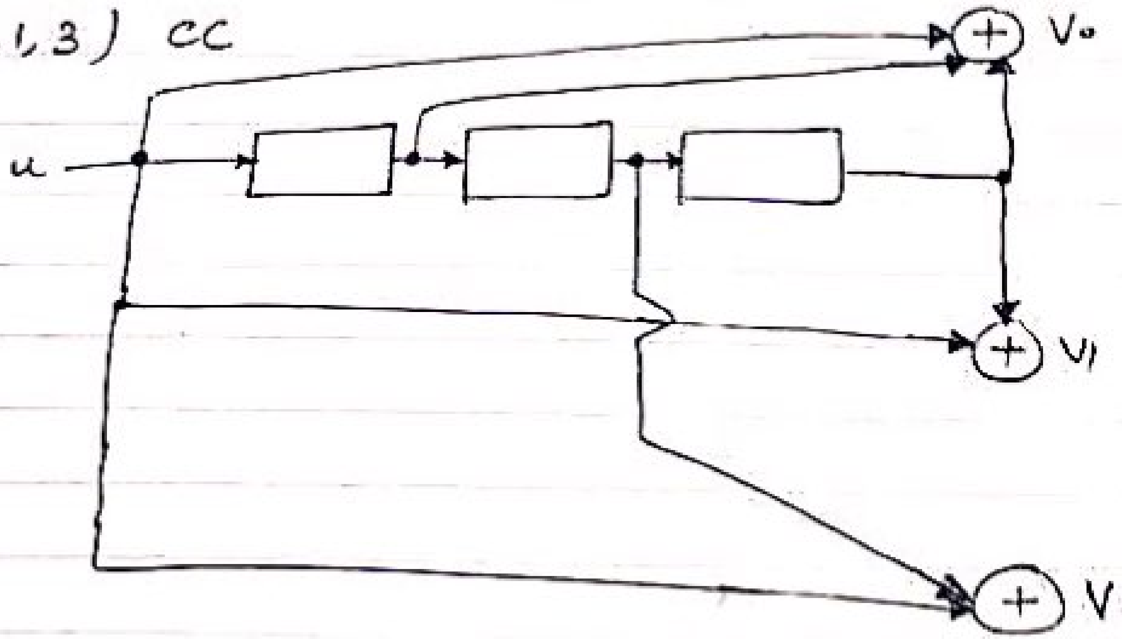
Ex : (2, 1, 3) C.C :-

- 3 registers.
- 8 states.

cont. State diagram



(3, 1, 3) CC



$f_0 = [1\,1\,0\,1]$

$f_1 = [1\,0\,0\,1]$

$f_2 = [1\,0\,1\,0]$

cont. state diagram

1/5

No. ....

(3, 1, 3) C.C.

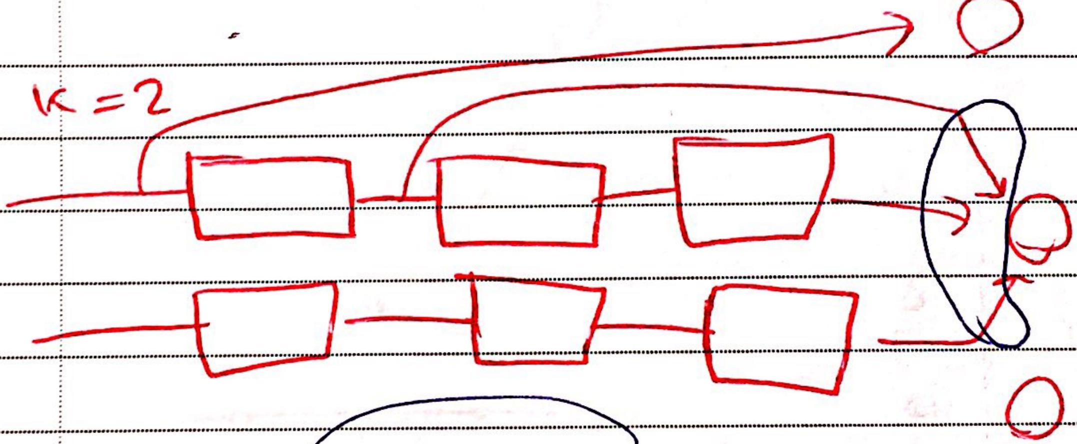
$d_{free} = 7$

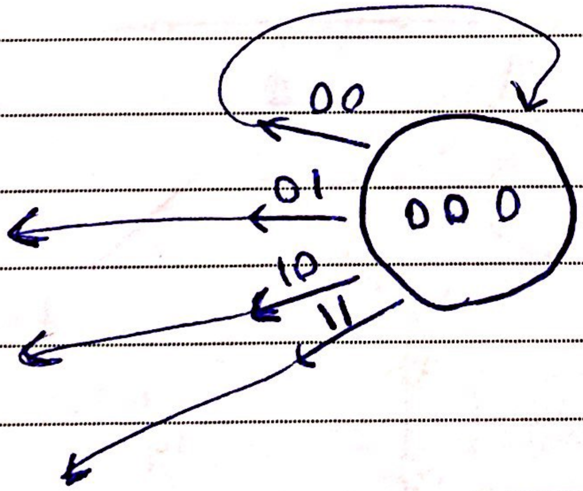
t = 3 errors every

4x3 = 12 bit

K > 1

K=2

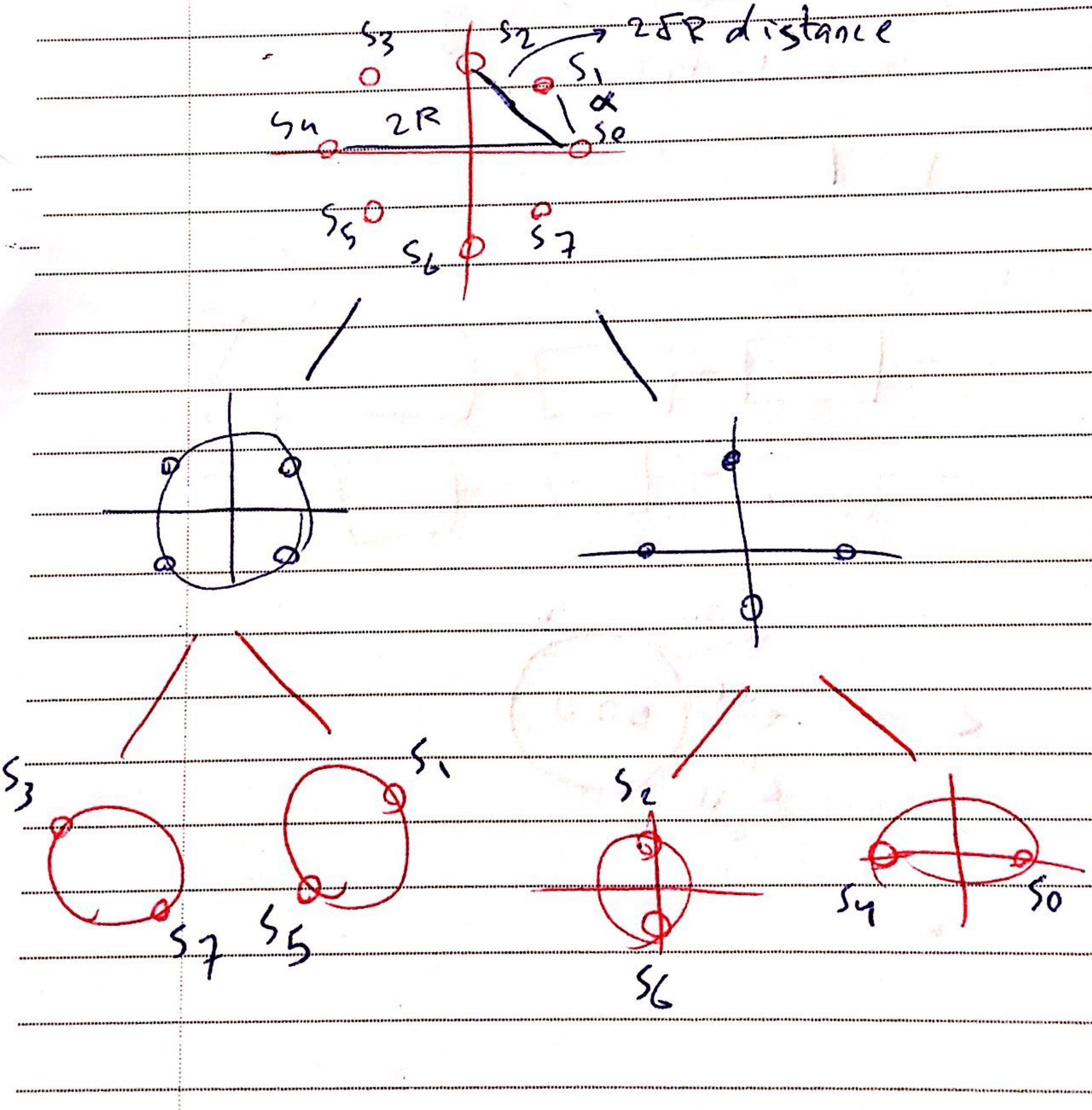




Trilles coded modulation

TCM :-

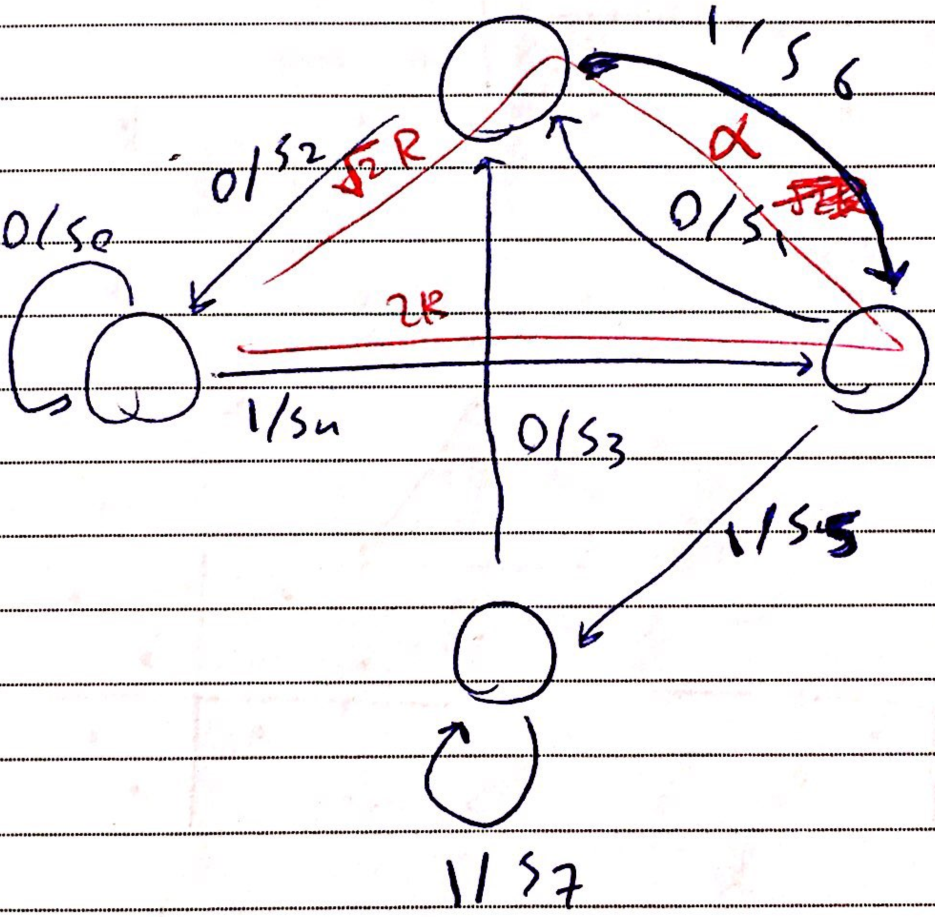
set partitioning



(3, 1, 2) C.C.

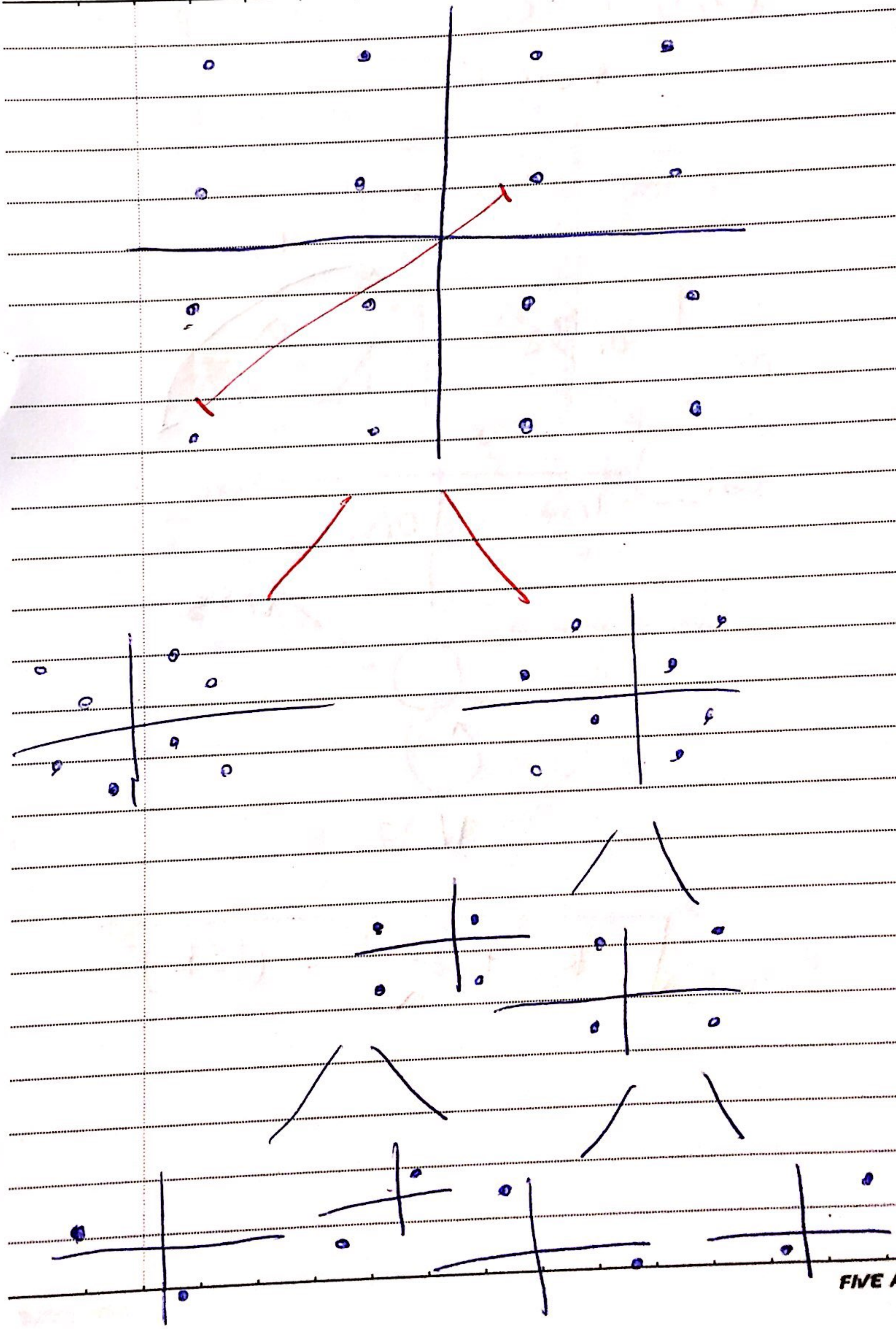
=

v [10 1]



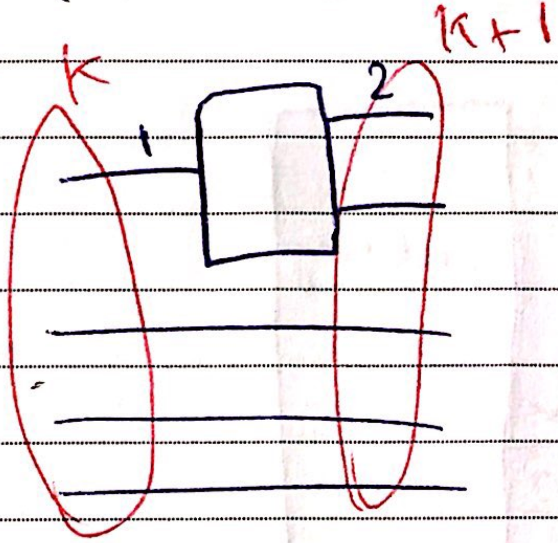
$$d_{free} = \sqrt{(2R)^2 + (\underset{\alpha}{\cancel{\sqrt{2}R}})^2 + (\sqrt{2}R)^2}$$

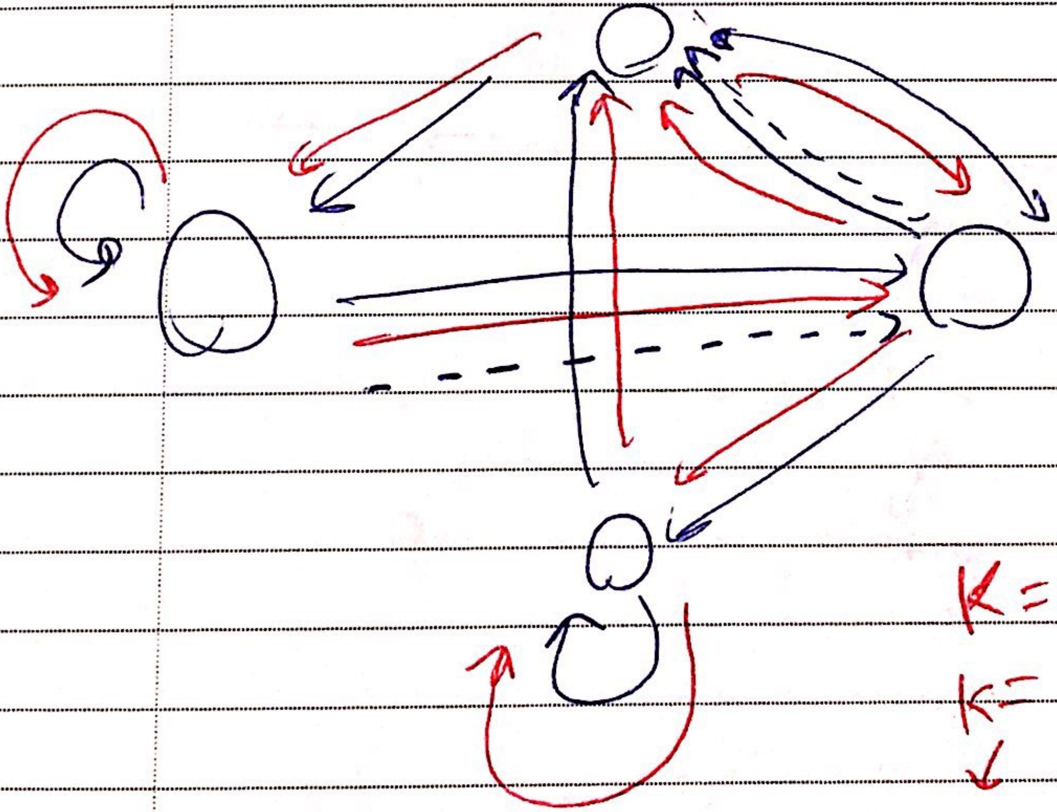
No.



No.

(2,1,2) C.C.





K=2

K=3 3

↓

3 parallel errors

parallel arrows have maximum distance in subsets

Turbo codes SOVA algorithm