Part V: Multiple Access Techniques for Wireless Communication Systems

November 4, 2017

What is the difference between:

- Duplexing
- Multiplexing
- Multiple-Access

Duplexing

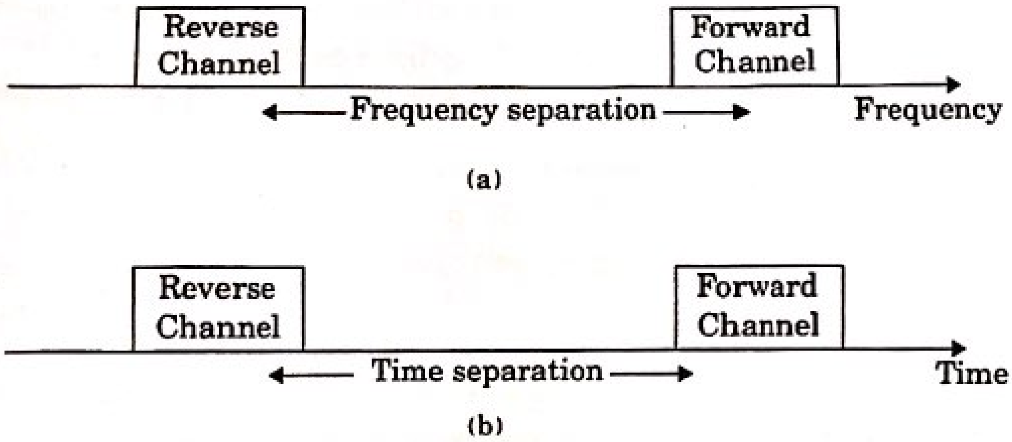
- Duplexing is needed to allow subscribers (users) send and receive information simultaneously.

Duplexing: FDD and TDD

- Frequency division duplexing (FDD)
  - Provides two distinct bands of frequencies for every user.
    - Forward band: from the base station to the mobile
    - Reverse band: from the mobile to the base
  - Consists of two simplex channels
  - Duplexer is used
  - The frequency split between the forward and reverse channel is constant.
- Time division duplexing (TDD)
  - Uses time to provide both a forward and reverse link.
  - If the time split between the forward and reverse time slot is small, then the transmission and reception of data appears simultaneous.
  - Allows communication on a single channel and simplifies the subscriber equipment since a duplexer is not required.
- Duplexer: https://en.wikipedia.org/wiki/Duplexer

Duplexing: FDD and TDD

(a) FDD (b) TDD



Multiple Access Techniques

- Multiple access techniques used to allow many mobile users to share simultaneously a finite amount of radio spectrum
  - High capacity is required.
  - must be done without severe degradation in the performance

Multiple Access Techniques

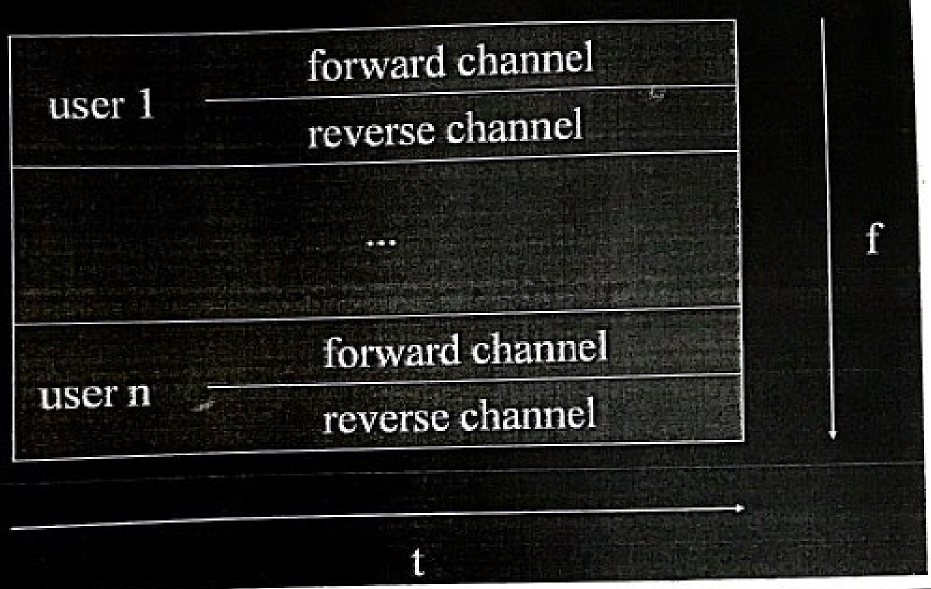
- Three major techniques:
  - Frequency division multiple access (FDMA)
  - Time division multiple access (TDMA)
  - Code division multiple access (CDMA)
- Two more
  - Space division multiple access (SDMA)
  - Packet radio (PR)
- These techniques can be grouped as narrowband and wideband systems, depending upon how the available bandwidth is allocated to the users.
- The duplexing technique of a multiple access system is usually described along with the particular multiple access scheme

Multiple Access Techniques: Narrowband Systems

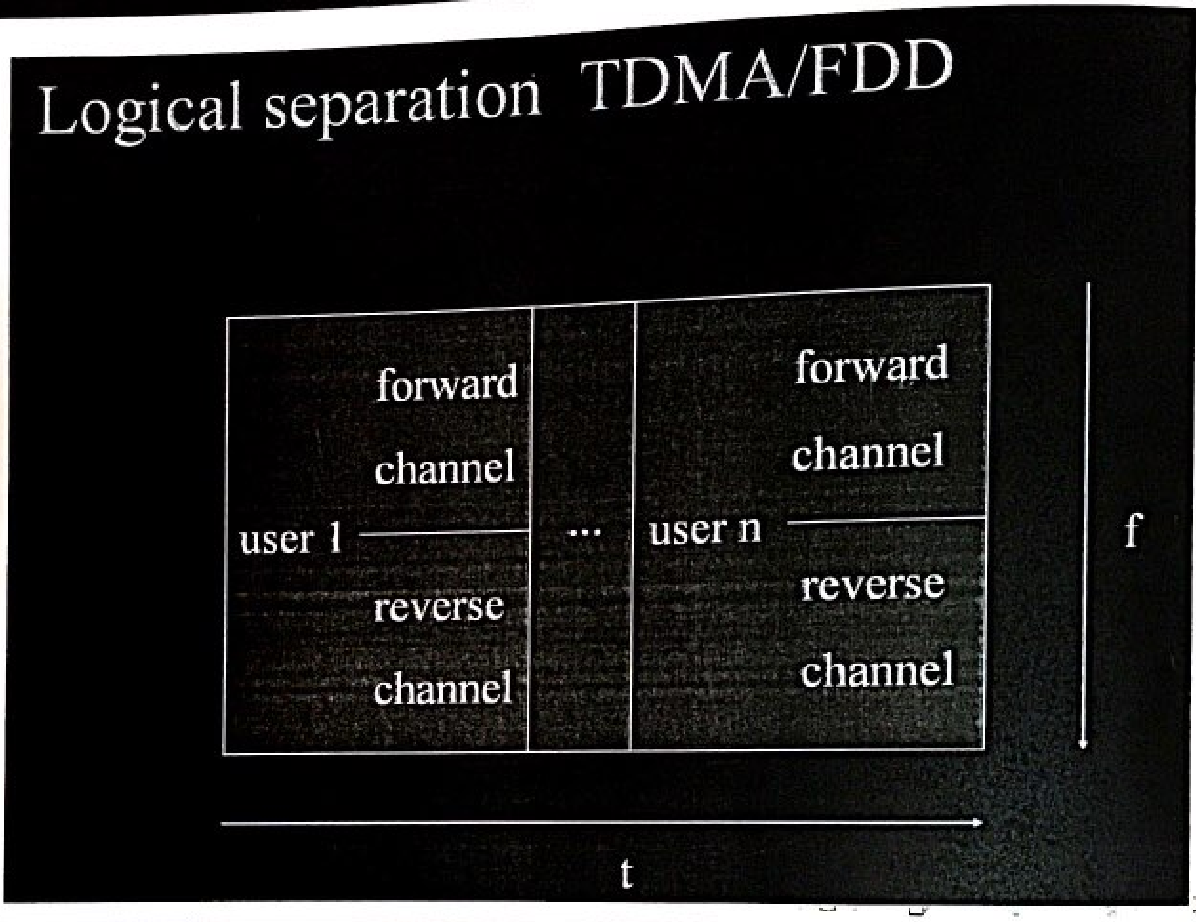
- The available Radio-Spectrum is divided into a large number of narrowband Chs, each with BW B_s
  - Each channel is relatively narrow as compared to the Ch coherence BW B_c
- Channels are usually operated using FDD
  - To reduce interference between Forward and Reverse channels; frequency separation is made as large as possible, which leads to inexpensive duplexers
- Narrowband FDMA:
  - a user is assigned a particular Ch which is not shared by other users (e.g, 1G)
  - If FDD is used, the system is called FDMA/FDD
- Narrowband TDMA:
  - multiple users can share same Ch, but a unique time-slot is assigned to each user
  - Either FDD or TDD is employed:TDMA/FDD or TDMA/TDD

Multiple Access Techniques

Logical separation FDMA/FDD

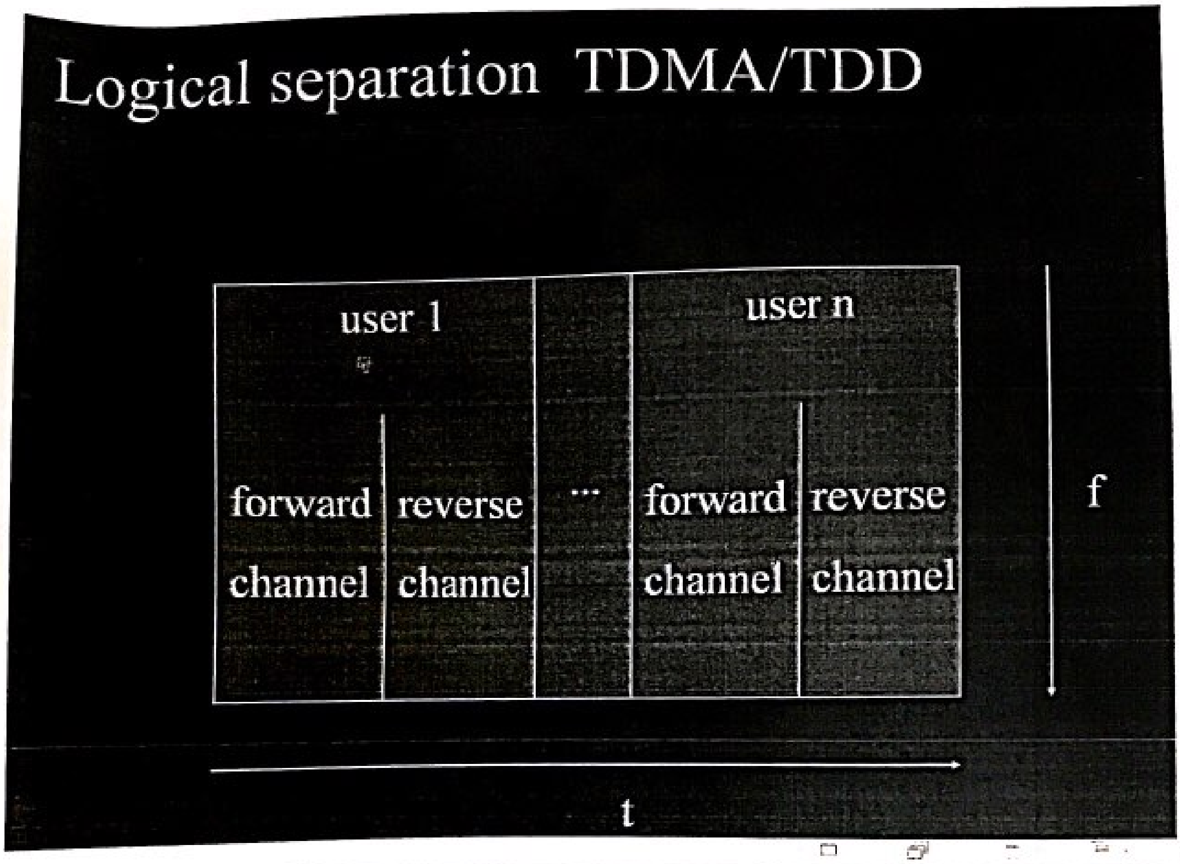


Multiple Access Techniques



* all users have the same forward channel freq. but diffrent time slots

Multiple Access Techniques



Multiple Access Techniques

b) Wideband systems

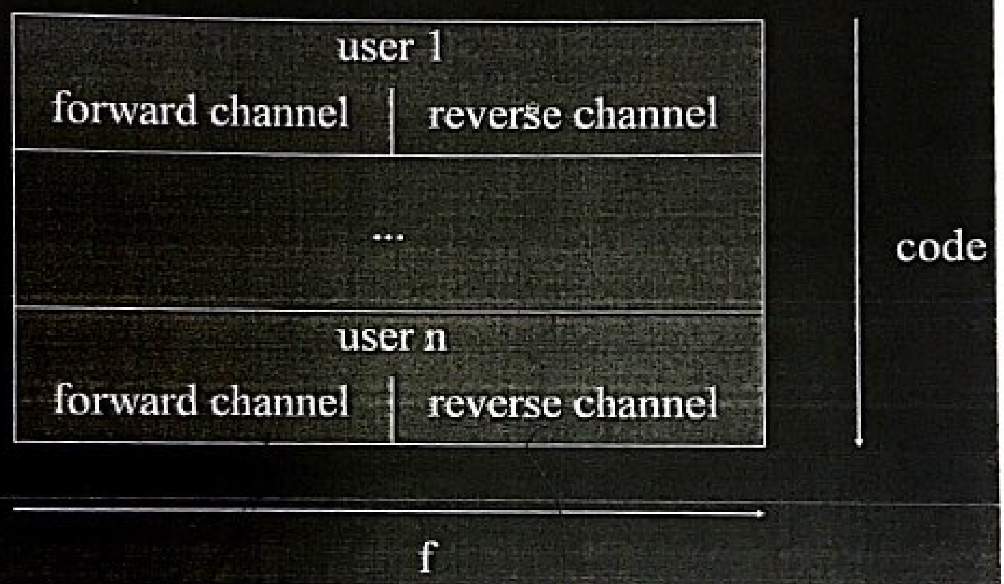
- **The transmission bandwidth of a single channel is much larger than the coherence bandwidth.**
  multipath fading does not greatly affect the received signal, frequency selective fades occur in only a small fraction of the bandwidth.
- **A large number of transmitters are allowed to transmit on the same channel.**

Wideband TDMA ---- allocates time slots to the many transmitters on the same channel and allows only one transmitter to access the channel at any instant of time,
TDMA/FDD, TDMA/TDD

3G ← **Wideband CDMA ---- allows all of the transmitters to access the channel at the same time.**
CDMA/FDD, CDMA/TDD

Multiple Access Techniques

Logical separation CDMA/FDD

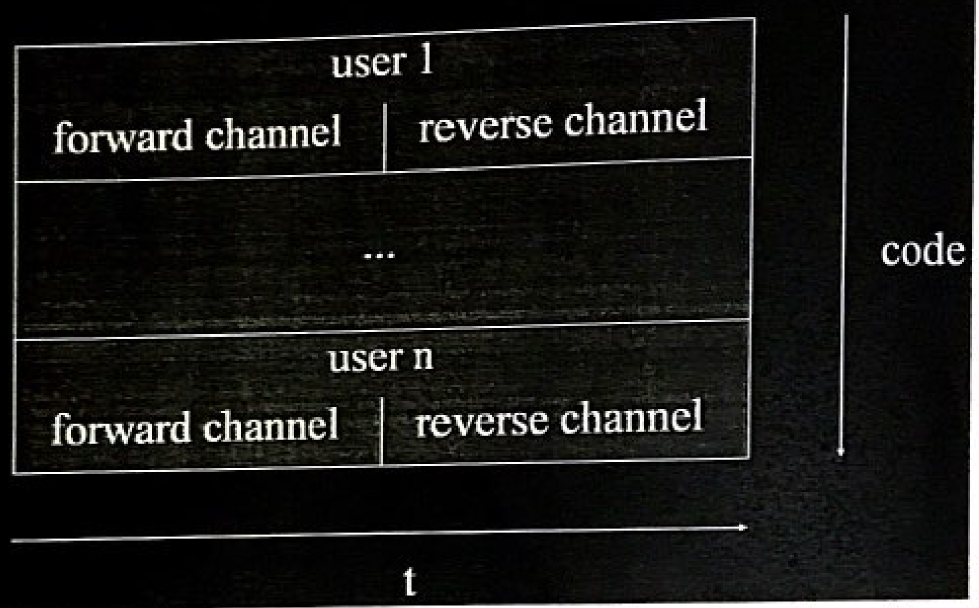


uplink (forward channel)

downlink (reverse channel)

Multiple Access Techniques

Logical separation CDMA/TDD



they send on the same freq.

Multiple Access Techniques

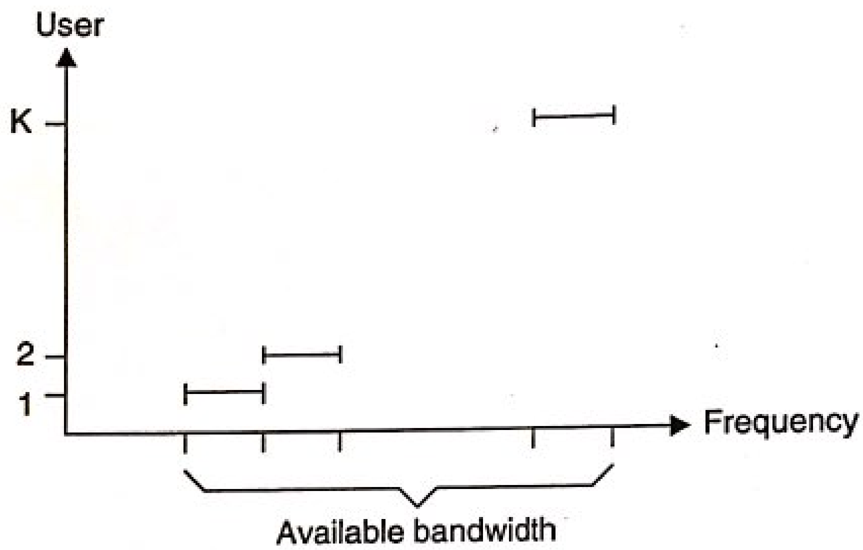
Table 9.1 Multiple Access Techniques Used in Different Wireless Communication Systems

	Cellular System	Multiple Access Technique		
(analog) 1G ←	Advanced Mobile Phone System (AMPS)	FDMA/FDD		} Analog
	Global System for Mobile (GSM)	FDMA/ TDMA/FDD		digital
	US Digital Cellular (USDC)	TDMA/FDD	→ 2G	
	Pacific Digital Cellular (PDC)	TDMA/FDD	→ 2G	
	CT2 (Cordless Telephone)	FDMA/TDD		
	Digital European Cordless Telephone (DECT)	FDMA/TDD		
	US Narrow band Spread Spectrum (IS-95)	CDMA/FDD		
	W-CDMA (3GPP)	CDMA/FDD CDMA/TDD	→ 3G	
	cdma2000 (3GPP2)	CDMA/FDD CDMA/TDD	→ 3G	

Frequency division multiple access (FDMA)

- Each user is allocated a unique frequency band or channel.
- These channels are assigned on demand, and can not be shared.

Analog modulation



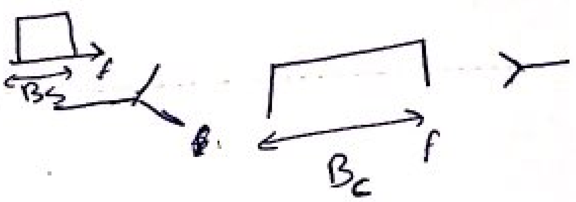
FDMA scheme in which different users are assigned different frequency bands.

FDMA Features I

↳ 1G

- The FDMA channel carries only one phone circuit at a time.
- If an FDMA channel is not in use, then it sits idle and cannot be used by other users to increase or share capacity. It is essentially a wasted resource.
- After the assignment of a voice channel (→ 3kHz bandwidth), the base station and the mobile transmit simultaneously and continuously.
- The bandwidths of FDMA channels are relatively narrow (30 kHz) as each channel supports only one circuit per carrier. That is, FDMA is usually implemented in narrowband systems.
- The symbol time is large as compared to the average delay spread. This implies that the amount of intersymbol interference is low and, thus, little or no equalization is required in FDMA narrowband systems.

$B_s = 30\,kHz \ll B_c$



FDMA Features II

- The complexity of FDMA mobile systems is lower when compared to TDMA systems, though this is changing as digital signal processing methods improve for TDMA.
- Since FDMA is a continuous transmission scheme, fewer bits are needed for overhead purposes (such as synchronization and framing bits) as compared to TDMA.
- FDMA systems have higher cell site system costs as compared to TDMA systems, because of the single channel per carrier design, and the need to use costly bandpass filters to eliminate spurious radiation at the base station.
- The FDMA mobile unit uses duplexers since both the transmitter and receiver operate at the same time. This results in an increase in the cost of FDMA subscriber units and base stations.
- FDMA requires tight RF filtering to minimize adjacent channel interference.

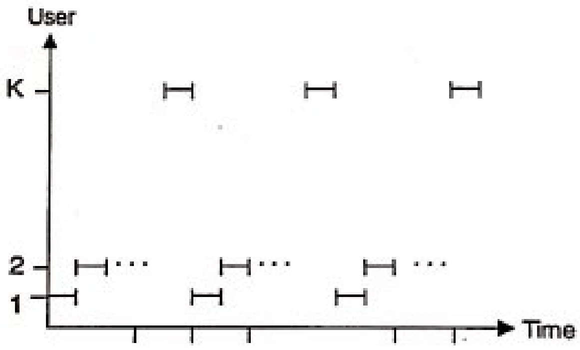
ما يدخل ال signal ل other users

فقط ل 1 user

Time Division Multiple Access(TDMA)

- Each user occupies a cyclically repeating time slot, a channel may be thought of as particular time slot that reoccurs every frame, where N time slots comprise a frame.
- Transmit data in a buffer-and-burst method, the transmission for any user is noncontinuous. digital data and digital modulation must be used with TDMA.

* signal must be pulses



TDMA scheme in which each user occupies a cyclically repeating time slot.

TDMA Frame Structure

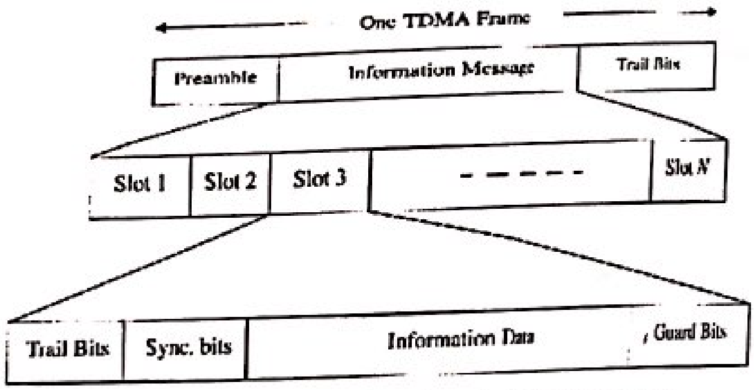


Figure 9.4 TDMA frame structure. The frame is cyclicaly repeated over time.

The transmission from various users is *<u>interlaced</u>* *into a repeating frame structure.*

- **Frame ---- consists of a number of <u>slots</u> (information message), together with a <u>preamble</u>, and <u>tail bits</u>.**
- **Preamble ---- contains the <u>address</u> and <u>synchronization</u> information that both the base station and the subscribers use to identify each other.**
- **Guard times ---- allow *synchronization* of the receivers between different <u>slots</u> and <u>frames</u>.**

TDMA Features I

- TDMA shares a single carrier frequency with several users, where each user makes use of nonoverlapping time slots. The number of time slots per frame depends on several factors, such as modulation technique, available bandwidth, etc.
- Data transmission for users of a TDMA system is not continuous, but occurs in bursts. This results in low battery consumption, since the subscriber transmitter can be turned off when not in use (which is most of the time).
- Because of discontinuous transmissions in TDMA, the handoff process is much simpler for a subscriber unit, since it is able to listen for other base stations during idle time slots. An enhanced link control, such as that provided by mobile assisted handoff (MAHO) can be carried out by a subscriber by listening on an idle slot in the TDMA frame.
- TDMA uses different time slots for transmission and reception, thus duplexers are not required. Even if FDD is used, a switch rather than a duplexer inside the subscriber unit is all that is required to switch between transmitter and receiver using TDMA.

TDMA Features II

- High synchronization overhead is required in TDMA systems because of burst transmissions. TDMA transmissions are slotted and this requires the receivers to be synchronized for each data burst. In addition, guard slots are necessary to separate users, and this results in the TDMA systems having larger overheads as compared to FDMA.
- TDMA has an advantage in that it is possible to allocate different numbers of time slots per frame to different users. Thus bandwidth can be supplied on demand to different users by concatenating or reassigning time slots based on priority.

TDMA Effeciency

- The frame efficiency, is the percentage of bits per frame which contain transmitted data.

$$\eta_f = \left(1 - \frac{b_{OH}}{b_T}\right) \times 100\% = \frac{b_T - b_{OH}}{b_T} \times 100\%$$

It is a measure of the percentage of transmitted data that contains information as opposed to providing overhead for the access scheme.

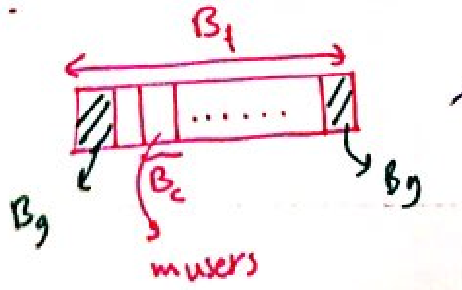
- The transmitted data may include source and channel coding bits, so the raw end-user efficiency of a system is generally less than frame efficiency.

Number of channels In TDMA system:

- Can be found by multiplying the number of TDMA slots per channel by the number of channels available

$$N = \frac{m\,(B_{tot} - 2B_{guard})}{B_c}$$

GSM:·



→ uplink N↗ N↙

$$\text{total users} = m\,\frac{B_t - 2B_g}{B_c} = \text{total number of actual channel}$$

N duplex → if you were asked about number of users you must use N duplex.

2N Simplex

Spread Spectrum Multiple Access (SSMA)

- **Transmission bandwidth is several <u>orders</u> of magnitude greater than the <u>minimum</u> required RF bandwidth.**

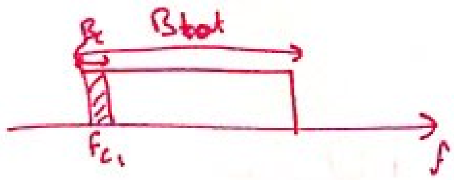
  Pseudo-noise (PN) sequence converts a narrowband signal to a wideband noise-like signal.

- **Provides <u>immunity</u> to multipath <u>interference</u> and <u>robust</u> multiple access capability.**

- **Bandwidth <u>efficient</u> in a muthple user environment.**

- **Two main types SSMA:**
  - **Frequency hopped multiple access (FH)**
  - **Direct sequence multiple access (DS)**

  Direct sequence multiple access is also called code division multiple access (CDMA).



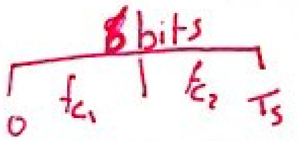
Frequency Hopped Multiple Access (FHMA)

Definition:

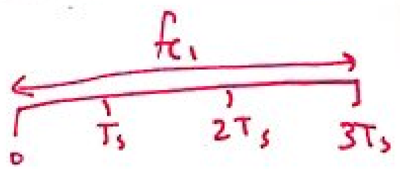
The carrier frequencies of the individual users are varied in a pseudorandom fashion within a wideband channel.

- Data is broken into uniform sized bursts then transmitted on different carrier frequencies.
- The instantaneous bandwidth of any one transmission burst is much smaller than the total spread bandwidth.
- The pseudorandom change of the carrier frequencies of the user randomizes the occupancy of a specific channel at any given time, multiple access allowed.
- In the FR receiver, a locally generated PN code is used to synchronize the receivers instantaneous frequency.
- At any given point in time, a frequency hopped signal only occupies a single, relatively narrow channel.

64 QAM



fast FH



slow FH

Frequency Hopped Multiple Access (FHMA)

FSK : freq. shift keying

Difference between FHMA and FDMA:

- **In FHMA, The frequency hopped signal changes channels at rapid intervals.**

Fast hopping and slow hopping:

- **fast frequency hopping ---- the rate of change of the carrier frequency is greater than the symbol rate**

  Can be thought of as an FDMA system which employs frequency diversity

- **slow frequency hopping ---- the channel changes at a rate less than or equal to the symbol rate**

Frequency Hopped Multiple Access (FHMA)

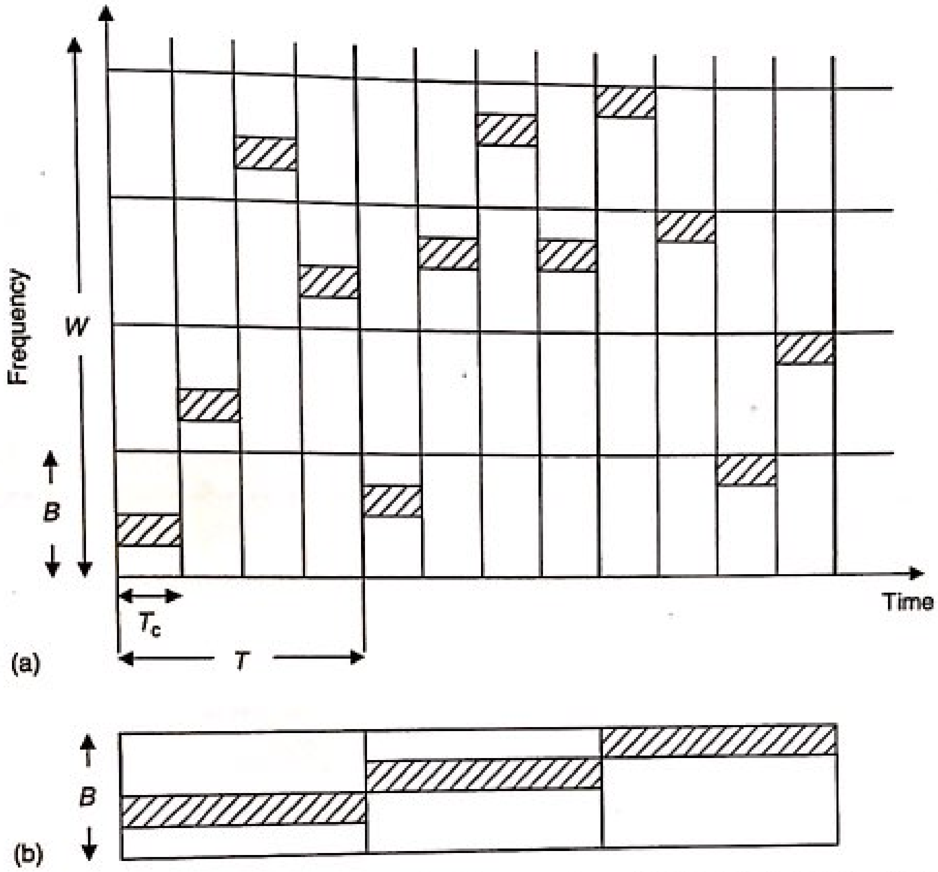


Figure 1.11. Illustration of FSK fast-frequency-hopped spread spectrum system

Frequency Hopped Multiple Access (FHMA)

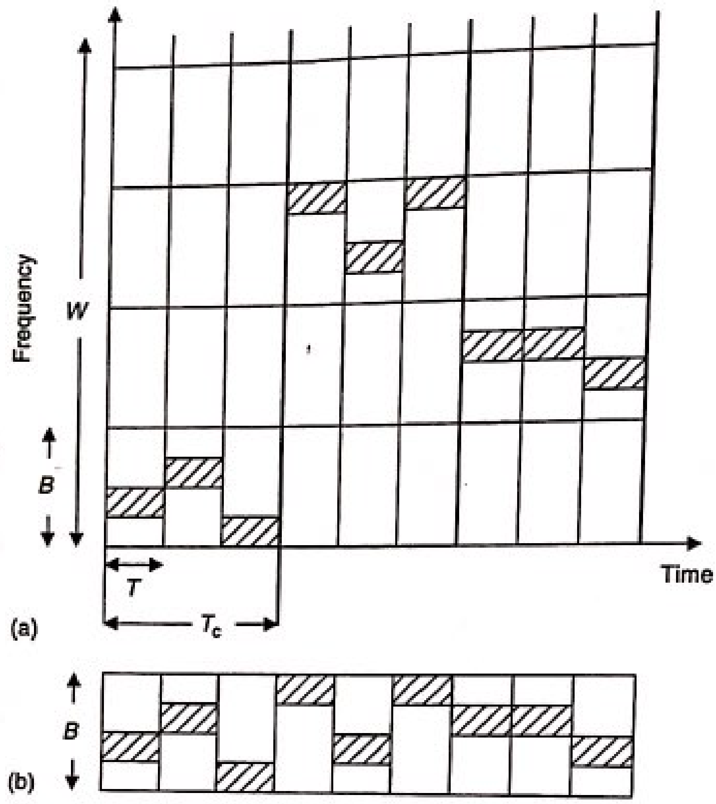


Figure 1.10. Illustration of FSK slow-frequency-hopped spread spectrum system. (a) transmitted signal; (b) dehopped signal (4-FSK modulation).

Frequency Hopped Multiple Access (FHMA)

- FHMA systems often employ energy efficient constant envelope modulation.
- Inexpensive receivers may be built to provide noncoherent detection of FHMA.

  linearity is not an issue.

- A frequency hopped system provides a level of security, especially when a large number of channels are used.

Code Division Multiple Access (CDMA)

- In CDMA, the narrowband message signal is multiplied by a very large bandwidth signal called the spreading signal.
- The spreading signal is a pseudo-noise code sequence that has a chip rate which is orders of magnitudes greater than the data rate of the message.
- All users use the same carrier frequency and may transmit simultaneously.
- Each user has its own pseudorandom codeword which is approximately orthogonal to all other code words.

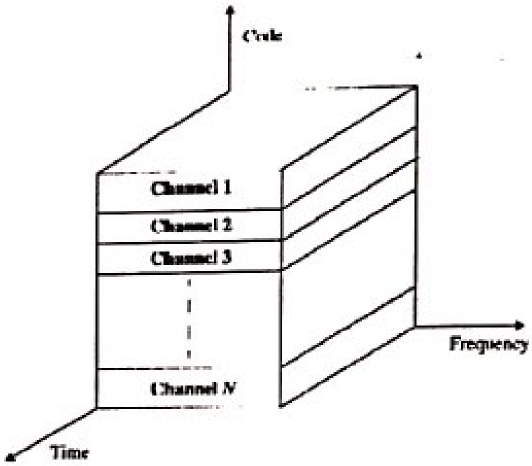


Figure 9.5 Spread spectrum multiple access in which each channel is assigned a unique PN code which is orthogonal or approximately orthogonal to PN codes used by other users.

Code Division Multiple Access (CDMA)

- **The receiver performs a time <u>correlation</u> operation to detect only the specific desired codeword.**
  All other codewords appear as noise due to decorrelation.
- **The receiver needs to know the codeword used by the transmitter.**
  Each user operates independently with no knowledge of the other users.

Code Division Multiple Access (CDMA)

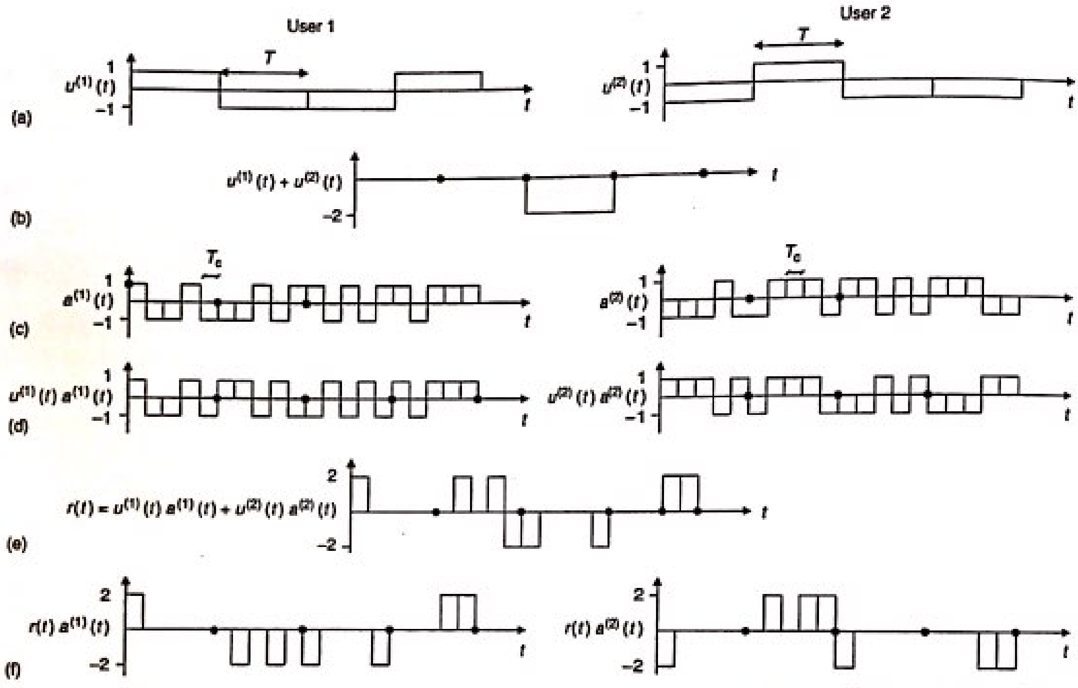


Figure 1.6. Example of the transmission over an adding channel, synchronous case.

$R_b = \frac{1}{T}$ ← symbol duration

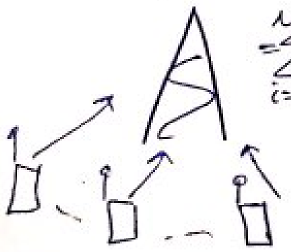
$R_c = \frac{1}{T_c}$ ← chip duration

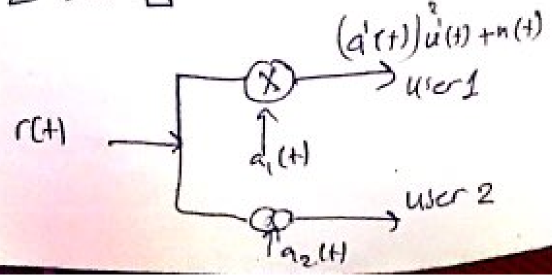
$R_c \gg R_b$

$R_c = 5R_b$

$r(t) = a^1(t)u^1(t) + \dots + a^N(t)u^N(t) + n(t)$

$= \sum_{i=1}^{N} a^i(t)\,u^i(t) + n(t)$





Code Division Multiple Access (CDMA)

near-far problem:

- The near-far problem occurs when many mobile users <u>share</u> the same channel.
  - In general, the strongest received mobile signal will capture the demodulator at a base station.
  - In CDMA, stronger received signal levels raise the noise floor at the base station demodulators for the weaker signals, thereby decreasing the probability that weaker signals will be received.
  - the power of multiple users at a receiver determines the noise floor after decorrelation.

$$c^1(t)r(t) = \underbrace{(c^1(t))^2 u^1(t)}_{D_1 \; P_t} + \underbrace{\sum_{i=1}^{N} c^1(t)\, c^i(t)\, u^i(t)}_{I_1} + \underbrace{n(t)}_{N_1}$$

I_1
must be zero
but not always zero

$$SINR = \frac{D_1}{I_1 + N_1}$$

Code Division Multiple Access (CDMA)

Power control:

- **Provided by <u>each base station</u> in a cellular system and assures that each mobile within the base station coverage area provides the <u>same signal level</u> to the base station receiver.**

  This solves the problem of a nearby subscriber overpowering the base station receiver and drowning out the signals of far away subscribers.

- **Power control is <u>implemented</u> at the base station by rapidly sampling the radio signal strength indicator (<u>RSSI</u>) levels of each mobile and then sending a power change command over the forward radio link.**

  out-of-cell mobiles provide interference which is not under the control of the receiving base station.

Code Division Multiple Access (CDMA)

Features of CDMA:

- **Many users of a CDMA system share the same frequency. Either TDD or FDD may be used.**
- **Unlike TDMA or FDMA, CDMA has a soft capacity limit. Increasing the number of users in a CDMA system raises the noise floor in a linear manner. Thus, there is no absolute limit on the number of users in CDMA. Rather, the system performance gradually degrades for all users as the number of users is increased, and improves as the number of users is decreased.**
- **Multipath fading may be substantially reduced because the signal is spread over a large spectrum. If the spread spectrum bandwidth is greater than the coherence bandwidth of the channel, the inherent frequency diversity will mitigate the effects of small-scale fading.**

Code Division Multiple Access (CDMA)

Features of CDMA (continued):

- **Channel <u>data rates</u> are very <u>high</u> in CDMA systems. Consequently, the symbol (chip) duration is very short and usually much less than the channel delay spread. Since PN sequences have low autocorrelation, multipath which is delayed by more than a chip will appear as noise. A <u>RAKE</u> receiver can be used to improve reception by <u>collecting</u> time delayed versions of the required signal.**

- **Since CDMA uses co-channel cells, it can use <u>macroscopic</u> spatial <u>diversity</u> to provide <u>soft handoff</u>. Soft handoff is performed by the MSC, which can simultaneously monitor a particular user from two or more base stations. The MSC may chose the <u>best version</u> of the signal at any time without switching frequencies.**

Code Division Multiple Access (CDMA)

Features of CDMA (continued):

- Self-jamming is a problem in CDMA system. Self-jamming arises from the fact that the spreading sequences of different users are not exactly orthogonal, hence in the despreading of a particular PN code, non-zero contributions to the receiver decision statistic for a desired user arise from the transmissions of other users in the system.
- The near-far problem occurs at a CDMA receiver if an undesired user has a high detected power as compared to the desired user.

[a] fundemental limits In information theory "shannon" opened the gate to design digital communication system over noisy channel

Communication system

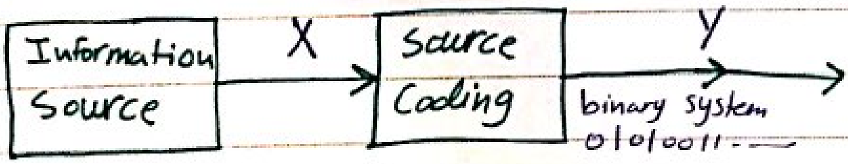
efficient

- acheived by Source Coding
  - related to compressing "compection"
  - to remove redundancy

reliable

- acheived by channel Coding
  - small error amount error free comm. system

[e.g] Voice: 0-3.4kHZ



- human speech
- audio
- image
- text, data
- radio

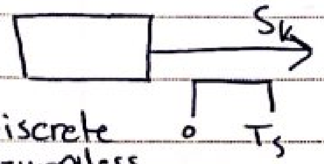
"to remove redundant information"

* what is minimum of Y from X to gaurntee lossless compression.

No.

* How to measure information "Source entropy"

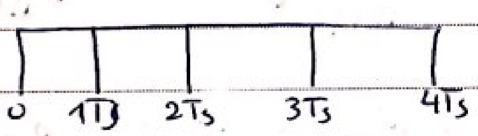
$S = \{S_0, S_1, S_2, \ldots, S_{k-1}\}$



Discrete memoryless Source

all possible symbols emitted by the source S

* memoryless source symbols independent to the last symbols.
* the source emits on symbol at each signaly period "T_s"



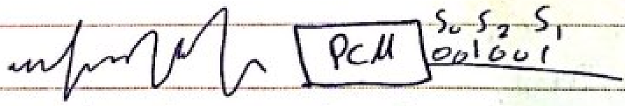
$P = \{P_0, P_1, \ldots, P_{k-1}\}$

→ propabilistic behaviour

$$\sum_{j=0}^{k-1} P_j = 1$$

P_j : probability that the source emits the symbol S_j

$I(S_j)$: amount of information in symbol (S_j) related to surprise level.

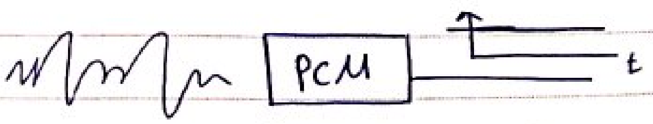


$I(S_j) = \log_2 \left(\frac{1}{P_j} \right)$ (bit) → primary digit (measuring the amount of information in every symbol)

$P_j > P_i$ $\qquad I(S_j) < I(S_i)$

$P_i \uparrow \quad I(S_i) \downarrow$

No.



$L = 1$

$S = \{S_0\}$

$P(S_0) = 1$

$I(S_0) = 0 \text{ bit}$

Properties :-

1) $I(S_0) = 0$, $P_k = 1$

2) $I(S_0) \geq 0$ $\quad 0 \leq P_k \leq 1$

$I(S_1) = \infty$ $\quad I(S_1) = 0$

3) $I(S_k) < I(S_j)$ if $P_{sk} > P_{sj}$

4) $I(S_k S_j) = \log \frac{1}{P(S_k \cap S_j)} = \log \frac{1}{P(S_k)\,P(S_j)} = I(S_k) + I(S_j)$

$P(S_k \cap S_j) = P(S_k)\,P(S_j)$ $\qquad P_k = P(Sk)$

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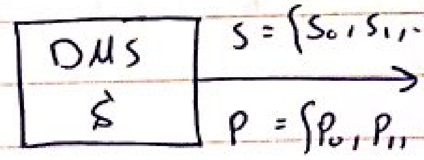
Entropy :- Source avg information



$S = \{S_0, S_1, \ldots, S_{k-1}\}$

$P = \{P_0, P_1, \ldots, P_{k-1}\}$

$I = \{I(S_0), I(S_1), \ldots, I(S_{k-1})\}$

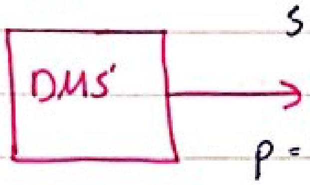
entropy $H(S) = E[I] = \sum_{j=0}^{k-1} P_j \, I(S_j)$

$$\boxed{H(S) = \sum_{j=0}^{k-1} P_j \log\left(\frac{1}{P_j}\right)}$$ (bit / symbol)

No. 31/7/2018

Entropy :-

$S = \{s_0, s_1, \ldots, s_{k-1}\}$ ← k symbols

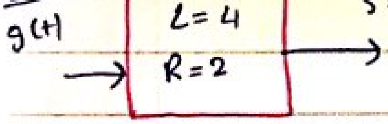


$P = \{P_0, P_1, \ldots, P_{k-1}\}$ ∴ $\sum_{j=0}^{k-1} P_j = 1$

$I = \{I(s_0), I(s_1), \ldots, I(s_{k-1})\}$ "bit"

$$\boxed{H(s) = \sum_{j=0}^{k-1} P_j \log_2\left(\frac{1}{P_j}\right)}$$ bit/sys.

Ex: ⓐ PCM



$S = \{s_0, s_1, s_2, s_3\}$

	$I(s_i)$
s_0 = "00"	2 bits
s_1 = "01"	2 bits
s_2 = "10"	2 bits
s_3 = "11"	2 bits

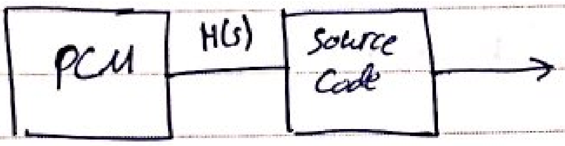
Average = 2 bit/symbol

ⓑ assume : $P = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right\}$ $I(s_k) = \log_2\left(\frac{1}{P_k}\right)$

$I = \{1, 2, 3, 3\}$

Avg information $H(s) = \left(\frac{1}{2}\right)(1) + \left(\frac{1}{4}\right)(2) + \left(\frac{1}{8}\right) \cdot 3 \cdot 2 = 1.75$ bit/sym

Redundant information $= 2 - 1.75 = 0.25$ bit/sym



$I \geq H(s)$

for lossless Compression

* Shannon's theories $R_b \leq C$; $C = B\log_2(1 + SNR)$

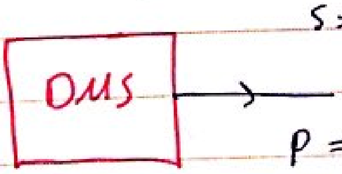
$I \geq H(s)$

$\Rightarrow \quad 0 \leq H(s) \leq \log_2(k)$

$P_i = 1$, $P_j = 0$, $j \neq i$

$P_0 = P_1 = \cdots = P_{k-1} = \frac{1}{k}$

Ex:- Entropy for a binary source.

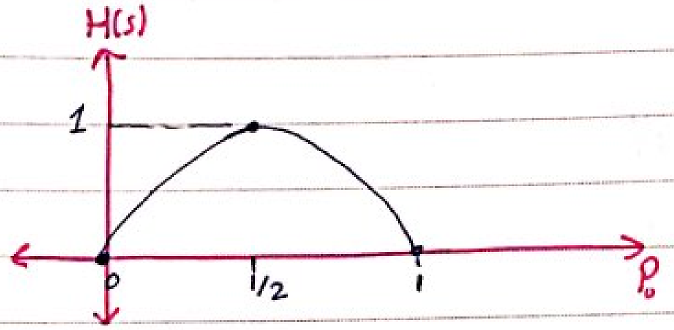


$S = \{S_0, S_1\}$, $k = 2$

$P = \{P_0, 1 - P_0\}$

$$H(s) = P_0 \log_2\left(\frac{1}{P_0}\right) + (1 - P_0)\log \frac{1}{1 - P_0}$$

$$\boxed{H(s) = -P_0 \log_2(P_0) - (1 - P_0)\log_2(1 - P_0)}$$



$P_0 = 0 \ , P_1 = 1 \Rightarrow H(s) = 0$

$P_0 = \frac{1}{2} \ , P_1 = \frac{1}{2} \Rightarrow H(s) = 1$

$P_0 = 1 \Rightarrow H(s) = 0$

$S = \{S_0, S_1\}$ $\quad S_0 \Rightarrow$ "0"

$P = \{\frac{1}{2}, \frac{1}{2}\}$ $\quad S_1 \rightarrow$ "1"

$I = \{1, 1\}$

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Extention of DMS:

$$\boxed{\text{DMS}} \longrightarrow \begin{array}{l} S = \{s_0, s_1, \dots, s_{k-1}\} \\ P = \{p_0, p_1, \dots, p_{k-1}\} \end{array}$$

Extension: we Concern with blocks, with length of n symbols.

Thus, the # of possible blocks $= k^n$

$$\boxed{S^{(n)}} \longrightarrow S^{(n)} = \{\sigma_0, \sigma_1, \sigma_2, \dots, \sigma_{k^n-1}\}$$

$$P^{(n)} = \{P_{\sigma_0}, P_{\sigma_1}, \dots, P_{\sigma_{k^n-1}}\}$$

$S = \{s_0, s_1\}$

$k = 2$

$n = 3$

$\sigma_0 = s_0 s_0$

$\sigma_1 = s_0 s_1$

$\sigma_2 = s_1 s_0$

$P_{\sigma_0} = p(s_0 s_0)$

$= P_{s_0} P_{s_0}$

$$I(\sigma_i) = \log_2\left(\frac{1}{P_{\sigma_i}}\right)$$

$$\boxed{H(S^{(n)}) = \sum_{j=0}^{k^n-1} P_{\sigma_i} \log_2\left(\frac{1}{P_{\sigma_i}}\right)} \text{ bit/block}$$

extended source Entropy

$$\boxed{H(S^{(n)}) = n\,H(S)}$$

smile...

Ex:- $S = \{S_0, S_1, S_2\}$, $k=3$

$P = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\}$

$I = \{2, 2, 1\}$ [bit]

Extend the source S by $n=2$

Find ⓐ $S^{(2)}$ ⓑ $P^{(2)}$ ⓒ $H(S^{(2)})$

Sol:-

\# of blocks in $S^{(n)}$: $k^n = 3^2 = 9$

$S^{(2)} = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8\}$

$S_0 S_0 = \sigma_0$

$S_0 S_1 = \sigma_1$

$S_0 S_2 = \sigma_2$

$S_1 S_0$

$S_1 S_1$

$S_1 S_2$

$S_2 S_0$

$S_2 S_1$

$S_2 S_2 = \sigma_8$

$$P^{(2)} = \{\underset{1/16}{P_{\sigma_0}}, \underset{1/16}{P_{\sigma_1}}, \underset{1/8}{P_{\sigma_2}}, \underset{1/16}{P_{\sigma_3}}, \underset{1/16}{P_{\sigma_4}}, \underset{1/8}{P_{\sigma_5}}, \underset{1/8}{P_{\sigma_6}}, \underset{1/8}{P_{\sigma_7}}, \underset{1/4}{P_{\sigma_8}}\}$$

No.

$$P_{\sigma_0} = P(S_0 \cap S_0) = P_{S_0} \cdot P_{S_0} = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$P_{\sigma_1} = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$I(\sigma_i) = \log_2 \left(\frac{1}{P_{\sigma_i}}\right)$$

$$I = \{4, 4, 3, 4, 4, 3, 3, 3, 2\} \text{ bit}$$

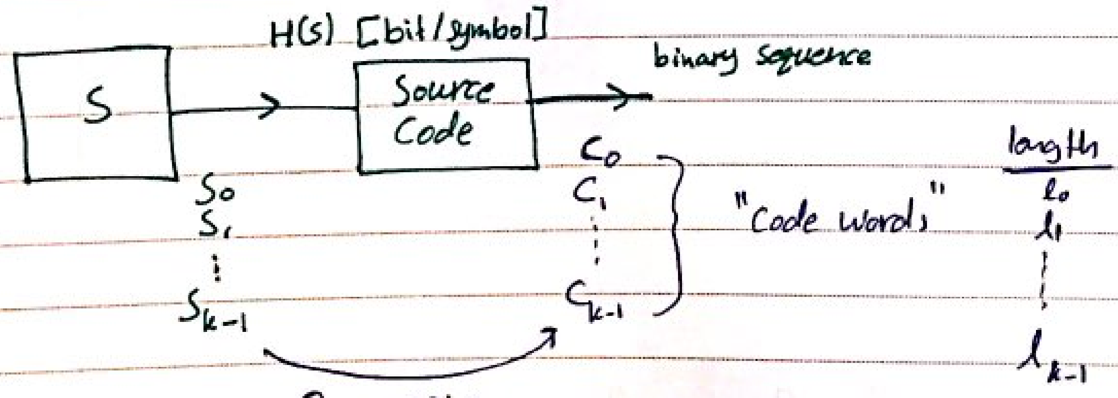
$$H(S^{(n)}) = 2 * 1.5 = \underset{\text{bits/block}}{3} = n\,H(S)$$

$$* \; H(S) = 2 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4} = 1.5 \text{ bit/symbol}$$

$\boxed{or}$

$$\frac{1}{16} \cdot 4 + \frac{1}{16} \cdot 4 + \frac{1}{8} \cdot 3 + \cdots + \frac{1}{4} \cdot 2 = 3 \text{ bits/block}$$

⊛ Source Coding Theorm & Huffman Coding :-



① mapping

② uniquely decodable (invertiable)

③ $\bar{L}$ is the minimum

$\bar{L} = E[l]$, $l = \{l_0, l_1, \dots, l_{k-1}\}$

average codeword length

$$\bar{L} = \sum_{i=1}^{k-1} P_i \, l_i \quad \text{bits / sys}$$

$\bar{L} \geq H(s)$ → To gourantee distortionless (lossless) compression.

* To acheive the bound ($\bar{L} \to H(s)$), the source code has to assign the shortest codeword to the most frequent symbol.

$$\eta = \frac{H(s)}{\bar{L}} * 100\% \quad , \eta \to 100\%$$

coding efficiency

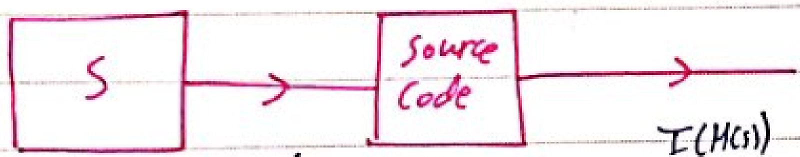
@ prefix Code :-

$S = \{S_0, S_1, S_2, S_3\}$

$P = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\}$

$\bar{L} = \{1, 2, 3, 3\}$ bits

$H(s) = 1.75$ bits/symbol



S	I
S_0	1
S_1	2
S_2	3
S_3	3

$I(H(s))$ $I(H(s))$

Code I	Code II
0	0
01	10
011	110
111	111

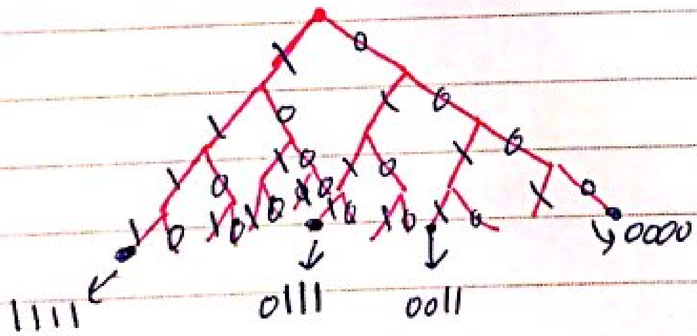
Code II → uniquly decodable

$\rightarrow S_1 S_3 S_1 S_0 \longrightarrow \underbrace{0}_{S_0}1\ \underbrace{111}_{S_3}010\ldots$

X

wrong information (if we used Code I)

Prefix Tree



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① prefix Code :-

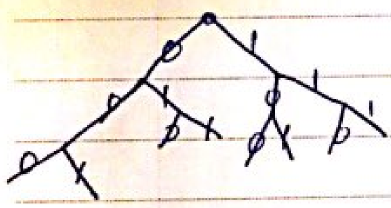
- No Codeword is a prefix of another Codeword

- $\sum_{i=0}^{k-1} \frac{1}{2^{l_i}} \leq 1$

- $\boxed{H(s) \leq \bar{L} \leq H(s)+1}$

under the Condition that the shortest Codeword is assigned to the symbol with the highest probability

prefix Tree



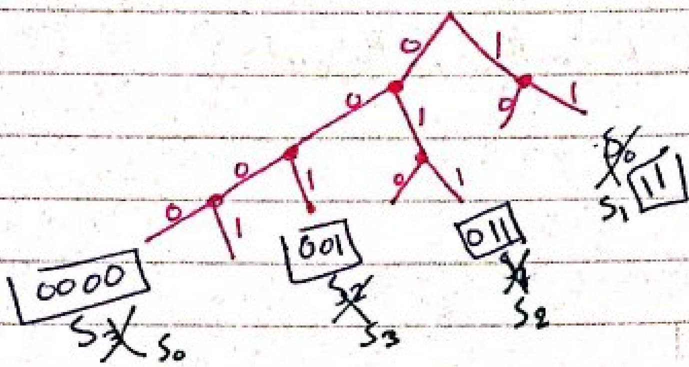
Ex:- $S = \{S_0, S_1, S_2, S_3\}$

$P = \{0.1, 0.6, 0.17, 0.13\}$

$H(s) = 1.592$ bits/symbol

$H(s)+1 = 2.592$

for e.g :-



		l
S_0	11	2
S_1	011	3
S_2	001	3
S_3	0000	4

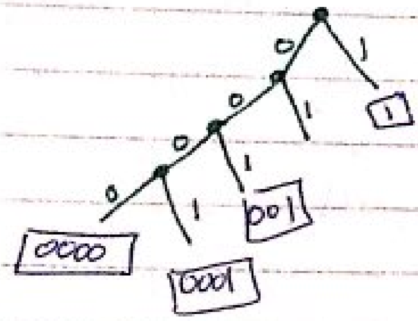
$\bar{L} = (0.1)(2) + (0.6)(3) + (0.17)(3) + (0.13)(4)$
$= 3.03$ bit / sym

$\bar{L} \in (H(s), H(s)+1)$

P		C	l
0.6	S_1	11	2
0.17	S_2	011	3
0.13	S_3	001	3
0.1	S_0	0000	4

$\bar{L} = 2.5$ bit / symbol

$\eta = \frac{H(s)}{\bar{L}} = \frac{1.592}{2.5} = 54.7\,\%$ (poor) another prefix code :-

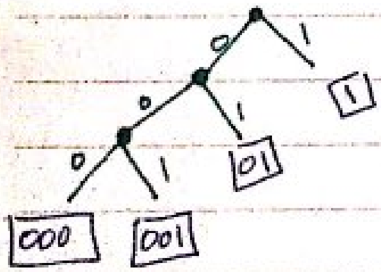


	c	l
S_1	1	1
S_2	001	3
S_3	0001	4
S_0	0000	4

$$\bar{L} = 2.03 \text{ bit / symbol}$$

$$\eta = \frac{1.592}{2.03} \times 100\% = 78.42\%$$

another prefix code :-



	c	
S_1	1	1
S_2	01	3
S_3	001	3
S_0	000	3

$$\bar{L} = 1.63 \text{ bit / sym}$$

$$\eta = \frac{1.592}{1.63} \times 100\% = 97.69\%$$

No.

Huffman Code :-

- it's the optimum prefix Code
- it gives you the minimum $\bar{L}$
- $H(s) \le \bar{L} < H(s) + 1$

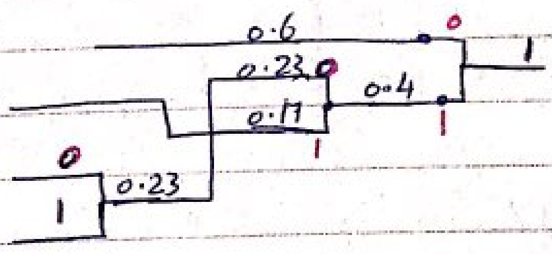
Ex :- $S = \{S_0, S_1, S_2, S_3\}$

$P = \{0.1, 0.6, 0.17, 0.13\}$

$I = \{3.32, 0.737, 2.56, 2.943\}$

$H(s) = 1.592$ bits/symbol

l	C	
1	0	0.6
2	11	0.17
3	100	0.13
3	101	0.1



$\bar{L} = (0.6)(1) + (0.17)(2) + (0.13)(3) + (0.1)(3) = 1.63$ bit/sym

$\eta = \dfrac{1.592}{1.63} = 97.67\%$

No.

Ex:- $S = \{S_0, S_1, S_2, S_3, S_4\}$

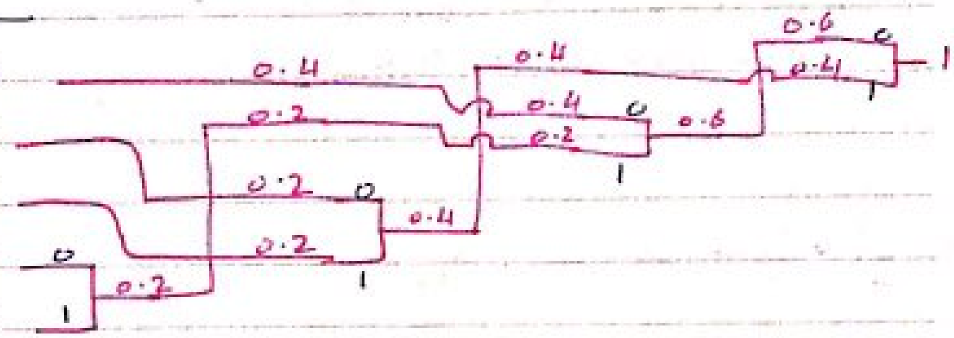
$P = \{0.4, 0.2, 0.2, 0.1, 0.1\}$

$I = \{1.322, 2.322, 2.322, 3.322, 3.322\}$

$H(s) = 2.12193$

Design a Huffman code.

ℓ	c	sym	P
2	00	S_0	0.4
2	10	S_1	0.2
2	11	S_2	0.2
3	010	S_3	0.1
3	011	S_4	0.1



as high as possible

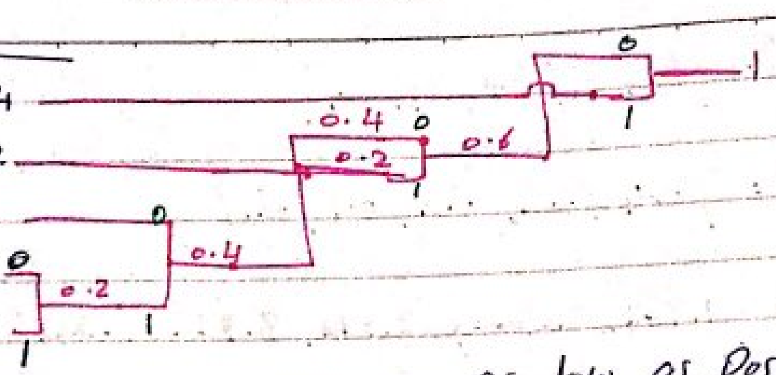
$\bar{L} = 2.2$ bit / codeword

$\eta = \frac{2.122}{2.2} \times 100\% = 96.45\%$

$Var(\ell) = 0.4(2-2.2)^2 + 0.2(2-2.2)^2 + 0.2(2-2.2)^2 + 0.1(3-2.2)^2$
$+ 0.1(3-2.2)^2 = 0.16$

No.

l	c		
1	1	S_0	0.4
2	01	S_1	0.2
3	000	S_2	0.2
4	0010	S_3	0.1
4	0011	S_4	0.1



as low as possible

$\bar{L} = 2.2$ bit/sym

- $\bar{L}$ value will be the same
- The huffman is not unique

$l = \{l_0, l_1, \ldots, l_{k-1}\}$

R.V

$\bar{L} = E[l] = \sum_{i=0}^{k-1} P_i l_i$

$var(l) = \sum_{i=0}^{k-1} P_i (l_i - \bar{L})^2$

length variance $\rightarrow$ measures the variability level

for the above example $\Rightarrow var(l) = 0.4(1-2.2)^2 + 0.2(2-2.2)^2$
$+ 0.2(3-2.2)^2 + 0.1(4-2.2)^2$
$+ 0.1(4-2.2)^2 = 1.36$

VLC : Variable length Code

$\boxed{S} \rightarrow \boxed{\text{Huffman source code}} \xrightarrow{l_0, l_1, \dots, l_{k-1}} \boxed{\text{buffer}}$

for data Transmission, we choose the one with lowest variance.

6/8/2018

Huffman Code for extended Source.

$\boxed{S} \rightarrow \boxed{S^{(n)}} \rightarrow \boxed{\text{Huffman Code}}$

$S = \{S_0, \dots, S_{k-1}\}$ $\quad \{\sigma_0, \sigma_1, \dots, \sigma_{k-1}\}$ $\quad \bar{L}^{(n)}$ bit/block

$P = \{P_0, \dots, P_{k-1}\}$ $\quad P^{(n)} = \{P_{\sigma_0}, \dots, P_{k-1}\}$

$H(S)^{(n)} - nH(S)$ $\quad \bar{L} = \frac{\bar{L}^{(n)}}{n}$ (bit/symbol)

$$H(S^{(n)}) \le \bar{L}^{(n)} \le H(S)^{(n)} + 1$$

$$nH(S) \le \bar{L}^{(n)} \le nH(S) + 1$$

$$H(S) \le \frac{\bar{L}^{(n)}}{n} \le H(S) + \frac{1}{n}$$

$$H(S) \le \bar{L} \le H(S) + \frac{1}{n}$$

$n \to \infty$, $\bar{L} \to H(S)$

Ex :- $S = \{S_0, S_1\}$; $P = \{0.8, 0.2\}$

Apply huffman coding to Ⓐ S Ⓑ $S^{(2)}$ Ⓒ $S^{(3)}$

①

P			C	l
0.8	S_0	0	0	1
0.2	S_1	1	1	1

$\bar{L} = 1$ bit/symbol

$H(S) = 0.72$ bit/symbol

$\eta = 72\%$

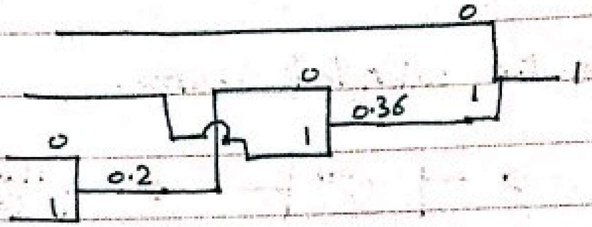
No.

② $S^{(2)} = \{ S_0S_0 , S_0S_1 , S_1S_0 , S_1S_1 \}$

$\{ \sigma_0 , \sigma_1 , \sigma_2 , \sigma_3 \}$

$\{ 0.64 , 0.16 , 0.16 , 0.04 \}$

l		P	
1	0	0.64	σ_0
2	11	0.16	σ_1
3	100	0.16	σ_2
3	101	0.04	σ_3



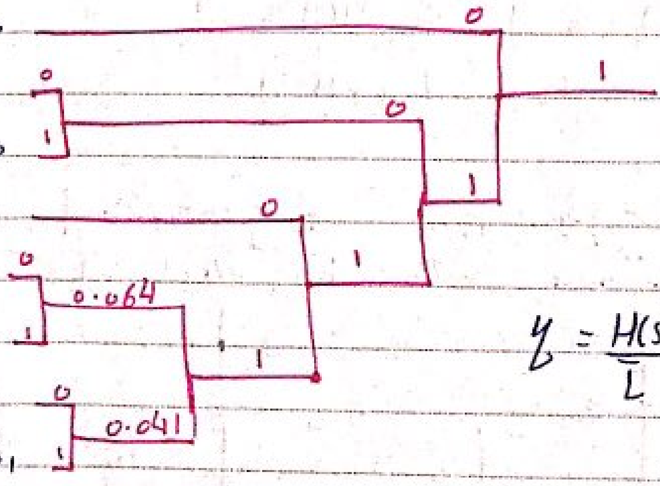
$\bar{L}^{(2)} = 1.56$ bit / (2 symbol)

$\bar{L} = 0.78$ bit / symbol

$\eta = \dfrac{H(s)}{\bar{L}} = \dfrac{0.72}{0.78} = 92.3\%$

③ $S^{(3)} = \{ \sigma_0 , \sigma_1 , \cdots , \sigma_7 \}$

l	C	P	
1	0	0.512	$S_0S_0S_0$
2	100	0.128	
3	101	0.128	$S_0S_1S_0$
3	111	0.128	S_0S_1
5	11100	0.032	
5	11101	0.032	
5	11110	0.032	
5	11111	0.008	$S_1S_1S_1$



$\eta = \dfrac{H(s)}{\bar{L}} = \dfrac{0.72}{0.728}$

$= 98.9\%$

smile

$\bar{L}^{(3)} = 2.184$ bits/3 symbol

$\bar{L} = 0.78$ bit/symbol

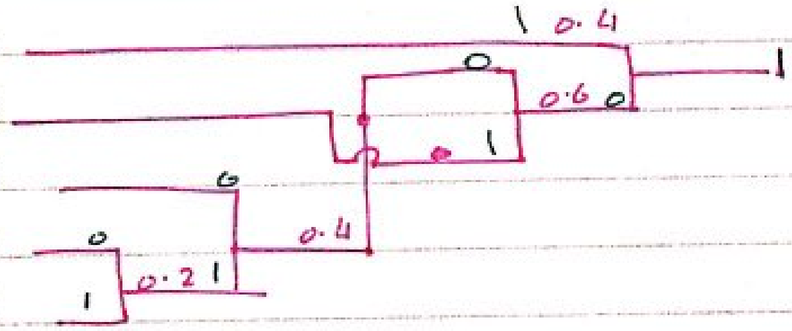
Huffman decoding

$S \rightarrow$ Huffman $\rightarrow \dots\dots$ Huffman decoding $\rightarrow$ symbols

$S = \{a_1, a_2, a_3, a_4, a_5\}$

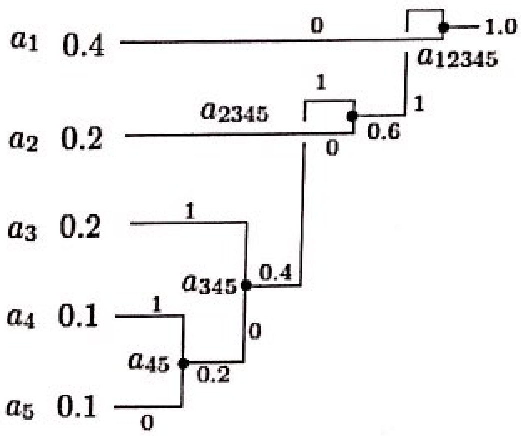
$P = \{0.4, 0.2, 0.2, 0.1, 0.1\}$

a_1	1	1	0.4
a_2	2	01	0.2
a_3	3	000	0.2
a_4	4	0010	0.1
a_5	4	0011	0.1



0010/1/1/01/111 011 00 $\rightarrow$ Huffman Decoder

a_4 $a_1 a_1 a_2$ $a_1 a_1 a_1$ $a_2 a_1$



$T_0 = \Lambda$			$T_1 = 1$			$T_2 = 11$			$T_3 = 110$		
000	$a_1a_1a_1$	0	$1\vert 000$	$a_2a_1a_1$	0	$11\vert 000$	a_5a_1	0	$110\vert 000$	$a_5a_1a_1$	0
001	a_1a_1	1	$1\vert 001$	a_2a_1	1	$11\vert 001$	a_5	1	$110\vert 001$	a_5a_1	1
010	a_1a_2	0	$1\vert 010$	a_2a_2	0	$11\vert 010$	a_4a_1	0	$110\vert 010$	a_5a_2	0
011	a_1	2	$1\vert 011$	a_2	2	$11\vert 011$	a_4	1	$110\vert 011$	a_5	2
100	a_2a_1	0	$1\vert 100$	a_5	0	$11\vert 100$	$a_3a_1a_1$	0	$110\vert 100$	$a_4a_1a_1$	0
101	a_2	1	$1\vert 101$	a_4	0	$11\vert 101$	a_3a_1	1	$110\vert 101$	a_4a_1	1
110	–	3	$1\vert 110$	a_3a_1	0	$11\vert 110$	a_3a_2	0	$110\vert 110$	a_4a_2	0
111	a_3	0	$1\vert 111$	a_3	1	$11\vert 111$	a_3	2	$110\vert 111$	a_4	2

Letter	Probability	Letter	Probability
A	0.057305	N	0.056035
B	0.014876	O	0.058215
C	0.025775	P	0.021034
D	0.026811	Q	0.000973
E	0.112578	R	0.048819
F	0.022875	S	0.060289
G	0.009523	T	0.078085
H	0.042915	U	0.018474
I	0.053475	V	0.009882
J	0.002031	W	0.007576
K	0.001016	X	0.002264
L	0.031403	Y	0.011702
M	0.015892	Z	0.001502

Letter	Probability	Letter	Probability
A	0.049855	N	0.048039
B	0.016100	O	0.050642
C	0.025835	P	0.015007
D	0.030232	Q	0.001509
E	0.097434	R	0.040492
F	0.019754	S	0.042657
G	0.012053	T	0.061142
H	0.035723	U	0.015794
I	0.048783	V	0.004988
J	0.000394	W	0.012207
K	0.002450	X	0.003413
L	0.025835	Y	0.008466
M	0.016494	Z	0.001050